Question 4

This question tested candidates' ability to:

- use the formulae for arc length and sector area of a circle;
- use the compound angle formula to evaluate a value for sin θ in surd form.

There were 1705 responses to this question. The mean mark was 4.20 with standard deviation 2.24. Two hundred and fifteen candidates obtained full marks.

Candidates performed best on Part (a), that is, finding the area and perimeter of a given shape, although some candidates did not seem to understand the instructions to give their answer in terms of π. For example, some candidates having arrived at the area of the figure as $16 + \frac{8\pi}{3}$ then went on to give a decimal answer. Other candidates mistakenly simplified this to $\frac{24\pi}{3}$. In Part (a) (ii), some candidates failed to recognize that one side of the square (or one radius of the sector) should not be included in the calculation for perimeter, and so added the whole perimeter of the square to the whole perimeter of the sector.

Part (b) presented difficulties for many candidates, in particular coming up with an appropriate split for $\frac{7\pi}{12}$; hence many candidates did not recognize the need for use of the compound angle formula. Some candidates who recognized the need to split then wrote it as $\cos\frac{\pi}{3} + \cos\frac{\pi}{4}$ for which no marks were awarded. Of note, a few candidates wrote that $\cos\frac{7\pi}{12} \equiv \cos\left(\frac{7\pi}{3} + \frac{7\pi}{4}\right)$ and used the fact that 2π is one revolution to come to the correct compound angle $\cos\left(\frac{\pi}{3} + \frac{\pi}{4}\right)$. The concept of exactness also appeared to be lost on many of the candidates who proceeded to write the solution, $\frac{\sqrt{2}-\sqrt{6}}{4}$ as a decimal, or some other incorrect simplification, such as $-\frac{\sqrt{4}}{4}$.

Part (c) was not marked. However, approximately $\frac{1}{3}$ of the candidates who attempted this part of the question knew that $\sec x = \frac{1}{\cos x}$.

Solutions

(a) (i) $16 + \frac{8\pi}{3}\text{ m}^2$ (ii) $16 + \frac{4\pi}{3}\text{ m}$

(b) $\frac{\sqrt{2}-\sqrt{6}}{4}$

Section 3: Introductory Calculus

Question 5

This question tested candidates' ability to:

- differentiate a quotient made up of simple polynomials;
- determine the equations of tangents and normal to curves;
- make use of the concept of derivative as a rate of change.
-

There were 1712 responses to this question. The mean mark was 5.89 with standard deviation 4.47. One hundred and thirty-three candidates obtained full marks.

In this question, candidates performed best on the mechanics of differentiation. Many though could not state the correct quotient rule for differentiation, and so performed the mechanics of differentiation in Part (a) as if for a product rule. And, having performed the mechanics of differentiation correctly, a marked number of candidates then could not apply the distributive law correctly with a negative 1 multiplier to expand the brackets, i.e. $-1(3x + 4)$ was often expanded as $-3x + 4$.

Part (b) was generally reasonably done. Some candidates experienced difficulty with the meaning/significance of $\frac{dy}{dx}$, in that having found it for the given curve, they could not translate it into finding the value of the gradient at the given point (2, 10). That said, making use of a value for gradient to find the equation of the tangent and the equation of the normal was fairly widely known.

Approximately 50 per cent of candidates did not appear to know the topic 'Rates of Change' and so could not do Part (c). Others, having determined that the area, A, of the square (s) could be written as $A = s^2$ then went on to differentiate this as $\frac{dy}{dx} = 2s$ instead of $\frac{dA}{ds} = 2s$, showing a lack of understanding of the meaning of differentiation as rate of change. Candidates' difficulty with notation was especially evident in this part of the question; very few candidates wrote their differential equations in terms of dt.

Solutions

(a) $\frac{-10}{(x-2)^2}$ (b) (i) $y = 17x - 24$

(ii) $10y = x + 172$

(c) $\frac{dA}{dt} = 40\text{ cm}^2\text{s}^{-1}$

Question 6

This question tested candidates' ability to:

- evaluate the definite integral of a function [of the form $(ax \pm b)^n$, $n \neq -1$];
- determine the equation of a curve given its gradient function and a point on the curve;
- find the area of a region bounded by a trigonometric curve and the x-axis;
- find the volume of revolution of the solid formed when a curve of degree 2 is rotated about the x-axis.

There were 1655 responses to this question. The mean mark was 6.23 with standard deviation 4.61. One hundred and two candidates obtained full marks.

Candidates performed best on Part (b), although some lost marks as they did not include or failed to find a value for the constant of integration. Part (c) was also relatively reasonably done, although some candidates experienced difficulties in correctly evaluating cos θ. In Part (a), in their integration of $(16 - 7x)^3$, although many candidates could perform the mechanics of integration to obtain 4 in the denominator, many candidates failed to multiply the coefficient of x in the polynomial by the 4 to obtain -28 for the denominator. A few candidates expanded the polynomial in order to integrate it.

Part (d) presented the most difficulty for candidates. Many candidates could set up the problem as $\pi\int_0^3 (x^2+2)^2\,dx$, although some did forget the 'π' in their formula, but a number of candidates were unable to expand $(x^2 + 2)^2$ correctly, very often obtaining $x^4 + 4$.

Solutions

(a) $\dfrac{935}{4}$ (b) $y = \dfrac{3x^2}{2} - 5x + 4$ (c) $\dfrac{2\sqrt{3}+3}{2}$ units^2

(d) $96.6\,\pi$ units^3

Section 4: Basic Mathematical Applications

Question 7

This question tested candidates' ability to:

- make use of the laws of probability, specifically the sum of all probabilities in a sample and the addition rule;
- calculate the probability of an event occurring;
- construct and use a tree diagram and calculate conditional probability.

Candidates performed best on Part (a). Many used their knowledge of Sets and Venn Diagrams from CSEC Mathematics to represent the given problem and successfully solve it. However, for candidates who used a formula method to solve, a common error observed was forgetting to subtract the intersection $P(L \cap D)$ (that is, owned both a laptop and a desktop computer) from the $P(L) + P(D)$.

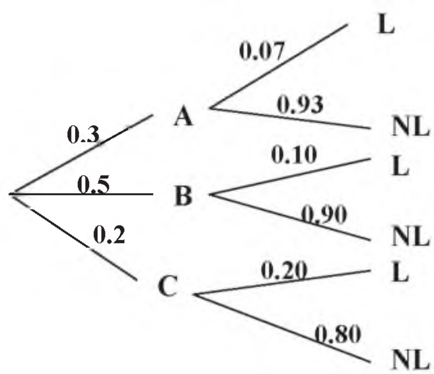
Parts (b) and (c) presented the most difficulty for candidates. In Part (b), a common error observed related to candidates presenting a solution representing sampling with replacement, although the question stated that the items were withdrawn without replacement.

In Part (c) (i), many candidates had difficulty drawing the tree diagram and it was clear that some did not know what this was as they drew diagrams of actual trees, or drew stem and leaf diagrams. Of those who did know what to do, many could only correctly draw the first branch, and had problems completing the second branches for 'late/not late'. Two common errors observed were candidates who correctly wrote the probability of a taxi being late from garage A as $P(L) = 0.07$ but then calculated $P(\bar{L}) = 0.23$, and candidates who incorrectly wrote the probability of a taxi being late from garage A as $P(L) = 0.7$ instead of $P(L) = 0.07$ as given. In Part (c) (ii) b, candidates recognized that conditional probability was involved but did not know the formula for conditional probability; they knew that some sort of division was necessary but did not know what to divide by, with many not using the probability they had computed in Part (c) (ii) a) as the denominator here.

Solutions

(a) 17% (b) (i) $\frac{1}{20}$ (ii) $\frac{1}{2}$

(c) (i)



(c) (ii) (a) 0.111 (b) $\frac{40}{111}$

<u>Question 8</u>

This question tested candidates' ability to:

- draw and interpret and make use of velocity-time graphs;
- apply rates of change to the motion of a particle.

There were 879 responses to this question. The mean mark was 6.96 with standard deviation 5.02. Seven candidates obtained full marks.

Candidates performed best on Parts (a) (i) and (ii) which required them to draw a velocity-time graph from given information, and also to determine the distance.

Part (a) (iii) however presented the most difficulty for candidates. This part dealt with a second car starting 3 seconds after the first car from the same starting point, moving at a constant (faster) velocity and meeting the first car sometime later. Most candidates did not know that the distances covered would have been the same at the meeting point, or that the second car would have taken 3 seconds less than the first car to reach that point. Candidates' experience of difficulty with Part (b) related mainly to notations and their failure to incorporate the constant of integration into their solutions.

Solutions

(a) (ii) 264 m (iii) 6 seconds

(b) (i) velocity = 32 ms^{-1} (ii) displacement = $\frac{52}{3}$ m

Paper 031 - School -Based Assessment (SBA)

Generally, many of the projects submitted were of a high quality and related to real-world situations. Many showed a high level of ingenuity, and were obviously scenarios in which students were interested (for example, modelling the trajectory of a basketball to maximize the number of points that could be gained from free throws). A number of the submissions showed evidence that students had applied a high degree of effort to the task, understood the focus of their topic selection and had conducted the requisite research thoroughly. Projects were generally well presented. Marks were generally in the range of 14 to 20, with many of the sample submissions receiving the full score of 20 marks.

There were nonetheless a number of observed weak areas related to sample submissions. These included:

- Some SBA project titles and aims/purpose were not clear, not concise and not well defined. Some titles were too vague, others too wordy. In some cases, students simply restated the project's title as its aim/purpose;
- In a number of cases, the project's aim did not relate to the given title and sometimes not linked to any syllabus objective;
- In some cases for Project B, students did not identify their variables which would be a key aspect of this project type related to the analysis and discussion of the results;
- In some cases for both Projects A and B assumptions were not always stated;
- Some students' data collection method(s) were flawed, with the resulting negative impact on analysis and findings;
- It appeared that all (or most) of the students at some centres conducted their research using the same topic or a limited number of topics. Although there is nothing untoward with this approach, teachers must be aware that the probability of students presenting identical results, analyses and conclusions is very remote. In essence, students must demonstrate individuality in their work;
- In presenting their data or mathematical formulations, many students were short on explanations of what was happening so it was sometimes difficult to follow their presentations;
- Many students had difficulty presenting an appropriate, coherent and full analysis of their data, hence finding(s) and conclusion(s) were deficient;
- The majority of students did not incorporate or suggest any future analysis in their submissions;
- Some students failed to adequately connect their finding(s), discussion and/or conclusion(s) to their original project aim(s)/purpose, their data collected and their analysis of the data. In some cases, the conclusions stated did not arrive out of the presented findings;
- A few submissions appeared to be downloaded from the internet, and there were cases where submitted samples were blatantly plagiarized from CXC/CSEC Additional Mathematics material. Teachers must be alert to any suspected plagiarism. Plagiarism must not be condoned and marked accordingly;

- Some students presented projects that were more applicable to the sciences with an inadequate application of Additional Mathematics concepts;
- Whilst projects were generally well presented, a marked number of them showed deficiencies in grammar, spelling and punctuation.

Other general issues encountered in moderating the SBA samples included:

- There was no CSEC Additional Mathematics rubric/mark scheme accompanying the samples submitted by some centres;
- Some teachers created their own rubric or made adjustments to the CSEC Additional Mathematics rubric;
- Some projects were incorrectly categorized and assessed, that is, a Project A being labelled and assessed as a Project B, and vice versa. This did create some problems as for example, Project B requires the collection and analysis of data from an experimental-type activity.

The following recommendations hold for continued improvement in this aspect of the Additional Mathematics examinations:

- All projects should have a clear and concise title, and well defined aim(s) or purpose;
- Where possible the SBA should identify with authentic situations;
- The variables that are being used or measured (Project B) must be clearly stated and described. Variables can be controlled, manipulated and responding;
- The type of sample and sample size if relevant must be clearly stated;
- Teachers must ensure that projects which integrate other subject areas utilize concepts as contained in the CSEC Additional Mathematics syllabus;
- If students collect their data in a group setting, they must demonstrate their individual effort in relation to analysis (interpretation) and finding(s)/conclusion(s);
- Projects involving dice or playing cards must be more expansive so that students can present a more in-depth analysis of the topic under consideration;
- As good practice, students should be encouraged to cite all sources and insert a reference/bibliography page;
- Teachers should guide students using the assessment criteria found in forms ‘Add Math 1 – 5A’ and ‘Add Math 1 – 5B’ which are both available on the CXC website. Teachers can give their students the rubric as a means of guidance in developing their projects.

Paper 032 - Alternative to School-Based Assessment (SBA)

This paper tested candidates’ ability to:

- formulate a mathematical problem and derive values to maximize its solution;
- make use of the laws of indices and logarithms to evaluate a specific n^{th} value of an AP.

There were 54 responses to this paper. The mean mark was 5.24 with standard deviation 4.22. The highest mark awarded was 18/20.

In Part (a), candidates were able to formulate the mathematical problem as directed. However, a number of candidates lost marks for not finding the second derivative $\frac{d^2A}{dx^2}$ (for both sports clubs) and confirming that it was a maximum. Candidates also made various arithmetic errors in their solutions so few were able to come up with the expected dimensions for both sports clubs.

All candidates who attempted Part (b) approached a solution using the given information that the series was an AP. A number of candidates were able to get one expression for the common difference d by subtracting two consecutive terms given. However, many failed to obtain a second expression for d and so could not equate these two to get an expression for a in terms of b. Many did not realize that they needed to substitute for a and so ended up with a final expression in ab, which, in some cases even though correct, would not have given them the requested value for n.

Solutions

(a)	(i)	Maximize $A = 6xy$	Subject to	$9x + 8y = 600$
	(ii)	Maximum area Q = 7500 m^2	(iii)	Maximum area P = 22 500 m^2
(b)		$n = 112$		

Appendix: Suggested Solutions to Selected Question Parts in Paper 02

1(c) Solve $3^{2x} - 9(3^{-2x}) = 8$ [1]

Let $3^{2x} = m$; Substitute into [1] gives

$$m - \frac{9}{m} = 8$$
$$m^2 - 9 - 8m = 0$$
$$m^2 - 8m - 9 = 0$$
$$(m - 9)(m + 1) = 0$$
$$m = 9 \quad \text{OR} \quad m = -1$$

that is, $3^{2x} = 9$ OR $3^{2x} = -1$ (INVALID)

From $3^{2x} = 9$

$$2x \log 3 = \log 9$$
$$2x = \frac{\log 9}{\log 3} = 2$$
$$x = 1$$

2(b) $\dfrac{2x-5}{3x+1} > 0$

For LHS > 0, numerator AND denominator must BOTH have the same sign

For $2x - 5 > 0$ AND $3x + 1 > 0$

$x > \dfrac{5}{2}$ AND $x > \dfrac{-1}{2}$

Both true when $x > \dfrac{5}{2}$ [1]

AND for $2x - 5 < 0$ AND $3x + 1 < 0$

$x < \dfrac{5}{2}$ AND $x < \dfrac{-1}{2}$

Both true when $x < \dfrac{-1}{2}$ [2]

From [1] AND [2], range of values when $x > \dfrac{5}{2}$ AND $x < \dfrac{-1}{2}$

5(c) $\dfrac{ds}{dt} = 4 \text{ cm s}^{-1}$

$A = s^2$; $\dfrac{dA}{ds} = 2s$

$$\frac{dA}{dt} = \frac{dA}{ds} \times \frac{ds}{dt}$$
$$= 2s \times 4 = 8s$$

When $s = 5$, $\dfrac{dA}{ds} = 8 \times 5 = 40 \text{ cm}^2 \text{ s}^{-1}$

7(b) Let R = red marble, B = blue marble, K = black marble

(i) P(all of same colour) = P(RRR) + P(BBB) + P(KKK)

$$= \frac{4}{10} \times \frac{3}{9} \times \frac{2}{8} + \frac{3}{10} \times \frac{2}{9} \times \frac{1}{8} + \frac{3}{10} \times \frac{2}{9} \times \frac{1}{8}$$

$$= \frac{1}{20}$$

(ii) P(exactly one Red) $= P(R\bar{R}\,\bar{R}) \times 3$

$$= \frac{4}{10} \times \frac{6}{9} \times \frac{5}{8} \times 3$$

$$= \frac{1}{2}$$

7(c) (ii) (a) $P(L) = P(A \cap L) + P(B \cap L) + P(C \cap L)$

$= 0.3 \times 0.07 + 0.5 \times 0.1 + 0.2 \times 0.2$

$= 0.021 + 0.05 + 0.04$

$= 0.111$

(b) $$P(C/L) = \frac{P(C \cap L)}{P(L)}$$

$$= \frac{0.2 \times 0.2}{0.111}$$

$$= \frac{0.04}{0.111}$$

$$= \frac{40}{111}$$

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

REPORT ON CANDIDATES' WORK IN THE CARIBBEAN SECONDARY EDUCATION CERTIFICATE® EXAMINATION

MAY/JUNE 2013

ADDITIONAL MATHEMATICS

GENERAL PROFICIENCY EXAMINATION

GENERAL COMMENTS

Additional Mathematics, the newest subject offering at the Caribbean Secondary Education Certificate (CSEC) level, was tested for the second time in the May/June 2013 examinations. The subject is intended to bridge a perceived gap between the CSEC General Mathematics and the CAPE Unit 1 Pure Mathematics. The intent is to offer CSEC Mathematics students a more seamless transition to the advanced thinking and skills needed for CAPE Unit 1 Mathematics courses, although a student who masters the content and skills set out in the CSEC Mathematics syllabus should be ready to meet the skills and thinking demands of the CAPE Unit 1 Mathematics course.

The examinations consist of four papers:

- Paper 01 – a 45-item multiple choice paper
- Paper 02 – a structured, 'essay-type' paper consisting of eight questions
- Paper 031– a School-Based Assessment (SBA) project component for candidates in schools
- Paper 032 – an alternative to the SBA for out-of-school candidates.

Questions in the examination targeted the content and specific objectives of the Additional Mathematics syllabus (CXC 37/G/SYLL 10) with the aim of ensuring appropriate breadth and depth of syllabus coverage.

Paper 01 tests content from Sections 1, 2 and 3 of the syllabus and consists of 45 items. Paper 02 tests content from all four sections of the syllabus. This year the paper consisted of four sections and each section contained two problem-solving type questions. The questions in Sections 1, 2 and 3 were all compulsory. The two questions in these sections were worth a total of 28, 24 and 28 marks respectively. Section 4 also contained two questions, one on Data Representation and Probability and the other on Kinematics, each worth 20 marks. Candidates were required to choose one question from this section. Paper 031 represents the SBA component of the examination. Candidates can do a project chosen from two project types, a mathematical modelling project (Project A) and a data handling/statistical analysis project (Project B). The SBA component is worth 20 marks. Alternatively, private candidates can sit an alternative paper to the SBA, Paper 032, which consists of an in-depth, extended question from Sections 1, 2 and/or 3 of the syllabus. This paper carries 20 marks.

This year saw a 69 per cent increase in candidates registered for the examination, up from 1720 candidates in 2012 to 3100 in 2013.

This year, a formula sheet was included as part of the question paper on Papers 01 and 02.

DETAILED COMMENTS

Paper 01 – Multiple Choice

This was a 45-item paper covering Sections 1, 2 and 3 of the syllabus. The mean score on this paper was 35.25, with standard deviation of 13.31, compared with 30.96 and 11.69 in 2012.

Paper 02 – Structured Essay Question

This paper consisted of 8 questions, of which questions 1 – 6 were compulsory. Candidates had to choose either Question 7 or Question 8. The total possible score on this paper was 100 marks. The mean and standard deviation for this paper were 47.82 and 27.2 respectively, compared to 37.32 and 22.95 in 2012.

Section 1: Algebra and Functions

Question 1

This question tested candidates' ability to:

- make use of the Factor Theorem and factorize a polynomial of degree 3, obtaining all its linear factors;
- determine the inverse of a given function and the composite of two functions;
- use logarithms to solve equations of the form $a^x = b$; simplify expressions using Laws of Logarithms.

There were approximately 2900 responses to this question. The mean mark was 8.02 and the standard deviation was 3.99. Three hundred and forty-four candidates obtained full marks (14 marks).

Candidates performed best on Parts (a) and (b) (ii), with many candidates earning full marks. Part (a) required candidates to show that a given linear expression was a factor of a given polynomial and then go on to find the other linear factors of the polynomial. Most candidates were able to do so either by using the Factor Theorem or via the longer process of long division. More often than not, candidates were then able to successfully factorize the polynomial expression completely. However, the quadratic factor $x^2 - 5x + 6$ was commonly incorrectly factorized as $(x - 1)(x - 6)$. In Part (b) (ii) candidates were required to find an expression for the composite of two functions. Generally, candidates were able to make the correct substitution, although some then made errors in the simplification process. Others still, having correctly simplified and obtained the correct result of $\frac{2x+1}{x+3}$, then proceeded to erroneously cancel out the x in both the numerator and the denominator.

Part (b) (i) and Part (c) presented the greatest difficulty to candidates. In Part (b) (i), which required candidates to obtain an expression for an inverse function, some candidates, having interchanged the variables x and y, experienced difficulties in making y the subject of the formula. Additionally, some candidates did not write their final obtained expression in terms of $f^{-1}(x)$, and simply left it in terms of y. In Part (c), although most candidates knew they needed to take logarithms, they made many errors in applying the mechanics of this to the given problem. For example, in bringing down the indices they did not use brackets, and so made errors in the expansion of the expressions obtained. A common error seen was:

$$\lg[5^{3x-2}] = \lg[7^{x+2}]$$
$$3x - 2\lg 5 = x + 2\lg 7$$
$$2x = 2\lg 7 + 2\lg 5.$$

With respect to candidates' responses on this question, there is consolidation needed in the following areas:

- The correct transposition of equations containing more than one variable. For example, transpose for y, $xy + 2x = 2y - 1$.
- Students must be alert to the fact that when finding $f^{-1}(x)$, the final result must contain $f^{-1}(x)$ equal to some expression in x.
- Students **must not** consider as equivalent, the *factors of a polynomial expression* and the *roots of a polynomial equation*.
- The correct application of the laws of logarithms. Examples of common misconceptions presented were: $(\log A)(\log B) = \log A + \log B$; $\log\left(\frac{A}{B}\right) = \frac{\log A}{\log B}$

Solutions: (a) (ii) $(x-3), (x-2)$

(b) (i) $f^{-1}(x) = \dfrac{2x+1}{2-x}$ (ii) $fg(x) = \dfrac{2x+1}{x+3}$

Question 2

This question tested candidates' ability to:

- express a quadratic function in the form a(x + h) + k where a, h and k are constants; and determine the function's minimum value;
- find the solution set of a quadratic inequality;
- identify geometric series and calculate the sum to infinity of geometric series.

There were 2959 responses to this question. The mean mark was 6.93 and the standard deviation was 3.59. Ninety-one candidates obtained full marks (14 marks).

Candidates performed best on Part (b) of this question, which required them to find the solution set of a quadratic inequality. Most candidates correctly factorized the quadratic and were able to identify the critical values as $\frac{-5}{2}$ and 1. However, correctly *stating* the solution to satisfy the inequality proved to be a challenge for a number of candidates, even in cases where they were able to correctly identify the regions which satisfied the inequality in a sketch.

Parts (a) and (c) presented some difficulty to candidates. In Part (a), candidates were required to write a given quadratic expression in the form $a(x + h)^2 + k$. Approximately 50 per cent of the candidates attempted this question by the method of completing the square, with some candidates encountering difficulties. Some of the candidates used the strategy involving the identity to find the solution as follows:

$$3x^2 + 6x - 1 \equiv a(x + h)^2 + k \rightarrow 3(x + 1)^2 - 4 \quad \text{where h} = 1 \text{ and k} = -4$$

Deductions: (i) Minimum value of function = –4 (ii) Value of x at minimum = –1

A few candidates attempted to solve for roots of function and abandoned the solution without completing the square.

In Part (c) most candidates were unable to identify two series from the given series, and generally the strategy of separating a given series into two appeared to be unknown to most candidates. Most candidates who attempted the question ignored the note given at the end of the question, and proceeded to find the sum to infinity as if for one series. Very few candidates attempted to separate the series into two GPs. There were also a few candidates who did not seem to know which formula to use to find the sum to infinity, and rather found the sum of the first two terms.

Solutions: (a) (i) $3(x + 1)^2 - 4$

(ii) Minimum value $f(x) = -4$

(iii) Value of x at minimum $= -1$

(b) Solution set: $\{x: x \leq \frac{-5}{2}\} \cup \{x: x \geq 1\}$

(c) $\frac{2}{5}$

Section 2: Coordinate Geometry, Vectors and Trigonometry

Question 3

This question tested candidates' ability to:

- write the equation of a circle, find the radius of a given circle; and find the equation of a tangent to a circle at a given point;
- apply properties of perpendicular vectors;
- derive a unit vector.

Candidates performed best on Part (a) (i), and Parts (b) and (c) of this question. In Part (a) (i) many candidates were able to write the equation of the circle (without consideration of the radius), and responses of $x^2 + y^2 - 4x - 2y + K = 0$ or $(x - 2)^2 + (y - 1)^2 = r^2$ were often seen. In Part (a) (ii) candidates also knew how to find the gradient of the radius and the fact that the tangent is perpendicular to the radius (normal).

In Part (b), a significant number of the candidates either knew that the dot product of perpendicular vectors is zero ($\vec{OP} \,.\, \underset{\sim}{OQ} = 0$) or used the formula to find the angles between two vectors and substituted 90° for this angle. In Part (c), the majority of the candidates knew how to find $\vec{AB}$, either from $\overline{AB} = \overline{AO} + \overline{OB}$ or $\overline{AB} = -\overline{OA} + \overline{OB}$. However, they made mistakes in writing down the vectors or in changing the signs, and responses of $-2i - 5j + 3i - 7j = i - 12j$ or $2i + 5j + 3i - 7j = 5i - 2j$ were often seen. Candidates also knew how to find the modulus of a vector.

Candidates did have difficulty with part of Part (a) (i), specifically to determine r or K in the formula. Many did not seem to know how to do this, or that they needed to do this, and $(x - 2)^2 + (y - 1)^2 = 0$ was often seen. In Part (b), correctly computing an expression for the dot product proved to be a problem and $10\lambda + 80$ was often seen. Additionally, some candidates confused the concepts of perpendicularity of lines and perpendicularity of vectors, thus equating the dot product to -1 was seen. In Part (c) finding the unit vector proved to be problematic even in cases where the vector $\vec{AB}$ and its modulus were correctly found.

With respect to candidates' responses in this question, the following areas of consolidation are needed:

- Candidates should be taught to derive expressions rather than memorize them. The formula $\frac{dy}{dx} = \frac{-(g + x)}{f + y}$ was used by some candidates to find the gradient of the tangent; however some of them did not know that g in their expression was half of h from the given formula for the equation of a circle and f was half of g from the given equation of the circle.

- Candidates should also be encouraged to use techniques appropriate to the topic being tested; for example, in the coordinate geometry question some candidates tried to use calculus to determine the gradient of the tangent. While this technique was a legitimate one, only a few of them were able to correctly do it. The concept of the product of gradients being equal to −1 should be used for perpendicularity of lines in coordinate geometry, while for vectors, equating the dot product to zero should be the preferred method of establishing perpendicularity.

- When the coordinates of the centre of a circle and a point on the circumference are given, candidates should be encouraged to use the formula $(x - a)^2 + (y - b)^2 = r^2$, where (a, b) are the coordinates of the centre and r is the radius.

- The definition of a unit vector and how to determine a unit in the direction of a given vector should be emphasized.

Solutions: (a) (i) $x^2 + y^2 - 4x - 2y - 95 = 0$

(ii) Equation of the line l: $y - 3 = \dfrac{-4}{3}(x - 10)$

(b) $\lambda = 8$

(c) $\overrightarrow{AB} = \dfrac{1}{13}(5\text{i} - 12\text{j})$

Question 4

This question tested candidates' ability to:

- use the formulae for arc length and sector area of a circle;

- find solutions of simple trigonometric equations for a given range, which involve the use of $\cos^2\theta + \sin^2\theta \equiv 1$.

- use the compound angle formula for tan θ.

Candidates performed best on Part (b), that is, solving a given trigonometric equation. Candidates were able to state the necessary trigonometric identity and perform the required substitution, factorize the quadratic equation obtained, identify the range of the trigonometric function and so correctly solve the trigonometric equation for the given range. Some candidates, though, having found the principal angle/value, did have difficulty calculating the other angles that would fit the solution in the given range. Candidates readily drew:

S	A
T	C

but seemed unsure as to how and why this aid was developed. Teachers are urged to explore why each of the trigonometric ratios is positive or negative.

Part (a) presented the most difficulties for candidates. A number of candidates incorrectly stated the formula for the perimeter of a sector, most often only writing this in terms of the arc length and not including $2r$. Many then attempted to equate coefficients in finding a value for r instead of using an algebraic method, and so did not obtain a correct value for r, nor the sector's area, as asked for in the question. In Part (c), whilst most candidates were able to identify and use $\dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$ ≡ tan (A ± B), some candidates had difficulty interpreting that the ± and the ∓ signs, and so often wrote: tan (A – B) = $\dfrac{\tan A - \tan B}{1 - \tan A \tan B}$

Teachers are encouraged to have students learn the formulae and to not rely solely on the formula sheet.

Solutions: (a) Area = $\dfrac{25\pi}{12}\text{m}^2$

(b) $\theta = 210^{\circ}$ and 330°

(c) $\alpha = 45^{\circ}$

Section 3: Introductory Calculus

Question 5

This question tested candidates' ability to:

- differentiate a simple polynomial made up of x^n, use the concept of stationary points; calculate the second derivative of polynomials; interpret the significance of the sign of the second derivative; and use the sign of the second derivative to determine the nature of stationary points;

- differentiate sin x, differentiate products of simple polynomials and trigonometric functions; apply the chain rule in the differentiation of composite functions.

In this question candidates performed best on Part (a) (i) and well on Part (a) (ii). In Part (a) (i) the concept of stationary points appeared to be familiar to most candidates, and approximately 70 per cent were able to successfully identify the coordinates of the stationary points. Incorrect solutions were obtained by candidates who equated the function rather than the first derivative to zero to obtain the coordinates of the stationary points. Common errors in algebraic manipulations involved solving $3x(x - 2) = 0$, and writing $3x = 0$, $x = -3$. In Part (a) (ii) most candidates were able to correctly find the second derivative and use the x-values from Part (a) (i) to deduce the nature of the stationary points. However, a common error seen was equating the second derivative to 0 and solving for x in order to determine the nature of the stationary points.

Part (b) presented the most difficulty to candidates. It required them to simultaneously use the product and the function of a function rule in differentiating a combination trigonometric and polynomial expression. A number of candidates stopped at one term of the product rule and differentiation of the term in x in the brackets was not done. Additionally, many candidates incorrectly stated the derivative of sin x as –cos x. Finally, some candidates had problems simplifying the expression obtained, with a number of them attempting to expand the algebraic term as simplification of the result.

Solutions: (a) (i) Coordinates stationary points = (0, 2) and (2, –2)

(ii) $\dfrac{d^2y}{dx^2} = 6x - 6$; (0, 2) is a maximum point, (2, –2) is a minimum point

(b) $(5x + 3)^2\,[(5x + 3)\cos x + 15\sin x]$

Question 6

This question tested candidates' ability to:

- use the rules of simple integration;
- integrate simple trigonometric functions and compute definite integrals;
- formulate the equation of a curve, given its gradient function and points on the curve; find the area of a finite region in the first quadrant bounded by a curve and the x- and y-axes;

There were 2800 responses to this question. The mean mark was 6.16 and the standard deviation was 4.67. Three hundred and nineteen candidates obtained full marks (14 marks).

Part (a) presented the most difficulty to candidates. In Part (a) a number of candidates did not remember to include the arbitrary constant in integrating the given function. In Part (c) some candidates could not obtain the correct equation of the curve. Even among those who knew to integrate the gradient function to obtain this equation, some did not include a constant of integration and hence obtained an incorrect equation. Additionally, a minority had difficulty obtaining the limits between which they should integrate.

Solutions: (a) $\frac{5}{3}x^3 + 4x + C$

(b) -2

(c) Area $= 26\frac{2}{3}$ units^2

Section 4: Basic Mathematical Applications

Question 7

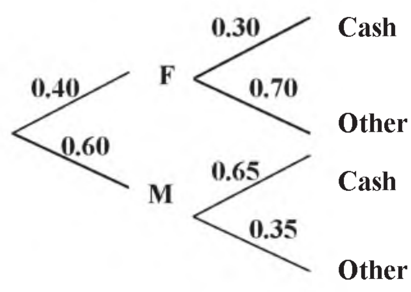
This question tested candidates' ability to:

- construct and use a tree diagram to solve problems involving probability; calculate conditional probability; calculate classical probability;
- state the relative advantage of a stem and leaf diagram versus a box and whisker plot to display data; construct a stem and leaf diagram; determine quartiles and measures of central tendency from stem and leaf diagram.

Candidates performed best on Part (a) (i) of this question, which required them to complete the construction of a tree diagram. Candidates knew that the sum of each branch equates to 1. However, having drawn the tree diagram, many candidates did not use it to answer Parts (a) (ii) and (iii). Part (a) (ii), on conditional probability, was badly done, and it appeared that most candidates did not even realise that the question required the use of conditional probability. In Part (a) (iii), most candidates were able to get the probability for Event V, and also to say which was the more likely event.

Parts (b) (i), (ii), (iv) and (v) presented the most difficulty to candidates. In Part (b) (i), which asked candidates to state an advantage of a stem and leaf diagram versus a box and whiskers plot to display the given data, a number of candidates stated their personal preference rather than an advantage, and wrote, for example, 'the stem and leaf is easier to draw and understand'. In Part (b) (ii), which required candidates to construct the stem and leaf diagram, a number of candidates drew tree diagrams. A number of candidates who did draw a stem and leaf diagram lost a mark for not including a key to the diagram. In Part (b) (iv), candidates did not know the formula for semi-interquartile range, or did not know how to work this out. In many cases they could not correctly find the lower and upper quartiles, and many did not know to divide this difference by two. In Part (b) (v), some candidates mistook the sample space for the marks. Many other candidates did not recognize the sample space being reduced, and so could not correctly give the probability of the second student scoring less than 50 marks on the exam.

Solutions: (a) (i)



(ii) P(Cash) = 0.51

(iii) Event T more likely

(b) (i) All data values are retained; shape of distribution easily seen

(ii)

Stem	**Leaf**
4	0 1 5
5	0 0 1 1 3 6 6 8 8
6	3 3 6 6 9
7	2 4 5 5 6
8	0 1 3 5 9
9	2 4 9

Key: 6 | 3 means 63

(iii) Median = 66

(iv) Semi-interquartile range = 13.5

(v) P (less than 50 marks) = $\frac{3}{30} \times \frac{2}{29} = \frac{3}{435}$

Question 8

This question tested candidates' ability to:

- draw, interpret and make use of velocity-time graphs;
- calculate and use velocity, acceleration and time in simple equations representing the motion of a particle in a straight line.

Candidates performed best on Parts (a) (i), (b) (i) and (iii) of this question, which required them to draw a velocity-time graph from given information; obtain from its velocity function, t-values when a particle is at instantaneous rest and obtaining values for $\frac{dv}{dt}$ at particular points in time.

Parts (a) (ii), (b) (ii) and (iv), however, presented the most difficulty to candidates. In Part (a) (ii), candidates were required to find the total distance travelled by the particle from their graph, as well as to find the average velocity of the particle. A few candidates did not use the graph and used instead the formula, distance = speed × time, which gave incorrect values. This formula will only give correct distances if the speed is constant. Other candidates had some problems correctly finding the area under the graph of all parts of the graph. A few candidates did not know how to calculate average velocity, that is, average velocity = $\frac{\text{distance}}{\text{time}}$, with some simply dividing their total distance by 2.

In Part (b) (ii), which required candidates to obtain the distance travelled by the particle between $t = 1$ and $t = 3$ by integrating v to obtain displacement. Whilst a number of candidates correctly obtained that at t = 1, displacement = 7, and at t = 3, displacement = −9, about one-half of them did not know that the distance between t = 1 and t = 3 is 16, as the displacement of −9 meant 9m (or whatever unit of length) to the other side of 0. They experienced difficulty in interpreting distance from the calculated displacement values. In Part (b) (iv), the interpretation of the $\frac{dv}{dt}$ values presented difficulty to about one-half of the candidates, with many, for example, believing that $\frac{dv}{dt} = 0$ implied that the particle was at rest.

Solutions: (a) (ii) a) 900 m

b) average velocity = 25.7 m s^{-1}

(b) (i) $t = 1$ or t = 5

(ii) distance travelled = 16 m (units)

(iii) a) at $t = 2$, $\frac{dv}{dt} = -6$

b) at $t = 3$, $\frac{dv}{dt} = 0$

(iv) a) at $t = 2$, particle is decelerating

b) at $t = 3$, acceleration is 0, or constant velocity

Paper 031 – School Based Assessment (SBA)

Many of the projects submitted were of a high quality and generally related to real-world situations. Many of the submissions received full marks.

The following were general observations on both Projects A and B:

- Some SBA project titles and aims/purpose were not clear, not concise and not well defined. Some titles were too vague, others too wordy.
- In far too many cases, candidates simply re-stated the project's title as its aim/purpose. This often meant that the reason for the project was not stated.
- In a number of cases, the project's aim/purpose did not relate to the given title and sometimes was not linked to any syllabus objective;
- Some submissions did not come with the required CSEC Additional Mathematics rubric;

- Some teachers created their own rubric or made adjustments to the CSEC Additional Mathematics rubric. This is unacceptable, as the project then has to be re-marked using the appropriate CSEC Additional Mathematics rubric;

- A few project submissions appeared to have been downloaded from the Internet;

- There were too many instances of incorrect spelling and grammar. However, candidates used the correct mathematical jargon and symbols appropriately most of the time;

- Some projects were incorrectly categorized and assessed, that is, a Project A being mislabelled and assessed as a Project B, and vice versa. This did create some problems as, for example, Project B requires the collection and analysis of data from an experimental-type activity.

Specifically for Project A:

Mathematical Formulation

- In stating how the project was going to be done generally most candidates correctly identified all the important elements of the problem and showed understanding of the relationship between the elements.

- The content used in some cases was up only to a Form 3 (or Grade 8) level Mathematics, for example some projects had only simple calculations with area and volume.

The Problem Solution

- Assumptions were often not clearly stated. Some candidates, though, did state limitations of the project.

- Explanations were generally not sufficient and also were not placed between every successive step in the problem solution.

- Calculations were often precise but solutions were not clearly stated.

Application of Solution

- Many candidates were unaware of how to show that the solution or proof of the given problem was valid.

Discussion of Findings/Conclusion

- In some instances the discussion was worthwhile. However, the discussion was not always related to the project's purpose and was often quite broad.

- Conclusions in many instances were not found to be valid. Some candidates stated many conclusions which were not related to the purpose of the project.

- There were only few instances where suggestions for future analyses were stated.

Specifically for Project B:

Method of Data Collection

- Although candidates indicated what they did, some were not specific enough about the type of data collected and method of sampling.

Presentation of Data

- Candidates had the minimum requirement of one table or chart. However, some tables and charts were not properly named or labeled.

- There was not a systematic layout or organization of the tables and charts.

Mathematical Knowledge (analysis of data)

- Mathematical concepts in many cases were not used appropriately from the Additional Mathematics syllabus.

- In many cases candidates used concepts and representations up to just Form 2 level Mathematics; for example, some candidates only used pie charts and bar charts in their representation of the data.

- Most of the calculations shown were accurate. However, in some cases no working was seen because some candidates used software to generate the values.

- Generally some analysis was attempted, but the analysis was often not coherent. This could be attributed to the fact that no proper summary table of calculation values was seen, so the markers had to search through the document constantly to link the calculated values with the analysis.

- The candidates were not too clear on two approaches to be used in the analysis. In many cases the two approaches used were of the same concepts; for example, mean, mode and median were presented as different approaches to the analysis, but all of these are measures of central tendency.

Discussion of Findings/Conclusions

- In most instances there was no statement of findings.

- Conclusions made were based on the reported findings but often were not related to the purpose of the project. As a result their conclusions were invalid.

- In most cases there were no suggestions for future analysis.

Recommendations

The following recommendations hold for continued improvement in this aspect of the Additional Mathematics examinations:

- Teachers are reminded not to use projects taken directly from the specimen papers or from the exemplar projects given in the Additional Mathematics syllabus.

- In the cases where group work is done, candidates are reminded to submit individual reports.

- All projects should have a clear and concise title, and well-defined aim(s) or purpose;

- Where possible the SBA should be related to authentic situations;

- The variables that are being used or measured (Project B) must be clearly stated and described. Variables can be controlled, manipulated and responding;
- The type of sample and sample size, if relevant, must be clearly stated;
- Teachers must ensure that projects which integrate other subject areas utilize concepts as contained in the CSEC Additional Mathematics syllabus;
- If candidates collect their data in a group setting, candidates **must** demonstrate their **individual** effort in relation to analysis (interpretation) and finding(s)/conclusion(s);
- Projects involving dice or playing cards must be more expansive so that candidates can present a more in-depth analysis of the topic under consideration, for example, this could include multiple and non-standard dice or multiple decks of cards;
- As good practice, candidates should be encouraged to cite all sources and insert a reference/bibliography page;
- Teachers should guide students using the assessment criteria found in forms 'Add Math 1 – 5A' and 'Add Math 1 – 5B' which are both available on the CXC website. Teachers can give their students the rubric as a means of guidance in developing their projects;
- Overall, it is clear that training workshops are needed in the area of project work for Additional Mathematics. Therefore, it is recommended that workshops be held throughout the Caribbean. Teachers need to be aware that there are TWO RUBRIC SHEETS 1-5A for project A (the mathematical modelling project) and 1-5B (the statistical project), and to use the appropriate sheet in marking students' projects. It was apparent that some teachers found difficulties in understanding some aspects of the rubric, or were unaware of the existence of the rubric. Teachers need to ensure that the content level in the project coincides with the Additional Mathematics syllabus.

Paper 03/2 – Alternative to the School Based Assessment (SBA)

This paper tested candidates' ability to:

- write the equation of a circle; find the points of intersection of a curve with a straight line;
- use the concept of stationary points; differentiate simple polynomials and quotients of simple polynomials.

In Part (a), use of the formula for a circle whose centre was the origin was generally known. Part (b) (i), which required candidates to find an expression for the height of a cuboid given its length, width and volume, as well as Part (b) (ii), which required candidates to write an expression for the total surface area of the cuboid, appeared to be widely known. The formula for total surface area, $2\times(l\times w + l\times h + w\times h)$ was also well known.

In Part (a), candidates had difficulty relating the sides of the square to their equations $x = \pm 5$ and $y = \pm 5$ and hence did not know they could use simultaneous equations to obtain the other coordinates. In Part (b), candidates for the most part did not know that the stationary point occurred when $\frac{dA}{dx} = 0$ and many of them could not differentiate the expression $\frac{2160}{x}$.

Teachers need to emphasize that a stationary point occurs when the gradient function, $\frac{dy}{dx} = 0$. Teachers also need to do more examples on differentiating functions of the form $\frac{a}{x^n}$, where a is a constant and n a positive integer.

Solutions : (a) $(5, \sqrt{11})$; $(5, -\sqrt{11})$; $(-5, \sqrt{11})$; $(-5, -\sqrt{11})$;

$(\sqrt{11}), 5)$; $(\sqrt{11}), -5)$; $(-\sqrt{11}), 5)$; $(-\sqrt{11}), -5)$

(b) (i) $h = \frac{360}{x^2}$

Generally, teachers need to ensure that the syllabus is completed with sufficient time to give candidates time to do practice exercise and past papers.

C A R I B B E A N E X A M I N A T I O N S C O U N C I L

REPORT ON CANDIDATES' WORK IN THE

CARIBBEAN SECONDARY EDUCATION CERTIFICATE® EXAMINATION

MAY/JUNE 2014

ADDITIONAL MATHEMATICS

GENERAL PROFICIENCY EXAMINATION

GENERAL COMMENTS

Additional Mathematics was tested for the third time in the May/June 2014 examinations. The subject is intended to bridge a perceived gap between the CSEC General Mathematics and the CAPE Unit l Pure Mathematics.

The intent is to offer CSEC Mathematics students a more seamless transition to the advanced thinking and skills needed for CAPE Unit 1 Mathematics courses, although a student who masters the content and skills set out in the CSEC Mathematics syllabus should be ready to meet the skills and thinking demands of the CAPE Unit 1 Mathematics course.

The examinations consist of four papers:

- Paper 01 – a 45-item multiple choice paper
- Paper 02 – a structured, 'essay-type' paper consisting of eight questions
- Paper 031– a School-Based Assessment (SBA) project component for candidates in schools
- Paper 032 – an alternative to the SBA for out-of-school candidates.

Questions in the examination targeted the content and specific objectives of the Additional Mathematics syllabus (CXC 37/G/SYLL 10) with the aim of ensuring a wide coverage of the syllabus. The questions were designed at the appropriate level to test the following skills: Conceptual Knowledge, Algorithmic Knowledge and Reasoning.

Paper 01 tested content from Sections 1, 2 and 3 of the syllabus and consisted of 45 items. Paper 02 tested content from all four sections of the syllabus. This year the paper consisted of four sections and each section contained two problem-solving questions. The questions in Section 1, 2 and 3 were all compulsory, and the questions in Section 1 were worth 14 marks each, those in Section 2 were worth 12 marks each and those in Section 3, 14 marks each. Section 4 contained two questions, one on Data Representation and Probability and the other on Kinematics and each question was worth 20 marks. Candidates were required to answer only one question from this section. Paper 031 is the SBA component of the examination. Candidates are required to complete one project chosen from two project types; a mathematical modelling project (Project A) and a data handling/statistical analysis project (Project B). The SBA component is worth 20 marks. Private candidates can sit an alternative paper to the SBA, Paper 032, which consists of one in-depth, extended question from Sections 1, 2 and /or 3 of the syllabus. This paper is worth 20 marks.

This year saw a 16 per cent increase in candidates registered for the examinations, up from 3100 candidates in 2013 to 3597 in 2014.

Again this year, a formula sheet was included as part of the question paper on Papers 01, 02 and 03/2.

DETAILED COMMENTS

Paper 01 – Multiple Choice

This was a 45 item paper covering Sections 1, 2 and 3 of the syllabus. The mean score on this paper was 39.41, with standard deviation of 12.71 respectively, compared to 35.25 and 13.31 in 2013.

Paper 02 – Structured Essay Questions

This paper consisted of eight questions, of which Questions 1 to 6 were compulsory. In addition, candidates had to answer either Question 7 or Question 8. The total possible score on this paper was 100 marks. The mean and standard deviation for this paper were 46.32 and 26.01 respectively, compared to 47.82 and 27.27 in 2013.