

The Derivation Formula of MMD-FR Linearity Criteria as well as Models and Reformulations of Different Models

I. DERIVATION FORMULA OF MMD-FR LINEARITY CRITERIA

This section presents the derivation formula of the MMD-FR linearity criteria, focusing on the formula derivation process

$$|\Delta f(t_{tra})| = \begin{cases} \frac{\Delta P_{Max}^D}{D'_t} \cdot (1 - e^{-\frac{D'_t}{2H_t^{Sys}} t_{tra}}) & \text{if } t_{tra} \in [t, t_{DB}] \\ \Delta f_{DB} + \left(\frac{\Delta P_t'^D}{D'_t} + \frac{2H_t^{Sys} \cdot PFR_t'}{D_t'^2} \right) \cdot (1 - e^{-\frac{D'_t}{2H_t^{Sys}} (t_{tra} - t_{DB})}) - \frac{PFR_t'}{D'_t} \cdot (t_{tra} - t_{DB}), & \text{if } t_{tra} \in [t_{DB}, t_c] \\ \Delta f_c + \left(\frac{\Delta P_t''^D}{D'_t} + \frac{2H_t^{Sys} \cdot PFR_t''}{D_t'^2} \right) \cdot (1 - e^{-\frac{D'_t}{2H_t^{Sys}} (t_{tra} - t_c)}) - \frac{PFR_t''}{D'_t} \cdot (t_{tra} - t_c), & \text{if } t_{tra} \in [t_c, t_g] \end{cases} \quad (1)$$

where $\Delta P_t'^D = \Delta P_{Max}^D - D'_t \cdot \Delta f_{DB}$, $\Delta P_t''^D = \Delta P_{Max}^D - (\sum_g PFR_{g,t}^G + \sum_w PFR_{w,t}^W) \cdot T_c / T_g - \sum_c PFR_{c,t}^{PB} - D'_t \cdot \Delta f_c$, $PFR_t' = (\sum_g PFR_{g,t}^G + \sum_w PFR_{w,t}^W) / T_g + \sum_c PFR_{c,t}^{PB} / T_c$, $PFR_t'' = (\sum_g PFR_{g,t}^G + \sum_w PFR_{w,t}^W) / T_g$.

Set $\partial |\Delta f(t_{tra})| / \partial t = 0$ to obtain the following maximum frequency deviation $|\Delta f_{nadir}|$:

$$t' = \begin{cases} t_{DB} - \frac{2H_t^{Sys}}{D'_t} \cdot \log\left(\frac{2\kappa'}{\Delta P_t'^D \cdot D'_t + 2\kappa'}\right) \\ t_c - \frac{2H_t^{Sys}}{D'_t} \cdot \log\left(\frac{2\kappa''}{\Delta P_t''^D \cdot D'_t + 2\kappa''}\right) \end{cases} \Rightarrow \begin{cases} 2\kappa'_t \cdot \log\left(\frac{2\kappa'}{\Delta P_t'^D \cdot D'_t + 2\kappa'}\right) \leq D_t'^2 \cdot (\Delta f_{max} - \Delta f_{DB}) - D'_t \cdot \Delta P_t'^D & \text{if } t_{tra} \in [t_{DB}, t_c] \\ 2\kappa''_t \cdot \log\left(\frac{2\kappa''}{\Delta P_t''^D \cdot D'_t + 2\kappa''}\right) \leq D_t'^2 \cdot (\Delta f_{max} - \Delta f_c) - D'_t \cdot \Delta P_t''^D & \text{if } t_{tra} \in [t_c, t_g] \end{cases} \quad (2)$$

Substituting (2) into (1), the following is obtained:

$$|\Delta f_{nadir}| = \begin{cases} \Delta f_{DB} + \frac{\Delta P_t'^D}{D'_t} + \frac{2\kappa'}{T_c \cdot D_t'^2} \cdot \log\left(\frac{2\kappa'}{T_c \cdot \Delta P_t'^D \cdot D'_t + 2\kappa'}\right) & \text{if } t_{tra} \in [t_{DB}, t_c] \\ \Delta f_c + \frac{\Delta P_t''^D}{D'_t} + \frac{2\kappa''}{T_g \cdot D_t'^2} \cdot \log\left(\frac{2\kappa''}{T_g \cdot \Delta P_t''^D \cdot D'_t + 2\kappa''}\right) & \text{if } t_{tra} \in [t_c, t_g] \end{cases} \quad (3)$$

Equation (3) is a piecewise function. The frequency nadir of the two segments depend on the PFR provision of SGs, wind

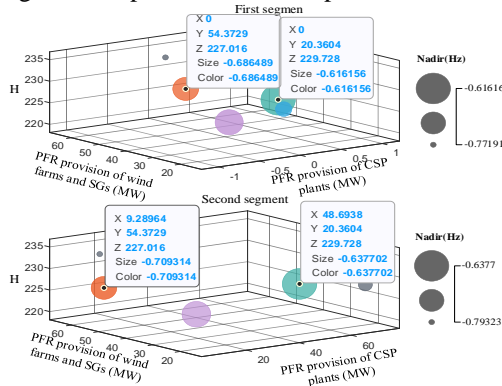


Fig. 1 Numerical simulation

for the Multi-Speed Dependent frequency Nadir, corresponding with the main text.

Substituting (2) and (3) from the main text into (1), the frequency deviation piecewise function is obtained as follows:

farms, CSP plants, and system inertia, making it impossible to obtain an analytical solution. Numerical analysis shows that the frequency nadir appears in the first segment, as illustrated in Fig. 1.

II. PROBLEM FORMULATION AND ANALYSIS

In this section, the mathematical formulation of deterministic FUCU (D-FCUC) and non-causal two-stage robust FUCU (NT-FCUC) under multi-uncertainty, for ease of understanding and discussion of validity of the proposed method.

A. D-FCUC Model

The D-FCUC model to minimize the total operational costs, including start-up and shut-down costs, operating costs of SGs, FRS provision costs, and curtailment penalties, all calculated from respective unit costs SU , VC , PC , SC , and $VoLL$, for given loads $P_{d,t}^D$, can be mathematically formulated as:

objective function:

$$OC^{sys} = \sum_{t \in T} \left[\sum_{g \in G} (SU_g^G \cdot x_{g,t}^{G,Su} + VC_g^G \cdot P_{g,t}^G) + \sum_{c \in CSP} (SU_c^{PB} \cdot x_{c,t}^{PB,Su} + VC_c^{PB} \cdot P_{c,t}^{PB}) + VoLL^D \cdot \sum_{d \in D} P_{d,t}^{D,Cur} + PC \cdot PFR_t^{Sys}(t_{QSS}) + SC \cdot SFR_t^{Sys}(t_{SFR}) \right] \quad (4)$$

subject to:

- Logic constraints of commitment states

$$\mathcal{LC} := \{x := \{0,1\} \in \mathbb{Z}^{2 \cdot (N^{CSP} + N^G) \times N^T} : \quad (5)$$

$$x_{g,t}^G, x_{g,t}^{G,Su}, x_{c,t}^{PB}, x_{c,t}^{PB,Su}; \quad g \in \mathcal{G}, c \in \mathcal{CSP}, t \in \mathcal{T}\}$$

• Operational constraints of CSP plants

$$\mathcal{OC} := \{y \in \mathbb{R}^{5 \cdot N^{CSP} \times N^T} : \quad (6)$$

$$\begin{aligned} \tilde{Q}_{c,t}^{SF} &= Q_{c,t}^{HT} / \eta_c^{cha} - Q_{c,t}^{TH} \cdot \eta_c^{dis} + P_{c,t}^{PB} / \eta_c^{PB} : (\theta_{c,t}^{SD}), \\ r_c^{TES,lim} \cdot E_c^{TES,Max} &\leq (1-t)^t \cdot E_c^{TES,In} + \sum_{\tau \in 1:t} (1-t)^{t-\tau} \\ &\cdot (Q_{c,t}^{HT} - Q_{c,t}^{TH}) \leq E_c^{TES,Max} : (\theta_{c,t}^{TMIN}, \theta_{c,t}^{TMAX}), \\ 0 \leq Q_{c,t}^{HT} &\leq Q_{c,t}^{HT,Max} : (\theta_{c,t}^{IMIN}, \theta_{c,t}^{IMAX}), \\ 0 \leq Q_{c,t}^{TH} &\leq Q_{c,t}^{TH,Max} : (\theta_{c,t}^{OMIN}, \theta_{c,t}^{OMAX}); \end{aligned} \quad (7)$$

$$\begin{aligned} P_{c,t}^{PB} + PFR_{c,t}^{PB} + SFR_{c,t}^{PB} &\leq P_{c,t}^{PB,Max} \cdot x_{c,t}^{PB} : (\theta_{c,t}^{MAX}), \\ P_{c,t}^{PB} - PFR_{c,t}^{PB} - SFR_{c,t}^{PB} &\geq P_{c,t}^{PB,Min} \cdot x_{c,t}^{PB} : (\theta_{c,t}^{MIN}); \end{aligned} \quad (8)$$

$$\begin{aligned} -RD_c^{PB} &\leq P_{c,t}^{PB} + PFR_{c,t}^{PB} + SFR_{c,t}^{PB} - P_{c,t-1}^{PB} + PFR_{c,t-1}^{PB} \\ &+ SFR_{c,t-1}^{PB} \leq RU_c^{PB} : (\theta_{c,t}^{UP}, \theta_{c,t}^{DN}); \quad c \in \mathcal{CSP}, t \in \mathcal{T} \end{aligned} \quad (9)$$

• Operational constraints of SGs

$$\mathcal{OS} := \{z \in \mathbb{R}^{3 \cdot N^G \times N^T} :$$

$$\begin{aligned} P_{g,t}^G + PFR_{g,t}^G + SFR_{g,t}^G &\leq x_{g,t} \cdot P_{g,t}^{G,Max} : (\alpha_{g,t}^{MAX}), \quad (10) \\ P_{g,t}^G - PFR_{g,t}^G - SFR_{g,t}^G &\geq x_{g,t} \cdot P_{g,t}^{G,Min} : (\alpha_{g,t}^{MIN}); \\ -RD_g^G &\leq P_{g,t}^G + PFR_{g,t}^G + SFR_{g,t}^G - P_{g,t-1}^G + PFR_{g,t-1}^G \\ &+ SFR_{g,t-1}^G \leq RU_g^G : (\alpha_{g,t}^{DN}, \alpha_{g,t}^{UP}); \quad g \in \mathcal{G}, t \in \mathcal{T} \end{aligned} \quad (11)$$

• Constraints Exclusively on Spatial Considerations:

$$\mathcal{ES} := \{(y, z, u) \in \mathbb{R}^{(2 \cdot N^G + 4 \cdot N^W + 2 \cdot N^{CSP} + 2 \cdot N^D) \times N^T} : \quad (12)$$

$$\begin{aligned} \sum_{g \in \mathcal{G}} P_{g,t}^G + \sum_{w \in \mathcal{W}} P_{w,t}^W + \sum_{c \in \mathcal{CSP}} P_{c,t}^{PB} &= \sum_{d \in \mathcal{D}} P_{d,t}^D : (\lambda_t^{SD}); \\ -F_l^{Max} &\leq \sum_{g \in \mathcal{G}} P_{g,t}^G \cdot \Gamma_{l,g}^G + \sum_{w \in \mathcal{W}} P_{w,t}^W \cdot \Gamma_{l,w}^W + \sum_{c \in \mathcal{CSP}} \times \end{aligned} \quad (13)$$

$$P_{c,t}^{CSP} \cdot \Gamma_{l,c}^{CSP} - \sum_{d \in \mathcal{D}} P_{d,t}^D \cdot \Gamma_{l,d}^D \leq F_l^{Max} : (\epsilon_{l,t}^{MIN}, \epsilon_{l,t}^{MAX}); \quad (14)$$

$$\begin{aligned} P_{d,t}^{D,Cur} + P_{d,t}^D &= \tilde{P}_{d,t}^D : (\beta_{d,t}^{SD,D}), \\ 0 \leq P_{d,t}^{D,Cur} &\leq \tilde{P}_{d,t}^D : (\beta_{d,t}^{MIN}, \beta_{d,t}^{MAX}); \end{aligned} \quad (15)$$

$$\begin{aligned} \tilde{P}_{w,t}^W &= P_{w,t}^W + PFR_{w,t}^W + SFR_{w,t}^W + P_{w,t}^{W,Cur} : (\beta_{w,t}^{SD,W}), \\ 0 \leq P_{w,t}^{W,Cur} &\leq \tilde{P}_{w,t}^W : (\beta_{w,t}^{MIN}, \beta_{w,t}^{MAX}); \end{aligned} \quad (16)$$

$$\begin{aligned} 0 \leq PFR_{c,t}^{PB} &\leq PFR_{c,t}^{PB,Max} \cdot u_{c,t}^{PB} : (\delta_{c,t}^{PMIN}, \delta_{c,t}^{PMAX}), \\ 0 \leq SFR_{c,t}^{PB} &\leq SFR_{c,t}^{PB,Max} \cdot v_{c,t}^{PB} : (\delta_{c,t}^{SMIN}, \delta_{c,t}^{SMAX}); \end{aligned} \quad (17)$$

$$(3)-(9); d \in \mathcal{D}, g \in \mathcal{G}, w \in \mathcal{W}, c \in \mathcal{CSP}, l \in \mathcal{L}, t \in \mathcal{T}\}$$

$$\mathcal{LF} := \{u := \{0,1\} \in \mathbb{Z}^{2 \cdot (N^G + N^W + N^{CSP}) \times N^T} :$$

$$\begin{aligned} u_{g,t}^G &\leq x_{g,t}^G, u_{c,t}^{PB} \leq x_{c,t}^{PB}, v_{g,t}^G \leq x_{g,t}^G, v_{c,t}^{PB} \leq x_{c,t}^{PB}, u_{w,t}^W, \\ v_{w,t}^W; \quad g &\in \mathcal{G}, w \in \mathcal{W}, c \in \mathcal{CSP}, t \in \mathcal{T} \end{aligned} \quad (18)$$

where all feasible UC of SGs and CSP plants are stated in (5), including as start-up and shut-down constraints as well as online and offline times in the \mathcal{LC} (see e.g. [22] for details on the formulation). The set of feasible operations for SGs \mathcal{OC} , as described in (6)-(9), present the direct normal irradiance (DNI)-thermal-electrical energy conversion in the solar

field (SF), thermal energy storage (TES), and power block (PB) of CSP plants, explicitly incorporating PFR and SFR provision into output limit to ensure hourly FRS deliverability under variable DNI and limited ramping in real-time ED, as describes in constraints (8)-(9). Feasible region of SGs \mathcal{OS} , explicitly including FRS provision capability, are listed in (10) and (11), with capacity limits in (10) and ramping limits in (11). Equation (12) enforces the active power balance and (13) limits the power flow of transmission lines in the set of feasible ED schedules \mathcal{ES} that is non-temporal and exclusively spatial. The non-negativity of unserved load and wind spillage are indicated in (14) and (15), respectively. Equation (16) highlights the maximal PFR and SFR provision capability and CSP plants, associated with FRS-status-related decisions, whose relationship with commitment-status-related decisions is encapsulated within (17). Similar constraints for SGs and winds are not elaborated individually.

B. NT-FCUC Model

The user-defined set of multi-uncertain parameters, characterized by a hyperrectangle with upper and lower bounds encompassing load demand $\tilde{P}_{d,t}^D$, wind power production $\tilde{P}_{w,t}^W$, and thermal energy $\tilde{Q}_{c,t}^{SF}$ absorbed by SF in CSP plants, manifests its worst-case realization only when reaching extreme upper $\tilde{P}_{d,t}^D$ or lower limits $\underline{P}_{w,t}^W, \underline{Q}_{c,t}^{SF}$. It can be recast into a more tractable formulation given as:

$$\begin{aligned} \xi_t &= (\tilde{P}_{d,t}^D = \bar{P}_{d,t}^D + P_{d,t}^{D+}, \tilde{P}_{w,t}^W = \bar{P}_{w,t}^W - P_{w,t}^{W-}, \\ \tilde{Q}_{c,t}^{CSP} &= \bar{Q}_{c,t}^{CSP} - Q_{c,t}^{CSP-}) \in \mathbb{R}^{(N^G + N^W + N^{CSP}) \times 1} \\ 0 \leq P_{d,t}^{D+} &\leq \bar{P}_{d,t}^D, 0 \leq P_{w,t}^{W-} \leq \bar{P}_{w,t}^W, 0 \leq Q_{c,t}^{CSP-} \leq \bar{Q}_{c,t}^{CSP}, \\ -\sum_{d \in \mathcal{D}} \left(\frac{P_{d,t}^{D+}}{\bar{P}_{d,t}^D} \right) - \sum_{w \in \mathcal{W}} \left(\frac{P_{w,t}^{W-}}{\bar{P}_{w,t}^W} \right) - \sum_{c \in \mathcal{CSP}} \left(\frac{Q_{c,t}^{CSP-}}{\bar{Q}_{c,t}^{CSP}} \right) &\geq -\Lambda_t, \\ d \in \mathcal{D}, w \in \mathcal{W}, c \in \mathcal{CSP}, t \in \mathcal{T} \setminus \{1\} \end{aligned} \quad (19)$$

Subsequently, compact form of NT-FCUC model, minimizing the worst-case total operational cost for multi-uncertainty set Ξ , is formulated in a conventional way as following:

$$\begin{aligned} \min_{x \in \mathcal{OC}, u \in \mathcal{LF}} \{a^T \cdot x + b^T \cdot u + \max_{\xi \in \Xi} \varepsilon(x, u, \xi, y, z)\} \\ s. t. E \cdot x + F \cdot u \leq n; \\ where \varepsilon(x, u, \xi, y, z) = \min_{\substack{(y \in \mathcal{OC}(x), z \in \mathcal{OS}(x), \\ u \in \mathcal{LF}) \in \mathcal{ES}(x, u)}} (c^T \cdot y + d^T \cdot z) \end{aligned} \quad (19)$$

s. t. $G \cdot y + H \cdot z \leq n - L \cdot x - M \cdot u - N \cdot \xi$ where due to structural changes, m becomes vector n . Also, the parameters represented by matrices E, F, G, H, L, M , and N can be calculated by constraints of NT-FCUC model. $\varepsilon(x, u, \xi, y, z)$ represents the feasible domain produced by HANDs and actual resolution of multi-uncertainty parameters ξ . The detailed robust counterpart is listed in the Appendix.

III. ROBUST COUNTERPART OF NT-FCUC

This section mainly presents the mathematical formulation of SP in NT-FCUC model.

objective function:

$$\begin{aligned}
& \sum_{t \in \mathcal{T}} \{ \sum_{d \in \mathcal{D}} [(\hat{P}_{d,t}^{\mathcal{D}} \cdot \lambda_t^{SD} + P_{d,t}^{\mathcal{D}+} \cdot \vartheta_{d,t}^{SD}) - (\hat{P}_{d,t}^{\mathcal{D}} \cdot \beta_{d,t}^{MAX} \\
& + P_{d,t}^{\mathcal{D}+} \cdot \vartheta_{d,t}^{MAX})] + \sum_{g \in \mathcal{G}} (x_{g,t} \cdot P_g^{\mathcal{G},Min} \cdot \alpha_{g,t}^{MIN} \\
& - x_{g,t} \cdot P_g^{\mathcal{G},Max} \cdot \alpha_{g,t}^{MAX} - PFR_g^{\mathcal{G},Max} \cdot u_{g,t}^{\mathcal{G}} \cdot \delta_{g,t}^{PMAX} \\
& - SFR_g^{\mathcal{G},Max} \cdot v_{g,t}^{\mathcal{G}} \cdot \delta_{g,t}^{SMAX} - M \cdot (1 - x_{g,t}^{\mathcal{G}}) \cdot \rho_{g,t}^{MIN-} \\
& - M \cdot x_{g,t}^{\mathcal{G}} \cdot \rho_{g,t}^{MAX+}) + \sum_{w \in \mathcal{W}} [\hat{P}_{w,t}^{\mathcal{W}} \cdot \beta_{w,t}^{SD} - P_{w,t}^{\mathcal{W}-} \\
& \cdot \vartheta_{w,t}^{SD} - (\hat{P}_{w,t}^{\mathcal{W}} \cdot \beta_{w,t}^{MAX} - P_{w,t}^{\mathcal{W}-} \cdot \vartheta_{w,t}^{MAX}) - PFR_w^{\mathcal{W},Max} \\
& \cdot u_{w,t}^{\mathcal{W}} \cdot \delta_{w,t}^{PMAX} - SFR_w^{\mathcal{W},Max} \cdot v_{w,t}^{\mathcal{W}} \cdot \delta_{w,t}^{SMAX} \\
& - M \cdot (1 - o_{w,t}^{\mathcal{W}}) \cdot \rho_{w,t}^{MIN-} - M \cdot o_{w,t}^{\mathcal{W}} \cdot \rho_{w,t}^{MAX+}] \\
& + \sum_{c \in \mathcal{CSP}} [\hat{Q}_{c,t}^{SF} \cdot \theta_{c,t}^{SD} - Q_{c,t}^{SF-} \cdot \vartheta_{c,t}^{SD} + P_c^{\mathcal{PB},Min} \\
& \cdot x_{c,t}^{\mathcal{PB}} \cdot \theta_{c,t}^{MIN} - P_c^{\mathcal{PB},Max} \cdot x_{c,t}^{\mathcal{PB}} \cdot \theta_{c,t}^{MAX} + [r_c^{\mathcal{TES},lim} \\
& \cdot E_c^{\mathcal{TES},Max} - (1 - \iota)^t \cdot E_c^{\mathcal{TES},in}] \cdot \theta_{c,t}^{TMIN} - [E_c^{\mathcal{TES},Max} \\
& - (1 - \iota)^t \cdot E_c^{\mathcal{TES},in}] \cdot \theta_{c,t}^{TMAX} - Q_{c,t}^{\mathcal{HT},Max} \cdot \theta_{c,t}^{IMAX} \\
& \cdot \zeta_{c,t}^{\mathcal{HT}} - Q_{c,t}^{\mathcal{TH},Max} \cdot \theta_{c,t}^{OMAX} \cdot (1 - \zeta_{c,t}^{\mathcal{HT}}) \\
& - PFR_{c,t}^{\mathcal{PB},Max} \cdot u_{c,t}^{\mathcal{PB}} \cdot \delta_{c,t}^{PMAX} - SFR_{c,t}^{\mathcal{PB},Max} \cdot v_{c,t}^{\mathcal{PB}} \\
& \cdot \delta_{c,t}^{SMAX} - M \cdot (1 - x_{c,t}^{\mathcal{PB}}) \cdot \rho_{c,t}^{MIN-} - M \cdot x_{c,t}^{\mathcal{PB}} \\
& \cdot \rho_{c,t}^{MAX+}] - \sum_{l \in \mathcal{L}} (F_l^{Max} \cdot \varepsilon_{l,t}^{MIN} + F_l^{Max} \cdot \varepsilon_{l,t}^{MAX}) \\
& - (D' \cdot \Delta f_{Max}^{QSS} - \Delta P_{t,Max}^{\mathcal{D}}) \cdot \pi_t^{QSS} + \Delta f_{Max}^{QSS} \cdot D \cdot \\
& \sum_{d \in \mathcal{D}} P_{d,t}^{\mathcal{D}+} \cdot \vartheta_{d,t}^{QSS} + \kappa' \cdot f_0 \cdot \rho_t \\
& + (SFR^{Min} - \Delta P_{Max}^{\mathcal{L}} \cdot \frac{t_{SFR} - t_{QSS} - 2\zeta_1}{20\zeta_2}) \cdot \nu_t^{MIN} \\
& - (SFR^{Max} - \Delta P_{Max}^{\mathcal{L}} \cdot \frac{t_{SFR} - t_{QSS} - 2\zeta_1}{20\zeta_2}) \cdot \nu_t^{MAX} \\
& + \Delta P_{t,Max}^{\mathcal{D}} \cdot \nu_t^{FRS} \} - \sum_{t \in \mathcal{T} \setminus \{1\}} \{ \sum_{g \in \mathcal{G}} \{ [P_g^{\mathcal{G},Max} \\
& - x_{g,t-1}^{\mathcal{G}} \cdot (P_g^{\mathcal{G},Min} - \Delta P_g^{\mathcal{G},Up}) - x_{g,t}^{\mathcal{G}} \cdot (P_g^{\mathcal{G},Max} \\
& - P_g^{\mathcal{G},Min})] \cdot \alpha_{g,t}^{UP} + [P_g^{\mathcal{G},Max} - x_{g,t}^{\mathcal{G}} \cdot (P_g^{\mathcal{G},Min} \\
& - \Delta P_g^{\mathcal{G},Dn}) - x_{g,t-1}^{\mathcal{G}} \cdot (P_g^{\mathcal{G},Max} - P_g^{\mathcal{G},Min})] \cdot \alpha_{g,t}^{DN} \} \\
& + \sum_{c \in \mathcal{CSP}} \{ [P_c^{\mathcal{PB},Max} - x_{c,t-1}^{\mathcal{PB}} \cdot (P_c^{\mathcal{PB},Max} - R_c^{\mathcal{PB},Up}) \\
& - x_{c,t}^{\mathcal{PB}} \cdot (P_c^{\mathcal{PB},Max} - R_c^{\mathcal{PB},Min})] \cdot \theta_{c,t}^{UP} + [P_c^{\mathcal{PB},Max} \\
& - x_{c,t}^{\mathcal{PB}} \cdot (P_c^{\mathcal{PB},Min} - R_c^{\mathcal{PB},Dn}) - x_{c,t-1}^{\mathcal{PB}} \cdot (P_c^{\mathcal{PB},Max} \\
& - R_c^{\mathcal{PB},Min})] \cdot \theta_{c,t}^{DN} \} \}
\end{aligned} \tag{20}$$

subject to:

$$\begin{aligned}
\vartheta_{d,t}^{SD} &= o_{d,t}^{\mathcal{D}} \cdot \lambda_t^{SD}; \vartheta_{d,t}^{MAX} = o_{d,t}^{\mathcal{D}} \cdot \beta_{d,t}^{MAX}; \\
\vartheta_{d,t}^{QSS} &= o_{d,t}^{\mathcal{D}} \cdot \pi_t^{QSS}; d \in \mathcal{D}, t \in \mathcal{T}
\end{aligned} \tag{21}$$

$$\begin{aligned}
\vartheta_{w,t}^{SD} &= o_{w,t}^{\mathcal{W}} \cdot \beta_{w,t}^{SD}; \vartheta_{w,t}^{MAX} = o_{w,t}^{\mathcal{W}} \cdot \beta_{w,t}^{MAX} \\
w &\in \mathcal{W}, t \in \mathcal{T}
\end{aligned} \tag{22}$$

$$\vartheta_{c,t}^{SD} = o_{c,t}^{SF} \cdot \theta_{c,t}^{SD}; \quad c \in \mathcal{CSP}, t \in \mathcal{T} \tag{23}$$

$$\begin{aligned}
-M \cdot (1 - o_{d,t}^{\mathcal{D}}) &\leq \vartheta_{d,t}^{SD} - \lambda_t^{SD} \leq 0; 0 \leq \vartheta_{d,t}^{SD} \leq M \cdot o_{d,t}^{\mathcal{D}}; \\
-M \cdot (1 - o_{d,t}^{\mathcal{D}}) &\leq \vartheta_{d,t}^{MAX} - \beta_{d,t}^{MAX} \leq 0; \\
0 &\leq \vartheta_{d,t}^{SD} \leq M \cdot o_{d,t}^{\mathcal{D}}; \\
-M \cdot (1 - o_{d,t}^{\mathcal{D}}) &\leq \vartheta_{d,t}^{QSS} - \pi_t^{QSS} \leq 0; 0 \leq \vartheta_{d,t}^{QSS} \leq M \cdot o_{d,t}^{\mathcal{D}}; \\
d &\in \mathcal{D}, t \in \mathcal{T}
\end{aligned} \tag{24}$$

$$\begin{aligned}
-M \cdot (1 - o_{w,t}^{\mathcal{W}}) &\leq \vartheta_{w,t}^{SD} - \beta_{w,t}^{SD} \leq 0; 0 \leq \vartheta_{w,t}^{SD} \leq M \cdot o_{w,t}^{\mathcal{W}}; \\
-M \cdot (1 - o_{w,t}^{\mathcal{W}}) &\leq \vartheta_{w,t}^{MAX} - \beta_{w,t}^{MAX} \leq 0; \\
0 &\leq \vartheta_{w,t}^{MAX} \leq M \cdot o_{w,t}^{\mathcal{W}}; w \in \mathcal{W}, t \in \mathcal{T}
\end{aligned} \tag{25}$$

$$\begin{aligned}
-M \cdot (1 - o_{c,t}^{SF}) &\leq \vartheta_{c,t}^{SD} - \theta_{c,t}^{SD} \leq 0; \\
0 &\leq \vartheta_{c,t}^{SD} \leq M \cdot o_{c,t}^{SF}; c \in \mathcal{CSP}, t \in \mathcal{T}
\end{aligned} \tag{26}$$

$$\begin{aligned}
\alpha_{g,t}^{MIN} - \alpha_{g,t}^{MAX} + \lambda_t^{SD} - \alpha_{g,t}^{UP} + \alpha_{g,t+1}^{UP} + \alpha_{g,t}^{DN} - \alpha_{g,t+1}^{DN} \\
+ \sum_{l \in \mathcal{L}} (\Gamma_{l,g}^{\mathcal{G}} \cdot \varepsilon_{l,t}^{MIN} - \Gamma_{l,g}^{\mathcal{G}} \cdot \varepsilon_{l,t}^{MAX}) = VC_g^{\mathcal{G}} : (P_g^{\mathcal{G}})
\end{aligned} \tag{27}$$

$$\begin{aligned}
& g \in \mathcal{G}, t \in \mathcal{T} \setminus \{1, N^T\} \\
\alpha_{g,t}^{MIN} - \alpha_{g,t}^{MAX} + \lambda_t^{SD} - \alpha_{g,t}^{UP} + \alpha_{g,t}^{DN} + \sum_{l \in \mathcal{L}} (\Gamma_{l,g}^{\mathcal{G}} \cdot \varepsilon_{l,t}^{MIN} \\
- \Gamma_{l,g}^{\mathcal{G}} \cdot \varepsilon_{l,t}^{MAX}) = VC_g^{\mathcal{G}} : (P_g^{\mathcal{G}}) \quad g \in \mathcal{G}, t = N^T
\end{aligned} \tag{28}$$

$$\begin{aligned}
\alpha_{g,t}^{MIN} - \alpha_{g,t}^{MAX} + \lambda_t^{SD} + \alpha_{g,t+1}^{UP} - \alpha_{g,t+1}^{DN} + \sum_{l \in \mathcal{L}} (\Gamma_{l,g}^{\mathcal{G}} \\
\cdot \varepsilon_{l,t}^{MIN} - \Gamma_{l,g}^{\mathcal{G}} \cdot \varepsilon_{l,t}^{MAX}) = VC_g^{\mathcal{G}} : (P_g^{\mathcal{G}}) \quad g \in \mathcal{G}, t = 1
\end{aligned} \tag{29}$$

$$\begin{aligned}
\lambda_t^{SD} + \beta_{w,t}^{SD} + \sum_{l \in \mathcal{L}} (\Gamma_{l,w}^{\mathcal{W}} \cdot \varepsilon_{l,t}^{MIN} - \Gamma_{l,w}^{\mathcal{W}} \cdot \varepsilon_{l,t}^{MAX}) = 0 \\
: (P_{w,t}^{\mathcal{W}}) \quad w \in \mathcal{W}, t \in \mathcal{T}
\end{aligned} \tag{30}$$

$$\beta_{w,t}^{SD, W} \leq VoLL^{\mathcal{W}} : (P_{w,t}^{\mathcal{W},Cur}) \quad w \in \mathcal{W}, t \in \mathcal{T} \tag{31}$$

$$\begin{aligned}
-\lambda_t^{SD} + \beta_t^{SD, D} - \sum_{l \in \mathcal{L}} (\Gamma_{l,d}^{\mathcal{D}} \cdot \varepsilon_{l,t}^{MIN} - \Gamma_{l,d}^{\mathcal{D}} \cdot \varepsilon_{l,t}^{MAX}) = 0 \\
: (P_{d,t}^{\mathcal{D}}) \quad d \in \mathcal{D}, t \in \mathcal{T}
\end{aligned} \tag{32}$$

$$\lambda_t^{SD} + \beta_t^{SD, D} \leq VoLL^{\mathcal{D}} : (P_{d,t}^{\mathcal{D},Cur}) \quad d \in \mathcal{D}, t \in \mathcal{T} \tag{33}$$

$$\begin{aligned}
\lambda_t^{SD} + \theta_{c,t}^{MIN} - \theta_{c,t}^{MAX} - \theta_{c,t}^{UP} + \theta_{c,t+1}^{UP} + \theta_{c,t}^{DN} - \theta_{c,t+1}^{DN} \\
+ \theta_{c,t}^{SD} / \eta_c^{PB} + \sum_{l \in \mathcal{L}} (\Gamma_{l,c}^{\mathcal{PB}} \cdot \varepsilon_{l,t}^{MIN} - \Gamma_{l,c}^{\mathcal{PB}} \cdot \varepsilon_{l,t}^{MAX}) \\
= 0 : (Q_{c,t}^{\mathcal{PB}}) \quad c \in \mathcal{CSP}, t \in \mathcal{T} \setminus \{1, N^T\}
\end{aligned} \tag{34}$$

$$\begin{aligned}
\lambda_t^{SD} + \theta_{c,t}^{MIN} - \theta_{c,t}^{MAX} + \theta_{c,t+1}^{UP} - \theta_{c,t+1}^{DN} + \theta_{c,t}^{SD} / \eta_c^{PB} \\
+ \sum_{l \in \mathcal{L}} (\Gamma_{l,c}^{\mathcal{PB}} \cdot \varepsilon_{l,t}^{MIN} - \Gamma_{l,c}^{\mathcal{PB}} \cdot \varepsilon_{l,t}^{MAX}) = 0 : (Q_{c,t}^{\mathcal{PB}}) \\
c \in \mathcal{CSP}, t = 1
\end{aligned} \tag{35}$$

$$\begin{aligned}
\lambda_t^{SD} + \theta_{c,t}^{MIN} - \theta_{c,t}^{MAX} - \theta_{c,t}^{UP} + \theta_{c,t}^{DN} + \theta_{c,t}^{SD} / \eta_c^{PB} \\
+ \sum_{l \in \mathcal{L}} (\Gamma_{l,c}^{\mathcal{PB}} \cdot \varepsilon_{l,t}^{MIN} - \Gamma_{l,c}^{\mathcal{PB}} \cdot \varepsilon_{l,t}^{MAX}) = 0 : (Q_{c,t}^{\mathcal{PB}}) \\
c \in \mathcal{CSP}, t = N^T
\end{aligned} \tag{36}$$

$$\begin{aligned}
-\eta_c^{dis} \cdot \theta_{c,t}^{SD} - \sum_{\tau \in t:N^T} (1 - \iota)^{\tau-t} \cdot (\theta_{c,\tau}^{TMIN} - \theta_{c,\tau}^{TMAX}) \\
+ \theta_{c,t}^{OMIN} - \theta_{c,t}^{OMAX} = 0 : (Q_{c,t}^{\mathcal{TH}}) \quad c \in \mathcal{CSP}, t \in \mathcal{T}
\end{aligned} \tag{37}$$

$$\begin{aligned}
\theta_{c,t}^{SD} / \eta_c^{cha} + \sum_{\tau \in t:N^T} (1 - \iota)^{\tau-t} \cdot (\theta_{c,\tau}^{TMIN} - \theta_{c,\tau}^{TMAX}) \\
+ \theta_{c,t}^{IMIN} - \theta_{c,t}^{IMAX} = 0 : (Q_{c,t}^{\mathcal{HT}}) \quad c \in \mathcal{CSP}, t \in \mathcal{T}
\end{aligned} \tag{38}$$

$$\omega_t - \sum_{g \in \mathcal{G}} (\rho_{g,t}^{MIN-} - \rho_{g,t}^{MAX-}) - \sum_{c \in \mathcal{CSP}} (\rho_{c,t}^{MIN-} - \rho_{c,t}^{MAX-}) - \sum_{w \in \mathcal{W}} (\rho_{w,t}^{MIN-} - \rho_{w,t}^{MAX-} - H_w^{\mathcal{W}} \cdot \hat{P}_{w,t}^{\mathcal{W}} \cdot \rho_t) - H_d^{\mathcal{D}} \cdot \Delta P_{t,Max}^{\mathcal{D}} / T_g \cdot \rho_t = 0 \quad (39)$$

$$\nu_t^{FRS} - \alpha_{g,t}^{MIN} - \alpha_{g,t}^{MAX} - \alpha_{g,t}^{UP} - \alpha_{g,t+1}^{UP} - \alpha_{g,t}^{DN} - \alpha_{g,t+1}^{DN} - \delta_{g,t}^{PMAX} + \pi_t^{QSS} - \omega_t / T_g - \frac{t_{SFR} - t_{QSS} - 2\zeta_1}{20\zeta_2} \cdot (\nu_t^{MIN} - \nu_t^{MAX}) \leq PC_g^{\mathcal{G}} : (PFR_{g,t}^{\mathcal{G}}) \quad g \in \mathcal{G}, t \in \mathcal{T} \setminus \{1, N^T\} \quad (40)$$

$$\nu_t^{FRS} - \alpha_{g,t}^{MIN} - \alpha_{g,t}^{MAX} - \alpha_{g,t}^{UP} - \alpha_{g,t}^{DN} - \delta_{g,t}^{PMAX} + \pi_t^{QSS} - \omega_t / T_g - \frac{t_{SFR} - t_{QSS} - 2\zeta_1}{20\zeta_2} \cdot (\nu_t^{MIN} - \nu_t^{MAX}) \leq PC_g^{\mathcal{G}} : (PFR_{g,t}^{\mathcal{G}}) \quad g \in \mathcal{G}, t = N^T \quad (41)$$

$$\nu_t^{FRS} - \alpha_{g,t}^{MIN} - \alpha_{g,t}^{MAX} - \alpha_{g,t+1}^{UP} - \alpha_{g,t+1}^{DN} - \delta_{g,t}^{PMAX} + \pi_t^{QSS} - \omega_t / T_g - \frac{t_{SFR} - t_{QSS} - 2\zeta_1}{20\zeta_2} \cdot (\nu_t^{MIN} - \nu_t^{MAX}) \leq PC_g^{\mathcal{G}} : (PFR_{g,t}^{\mathcal{G}}) \quad g \in \mathcal{G}, t = 1 \quad (42)$$

$$\nu_t^{FRS} + \beta_{w,t}^{SD} + \delta_{w,t}^{PMIN} - \delta_{w,t}^{PMAX} + \pi_t^{QSS} - \omega_t / T_g - \frac{t_{SFR} - t_{QSS} - 2\zeta_1}{20\zeta_2} \cdot (\nu_t^{MIN} - \nu_t^{MAX}) \leq PC_w^{\mathcal{W}} : (PFR_{w,t}^{\mathcal{W}}) \quad w \in \mathcal{W}, t \in \mathcal{T} \quad (43)$$

$$\nu_t^{FRS} - \theta_{c,t}^{MIN} - \theta_{c,t}^{MAX} - \theta_{c,t}^{UP} - \theta_{c,t+1}^{UP} - \theta_{c,t}^{DN} - \theta_{c,t+1}^{DN} - \delta_{c,t}^{PMAX} + \pi_t^{QSS} - \omega_t / T_g - \frac{t_{SFR} - t_{QSS} - 2\zeta_1}{20\zeta_2} \cdot (\nu_t^{MIN} - \nu_t^{MAX}) \leq PC_c^{PB} : (PFR_{c,t}^{CSP}) \quad c \in \mathcal{CSP}, t \in \mathcal{T} \setminus \{1, N^T\} \quad (44)$$

$$\nu_t^{FRS} - \theta_{c,t}^{MIN} - \theta_{c,t}^{MAX} - \theta_{c,t}^{UP} - \theta_{c,t}^{DN} - \delta_{c,t}^{PMAX} + \pi_t^{QSS} - \omega_t / T_g - \frac{t_{SFR} - t_{QSS} - 2\zeta_1}{20\zeta_2} \cdot (\nu_t^{MIN} - \nu_t^{MAX}) \leq PC_c^{CSP} : (PFR_{c,t}^{PB}) \quad c \in \mathcal{CSP}, t = N^T \quad (45)$$

$$\nu_t^{FRS} - \theta_{c,t}^{MIN} - \theta_{c,t}^{MAX} - \theta_{c,t+1}^{UP} - \theta_{c,t+1}^{DN} - \delta_{c,t}^{PMAX} + \pi_t^{QSS} - \omega_t / T_g - \frac{t_{SFR} - t_{QSS} - 2\zeta_1}{20\zeta_2} \cdot (\nu_t^{MIN} - \nu_t^{MAX}) \leq PC_c^{CSP} : (PFR_{c,t}^{PB}) \quad c \in \mathcal{CSP}, t = 1 \quad (46)$$

$$\nu_t^{FRS} - \theta_{c,t}^{MIN} - \theta_{c,t}^{MAX} - \theta_{c,t+1}^{UP} - \theta_{c,t+1}^{DN} - \delta_{c,t}^{PMAX} + \pi_t^{QSS} - \omega_t / T_g - \frac{t_{SFR} - t_{QSS} - 2\zeta_1}{20\zeta_2} \cdot (\nu_t^{MIN} - \nu_t^{MAX}) \leq PC_c^{CSP} : (PFR_{c,t}^{PB}) \quad c \in \mathcal{CSP}, t = 1 \quad (47)$$

$$-\alpha_{g,t}^{MIN} - \alpha_{g,t}^{MAX} - \alpha_{g,t}^{UP} - \alpha_{g,t+1}^{UP} - \alpha_{g,t}^{DN} - \alpha_{g,t+1}^{DN} - \delta_{g,t}^{SMAX} + \nu_t^{FRS} \leq SC_g^{\mathcal{G}} : (SFR_{g,t}^{\mathcal{G}}) \quad g \in \mathcal{G}, t \in \mathcal{T} \setminus \{1, N^T\} \quad (48)$$

$$-\alpha_{g,t}^{MIN} - \alpha_{g,t}^{MAX} - \alpha_{g,t}^{UP} - \alpha_{g,t}^{DN} - \delta_{g,t}^{SMAX} + \nu_t^{FRS} \leq SC_g^{\mathcal{G}} : (SFR_{g,t}^{\mathcal{G}}) \quad g \in \mathcal{G}, t = N^T \quad (49)$$

$$-\alpha_{g,t}^{MIN} - \alpha_{g,t}^{MAX} - \alpha_{g,t+1}^{UP} - \alpha_{g,t+1}^{DN} - \delta_{g,t}^{SMAX} + \nu_t^{FRS} \leq SC_g^{\mathcal{G}} : (SFR_{g,t}^{\mathcal{G}}) \quad g \in \mathcal{G}, t = 1 \quad (50)$$

$$\beta_t^{w,(SD)} - \delta_t^{w,(SMAX)} + \nu_t^{FRS} \leq SC_w^{\mathcal{W}} : (SFR_{w,t}^{\mathcal{W}}) \quad w \in \mathcal{W}, t \in \mathcal{T} \quad (51)$$

$$-\theta_{c,t}^{MAX} - \theta_{c,t}^{MIN} - \theta_{c,t}^{UP} - \theta_{c,t+1}^{UP} - \theta_{c,t}^{DN} - \theta_{c,t+1}^{DN} + \delta_{c,t}^{SMIN} + \nu_t^{FRS} = SC_c^{PB} : (SFR_{c,t}^{PB}) \quad c \in \mathcal{CSP}, t \in \mathcal{T} \setminus \{1, N^T\} \quad (52)$$

$$-\theta_{c,t}^{MAX} - \theta_{c,t}^{MIN} - \theta_{c,t}^{UP} - \theta_{c,t}^{DN} - \delta_{c,t}^{SMAX} + \nu_t^{FRS} \leq SC_c^{PB} : (SFR_{c,t}^{PB}) \quad c \in \mathcal{CSP}, t = N^T \quad (53)$$

$$-\theta_{c,t}^{MAX} - \theta_{c,t}^{MIN} - \theta_{c,t+1}^{UP} - \theta_{c,t+1}^{DN} - \delta_{c,t}^{SMAX} + \nu_t^{FRS} \leq SC_c^{PB} : (SFR_{c,t}^{PB}) \quad c \in \mathcal{CSP}, t = 1 \quad (54)$$

$$H_g^{\mathcal{G}} \cdot P_{g,Max}^{\mathcal{G}} \cdot \rho_t + \rho_{g,t}^{MIN-} - \rho_{g,t}^{MAX-} + \rho_{g,t}^{MIN+} - \rho_{g,t}^{MAX+} = 0 : (\varpi_{g,t}^{\mathcal{G}}) \quad g \in \mathcal{G}, t \in \mathcal{T} \quad (55)$$

$$H_c^{CSP} \cdot P_{c,Max}^{PB} \cdot \rho_t + \rho_{c,t}^{MIN-} - \rho_{c,t}^{MAX-} + \rho_{c,t}^{MIN+} - \rho_{c,t}^{MAX+} = 0 : (\varpi_{c,t}^{PB}) \quad c \in \mathcal{CSP}, t \in \mathcal{T} \quad (56)$$

$$-P_{w,t}^{\mathcal{W}-} \cdot \rho_t + \rho_{w,t}^{MIN-} - \rho_{w,t}^{MAX-} + \rho_{w,t}^{MIN+} - \rho_{w,t}^{MAX+} = 0 : (\varpi_{w,t}^{\mathcal{W}}) \quad c \in \mathcal{W}, t \in \mathcal{T} \quad (57)$$

$$\alpha, \beta, \lambda, \varepsilon, \delta, \nu, \theta, \rho, \pi \geq 0 \quad (58)$$

IV. ROBUST COUNTERPART OF MTAR-FCUC

This section mainly presents the mathematical formulation of SP in MTAR-FCUC model.

objective function:

$$\begin{aligned} & \sum_{t \in \mathcal{T}} \{ \sum_{d \in \mathcal{D}} [(\hat{P}_{d,t}^{\mathcal{D}} \cdot \beta_{d,t}^{SD,\mathcal{D}} + P_{d,t}^{\mathcal{D}+} \cdot \vartheta_{d,t}^{SD,\mathcal{D}})] + \sum_{g \in \mathcal{G}} [P_{g,t}^{\mathcal{G}} \cdot \alpha_{g,t}^{MIN} - \bar{P}_{g,t}^{\mathcal{G}} \cdot \alpha_{g,t}^{MAX} + \underline{PFR}_{g,t}^{\mathcal{G}} \cdot \delta_{g,t}^{PMIN} - \overline{PFR}_{g,t}^{\mathcal{G}} \cdot \delta_{g,t}^{PMAX} + \underline{SFR}_{g,t}^{\mathcal{G}} \cdot \delta_{g,t}^{SMIN} - \overline{SFR}_{g,t}^{\mathcal{G}} \cdot \delta_{g,t}^{SMAX} - M \cdot (1 - x_{g,t}^{\mathcal{G}}) \cdot \rho_{g,t}^{MIN-} - M \cdot x_{g,t}^{\mathcal{G}} \cdot \rho_{g,t}^{MAX+}] + \sum_{w \in \mathcal{W}} [\hat{P}_{w,t}^{\mathcal{W}} \cdot \beta_{w,t}^{SD,\mathcal{W}} - P_{w,t}^{\mathcal{W}-} \cdot \vartheta_{w,t}^{SD,\mathcal{W}} - (\hat{P}_{w,t}^{\mathcal{W}} \cdot \beta_{w,t}^{MAX} - P_{w,t}^{\mathcal{W}-} \cdot \vartheta_{w,t}^{MAX}) - PFR_{w,Max}^{\mathcal{W}} \cdot u_{w,t}^{\mathcal{W}} \cdot \delta_{w,t}^{PMAX} - SFR_{w,Max}^{\mathcal{W}} \cdot v_{w,t}^{\mathcal{W}} \cdot \delta_{w,t}^{SMAX} - M \cdot (1 - o_{w,t}^{\mathcal{W}}) \cdot \rho_{w,t}^{MIN-} - M \cdot o_{w,t}^{\mathcal{W}} \cdot \rho_{w,t}^{MAX+}] + \sum_{c \in \mathcal{CSP}} \{ \hat{Q}_{c,t}^{SF} \cdot \theta_{c,t}^{SD} - Q_{c,t}^{SF-} \cdot \vartheta_{c,t}^{SD} + \underline{P}_{c,t}^{PB} \cdot \theta_{c,t}^{MIN} - \overline{P}_{c,t}^{PB} \cdot \theta_{c,t}^{MAX} + Q_{c,t}^{\mathcal{HT}} \cdot \theta_{c,t}^{IMIN} - \overline{Q}_{c,t}^{\mathcal{HT}} \cdot \theta_{c,t}^{IMAX} + Q_{c,t}^{\mathcal{TH}} \cdot \theta_{c,t}^{OMIN} - \overline{Q}_{c,t}^{\mathcal{TH}} \cdot \theta_{c,t}^{OMAX} + \underline{PFR}_{c,t}^{PB} \cdot \delta_{c,t}^{PMIN} - \overline{PFR}_{c,t}^{PB} \cdot \delta_{c,t}^{PMAX} + \underline{SFR}_{c,t}^{PB} \cdot \delta_{c,t}^{SMIN} - \overline{SFR}_{c,t}^{PB} \cdot \delta_{c,t}^{SMAX} - M \cdot (1 - x_{c,t}^{PB}) \cdot \rho_{c,t}^{MIN-} - M \cdot x_{c,t}^{PB} \cdot \rho_{c,t}^{MAX+} \} - \sum_{l \in \mathcal{L}} (F_l^{Max} \cdot \varepsilon_{l,t}^{MIN} + F_l^{Max} \cdot \varepsilon_{l,t}^{MAX}) - (D' \cdot \Delta f_{Max}^{QSS} - \Delta P_{Max}^{\mathcal{D}}) \cdot \pi_t^{QSS} + \Delta f_{Max}^{QSS} \cdot D \cdot \sum_{d \in \mathcal{D}} P_{d,t}^{\mathcal{D}+} \cdot \vartheta_{d,t}^{QSS} + \kappa' \cdot f_0 \cdot \rho_t - \sum_{w \in \mathcal{W}} H_w^{\mathcal{W}} \cdot (\hat{P}_{w,t}^{\mathcal{W}} \cdot \rho_t - P_{w,t}^{\mathcal{W}-} \cdot \vartheta_{w,t}) + (SFR_{Min}^{Min} - \Delta P_{Max}^{\mathcal{D}} \cdot \frac{t_{SFR} - t_{QSS} - 2\zeta_1}{20\zeta_2}) \cdot \nu_t^{MIN} - (SFR_{Max}^{Max} - \Delta P_{Max}^{\mathcal{D}} \cdot \frac{t_{SFR} - t_{QSS} - 2\zeta_1}{20\zeta_2}) \cdot \nu_t^{MAX} + \Delta P_{Max}^{\mathcal{D}} \cdot \nu_t^{FRS} \} \end{aligned} \quad (59)$$

subject to:

$$\vartheta_{d,t}^{SD,D} = o_{d,t}^D \cdot \beta_{d,t}^{SD,D}; \vartheta_{d,t}^{QSS} = o_{d,t}^D \cdot \pi_t^{QSS}; d \in \mathcal{D}, t \in \mathcal{T} \quad (60)$$

$$\vartheta_{w,t}^{SD,W} = o_{w,t}^W \cdot \beta_{w,t}^{SD,W}; w \in \mathcal{W}, t \in \mathcal{T} \quad (61)$$

$$\vartheta_{w,t}^{Naidr,W} = o_{w,t}^W \cdot \rho_t; w \in \mathcal{W}, t \in \mathcal{T} \quad (62)$$

$$\vartheta_{c,t}^{SD} = o_{c,t}^{SF} \cdot \theta_{c,t}^{SD}; c \in \mathcal{CSP}, t \in \mathcal{T} \quad (63)$$

$$\begin{aligned} -M \cdot (1 - o_{d,t}^D) &\leq \vartheta_{d,t}^{MAX} - \beta_{d,t}^{SD,D} \leq 0; 0 \leq \vartheta_{d,t}^{SD,D} \leq M \cdot o_{d,t}^D; \\ -M \cdot (1 - o_{d,t}^D) &\leq \vartheta_{d,t}^{QSS} - \pi_t^{QSS} \leq 0; 0 \leq \vartheta_{d,t}^{QSS} \leq M \cdot o_{d,t}^D; \end{aligned} \quad (64)$$

$$\begin{aligned} -M \cdot (1 - o_{w,t}^W) &\leq \vartheta_{w,t}^{SD} - \beta_{w,t}^{SD,W} \leq 0; 0 \leq \vartheta_{w,t}^{SD,W} \leq M \cdot o_{w,t}^W; \\ -M \cdot (1 - o_{w,t}^W) &\leq \vartheta_{w,t}^{Naidr,W} - \rho_t \leq 0; 0 \leq \vartheta_{w,t}^{Naidr,W} \leq M \cdot o_{w,t}^W; \end{aligned} \quad (65)$$

$$\begin{aligned} -M \cdot (1 - o_{c,t}^{SF}) &\leq \vartheta_{c,t}^{SD} - \theta_{c,t}^{SD} \leq 0; 0 \leq \vartheta_{c,t}^{SD} \leq M \cdot o_{c,t}^{SF}; \\ c &\in \mathcal{CSP}, t \in \mathcal{T} \end{aligned} \quad (66)$$

$$\begin{aligned} \lambda_t^{SD} + \alpha_{g,t}^{MIN} - \alpha_{g,t}^{MAX} + \sum_{l \in \mathcal{L}} (\Gamma_{l,g}^G \cdot \varepsilon_{l,t}^{MIN} - \Gamma_{l,g}^G \cdot \varepsilon_{l,t}^{MAX}) \\ = VC_g^G : (P_{g,t}^G) \quad g \in \mathcal{G}, t \in \mathcal{T} \end{aligned} \quad (67)$$

$$\begin{aligned} \lambda_t^{SD} + \beta_{w,t}^{SD,W} + \sum_{l \in \mathcal{L}} (\Gamma_{l,w}^W \cdot \varepsilon_{l,t}^{MIN} - \Gamma_{l,w}^W \cdot \varepsilon_{l,t}^{MAX}) = 0 \\ : (P_{w,t}^W) \quad w \in \mathcal{W}, t \in \mathcal{T} \end{aligned} \quad (68)$$

$$\beta_{w,t}^{SD,W} \leq VoLL^W : (P_{w,t}^{W,Cur}) \quad w \in \mathcal{W}, t \in \mathcal{T} \quad (69)$$

$$\begin{aligned} \beta_{d,t}^{SD,D} - \lambda_t^{SD} - \sum_{l \in \mathcal{L}} (\Gamma_{l,d}^D \cdot \varepsilon_{l,t}^{MIN} - \Gamma_{l,d}^D \cdot \varepsilon_{l,t}^{MAX}) = 0 \\ : (P_{d,t}^D) \quad d \in \mathcal{D}, t \in \mathcal{T} \end{aligned} \quad (70)$$

$$\beta_{d,t}^{SD,D} \leq VoLL^D : (P_{d,t}^{D,Cur}) \quad d \in \mathcal{D}, t \in \mathcal{T} \quad (71)$$

$$\begin{aligned} \eta_c^{PB} \cdot \lambda_t^{SD} + \eta_c^{PB} \cdot \theta_{c,t}^{MIN} - \eta_c^{PB} \cdot \theta_{c,t}^{MAX} + \theta_{c,t}^{SD} + \\ \eta_c^{PB} \cdot \sum_{l \in \mathcal{L}} (\Gamma_{l,c}^{PB} \cdot \varepsilon_{l,t}^{MIN} - \Gamma_{l,c}^{PB} \cdot \varepsilon_{l,t}^{MAX}) = 0 \\ : (Q_{c,t}^{PB}) \quad c \in \mathcal{CSP}, t \in \mathcal{T} \end{aligned} \quad (72)$$

$$\begin{aligned} -\eta_c^{dis} \cdot \theta_{c,t}^{SD} - \sum_{\tau \in t:N^T} (1 - \iota)^{\tau-t} \cdot (\theta_{c,\tau}^{TMIN} - \theta_{c,\tau}^{TMAX}) + \\ \theta_{c,t}^{OMIN} - \theta_{c,t}^{OMAX} = 0 : (Q_{c,t}^{TH}) \quad c \in \mathcal{CSP}, t \in \mathcal{T} \end{aligned} \quad (73)$$

$$\begin{aligned} \theta_{c,t}^{SD} / \eta_c^{cha} + \sum_{\tau \in t:N^T} (1 - \iota)^{\tau-t} \cdot (\theta_{c,\tau}^{TMIN} - \theta_{c,\tau}^{TMAX}) + \\ \theta_{c,t}^{IMIN} - \theta_{c,t}^{IMAX} = 0 : (Q_{c,t}^{HT}) \quad c \in \mathcal{CSP}, t \in \mathcal{T} \end{aligned} \quad (74)$$

$$\begin{aligned} \omega_t - \sum_{g \in \mathcal{G}} (\rho_{g,t}^{MIN-} - \rho_{g,t}^{MAX-}) - \sum_{c \in \mathcal{CSP}} (\rho_{c,t}^{MIN-} - \rho_{c,t}^{MAX-}) \\ - \sum_{w \in \mathcal{W}} [\rho_{w,t}^{MIN-} - \rho_{w,t}^{MAX-} - H_{w,t}^W \cdot (\bar{P}_{w,t}^W \cdot \rho_t - \\ P_{w,t}^{W-} \cdot \vartheta_{w,t}^{Naidr,W})] - H_d^D \cdot \Delta P_{t,Max}^D / T_g \cdot \rho_t = 0 \end{aligned} \quad (75)$$

$$\begin{aligned} \nu_t^{FRS} + \delta_{c,t}^{PMIN} - \delta_{c,t}^{PMAX} + \pi_t^{QSS} - \omega_t / T_g - \\ \frac{t_{SFR} - t_{QSS} - 2\zeta_1}{20\zeta_2} \cdot (\nu_t^{MIN} - \nu_t^{MAX}) = PC_g^G \\ : (PFR_{g,t}^G) \quad g \in \mathcal{G}, t \in \mathcal{T} \setminus \{1, N^T\} \end{aligned} \quad (76)$$

$$\begin{aligned} \beta_{w,t}^{SD,W} + \nu_t^{FRS} - \delta_{w,t}^{PMAX} + \pi_t^{QSS} - \omega_t / T_g - \\ \frac{t_{SFR} - t_{QSS} - 2\zeta_1}{20\zeta_2} \cdot (\nu_t^{MIN} - \nu_t^{MAX}) \leq PC_w^W \\ : (PFR_{w,t}^W) \quad w \in \mathcal{W}, t \in \mathcal{T} \end{aligned} \quad (77)$$

$$\begin{aligned} \nu_t^{FRS} + \delta_{c,t}^{PMIN} - \delta_{c,t}^{PMAX} + \pi_t^{QSS} - \omega_t / T_g - \\ \frac{t_{SFR} - t_{QSS} - 2\zeta_1}{20\zeta_2} \cdot (\nu_t^{MIN} - \nu_t^{MAX}) = PC_c^{PB} \\ : (PFR_{c,t}^{PB}) \quad c \in \mathcal{CSP}, t \in \mathcal{T} \end{aligned} \quad (78)$$

$$\begin{aligned} \delta_{g,t}^{SMIN} - \delta_{g,t}^{SMAX} + \nu_t^{FRS} = SC_g^G : (SFR_{g,t}^G) \\ g \in \mathcal{G}, t \in \mathcal{T} \end{aligned} \quad (79)$$

$$\begin{aligned} \beta_{w,t}^{SD,W} - \delta_{w,t}^{SMAX} + \nu_t^{FRS} \leq SC_w^W : (PFR_{w,t}^W) \\ w \in \mathcal{W}, t \in \mathcal{T} \end{aligned} \quad (80)$$

$$\begin{aligned} \delta_{c,t}^{SMIN} - \delta_{c,t}^{SMAX} + \nu_t^{FRS} = SC_c^{PB} : (SFR_{c,t}^{PB}) \\ c \in \mathcal{CSP}, t \in \mathcal{T} \end{aligned} \quad (81)$$

$$\begin{aligned} H_g^G \cdot P_{g,t}^{G,Max} \cdot \rho_t + \rho_{g,t}^{MIN-} - \rho_{g,t}^{MAX-} - \rho_{g,t}^{MAX+} \leq 0 \\ : (\varpi_{g,t}^G) \quad g \in \mathcal{G}, t \in \mathcal{T} \end{aligned} \quad (82)$$

$$\begin{aligned} H_c^{PB} \cdot P_{c,t}^{PB,Max} \cdot \rho_t + \rho_{c,t}^{MIN-} - \rho_{c,t}^{MAX-} - \rho_{c,t}^{MAX+} \leq 0 \\ : (\varpi_{c,t}^{PB}) \quad c \in \mathcal{CSP}, t \in \mathcal{T} \end{aligned} \quad (83)$$

$$\begin{aligned} -P_{w,t}^{W-} \cdot \rho_t + \rho_{w,t}^{MIN-} - \rho_{w,t}^{MAX-} - \rho_{w,t}^{MAX+} \leq 0 : (\varpi_{w,t}^W) \\ c \in \mathcal{W}, t \in \mathcal{T} \end{aligned} \quad (84)$$

$$\alpha, \beta, \lambda, \varepsilon, \delta, \nu, \theta, \rho, \pi \geq 0 \quad (85)$$

V. THE TOPOLOGY OF THE AUGMENTED IEEE RTS-30 SYSTEM

The topology of the augmented IEEE RTS-30 system, as mentioned in the text, is illustrated below:

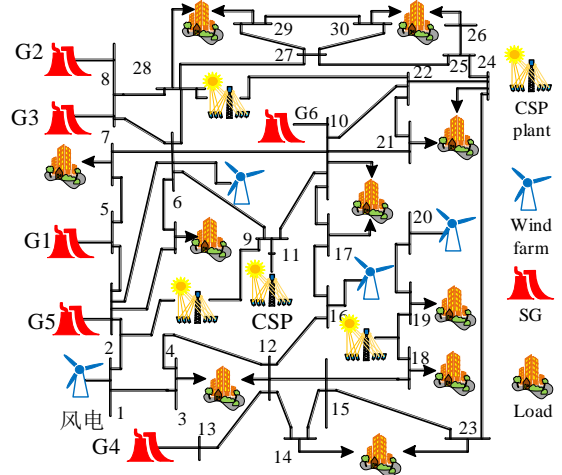


Fig. 1 The topology of the augmented IEEE RTS-30 system

VI. PARAMETERS OF THE AUGMENTED IEEE RTS-30 SYSTEM

The parameters of SGs and lines in the IEEE RTS-30 system are summarized in TABLE I, TABLE II.

TABLE I
PARAMETERS OF SGs

SGs No.	Maximum/Minimum (MW)	Ramp Level (MW/h)	Minimum Down Time (h)	Minimum Up Time (h)	Start-up and Shut-down Cost (\$)	Inertia Constant (s)
G1	200/50	66.67	1	1	70/50	6
G2	160/40	53.33	2	2	74/60	7.8
G3	150/37.5	50	1	1	50/30	6
G4	100/25	33.33	1	2	110/85	7.8
G5	110/27.5	36.67	2	1	72/52	7.8
G6	150/37.5	50	1	1	40/30	7.8

TABLE II
PARAMETERS OF LINES

Line No.	From node	To node	X(p.u.)	Rating(MWA)
1	1	2	0.0575	221
2	1	3	0.1652	221
3	2	4	0.1737	170
4	3	4	0.0379	221
5	2	5	0.1983	221
6	2	6	0.1763	170
7	4	6	0.0414	153
8	5	7	0.116	119
9	6	7	0.082	221
10	6	8	0.042	108.8
11	6	9	0.208	110.5
12	6	10	0.556	170
13	9	10	0.208	110.5
14	9	11	0.11	280.5
15	4	12	0.256	110.5
16	12	13	0.14	170
17	12	14	0.2559	88.4
18	12	15	0.1304	122.4
19	12	16	0.1987	88.4
20	14	15	0.1997	85
21	16	17	0.1923	27.2
22	15	18	0.2185	170
23	18	19	0.1292	170
24	19	20	0.068	170
25	10	20	0.209	122.4
26	10	17	0.0845	127.5
27	10	21	0.0749	119
28	10	22	0.1499	119
29	21	22	0.0236	54.4
30	15	23	0.202	27.2
31	22	24	0.179	27.2
32	23	24	0.27	27.2
33	24	25	0.3292	136
34	25	26	0.38	119
35	25	27	0.2087	170
36	28	27	0.396	110.5
37	27	29	0.4153	204
38	27	30	0.6027	136
39	29	30	0.4533	136
40	8	28	0.2	54.4
41	6	28	0.0599	54.4

VII. PARAMETERS OF THE AUGMENTED IEEE-118 SYSTEM

The parameters of SGs, CSP plants, and lines in the IEEE-118 system are summarized in TABLE III, TABLE IV, and TABLE V.

TABLE III
PARAMETERS OF SGs

SGs No.	Maximum/Minimum (MW)	Ramp Level (MW/h)	Minimum Down Time (h)	Minimum Up Time (h)	Start-up and Shut-down Cost (\$)	Inertia Constant (s)
G1	100/20	50	1	1	70/50	6.5
G2	100/20	50	2	2	74/60	7.8

G3	100/20	50	3	3	50/30	7.8
G4	100/20	50	2	2	110/85	6.5
G5	185/37	92.5	2	1	40/30	6.5
G6	320/64	160	4	4	95/80	7.8
G7	100/20	50	2	2	75.51/51.47	6.5
G8	107/21.4	53.5	2	2	75.99/51.92	6.5
G9	100/20	50	3	2	73.6/53.34	7.8
G10	100/20	50	2	3	70.67/50.4	6.5
G11	119/23.8	59.5	2	2	8162	6.5
G12	304/60.8	152	3	3	99/79	7.8
G13	148/29.6	74	2	2	55/40	6.5
G14	100/20	50	3	3	77.97/53.34	6.5
G15	100/20	50	2	2	70.63/50.4	7.8
G16	255/51	127.5	2	3	81/62	6.5
G17	260/52	130	4	2	85/59	6.5
G18	100/20	50	3	2	77.7/57.56	7.8

TABLE IV
PARAMETERS OF LINES

Line No.	From node	To node	X(p.u.)	Rating(MWA)
1	1	2	0.0999	510
2	1	3	0.0424	750
3	4	5	0.00798	228
4	3	5	0.108	518
5	5	6	0.054	214
6	6	7	0.0208	136
7	8	9	0.0305	110
8	8	5	0.0267	759
9	9	10	0.0322	110
10	4	11	0.0688	133
11	5	11	0.0682	145
12	11	12	0.0196	338
13	2	12	0.0616	510
14	3	12	0.16	344
15	7	12	0.034	136
16	11	13	0.0731	286
17	12	14	0.0707	289
18	13	15	0.2444	286
19	14	15	0.195	355
20	12	16	0.0834	337
21	15	17	0.0437	144
22	16	17	0.1801	337
23	17	18	0.0505	314
24	18	19	0.0493	133
25	19	20	0.117	143
26	15	19	0.0394	346
27	20	21	0.0849	143
28	21	22	0.097	143
29	22	23	0.159	192
30	23	24	0.0492	506
31	23	25	0.08	265
32	26	25	0.0382	202
33	25	27	0.163	173
34	27	28	0.0855	120
35	28	29	0.0943	120
36	30	17	0.0388	164
37	8	30	0.0504	859
38	26	30	0.086	202
39	17	31	0.1563	132
40	29	31	0.0331	120
41	23	32	0.1153	291
42	31	32	0.0958	128
43	27	32	0.0755	134
44	15	33	0.1244	263
45	19	34	0.247	235
46	35	36	0.01102	137

47	35	37	0.0497	137
48	33	37	0.142	263
49	34	36	0.0268	184
50	34	37	0.0094	163
51	38	37	0.0375	179
52	37	39	0.106	228
53	37	40	0.168	227
54	30	38	0.054	805
55	39	40	0.0605	228
56	40	41	0.0487	196
57	40	42	0.183	196
58	41	42	0.135	196
59	43	44	0.2454	212
60	34	43	0.1681	212
61	44	45	0.0901	212
62	45	46	0.1356	129
63	46	47	0.127	134
64	46	48	0.189	115
65	47	49	0.0625	176
66	42	49	0.323	163
67	42	49	0.323	163
68	45	49	0.0186	193
69	48	49	0.0505	115
70	49	50	0.0752	141
71	49	51	0.137	148
72	51	52	0.0588	122
73	52	53	0.1635	122
74	53	54	0.122	122
75	49	54	0.289	142
76	49	54	0.291	142
77	54	55	0.0707	129
78	54	56	0.00955	170
79	55	56	0.0151	164
80	56	57	0.0966	141
81	50	57	0.134	141
82	56	58	0.0966	136
83	51	58	0.0719	136
84	54	59	0.2293	142
85	56	59	0.251	141
86	56	59	0.239	142
87	55	59	0.2158	149
88	59	60	0.145	128
89	59	61	0.15	131
90	60	61	0.0135	144
91	60	62	0.0561	128
92	61	62	0.0376	148
93	63	59	0.0386	205
94	63	64	0.02	205
95	64	61	0.0268	200
96	38	65	0.0986	755
97	64	65	0.0302	295
98	49	66	0.0919	170
99	49	66	0.0919	170
100	62	66	0.218	134
101	62	67	0.117	134
102	65	66	0.037	219
103	66	67	0.1015	134
104	65	68	0.016	948
105	47	69	0.2778	199
106	49	69	0.324	199
107	68	69	0.037	277
108	69	70	0.127	166
109	24	70	0.4115	282
110	70	71	0.0355	253
111	24	72	0.196	302
112	71	72	0.18	266
113	71	73	0.0454	159
114	70	74	0.1323	206
115	70	75	0.141	213
116	69	75	0.122	173
117	74	75	0.0406	156
118	76	77	0.148	179
119	69	77	0.101	395
120	75	77	0.1999	216
121	77	78	0.0124	142

122	78	79	0.0244	142
123	77	80	0.0485	182
124	77	80	0.105	143
125	79	80	0.0704	142
126	68	81	0.0202	781
127	81	80	0.037	781
128	77	82	0.0853	380
129	82	83	0.03665	331
130	83	84	0.132	219
131	83	85	0.148	254
132	84	85	0.0641	219
133	85	86	0.123	161
134	86	87	0.2074	161
135	85	88	0.102	138
136	85	89	0.173	160
137	88	89	0.0712	228
138	89	90	0.188	148
139	89	90	0.0997	181
140	90	91	0.0836	122
141	89	92	0.0505	192
142	89	92	0.1581	136
143	91	92	0.1272	205
144	92	93	0.0848	216
145	92	94	0.158	216
146	93	94	0.0732	216
147	94	95	0.0434	259
148	80	96	0.182	232
149	82	96	0.053	159
150	94	96	0.0869	278
151	80	97	0.0934	208
152	80	98	0.108	227
153	80	99	0.206	280
154	92	100	0.295	147
155	94	100	0.058	215
156	95	96	0.0547	259
157	96	97	0.0885	260
158	98	100	0.179	281
159	99	100	0.0813	209
160	100	101	0.1262	168
161	92	102	0.0559	209
162	101	102	0.112	209
163	100	103	0.0525	256
164	100	104	0.204	168
165	103	104	0.1584	137
166	103	105	0.1625	143
167	100	106	0.229	164
168	104	105	0.0378	141
169	105	106	0.0547	124
170	105	107	0.183	150
171	105	108	0.0703	187
172	106	107	0.183	151
173	108	109	0.0288	187
174	103	110	0.1813	170
175	109	110	0.0762	187
176	110	111	0.0755	295
177	110	112	0.064	110
178	17	113	0.0301	237
179	32	113	0.203	142
180	32	114	0.0612	122
181	27	115	0.0741	122
182	114	115	0.0104	122
183	68	116	0.00405	110
184	12	117	0.14	110
185	75	118	0.0481	217
186	76	118	0.0544	217

TABLE V
PARAMETERS OF CSP PLANTS

SGs No.	Maximum/Minimum (MW)	Ramp Level (MW/h)	Minimum Down Time (h)	Minimum Up Time (h)	Start-up and Shut-down Cost (\$)	Inertia Constant (s)
CSP1	100/10	40	1	1	61.5	7.8
CSP2	200/20	120	2	2	110.5	7.8

CSP3	100/10	80	1	1	61.5	7.8
CSP4	100/10	80	1	2	61.5	7.8
CSP5	100/10	80	2	1	61.5	7.8
CSP6	100/10	40	1	1	61.5	7.8
CSP7	100/10	40	1	1	61.5	7.8
CSP8	200/20	120	2	2	110.5	7.8
CSP9	100/10	80	1	1	61.5	7.8
CSP10	200/20	120	2	2	110.5	7.8
CSP11	100/10	40	1	1	61.5	7.8
CSP12	200/20	120	2	2	110.5	7.8
CSP13	100/10	80	1	1	61.5	7.8
CSP14	100/10	80	1	2	61.5	7.8
CSP15	100/10	80	2	1	61.5	7.8
CSP16	100/10	40	1	1	61.5	7.8
CSP17	100/10	40	1	1	61.5	7.8
CSP18	200/20	120	2	2	110.5	7.8
