The Derivation Formula of MMD-FR Linearity Criteria as Well as Models and Reformulations of Different models

I. Derivation Formula of MMD-FR Linearity Criteria

This section presents the derivation formula of the MMD-FR linearity criteria, focusing on the formula derivation process for the Multi-Speed Dependent frequency Nadir, corresponding with the main text.

Substituting (2) and (3) from the main text into (1), the frequency deviation piecewise function is obtained as follows:

$$\left|\Delta f(t_{tra})\right| = \begin{cases} \dfrac{\Delta P_{\text{Max}}^{\mathcal{D}}}{D_t'} \cdot (1 - e^{-\frac{D_t'}{2H_t^{Sys}} \cdot t_{tra}}) & \text{if } t_{tra} \in [t, t_{DB}) \\ \Delta f_{DB} + (\dfrac{\Delta P_t'^{\mathcal{D}}}{D_t'} + \dfrac{2H_t^{Sys} \cdot PFR_t'}{D_t'^2}) \cdot (1 - e^{-\frac{D_t'}{2H_t^{Sys}} \cdot (t_{tra} - t_{\text{DB}})}) - \dfrac{PFR_t'}{D_t'} \cdot (t_{tra} - t_{\text{DB}}), & \text{if } t_{tra} \in [t_{DB}, t_c) \\ \Delta f_c + (\dfrac{\Delta P_t''^{\mathcal{D}}}{D_t'} + \dfrac{2H_t^{Sys} \cdot PFR_t''}{D_t'^2}) \cdot (1 - e^{-\frac{D_t'}{2H_t^{Sys}} \cdot (t_{tra} - t_{\text{c}})}) - \dfrac{PFR_t''}{D_t'} \cdot (t_{tra} - t_{\text{c}}), & \text{if } t_{tra} \in [t_c, t_g) \end{cases} \tag{1}$$

where $\Delta P_t'^{\mathcal{D}} = \Delta P_{\text{Max}}^{\mathcal{D}} - D_t' \cdot \Delta f_{DB}$, $\Delta P_t''^{\mathcal{D}} = \Delta P_{\text{Max}}^{\mathcal{D}} - (\sum_g PFR_{g,t}^{\mathcal{G}} + \sum_w PFR_{w,t}^{\mathcal{W}}) \cdot T_c / T_g - \sum_c PFR_{\text{c},t}^{\mathcal{PB}} - D_t' \cdot \Delta f_{\text{c}}$, $PFR_t' = (\sum_g PFR_{g,t}^{\mathcal{G}} + \sum_w PFR_{w,t}^{\mathcal{W}})/T_g + \sum_c PFR_{\text{c},t}^{\mathcal{PB}}/T_c$, $PFR'' = (\sum_g PFR_{g,t}^{\mathcal{G}} + \sum_w PFR_{w,t}^{\mathcal{W}})/T_g$ 。

Set $\partial|\Delta f(t_{tra})|/\partial t = 0$ to obtain the following maximum frequency deviation $|\Delta f_{nadir}|$:

$$t' = \begin{cases} t_{DB} - \dfrac{2H_t^{Sys}}{D_t'} \cdot \log(\dfrac{2\kappa'}{\Delta P_t'^{\mathcal{D}} \cdot D_t' + 2\kappa'}) \\ t_c - \dfrac{2H_t^{Sys}}{D_t'} \cdot \log(\dfrac{2\kappa''}{\Delta P_t''^{\mathcal{D}} \cdot D_t' + 2\kappa''}) \end{cases} \Rightarrow \begin{cases} 2\kappa_t' \cdot \log(\dfrac{2\kappa'}{\Delta P_t'^{\mathcal{D}} \cdot D_t' + 2\kappa'}) \le D_t'^2 \cdot (\Delta f_{\max} - \Delta f_{DB}) - D_t' \cdot \Delta P_t'^{\mathcal{D}} \text{ if } t_{tra} \in [t_{DB}, t_c) \\ 2\kappa_t'' \cdot \log(\dfrac{2\kappa''}{\Delta P_t''^{\mathcal{D}} \cdot D_t' + 2\kappa''}) \le D_t'^2 \cdot (\Delta f_{\max} - \Delta f_c) - D_t' \cdot \Delta P_t''^{\mathcal{D}} \text{ if } t_{tra} \in [t_c, t_g) \end{cases} \tag{2}$$

Substituting (2) into (1), the following is obtained:

$$|\Delta f_{nadir}| = \begin{cases} \Delta f_{DB} + \dfrac{\Delta P_t^{\mathcal{D}'}}{D_t'} + \dfrac{2\kappa'}{T_c \cdot D_t'^2} \cdot \log(\dfrac{2\kappa'}{T_c \cdot \Delta P_t^{\mathcal{D}'} \cdot D_t' + 2\kappa'}) & \text{if } t_{tra} \in [t_{DB}, t_c) \\ \Delta f_c + \dfrac{\Delta P_t^{\mathcal{D}''}}{D_t'} + \dfrac{2\kappa''}{T_g \cdot D_t'^2} \cdot \log(\dfrac{2\kappa''}{T_g \cdot \Delta P_t^{\mathcal{D}''} \cdot D_t' + 2\kappa''}) & \text{if } t_{tra} \in [t_c, t_g) \end{cases} \tag{3}$$

Equation (3) is a piecewise function. The frequency nadir of the two segments depend on the PFR provision of SGs, wind farms, CSP plants, and system inertia, making it impossible to obtain an analytical solution. Numerical analysis shows that the frequency nadir appears in the first segment, as illustrated in Fig. 1.

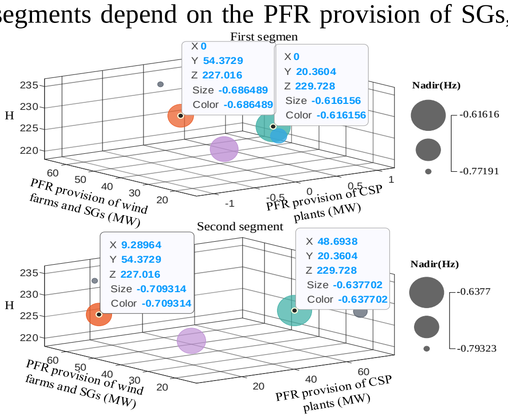

Fig. 1 Numerical simulation

II. Problem Formulation and Analysis

In this section, the mathematical formulation of deterministic FUCU (D-FCUC) and non-causal two-stage robust FUCU (NT-FCUC) under multi-uncertainty, for ease of understanding and discussion of validity of the proposed method.

A. D-FCUC Model

The D-FCUC model to minimize the total operational costs, including start-up and shut-down costs, operating costs of SGs, FRS provision costs, and curtailment penalties, all calculated from respective unit costs SU, VC, PC, SC, and $VoLL$, for given loads $P_{d,t}^{\mathcal{D}}$, can be mathematically formulated as:

objective function:

$$OC^{\text{sys}} = \sum_{t \in \mathcal{T}} [\sum_{g \in \mathcal{G}} (SU_g^{\mathcal{G}} \cdot x_{g,t}^{\mathcal{G},\text{Su}} + VC_g^{\mathcal{G}} \cdot P_{g,t}^{\mathcal{G}}) + \sum_{c \in \mathcal{CSP}} (SU_{\text{c}}^{\mathcal{PB}} \cdot x_{\text{c},t}^{\mathcal{PB},\text{Su}} + VC_{\text{c}}^{\mathcal{PB}} \cdot P_{\text{c},t}^{\mathcal{PB}}) + VoLL^{\mathcal{D}} \cdot \sum_{d \in \mathcal{D}} P_{d,t}^{\mathcal{D},\text{Cur}} + PC \cdot PFR_t^{\text{Sys}}(t_{QSS}) + SC \cdot SFR_t^{\text{Sys}}(t_{SFR})] \tag{4}$$

subject to:

- *Logic constraints of commitment states*

$$\mathcal{LC} := \Big\{ \boldsymbol{x} := \{0,1\} \in \mathbb{Z}^{2\cdot(N^{\mathcal{CSP}}+N^{\mathcal{G}})\times N^{\mathcal{T}}} : \\ x_{g,t}^{\mathcal{G}}, x_{g,t}^{\mathcal{G},\mathrm{Su}}, x_{c,t}^{\mathcal{PB}}, x_{c,t}^{\mathcal{PB},\mathrm{Su}}; \quad g\in\mathcal{G}, c\in\mathcal{CSP}, t\in\mathcal{T} \Big\} \tag{5}$$

- *Operational constraints of CSP plants*

$$\mathcal{OC} := \Big\{ \boldsymbol{y} \in \mathbb{R}^{5\cdot N^{\mathcal{CSP}}\times N^{\mathcal{T}}} : \tag{6}$$

$$\tilde{Q}_{c,t}^{\mathcal{SF}} = Q_{c,t}^{\mathcal{HT}} / \eta_c^{\mathrm{cha}} - Q_{c,t}^{\mathcal{TH}} \cdot \eta_c^{\mathrm{dis}} + P_{c,t}^{\mathcal{PB}} / \eta_c^{\mathcal{PB}} : (\theta_{c,t}^{\mathrm{SD}}),$$

$$r_c^{\mathcal{TES},\lim} \cdot E_c^{\mathcal{TES},\mathrm{Max}} \le (1-\iota)^{\mathrm{t}} \cdot E_c^{\mathcal{TES},\mathrm{In}} + \sum_{\tau\in 1:t} (1-\iota)^{\mathrm{t}-\tau} \\ \cdot (Q_{c,t}^{\mathcal{HT}} - Q_{c,t}^{\mathcal{TH}}) \le E_c^{\mathcal{TES},\mathrm{Max}} : (\theta_{c,t}^{\mathrm{TMIN}}, \theta_{c,t}^{\mathrm{TMAX}}), \\ 0 \le Q_{c,t}^{\mathcal{HT}} \le Q_{c,t}^{\mathcal{HT},\mathrm{Max}} : (\theta_{c,t}^{\mathrm{IMIN}}, \theta_{c,t}^{\mathrm{IMAX}}), \\ 0 \le Q_{c,t}^{\mathcal{TH}} \le Q_{c,t}^{\mathcal{TH},\mathrm{Max}} : (\theta_{c,t}^{\mathrm{OMIN}}, \theta_{c,t}^{\mathrm{OMAX}}); \tag{7}$$

$$P_{c,t}^{\mathcal{PB}} + PFR_{c,t}^{\mathcal{PB}} + SFR_{c,t}^{\mathcal{PB}} \le P_c^{\mathcal{PB},\mathrm{Max}} \cdot x_{c,t}^{\mathcal{PB}} : (\theta_{c,t}^{\mathrm{MAX}}). \\ P_{c,t}^{\mathcal{PB}} - PFR_{c,t}^{\mathcal{PB}} - SFR_{c,t}^{\mathcal{PB}} \ge P_c^{\mathcal{PB},\mathrm{Min}} \cdot x_{c,t}^{\mathcal{PB}} : (\theta_{c,t}^{\mathrm{MIN}}); \tag{8}$$

$$-RD_c^{\mathcal{PB}} \le P_{c,t}^{\mathcal{PB}} + PFR_{c,t}^{\mathcal{PB}} + SFR_{c,t}^{\mathcal{PB}} - P_{c,t-1}^{\mathcal{PB}} + PFR_{c,t-1}^{\mathcal{PB}} \\ + SFR_{c,t-1}^{\mathcal{PB}} \le RU_c^{\mathcal{PB}} : (\theta_{c,t}^{\mathrm{UP}}, \theta_{c,t}^{\mathrm{DN}}); \quad c\in\mathcal{CSP}, t\in\mathcal{T} \Big\} \tag{9}$$

- *Operational constraints of SGs*

$$\mathcal{OS} := \Big\{ \boldsymbol{z} \in \mathbb{R}^{3\cdot N^{\mathcal{G}}\times N^{\mathcal{T}}} :$$

$$P_{g,t}^{\mathcal{G}} + PFR_{g,t}^{\mathcal{G}} + SFR_{g,t}^{\mathcal{G}} \le x_{g,t} \cdot P_g^{\mathcal{G},\mathrm{Max}} : (\alpha_{g,t}^{\mathrm{MAX}}), \\ P_{g,t}^{\mathcal{G}} - PFR_{g,t}^{\mathcal{G}} - SFR_{g,t}^{\mathcal{G}} \ge x_{g,t} \cdot P_g^{\mathcal{G},\mathrm{Min}} : (\alpha_{g,t}^{\mathrm{MIN}}); \tag{10}$$

$$-RD_g^{\mathcal{G}} \le P_{g,t}^{\mathcal{G}} + PFR_{g,t}^{\mathcal{G}} + SFR_{g,t}^{\mathcal{G}} - P_{g,t-1}^{\mathcal{G}} + PFR_{g,t-1}^{\mathcal{G}} \\ + SFR_{g,t-1}^{\mathcal{G}} \le RU_g^{\mathcal{G}} : (\alpha_{g,t}^{\mathrm{DN}}, \alpha_{g,t}^{\mathrm{UP}}); \quad g\in\mathcal{G}, t\in\mathcal{T} \Big\} \tag{11}$$

- *Constraints Exclusively on Spatial Considerations:*

$$\mathcal{ES} := \Big\{ (\boldsymbol{y},\boldsymbol{z},\boldsymbol{u}) \in \mathbb{R}^{(2\cdot N^{\mathcal{G}}+4\cdot N^{\mathcal{W}}+2\cdot N^{\mathcal{CSP}}+2\cdot N^{\mathcal{D}})\times N^{\mathcal{T}}} :$$

$$\sum_{g\in\mathcal{G}} P_{g,t}^{\mathcal{G}} + \sum_{w\in\mathcal{W}} P_{w,t}^{\mathcal{W}} + \sum_{c\in\mathcal{CSP}} P_{c,t}^{\mathcal{PB}} = \sum_{d\in\mathcal{D}} P_{d,t}^{\mathcal{D}} : (\lambda_t^{\mathrm{SD}}); \tag{12}$$

$$-F_l^{\mathrm{Max}} \le \sum_{g\in\mathcal{G}} P_{g,t}^{\mathcal{G}} \cdot \Gamma_{l,g}^{\mathcal{G}} + \sum_{w\in\mathcal{W}} P_{w,t}^{\mathcal{W}} \cdot \Gamma_{l,w}^{\mathcal{W}} + \sum_{c\in\mathcal{CSP}} \times \\ P_{c,t}^{\mathcal{CSP}} \cdot \Gamma_{l,c}^{\mathcal{CSP}} - \sum_{d\in\mathcal{D}} P_{d,t}^{\mathcal{D}} \cdot \Gamma_{l,d}^{\mathcal{D}} \le F_l^{\mathrm{Max}} : (\varepsilon_{l,t}^{\mathrm{MIN}}, \varepsilon_{l,t}^{\mathrm{MAX}}); \tag{13}$$

$$P_{d,t}^{\mathcal{D},Cur} + P_{d,t}^{\mathcal{D}} = \tilde{P}_{d,t}^{\mathcal{D}} : (\beta_{d,t}^{\mathrm{SD},\mathcal{D}}), \\ 0 \le P_{d,t}^{\mathcal{D},Cur} \le \tilde{P}_{d,t}^{\mathcal{D}} : (\beta_{d,t}^{\mathrm{MIN}}, \beta_{d,t}^{\mathrm{MAX}}); \tag{14}$$

$$\tilde{P}_{w,t}^{\mathcal{W}} = P_{w,t}^{\mathcal{W}} + PFR_{w,t}^{\mathcal{W}} + SFR_{w,t}^{\mathcal{W}} + P_{w,t}^{\mathcal{W},\mathrm{Cur}} : (\beta_{w,t}^{\mathrm{SD},\mathcal{W}}), \\ 0 \le P_{w,t}^{\mathcal{W},\mathrm{Cur}} \le \tilde{P}_{w,t}^{\mathcal{W}} : (\beta_{w,t}^{\mathrm{MIN}}, \beta_{w,t}^{\mathrm{MAX}}); \tag{15}$$

$$0 \le PFR_{c,t}^{\mathcal{PB}} \le PFR_c^{\mathcal{PB},\mathrm{Max}} \cdot u_{c,t}^{\mathcal{PB}} : (\delta_{c,t}^{\mathrm{PMIN}}, \delta_{c,t}^{\mathrm{PMAX}}), \\ 0 \le SFR_{c,t}^{\mathcal{PB}} \le SFR_c^{\mathcal{PB},\mathrm{Max}} \cdot v_{c,t}^{\mathcal{PB}} : (\delta_{c,t}^{\mathrm{SMIN}}, \delta_{c,t}^{\mathrm{SMAX}}); \tag{16}$$

$$\text{(3)-(9)}; d\in\mathcal{D}, g\in\mathcal{G}, w\in\mathcal{W}, c\in\mathcal{CSP}, l\in\mathcal{L}, t\in\mathcal{T} \Big\}$$

$$\mathcal{LF} := \Big\{ \boldsymbol{u} := \{0,1\} \in \mathbb{Z}^{2\cdot(N^{\mathcal{G}}+N^{\mathcal{W}}+N^{\mathcal{CSP}})\times N^{\mathcal{T}}} : \\ u_{g,t}^{\mathcal{G}} \le x_{g,t}^{\mathcal{G}}, u_{c,t}^{\mathcal{PB}} \le x_{c,t}^{\mathcal{PB}}, v_{g,t}^{\mathcal{G}} \le x_{g,t}^{\mathcal{G}}, v_{c,t}^{\mathcal{PB}} \le x_{c,t}^{\mathcal{PB}}, u_{w,t}^{\mathcal{W}}, \\ v_{w,t}^{\mathcal{W}}; \quad g\in\mathcal{G}, w\in\mathcal{W}, c\in\mathcal{CSP}, t\in\mathcal{T} \Big\} \tag{17}$$

where all feasible UC of SGs and CSP plants are stated in (5), including as start-up and shut-down constraints as well as online and offline times in the $\mathcal{LC}$ (see e.g. [22] for details on the formulation). The set of feasible operations for SGs $\mathcal{OC}$, as described in (6)-(9), present the direct normal irradiance (DNI)-thermal-electrical energy conversion in the solar field (SF), thermal energy storage (TES), and power block (PB) of CSP plants, explicitly incorporating PFR and SFR provision into output limit to ensure hourly FRS deliverability under variable DNI and limited ramping in real-time ED, as describes in constraints (8)-(9). Feasible region of SGs $\mathcal{OS}$, explicitly including FRS provision capability, are listed in (10) and (11), with capacity limits in (10) and ramping limits in (11). Equation (12) enforces the active power balance and (13) limits the power flow of transmission lines in the set of feasible ED schedules $\mathcal{ES}$ that is non-temporal and exclusively spatial. The non-negativity of unserved load and wind spillage are indicated in (14) and (15), respectively. Equation (16) highlights the maximal PFR and SFR provision capability and CSP plants, associated with FRS-status-related decisions, whose relationship with commitment-status-related decisions is encapsulated within (17). Similar constraints for SGs and winds are not elaborated individually.

B. NT-FCUC Model

The user-defined set of multi-uncertain parameters, characterized by a hyperrectangle with upper and lower bounds encompassing load demand $\tilde{P}_{d,t}^{\mathcal{D}}$, wind power production $\tilde{P}_{w,t}^{\mathcal{W}}$, and thermal energy $\tilde{Q}_{c,t}^{\mathcal{SF}}$ absorbed by SF in CSP plants, manifests its worst-case realization only when reaching extreme upper $\overline{P}_{d,t}^{\mathcal{D}}$ or lower limits $\underline{P}_{w,t}^{\mathcal{W}}, \underline{Q}_{c,t}^{\mathcal{SF}}$. It can be recast into a more tractable formulation given as:

$$\boldsymbol{\xi}_t = (\tilde{P}_{d,t}^{\mathcal{D}} = \widehat{P}_{d,t}^{\mathcal{D}} + P_{d,t}^{\mathcal{D}+}, \tilde{P}_{w,t}^{\mathcal{W}} = \widehat{P}_{w,t}^{\mathcal{W}} - P_{w,t}^{\mathcal{W}-}, \\ \tilde{Q}_{c,t}^{\mathcal{CSP}} = \widehat{Q}_{c,t}^{\mathcal{CSP}} - Q_{c,t}^{\mathcal{CSP}-}) \in \mathbb{R}^{(N^{\mathcal{G}}+N^{\mathcal{W}}+N^{\mathcal{CSP}})\times 1} \\ 0 \le P_{d,t}^{\mathcal{D}+} \le \overline{P}_{d,t}^{\mathcal{D}}, 0 \le P_{w,t}^{\mathcal{W}-} \le \underline{P}_{w,t}^{\mathcal{W}}, 0 \le Q_{c,t}^{\mathcal{CSP}-} \le \underline{Q}_{c,t}^{\mathcal{CSP}}, \\ -\sum_{d\in\mathcal{D}} \Big(\frac{P_{d,t}^{\mathcal{D}+}}{\overline{P}_{d,t}^{\mathcal{D}}}\Big) - \sum_{w\in\mathcal{W}} \Big(\frac{P_{w,t}^{\mathcal{W}-}}{\underline{P}_{w,t}^{\mathcal{W}}}\Big) - \sum_{c\in\mathcal{CSP}} \Big(\frac{Q_{c,t}^{\mathcal{CSP}-}}{\underline{Q}_{c,t}^{\mathcal{CSP}}}\Big) \ge -\Lambda_t, \\ d\in\mathcal{D}, w\in\mathcal{W}, c\in\mathcal{CSP}, t\in\mathcal{T}\setminus\{1\} \tag{18}$$

Subsequently, compact form of NT-FCUC model, minimizing the worst-case total operational cost for multi-uncertainty set Ξ, is formulated in a conventional way as following:

$$\min_{x\in\mathcal{OC}, \boldsymbol{u}\in\mathcal{LF}} \{\boldsymbol{a}^{\mathrm{T}} \cdot \boldsymbol{x} + \boldsymbol{b}^{\mathrm{T}} \cdot \boldsymbol{u} + \max_{\xi\in\Xi} \varepsilon(\boldsymbol{x},\boldsymbol{u},\boldsymbol{\xi},\boldsymbol{y},\boldsymbol{z}) \\ s.\,t.\ \boldsymbol{E} \cdot \boldsymbol{x} + \boldsymbol{F} \cdot \boldsymbol{u} \le \boldsymbol{n}; \\ where\ \ \varepsilon(\boldsymbol{x},\boldsymbol{u},\boldsymbol{\xi},\boldsymbol{y},\boldsymbol{z}) = \min_{\substack{(\boldsymbol{y}\in\mathcal{OC}(\boldsymbol{x}), \boldsymbol{z}\in\mathcal{OS}(\boldsymbol{x}), \\ \boldsymbol{u}\in\mathcal{LF})\in\mathcal{ES}(\boldsymbol{x},\boldsymbol{u})}} (\boldsymbol{c}^{\mathrm{T}} \cdot \boldsymbol{y} + \boldsymbol{d}^{\mathrm{T}} \cdot \boldsymbol{z}) \\ s.\,t.\ \boldsymbol{G} \cdot \boldsymbol{y} + \boldsymbol{H} \cdot \boldsymbol{z} \le \boldsymbol{n} - \boldsymbol{L} \cdot \boldsymbol{x} - \boldsymbol{M} \cdot \boldsymbol{u} - \boldsymbol{N} \cdot \boldsymbol{\xi} \tag{19}$$

where due to structural changes, $\boldsymbol{m}$ becomes vector $\boldsymbol{n}$. Also, the parameters represented by matrices $\boldsymbol{E}$, $\boldsymbol{F}$, $\boldsymbol{G}$, $\boldsymbol{H}$, $\boldsymbol{L}$, $\boldsymbol{M}$, and $\boldsymbol{N}$ can be calculated by constraints of NT-FCUC model. $\varepsilon(x,u,\xi,y,z)$ represents the feasible domain produced by HANDs and actual resolution of multi-uncertainty parameters $\boldsymbol{\xi}$. The detailed robust counterpart is listed in the Appendix.

III. Robust Counterpart of NT-FCUC

This section mainly presents the mathematical formulation of SP in NT-FCUC model.

objective function:

$$
\begin{aligned}
&\sum_{t\in\mathcal{T}}\{\sum_{d\in\mathcal{D}}[(\widehat{P}_{d,t}^{\mathcal{D}}\cdot\lambda_t^{SD}+P_{d,t}^{\mathcal{D}+}\cdot\vartheta_{d,t}^{SD})-(\widehat{P}_{d,t}^{\mathcal{D}}\cdot\beta_{d,t}^{MAX}\\
&\quad+P_{d,t}^{\mathcal{D}+}\cdot\vartheta_{d,t}^{MAX})]+\sum_{g\in\mathcal{G}}(x_{g,t}\cdot P_g^{\mathcal{G},Min}\cdot\alpha_{g,t}^{MIN}\\
&\quad-x_{g,t}\cdot P_g^{\mathcal{G},Max}\cdot\alpha_{g,t}^{MAX}-PFR_g^{\mathcal{G},Max}\cdot u_{g,t}^{\mathcal{G}}\cdot\delta_{g,t}^{PMAX}\\
&\quad-SFR_g^{\mathcal{G},Max}\cdot v_{g,t}^{\mathcal{G}}\cdot\delta_{g,t}^{SMAX}-M\cdot(1-x_{g,t}^{\mathcal{G}})\cdot\rho_{g,t}^{MIN-}\\
&\quad-M\cdot x_{g,t}^{\mathcal{G}}\cdot\rho_{g,t}^{MAX+})+\sum_{w\in\mathcal{W}}[\widehat{P}_{w,t}^{\mathcal{W}}\cdot\beta_{w,t}^{SD}-P_{w,t}^{\mathcal{W}-}\\
&\quad\cdot\vartheta_{w,t}^{SD}-(\widehat{P}_{w,t}^{\mathcal{W}}\cdot\beta_{w,t}^{MAX}-P_{w,t}^{\mathcal{W}-}\cdot\vartheta_{w,t}^{MAX})-PFR_w^{\mathcal{W},Max}\\
&\quad\cdot u_{w,t}^{\mathcal{W}}\cdot\delta_{w,t}^{PMAX}-SFR_w^{\mathcal{W},Max}\cdot v_{w,t}^{\mathcal{W}}\cdot\delta_{w,t}^{SMAX}\\
&\quad-M\cdot(1-o_{w,t}^{\mathcal{W}})\cdot\rho_{w,t}^{MIN-}-M\cdot o_{w,t}^{\mathcal{W}}\cdot\rho_{w,t}^{MAX+}]\\
&\quad+\sum_{c\in\mathcal{CSP}}[\widehat{Q}_{c,t}^{\mathcal{SF}}\cdot\theta_{c,t}^{SD}-Q_{c,t}^{\mathcal{SF}-}\cdot\vartheta_{c,t}^{SD}+P_c^{\mathcal{PB},Min}\\
&\quad\cdot x_{c,t}^{\mathcal{PB}}\cdot\theta_{c,t}^{MIN}-P_c^{\mathcal{PB},Max}\cdot x_{c,t}^{\mathcal{PB}}\cdot\theta_{c,t}^{MAX}+[r_c^{\mathcal{TES},lim}\\
&\quad\cdot E_c^{\mathcal{TES},Max}-(1-\iota)^t\cdot E_c^{\mathcal{TES},in}]\cdot\theta_{c,t}^{TMIN}-[E_c^{\mathcal{TES},Max}\\
&\quad-(1-\iota)^t\cdot E_c^{\mathcal{TES},in}]\cdot\theta_{c,t}^{TMAX}-Q_{c,t}^{\mathcal{HT},Max}\cdot\theta_{c,t}^{IMAX}\\
&\quad\cdot\varsigma_{c,t}^{\mathcal{HT}}-Q_{c,t}^{\mathcal{TH},Max}\cdot\theta_{c,t}^{OMAX}\cdot(1-\varsigma_{c,t}^{\mathcal{HT}})\\
&\quad-PFR_{c,t}^{\mathcal{PB},Max}\cdot u_{c,t}^{\mathcal{PB}}\cdot\delta_{c,t}^{PMAX}-SFR_{c,t}^{\mathcal{PB},Max}\cdot v_{c,t}^{\mathcal{PB}}\\
&\quad\cdot\delta_{c,t}^{SMAX}-M\cdot(1-x_{c,t}^{\mathcal{PB}})\cdot\rho_{c,t}^{MIN-}-M\cdot x_{c,t}^{\mathcal{PB}}\\
&\quad\cdot\rho_{c,t}^{MAX+}]-\sum_{l\in\mathcal{L}}(F_l^{Max}\cdot\varepsilon_{l,t}^{MIN}+F_l^{Max}\cdot\varepsilon_{l,t}^{MAX})\\
&\quad-(D'\cdot\Delta f_{Max}^{QSS}-\Delta P_{t,Max}^{\mathcal{D}})\cdot\pi_t^{QSS}+\Delta f_{Max}^{QSS}\cdot D\cdot\\
&\quad\sum_{d\in\mathcal{D}}P_{d,t}^{\mathcal{D}+}\cdot\vartheta_{d,t}^{QSS}+\kappa'\cdot f_0\cdot\rho_t\\
&\quad+(SFR^{Min}-\Delta P_{Max}^{\mathcal{L}}\cdot\frac{t_{SFR}-t_{QSS}-2\zeta_1}{20\zeta_2})\cdot\nu_t^{MIN}\\
&\quad-(SFR^{Max}-\Delta P_{Max}^{\mathcal{L}}\cdot\frac{t_{SFR}-t_{QSS}-2\zeta_1}{20\zeta_2})\cdot\nu_t^{MAX}\\
&\quad+\Delta P_{t,Max}^{\mathcal{D}}\cdot\nu_t^{FRS}\}-\sum_{t\in\mathcal{T}\setminus\{1\}}\{\sum_{g\in\mathcal{G}}\{[P_g^{\mathcal{G},Max}\\
&\quad-x_{g,t-1}^{\mathcal{G}}\cdot(P_g^{\mathcal{G},Min}-\Delta P_g^{\mathcal{G},Up})-x_{g,t}^{\mathcal{G}}\cdot(P_g^{\mathcal{G},Max}\\
&\quad-P_g^{\mathcal{G},Min})]\cdot\alpha_{g,t}^{UP}+[P_g^{\mathcal{G},Max}-x_{g,t}^{\mathcal{G}}\cdot(P_g^{\mathcal{G},Min}\\
&\quad-\Delta P_g^{\mathcal{G},Dn})-x_{g,t-1}^{\mathcal{G}}\cdot(P_g^{\mathcal{G},Max}-P_g^{\mathcal{G},Min})]\cdot\alpha_{g,t}^{DN}\}\\
&\quad+\sum_{c\in\mathcal{CSP}}\{[P_c^{\mathcal{PB},Max}-x_{c,t-1}^{\mathcal{PB}}\cdot(P_c^{\mathcal{PB},Max}-R_c^{\mathcal{PB},Up})\\
&\quad-x_{c,t}^{\mathcal{PB}}\cdot(P_c^{\mathcal{PB},Max}-R_c^{\mathcal{PB},Min})]\cdot\theta_{c,t}^{UP}+[P_c^{\mathcal{PB},Max}\\
&\quad-x_{c,t}^{\mathcal{PB}}\cdot(P_c^{\mathcal{PB},Min}-R_c^{\mathcal{PB},Dn})-x_{c,t-1}^{\mathcal{PB}}\cdot(P_c^{\mathcal{PB},Max}\\
&\quad-R_c^{\mathcal{PB},Min})]\cdot\theta_{c,t}^{DN}\}\}
\end{aligned}
\tag{20}
$$

subject to:

$$
\begin{aligned}
&\vartheta_{d,t}^{SD}=o_{d,t}^{\mathcal{D}}\cdot\lambda_t^{SD};\vartheta_{d,t}^{MAX}=o_{d,t}^{\mathcal{D}}\cdot\beta_{d,t}^{MAX};\\
&\vartheta_{d,t}^{QSS}=o_{d,t}^{\mathcal{D}}\cdot\pi_t^{QSS};d\in\mathcal{D},t\in\mathcal{T}
\end{aligned}
\tag{21}
$$

$$
\begin{aligned}
&\vartheta_{w,t}^{SD}=o_{w,t}^{\mathcal{W}}\cdot\beta_{w,t}^{SD};\vartheta_{w,t}^{MAX}=o_{w,t}^{\mathcal{W}}\cdot\beta_{w,t}^{MAX}\\
&w\in\mathcal{W},t\in\mathcal{T}
\end{aligned}
\tag{22}
$$

$$
\vartheta_{c,t}^{SD}=o_{c,t}^{\mathcal{SF}}\cdot\theta_{c,t}^{SD};\quad c\in\mathcal{CSP},t\in\mathcal{T}
\tag{23}
$$

$$
\begin{aligned}
&-M\cdot(1-o_{d,t}^{\mathcal{D}})\le\vartheta_{d,t}^{SD}-\lambda_t^{SD}\le 0;0\le\vartheta_{d,t}^{SD}\le M\cdot o_{d,t}^{\mathcal{D}};\\
&-M\cdot(1-o_{d,t}^{\mathcal{D}})\le\vartheta_{d,t}^{MAX}-\beta_{d,t}^{MAX}\le 0;\\
&0\le\vartheta_{d,t}^{SD}\le M\cdot o_{d,t}^{\mathcal{D}};\\
&-M\cdot(1-o_{d,t}^{\mathcal{D}})\le\vartheta_{d,t}^{QSS}-\pi_t^{QSS}\le 0;0\le\vartheta_{d,t}^{QSS}\le M\cdot o_{d,t}^{\mathcal{D}};\\
&d\in\mathcal{D},t\in\mathcal{T}
\end{aligned}
\tag{24}
$$

$$
\begin{aligned}
&-M\cdot(1-o_{w,t}^{\mathcal{W}})\le\vartheta_{w,t}^{SD}-\beta_{w,t}^{SD}\le 0;0\le\vartheta_{w,t}^{SD}\le M\cdot o_{w,t}^{\mathcal{W}};\\
&-M\cdot(1-o_{w,t}^{\mathcal{W}})\le\vartheta_{w,t}^{MAX}-\beta_{w,t}^{MAX}\le 0;\\
&0\le\vartheta_{w,t}^{MAX}\le M\cdot o_{w,t}^{\mathcal{W}};w\in\mathcal{W},t\in\mathcal{T}
\end{aligned}
\tag{25}
$$

$$
\begin{aligned}
&-M\cdot(1-o_{c,t}^{\mathcal{SF}})\le\vartheta_{c,t}^{SD}-\theta_{c,t}^{SD}\le 0;\\
&0\le\vartheta_{c,t}^{SD}\le M\cdot o_{c,t}^{\mathcal{SF}};c\in\mathcal{CSP},t\in\mathcal{T}
\end{aligned}
\tag{26}
$$

$$
\begin{aligned}
&\alpha_{g,t}^{MIN}-\alpha_{g,t}^{MAX}+\lambda_t^{SD}-\alpha_{g,t}^{UP}+\alpha_{g,t+1}^{UP}+\alpha_{g,t}^{DN}-\alpha_{g,t+1}^{DN}\\
&+\sum_{l\in\mathcal{L}}(\Gamma_{l,g}^{\mathcal{G}}\cdot\varepsilon_{l,t}^{MIN}-\Gamma_{l,g}^{\mathcal{G}}\cdot\varepsilon_{l,t}^{MAX})=VC_g^{\mathcal{G}}:(P_{g,t}^{\mathcal{G}})\\
&g\in\mathcal{G},t\in\mathcal{T}\setminus\{1,N^{\mathcal{T}}\}
\end{aligned}
\tag{27}
$$

$$
\begin{aligned}
&\alpha_{g,t}^{MIN}-\alpha_{g,t}^{MAX}+\lambda_t^{SD}-\alpha_{g,t}^{UP}+\alpha_{g,t}^{DN}+\sum_{l\in\mathcal{L}}(\Gamma_{l,g}^{\mathcal{G}}\cdot\varepsilon_{l,t}^{MIN}\\
&-\Gamma_{l,g}^{\mathcal{G}}\cdot\varepsilon_{l,t}^{MAX})=VC_g^{\mathcal{G}}:(P_{g,t}^{\mathcal{G}})\quad g\in\mathcal{G},t=N^{\mathcal{T}}
\end{aligned}
\tag{28}
$$

$$
\begin{aligned}
&\alpha_{g,t}^{MIN}-\alpha_{g,t}^{MAX}+\lambda_t^{SD}+\alpha_{g,t+1}^{UP}-\alpha_{g,t+1}^{DN}+\sum_{l\in\mathcal{L}}(\Gamma_{l,g}^{\mathcal{G}}\\
&\cdot\varepsilon_{l,t}^{MIN}-\Gamma_{l,g}^{\mathcal{G}}\cdot\varepsilon_{l,t}^{MAX})=VC_g^{\mathcal{G}}:(P_{g,t}^{\mathcal{G}})\quad g\in\mathcal{G},t=1
\end{aligned}
\tag{29}
$$

$$
\begin{aligned}
&\lambda_t^{SD}+\beta_{w,t}^{SD}+\sum_{l\in\mathcal{L}}(\Gamma_{l,w}^{\mathcal{W}}\cdot\varepsilon_{l,t}^{MIN}-\Gamma_{l,w}^{\mathcal{W}}\cdot\varepsilon_{l,t}^{MAX})=0\\
&:(P_{w,t}^{\mathcal{W}})\quad w\in\mathcal{W},t\in\mathcal{T}
\end{aligned}
\tag{30}
$$

$$
\beta_{w,t}^{SD.W}\le VoLL^{\mathcal{W}}:(P_{w,t}^{\mathcal{W},Cur})\quad w\in\mathcal{W},t\in\mathcal{T}
\tag{31}
$$

$$
\begin{aligned}
&-\lambda_t^{SD}+\beta_t^{SD.D}-\sum_{l\in\mathcal{L}}(\Gamma_{l,d}^{\mathcal{D}}\cdot\varepsilon_{l,t}^{MIN}-\Gamma_{l,d}^{\mathcal{D}}\cdot\varepsilon_{l,t}^{MAX})=0\\
&:(P_{d,t}^{\mathcal{D}})\quad d\in\mathcal{D},t\in\mathcal{T}
\end{aligned}
\tag{32}
$$

$$
\lambda_t^{SD}+\beta_t^{SD.D}\le VoLL^{\mathcal{D}}:(P_{d,t}^{\mathcal{D},Cur})\quad d\in\mathcal{D},t\in\mathcal{T}
\tag{33}
$$

$$
\begin{aligned}
&\lambda_t^{SD}+\theta_{c,t}^{MIN}-\theta_{c,t}^{MAX}-\theta_{c,t}^{UP}+\theta_{c,t+1}^{UP}+\theta_{c,t}^{DN}-\theta_{c,t+1}^{DN}\\
&+\theta_{c,t}^{SD}/\eta_c^{PB}+\sum_{l\in\mathcal{L}}(\Gamma_{l,c}^{\mathcal{PB}}\cdot\varepsilon_{l,t}^{MIN}-\Gamma_{l,c}^{\mathcal{PB}}\cdot\varepsilon_{l,t}^{MAX})\\
&=0:(Q_{c,t}^{\mathcal{PB}})\quad c\in\mathcal{CSP},t\in\mathcal{T}\setminus\{1,N^{\mathcal{T}}\}
\end{aligned}
\tag{34}
$$

$$
\begin{aligned}
&\lambda_t^{SD}+\theta_{c,t}^{MIN}-\theta_{c,t}^{MAX}+\theta_{c,t+1}^{UP}-\theta_{c,t+1}^{DN}+\theta_{c,t}^{SD}/\eta_c^{\mathcal{PB}}\\
&+\sum_{l\in\mathcal{L}}(\Gamma_{l,c}^{\mathcal{PB}}\cdot\varepsilon_{l,t}^{MIN}-\Gamma_{l,c}^{\mathcal{PB}}\cdot\varepsilon_{l,t}^{MAX})=0:(Q_{c,t}^{\mathcal{PB}})\\
&c\in\mathcal{CSP},t=1
\end{aligned}
\tag{35}
$$

$$
\begin{aligned}
&\lambda_t^{SD}+\theta_{c,t}^{MIN}-\theta_{c,t}^{MAX}-\theta_{c,t}^{UP}+\theta_{c,t}^{DN}+\theta_{c,t}^{SD}/\eta_c^{\mathcal{PB}}\\
&+\sum_{l\in\mathcal{L}}(\Gamma_{l,c}^{\mathcal{PB}}\cdot\varepsilon_{l,t}^{MIN}-\Gamma_{l,c}^{\mathcal{PB}}\cdot\varepsilon_{l,t}^{MAX})=0:(Q_{c,t}^{\mathcal{PB}})\\
&c\in\mathcal{CSP},t=N^{\mathcal{T}}
\end{aligned}
\tag{36}
$$

$$
\begin{aligned}
&-\eta_c^{dis}\cdot\theta_{c,t}^{SD}-\sum_{\tau\in t:N^{\mathcal{T}}}(1-\iota)^{\tau-t}\cdot(\theta_{c,\tau}^{TMIN}-\theta_{c,\tau}^{TMAX})\\
&+\theta_{c,t}^{OMIN}-\theta_{c,t}^{OMAX}=0:(Q_{c,t}^{\mathcal{TH}})\quad c\in\mathcal{CSP},t\in\mathcal{T}
\end{aligned}
\tag{37}
$$

$$
\begin{aligned}
&\theta_{c,t}^{SD}/\eta_c^{cha}+\sum_{\tau\in t:N^{\mathcal{T}}}(1-\iota)^{\tau-t}\cdot(\theta_{c,\tau}^{TMIN}-\theta_{c,\tau}^{TMAX})\\
&+\theta_{c,t}^{IMIN}-\theta_{c,t}^{IMAX}=0:(Q_{c,t}^{\mathcal{HT}})\quad c\in\mathcal{CSP},t\in\mathcal{T}
\end{aligned}
\tag{38}
$$

$$
\begin{aligned}
&\omega_t - \sum_{g\in\mathcal{G}}(\rho_{g,t}^{MIN-} - \rho_{g,t}^{MAX-}) - \sum_{c\in\mathcal{CSP}}(\rho_{c,t}^{MIN-} - \rho_{c,t}^{MAX-}) - \\
&\sum_{w\in\mathcal{W}}(\rho_{w,t}^{MIN-} - \rho_{w,t}^{MAX-} - H_w^{\mathcal{W}} \cdot \widehat{P}_{w,t}^{\mathcal{W}} \cdot \rho_t) - H_d^{\mathcal{D}} \cdot \\
&\Delta P_{t,Max}^{\mathcal{D}} / T_g \cdot \rho_t = 0
\end{aligned} \tag{39}
$$

$$
\begin{aligned}
&\nu_t^{FRS} - \alpha_{g,t}^{MIN} - \alpha_{g,t}^{MAX} - \alpha_{g,t}^{UP} - \alpha_{g,t+1}^{UP} - \alpha_{g,t}^{DN} - \alpha_{g,t+1}^{DN} - \\
&\delta_{g,t}^{PMAX} + \pi_t^{QSS} - \omega_t / T_g - \frac{t_{SFR} - t_{QSS} - 2\zeta_1}{20\zeta_2} \cdot (\nu_t^{MIN} - \\
&\nu_t^{MAX}) \leq PC_g^{\mathcal{G}} : (PFR_{g,t}^{\mathcal{G}}) \quad g \in \mathcal{G}, t \in \mathcal{T} \setminus \{1, N^{\mathcal{T}}\}
\end{aligned} \tag{40}
$$

$$
\begin{aligned}
&\nu_t^{FRS} - \alpha_{g,t}^{MIN} - \alpha_{g,t}^{MAX} - \alpha_{g,t}^{UP} - \alpha_{g,t}^{DN} - \delta_{g,t}^{PMAX} + \pi_t^{QSS} - \\
&\omega_t / T_g - \frac{t_{SFR} - t_{QSS} - 2\zeta_1}{20\zeta_2} \cdot (\nu_t^{MIN} - \nu_t^{MAX}) \leq PC_g^{\mathcal{G}} \\
&: (PFR_{g,t}^{\mathcal{G}}) \quad g \in \mathcal{G}, t = N^{\mathcal{T}}
\end{aligned} \tag{41}
$$

$$
\begin{aligned}
&\nu_t^{FRS} - \alpha_{g,t}^{MIN} - \alpha_{g,t}^{MAX} - \alpha_{g,t+1}^{UP} - \alpha_{g,t+1}^{DN} - \delta_{g,t}^{PMAX} + \pi_t^{QSS} - \\
&\omega_t / T_g - \frac{t_{SFR} - t_{QSS} - 2\zeta_1}{20\zeta_2} \cdot (\nu_t^{MIN} - \nu_t^{MAX}) \leq PC_g^{\mathcal{G}} \\
&: (PFR_{g,t}^{\mathcal{G}}) \quad g \in \mathcal{G}, t = 1
\end{aligned} \tag{42}
$$

$$
\begin{aligned}
&\nu_t^{FRS} + \beta_{w,t}^{SD} + \delta_{w,t}^{PMIN} - \delta_{w,t}^{PMAX} + \pi_t^{QSS} - \omega_t / T_g - \\
&\frac{t_{SFR} - t_{QSS} - 2\zeta_1}{20\zeta_2} \cdot (\nu_t^{MIN} - \nu_t^{MAX}) \leq PC_w^{\mathcal{W}} \\
&: (PFR_{w,t}^{\mathcal{W}}) \quad w \in \mathcal{W}, t \in \mathcal{T}
\end{aligned} \tag{43}
$$

$$
\begin{aligned}
&\nu_t^{FRS} - \theta_{c,t}^{MIN} - \theta_{c,t}^{MAX} - \theta_{c,t}^{UP} - \theta_{c,t+1}^{UP} - \theta_{c,t}^{DN} - \theta_{c,t+1}^{DN} - \\
&\delta_{c,t}^{PMAX} + \pi_t^{QSS} - \omega_t / T_g - \frac{t_{SFR} - t_{QSS} - 2\zeta_1}{20\zeta_2} \cdot \\
&(\nu_t^{MIN} - \nu_t^{MAX}) \leq PC_c^{\mathcal{PB}} : (PFR_{c,t}^{\mathcal{CSP}}) \\
&c \in \mathcal{CSP}, t \in \mathcal{T} \setminus \{1, N^{\mathcal{T}}\}
\end{aligned} \tag{44}
$$

$$
\begin{aligned}
&\nu_t^{FRS} - \theta_{c,t}^{MIN} - \theta_{c,t}^{MAX} - \theta_{c,t}^{UP} - \theta_{c,t}^{DN} - \delta_{c,t}^{PMAX} + \\
&\pi_t^{QSS} - \omega_t / T_g - \frac{t_{SFR} - t_{QSS} - 2\zeta_1}{20\zeta_2} \cdot (\nu_t^{MIN} - \nu_t^{MAX}) \\
&\leq PC_c^{\mathcal{CSP}} : (PFR_{c,t}^{\mathcal{PB}}) \quad c \in \mathcal{CSP}, t = N^{\mathcal{T}}
\end{aligned} \tag{45}
$$

$$
\begin{aligned}
&\nu_t^{FRS} - \theta_{c,t}^{MIN} - \theta_{c,t}^{MAX} - \theta_{c,t+1}^{UP} - \theta_{c,t+1}^{DN} - \delta_{c,t}^{PMAX} + \\
&\pi_t^{QSS} - \omega_t / T_g - \frac{t_{SFR} - t_{QSS} - 2\zeta_1}{20\zeta_2} \cdot (\nu_t^{MIN} - \\
&\nu_t^{MAX}) \leq PC_c^{\mathcal{CSP}} : (PFR_{c,t}^{\mathcal{PB}}) \quad c \in \mathcal{CSP}, t = 1
\end{aligned} \tag{46}
$$

$$
\begin{aligned}
&\nu_t^{FRS} - \theta_{c,t}^{MIN} - \theta_{c,t}^{MAX} - \theta_{c,t+1}^{UP} - \theta_{c,t+1}^{DN} - \delta_{c,t}^{PMAX} + \pi_t^{QSS} - \\
&- \omega_t / T_g - \frac{t_{SFR} - t_{QSS} - 2\zeta_1}{20\zeta_2} \cdot (\nu_t^{MIN} - \nu_t^{MAX}) \leq PC_c^{\mathcal{CSP}} \\
&: (PFR_{c,t}^{\mathcal{PB}}) \quad c \in \mathcal{CSP}, t = 1
\end{aligned} \tag{47}
$$

$$
\begin{aligned}
&-\alpha_{g,t}^{MIN} - \alpha_{g,t}^{MAX} - \alpha_{g,t}^{UP} - \alpha_{g,t+1}^{UP} - \alpha_{g,t}^{DN} - \alpha_{g,t+1}^{DN} - \\
&\delta_{g,t}^{SMAX} + \nu_t^{FRS} \leq SC_g^{\mathcal{G}} : (SFR_{g,t}^{\mathcal{G}}) \\
&g \in \mathcal{G}, t \in \mathcal{T} \setminus \{1, N^{\mathcal{T}}\}
\end{aligned} \tag{48}
$$

$$
\begin{aligned}
&-\alpha_{g,t}^{MIN} - \alpha_{g,t}^{MAX} - \alpha_{g,t}^{UP} - \alpha_{g,t}^{DN} - \delta_{g,t}^{SMAX} + \nu_t^{FRS} \\
&\leq SC_g^{\mathcal{G}} : (SFR_{g,t}^{\mathcal{G}}) \quad g \in \mathcal{G}, t = N^{\mathcal{T}}
\end{aligned} \tag{49}
$$

$$
\begin{aligned}
&-\alpha_{g,t}^{MIN} - \alpha_{g,t}^{MAX} - \alpha_{g,t+1}^{UP} - \alpha_{g,t+1}^{DN} - \delta_{g,t}^{SMAX} + \nu_t^{FRS} \\
&\leq SC_g^{\mathcal{G}} : (SFR_{g,t}^{\mathcal{G}}) \quad g \in \mathcal{G}, t = 1
\end{aligned} \tag{50}
$$

$$
\begin{aligned}
&\beta_t^{w,(SD)} - \delta_t^{w,(SMAX)} + \nu_t^{FRS} \leq SC_w^{\mathcal{W}} : (SFR_{w,t}^{\mathcal{W}}) \\
&w \in \mathcal{W}, t \in \mathcal{T}
\end{aligned} \tag{51}
$$

$$
\begin{aligned}
&-\theta_{c,t}^{MAX} - \theta_{c,t}^{MIN} - \theta_{c,t}^{UP} - \theta_{c,t+1}^{UP} - \theta_{c,t}^{DN} - \theta_{c,t+1}^{DN} + \\
&\delta_{c,t}^{SMIN} + \nu_t^{FRS} = SC_c^{\mathcal{PB}} : (SFR_{c,t}^{\mathcal{PB}}) \\
&c \in \mathcal{CSP}, t \in \mathcal{T} \setminus \{1, N^{\mathcal{T}}\}
\end{aligned} \tag{52}
$$

$$
\begin{aligned}
&-\theta_{c,t}^{MAX} - \theta_{c,t}^{MIN} - \theta_{c,t}^{UP} - \theta_{c,t}^{DN} - \delta_{c,t}^{SMAX} + \nu_t^{FRS} \\
&\leq SC_c^{\mathcal{PB}} : (SFR_{c,t}^{\mathcal{PB}}) \quad c \in \mathcal{CSP}, t = N^{\mathcal{T}}
\end{aligned} \tag{53}
$$

$$
\begin{aligned}
&-\theta_{c,t}^{MAX} - \theta_{c,t}^{MIN} - \theta_{c,t+1}^{UP} - \theta_{c,t+1}^{DN} - \delta_{c,t}^{SMAX} + \nu_t^{FRS} \\
&\leq SC_c^{\mathcal{PB}} : (SFR_{c,t}^{\mathcal{PB}}) \quad c \in \mathcal{CSP}, t = 1
\end{aligned} \tag{54}
$$

$$
\begin{aligned}
&H_g^{\mathcal{G}} \cdot P_g^{\mathcal{G},Max} \cdot \rho_t + \rho_{g,t}^{MIN-} - \rho_{g,t}^{MAX-} + \rho_{g,t}^{MIN+} - \rho_{g,t}^{MAX+} \\
&= 0 : (\varpi_{g,t}^{\mathcal{G}}) \quad g \in \mathcal{G}, t \in \mathcal{T}
\end{aligned} \tag{55}
$$

$$
\begin{aligned}
&H_c^{\mathcal{CSP}} \cdot P_c^{\mathcal{PB},Max} \cdot \rho_t + \rho_{c,t}^{MIN-} - \rho_{c,t}^{MAX-} + \rho_{c,t}^{MIN+} - \\
&\rho_{c,t}^{MAX+} = 0 : (\varpi_{c,t}^{\mathcal{PB}}) \quad c \in \mathcal{CSP}, t \in \mathcal{T}
\end{aligned} \tag{56}
$$

$$
\begin{aligned}
&-P_{w,t}^{\mathcal{W}-} \cdot \rho_t + \rho_{w,t}^{MIN-} - \rho_{w,t}^{MAX-} + \rho_{w,t}^{MIN+} - \rho_{w,t}^{MAX+} \\
&= 0 : (\varpi_{w,t}^{\mathcal{W}}) \quad c \in \mathcal{W}, t \in \mathcal{T}
\end{aligned} \tag{57}
$$

$$
\alpha, \beta, \lambda, \varepsilon, \delta, \nu, \theta, \rho, \pi \geq 0 \tag{58}
$$

IV. Robust Counterpart of MTAR-FCUC

This section mainly presents the mathematical formulation of SP in MTAR-FCUC model.

objective function:

$$
\begin{aligned}
&\sum_{t\in\mathcal{T}}\{\sum_{d\in\mathcal{D}}[(\widehat{P}_{d,t}^{\mathcal{D}} \cdot \beta_{d,t}^{SD,\mathcal{D}} + P_{d,t}^{\mathcal{D}+} \cdot \vartheta_{d,t}^{SD,\mathcal{D}})] + \sum_{g\in\mathcal{G}}[\underline{P}_{g,t}^{\mathcal{G}} \cdot \\
&\alpha_{g,t}^{MIN} - \overline{P}_{g,t}^{\mathcal{G}} \cdot \alpha_{g,t}^{MAX} + \underline{PFR}_{g,t}^{\mathcal{G}} \cdot \delta_{g,t}^{PMIN} - \overline{PFR}_{g,t}^{\mathcal{G}} \cdot \\
&\delta_{g,t}^{PMAX} + \underline{SFR}_{g,t}^{\mathcal{G}} \cdot \delta_{g,t}^{SMIN} - \overline{SFR}_{g,t}^{\mathcal{G}} \cdot \delta_{g,t}^{SMAX} \\
&- M \cdot (1 - x_{g,t}^{\mathcal{G}}) \cdot \rho_{g,t}^{MIN-} - M \cdot x_{g,t}^{\mathcal{G}} \cdot \rho_{g,t}^{MAX+}] \\
&+ \sum_{w\in\mathcal{W}}[\widehat{P}_{w,t}^{\mathcal{W}} \cdot \beta_{w,t}^{SD,\mathcal{W}} - P_{w,t}^{\mathcal{W}-} \cdot \vartheta_{w,t}^{SD,\mathcal{W}} - (\widehat{P}_{w,t}^{\mathcal{W}} \cdot \beta_{w,t}^{MAX} \\
&- P_{w,t}^{\mathcal{W}-} \cdot \vartheta_{w,t}^{MAX}) - PFR_w^{\mathcal{W},Max} \cdot u_{w,t}^{\mathcal{W}} \cdot \delta_{w,t}^{PMAX} - \\
&SFR_w^{\mathcal{W},Max} \cdot v_{w,t}^{\mathcal{W}} \cdot \delta_{w,t}^{SMAX} - M \cdot (1 - o_{w,t}^{\mathcal{W}}) \cdot \rho_{w,t}^{-MIN} \\
&- M \cdot o_{w,t}^{\mathcal{W}} \cdot \rho_{w,t}^{MAX})] + \sum_{c\in\mathcal{CSP}}\{\widehat{Q}_{c,t}^{\mathcal{SF}} \cdot \theta_{c,t}^{SD} - Q_{c,t}^{\mathcal{SF}-} \cdot \\
&\vartheta_{c,t}^{SD} + \underline{P}_{c,t}^{\mathcal{PB}} \cdot \theta_{c,t}^{MIN} - \overline{P}_{c,t}^{\mathcal{PB}} \cdot \theta_{c,t}^{MAX} + \underline{Q}_{c,t}^{\mathcal{HT}} \cdot \theta_{c,t}^{IMIN} - \\
&\overline{Q}_{c,t}^{\mathcal{HT}} \cdot \theta_{c,t}^{IMAX} + \underline{Q}_{c,t}^{\mathcal{TH}} \cdot \theta_{c,t}^{OMIN} - \overline{Q}_{c,t}^{\mathcal{TH}} \cdot \theta_{c,t}^{OMAX} + \\
&\underline{PFR}_{c,t}^{\mathcal{PB}} \cdot \delta_{c,t}^{PMIN} - \overline{PFR}_{c,t}^{\mathcal{PB}} \cdot \delta_{c,t}^{PMAX} + \underline{SFR}_{c,t}^{\mathcal{PB}} \cdot \delta_{c,t}^{SMIN} \\
&- \overline{SFR}_{c,t}^{\mathcal{PB}} \cdot \delta_{c,t}^{SMAX} - M \cdot (1 - x_{c,t}^{\mathcal{PB}}) \cdot \rho_{c,t}^{MIN-} - M \cdot \\
&x_{c,t}^{\mathcal{PB}} \cdot \rho_{c,t}^{MAX+}\} - \sum_{l\in\mathcal{L}}(F_l^{Max} \cdot \varepsilon_{l,t}^{MIN} + F_l^{Max} \cdot \varepsilon_{l,t}^{MAX}) - \\
&(D' \cdot \Delta f_{Max}^{QSS} - \Delta P_{Max}^{\mathcal{D}}) \cdot \pi_t^{QSS} + \Delta f_{Max}^{QSS} \cdot D \cdot \sum_{d\in\mathcal{D}} P_{d,t}^{\mathcal{D}+} \cdot \\
&\vartheta_{d,t}^{QSS} + \kappa' \cdot f_0 \cdot \rho_t - \sum_{w\in\mathcal{W}} H_w^{\mathcal{W}} \cdot (\widehat{P}_{w,t}^{\mathcal{W}} \cdot \rho_t - P_{w,t}^{\mathcal{W}-} \cdot \vartheta_{w,t}) \\
&+ (SFR^{Min} - \Delta P_{Max}^{\mathcal{D}} \cdot \frac{t_{SFR} - t_{QSS} - 2\zeta_1}{20\zeta_2}) \cdot \\
&\nu_t^{MIN} - (SFR^{Max} - \Delta P_{Max}^{\mathcal{D}} \cdot \frac{t_{SFR} - t_{QSS} - 2\zeta_1}{20\zeta_2}) \cdot \\
&\nu_t^{MAX} + \Delta P_{Max}^{\mathcal{D}} \cdot \nu_t^{FRS}\}
\end{aligned} \tag{59}
$$

subject to:

$$\vartheta_{d,t}^{SD,D} = o_{d,t}^{\mathcal{D}} \cdot \beta_{d,t}^{SD,\mathcal{D}}; \vartheta_{d,t}^{QSS} = o_{d,t}^{\mathcal{D}} \cdot \pi_t^{QSS}; d \in \mathcal{D}, t \in \mathcal{T} \tag{60}$$

$$\vartheta_{w,t}^{SD,\mathcal{W}} = o_{w,t}^{\mathcal{W}} \cdot \beta_{w,t}^{SD,\mathcal{W}}; w \in \mathcal{W}, t \in \mathcal{T} \tag{61}$$

$$\vartheta_{w,t}^{Naidr,\mathcal{W}} = o_{w,t}^{\mathcal{W}} \cdot \rho_t; w \in \mathcal{W}, t \in \mathcal{T} \tag{62}$$

$$\vartheta_{c,t}^{SD} = o_{c,t}^{\mathcal{SF}} \cdot \theta_{c,t}^{SD}; c \in \mathcal{CSP}, t \in \mathcal{T} \tag{63}$$

$$\begin{aligned} &-M \cdot (1 - o_{d,t}^{\mathcal{D}}) \le \vartheta_{d,t}^{SD,\mathcal{D}} - \beta_{d,t}^{SD,\mathcal{D}} \le 0; 0 \le \vartheta_{d,t}^{SD,\mathcal{D}} \le M \cdot o_{d,t}^{\mathcal{D}};\\ &-M \cdot (1 - o_{d,t}^{\mathcal{D}}) \le \vartheta_{d,t}^{QSS} - \pi_t^{QSS} \le 0; 0 \le \vartheta_{d,t}^{QSS} \le M \cdot o_{d,t}^{\mathcal{D}};\\ &d \in \mathcal{D}, t \in \mathcal{T} \end{aligned} \tag{64}$$

$$\begin{aligned} &-M \cdot (1 - o_{w,t}^{\mathcal{W}}) \le \vartheta_{w,t}^{SD} - \beta_{w,t}^{SD,\mathcal{W}} \le 0; 0 \le \vartheta_{w,t}^{SD,\mathcal{W}} \le M \cdot o_{w,t}^{\mathcal{W}};\\ &-M \cdot (1 - o_{w,t}^{\mathcal{W}}) \le \vartheta_{w,t}^{Naidr,\mathcal{W}} - \rho_t \le 0; 0 \le \vartheta_{w,t}^{Naidr,\mathcal{W}} \le M \cdot\\ &o_{w,t}^{\mathcal{W}}; w \in \mathcal{W}, t \in \mathcal{T} \end{aligned} \tag{65}$$

$$\begin{aligned} &-M \cdot (1 - o_{c,t}^{\mathcal{SF}}) \le \vartheta_{c,t}^{SD} - \theta_{c,t}^{SD} \le 0; 0 \le \vartheta_{c,t}^{SD} \le M \cdot o_{c,t}^{\mathcal{SF}};\\ &c \in \mathcal{CSP}, t \in \mathcal{T} \end{aligned} \tag{66}$$

$$\begin{aligned} &\lambda_t^{SD} + \alpha_{g,t}^{MIN} - \alpha_{g,t}^{MAX} + \sum_{l \in \mathcal{L}} (\Gamma_{l,g}^{\mathcal{G}} \cdot \varepsilon_{l,t}^{MIN} - \Gamma_{l,g}^{\mathcal{G}} \cdot \varepsilon_{l,t}^{MAX})\\ &= VC_g^{\mathcal{G}} : (P_{g,t}^{\mathcal{G}}) \quad g \in \mathcal{G}, t \in \mathcal{T} \end{aligned} \tag{67}$$

$$\begin{aligned} &\lambda_t^{SD} + \beta_{w,t}^{SD,\mathcal{W}} + \sum_{l \in \mathcal{L}} (\Gamma_{l,w}^{\mathcal{W}} \cdot \varepsilon_{l,t}^{MIN} - \Gamma_{l,w}^{\mathcal{W}} \cdot \cdot \varepsilon_{l,t}^{MAX}) = 0\\ &: (P_{w,t}^{\mathcal{W}}) \quad w \in \mathcal{W}, t \in \mathcal{T} \end{aligned} \tag{68}$$

$$\beta_{w,t}^{SD,\mathcal{W}} \le VoLL^{\mathcal{W}} : (P_{w,t}^{\mathcal{W},Cur}) \quad w \in \mathcal{W}, t \in \mathcal{T} \tag{69}$$

$$\begin{aligned} &\beta_{d,t}^{SD,\mathcal{D}} - \lambda_t^{SD} - \sum_{l \in \mathcal{L}} (\Gamma_{l,d}^{\mathcal{D}} \cdot \varepsilon_{l,t}^{MIN} - \Gamma_{l,d}^{\mathcal{D}} \cdot \varepsilon_{l,t}^{MAX}) = 0\\ &: (P_{d,t}^{\mathcal{D}}) \quad d \in \mathcal{D}, t \in \mathcal{T} \end{aligned} \tag{70}$$

$$\beta_{d,t}^{SD,\mathcal{D}} \le VoLL^{\mathcal{D}} : (P_{d,t}^{\mathcal{D},Cur}) \quad d \in \mathcal{D}, t \in \mathcal{T} \tag{71}$$

$$\begin{aligned} &\eta_c^{\mathcal{PB}} \cdot \lambda_t^{SD} + \eta_c^{\mathcal{PB}} \cdot \theta_{c,t}^{MIN} - \eta_c^{\mathcal{PB}} \cdot \theta_{c,t}^{MAX} + \theta_{c,t}^{SD} +\\ &\eta_c^{\mathcal{PB}} \cdot \sum_{l \in \mathcal{L}} (\Gamma_{l,c}^{\mathcal{PB}} \cdot \varepsilon_{l,t}^{MIN} - \Gamma_{l,c}^{\mathcal{PB}} \cdot \varepsilon_{l,t}^{MAX}) = 0\\ &: (Q_{c,t}^{\mathcal{PB}}) \quad c \in \mathcal{CSP}, t \in \mathcal{T} \end{aligned} \tag{72}$$

$$\begin{aligned} &-\eta_c^{dis} \cdot \theta_{c,t}^{SD} - \sum_{\tau \in t:N^{\mathcal{T}}} (1 - \iota)^{\tau - t} \cdot (\theta_{c,\tau}^{TMIN} - \theta_{c,\tau}^{TMAX}) +\\ &\theta_{c,t}^{OMIN} - \theta_{c,t}^{OMAX} = 0 : (Q_{c,t}^{\mathcal{TH}}) \quad c \in \mathcal{CSP}, t \in \mathcal{T} \end{aligned} \tag{73}$$

$$\begin{aligned} &\theta_{c,t}^{SD} / \eta_c^{cha} + \sum_{\tau \in t:N^{\mathcal{T}}} (1 - \iota)^{\tau - t} \cdot (\theta_{c,\tau}^{TMIN} - \theta_{c,\tau}^{TMAX}) +\\ &\theta_{c,t}^{IMIN} - \theta_{c,t}^{IMAX} = 0 : (Q_{c,t}^{\mathcal{HT}}) \quad c \in \mathcal{CSP}, t \in \mathcal{T} \end{aligned} \tag{74}$$

$$\begin{aligned} &\omega_t - \sum_{g \in \mathcal{G}} (\rho_{g,t}^{MIN-} - \rho_{g,t}^{MAX-}) - \sum_{c \in \mathcal{CSP}} (\rho_{c,t}^{MIN-} - \rho_{c,t}^{MAX-})\\ &- \sum_{w \in \mathcal{W}} [\rho_{w,t}^{MIN-} - \rho_{w,t}^{MAX-} - H_w^{\mathcal{W}} \cdot (\widehat{P}_{w,t}^{\mathcal{W}} \cdot \rho_t -\\ &P_{w,t}^{\mathcal{W}-} \cdot \vartheta_{w,t}^{Naidr,\mathcal{W}})] - H_d^{\mathcal{D}} \cdot \Delta P_{t,Max}^{\mathcal{D}} / T_g \cdot \rho_t = 0 \end{aligned} \tag{75}$$

$$\begin{aligned} &\nu_t^{FRS} + \delta_{g,t}^{PMIN} - \delta_{g,t}^{PMAX} + \pi_t^{QSS} - \omega_t / T_g -\\ &\frac{t_{SFR} - t_{QSS} - 2\zeta_1}{20\zeta_2} \cdot (\nu_t^{MIN} - \nu_t^{MAX}) = PC_g^{\mathcal{G}}\\ &: (PFR_{g,t}^{\mathcal{G}}) \quad g \in \mathcal{G}, t \in \mathcal{T} \setminus \{1, N^{\mathcal{T}}\} \end{aligned} \tag{76}$$

$$\begin{aligned} &\beta_{w,t}^{SD,\mathcal{W}} + \nu_t^{FRS} - \delta_{w,t}^{PMAX} + \pi_t^{QSS} - \omega_t / T_g -\\ &\frac{t_{SFR} - t_{QSS} - 2\zeta_1}{20\zeta_2} \cdot (\nu_t^{MIN} - \nu_t^{MAX}) \le PC_w^{\mathcal{W}}\\ &: (PFR_{w,t}^{\mathcal{W}}) \quad w \in \mathcal{W}, t \in \mathcal{T} \end{aligned} \tag{77}$$

$$\begin{aligned} &\nu_t^{FRS} + \delta_{c,t}^{PMIN} - \delta_{c,t}^{PMAX} + \pi_t^{QSS} - \omega_t / T_g -\\ &\frac{t_{SFR} - t_{QSS} - 2\zeta_1}{20\zeta_2} \cdot (\nu_t^{MIN} - \nu_t^{MAX}) = PC_c^{\mathcal{PB}}\\ &: (PFR_{c,t}^{\mathcal{PB}}) \quad c \in \mathcal{CSP}, t \in \mathcal{T} \end{aligned} \tag{78}$$

$$\begin{aligned} &\delta_{g,t}^{SMIN} - \delta_{g,t}^{SMAX} + \nu_t^{FRS} = SC_g^{\mathcal{G}} : (SFR_{g,t}^{\mathcal{G}})\\ &g \in \mathcal{G}, t \in \mathcal{T} \end{aligned} \tag{79}$$

$$\begin{aligned} &\beta_{w,t}^{SD,\mathcal{W}} - \delta_{w,t}^{SMAX} + \nu_t^{FRS} \le SC_w^{\mathcal{W}} : (PFR_{w,t}^{\mathcal{W}})\\ &w \in \mathcal{W}, t \in \mathcal{T} \end{aligned} \tag{80}$$

$$\begin{aligned} &\delta_{c,t}^{SMIN} - \delta_{c,t}^{SMAX} + \nu_t^{FRS} = SC_c^{\mathcal{PB}} : (SFR_{c,t}^{\mathcal{PB}})\\ &c \in \mathcal{CSP}, t \in \mathcal{T} \end{aligned} \tag{81}$$

$$\begin{aligned} &H_g^{\mathcal{G}} \cdot P_g^{\mathcal{G},Max} \cdot \rho_t + \rho_{g,t}^{MIN-} - \rho_{g,t}^{MAX-} - \rho_{g,t}^{MAX+} \le 0\\ &: (\varpi_{g,t}^{\mathcal{G}}) \quad g \in \mathcal{G}, t \in \mathcal{T} \end{aligned} \tag{82}$$

$$\begin{aligned} &H_c^{\mathcal{PB}} \cdot P_c^{\mathcal{PB},Max} \cdot \rho_t + \rho_{c,t}^{MIN-} - \rho_{c,t}^{MAX-} - \rho_{c,t}^{MAX+} \le 0\\ &: (\varpi_{c,t}^{\mathcal{PB}}) \quad c \in \mathcal{CSP}, t \in \mathcal{T} \end{aligned} \tag{83}$$

$$\begin{aligned} &-P_{w,t}^{\mathcal{W}-} \cdot \rho_t + \rho_{w,t}^{MIN-} - \rho_{w,t}^{MAX-} - \rho_{w,t}^{MAX+} \le 0 : (\varpi_{w,t}^{\mathcal{W}})\\ &c \in \mathcal{W}, t \in \mathcal{T} \end{aligned} \tag{84}$$

$$\alpha, \beta, \lambda, \varepsilon, \delta, \nu, \theta, \rho, \pi \ge 0 \tag{85}$$