

The Derivation Formula of MMD-FR Linearity Criteria and Parameters of Augmented IEEE RTS-30 and IEEE-118 System as well as Models and Reformulations of Different Models

I. DERIVATION FORMULA OF MMD-FR LINEARITY CRITERIA

This section presents the derivation formula of the MMD-FR linearity criteria, focusing on the formula derivation process

$$|\Delta f(t_{tra})| = \begin{cases} \frac{\Delta P_{Max}^D}{D_t'} \cdot (1 - e^{-\frac{D_t'}{2H_t^{Sys}} \cdot t_{tra}}) & \text{if } t_{tra} \in [t, t_{DB}) \\ \Delta f_{DB} + \left(\frac{\Delta P_t'^D}{D_t'} + \frac{2H_t^{Sys} \cdot PFR_t'}{D_t'^2} \right) \cdot (1 - e^{-\frac{D_t'}{2H_t^{Sys}} \cdot (t_{tra} - t_{DB})}) - \frac{PFR_t'}{D_t'} \cdot (t_{tra} - t_{DB}), & \text{if } t_{tra} \in [t_{DB}, t_c) \\ \Delta f_c + \left(\frac{\Delta P_t''^D}{D_t'} + \frac{2H_t^{Sys} \cdot PFR_t''}{D_t'^2} \right) \cdot (1 - e^{-\frac{D_t'}{2H_t^{Sys}} \cdot (t_{tra} - t_c)}) - \frac{PFR_t''}{D_t'} \cdot (t_{tra} - t_c), & \text{if } t_{tra} \in [t_c, t_g) \end{cases} \quad (1)$$

where $\Delta P_t'^D = \Delta P_{Max}^D - D_t' \cdot \Delta f_{DB}$, $\Delta P_t''^D = \Delta P_{Max}^D - (\sum_g PFR_{g,t}^G + \sum_w PFR_{w,t}^W) \cdot T_c / T_g - \sum_c PFR_{c,t}^{PB} - D_t' \cdot \Delta f_c$, $PFR_t' = (\sum_g PFR_{g,t}^G + \sum_w PFR_{w,t}^W) / T_g + \sum_c PFR_{c,t}^{PB} / T_c$, $PFR_t'' = (\sum_g PFR_{g,t}^G + \sum_w PFR_{w,t}^W) / T_g$.

Set $\partial |\Delta f(t_{tra})| / \partial t = 0$ to obtain the following maximum frequency deviation $|\Delta f_{nadir}|$:

$$t' = \begin{cases} t_{DB} - \frac{2H_t^{Sys}}{D_t'} \cdot \log\left(\frac{2\kappa'}{\Delta P_t'^D \cdot D_t' + 2\kappa'}\right) \\ t_c - \frac{2H_t^{Sys}}{D_t'} \cdot \log\left(\frac{2\kappa''}{\Delta P_t''^D \cdot D_t' + 2\kappa''}\right) \end{cases} \Rightarrow \begin{cases} 2\kappa' \cdot \log\left(\frac{2\kappa'}{\Delta P_t'^D \cdot D_t' + 2\kappa'}\right) \leq D_t'^2 \cdot (\Delta f_{max} - \Delta f_{DB}) - D_t' \cdot \Delta P_t'^D & \text{if } t_{tra} \in [t_{DB}, t_c) \\ 2\kappa'' \cdot \log\left(\frac{2\kappa''}{\Delta P_t''^D \cdot D_t' + 2\kappa''}\right) \leq D_t'^2 \cdot (\Delta f_{max} - \Delta f_c) - D_t' \cdot \Delta P_t''^D & \text{if } t_{tra} \in [t_c, t_g) \end{cases} \quad (2)$$

Substituting (2) into (1), the following is obtained:

$$\Delta f_{nadir} = \begin{cases} \Delta f_{DB} + \frac{\Delta P_t'^D}{D_t'} + \frac{2\kappa'}{T_c \cdot D_t'^2} \cdot \log\left(\frac{2\kappa'}{T_c \cdot \Delta P_t'^D \cdot D_t' + 2\kappa'}\right) & \text{if } t_{tra} \in [t_{DB}, t_c) \\ \Delta f_c + \frac{\Delta P_t''^D}{D_t'} + \frac{2\kappa''}{T_g \cdot D_t'^2} \cdot \log\left(\frac{2\kappa''}{T_g \cdot \Delta P_t''^D \cdot D_t' + 2\kappa''}\right) & \text{if } t_{tra} \in [t_c, t_g) \end{cases} \quad (3)$$

Equation (3) is a piecewise function. The frequency nadir of the two segments depend on the PFR provision of SGs, wind

for the Multi-Speed Dependent frequency Nadir, corresponding with the main text.

Substituting (2) and (3) from the main text into (1), the frequency deviation piecewise function is obtained as follows:

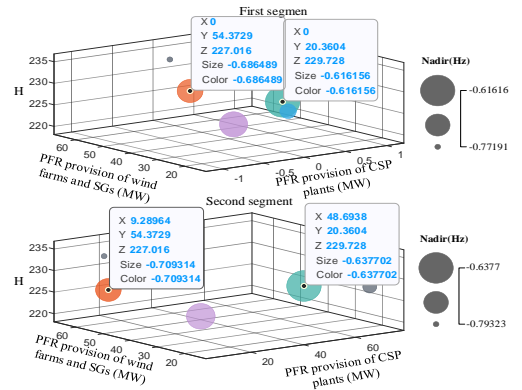


Fig. 1 Numerical simulation farms, CSP plants, and system inertia, making it impossible to obtain an analytical solution. Numerical analysis shows that the frequency nadir appears in the first segment, as illustrated in Fig. 1.

II. PROBLEM FORMULATION AND ANALYSIS

In this section, the mathematical formulation of deterministic FUCU (D-FCUC) and non-causal two-stage robust FUCU (NT-FCUC) under multi-uncertainty, for ease of understanding and discussion of validity of the proposed method.

A. D-FCUC Model

The D-FCUC model to minimize the total operational costs, including start-up and shut-down costs, operating costs of SGs, FRS provision costs, and curtailment penalties, all calculated from respective unit costs SU, VC, PC, SC , and $VoLL$, for given loads $P_{d,t}^D$, can be mathematically formulated as:

objective function:

$$OC^{sys} = \sum_{t \in T} [\sum_{g \in \mathcal{G}} (SU_g^G \cdot x_{g,t}^{G,Su} + VC_g^G \cdot P_{g,t}^G) + \sum_{c \in \mathcal{CSP}} (SU_c^{PB} \cdot x_{c,t}^{PB,Su} + VC_c^{PB} \cdot P_{c,t}^{PB}) + VoLL^D \cdot \sum_{d \in \mathcal{D}} P_{d,t}^{D,Cur} + PC \cdot PFR_t^{Sys}(t_{QSS}) + SC \cdot SFR_t^{Sys}(t_{SFR})] \quad (4)$$

subject to:

• *Logic constraints of commitment states*

$$\mathcal{LC} := \{x := \{0,1\} \in \mathbb{Z}^{2 \cdot (N^{CSP} + N^G) \times N^T} : x_{g,t}^G, x_{g,t}^{G,Su}, x_{c,t}^{PB}, x_{c,t}^{PB,Su}; \quad g \in \mathcal{G}, c \in \mathcal{CSP}, t \in T\} \quad (5)$$

• *Operational constraints of CSP plants*

$$\begin{aligned} \mathcal{OC} := \{y \in \mathbb{R}^{5 \cdot N^{CSP} \times N^T} : \\ \tilde{Q}_{c,t}^{SF} = Q_{c,t}^{HT} / \eta_c^{cha} - Q_{c,t}^{TH} \cdot \eta_c^{dis} + P_{c,t}^{PB} / \eta_c^{PB} : (\theta_{c,t}^{SD}), \\ r_c^{TES,lim} \cdot E_c^{TES,Max} \leq (1-t) \cdot E_c^{TES,ln} + \sum_{\tau \in [t]} (1-t)^{t-\tau} \\ \cdot (Q_{c,t}^{HT} - Q_{c,t}^{TH}) \leq E_c^{TES,Max} : (\theta_{c,t}^{TMIN}, \theta_{c,t}^{TMAX}), \\ 0 \leq Q_{c,t}^{HT} \leq Q_{c,t}^{HT,Max} : (\theta_{c,t}^{IMIN}, \theta_{c,t}^{IMAX}), \\ 0 \leq Q_{c,t}^{TH} \leq Q_{c,t}^{TH,Max} : (\theta_{c,t}^{OMIN}, \theta_{c,t}^{OMAX}); \\ P_{c,t}^{PB} + PFR_{c,t}^{PB} + SFR_{c,t}^{PB} \leq P_{c,t}^{PB,Max} \cdot x_{c,t}^{PB} : (\theta_{c,t}^{MAX}). \\ P_{c,t}^{PB} - PFR_{c,t}^{PB} - SFR_{c,t}^{PB} \geq P_{c,t}^{PB,Min} \cdot x_{c,t}^{PB} : (\theta_{c,t}^{MIN}); \\ -RD_c^{PB} \leq P_{c,t}^{PB} + PFR_{c,t}^{PB} + SFR_{c,t}^{PB} - P_{c,t-1}^{PB} + PFR_{c,t-1}^{PB} \\ + SFR_{c,t-1}^{PB} \leq RU_c^{PB} : (\theta_{c,t}^{UP}, \theta_{c,t}^{DN}); \quad c \in \mathcal{CSP}, t \in T\} \end{aligned} \quad (6)$$

• *Operational constraints of SGs*

$$\begin{aligned} \mathcal{OS} := \{z \in \mathbb{R}^{3 \cdot N^G \times N^T} : \\ P_{g,t}^G + PFR_{g,t}^G + SFR_{g,t}^G \leq x_{g,t}^G \cdot P_{g,t}^{G,Max} : (\alpha_{g,t}^{MAX}), \\ P_{g,t}^G - PFR_{g,t}^G - SFR_{g,t}^G \geq x_{g,t}^G \cdot P_{g,t}^{G,Min} : (\alpha_{g,t}^{MIN}); \\ -RD_g^G \leq P_{g,t}^G + PFR_{g,t}^G + SFR_{g,t}^G - P_{g,t-1}^G + PFR_{g,t-1}^G \\ + SFR_{g,t-1}^G \leq RU_g^G : (\alpha_{g,t}^{DN}, \alpha_{g,t}^{UP}); \quad g \in \mathcal{G}, t \in T\} \end{aligned} \quad (11)$$

• *Constraints Exclusively on Spatial Considerations:*

$$\mathcal{ES} := \{(y, z, u) \in \mathbb{R}^{(2 \cdot N^G + 4 \cdot N^W + 2 \cdot N^{CSP} + 2 \cdot N^D) \times N^T} : \\ \sum_{g \in \mathcal{G}} P_{g,t}^G + \sum_{w \in \mathcal{W}} P_{w,t}^W + \sum_{c \in \mathcal{CSP}} P_{c,t}^{PB} = \sum_{d \in \mathcal{D}} P_{d,t}^D : (\lambda_t^{SD}); \quad (12)$$

$$\begin{aligned} -F_l^{Max} \leq \sum_{g \in \mathcal{G}} P_{g,t}^G \cdot \Gamma_{l,g}^G + \sum_{w \in \mathcal{W}} P_{w,t}^W \cdot \Gamma_{l,w}^W + \sum_{c \in \mathcal{CSP}} P_{c,t}^{PB} \cdot \Gamma_{l,c}^{CSP} - \sum_{d \in \mathcal{D}} P_{d,t}^D \cdot \Gamma_{l,d}^D \leq F_l^{Max} : (\epsilon_{l,t}^{MIN}, \epsilon_{l,t}^{MAX}); \end{aligned} \quad (13)$$

$$\begin{aligned} P_{d,t}^{D,Cur} + P_{d,t}^D = \tilde{P}_{d,t}^D : (\beta_{d,t}^{SD,D}), \\ 0 \leq P_{d,t}^{D,Cur} \leq \tilde{P}_{d,t}^D : (\beta_{d,t}^{MIN}, \beta_{d,t}^{MAX}); \end{aligned} \quad (14)$$

$$\begin{aligned} \tilde{P}_{w,t}^W = P_{w,t}^W + PFR_{w,t}^W + SFR_{w,t}^W + P_{w,t}^{W,Cur} : (\beta_{w,t}^{SD,W}), \\ 0 \leq P_{w,t}^{W,Cur} \leq \tilde{P}_{w,t}^W : (\beta_{w,t}^{MIN}, \beta_{w,t}^{MAX}); \end{aligned} \quad (15)$$

$$\begin{aligned} 0 \leq PFR_{c,t}^{PB} \leq PFR_c^{PB,Max} \cdot u_{c,t}^{PB} : (\delta_{c,t}^{PMIN}, \delta_{c,t}^{PMAX}), \\ 0 \leq SFR_{c,t}^{PB} \leq SFR_c^{PB,Max} \cdot v_{c,t}^{PB} : (\delta_{c,t}^{SMIN}, \delta_{c,t}^{SMAX}); \end{aligned} \quad (16)$$

$$(3)-(9); d \in \mathcal{D}, g \in \mathcal{G}, w \in \mathcal{W}, c \in \mathcal{CSP}, l \in \mathcal{L}, t \in T\}$$

$$\mathcal{LF} := \{u := \{0,1\} \in \mathbb{Z}^{2 \cdot (N^G + N^W + N^{CSP}) \times N^T} :$$

$$\begin{aligned} u_{g,t}^G \leq x_{g,t}^G, u_{c,t}^{PB} \leq x_{c,t}^{PB}, v_{g,t}^G \leq x_{g,t}^G, v_{c,t}^{PB} \leq x_{c,t}^{PB}, u_{w,t}^W, \\ v_{w,t}^W; \quad g \in \mathcal{G}, w \in \mathcal{W}, c \in \mathcal{CSP}, t \in T\} \end{aligned} \quad (17)$$

where all feasible UC of SGs and CSP plants are stated in (5), including as start-up and shut-down constraints as well as online and offline times in the \mathcal{LC} (see e.g. [22] for details on the formulation). The set of feasible operations for SGs \mathcal{OC} , as described in (6)-(9), present the direct normal irradiance (DNI)-thermal-electrical energy conversion in the solar field (SF), thermal energy storage (TES), and power block (PB) of CSP plants, explicitly incorporating PFR and SFR provision into output limit to ensure hourly FRS deliverability under variable DNI and limited ramping in real-time ED, as describes in constraints (8)-(9). Feasible region of SGs \mathcal{OS} , explicitly including FRS provision capability, are listed in (10) and (11), with capacity limits in (10) and ramping limits in (11). Equation (12) enforces the active power balance and (13) limits the power flow of transmission lines in the set of feasible ED schedules \mathcal{ES} that is non-temporal and exclusively spatial. The non-negativity of unserved load and wind spillage are indicated in (14) and (15), respectively. Equation (16) highlights the maximal PFR and SFR provision capability and CSP plants, associated with FRS-status-related decisions, whose relationship with commitment-status-related decisions is encapsulated within (17). Similar constraints for SGs and winds are not elaborated individually.

B. NT-FCUC Model

The user-defined set of multi-uncertain parameters, characterized by a hyperrectangle with upper and lower bounds encompassing load demand $\tilde{P}_{d,t}^D$, wind power production $\tilde{P}_{w,t}^W$, and thermal energy $\tilde{Q}_{c,t}^{SF}$ absorbed by SF in CSP plants, manifests its worst-case realization only when reaching extreme upper $\tilde{P}_{d,t}^D$ or lower limits $\underline{P}_{w,t}^W, \underline{Q}_{c,t}^{SF}$. It can be recast into a more tractable formulation given as:

$$\begin{aligned} \xi_t = (\tilde{P}_{d,t}^D = \hat{P}_{d,t}^D + P_{d,t}^{D+}, \tilde{P}_{w,t}^W = \hat{P}_{w,t}^W - P_{w,t}^{W-}, \\ \tilde{Q}_{c,t}^{CSP} = \hat{Q}_{c,t}^{CSP} - Q_{c,t}^{CSP-}) \in \mathbb{R}^{(N^G + N^W + N^{CSP}) \times 1} \\ 0 \leq P_{d,t}^{D+} \leq \tilde{P}_{d,t}^D, 0 \leq P_{w,t}^{W-} \leq \tilde{P}_{w,t}^W, 0 \leq Q_{c,t}^{CSP-} \leq \hat{Q}_{c,t}^{CSP}, \\ -\sum_{d \in \mathcal{D}} \left(\frac{P_{d,t}^{D+}}{\tilde{P}_{d,t}^D} \right) - \sum_{w \in \mathcal{W}} \left(\frac{P_{w,t}^{W-}}{\tilde{P}_{w,t}^W} \right) - \sum_{c \in \mathcal{CSP}} \left(\frac{Q_{c,t}^{CSP-}}{\hat{Q}_{c,t}^{CSP}} \right) \geq -\Lambda_t, \\ d \in \mathcal{D}, w \in \mathcal{W}, c \in \mathcal{CSP}, t \in T \setminus \{1\} \end{aligned} \quad (18)$$

Subsequently, compact form of NT-FCUC model, minimizing the worst-case total operational cost for multi-uncertainty set Ξ , is formulated in a conventional way as following:

$$\begin{aligned} & \min_{x \in \mathcal{OC}, u \in \mathcal{LF}} \{ \mathbf{a}^\top \cdot \mathbf{x} + \mathbf{b}^\top \cdot \mathbf{u} + \max_{\xi \in \Xi} \varepsilon(\mathbf{x}, \mathbf{u}, \xi, \mathbf{y}, \mathbf{z}) \} \\ & s. t. \mathbf{E} \cdot \mathbf{x} + \mathbf{F} \cdot \mathbf{u} \leq \mathbf{n}; \\ & \text{where } \varepsilon(\mathbf{x}, \mathbf{u}, \xi, \mathbf{y}, \mathbf{z}) = \min_{\substack{(\mathbf{y} \in \mathcal{OC}(\mathbf{x}), \mathbf{z} \in \mathcal{OS}(\mathbf{x}), \\ \mathbf{u} \in \mathcal{LF}) \in \mathcal{ES}(\mathbf{x}, \mathbf{u})}} (\mathbf{c}^\top \cdot \mathbf{y} + \mathbf{d}^\top \cdot \mathbf{z}) \end{aligned} \quad (19)$$

$$s. t. \mathbf{G} \cdot \mathbf{y} + \mathbf{H} \cdot \mathbf{z} \leq \mathbf{n} - \mathbf{L} \cdot \mathbf{x} - \mathbf{M} \cdot \mathbf{u} - \mathbf{N} \cdot \xi$$

where due to structural changes, \mathbf{m} becomes vector \mathbf{n} . Also, the parameters represented by matrices $\mathbf{E}, \mathbf{F}, \mathbf{G}, \mathbf{H}, \mathbf{L}, \mathbf{M}$, and \mathbf{N} can be calculated by constraints of NT-FCUC model. $\varepsilon(\mathbf{x}, \mathbf{u}, \xi, \mathbf{y}, \mathbf{z})$ represents the feasible domain produced by HANDs and actual resolution of multi-uncertainty parameters ξ . The detailed robust counterpart is listed in the Appendix.

III. ROBUST COUNTERPART OF NT-FCUC

This section mainly presents the mathematical formulation of SP in NT-FCUC model.

objective function:

$$\begin{aligned} & \sum_{t \in \mathcal{T}} \{ \sum_{d \in \mathcal{D}} [(\hat{P}_{d,t}^D \cdot \lambda_t^{SD} + P_{d,t}^{D+} \cdot \vartheta_{d,t}^{SD}) - (\hat{P}_{d,t}^D \cdot \beta_{d,t}^{MAX} \\ & + P_{d,t}^{D+} \cdot \vartheta_{d,t}^{MAX})] + \sum_{g \in \mathcal{G}} (x_{g,t} \cdot P_g^{G,Min} \cdot \alpha_{g,t}^{MIN} \\ & - x_{g,t} \cdot P_g^{G,Max} \cdot \alpha_{g,t}^{MAX} - PFR_g^{G,Max} \cdot u_{g,t}^G \cdot \delta_{g,t}^{PMAX} \\ & - SFR_g^{G,Max} \cdot v_{g,t}^G \cdot \delta_{g,t}^{SMAX} - M \cdot (1 - x_{g,t}^G) \cdot \rho_{g,t}^{MIN-} \\ & - M \cdot x_{g,t}^G \cdot \rho_{g,t}^{MAX+}) + \sum_{w \in \mathcal{W}} [\hat{P}_{w,t}^W \cdot \beta_{w,t}^{SD} - P_{w,t}^{W-} \\ & \cdot \vartheta_{w,t}^{SD} - (\hat{P}_{w,t}^W \cdot \beta_{w,t}^{MAX} - P_{w,t}^{W-} \cdot \vartheta_{w,t}^{MAX}) - PFR_w^{W,Max} \\ & \cdot u_{w,t}^W \cdot \delta_{w,t}^{PMAX} - SFR_w^{W,Max} \cdot v_{w,t}^W \cdot \delta_{w,t}^{SMAX} \\ & - M \cdot (1 - o_{w,t}^W) \cdot \rho_{w,t}^{MIN-} - M \cdot o_{w,t}^W \cdot \rho_{w,t}^{MAX+}] \\ & + \sum_{c \in \mathcal{CSP}} [\hat{Q}_{c,t}^{SF} \cdot \theta_{c,t}^{SD} - Q_{c,t}^{SF-} \cdot \vartheta_{c,t}^{SD} + P_c^{PB,Min} \\ & \cdot x_{c,t}^{PB} \cdot \theta_{c,t}^{MIN} - P_c^{PB,Max} \cdot x_{c,t}^{PB} \cdot \theta_{c,t}^{MAX} + [r_c^{TES,lim} \\ & \cdot E_c^{TES,Max} - (1 - \iota)^t \cdot E_c^{TES,in}] \cdot \theta_{c,t}^{TMIN} - [E_c^{TES,Max} \\ & - (1 - \iota)^t \cdot E_c^{TES,in}] \cdot \theta_{c,t}^{TMAX} - Q_{c,t}^{HT,Max} \cdot \theta_{c,t}^{IMAX} \\ & \cdot \zeta_{c,t}^{HT} - Q_{c,t}^{TH,Max} \cdot \theta_{c,t}^{OMAX} \cdot (1 - \zeta_{c,t}^{HT}) \\ & - PFR_{c,t}^{PB,Max} \cdot u_{c,t}^{PB} \cdot \delta_{c,t}^{PMAX} - SFR_{c,t}^{PB,Max} \cdot v_{c,t}^{PB} \\ & \cdot \delta_{c,t}^{SMAX} - M \cdot (1 - x_{c,t}^{PB}) \cdot \rho_{c,t}^{MIN-} - M \cdot x_{c,t}^{PB} \\ & \cdot \rho_{c,t}^{MAX+}] - \sum_{l \in \mathcal{L}} (F_l^{MAX} \cdot \varepsilon_{l,t}^{MIN} + F_l^{MAX} \cdot \varepsilon_{l,t}^{MAX}) \\ & - (D' \cdot \Delta f_{Max}^{QSS} - \Delta P_{t,Max}^D) \cdot \pi_t^{QSS} + \Delta f_{Max}^{QSS} \cdot D. \end{aligned} \quad (20)$$

$$\begin{aligned} & \sum_{d \in \mathcal{D}} P_{d,t}^{D+} \cdot \vartheta_{d,t}^{QSS} + \kappa' \cdot f_0 \cdot \rho_t \\ & + (SFR^{Min} - \Delta P_{Max}^L \cdot \frac{t_{SFR} - t_{QSS} - 2\zeta_1}{20\zeta_2}) \cdot \nu_t^{MIN} \\ & - (SFR^{Max} - \Delta P_{Max}^L \cdot \frac{t_{SFR} - t_{QSS} - 2\zeta_1}{20\zeta_2}) \cdot \nu_t^{MAX} \\ & + \Delta P_{t,Max}^D \cdot \nu_t^{FRS} \} - \sum_{t \in \mathcal{T} \setminus \{1\}} \{ \sum_{g \in \mathcal{G}} \{ [P_g^{G,Max} \\ & - x_{g,t-1}^G \cdot (P_g^{G,Min} - \Delta P_g^{G,Up}) - x_{g,t}^G \cdot (P_g^{G,Max} \\ & - P_g^{G,Min})] \cdot \alpha_{g,t}^{UP} + [P_g^{G,Max} - x_{g,t}^G \cdot (P_g^{G,Min} \\ & - \Delta P_g^{G,Dn}) - x_{g,t-1}^G \cdot (P_g^{G,Max} - P_g^{G,Min})] \cdot \alpha_{g,t}^{DN} \} \\ & + \sum_{c \in \mathcal{CSP}} \{ [P_c^{PB,Max} - x_{c,t-1}^{PB} \cdot (P_c^{PB,Max} - R_c^{PB,Up}) \\ & - x_{c,t}^{PB} \cdot (P_c^{PB,Max} - R_c^{PB,Min})] \cdot \theta_{c,t}^{UP} + [P_c^{PB,Max} \\ & - x_{c,t}^{PB} \cdot (P_c^{PB,Min} - R_c^{PB,Dn}) - x_{c,t-1}^{PB} \cdot (P_c^{PB,Max} \\ & - R_c^{PB,Min})] \cdot \theta_{c,t}^{DN} \} \} \end{aligned}$$

subject to:

$$\vartheta_{d,t}^{SD} = o_{d,t}^D \cdot \lambda_t^{SD}; \vartheta_{d,t}^{MAX} = o_{d,t}^D \cdot \beta_{d,t}^{MAX}; \quad (21)$$

$$\vartheta_{d,t}^{QSS} = o_{d,t}^D \cdot \pi_t^{QSS}; d \in \mathcal{D}, t \in \mathcal{T} \quad (22)$$

$$\vartheta_{w,t}^{SD} = o_{w,t}^W \cdot \beta_{w,t}^{SD}; \vartheta_{w,t}^{MAX} = o_{w,t}^W \cdot \beta_{w,t}^{MAX} \quad (23)$$

$$\begin{aligned} & w \in \mathcal{W}, t \in \mathcal{T} \\ & \vartheta_{c,t}^{SD} = o_{c,t}^{SF} \cdot \theta_{c,t}^{SD}; \quad c \in \mathcal{CSP}, t \in \mathcal{T} \end{aligned} \quad (24)$$

$$\begin{aligned} & -M \cdot (1 - o_{d,t}^D) \leq \vartheta_{d,t}^{SD} - \lambda_t^{SD} \leq 0; 0 \leq \vartheta_{d,t}^{SD} \leq M \cdot o_{d,t}^D; \\ & -M \cdot (1 - o_{d,t}^D) \leq \vartheta_{d,t}^{MAX} - \beta_{d,t}^{MAX} \leq 0; \\ & 0 \leq \vartheta_{d,t}^{SD} \leq M \cdot o_{d,t}^D; \end{aligned} \quad (25)$$

$$\begin{aligned} & -M \cdot (1 - o_{d,t}^D) \leq \vartheta_{d,t}^{QSS} - \pi_t^{QSS} \leq 0; 0 \leq \vartheta_{d,t}^{QSS} \leq M \cdot o_{d,t}^D; \\ & d \in \mathcal{D}, t \in \mathcal{T} \end{aligned} \quad (26)$$

$$\begin{aligned} & -M \cdot (1 - o_{w,t}^W) \leq \vartheta_{w,t}^{SD} - \beta_{w,t}^{SD} \leq 0; 0 \leq \vartheta_{w,t}^{SD} \leq M \cdot o_{w,t}^W; \\ & -M \cdot (1 - o_{w,t}^W) \leq \vartheta_{w,t}^{MAX} - \beta_{w,t}^{MAX} \leq 0; \end{aligned} \quad (27)$$

$$\begin{aligned} & 0 \leq \vartheta_{w,t}^{MAX} \leq M \cdot o_{w,t}^W; w \in \mathcal{W}, t \in \mathcal{T} \\ & -M \cdot (1 - o_{c,t}^{SF}) \leq \vartheta_{c,t}^{SD} - \theta_{c,t}^{SD} \leq 0; \end{aligned} \quad (28)$$

$$\begin{aligned} & 0 \leq \vartheta_{c,t}^{SD} \leq M \cdot o_{c,t}^{SF}; c \in \mathcal{CSP}, t \in \mathcal{T} \\ & \alpha_{g,t}^{MIN} - \alpha_{g,t}^{MAX} + \lambda_t^{SD} - \alpha_{g,t}^{UP} + \alpha_{g,t+1}^{UP} + \alpha_{g,t}^{DN} - \alpha_{g,t+1}^{DN} \\ & + \sum_{l \in \mathcal{L}} (\Gamma_{l,g}^G \cdot \varepsilon_{l,t}^{MIN} - \Gamma_{l,g}^G \cdot \varepsilon_{l,t}^{MAX}) = VC_g^G : (P_g^G) \end{aligned} \quad (29)$$

$$\begin{aligned} & g \in \mathcal{G}, t \in \mathcal{T} \setminus \{1, N^T\} \\ & \alpha_{g,t}^{MIN} - \alpha_{g,t}^{MAX} + \lambda_t^{SD} - \alpha_{g,t}^{UP} + \alpha_{g,t}^{DN} + \sum_{l \in \mathcal{L}} (\Gamma_{l,g}^G \cdot \varepsilon_{l,t}^{MIN} \\ & - \Gamma_{l,g}^G \cdot \varepsilon_{l,t}^{MAX}) = VC_g^G : (P_g^G) \quad g \in \mathcal{G}, t = N^T \end{aligned} \quad (30)$$

$$\begin{aligned} & \alpha_{g,t}^{MIN} - \alpha_{g,t}^{MAX} + \lambda_t^{SD} + \alpha_{g,t+1}^{UP} - \alpha_{g,t+1}^{DN} + \sum_{l \in \mathcal{L}} (\Gamma_{l,g}^G \\ & \cdot \varepsilon_{l,t}^{MIN} - \Gamma_{l,g}^G \cdot \varepsilon_{l,t}^{MAX}) = VC_g^G : (P_g^G) \quad g \in \mathcal{G}, t = 1 \end{aligned} \quad (31)$$

$$\begin{aligned} & \lambda_t^{SD} + \beta_{w,t}^{SD} + \sum_{l \in \mathcal{L}} (\Gamma_{l,w}^W \cdot \varepsilon_{l,t}^{MIN} - \Gamma_{l,w}^W \cdot \varepsilon_{l,t}^{MAX}) = 0 \\ & : (P_{w,t}^W) \quad w \in \mathcal{W}, t \in \mathcal{T} \end{aligned} \quad (32)$$

$$\begin{aligned} & \beta_{w,t}^{SD,W} \leq VoLL^W : (P_{w,t}^{W,Cur}) \quad w \in \mathcal{W}, t \in \mathcal{T} \end{aligned} \quad (33)$$

$$-\lambda_t^{SD} + \beta_t^{SD,D} - \sum_{l \in \mathcal{L}} (\Gamma_{l,d}^D \cdot \varepsilon_{l,t}^{MIN} - \Gamma_{l,d}^D \cdot \varepsilon_{l,t}^{MAX}) = 0 \quad (32)$$

$$: (P_{d,t}^D) \quad d \in \mathcal{D}, t \in \mathcal{T} \quad (33)$$

$$\lambda_t^{SD} + \beta_t^{SD,D} \leq VoLL^D : (P_{d,t}^{D,Cur}) \quad d \in \mathcal{D}, t \in \mathcal{T} \quad (34)$$

$$\lambda_t^{SD} + \theta_{c,t}^{MIN} - \theta_{c,t}^{MAX} - \theta_{c,t}^{UP} + \theta_{c,t+1}^{UP} + \theta_{c,t}^{DN} - \theta_{c,t+1}^{DN} + \theta_{c,t}^{SD} / \eta_c^{PB} + \sum_{l \in \mathcal{L}} (\Gamma_{l,c}^{PB} \cdot \varepsilon_{l,t}^{MIN} - \Gamma_{l,c}^{PB} \cdot \varepsilon_{l,t}^{MAX}) = 0 : (Q_{c,t}^{PB}) \quad c \in \mathcal{CSP}, t \in \mathcal{T} \setminus \{1, N^T\}$$

$$\lambda_t^{SD} + \theta_{c,t}^{MIN} - \theta_{c,t}^{MAX} + \theta_{c,t+1}^{UP} - \theta_{c,t+1}^{DN} + \theta_{c,t}^{SD} / \eta_c^{PB} + \sum_{l \in \mathcal{L}} (\Gamma_{l,c}^{PB} \cdot \varepsilon_{l,t}^{MIN} - \Gamma_{l,c}^{PB} \cdot \varepsilon_{l,t}^{MAX}) = 0 : (Q_{c,t}^{PB}) \quad c \in \mathcal{CSP}, t = 1 \quad (35)$$

$$\lambda_t^{SD} + \theta_{c,t}^{MIN} - \theta_{c,t}^{MAX} - \theta_{c,t}^{UP} + \theta_{c,t}^{DN} + \theta_{c,t}^{SD} / \eta_c^{PB} + \sum_{l \in \mathcal{L}} (\Gamma_{l,c}^{PB} \cdot \varepsilon_{l,t}^{MIN} - \Gamma_{l,c}^{PB} \cdot \varepsilon_{l,t}^{MAX}) = 0 : (Q_{c,t}^{PB}) \quad c \in \mathcal{CSP}, t = N^T \quad (36)$$

$$-\eta_c^{dis} \cdot \theta_{c,t}^{SD} - \sum_{\tau \in \mathcal{T}: N^T} (1 - \iota)^{\tau-t} \cdot (\theta_{c,\tau}^{TMIN} - \theta_{c,\tau}^{TMAX}) + \theta_{c,t}^{OMIN} - \theta_{c,t}^{OMAX} = 0 : (Q_{c,t}^{TH}) \quad c \in \mathcal{CSP}, t \in \mathcal{T} \quad (37)$$

$$\theta_{c,t}^{SD} / \eta_c^{cha} + \sum_{\tau \in \mathcal{T}: N^T} (1 - \iota)^{\tau-t} \cdot (\theta_{c,\tau}^{TMIN} - \theta_{c,\tau}^{TMAX}) + \theta_{c,t}^{IMIN} - \theta_{c,t}^{IMAX} = 0 : (Q_{c,t}^{HT}) \quad c \in \mathcal{CSP}, t \in \mathcal{T} \quad (38)$$

$$\omega_t - \sum_{g \in \mathcal{G}} (\rho_{g,t}^{MIN-} - \rho_{g,t}^{MAX-}) - \sum_{c \in \mathcal{CSP}} (\rho_{c,t}^{MIN-} - \rho_{c,t}^{MAX-}) - \sum_{w \in \mathcal{W}} (\rho_{w,t}^{MIN-} - \rho_{w,t}^{MAX-} - H_w^W \cdot \hat{P}_{w,t}^W \cdot \rho_t) - H_d^D \cdot \Delta P_{t,Max}^D / T_g \cdot \rho_t = 0 \quad (39)$$

$$\nu_t^{FRS} - \alpha_{g,t}^{MIN} - \alpha_{g,t}^{MAX} - \alpha_{g,t}^{UP} - \alpha_{g,t+1}^{UP} - \alpha_{g,t}^{DN} - \alpha_{g,t+1}^{DN} - \delta_{g,t}^{PMAX} + \pi_t^{QSS} - \omega_t / T_g - \frac{t_{SFR} - t_{QSS} - 2\zeta_1}{20\zeta_2} \cdot (\nu_t^{MIN} - \nu_t^{MAX}) \leq PC_g^G : (PFR_{g,t}^G) \quad g \in \mathcal{G}, t \in \mathcal{T} \setminus \{1, N^T\} \quad (40)$$

$$\nu_t^{FRS} - \alpha_{g,t}^{MIN} - \alpha_{g,t}^{MAX} - \alpha_{g,t}^{UP} - \alpha_{g,t}^{DN} - \delta_{g,t}^{PMAX} + \pi_t^{QSS} - \omega_t / T_g - \frac{t_{SFR} - t_{QSS} - 2\zeta_1}{20\zeta_2} \cdot (\nu_t^{MIN} - \nu_t^{MAX}) \leq PC_g^G : (PFR_{g,t}^G) \quad g \in \mathcal{G}, t = N^T \quad (41)$$

$$\nu_t^{FRS} - \alpha_{g,t}^{MIN} - \alpha_{g,t}^{MAX} - \alpha_{g,t+1}^{UP} - \alpha_{g,t+1}^{DN} - \delta_{g,t}^{PMAX} + \pi_t^{QSS} - \omega_t / T_g - \frac{t_{SFR} - t_{QSS} - 2\zeta_1}{20\zeta_2} \cdot (\nu_t^{MIN} - \nu_t^{MAX}) \leq PC_g^G : (PFR_{g,t}^G) \quad g \in \mathcal{G}, t = 1 \quad (42)$$

$$\nu_t^{FRS} + \beta_{w,t}^{SD} + \delta_{w,t}^{PMIN} - \delta_{w,t}^{PMAX} + \pi_t^{QSS} - \omega_t / T_g - \frac{t_{SFR} - t_{QSS} - 2\zeta_1}{20\zeta_2} \cdot (\nu_t^{MIN} - \nu_t^{MAX}) \leq PC_w^W : (PFR_{w,t}^W) \quad w \in \mathcal{W}, t \in \mathcal{T} \quad (43)$$

$$\nu_t^{FRS} - \theta_{c,t}^{MIN} - \theta_{c,t}^{MAX} - \theta_{c,t}^{UP} - \theta_{c,t+1}^{UP} - \theta_{c,t}^{DN} - \theta_{c,t+1}^{DN} - \delta_{c,t}^{PMAX} + \pi_t^{QSS} - \omega_t / T_g - \frac{t_{SFR} - t_{QSS} - 2\zeta_1}{20\zeta_2} \cdot (\nu_t^{MIN} - \nu_t^{MAX}) \leq PC_c^{PB} : (PFR_{c,t}^{CP}) \quad c \in \mathcal{CSP}, t \in \mathcal{T} \setminus \{1, N^T\} \quad (44)$$

$$\nu_t^{FRS} - \theta_{c,t}^{MIN} - \theta_{c,t}^{MAX} - \theta_{c,t}^{UP} - \theta_{c,t}^{DN} - \delta_{c,t}^{PMAX} + \pi_t^{QSS} - \omega_t / T_g - \frac{t_{SFR} - t_{QSS} - 2\zeta_1}{20\zeta_2} \cdot (\nu_t^{MIN} - \nu_t^{MAX}) \leq PC_c^{CP} : (PFR_{c,t}^{PB}) \quad c \in \mathcal{CSP}, t = N^T \quad (45)$$

$$\nu_t^{FRS} - \theta_{c,t}^{MIN} - \theta_{c,t}^{MAX} - \theta_{c,t+1}^{UP} - \theta_{c,t+1}^{DN} - \delta_{c,t}^{PMAX} + \pi_t^{QSS} - \omega_t / T_g - \frac{t_{SFR} - t_{QSS} - 2\zeta_1}{20\zeta_2} \cdot (\nu_t^{MIN} - \nu_t^{MAX}) \leq PC_c^{CP} : (PFR_{c,t}^{PB}) \quad c \in \mathcal{CSP}, t = 1 \quad (46)$$

$$\nu_t^{FRS} - \theta_{c,t}^{MIN} - \theta_{c,t}^{MAX} - \theta_{c,t+1}^{UP} - \theta_{c,t+1}^{DN} - \delta_{c,t}^{PMAX} + \pi_t^{QSS} - \omega_t / T_g - \frac{t_{SFR} - t_{QSS} - 2\zeta_1}{20\zeta_2} \cdot (\nu_t^{MIN} - \nu_t^{MAX}) \leq PC_c^{CP} : (PFR_{c,t}^{PB}) \quad c \in \mathcal{CSP}, t = 1 \quad (47)$$

$$-\alpha_{g,t}^{MIN} - \alpha_{g,t}^{MAX} - \alpha_{g,t}^{UP} - \alpha_{g,t+1}^{UP} - \alpha_{g,t}^{DN} - \alpha_{g,t+1}^{DN} - \delta_{g,t}^{SMAX} + \nu_t^{FRS} \leq SC_g^G : (SFR_{g,t}^G) \quad g \in \mathcal{G}, t \in \mathcal{T} \setminus \{1, N^T\} \quad (48)$$

$$-\alpha_{g,t}^{MIN} - \alpha_{g,t}^{MAX} - \alpha_{g,t}^{UP} - \alpha_{g,t}^{DN} - \delta_{g,t}^{SMAX} + \nu_t^{FRS} \leq SC_g^G : (SFR_{g,t}^G) \quad g \in \mathcal{G}, t = N^T \quad (49)$$

$$-\alpha_{g,t}^{MIN} - \alpha_{g,t}^{MAX} - \alpha_{g,t+1}^{UP} - \alpha_{g,t+1}^{DN} - \delta_{g,t}^{SMAX} + \nu_t^{FRS} \leq SC_g^G : (SFR_{g,t}^G) \quad g \in \mathcal{G}, t = 1 \quad (50)$$

$$\beta_t^{w,(SD)} - \delta_t^{w,(SMAX)} + \nu_t^{FRS} \leq SC_w^W : (SFR_{w,t}^W) \quad w \in \mathcal{W}, t \in \mathcal{T} \quad (51)$$

$$-\theta_{c,t}^{MAX} - \theta_{c,t}^{MIN} - \theta_{c,t}^{UP} - \theta_{c,t+1}^{UP} - \theta_{c,t}^{DN} - \theta_{c,t+1}^{DN} + \delta_{c,t}^{SMIN} + \nu_t^{FRS} = SC_c^{PB} : (SFR_{c,t}^{PB}) \quad c \in \mathcal{CSP}, t \in \mathcal{T} \setminus \{1, N^T\} \quad (52)$$

$$-\theta_{c,t}^{MAX} - \theta_{c,t}^{MIN} - \theta_{c,t}^{UP} - \theta_{c,t}^{DN} - \delta_{c,t}^{SMAX} + \nu_t^{FRS} \leq SC_c^{PB} : (SFR_{c,t}^{PB}) \quad c \in \mathcal{CSP}, t = N^T \quad (53)$$

$$-\theta_{c,t}^{MAX} - \theta_{c,t}^{MIN} - \theta_{c,t+1}^{UP} - \theta_{c,t+1}^{DN} - \delta_{c,t}^{SMAX} + \nu_t^{FRS} \leq SC_c^{PB} : (SFR_{c,t}^{PB}) \quad c \in \mathcal{CSP}, t = 1 \quad (54)$$

$$H_g^G \cdot P_g^{G,Max} \cdot \rho_t + \rho_{g,t}^{MIN-} - \rho_{g,t}^{MAX-} + \rho_{g,t}^{MIN+} - \rho_{g,t}^{MAX+} = 0 : (\varpi_{g,t}^G) \quad g \in \mathcal{G}, t \in \mathcal{T} \quad (55)$$

$$H_c^{CSP} \cdot P_c^{PB,Max} \cdot \rho_t + \rho_{c,t}^{MIN-} - \rho_{c,t}^{MAX-} + \rho_{c,t}^{MIN+} - \rho_{c,t}^{MAX+} = 0 : (\varpi_{c,t}^{PB}) \quad c \in \mathcal{CSP}, t \in \mathcal{T} \quad (56)$$

$$-P_{w,t}^W \cdot \rho_t + \rho_{w,t}^{MIN-} - \rho_{w,t}^{MAX-} + \rho_{w,t}^{MIN+} - \rho_{w,t}^{MAX+} = 0 : (\varpi_{w,t}^W) \quad c \in \mathcal{W}, t \in \mathcal{T} \quad (57)$$

$$\alpha, \beta, \lambda, \varepsilon, \delta, \nu, \theta, \rho, \pi \geq 0 \quad (58)$$

IV. ROBUST COUNTERPART OF MTAR-FCUC

This section mainly presents the mathematical formulation

of SP in MTAR-FCUC model.

objective function:

$$\begin{aligned}
& \sum_{t \in \mathcal{T}} \{ \sum_{d \in \mathcal{D}} [(\hat{P}_{d,t}^{\mathcal{D}} \cdot \beta_{d,t}^{SD,\mathcal{D}} + P_{d,t}^{\mathcal{D}+} \cdot \vartheta_{d,t}^{SD,\mathcal{D}})] + \sum_{g \in \mathcal{G}} [\underline{P}_{g,t}^{\mathcal{G}} \cdot \\
& \alpha_{g,t}^{MIN} - \overline{P}_{g,t}^{\mathcal{G}} \cdot \alpha_{g,t}^{MAX} + \underline{PFR}_{g,t}^{\mathcal{G}} \cdot \delta_{g,t}^{PMIN} - \overline{PFR}_{g,t}^{\mathcal{G}} \cdot \delta_{g,t}^{SMAX} \\
& \delta_{g,t}^{PMAX} + \underline{SFR}_{g,t}^{\mathcal{G}} \cdot \delta_{g,t}^{SMIN} - \overline{SFR}_{g,t}^{\mathcal{G}} \cdot \delta_{g,t}^{SMAX} \\
& - M \cdot (1 - x_{g,t}^{\mathcal{G}}) \cdot \rho_{g,t}^{MIN-} - M \cdot x_{g,t}^{\mathcal{G}} \cdot \rho_{g,t}^{MAX+}] \\
& + \sum_{w \in \mathcal{W}} [\hat{P}_{w,t}^{\mathcal{W}} \cdot \beta_{w,t}^{SD,\mathcal{W}} - P_{w,t}^{\mathcal{W}-} \cdot \vartheta_{w,t}^{SD,\mathcal{W}} - (\hat{P}_{w,t}^{\mathcal{W}} \cdot \beta_{w,t}^{MAX} \\
& - P_{w,t}^{\mathcal{W}-} \cdot \vartheta_{w,t}^{MAX}) - PFR_{w,t}^{\mathcal{W},Max} \cdot u_{w,t}^{\mathcal{W}} \cdot \delta_{w,t}^{PMAX} - \\
& SFR_{w,t}^{\mathcal{W},Max} \cdot v_{w,t}^{\mathcal{W}} \cdot \delta_{w,t}^{SMAX} - M \cdot (1 - o_{w,t}^{\mathcal{W}}) \cdot \rho_{w,t}^{MIN-} \\
& - M \cdot o_{w,t}^{\mathcal{W}} \cdot \rho_{w,t}^{MAX}] \} + \sum_{c \in \mathcal{CSP}} \{ \bar{Q}_{c,t}^{SF} \cdot \theta_{c,t}^{SD} - Q_{c,t}^{SF-} \cdot \\
& \vartheta_{c,t}^{SD} + \underline{P}_{c,t}^{\mathcal{PB}} \cdot \theta_{c,t}^{MIN} - \overline{P}_{c,t}^{\mathcal{PB}} \cdot \theta_{c,t}^{MAX} + \underline{Q}_{c,t}^{\mathcal{HT}} \cdot \theta_{c,t}^{IMIN} - \\
& \overline{Q}_{c,t}^{\mathcal{HT}} \cdot \theta_{c,t}^{IMAX} + \underline{Q}_{c,t}^{\mathcal{TH}} \cdot \theta_{c,t}^{OMIN} - \overline{Q}_{c,t}^{\mathcal{TH}} \cdot \theta_{c,t}^{OMAX} + \\
& \underline{PFR}_{c,t}^{\mathcal{PB}} \cdot \delta_{c,t}^{PMIN} - \overline{PFR}_{c,t}^{\mathcal{PB}} \cdot \delta_{c,t}^{PMAX} + \underline{SFR}_{c,t}^{\mathcal{PB}} \cdot \delta_{c,t}^{SMIN} \\
& - \overline{SFR}_{c,t}^{\mathcal{PB}} \cdot \delta_{c,t}^{SMAX} - M \cdot (1 - x_{c,t}^{\mathcal{PB}}) \cdot \rho_{c,t}^{MIN-} - M \cdot \\
& x_{c,t}^{\mathcal{PB}} \cdot \rho_{c,t}^{MAX+} \} - \sum_{l \in \mathcal{L}} (F_l^{Max} \cdot \varepsilon_{l,t}^{MIN} + F_l^{Max} \cdot \varepsilon_{l,t}^{MAX}) - \\
& (D' \cdot \Delta f_{Max}^{QSS} - \Delta P_{Max}^{\mathcal{D}}) \cdot \pi_t^{QSS} + \Delta f_{Max}^{QSS} \cdot D \cdot \sum_{d \in \mathcal{D}} P_{d,t}^{\mathcal{D}+} \cdot \\
& \vartheta_{d,t}^{QSS} + \kappa' \cdot f_0 \cdot \rho_t - \sum_{w \in \mathcal{W}} H_w^{\mathcal{W}} \cdot (\hat{P}_{w,t}^{\mathcal{W}} \cdot \rho_t - P_{w,t}^{\mathcal{W}-} \cdot \vartheta_{w,t}) \\
& + (SFR^{Min} - \Delta P_{Max}^{\mathcal{D}} \cdot \frac{t_{SFR} - t_{QSS} - 2\zeta_1}{20\zeta_2}) \cdot \\
& \nu_t^{MIN} - (SFR^{Max} - \Delta P_{Max}^{\mathcal{D}} \cdot \frac{t_{SFR} - t_{QSS} - 2\zeta_1}{20\zeta_2}) \cdot \\
& \nu_t^{MAX} + \Delta P_{Max}^{\mathcal{D}} \cdot \nu_t^{FRS} \}
\end{aligned} \quad (59)$$

subject to:

$$\vartheta_{d,t}^{SD,\mathcal{D}} = o_{d,t}^{\mathcal{D}} \cdot \beta_{d,t}^{SD,\mathcal{D}}; \vartheta_{d,t}^{QSS} = o_{d,t}^{\mathcal{D}} \cdot \pi_t^{QSS}; d \in \mathcal{D}, t \in \mathcal{T} \quad (60)$$

$$\vartheta_{w,t}^{SD,\mathcal{W}} = o_{w,t}^{\mathcal{W}} \cdot \beta_{w,t}^{SD,\mathcal{W}}; w \in \mathcal{W}, t \in \mathcal{T} \quad (61)$$

$$\vartheta_{w,t}^{Naidr,\mathcal{W}} = o_{w,t}^{\mathcal{W}} \cdot \rho_t; w \in \mathcal{W}, t \in \mathcal{T} \quad (62)$$

$$\vartheta_{c,t}^{SD} = o_{c,t}^{SF} \cdot \theta_{c,t}^{SD}; c \in \mathcal{CSP}, t \in \mathcal{T} \quad (63)$$

$$\begin{aligned}
& -M \cdot (1 - o_{d,t}^{\mathcal{D}}) \leq \vartheta_{d,t}^{MAX} - \beta_{d,t}^{SD,\mathcal{D}} \leq 0; 0 \leq \vartheta_{d,t}^{SD,\mathcal{D}} \leq M \cdot o_{d,t}^{\mathcal{D}}; \\
& -M \cdot (1 - o_{d,t}^{\mathcal{D}}) \leq \vartheta_{d,t}^{QSS} - \pi_t^{QSS} \leq 0; 0 \leq \vartheta_{d,t}^{QSS} \leq M \cdot o_{d,t}^{\mathcal{D}}; \quad (64)
\end{aligned}$$

$$d \in \mathcal{D}, t \in \mathcal{T}$$

$$\begin{aligned}
& -M \cdot (1 - o_{w,t}^{\mathcal{W}}) \leq \vartheta_{w,t}^{SD} - \beta_{w,t}^{SD,\mathcal{W}} \leq 0; 0 \leq \vartheta_{w,t}^{SD,\mathcal{W}} \leq M \cdot o_{w,t}^{\mathcal{W}}; \\
& -M \cdot (1 - o_{w,t}^{\mathcal{W}}) \leq \vartheta_{w,t}^{Naidr,\mathcal{W}} - \rho_t \leq 0; 0 \leq \vartheta_{w,t}^{Naidr,\mathcal{W}} \leq M \cdot o_{w,t}^{\mathcal{W}}; \quad (65)
\end{aligned}$$

$$o_{w,t}^{\mathcal{W}}; w \in \mathcal{W}, t \in \mathcal{T}$$

$$-M \cdot (1 - o_{c,t}^{SF}) \leq \vartheta_{c,t}^{SD} - \theta_{c,t}^{SD} \leq 0; 0 \leq \vartheta_{c,t}^{SD} \leq M \cdot o_{c,t}^{SF}; \quad (66)$$

$$c \in \mathcal{CSP}, t \in \mathcal{T}$$

$$\begin{aligned}
& \lambda_t^{SD} + \alpha_{g,t}^{MIN} - \alpha_{g,t}^{MAX} + \sum_{l \in \mathcal{L}} (\Gamma_{l,g}^{\mathcal{G}} \cdot \varepsilon_{l,t}^{MIN} - \Gamma_{l,g}^{\mathcal{G}} \cdot \varepsilon_{l,t}^{MAX}) \\
& = VC_g^{\mathcal{G}} : (P_{g,t}^{\mathcal{G}}) \quad g \in \mathcal{G}, t \in \mathcal{T} \quad (67)
\end{aligned}$$

$$\begin{aligned}
& \lambda_t^{SD} + \beta_{w,t}^{SD,\mathcal{W}} + \sum_{l \in \mathcal{L}} (\Gamma_{l,w}^{\mathcal{W}} \cdot \varepsilon_{l,t}^{MIN} - \Gamma_{l,w}^{\mathcal{W}} \cdot \varepsilon_{l,t}^{MAX}) = 0 \\
& : (P_{w,t}^{\mathcal{W}}) \quad w \in \mathcal{W}, t \in \mathcal{T} \quad (68)
\end{aligned}$$

$$\beta_{w,t}^{SD,\mathcal{W}} \leq VoLL^{\mathcal{W}} : (P_{w,t}^{\mathcal{W},Cur}) \quad w \in \mathcal{W}, t \in \mathcal{T} \quad (69)$$

$$\begin{aligned}
& \beta_{d,t}^{SD,\mathcal{D}} - \lambda_t^{SD} - \sum_{l \in \mathcal{L}} (\Gamma_{l,d}^{\mathcal{D}} \cdot \varepsilon_{l,t}^{MIN} - \Gamma_{l,d}^{\mathcal{D}} \cdot \varepsilon_{l,t}^{MAX}) = 0 \\
& : (P_{d,t}^{\mathcal{D}}) \quad d \in \mathcal{D}, t \in \mathcal{T} \quad (70)
\end{aligned}$$

$$\beta_{d,t}^{SD,\mathcal{D}} \leq VoLL^{\mathcal{D}} : (P_{d,t}^{\mathcal{D},Cur}) \quad d \in \mathcal{D}, t \in \mathcal{T} \quad (71)$$

$$\begin{aligned}
& \eta_c^{\mathcal{PB}} \cdot \lambda_t^{SD} + \eta_c^{\mathcal{PB}} \cdot \theta_{c,t}^{MIN} - \eta_c^{\mathcal{PB}} \cdot \theta_{c,t}^{MAX} + \theta_{c,t}^{SD} + \\
& \eta_c^{\mathcal{PB}} \cdot \sum_{l \in \mathcal{L}} (\Gamma_{l,c}^{\mathcal{PB}} \cdot \varepsilon_{l,t}^{MIN} - \Gamma_{l,c}^{\mathcal{PB}} \cdot \varepsilon_{l,t}^{MAX}) = 0 \\
& : (Q_{c,t}^{\mathcal{PB}}) \quad c \in \mathcal{CSP}, t \in \mathcal{T} \quad (72)
\end{aligned}$$

$$\begin{aligned}
& -\eta_c^{dis} \cdot \theta_{c,t}^{SD} - \sum_{\tau \in t:N^T} (1 - \iota)^{\tau-t} \cdot (\theta_{c,\tau}^{TMIN} - \theta_{c,\tau}^{TMAX}) + \\
& \theta_{c,t}^{OMIN} - \theta_{c,t}^{OMAX} = 0 : (Q_{c,t}^{\mathcal{TH}}) \quad c \in \mathcal{CSP}, t \in \mathcal{T} \quad (73)
\end{aligned}$$

$$\begin{aligned}
& \theta_{c,t}^{SD} / \eta_c^{cha} + \sum_{\tau \in t:N^T} (1 - \iota)^{\tau-t} \cdot (\theta_{c,\tau}^{TMIN} - \theta_{c,\tau}^{TMAX}) + \\
& \theta_{c,t}^{IMIN} - \theta_{c,t}^{IMAX} = 0 : (Q_{c,t}^{\mathcal{HT}}) \quad c \in \mathcal{CSP}, t \in \mathcal{T} \quad (74)
\end{aligned}$$

$$\begin{aligned}
& \omega_t - \sum_{g \in \mathcal{G}} (\rho_{g,t}^{MIN-} - \rho_{g,t}^{MAX-}) - \sum_{c \in \mathcal{CSP}} (\rho_{c,t}^{MIN-} - \rho_{c,t}^{MAX-}) \\
& - \sum_{w \in \mathcal{W}} [\rho_{w,t}^{MIN-} - \rho_{w,t}^{MAX-} - H_w^{\mathcal{W}} \cdot (\hat{P}_{w,t}^{\mathcal{W}} \cdot \rho_t - \\
& P_{w,t}^{\mathcal{W}-} \cdot \vartheta_{w,t}^{Naidr,\mathcal{W}})] - H_d^{\mathcal{D}} \cdot \Delta P_{t,Max}^{\mathcal{D}} / T_g \cdot \rho_t = 0 \quad (75)
\end{aligned}$$

$$\begin{aligned}
& \nu_t^{FRS} + \delta_{g,t}^{PMIN} - \delta_{g,t}^{PMAX} + \pi_t^{QSS} - \omega_t / T_g - \\
& \frac{t_{SFR} - t_{QSS} - 2\zeta_1}{20\zeta_2} \cdot (\nu_t^{MIN} - \nu_t^{MAX}) = PC_g^{\mathcal{G}} \\
& : (PFR_{g,t}^{\mathcal{G}}) \quad g \in \mathcal{G}, t \in \mathcal{T} \setminus \{1, N^T\} \quad (76)
\end{aligned}$$

$$\begin{aligned}
& \beta_{w,t}^{SD,\mathcal{W}} + \nu_t^{FRS} - \delta_{w,t}^{PMAX} + \pi_t^{QSS} - \omega_t / T_g - \\
& \frac{t_{SFR} - t_{QSS} - 2\zeta_1}{20\zeta_2} \cdot (\nu_t^{MIN} - \nu_t^{MAX}) \leq PC_w^{\mathcal{W}} \\
& : (PFR_{w,t}^{\mathcal{W}}) \quad w \in \mathcal{W}, t \in \mathcal{T} \quad (77)
\end{aligned}$$

$$\begin{aligned}
& \nu_t^{FRS} + \delta_{c,t}^{PMIN} - \delta_{c,t}^{PMAX} + \pi_t^{QSS} - \omega_t / T_g - \\
& \frac{t_{SFR} - t_{QSS} - 2\zeta_1}{20\zeta_2} \cdot (\nu_t^{MIN} - \nu_t^{MAX}) = PC_c^{\mathcal{PB}} \\
& : (PFR_{c,t}^{\mathcal{PB}}) \quad c \in \mathcal{CSP}, t \in \mathcal{T} \quad (78)
\end{aligned}$$

$$\begin{aligned}
& \delta_{g,t}^{SMIN} - \delta_{g,t}^{SMAX} + \nu_t^{FRS} = SC_g^{\mathcal{G}} : (SFR_{g,t}^{\mathcal{G}}) \\
& g \in \mathcal{G}, t \in \mathcal{T} \quad (79)
\end{aligned}$$

$$\begin{aligned}
& \beta_{w,t}^{SD,\mathcal{W}} - \delta_{w,t}^{SMAX} + \nu_t^{FRS} \leq SC_w^{\mathcal{W}} : (PFR_{w,t}^{\mathcal{W}}) \\
& w \in \mathcal{W}, t \in \mathcal{T} \quad (80)
\end{aligned}$$

$$\begin{aligned}
& \delta_{c,t}^{SMIN} - \delta_{c,t}^{SMAX} + \nu_t^{FRS} = SC_c^{\mathcal{PB}} : (SFR_{c,t}^{\mathcal{PB}}) \\
& c \in \mathcal{CSP}, t \in \mathcal{T} \quad (81)
\end{aligned}$$

$$\begin{aligned}
& H_g^{\mathcal{G}} \cdot P_g^{\mathcal{G},Max} \cdot \rho_t + \rho_{g,t}^{MIN-} - \rho_{g,t}^{MAX-} - \rho_{g,t}^{MAX+} \leq 0 \\
& : (\varpi_{g,t}^{\mathcal{G}}) \quad g \in \mathcal{G}, t \in \mathcal{T} \quad (82)
\end{aligned}$$

$$\begin{aligned}
& H_c^{\mathcal{PB}} \cdot P_c^{\mathcal{PB},Max} \cdot \rho_t + \rho_{c,t}^{MIN-} - \rho_{c,t}^{MAX-} - \rho_{c,t}^{MAX+} \leq 0 \\
& : (\varpi_{c,t}^{\mathcal{PB}}) \quad c \in \mathcal{CSP}, t \in \mathcal{T} \quad (83)
\end{aligned}$$

$$\begin{aligned}
& -P_{w,t}^{\mathcal{W}-} \cdot \rho_t + \rho_{w,t}^{MIN-} - \rho_{w,t}^{MAX-} - \rho_{w,t}^{MAX+} \leq 0 : (\varpi_{w,t}^{\mathcal{W}}) \\
& c \in \mathcal{W}, t \in \mathcal{T} \quad (84)
\end{aligned}$$

$$\alpha, \beta, \lambda, \varepsilon, \delta, \nu, \theta, \rho, \pi \geq 0 \quad (85)$$

V. THE TOPOLOGY OF THE AUGMENTED IEEE RTS-30 SYSTEM

The topology of the augmented IEEE RTS-30 system, as mentioned in the text, is illustrated below:

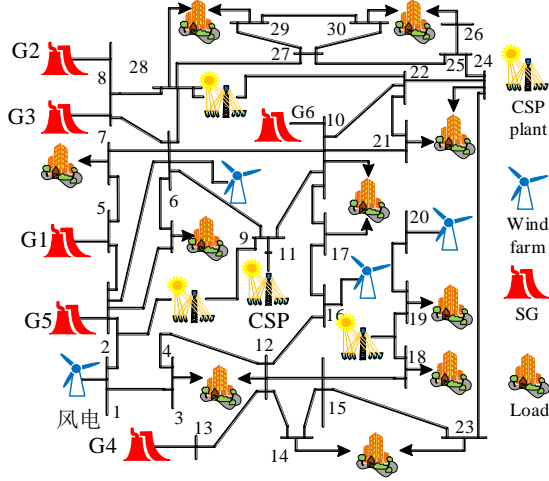


Fig. 1 The topology of the augmented IEEE RTS-30 system

VI. PARAMETERS OF THE AUGMENTED IEEE RTS-30 SYSTEM

The parameters of SGs and lines in the IEEE RTS-30 system are summarized in TABLE I, TABLE II.

TABLE I
PARAMETERS OF SGs

SGs No.	Maxi-mum/Minimum (MW)	Ramp Level (MW/h)	Minimum Down Time (h)	Minimum Up Time (h)	Start-up and Shut-down Cost (\$)	Inertia Constant (s)
G1	200/50	66.67	1	1	70/50	6
G2	160/40	53.33	2	2	74/60	7.8
G3	150/37.5	50	1	1	50/30	6
G4	100/25	33.33	1	2	110/85	7.8
G5	110/27.5	36.67	2	1	72/52	7.8
G6	150/37.5	50	1	1	40/30	7.8

TABLE II
PARAMETERS OF LINES

Line No.	From node	To node	X(p.u.)	Rating(MWA)
1	1	2	0.0575	221
2	1	3	0.1652	221
3	2	4	0.1737	170
4	3	4	0.0379	221
5	2	5	0.1983	221
6	2	6	0.1763	170
7	4	6	0.0414	153
8	5	7	0.116	119
9	6	7	0.082	221
10	6	8	0.042	108.8
11	6	9	0.208	110.5
12	6	10	0.556	170
13	9	10	0.208	110.5
14	9	11	0.11	280.5
15	4	12	0.256	110.5
16	12	13	0.14	170
17	12	14	0.2559	88.4

18	12	15	0.1304	122.4
19	12	16	0.1987	88.4
20	14	15	0.1997	85
21	16	17	0.1923	27.2
22	15	18	0.2185	170
23	18	19	0.1292	170
24	19	20	0.068	170
25	10	20	0.209	122.4
26	10	17	0.0845	127.5
27	10	21	0.0749	119
28	10	22	0.1499	119
29	21	22	0.0236	54.4
30	15	23	0.202	27.2
31	22	24	0.179	27.2
32	23	24	0.27	27.2
33	24	25	0.3292	136
34	25	26	0.38	119
35	25	27	0.2087	170
36	28	27	0.396	110.5
37	27	29	0.4153	204
38	27	30	0.6027	136
39	29	30	0.4533	136
40	8	28	0.2	54.4
41	6	28	0.0599	54.4

VII. PARAMETERS OF THE AUGMENTED IEEE-118 SYSTEM

The parameters of SGs, CSP plants, and lines in the IEEE -118 system are summarized in TABLE III, TABLE IV, and TABLE V.

TABLE III
PARAMETERS OF SGs

SGs No.	Maxi-mum/Minimum (MW)	Ramp Level (MW/h)	Minimum Down Time (h)	Minimum Up Time (h)	Start-up and Shut-down Cost (\$)	Inertia Constant (s)
G1	100/20	50	1	1	70/50	6.5
G2	100/20	50	2	2	74/60	7.8
G3	100/20	50	3	3	50/30	7.8
G4	100/20	50	2	2	110/85	6.5
G5	185/37	92.5	2	1	40/30	6.5
G6	320/64	160	4	4	95/80	7.8
G7	100/20	50	2	2	75.51/51.47	6.5
G8	107/21.4	53.5	2	2	75.99/51.92	6.5
G9	100/20	50	3	2	73.6/53.34	7.8
G10	100/20	50	2	3	70.67/50.4	6.5
G11	119/23.8	59.5	2	2	81/62	6.5
G12	304/60.8	152	3	3	99/79	7.8
G13	148/29.6	74	2	2	55/40	6.5
G14	100/20	50	3	3	77.97/53.34	6.5
G15	100/20	50	2	2	70.63/50.4	7.8
G16	255/51	127.5	2	3	81/62	6.5
G17	260/52	130	4	2	85/59	6.5
G18	100/20	50	3	2	77.7/57.56	7.8

TABLE IV
PARAMETERS OF LINES

Line No.	From node	To node	X(p.u.)	Rating(MWA)
1	1	2	0.0999	510
2	1	3	0.0424	750
3	4	5	0.00798	228
4	3	5	0.108	518
5	5	6	0.054	214
6	6	7	0.0208	136
7	8	9	0.0305	110
8	8	5	0.0267	759

9	9	10	0.0322	110	81	50	57	0.134	141
10	4	11	0.0688	133	82	56	58	0.0966	136
11	5	11	0.0682	145	83	51	58	0.0719	136
12	11	12	0.0196	338	84	54	59	0.2293	142
13	2	12	0.0616	510	85	56	59	0.251	141
14	3	12	0.16	344	86	56	59	0.239	142
15	7	12	0.034	136	87	55	59	0.2158	149
16	11	13	0.0731	286	88	59	60	0.145	128
17	12	14	0.0707	289	89	59	61	0.15	131
18	13	15	0.2444	286	90	60	61	0.0135	144
19	14	15	0.195	355	91	60	62	0.0561	128
20	12	16	0.0834	337	92	61	62	0.0376	148
21	15	17	0.0437	144	93	63	59	0.0386	205
22	16	17	0.1801	337	94	63	64	0.02	205
23	17	18	0.0505	314	95	64	61	0.0268	200
24	18	19	0.0493	133	96	38	65	0.0986	755
25	19	20	0.117	143	97	64	65	0.0302	295
26	15	19	0.0394	346	98	49	66	0.0919	170
27	20	21	0.0849	143	99	49	66	0.0919	170
28	21	22	0.097	143	100	62	66	0.218	134
29	22	23	0.159	192	101	62	67	0.117	134
30	23	24	0.0492	506	102	65	66	0.037	219
31	23	25	0.08	265	103	66	67	0.1015	134
32	26	25	0.0382	202	104	65	68	0.016	948
33	25	27	0.163	173	105	47	69	0.2778	199
34	27	28	0.0855	120	106	49	69	0.324	199
35	28	29	0.0943	120	107	68	69	0.037	277
36	30	17	0.0388	164	108	69	70	0.127	166
37	8	30	0.0504	859	109	24	70	0.4115	282
38	26	30	0.086	202	110	70	71	0.0355	253
39	17	31	0.1563	132	111	24	72	0.196	302
40	29	31	0.0331	120	112	71	72	0.18	266
41	23	32	0.1153	291	113	71	73	0.0454	159
42	31	32	0.0958	128	114	70	74	0.1323	206
43	27	32	0.0755	134	115	70	75	0.141	213
44	15	33	0.1244	263	116	69	75	0.122	173
45	19	34	0.247	235	117	74	75	0.0406	156
46	35	36	0.01102	137	118	76	77	0.148	179
47	35	37	0.0497	137	119	69	77	0.101	395
48	33	37	0.142	263	120	75	77	0.1999	216
49	34	36	0.0268	184	121	77	78	0.0124	142
50	34	37	0.0094	163	122	78	79	0.0244	142
51	38	37	0.0375	179	123	77	80	0.0485	182
52	37	39	0.106	228	124	77	80	0.105	143
53	37	40	0.168	227	125	79	80	0.0704	142
54	30	38	0.054	805	126	68	81	0.0202	781
55	39	40	0.0605	228	127	81	80	0.037	781
56	40	41	0.0487	196	128	77	82	0.0853	380
57	40	42	0.183	196	129	82	83	0.03665	331
58	41	42	0.135	196	130	83	84	0.132	219
59	43	44	0.2454	212	131	83	85	0.148	254
60	34	43	0.1681	212	132	84	85	0.0641	219
61	44	45	0.0901	212	133	85	86	0.123	161
62	45	46	0.1356	129	134	86	87	0.2074	161
63	46	47	0.127	134	135	85	88	0.102	138
64	46	48	0.189	115	136	85	89	0.173	160
65	47	49	0.0625	176	137	88	89	0.0712	228
66	42	49	0.323	163	138	89	90	0.188	148
67	42	49	0.323	163	139	89	90	0.0997	181
68	45	49	0.0186	193	140	90	91	0.0836	122
69	48	49	0.0505	115	141	89	92	0.0505	192
70	49	50	0.0752	141	142	89	92	0.1581	136
71	49	51	0.137	148	143	91	92	0.1272	205
72	51	52	0.0588	122	144	92	93	0.0848	216
73	52	53	0.1635	122	145	92	94	0.158	216
74	53	54	0.122	122	146	93	94	0.0732	216
75	49	54	0.289	142	147	94	95	0.0434	259
76	49	54	0.291	142	148	80	96	0.182	232
77	54	55	0.0707	129	149	82	96	0.053	159
78	54	56	0.00955	170	150	94	96	0.0869	278
79	55	56	0.0151	164	151	80	97	0.0934	208
80	56	57	0.0966	141	152	80	98	0.108	227
					153	80	99	0.206	280
					154	92	100	0.295	147
					155	94	100	0.058	215

156	95	96	0.0547	259
157	96	97	0.0885	260
158	98	100	0.179	281
159	99	100	0.0813	209
160	100	101	0.1262	168
161	92	102	0.0559	209
162	101	102	0.112	209
163	100	103	0.0525	256
164	100	104	0.204	168
165	103	104	0.1584	137
166	103	105	0.1625	143
167	100	106	0.229	164
168	104	105	0.0378	141
169	105	106	0.0547	124
170	105	107	0.183	150
171	105	108	0.0703	187
172	106	107	0.183	151
173	108	109	0.0288	187
174	103	110	0.1813	170
175	109	110	0.0762	187
176	110	111	0.0755	295
177	110	112	0.064	110
178	17	113	0.0301	237
179	32	113	0.203	142
180	32	114	0.0612	122
181	27	115	0.0741	122
182	114	115	0.0104	122
183	68	116	0.00405	110
184	12	117	0.14	110
185	75	118	0.0481	217
186	76	118	0.0544	217

TABLE V
PARAMETERS OF CSP PLANTS

SGs No.	Maximum/Minimum (MW)	Ramp Level (MW/h)	Minimum Down Time (h)	Minimum Up Time (h)	Start-up and Shut-down Cost (\$)	Inertia Constant (s)
CSP1	100/10	40	1	1	61.5	7.8
CSP2	200/20	120	2	2	110.5	7.8
CSP3	100/10	80	1	1	61.5	7.8
CSP4	100/10	80	1	2	61.5	7.8
CSP5	100/10	80	2	1	61.5	7.8
CSP6	100/10	40	1	1	61.5	7.8
CSP7	100/10	40	1	1	61.5	7.8
CSP8	200/20	120	2	2	110.5	7.8
CSP9	100/10	80	1	1	61.5	7.8
CSP10	200/20	120	2	2	110.5	7.8
CSP11	100/10	40	1	1	61.5	7.8
CSP12	200/20	120	2	2	110.5	7.8
CSP13	100/10	80	1	1	61.5	7.8
CSP14	100/10	80	1	2	61.5	7.8
CSP15	100/10	80	2	1	61.5	7.8
CSP16	100/10	40	1	1	61.5	7.8
CSP17	100/10	40	1	1	61.5	7.8
CSP18	200/20	120	2	2	110.5	7.8