Squarefree integers

Carl Friedrich Gauss, Wednesday 14th September 1831

Introduction

When you've written this page, you will be unstoppable, at least as far as typesetting mathematics is concerned. You will need to implement:

- Text mode stuff: sections, paragraphs, text formatting, labels and references, lists;
- Math mode stuff: definitions and results, aligned equations, etc.

So let's get on with it!

1 Squarefree integers

1.1 Definition and an elementary result

Definition 1.1. An integer a is **squarefree** if it is divisible by no perfect square other than 1. That is, if n^2 divides a then $n^2 = 1$.

Proposition 1.2. A non-zero non-unit a is squarefree if and only if

$$a = p_1 \times p_2 \times \cdots \times p_n$$

for distinct primes $p_1, p_2, \ldots, p_n$.

Proof. We leave the proof as an exercise to the reader. □

1.2 Some examples

Example 1.3. Some concrete examples include:

(i) 5610 is squarefree by Proposition 1.2, since

$$\begin{aligned} 5610 &= 10 \times 561 \\ &= (2 \times 5) \times (11 \times 17) \end{aligned}$$

(ii) 12 is not squarefree since $4 \mid 12$ and $4 = 2^2$.