Section 1: Algebra and Functions

Question 1

This question tested candidates' ability to:

- determine whether a given function is one-to-one; find the composite of two functions and determine its domain; determine the inverse of a given function and sketch the given graph and its inverse on the same pair of axes;
- apply the Remainder Theorem to evaluate an unknown coefficient;
- convert a real-world worded problem to obtain a quadratic equation and solve it in order to find the dimensions of a room.

There were 3 462 responses to this question. The mean mark was 7.50 with standard deviation 3.95. Approximately 5 per cent of candidates obtained full marks.

Candidates performed best in Parts (b) and (c) of this question, with many candidates earning full marks for these parts of the question.

Part (b) required candidates to evaluate the unknown coefficient of x. Most candidates successfully applied the Remainder Theorem to find this coefficient. Generally candidates were able to make the correct substitution and equate this to the remainder; however some candidates made errors while simplifying the expression.

In Part (c) candidates were required to formulate equations representing the length and width of the room. Most candidates then solved these equations simultaneously to find the required values. However, some candidates used trial and error to arrive at their answers, which at times were correct. Generally many candidates recognised that the positive solution from the quadratic equation was the correct value to be used to find the required length; however some used both positive and negative values to obtain the length without indicating which pair was valid.

Part (a) (i) and (a) (ii) presented the greatest difficulty to candidates.

Part (a) (i) required candidates to show that a given function was NOT one-to-one. Few candidates were able to show that the function was not one-to-one although some simplified the function in a cubic expression correctly using the difference of squares method.

Candidates who obtained full marks were able to do so by either showing that $f(–a) = f(a)$ for $a \in R$ or making a sketch of the function and applying the horizontal line test. Most candidates, when doing Part (a) (ii) a), were able to correctly find the value of the composite function. However, they were unable to state the domain and when they did, it was done incorrectly.

Part (a) (ii) b) also presented some difficulty to candidates. The inverse of the function was at times incorrectly stated; for example, it was often represented as $2x + 3$. Many candidates also did not even attempt to sketch the two graphs $g(x)$ and their inverse in this part of the question, even though they performed well in other parts of the question.

With respect to candidates' responses on this question, consolidation is needed in the following areas:

- More graph work and analysis of graphs need to be done. Students should be given extensive practice in drawing graphs or, in cases where a sketch is required, in representing the data in an efficient and effective way. They also need to differentiate between a quadratic graph and a linear graph.

- Students must not consider as equivalent, the quotient of a polynomial expression and the remainder of a polynomial expression as it applies to the Remainder Theorem.

- Students should be shown the importance of applying mathematical solutions to real-world situations. For example, they correctly found the length and width of the room but did not state these values to represent such. This occurrence had little impact here since once the correct values were seen they were credited. However, if these values were to be used for further analysis of the question some difficulty might have arisen.

Solutions are: (a) (i) f(-a) = f(a) thus f is many- to- one

(ii) a) $-\frac{1}{4}x^2 + 3x - 8 \quad x \in \Re$

b) $g^{-1}(x) = 2(x+3) \quad x \in \Re$

(b) a =3

(c) width x = 6 cm and length y = 18 cm

Question 2

This question tested candidates' ability to:

- express a quadratic function *f(x)* in the form $k + a(x+h)^2$ where *a, h* and *k* are constants; determine the function's maximum value and the value of x for which the maximum occurred;
- find the solution set of a quadratic inequality;
- identify a geometric series and calculate the sum to infinity of the geometric series.

There were 3 522 responses to this question. The mean mark was 8.19 with standard deviation 4.27.

Candidates performed best in Part (c) of this question. However approximately 20 per cent of the candidates incorrectly used the common ratio as 10. To show that the series was geometric approximately 25 per cent of the candidates used the first two terms only. A few even treated the series as an arithmetic progression and obtained a common difference of 0.18 between the first two terms.

Parts (a) and (b) were equally challenging to the candidates. In Part (a) (i) candidates demonstrated difficulty in factorising from the given function and in completing the square correctly. Most of the candidates used the alternative approach in which they recalled expressions for the required h and k based on their previous knowledge. Some candidates made errors in recalling these expressions and the majority of candidates who were able to present a correct expression made errors in computing the numerical values of h and k. In Part (a) (ii) most candidates were able to deduce their correct maximum value however some candidates provided a minimum value of $f(x)$ although the question was clearly stated. Some candidates provided the coordinates of the maximum point instead of providing the function's maximum value. In Part (a) (iii) some candidates did not see the connection of this question to their response in Part (a) (i).

Some of the candidates could not obtain the correct set of values of x although they correctly factorised and determined the correct critical values. A few candidates failed to change the sign of the inequality when they multiplied by −1. For the most part, however, the attempt of factorisation was well done.

This question was generally well done by the majority of candidates.

Solutions are: (a) (i) $9 - 2(x+3)^2$

(ii) Maximum value $f(x) = 9$

(iii) Value of x at maximum = −3

(b) Solution set: $\{x: x \le \frac{-1}{2}\} \cup \{x: x \ge 3\}$

(c) (i) common ratio of $\frac{1}{10}$

(ii) $\frac{2}{9}$

Section 2: Coordinate Geometry, Vectors and Trigonometry

Question 3

This question tested candidates' ability to:

- apply the perpendicular property for two lines; determine the equation of a circle, given its radius, and whose centre is the point of intersection of two given lines;
- apply properties of perpendicular vectors and determine the magnitude of a displacement vector.

Candidates demonstrated good recall of knowledge of Coordinate Geometry and vectors.

In Part (a) (i) candidates correctly expressed the condition for two straight lines perpendicular to each other as $m_1 \times m_2 = -1$, $m_2 = -1/m_1$ or the equivalent in words.

In Part (a) (ii) candidates correctly recalled the equation of the circle in the form $(x - a)^2 + (y - b)^2 = r^2$ where (a, b) are the coordinates of the centre of the circle and r is the radius. This form of the circle equation was most suitable for the solution required. Some candidates correctly recalled the expanded form $x^2 + y^2 - 2fx - 2gy + c = 0$.

In Part (b) (i), candidates recognised that the dot product $\overrightarrow{RT} \bullet \overrightarrow{RS} = 0$ was the condition required for proving that $<TRS = 90^0$. Other conditions presented in candidates' responses were for angle $RTS = 90^0$, cos $<RTS = \underline{\overrightarrow{RT}.}\ \underline{\overrightarrow{RS}} = 0$, requiring knowledge of the length of a vector.

$|\overrightarrow{RT}.||\overrightarrow{RS}|$

- The product gradient RT × gradient RS = –1
- Graphical method
- An unknown method involving ratios

In Part (b) (ii) candidates correctly recalled Pythagoras' Theorem to determine the length of the hypotenuse. Candidates faced challenges throughout their responses in terms of correctly applying the content to data.

In Part (a) (i) candidates were unable to manipulate the two equations, given in the form $-x + 3y = 6$ and $kx + 2y = 12$, to obtain the gradients. Only a few candidates correctly identified $-k/2$ as one of the gradients. As a consequence candidates were unable to formulate the equation $-1/3 \times -k/2 = -1$, resulting in incorrect values for k.

The layout of both the given equations, $x + 3y = 6$ and $kx + 2y = 12$ prompted candidates to obtain the value of k by solving the equations simultaneously, not recognising that two equations containing three unknowns cannot be solved. This attempt by candidates was abandoned at some point in the procedure.

The follow through allowed in Part (a) (ii) for the incorrect value of k made it possible for candidates to be awarded both marks for solving the simultaneous equations to obtain the coordinates of the centre of the circle. Some candidates wrote the equation of the circle incorrectly due to the non-reversal of signs when substituting the coordinates of the centre of the circle in the correct equation of the circle.

The use of the word "show" in the stem of Part (b) (i) led to the use of a variety of strategies in the proof.

In both Parts (b) (i) and (b) (ii) incorrect selection of vectors for substitution in the dot product or the formula for the length of a straight line was frequently seen. Position vectors were commonly seen rather than the correct translation vectors. Incorrect terminology in writing vectors was also seen e.g. $\overrightarrow{RT} \bullet \overrightarrow{RS}$ was written $R \times S$).

Consolidation in Coordinate Geometry is required in the area of expressing equations of straight lines given the form $ax + by = c$ to the form $y = mx + c$ in order to better facilitate the identification of the gradient. Competency in developing proofs using a vector method also needs to be emphasized.

Solutions are: (a) (i) $k = -6$

(ii) Equation of the circle $(x+\frac{6}{5})^2 + (y-\frac{12}{5})^2 = 5^2$

(b) (i) RT.RS = 0 for angle TRS $= 90^0$

(ii) $|\overline{ST}| = \sqrt{26}$

<u>Question 4</u>

This question tested candidates' ability to:

- determine the area of a sector of a circle and find the area of a shaded region using the area of the sector and that of a triangle;
- use the compound angle formula for $\cos(x + \frac{\pi}{6})$
- prove a trigonometric identity.

Candidates performed best on Part (b), which required proving the compound angle formula. Candidates were able to apply the formula given in the formula sheet and correctly substitute the values given. Candidates were also successful in factorizing the $\frac{1}{2}$ to give the result. Some candidates, though, used both sides of the proof to attain the answer.

Teachers are urged to show the students the method of substituting correctly and how to use either the left hand side or the right hand side of the equation to achieve the proof.

Part (a) presented the most difficulty for candidates. A number of candidates incorrectly calculated the area of the triangle and subsequently calculated an incorrect area of the shaded region, H. However, most candidates knew that the area of the shaded region, *H*, was calculated by subtracting the area of the sector AOB from the area of the triangle OAC. Most candidates also attained full marks for Part (i) in finding the area of the sector. Teachers are encouraged to have students understand the steps needed to complete questions of this nature, that is, the correct use of the formula Area $= \frac{1}{2}ab\sin C$.

In Part (c), most candidates received marks for substituting correctly for $\tan\theta = \frac{sin\theta}{cos\theta}$ and $\sin^2\theta = 1 - \cos^2\theta$. However, simplification after the substitution, to get the required result $1 + \frac{1}{cos\theta}$, proved to be most difficult. Candidates did not recognize the difference of squares $1 - \cos^2\theta = (1 + \cos\theta)(1 - \cos\theta)$ and, as a result, lost marks at that part of the proof.

Teachers are encouraged to reinforce the use of the difference of squares examples including $1 - \cos^2\theta$ and $1 - \sin^2\theta$.

Solutions are: (a) (i) Area of sector = 28.35 cm^2

(ii) Area of shaded region = 5.76 cm^2

Section 3: Introductory Calculus

<u>Question 5</u>

This question tested candidates' ability to:

- determine the equations of the tangent to a curve;
- use the concept of stationary points; locate the maximum and minimum stationary points, by considering the sign changes of the derivative; calculate the second derivative of polynomials; and use the sign of the second derivative to determine the nature of stationary points.

Candidates' demonstration of the understanding of the Calculus and Coordinate Geometry content in this section of the syllabus was commendable. Candidates recognised that in Part (a)

- the first derivative would give rise to an expression for the gradient of the tangent, $-2x + 4$.
- substituting the x-coordinate of the given point in $-2x + 4$ would give the value of gradient of the tangent.
- the general linear equation $y = mx + c$ or $y - y_1 = m(x - x_1)$ would give rise to the equation of the tangent of the curve.

For Part (b) (ii)

- differentiating the function $f(x) = 2x^3 - 9x^2 - 24x + 7$ to obtain the first derivative $f'(x) = 6x^2 - 18x - 24$ and equating the resulting quadratic equation to zero would lead to solutions which are the stationary points of the curve.
- further differentiating the first derivative to obtain the second derivative $f''(x) = 12x - 18$ would enable them to determine the nature of EACH of the stationary points.

In Part (a) the candidates showed competence in correctly obtaining the first derivative $dy/dx = \mathbf{4 - 2x}$ to represent the gradient of the given quadratic curve $y = 3 + 4x - 3x^2$ and substituting $x = 3$ from the given point to obtain $m = -2$.

In Part (b) (i) candidates solved the quadratic equation $6x^2 - 18x - 24 = 0$ using the methods of the factorisation, completing the square and use of the quadratic formula.

In Part (b) (ii) candidates were able to obtain the second derivative $f''(x) = 12x - 18$. The follow through x values from Part (b) (i) was an added advantage to candidates who were able to substitute in $f''(x) = 12x - 18$ to differentiate between the maximum and minimum stationary points of the function. Many candidates obtained full marks in this part of the question.

Other strategies used in candidates' responses to differentiate between the maximum and minimum stationary points were as follows:

- Sketching the graph of $f(x)$
- Examining the coefficient of x^3 to determine the shape of the graph and the direction of the stationary points.

Areas of ambiguity in Part (a) were

- whether the equation of the tangent to the curve or the normal was required. Some candidates converted the gradient m = −2 to m = $\frac{1}{2}$ in order to substitute into general equation thus finding the equation of the normal to the curve at P (3, 6) instead of the equation of the tangent.

- the use of the given point on the curve P (3, 6). Some candidates used the gradient expression to formulate the equation $-2x + 4 = 0$ to find a new value of $x = 2$ for substituting in the gradient expression. Others used the given function to formulate a quadratic equation in order to find x values for substituting in the gradient expression.

- multiplying through the curve $y = 3 + 4x - x^2$ by –1 before differentiating. This gave rise to the incorrect gradient expression $2x + 4$.

Weakness in basic algebraic manipulations was evident in this part of the question, including

- transposition of terms in an equation
- factorisation of quadratic expressions.

Transposing the equation of the tangent obtained to the required format posed a challenge to candidates who often expressed their equation in the form $y = mx + c$. Attempts at transposing the equation showed incorrect signs associated with the coefficients of x and y.

In Part (b) (i), factorising the quadratic equation $6x^2 - 18x - 24 = 0$ was a challenge to many candidates. Incorrect factors were obtained. Use of the quadratic formula and completion of the square method were often unsuccessful and abandoned before arriving at solutions to the equation.

Some candidates who obtained the correct x-coordinates did not proceed to calculate the corresponding y–coordinates to obtain ALL the stationary points.

With respect to candidates' responses in this question, there was good demonstration of understanding of the content areas throughout the question. Deficiencies in algebraic manipulations for weaker candidates can be a drawback to these candidates' performance. These deficiencies can be eliminated by consistent and appropriate worksheets with specific focus on developing skills in algebraic manipulations with accompanying effective feedback to candidates.

Solutions are: (a) Equation of the tangent to the curve at (3, 6) is $2x + y - 12 = 0$.

(b) (i) Stationary points are (–1, 20) and (4, –105)

(ii) (–1, 20) maximum point, (4, –105) minimum

<u>Question 6</u>

This question tested candidates' ability to:

- evaluate a definite integral;
- compute a definite trigonometric integral leaving the answer in surd;
- formulate the equation of a curve given its gradient function and points on the curve; find the area of a finite region in the first quadrant bounded by a curve and the x-axis and the lines $x = 3$ and $x = 4$.

Candidates performed best on Part (b), except for those candidates that did not know the surd form for $\cos\frac{\pi}{3}\, and \sin\frac{\pi}{3}$. Part (c) (i) was fairly well done; however, there were many candidates who were unable to correctly calculate the value of the constant of integration, c, despite the fact that they were able to correctly integrate the function and substituted the point (2, –5). In Part (c) (i), some candidates could not obtain the correct equation of the curve. Even among those who knew to integrate the gradient function to obtain this equation, some did not include a constant of integration and hence obtained an incorrect equation.

Part (a) and Part (c) (ii) presented the most difficulty to candidates. In Part (a), many candidates used the incorrect formula. "the integral of a product is the product of the integrals". Others did not integrate at all, but simply substituted the limits in the function.

In Part (c) (ii), generally candidates knew that area is the integral of a function between limits. However, many used the function as $y = 6x^2 - 1$, which was actually the given gradient function, $\frac{dy}{dx} = 6x^2 - 1$, instead of the equation of the curve obtained in Part (c) (i).

Solutions are: (a) 48

(b) $2\sqrt{3}+1$

(c) (i) $2x^3 - x - 19$ Area = 65 units^2

Section 4: Basic Mathematical Applications

Question 7

This question tested candidates' ability to:

- use the formula $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ to calculate probabilities;
- construct and use possibility space diagrams to solve problems involving probability;
- use the given stem and leaf diagram to determine quartiles and the median and to construct a box- and-whisker plot.

Candidates performed best on Part (b) (i), which required them to complete the possibility space diagram, and they also able to interpret the correct sample space of 36. For Part (a), most candidates used previous knowledge of sets and Venn diagrams to evaluate the correct probabilities.

Candidates performed well on the interpretation of the stem and leaf diagram to calculate the median and quartiles for the data given, recognizing that the median was the 26th observation, 71. The majority of the candidates also knew to construct a box-and-whisker plot to illustrate the data given. However, the comment on the skewness of the distribution proved to be very difficult. Candidates opted to omit the answer of negatively skewed and some did not provide a mathematical comment on the plot.

Teachers are advised to stress on the skewness of a box-and-whisker plot to describe the distribution of the data. The box-and-whisker plot should be accurately drawn on graph paper as points were deducted for incorrect scale.

Part (b) (iii) was also difficult in that most candidates who attempted to calculate the difference between the two dice forgot to multiply by 2 since the answers could be reversed. Very few candidates used a possibility space diagram to calculate the probabilities, but instead listed the possible combinations.

Generally, candidates displayed knowledge on the Statistics option but made some simple mistakes. For example, candidates need to know that probabilities are between 0 and 1 inclusive and that any answer above 1 is incorrect.

Candidates performed best on Part (a) (i) of this question, which required them to complete the construction of a tree diagram. Candidates knew that the sum of each branch equated to 1. However, having drawn the tree diagram, many candidates did not use it to answer Parts (a) (ii) and (iii). Part (a) (ii), on the conditional probability, was badly done, and it appeared that most candidates did not even know that the question required the use of conditional probability. In Part (a) (iii), most candidates were able to get the probability for Event V, and also to say which was the more likely event.

Parts (b) (i), (ii), (iv) and (v) presented the most difficulty to candidates. In Part (b) (i), which asked candidates to state an advantage of a stem and leaf diagram versus a box and whiskers plot to display the given data, a number of candidates stated their personal preference rather than an advantage, for example, ‘the stem and leaf is easier to draw and understand’. In Part (b) (ii), which required candidates to construct the actual stem and leaf diagram, a number of candidates drew actual tree diagrams. For candidates who did draw a stem and leaf diagram, a number of them lost a mark for not including a key to the diagram. In Part (b) (iv) candidates did not know the formula for semi-interquartile range, or did not know how to work this out. In many cases they could not correctly find the lower and upper quartiles, and many did not know to divide this difference by two. In Part (b) (v), some candidates mistook the sample space for the marks. Many other candidates did not recognize the sample space being reduced, and so could not correctly give the probability for the second student scoring less than 50 marks on the exam.

Solutions are: (a) (i) $\frac{3}{20}$

(ii) $\frac{11}{60}$

(b) (i)

6	7	8	9	10	11	12
5	6	7	8	9	10	11
4	5	6	7	8	9	10
3	4	5	6	7	8	9
2	3	4	5	6	7	8
1	2	3	4	5	6	7
	1	**2**	**3**	**4**	**5**	**6**

(ii) a) $P(S > 9) = \frac{1}{6}$

b) $P(S \le 4) = \frac{1}{6}$

(iii) $a = \frac{5}{18}$ $b = \frac{1}{6}$ $c = \frac{1}{18}$

(c) (i) median = 71 lower quartile = 56 upper quartile = 79

(ii) Draw box-and-whisker plot
Shape is negatively skewed

Question 8

This question tested candidates' ability to:

- interpret and make use of velocity-time graphs to find distance , deceleration and time;
- apply the appropriate rate of change such as $a = \frac{dv}{dt}$ and use such to calculate time when velocity is maximum, maximum velocity and distance moved by particle from rest to where the particle attains maximum velocity.

Candidates performed best on Parts (a) (i) and (iii) and (b) (i) and (ii) of this question which required them to calculate the distance from a velocity-time graph, the time it took to decelerate to $10ms^{-2}$, obtain the acceleration when the velocity is maximum and calculate the maximum velocity.

Parts (a) (ii), (iv) and (b) (iii), however, presented the most difficulty to candidates. In Part (a) (ii), candidates were required to find the time travelled by the particle when it stated to decelerate, as well as to find the average acceleration of the particle. Most candidates looked at the graph and estimated the time as 10 seconds instead of using the formula distance = total area under the curve to calculate the correct time. A few candidates were able to correctly estimate that the car took 8 seconds to decelerate and then found the average velocity using average velocity = $\frac{\text{dis tan ce}}{\text{time}}$.

In Part (a) (iv), many candidates were unable to correctly find the additional time after 90 seconds for the particle to come to its final rest. Candidates did not use the area of the triangle to calculate the time, but rather some divided the 30 metres it further travelled by 10 ms^{-1}.

Part (b) (iii), required candidates to calculate the distance moved by the particle from rest to where the particle attained maximum velocity, that is they were expected to evaluate $\int_0^5 0.72t^2 - 0.096t^3 dt$.

Some candidates used the first function with limits between 5 and 10. Others used the second function with limits between 0 and 5. Still others integrated the first function with limits between 0 and "his" time that represented where the particle reached maximum velocity.

Solutions are:	(a)	(i)	750 m
		(ii)	deceleration = 1.875 m s^{-2}
		(iii)	52 seconds
		(iv)	15 m s^{-1}
	(b)	(i)	t = 5 seconds
		(ii)	max velocity = 6 m s^{-1}
		(iii)	distance travelled = 15 m

Paper 03/2-Alternate to the School Based Assessment (SBA)

This paper tested candidate ability to:

- construct an geometric series from circumscribed rectangles and obtain the sum of its first *n* terms.
- apply logarithms to a problem involving the transformation of a given relationship to a linear form and use its graph to unknown constants.

For Part (a), candidates were required to use the method of circumscribed rectangles to:

- determine the width of each of n subdivisions
- determine the point of each subdivision
- determine the height of each circumscribed rectangle
- determine area of each circumscribed rectangle
- determine the area of the circumscribed rectangles
- compute the area of the circumscribed rectangles as the progression increases
- determine the limit to which these values approaches

For Part (b), given a logarithm function:

- convert to the linear form
- find the values a and n by finding gradient and y intercept

There were approximately 120 responses to this paper. No candidate received the full 20 marks. The highest mark attained was 17. The mean score was 8.69 with a standard deviation of 7.75

Candidates, in general, were unable to do Part (a) of the question. About 50 % substituted into S_n to get values for a but were unable to identify that the limit approached 0.5.

For Part (b) most candidates were successful in finding the logarithms of the table and plotting the graph. The scale did not accommodate all the points to be plotted. Therefore, some of the candidates were unable to make the necessary adjustments. They were however able to convert the equation to the logarithm format, as well as to calculate the values of *a* and *n* from the graph. In general, the question was poorly done.

Solutions are: (a) (i) $\frac{1}{n}$.

(ii) $x_1 = \frac{1}{n}$, $x_2 = \frac{2}{n}$, $x_3 = \frac{3}{n}$ ….. . $x_{n-1} = \frac{n-1}{n}$.

(iii) $h_1 = \frac{1}{n}$, $h_2 = \frac{2}{n}$, $h_3 = \frac{3}{n}$,…….. $h_n = 1$

(iv) $A_1 = \frac{1}{n^2}$, $A_2 = \frac{2}{n^2}$, $A_3 = \frac{3}{n^2}$ ….. $A_n = \frac{1}{n}$.

(vi) 0.55 0.525 0.51 0.505

(b) Approaches 0.5 as n gets larger

log *y* = log *a* +*n* log*x*
n is approximately 4
a is approximately 3 .

Paper 031 – School-Based Assessment (SBA)

Many of the projects submitted were of a high quality and generally related to real-world situations. The mean mark awarded on the SBA was 33.06 (out of a possible score of 40 marks), with standard deviation of 6.30.

A number of observed weak areas related to sample submissions, related to the submissions themselves as well as to the marking of these samples. The following were general observations on both Project A and Project B:

- Some SBA project titles and aim(s) or purpose were not clear, not concise and not well defined. Some titles were too vague, others too wordy.
- In far too many cases, students simply re-stated the project's title as its aim or purpose. This often meant that the reason for the project was not stated.
- In a number of cases, the project's aim or purpose did not relate to the given title and sometimes was not linked to any syllabus objective;
- Some submissions did not come with the required CSEC Additional Mathematics rubric;
- Some teachers created their own rubric or made adjustments to the CSEC Additional Mathematics rubric. This is unacceptable, as the project then has to be re-marked using the appropriate CSEC Additional Mathematics rubric;
- A few project submissions appeared to have been downloaded from the Internet;
- There were few instances of incorrect spelling and grammar. However, candidates used the correct mathematical jargon and symbols appropriately most of the time;
- Some projects were incorrectly categorized and assessed, that is, a Project A being labelled an assessed as a Project B, and vice versa. This did create some problems as, for example, Project B requires the collection and analysis of data from an experimental-type activity.

Comments specifically for Project A:

Mathematical Formulation

- In stating how the project was going to be done most students correctly identified all the important elements of the problem and showed understanding of the relationship between the elements.

The Problem Solution

- Assumptions were often not clearly stated. Some students, though, did state limitations of the project.
- Explanations were generally not sufficient and also were not placed between every successive step in the problem solution.
- Calculations were often precise but solutions were not clearly stated.

Application of Solution

- Many students were unaware of how to show that the solution or proof of the given problem was valid; was necessary to see how the candidate's solution could be applied elsewhere or to be able to substitute their values to check for validity.

Discussion of Findings/Conclusion

- In some instances the discussion was worthwhile. However, the discussion was not always related to the project's purpose and was often quite broad.
- Conclusions in many instances were not found to be valid. Some candidates stated many conclusions which were not related to the purpose of the project.
- There were few instances where suggestions for analyses were not stated or included in the project.

Comments specifically for Project B:

Method of Data Collection

- Although students indicated what they did, some were not specific enough about the type of data collected and method of sampling. Since the marking rubric was not specific as to what is needed, the candidates were not penalized.

Presentation of Data

- Too many students had the minimum required one table or chart; however, some tables and charts were not properly named or labelled, for example, graphs produced using software programs.
- There was not a systemic layout or organization of the tables and charts

Mathematical Knowledge (Analysis of Data)

- In this section, the student was expected to outline HOW the project was going to be done.
- Mathematical concepts in many cases were not used appropriately from the Additional Mathematics syllabus. Many projects utilized content at Form 3 level or below.
- Most of the calculations shown were accurate. However, in some cases no working was seen because some students used software to generate the values.
- Generally some analysis was attempted, but the analysis was often not coherent. This could be attributed to the fact that no proper summary table of calculation values was seen, so the markers had to search through the document constantly to link the calculated values with the analysis.
- The students were often not very clear on which two approaches were to be used in the analysis. In many cases the two approaches used were of the same concepts, for example, mean, mode and median were presented as different approaches to the analysis, but all of these are measures of central tendency.

Discussion of Findings/Conclusions

- In most instances there was no statement of findings, in some cases if it was in the project, it was not properly placed.
- Conclusions made were based on the reported findings but often were not related to the purpose of the project. As a result this made their conclusions invalid.
- In few cases there were no suggestions for future analysis. This was an improvement for last year 2013.

Plagiarism

Plagiarism continued to be a problem in the Additional Mathematics projects bring submitted for moderation. There were cases where projects were taken directly from the specimen papers as well from the exemplar projects given in the Additional Mathematics syllabus. This was noted among last year's submissions, and the practice continued in this year's submissions. The concern here is that the teachers associated appeared to be accepting these as legitimate projects, when it is clear that they are not the candidates' own work. This is must be stopped.

There were other instances where candidates had obviously done the project as a group. However, they submitted the same data, same mathematical formulations/solution, same diagrams, graphs and tables, same discussions and same conclusion, in some cases even printing and submitting identical copies. This practice must be stopped. Even where the project is done as a group, ALL aspects of the write-up must show candidates' individual effort. Only the raw data collected must be the same.

The following recommendations hold for continued improvement in this aspect of the Additional Mathematics examinations:

- All projects should have a clear and concise title, and well-defined aim(s) or purpose;
- Where possible the SBA should be relevant to authentic situations;

- The variables that are being used (Project B) must be clearly stated and described. Variable can be controlled, manipulated and responding;
- The type of sample and sample size, if relevant, must be clearly stated;
- Teachers must ensure that projects which integrate other subject areas utilize concepts as contained in the CSEC Additional Mathematics syllabus;
- If students collect their data in a group setting, they MUST demonstrate their INDIVIDUAL effort in relation to analysis (interpretation) and finding(s)/conclusion(s);
- Projects involving dice or playing cards must be more expansive so that candidates can present a more in-depth analysis of the topic under consideration;
- As good practice, students should be encouraged to cite all sources and insert a reference/bibliography page;
- Teachers should guide students using the assessment criteria found in forms 'Add Math 1- 5A' and 'Add Math 1-5B' which are both available on the CXC website. Teachers can give their students the rubric as a means of guidance in developing their projects.

It is clear that training workshops are needed in the area of project work for Additional Mathematics. Therefore, it is recommended that workshops be held throughout the Caribbean. Teachers need to be aware that there are TWO RUBRIC SHEETS 1-5A for Project A (the mathematical modelling project) and 1-5B for Project B (the statistical project), and to use the appropriate sheet in marking students' projects. It was apparent that some teachers found difficulties in understanding some aspects of the rubric, or were unaware of the existence of the rubric. Teachers need to ensure that the content level in the project coincides with the Additional Mathematics syllabus.

Paper 032 – Alternative to School-Based Assessment (SBA)

This paper tested candidates' ability to:

- construct a geometric series from circumscribed rectangles and obtain the sum of its first *n* terms.
- apply logarithms to a problem involving the transformation of a given relationship to a linear form and use its graph to unknown constants.

For Part (a), candidates were required to use the method of circumscribed rectangles to:

- determine the width of each of n subdivisions
- determine the point of each subdivision
- determine the height of each circumscribed rectangle
- determine the area of each circumscribed rectangle
- determine the area of the circumscribed rectangles
- compute the area of the circumscribed rectangles as the progression increases
- determine the limit to which these values approaches

For Part (b), given a logarithm function:

- convert to the linear form
- find the values *a* and *n* by finding gradient and y intercept

There were approximately 120 responses to this paper. No candidate received the full 20 marks. The highest mark attained was 17. The mean score was 8.69 with a standard deviation of 7.75

Candidates, in general, were unable to do Part (a) of the question. About 50 per cent substituted into S_n to get values for *a* but were unable to identify that the limit approached 0.5.

For Part (b) most candidates were successful in finding the logarithms of the table and plotting the graph. The scale did not accommodate all the points to be plotted. Therefore, some of the candidates were unable to make the necessary adjustments. They were however able to convert the equation to the logarithm format, as well as to calculate the values of *a* and *n* from the graph. In general, the question was poorly done.

Solutions are: (a) (i) $\frac{1}{n}$.

(ii) $x_1 = \frac{1}{n}$, $x_2 = \frac{2}{n}$, $x_3 = \frac{3}{n}$ ….. . $x_{n-1} = \frac{n-1}{n}$.

(iii) $h_1 = \frac{1}{n}$, $h_2 = \frac{2}{n}$, $h_3 = \frac{3}{n}$,…….. $h_n = 1$

(iv) $A_1 = \frac{1}{n^2}$, $A_2 = \frac{2}{n^2}$, $A_3 = \frac{3}{n^2}$ ….. $A_n = \frac{1}{n}$.

(vi) 0.55 0.525 0.51 0.505

(b) Approaches 0.5 as n gets larger

log *y* = log *a* +*n* log*x*
n is approximately 4
a is approximately 3 .

C A R I B B E A N E X A M I N A T I O N S C O U N C I L

REPORT ON CANDIDATES' WORK IN THE CARIBBEAN SECONDARY EDUCATION CERTIFICATE® EXAMINATION

MAY/JUNE 2015

ADDITIONAL MATHEMATICS GENERAL PROFICIENCY EXAMINATION

GENERAL COMMENTS

Additional Mathematics was tested for the fourth time in the May/June 2015 CSEC examinations. The subject is intended to bridge a gap between CSEC General Proficiency Mathematics and the CAPE Unit 1 Pure Mathematics.

The intention is to offer candidates progressing form the CSEC level in Mathematics to the CAPE Unit 1 level in Pure Mathematics/Applied Mathematics a seamless transition to the advanced thinking required for these courses, although a candidate who masters the content set out in the CSEC syllabus for General Proficiency Mathematics should be armed with the tools needed to be successful at the CAPE proficiency level.

As established, the examination consists of three papers:

- Paper 01 – a 45-item multiple choice paper
- Paper 02 – a structured, ‘essay-type’ paper consisting of eight questions
- Paper 031 – a School-Based Assessment (SBA) project component for candidates in approved educational institutions
- Paper 032 – an alternative to SBA for out-of-school (private) candidates

Questions in the examination targeted the content and specific objectives of the Additional Mathematics syllabus (CXC 37/G/SYLL 10) with the aim of ensuring a wide coverage of the syllabus. The questions were designed at the appropriate level to test the following skills: Conceptual Knowledge, Algorithmic Knowledge and Reasoning.

Paper 01 tests content from Sections 1, 2 and 3 of the syllabus and consists of 45 items. Paper 02 tests content from all four sections of the syllabus. This year the paper consisted of four sections and each section contained two problem-solving questions. The questions in Section 1, 2 and 3 were all compulsory and each question in these sections were worth 14, 12 and 14 marks respectively. Section 4 contained two questions, one on Data Representation and Probability and the other on Kinematics and each question was worth 20 marks. Candidates were required to answer only one question from this section. Paper 031 is the SBA component of the examination. Candidates are required to do one project chosen from two project types: a mathematical modelling project (Project A) and a data handling/statistical analysis project (Project B). The SBA component is worth 20 marks. Private candidates can sit an alternate paper to the SBA, Paper 032, which consists of one in-depth, extended question from Sections 1, 2 and/or 3 of the syllabus. This paper is worth 20 marks.

The percentage of candidates earning Grades I–III was 73 per cent, compared with 77 per cent in 2014.

This year saw an 11.6 per cent increase in candidates registered for the examination, up from 3609 candidates in 2014 to 4016 in 2015.

Again this year, a formula sheet was included as part of the question paper on Papers 01, 02 and 032.

DETAILED COMMENTS

Paper 01 – Multiple Choice

This was a 45 item paper covering Sections 1, 2 and 3 of the syllabus. The mean score on this paper was 39.54 with a standard deviation of 13.36 respectively, compared with 39.41 and 12.71 in 2014.

Paper 02 – Structured Essay Questions

This paper consisted of eight questions, of which Questions 1 to 6 were compulsory. Candidates had to choose one of Questions 7 and 8. The total possible score on this paper was 100 marks. The mean and standard deviation for this paper were 47.42 and 26.98 respectively, compared with 46.32 and 27.27 in 2014.

Section 1: Algebra and Functions

Question 1

This question tested candidates' ability to:

- Determine the value of $g(f(2))$ given the two functions g and f.
- Determine the inverse of a function given in the form $\frac{m(x)}{n(x)}$, where $m(x)$ and $n(x)$ are linear functions.
- Completely factorize a cubic equation given one of its linear factors.
- Use the laws of indices to solve an indicial equation.
  Use the laws of logarithms to solve a logarithmic equation.

There were about 4016 responses to this question. The mean mark was 8.41 with a standard deviation of 4.19.

Candidates performed best on Parts (a) and (b), as they should at this level. Candidates who did poorly in these parts generally performed poorly throughout the entire question.

In Part (a), most candidates were able to achieve full marks. Some candidates attempted to substitute $f(x)$ into $g(x)$ and made mistakes while doing the algebra. Few candidates were seen calculating $f(2)$ first and then substituting this value into $g(x)$ which resulted in fewer mistakes.

Common Errors

Simple arithmetic miscalculations were made, for example, 20 – 3 was given as '−17' instead of *17*.

The majority of candidates performed very well on Part (b). Even weaker candidates were able to achieve a mark by interchanging x and y.

Common Errors

Candidates misread the original function, which led to incorrect factorization of y.

Candidates 'forgot' to interchange x and y.

Part (c) was done relatively well. The majority of candidates recognized the application of the factor theorem and correctly used the long division method to obtain a quadratic equation, which was then factorized. Some used a comparison of coefficients approach and a trial and error approach while a few used the synthetic division method in answering this question. The candidates who were most successful used the long division method.

Common Errors

Candidates did not factorize their quadratic quotient to obtain the other two linear factors.

Many proved (x – 2) was a factor of $k(x)$, but this was given in the question.

It appears as though some candidates used programmable calculators to find the roots of the cubic equation and were therefore unable to provide the necessary supporting statements. Simple algebraic/arithmetic errors were seen in the long division method.

In Part (d) (i), candidates recognized the need to change each side to a common base, but many of them were unable to correctly apply laws of indices and solve the resulting simple linear equation presented by the equating of indices. Some candidates applied logarithms to both sides, but few were successful as they applied the laws of logarithms incorrectly.

Common Errors

Equating indices of different bases
Expanding brackets incorrectly when simplifying the indices
Inability to recognize ¼ = 4^{-1} (negative indices)

Part (d) (ii) presented the most difficulty to candidates. Candidates did not demonstrate a sound knowledge in logarithms. Most showed confusion when 'dropping logs', as they simply removed the log function and proceeded to solve the question as is.

Some candidates who correctly applied the laws of logarithms, transposed incorrectly resulting in an incorrect quadratic equation while those who obtained the correct quadratic equation lost a mark as a result of their inability to factorize correctly.

Common Errors

$\log_a M + \log_a N = \log_a (M + N)$ instead of $\log_a M + \log_a N = \log_a (MN)$
$3^0 = 0$ *instead of* $3^0 = 1$

Factorization Errors

With respect to candidates' responses on this question, it is recommended that candidates be given more practice questions. This will allow them to understand and apply appropriate laws where and when needed.

Solutions

(a) $gf(2) = 33$

(b) $h^{-1}(x) = \dfrac{2x+5}{x-3} \quad x \in \mathbf{R}, x \neq 3$

(c) $2x^3 - 5x^2 + x + 2 = (x-2)(x-1)(2x+1)$

(d) (i) $x = -2.5$ (ii) $x = 2$ and $x = 3$

Question 2

This question tested candidates' ability to:

- Express a quadratic function *f(x)* in the form $a(x+b)^2+c$ where *a, b* and *c* are constant
- Determine the coordinates of the function's minimum point.
- Use the sum and product of the roots of a quadratic equation to determine the value of $\frac{1}{\alpha}+\frac{1}{\beta}$, where α and β are the roots of the equation.
- Determine, algebraically, the point of intersection of a non-linear curve and a straight line.
- Use the formula for the n^{th} term of an arithmetic progression to calculate a particular term given the first term and the common difference.

There were 3522 responses to this question. The mean mark was 8.79 with a standard deviation of 4.18.

The overall performance on this question was good. About 55 per cent of the candidates were able to score full marks.

In Part (a), the majority of candidates attempted to complete the square but only about 60 per cent were able to do so successfully. The majority of the candidates used the classical method to complete the squares and about 95 per cent of them were able to identify that a = 3, however, finding correct values for b and c proved to be difficult due to numerical errors they made along the way. Some candidates used other techniques to arrive at the values of their constants; $3x^2 - 9x + 4 \equiv a(x+h)^2 + k$, where $a = 3$, $h=\frac{b}{2a} = -\frac{3}{2}$ and $k=c-ah^2$ or $\frac{4ac-b^2}{4a} = -\frac{11}{4}$ $\rightarrow$ $3x^2 - 9x + 4 \equiv 3\left(x-\frac{3}{2}\right)^2-\frac{11}{4}$ being the more common one. Some of the candidates using this approach did not remember the formulae correctly; $h=-\frac{b}{2a} = \frac{3}{2}$ and $k = \frac{b^2-4ac}{4a} = \frac{11}{4}$ were some of the common mistakes made.
The majority of candidates was not able to provide the coordinates of the minimum point from their completed square form. They gave instead either the minimum value of the function or the x value at the minimum point only.

In Part (b), deriving the sum and the product of the roots from a given equation was well known but some candidates confused the signs in determining their answers; $\alpha+\beta=\frac{b}{a}$ and $\alpha\beta=\frac{-c}{a}$ were sometimes used. Other candidates used the numerical values they obtained by solving for α and β instead of α + β and αβ. It was also difficult for some candidates to express $\frac{1}{\alpha}+\frac{1}{\beta}$ in terms of $\alpha+\beta$ and $\alpha\beta$.
In Part (c), many candidates experienced great difficulty in simplifying the equation they obtained after eliminating one of the variables from the given equations. Some also did not proceed to find the coordinates of the points leaving only the x values they obtained. In many cases where they proceeded to find y, errors were often made.

In Part (d), candidates displayed a lack of knowledge about arithmetic progressions and geometric progressions when given in a real world context. Many attempted a trial and error approach rather

than using an arithmetic progression where $T_n = a + (n-1)d$ and some even used a geometric progression. Some candidates used the first term as 38 400 instead of
36 000 while others used the formula for the sum of terms, $S_n = \frac{n}{2}(2a + (n-1)d)$.

Candidates need to be more vigilant when using equations and numerical figures given in the question paper in their solutions. Many of such errors were seen throughout the question.

Solutions

(a) (i) $3\left(x - \frac{3}{2}\right)^2 - \frac{11}{4}$ (ii) Minimum point is $\left(\frac{3}{2},\ -\frac{11}{4}\right)$

(b) $\frac{1}{\alpha} + \frac{1}{\beta} = -\frac{3}{2}$

(c) Points of intersection are (−3, 37) and $\left(-\frac{5}{2},\ \frac{63}{2}\right)$

(d) Annual salary for the 9th year = \$55 200

Section 2: Coordinate Geometry, Vectors and Trigonometry

Question 3

This question tested candidates' ability to:

- Determine the coordinates of the centre of a circle and (ii) the length of its radius given the equation in the form $x^2 + y^2 + 2fx + 2gy + c = 0$; (iii) determine the equation of the normal to a circle at a given point on the circle.
- Determine the unit vector in the direction of $\overrightarrow{AB}$ given two vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$ written in the form ***ai*** + ***bj***; (ii) calculate the acute angle between the vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$.

There were 2923 responses to this question. The mean mark was 6.03 with a standard deviation of 4.03.

Candidates' recall of knowledge of coordinate geometry and vectors was good.

In Part (a) (i), candidates were able to determine the coordinates of the centre of the circle by either completing the square or by using coordinates of centre $= \left(\frac{-2f}{2}, \frac{-2g}{2}\right)$, from the equation of the circle. The approach involving the coefficient method was the most popular one used by candidates.

In Part (a) (ii), finding the length of the radius was a challenge to most candidates. This was often the only mark lost in this section.

In Part (a) (iii), candidates were able to apply the gradient algorithm using the coordinates of the centre they obtained. However, some of them misinterpreted this gradient to be that of the gradient of the tangent and proceeded to obtain the gradient of the normal as the negative reciprocal of this result.

There was a unique solution of determining the **equation of the normal:**
$(y_1 + g)x - (x_1 + f)y - gx_1 + fy_1 = 0$

In Part (b) (i), candidates correctly obtained the vector $\overrightarrow{AB}$. Most candidates correctly calculated the modulus of $\overrightarrow{AB}$ but were unable to write the **unit vector** $\overrightarrow{AB}$

In Part (b) (ii), candidates were able to determine the dot product and the moduli of $\overrightarrow{OA}$ and $\overrightarrow{OB}$. However, errors in computations led to an incorrect value of the required angle.

Solutions

(a) (i) coordinates of centre are (6, 11)

(ii) radius = $\sqrt{5}$

(b) (i) unit vector $\overrightarrow{AB}$ = $\frac{1}{\sqrt{29}}(2i - 5j)$

(ii) $A\hat{O}B = 20.8^0$

Question 4

This question tested candidates' ability to:

- Determine the area in a circle between a chord and an arc given the formula for the area of a sector.
- Use the trigonometric identity $\sin^2\theta = 1 - \cos^2\theta$ to solve a quadratic equation with trigonometric arguments.
- Prove a trigonometric identity using the expansions for sin2θ and cos2θ.

Candidates performed best on Part (a), which required finding the area of the shaded region. Most candidates knew the formula for finding the required area. However, many candidates substituted 30° for the angle in the formula that involved radians.

In Part (b), many candidates knew that they had to substitute $\sin^2\theta = 1 - \cos^2\theta$. Most of the candidates who substituted correctly were able to simplify and form the quadratic equation. Many of them were able to solve the quadratic equation and obtain the required angles.

In Part (c), most candidates received marks for substituting correctly for $\sin 2\theta = 2\sin\theta\cos\theta$. However, many did not realize that the substitution of $\cos 2\theta = 2\cos^2\theta - 1$ was necessary to lead to the proof. Some of those who substituted correctly had difficulty in factorizing and simplifying and as a result were unable to complete the proof.

Solutions

(a) Area of shaded region = $\frac{4}{3}\pi - 4 \ cm^2$

(b) $\theta = 104.5^\circ, 255.5^\circ$