

Algorithms Lab HS22
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cadmo.ethz.ch/education/lectures/HS22/algolab

Exercise - San Francisco

After traveling half the globe for more than sixty days, Phileas arrives in San Francisco together with Passepartout and Fix. Their plan is to use the *Pacific Railroad* for their journey to New York. From ocean to ocean—as the Americans say; these four words being a synonym for the 'great trunk line' which crosses the entire width of the United States.

However, unforeseen events start to unroll. The train is ambushed by a band of Sioux warriors and Passepartout is captured. Phileas, a true gentleman, cannot leave a man behind. He pursues the band across the vast white plains and finally manages to catch up with them.

After a lengthy brawl, the chief of the tribe, being a reasonable man, agrees with Phileas upon the following. The two will play an ancient Sioux board game: if Phileas manages to score at least as many points as the chief, he may take Passepartout and they are free to leave; otherwise, they both become slaves for an unspecified amount of time—at the current stage of their journey clearly a suboptimal event.

The game is a single player game, played with a single marble on a wooden board with n carved holes and m carved canals between these holes. An arrow is engraved in each canal to indicate the direction in which the marble may be moved through the canal. Furthermore, each canal carries a nonnegative number of *points*, which the player *scores* whenever rolling the marble through the canal (every canal can be used and scored multiple times throughout a game). Given only a limited number of moves, the goal of the game is to maximise the score, of course.

The chief explains the rules of the game to Phileas. There is a unique starting hole, called Angvariationu-toke (a Sioux word for 'another day'). The marble, called Canowicakte (a Sioux word for 'forest hunter'), starts at Angvariationu-toke. In each move, the player rolls the marble from the current hole to a neighboring hole through one of the incident canals, while respecting the direction of the engraved arrow. Doing so, (s)he scores as many points as the canal carries. A hole with no outgoing canal is called Weayaya (a Sioux word for 'setting sun') and from such a hole the player may take the marble back to Angvariationu-toke as a free action. Such a free action does not count as a move and it yields no score.

The chief makes the bold claim that he can achieve a score of x in k moves. Phileas' goal is to beat the chief dramatically: either find the minimum number of moves in order to score at least as much as the chief, or prove that it is impossible to achieve the score of x in k moves. It may be noted that the same canal can be scored more than once.

Input The first line of the input contains the number $t \leq 30$ of test cases. Each of the t test cases is described as follows.

- The first line contains four integers n m x k, separated by a space. They denote
 - n, the number of holes in the game board $(2 \le n \le 10^3)$;
 - m, the number of canals between the holes $(1 \le m \le 4 \cdot 10^3)$;
 - x, the claimed score of the chief $(1 \le x \le 10^{14})$;

- k, the maximum number of moves allowed $(1 \le k \le 4 \cdot 10^3)$.

Hole 0 always corresponds to Angvariationu-toke.

• The following m lines define the canals. Each line consists of three integers u v p, separated by a space, and such that $0 \le u, v \le n-1$ and $0 \le p < 2^{31}$. This means that the arrow engraved in the canal points from u to v. The player can roll the marble from hole u to hole v, thereby scoring p points. Note that (1) there can be more than one canal from hole u to hole v and (2) possibly u = v.

Output For each test case output one line containing a single integer that denotes the minimum number of moves to get at least x points. If it is not possible to score at least x points in k moves, output 'Impossible'.

Points There are three groups of test sets, worth 100 points in total.

- 1. For the first group of test sets, worth 25 points, and the corresponding hidden test sets, worth 5 points, you may assume $n \le 40$ and $k \le 20$. Furthermore, you may assume that all routes from Angvariationu-toke to a Weayaya hole use exactly k canals.
- 2. For the second group of test sets, worth 25 points, and the corresponding hidden test sets, worth 5 points, you may assume that all routes from Angvariationu-toke to a Weayaya hole use exactly k canals.
- 3. For the third group of test sets, worth 30 points, and the corresponding hidden test sets, worth 10 points, there are no additional assumptions.

Corresponding sample test sets are contained in testi.in/out, for $i \in \{1,2,3\}$.

Sample Input	Sample Output
3 6 6 7 3 0 1 1	3 5 Impossible
0 2 1 1 4 2	
2 3 1	
3 5 5	
4 5 2	
6 8 7 5	
0 1 0	
0 2 2 0 2 1	
0 5 1	
1 3 0	
2 4 0	
3 5 4	
4 5 0	
4 4 1 100	
0 1 0 1 2 0	
1 2 0 2 3 0	
3 1 0	