

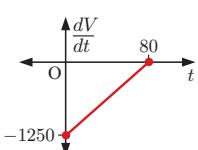
- f** $\frac{dP}{dt} \Big|_{t=4} = 4$ Profit is increasing at \$4000 per year after 4 years.
 $\frac{dP}{dt} \Big|_{t=10} = 28$ Profit is increasing at \$28 000 per year after 10 years.
 $\frac{dP}{dt} \Big|_{t=25} = 88$ Profit is increasing at \$88 000 per year after 25 years.

- 2 a** **i** $Q(0) = 100$ **ii** $Q(25) = 50$ **iii** $Q(100) = 0$
b **i** decr. 1 unit per year **ii** decr. $\frac{1}{\sqrt{2}}$ units per year
c $Q'(t) = -\frac{5}{\sqrt{t}} < 0$

- 3 a** 0.5 m
b $t = 4$: 9.17 m, $t = 8$: 12.5 m, $t = 12$: 14.3 m
c $t = 0$: 3.9 m year^{-1} , $t = 5$: 0.975 m year^{-1} ,
 $t = 10$: 0.433 m year^{-1}
d As $\frac{dH}{dt} = \frac{97.5}{(t+5)^2} > 0$ for all $t \geq 0$, the tree is always growing.

- 4 a** $C'(x) = 0.0009x^2 + 0.04x + 4$ dollars per pair
b $C'(220) = \$56.36$ per pair. This estimates the additional cost of making one more pair of jeans if 220 pairs are currently being made.
c \$56.58 This is the actual increase in cost to make an extra pair of jeans (221 rather than 220).
d $C''(x) = 0.0018x + 0.04$
 $C''(x) = 0$ when $x = -22.2$. This is where the rate of change is a minimum, however it is out of the bounds of the model (you cannot make < 0 jeans!).

- 5 a** **i** €4500 **ii** €4000
b **i** decrease of €210.22 per km h^{-1}
ii increase of €11.31 per km h^{-1}
c $\frac{dC}{dv} = 0$ at $v = \sqrt[3]{500\ 000} \approx 79.4 \text{ km h}^{-1}$
d $\frac{dV}{dt} = -1250 \left(1 - \frac{t}{80}\right) \text{ L min}^{-1}$
b at $t = 0$ when the tap was first opened
c $\frac{d^2V}{dt^2} = \frac{125}{8} \text{ L min}^{-2}$



This shows that the rate of change of V is constantly increasing, so the outflow is decreasing at a constant rate.

- 7 a** The near part of the lake is 2 km from the sea, the furthest part is 3 km.
b $\frac{dy}{dx} = \frac{3}{10}x^2 - x + \frac{3}{5}$
 $\frac{dy}{dx} \Big|_{x=\frac{1}{2}} = 0.175$, height of hill is increasing as gradient is positive.
 $\frac{dy}{dx} \Big|_{x=1\frac{1}{2}} = -0.225$, height of hill is decreasing as gradient is negative.
 \therefore top of the hill is between $x = \frac{1}{2}$ and $x = 1\frac{1}{2}$.
c 2.55 km from the sea, 63.1 m deep

- 8 a** $k = \frac{1}{50} \ln 2 \approx 0.0139$
b **i** 20 grams **ii** 14.3 grams **iii** 1.95 grams
c 9 days and 6 minutes (216 hours)
d **i** -0.0693 g h^{-1} **ii** $-2.64 \times 10^{-7} \text{ g h}^{-1}$
e Hint: You should find $\frac{dW}{dt} = -\frac{1}{50} \ln 2 \times 20e^{-\frac{1}{50} \ln 2 t}$

- 9 a** $k = \frac{1}{15} \ln \left(\frac{19}{3}\right) \approx 0.123$ **b** 100°C

- c** $c = -k \approx -0.123$
d **i** decreasing at $11.7^\circ\text{C min}^{-1}$
ii decreasing at $3.42^\circ\text{C min}^{-1}$
iii decreasing at $0.998^\circ\text{C min}^{-1}$

- 10 a** 43.9 cm **b** 10.4 years
c **i** growing at 5.45 cm per year
ii growing at 1.88 cm per year

- 11 a** $A(0) = 0$
b **i** $k = \frac{\ln 2}{3} \approx 0.231$
ii 0.728 litres of alcohol produced per hour

- 12** $\frac{21}{\sqrt{2}} \text{ cm}^2$ per radian

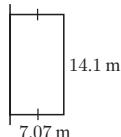
- 13 a** rising at 2.73 m per hour **b** rising

- 14 b** **i** 0 **ii** 1 **iii** ≈ 1.11

EXERCISE 14E

- 1** 250 items

- 2 b** $L_{\min} \approx 28.3 \text{ m}$, $x \approx 7.07 \text{ m}$



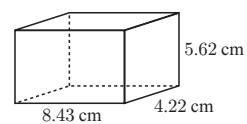
- 3** 10 blankets **4** 14.8 km h^{-1}

- 5** at 4.41 months old

- 6 a** Hint: $V = 200 = 2x \times x \times h$

- b** Hint: Show $h = \frac{100}{x^2}$ and substitute into the surface area equation.

- c** $SA_{\min} \approx 213 \text{ cm}^2$, $x \approx 4.22 \text{ cm}$



- 7** 20 kettles **8** $C\left(\frac{1}{\sqrt{2}}, e^{-\frac{1}{2}}\right)$

- 9 a** Recall that $V_{\text{cylinder}} = \pi r^2 h$ and that $1 \text{ L} = 1000 \text{ cm}^3$.

- b** Recall that $SA_{\text{cylinder}} = 2\pi r^2 + 2\pi r h$.

- c** radius $\approx 5.42 \text{ cm}$, height $\approx 10.8 \text{ cm}$

- 10 b** $\theta \approx 1.91$, $A \approx 237 \text{ cm}^2$ **11 b** $6 \text{ cm} \times 6 \text{ cm}$

- 12 a** $0 \leq x \leq 63.7$

- b** $l = 100 \text{ m}$, $x = \frac{100}{\pi} \approx 31.83 \text{ m}$, $A = \frac{20\ 000}{\pi} \approx 6366 \text{ m}^2$

- 13** after 13.8 weeks **14** after 40 minutes

- 15 c** $\theta = 30^\circ$, $A \approx 130 \text{ cm}^2$

- 16 a** Hint: Show that $AC = \frac{\theta}{360} \times 2\pi \times 10$

- b** Hint: Show that $2\pi r = AC$

- c** Hint: Use the result from **b** and Pythagoras' theorem.

- d** $V = \frac{1}{3}\pi \left(\frac{\theta}{36}\right)^2 \sqrt{100 - \left(\frac{\theta}{36}\right)^2}$ **e** $\theta \approx 294^\circ$

- 17** 1 hour 34 min 53 s when $\theta \approx 36.9^\circ$ **18** 9.87 m

EXERCISE 14F

- 1** a is decreasing at 7.5 units per second

- 2** increasing at 1 cm per minute

- 3 a** $4\pi \text{ m}^2$ per second **b** $8\pi \text{ m}^2$ per second

- 4** increasing at $6\pi \text{ m}^2$ per minute

5 decreasing at 0.16 m^3 per minute

6 $\frac{20}{3} \text{ cm per minute}$

7 $\frac{25\sqrt{3}}{6} \approx 7.22 \text{ cm per minute}$

8 decreasing at $\frac{250}{13} \approx 19.2 \text{ ms}^{-1}$

9 a 0.2 ms^{-1} b $\frac{4}{45} \text{ ms}^{-1}$

10 decreasing at $\frac{\sqrt{2}}{100} \text{ radians per second}$

11 increasing at $0.12 \text{ radians per minute}$

REVIEW SET 14A

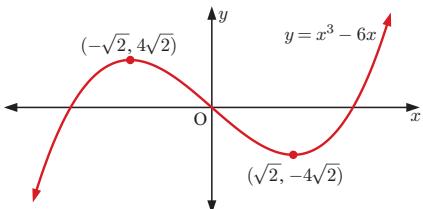
1 a $y = 4x + 2$ b $y = 4x + 4 \ln 2 - 4$ c $y = e^2$

2 $a = \frac{5}{2}$, $b = -\frac{3}{2}$

3 a $a = -6$

b local max. $(-\sqrt{2}, 4\sqrt{2})$, local min. $(\sqrt{2}, -4\sqrt{2})$

c



4 a $y = \frac{1}{5}x - \frac{11}{5}$ (or $x - 5y = 11$) b $y = -4x + 14$

5 $\frac{3267}{152} \text{ units}^2$

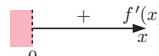
6 $a = 64$

7 P(0, 7.5), Q(3, 0)

9 $3x - 4y = -5$

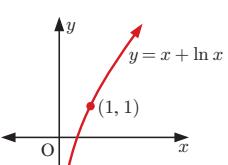
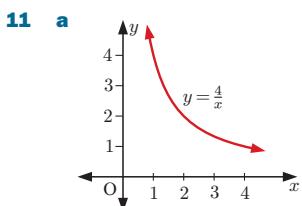
10 a $x > 0$

b Sign diagram of $f'(x)$



$f(x)$ is increasing for all $x > 0$.

d normal is $x + 2y = 3$



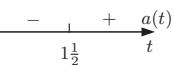
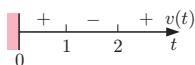
b $y = -\frac{4}{k^2}x + \frac{8}{k}$

c A($2k, 0$), B $\left(0, \frac{8}{k}\right)$

d Area = 8 units 2

e $k = 2$

12 a $v(t) = (6t^2 - 18t + 12) \text{ cm s}^{-1}$, $a(t) = (12t - 18) \text{ cm s}^{-2}$



b $s(0) = -5 \text{ cm}$ (5 cm to the left of origin)

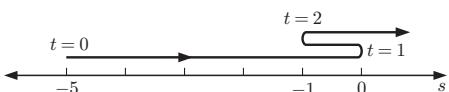
$v(0) = 12 \text{ cm s}^{-1}$ towards origin

$a(0) = -18 \text{ cm s}^{-2}$ (reducing speed)

c At $t = 2$, particle is 1 cm to the left of the origin, is stationary and is accelerating towards the origin.

d $t = 1$, $s = 0$ and $t = 2$, $s = -1$

e



f $1 \leq t \leq 1\frac{1}{2}$ and $t \geq 2$

13 b $k = 9$

14 a $x(0) = 3 \text{ cm}$, $x'(0) = 2 \text{ cm s}^{-1}$, $x''(0) = 0 \text{ cm s}^{-2}$

b $t = \frac{\pi}{4} \text{ s}$ and $\frac{3\pi}{4} \text{ s}$

c 4 cm

15 6 cm from each end

16 a $y = \frac{1}{x^2}$, $x > 0$

b base is 1.26 m square, height 0.630 m

17 a $v(t) = 15 + \frac{120}{(t+1)^3} \text{ cm s}^{-1}$, $a(t) = \frac{-360}{(t+1)^4} \text{ cm s}^{-2}$

b At $t = 3$, particle is 41.25 cm to the right of the origin, moving to the right at 16.88 cm s^{-1} and decelerating at 1.41 cm s^{-2} .

c speed is never increasing

18 A $\left(\frac{1}{2}, \frac{1}{e}\right)$

19 $\frac{20\sqrt{10}}{3} \approx 21.1 \text{ m per minute}$

20 a $V(r) = \frac{8}{9}\pi r^3 \text{ m}^3$

b $\frac{dr}{dt} = -\frac{8}{375\pi} \approx -0.00679 \text{ m min}^{-1}$

REVIEW SET 14B

1 a $x = 1$ b $ex - 2y = e - \frac{2}{e}$ c $y = 16x - \frac{127}{2}$

2 $a = -14$, $b = 21$

3 a $f(3) = 2$, $f'(3) = -1$ b $f(x) = x^2 - 7x + 14$

4 a $2x + 3y = \frac{2\pi}{3} + 2\sqrt{3}$ b $\sqrt{2}y - 4x = 1 - 2\pi$

5 $p = 1$, $q = -8$ 6 $(-2, -25)$ 7 $a = \frac{1}{2}$

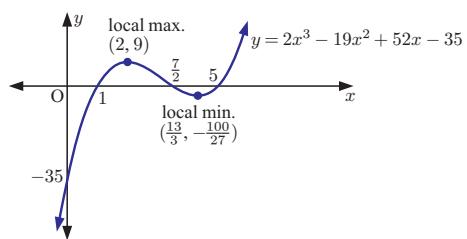
8 a local minimum at $(0, 1)$ c $f''(x) = e^x$

9 $(0, \ln 4 - 1)$

10 a y -intercept = -35 b $x = 1, \frac{7}{2}, 5$

c local maximum at $(2, 9)$, local minimum at $(\frac{13}{3}, -\frac{100}{27})$

d



11 BC = $\frac{8\sqrt{10}}{3}$ units

12 a 60 cm b i 4.24 years ii 201 years

c i 16 cm per year ii 1.95 cm per year

13 a $v(t) = -8e^{-\frac{t}{10}} - 40 \text{ ms}^{-1}$

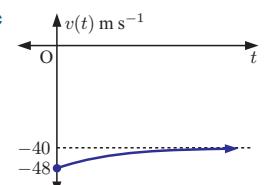
$a(t) = \frac{4}{5}e^{-\frac{t}{10}} \text{ ms}^{-2}$ { $t \geq 0$ }

b $s(0) = 80 \text{ m}$

$v(0) = -48 \text{ ms}^{-1}$

$a(0) = 0.8 \text{ ms}^{-2}$

d $t = 10 \ln 2$ seconds



14 a i \$535

b i $-\$0.267 \text{ per km h}^{-1}$

b ii $\$2.33 \text{ per km h}^{-1}$

c 51.3 km h $^{-1}$

15 a $v(t) = 3 - \frac{1}{2\sqrt{t+1}}$



a $a(t) = \frac{1}{4(t+1)^{\frac{3}{2}}}$



b $x(0) = -1, v(0) = 2.5, a(0) = 0.25$

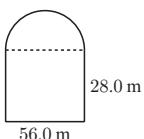
Particle is 1 cm to the left of the origin, is travelling to the right at 2.5 cm s^{-1} , and accelerating at 0.25 cm s^{-2} .

c Particle is 21 cm to the right of the origin, is travelling to the right at 2.83 cm s^{-1} , and accelerating at $0.00926 \text{ cm s}^{-2}$.

d never changes direction **e** never decreasing

16 b $A = 200x - 2x^2 - \frac{1}{2}\pi x^2$

c



17 a $v(0) = 0 \text{ cm s}^{-1}, v(\frac{1}{2}) = -\pi \text{ cm s}^{-1}, v(1) = 0 \text{ cm s}^{-1}, v(\frac{3}{2}) = \pi \text{ cm s}^{-1}, v(2) = 0 \text{ cm s}^{-1}$

b $0 \leq t \leq 1, 2 \leq t \leq 3, 4 \leq t \leq 5, \text{ etc.}$

So, for $2n \leq t \leq 2n+1, n \in \{0, 1, 2, 3, \dots\}$

18 $x = \frac{k}{2} \left(1 - \frac{1}{\sqrt{3}}\right)$ **19** 3.60 m s^{-1}

20 increasing at $0.128 \text{ radians per second}$

21 a $\frac{\sqrt{3}}{2}\pi \text{ cm s}^{-1}$ **b** 0 cm s^{-1}

22 a **i** $y = -\frac{a^2}{4b} + \frac{a}{2b}x$ **ii** when $y = 0, x = \frac{a}{2}$

b **i** $y = -\frac{2b}{a}x + b$ **ii** when $x = 0, y = b$

iii Hint: Let P'' be the point on the line $y = -b$ where the distance to P is shortest.

Show that $FP = P''P$.

c **i** Hint: Show that $\triangle FPP' \cong \triangle P''PP'$.

ii Hint: Show that the tangents meet at $\left(\frac{a+c}{2}, \frac{ac}{4b}\right)$.

EXERCISE 15A.1

1 a i 0.6 units^2 **ii** 0.4 units^2 **b** 0.5 units^2

2 a 0.737 units^2 **b** 0.653 units^2

3	n	A_L	A_U
10	2.1850	2.4850	
25	2.2736	2.3936	
50	2.3034	2.3634	
100	2.3184	2.3484	
500	2.3303	2.3363	

converges to $\frac{7}{3}$

4 a i	n	A_L	A_U
5	0.16000	0.36000	
10	0.20250	0.30250	
50	0.24010	0.26010	
100	0.24503	0.25503	
500	0.24900	0.25100	
1000	0.24950	0.25050	
10 000	0.24995	0.25005	

ii	n	A_L	A_U
5	0.40000	0.60000	
10	0.45000	0.55000	
50	0.49000	0.51000	
100	0.49500	0.50500	
500	0.49900	0.50100	
1000	0.49950	0.50050	
10 000	0.49995	0.50005	

iii	n	A_L	A_U
5	0.54974	0.74974	
10	0.61051	0.71051	
50	0.65610	0.67610	
100	0.66146	0.67146	
500	0.66565	0.66765	
1000	0.66616	0.66716	
10 000	0.66662	0.66672	

iv	n	A_L	A_U
5	0.61867	0.81867	
10	0.68740	0.78740	
50	0.73851	0.75851	
100	0.74441	0.75441	
500	0.74893	0.75093	
1000	0.74947	0.75047	
10 000	0.74995	0.75005	

b **i** $\frac{1}{4}$ **ii** $\frac{1}{2}$ **iii** $\frac{2}{3}$ **iv** $\frac{3}{4}$

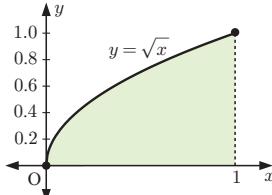
c area $= \frac{1}{a+1}$

b $n = 10000$

5 a	n	Rational bounds for π
10		$2.9045 < \pi < 3.3045$
50		$3.0983 < \pi < 3.1783$
100		$3.1204 < \pi < 3.1604$
200		$3.1312 < \pi < 3.1512$
1000		$3.1396 < \pi < 3.1436$
10 000		$3.1414 < \pi < 3.1418$

EXERCISE 15A.2

1 a



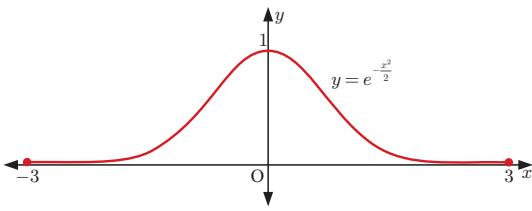
b	n	A_L	A_U
5	0.5497	0.7497	
10	0.6105	0.7105	
50	0.6561	0.6761	
100	0.6615	0.6715	
500	0.6656	0.6676	

c $\int_0^1 \sqrt{x} dx \approx 0.67$

2 a $A_L = \frac{2}{n} \sum_{i=0}^{n-1} \sqrt{1+x_i^3}, A_U = \frac{2}{n} \sum_{i=1}^n \sqrt{1+x_i^3}$

b	n	A_L	A_U
50	3.2016	3.2816	
100	3.2214	3.2614	
500	3.2373	3.2453	

c $\int_0^2 \sqrt{1+x^3} dx \approx 3.24$

3 a

- b** upper ≈ 1.2506 , lower ≈ 1.2493
c upper ≈ 1.2506 , lower ≈ 1.2493

d $\int_{-3}^3 e^{-\frac{x^2}{2}} dx \approx 2.4999$ compared to $\sqrt{2\pi} \approx 2.5066$

4 a 18**b** 4.5**c** 2π **EXERCISE 15B**

1 a i $\frac{x^2}{2}$ ii $\frac{x^3}{3}$ iii $\frac{x^6}{6}$ iv $-\frac{1}{x}$
 v $-\frac{1}{3x^3}$ vi $\frac{3}{4}x^{\frac{4}{3}}$ vii $2\sqrt{x}$

b The antiderivative of x^n is $\frac{x^{n+1}}{n+1}$ ($n \neq -1$).

2 a i $\frac{1}{2}e^{2x}$ ii $\frac{1}{5}e^{5x}$ iii $2e^{\frac{1}{2}x}$ iv $100e^{0.01x}$
 v $\frac{1}{\pi}e^{\pi x}$ vi $3e^{\frac{x}{3}}$

b The antiderivative of e^{kx} is $\frac{1}{k}e^{kx}$.

3 a $\frac{d}{dx}(x^3 + x^2) = 3x^2 + 2x$
 \therefore the antiderivative of $6x^2 + 4x = 2x^3 + 2x^2$
b $\frac{d}{dx}(e^{3x+1}) = 3e^{3x+1}$
 \therefore the antiderivative of $e^{3x+1} = \frac{1}{3}e^{3x+1}$
c $\frac{d}{dx}(x\sqrt{x}) = \frac{3}{2}\sqrt{x}$
 \therefore the antiderivative of $\sqrt{x} = \frac{2}{3}x\sqrt{x}$
d $\frac{d}{dx}(2x+1)^4 = 8(2x+1)^3$
 \therefore the antiderivative of $(2x+1)^3 = \frac{1}{8}(2x+1)^4$

EXERCISE 15C

1 a $\frac{1}{4}$ units² b $2\frac{1}{3}$ units² c $\frac{2}{3}$ units²
3 a $3\frac{3}{4}$ units² b $24\frac{2}{3}$ units² c $\frac{-2+4\sqrt{2}}{3}$ units²
d ≈ 3.48 units² e 2 units²

- 4 c** i $\int_0^1 (-x^2) dx = -\frac{1}{3}$, the area between $y = -x^2$ and the x -axis from $x = 0$ to $x = 1$ is $\frac{1}{3}$ units².
 ii $\int_0^1 (x^2 - x) dx = -\frac{1}{6}$, the area between $y = x^2 - x$ and the x -axis from $x = 0$ to $x = 1$ is $\frac{1}{6}$ units².
 iii $\int_{-2}^0 3x dx = -6$, the area between $y = 3x$ and the x -axis from $x = -2$ to $x = 0$ is 6 units²
d $-\pi$

EXERCISE 15D

1 $\frac{dy}{dx} = 7x^6$, $\int x^6 dx = \frac{1}{7}x^7 + c$
2 $\frac{dy}{dx} = 3x^2 + 2x$, $\int (3x^2 + 2x) dx = x^3 + x^2 + c$

3 $\frac{dy}{dx} = 2e^{2x+1}$, $\int e^{2x+1} dx = \frac{1}{2}e^{2x+1} + c$

4 $\frac{dy}{dx} = 8(2x+1)^3$, $\int (2x+1)^3 dx = \frac{1}{8}(2x+1)^4 + c$

5 $\frac{dy}{dx} = \frac{3}{2}\sqrt{x}$, $\int \sqrt{x} dx = \frac{2}{3}x\sqrt{x} + c$

6 $\frac{dy}{dx} = -\frac{1}{2x\sqrt{x}}$, $\int \frac{1}{x\sqrt{x}} dx = -\frac{2}{\sqrt{x}} + c$

7 $\frac{dy}{dx} = -2 \sin 2x$, $\int \sin 2x dx = -\frac{1}{2} \cos 2x + c$

8 $\frac{dy}{dx} = -5 \cos(1 - 5x)$,

$\int \cos(1 - 5x) dx = -\frac{1}{5} \sin(1 - 5x) + c$

9 $\int (2x-1)(x^2-x)^2 dx = \frac{1}{3}(x^2-x)^3 + c$

11 $\frac{dy}{dx} = \frac{-2}{\sqrt{1-4x}}$, $\int \frac{1}{\sqrt{1-4x}} dx = -\frac{1}{2}\sqrt{1-4x} + c$

EXERCISE 15E.1

1 a i $\frac{x^5}{5} - \frac{x^3}{3} - \frac{x^2}{2} + 2x + c$ b $x^5 - x^4 - 2x^3 - 7x + c$

c $\frac{2}{3}x^{\frac{3}{2}} + e^x + c$

e $\frac{2}{5}x^{\frac{5}{2}} - 2x + c$

g $\frac{1}{8}x^4 - \frac{1}{5}x^5 + \frac{3}{4}x^{\frac{4}{3}} + c$

i $5e^x + \frac{1}{12}x^4 - \frac{2}{3}x^{\frac{3}{2}} + c$

2 a $-3 \cos x - 2x + c$ b $2x^2 - 2 \sin x + c$

c $-\cos x - 2 \sin x + e^x + c$

e $\frac{1}{9}x^3 - \frac{1}{6}x^2 + \sin x + c$

g $\frac{1}{3}x^3 + \frac{3}{2}x^2 - 2x + c$

i $2e^x + \frac{1}{x} + c$

3 a $\frac{2}{3}x^{\frac{3}{2}} + \frac{1}{2}x^2 - 2x + c$ b $\frac{2}{3}x^{\frac{3}{2}} - 2x^{\frac{1}{2}} + c$

c $\frac{4}{3}x^3 + 2x^2 + x + c$

e $\frac{4}{3}x^{\frac{3}{2}} - 2x^{\frac{1}{2}} + c$

g $2x^{\frac{1}{2}} + 8x^{-\frac{1}{2}} - \frac{20}{3}x^{-\frac{3}{2}} + c$

i $\frac{1}{4}x^4 + x^3 + \frac{3}{2}x^2 + x + c$

4 a $\frac{2}{3}x^{\frac{3}{2}} + \frac{1}{2} \sin x + c$ b $2e^t + 4 \cos t + c$

c $3 \sin t + \cos t + c$

5 a $y = 6x + c$ b $y = \frac{4}{3}x^3 + c$

c $y = \frac{10}{3}x\sqrt{x} - \frac{1}{3}x^3 + c$

e $y = 2e^x - 5x + c$

6 a $f(x) = x - 2x^2 + \frac{4}{3}x^3 + c$

b $f(x) = \frac{2}{3}x^{\frac{3}{2}} - 4\sqrt{x} + c$

c $f(x) = x + \frac{5}{x} + c$

EXERCISE 15E.2

1 a $f(x) = x^2 - x + 3$ b $f(x) = x^3 + x^2 - 7$

c $f(x) = e^x + 2\sqrt{x} - 1 - e$

d $f(x) = \frac{1}{2}x^2 - 4\sqrt{x} + \frac{11}{2}$

2 a $f(x) = \frac{x^3}{3} - 4 \sin x + 3$

b $f(x) = 2 \sin x + 3 \cos x - 2\sqrt{2}$

- 3** **a** $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 + x + \frac{1}{3}$
b $f(x) = 4x^{\frac{5}{2}} + 4x^{\frac{3}{2}} - 4x + 5$
c $f(x) = -\cos x - x + 4$ **d** $f(x) = \frac{1}{3}x^3 - \frac{16}{3}x + 5$

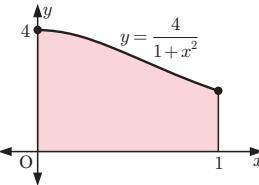
EXERCISE 15F

- 1** **a** $\frac{1}{8}(2x+5)^4 + c$ **b** $\frac{1}{2(3-2x)} + c$
c $\frac{-2}{3(2x-1)^3} + c$ **d** $\frac{1}{32}(4x-3)^8 + c$
e $\frac{2}{9}(3x-4)^{\frac{3}{2}} + c$ **f** $-4\sqrt{1-5x} + c$
g $-\frac{3}{5}(1-x)^5 + c$ **h** $-2\sqrt{3-4x} + c$
2 **a** $-\frac{1}{3}\cos(3x) + c$ **b** $-\frac{1}{2}\sin(-4x) + x + c$
c $6\sin\left(\frac{x}{2}\right) + c$ **d** $-\frac{3}{2}\cos(2x) + e^{-x} + c$
e $-\cos\left(2x + \frac{\pi}{6}\right) + c$ **f** $3\sin\left(\frac{\pi}{4} - x\right) + c$
g $\frac{1}{2}\sin(2x) - \frac{1}{2}\cos(2x) + c$
h $-\frac{2}{3}\cos(3x) + \frac{5}{4}\sin(4x) + c$
i $\frac{1}{16}\sin(8x) + 3\cos x + c$

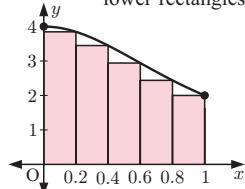
- 3** $y = \frac{1}{3}(2x-7)^{\frac{3}{2}} + 2$ **4** $(-8, -19)$
5 **a** $\frac{1}{2}(2x-1)^3 + c$ **b** $\frac{1}{5}x^5 - \frac{1}{2}x^4 + \frac{1}{3}x^3 + c$
c $-\frac{1}{12}(1-3x)^4 + c$ **d** $x - \frac{2}{3}x^3 + \frac{1}{5}x^5 + c$
e $-\frac{8}{3}(5-x)^{\frac{3}{2}} + c$ **f** $\frac{1}{7}x^7 + \frac{3}{5}x^5 + x^3 + x + c$
6 **a** $2e^x + \frac{5}{2}e^{2x} + c$ **b** $\frac{3}{5}e^{5x-2} + c$
c $-\frac{1}{3}e^{7-3x} + c$ **d** $\frac{1}{2}e^{2x} + 2x - \frac{1}{2}e^{-2x} + c$
e $-\frac{1}{2}e^{-2x} - 4e^{-x} + 4x + c$ **f** $\frac{1}{2}x^2 + 5(1-x)^{-1} + c$
7 $y = x - 2e^x + \frac{1}{2}e^{2x} + \frac{11}{2}$
8 $p = -\frac{1}{4}$, $f(x) = \frac{1}{2}\cos(\frac{1}{2}x) + \frac{1}{2}$
10 $f(x) = -e^{-2x} + 4$
11 $f(x) = \frac{2}{3}x^{\frac{3}{2}} - \frac{1}{8}e^{-4x} + \frac{1}{8}e^{-4} - \frac{2}{3}$

EXERCISE 15G

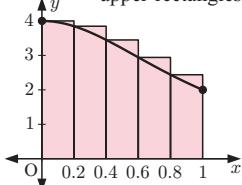
- 1** **a** $\int_1^4 \sqrt{x} dx = \frac{14}{3}$, $\int_1^4 (-\sqrt{x}) dx = -\frac{14}{3}$
b $\int_0^1 x^7 dx = \frac{1}{8}$, $\int_0^1 (-x^7) dx = -\frac{1}{8}$
2 **a** $\frac{1}{3}$ **b** $\frac{7}{3}$ **c** $\frac{8}{3}$ **d** 1
3 **a** -4 **b** 6.25 **c** 2.25 **4** **a** $\frac{1}{3}$ **b** $\frac{2}{3}$ **c** 1
5 **a** $\frac{1}{4}$ **b** $\frac{2}{3}$ **c** $e-1$ (≈ 1.72) **d** $\frac{1}{2}$
e $1\frac{1}{2}$ **f** $6\frac{2}{3}$ **g** $\ln 3$ (≈ 1.10) **h** $\frac{1}{2}$
i ≈ 1.52 **j** 2 **k** $e-1$ (≈ 1.72) **l** $\frac{1}{3}$
6 $m = -1$ or $\frac{4}{3}$ **7** **a** $\frac{\pi}{8} + \frac{1}{4}$ **b** $\frac{\pi}{4}$
8 **a** 6.5 **b** -9 **c** 0 **d** -2.5
9 **a** 2π **b** -4 **c** $\frac{\pi}{2}$ **d** $\frac{5\pi}{2} - 4$
10 **a** $\int_2^7 f(x) dx$ **b** $\int_1^9 g(x) dx$
11 **a** -5 **b** 4
12 **a** 4 **b** 0 **c** -8 **d** $k = -\frac{7}{4}$ **13** 0

REVIEW SET 15A**1** **a**

lower rectangles



upper rectangles



b n	A_L	A_U
5	2.9349	3.3349
50	3.1215	3.1615
100	3.1316	3.1516
500	3.1396	3.1436

c $\int_0^1 \frac{4}{1+x^2} dx \approx 3.1416$

2 **a** 2π **b** 4**3** **a** $8\sqrt{x} + c$ **b** $-\frac{1}{4}\cos(4x-5) + c$ **c** $-\frac{1}{3}e^{4-3x} + c$ **4** **a** $12\frac{4}{9}$ **b** $\sqrt{2}$

5 $\frac{dy}{dx} = \frac{x}{\sqrt{x^2-4}}$, $\int \frac{x}{\sqrt{x^2-4}} dx = \sqrt{x^2-4} + c$

6 $b = \frac{\pi}{4}, \frac{3\pi}{4}$

7 **a** $y = \frac{1}{5}x^5 - \frac{2}{3}x^3 + x + c$ **b** $y = 400x + 40e^{-\frac{x}{2}} + c$

8 $f(x) = 3x^3 + 5x^2 + 6x - 1$ **9** $a = \ln \sqrt{2}$

10 **a** $f(x) = \frac{1}{4}x^4 + \frac{1}{3}x^3 - \frac{10}{3}x + 3$ **b** $3x + 26y = 84$

11 **a** $e^{3x} + 6e^{2x} + 12e^x + 8$ **b** $\frac{1}{3}e^3 + 3e^2 + 12e - 7\frac{1}{3}$

REVIEW SET 15B

1 **a** $A = \frac{17}{4}$, $B = \frac{25}{4}$ **b** $\int_0^2 (4-x^2) dx \approx \frac{21}{4}$

2 **a** $-2e^{-x} + 3x + c$ **b** $\frac{2}{3}x\sqrt{x} - 2\sqrt{x} + c$
c $9x + 3e^{2x-1} + \frac{1}{4}e^{4x-2} + c$

3 $f(x) = \frac{1}{3}x^3 - \frac{3}{2}x^2 + 2x + 2\frac{1}{6}$ **4** $\frac{2}{3}(\sqrt{5} - \sqrt{2})$

5 $\frac{d}{dx}(3x^2+x)^3 = 3(3x^2+x)^2(6x+1)$
 $\int (3x^2+x)^2(6x+1) dx = \frac{1}{3}(3x^2+x)^3 + c$

6 **a** 6 **b** 3 **7** $f\left(\frac{\pi}{2}\right) = 3 - \frac{\pi}{2}$ **8** $e^{-\pi}$

9 $\frac{1}{2(n+1)}(2x+3)^{n+1} + c$, $n \neq -1$

10 $a = \frac{1}{3}$, $f'(x) = 2\sqrt{x} + \frac{1}{3\sqrt{x}}$ is never 0 as $\sqrt{x} \geq 0$ for all x
 $\therefore f'(x) > 0$ for all x

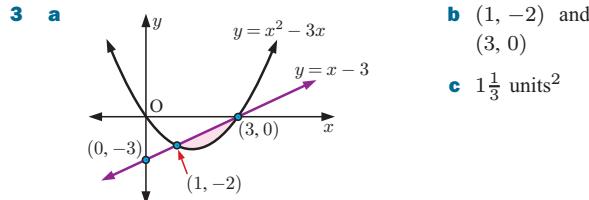
11 $a = 0$ or ± 3

EXERCISE 16A

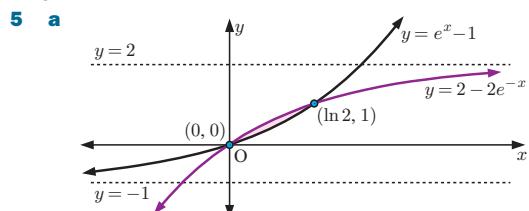
- 1** **a** 30 units² **b** $\frac{9}{2}$ units² **c** $\frac{27}{2}$ units² **d** 2 units²
2 **a** $\frac{1}{3}$ units² **b** 2 units² **c** $63\frac{3}{4}$ units²
d $(e - 1)$ units² **e** $20\frac{5}{6}$ units² **f** 18 units²
g $\frac{1}{2}$ units² **h** $4\frac{1}{2}$ units² **i** $(2e - \frac{2}{e})$ units²
3 $\frac{2}{3}$ units²

EXERCISE 16B

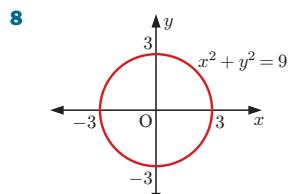
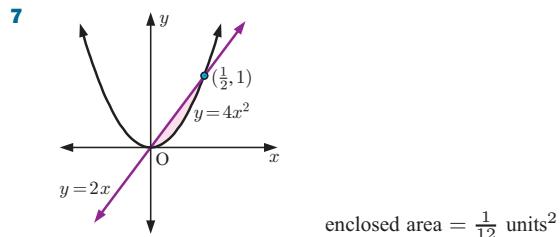
- 1** **a** $4\frac{1}{2}$ units² **b** $(1 + e^{-2})$ units² **c** $1\frac{5}{27}$ units²
d 2 units² **e** $2\frac{1}{4}$ units² **f** $(\frac{\pi}{2} - 1)$ units²
2 $10\frac{2}{3}$ units²



4 $\frac{1}{3}$ units²



6 $\frac{1}{2}$ units²



a Rearranging $x^2 + y^2 = 9$ gives $y = \pm\sqrt{9 - x^2}$. The upper half has $y \geq 0$, so $y = \sqrt{9 - x^2}$.

b $\frac{9\pi}{4}$

9 **a** $40\frac{1}{2}$ units² **b** 8 units² **c** 8 units²

10 **a** C_1 is $y = \sin x$, C_2 is $y = 3 \sin x$ **b** 4 units²

11 **a** $\int_3^5 f(x) dx = -$ (area between $x = 3$ and $x = 5$)

b $\int_1^3 f(x) dx - \int_3^5 f(x) dx + \int_5^7 f(x) dx$

12 **a** C_1 is $y = \frac{1}{2} + \frac{1}{2} \cos(2x)$, C_2 is $y = \cos(2x)$

b A(0, 1), B($\frac{\pi}{4}$, 0), C($\frac{\pi}{2}$, 0), D($\frac{3\pi}{4}$, 0), E(π , 1)

c Area = $\int_0^\pi (\frac{1}{2} + \frac{1}{2} \cos(2x) - \cos(2x)) dx$

- 13** If $h(x) \geq 0$ on $a \leq x \leq b$, the area between $y = h(x)$ and the x -axis is $\int_a^b h(x) dx$. If $h(x) < 0$ on $a \leq x \leq b$, the area between $y = h(x)$ and the x -axis is $\int_a^b -h(x) dx$.
 \therefore the area between $y = h(x)$ and the x -axis on $a \leq x \leq b$ is $\int_a^b |h(x)| dx$.

Letting $h(x) = f(x) - g(x)$, the area between $y = f(x) - g(x)$ and the x -axis $y = 0$ on $a \leq x \leq b$ is $\int_a^b |f(x) - g(x)| dx$.

Equivalently, the area between $y = f(x)$ and $y = g(x)$ on $a \leq x \leq b$ is $\int_a^b |f(x) - g(x)| dx$.

14 $b \approx 1.3104$

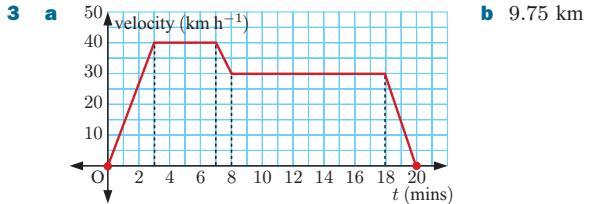
15 $a = \sqrt{3}$

EXERCISE 16C.1

1 110 m

- 2** **a** **i** travelling forwards
ii travelling backwards (opposite direction)

b 16 km **c** 8 km from starting point (on positive side)

**EXERCISE 16C.2**

1 **a** $s(t) = t - t^2 + 2$ cm **b** $\frac{1}{2}$ cm **c** 0 cm

2 **a** $s(t) = \frac{1}{3}t^3 - \frac{1}{2}t^2 - 2t$ cm **b** $5\frac{1}{6}$ cm

c 1 $\frac{1}{2}$ cm left of its starting point

3 $\frac{\sqrt{3}+2}{4}$ m

4 **a** $s(t) = 32t + 2t^2 + 16$ m

b no change of direction

so displacement = $s(t_1) - s(0) = \int_0^{t_1} (32 + 4t) dt$

c acceleration = 4 m s^{-2}

5 **a** 41 units **b** 34 units **6** **b** 2 m

7 **a** 40 ms^{-1} **b** 47.8 ms^{-1} **c** 1.39 seconds

d as $t \rightarrow \infty$, $v(t) \rightarrow 50$ from below

e $a(t) = 5e^{-0.5t}$ and as $e^x > 0$ for all x , $a(t) > 0$ for all t .

f

g ≈ 134.5 m



8 **a** $v(t) = -\frac{1}{(t+1)^2} + 1 \text{ ms}^{-1}$

b $s(t) = \frac{1}{t+1} + t - 1$ m

- c** The particle is $\frac{4}{3}$ m to the right of the origin, moving to the right at $\frac{8}{9} \text{ m s}^{-1}$, and accelerating at $\frac{2}{27} \text{ m s}^{-2}$.

9 a $v(t) = \frac{t^2}{20} - 3t + 45 \text{ m s}^{-1}$

- b** $\int_0^{60} v(t) dt = 900$. The train travels a total of 900 m in the first 60 seconds.

- 10 a** Show that $v(t) = 100 - 80e^{-\frac{1}{20}t} \text{ m s}^{-1}$ and as $t \rightarrow \infty$, $v(t) \rightarrow 100 \text{ m s}^{-1}$.

b 370.4 m

REVIEW SET 16A

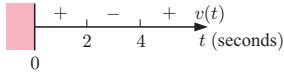
1 $A = \int_a^b [f(x) - g(x)] dx + \int_b^c [g(x) - f(x)] dx + \int_c^d [f(x) - g(x)] dx$

2 a $2 + \pi$ **b** -2 **c** π

3 No, total area shaded $= \int_{-1}^1 f(x) dx - \int_1^3 f(x) dx$.

4 $k = \sqrt[3]{16}$ **5** 4.5 units²

6 a



- b** The particle moves in the positive direction initially, then at $t = 2$, $6\frac{2}{3}$ m from its starting point, it changes direction. It changes direction again at $t = 4$, $5\frac{1}{3}$ m from its starting point, and at $t = 5$, it is $6\frac{2}{3}$ m from its starting point again.

c $6\frac{2}{3}$ m **d** $9\frac{1}{3}$ m

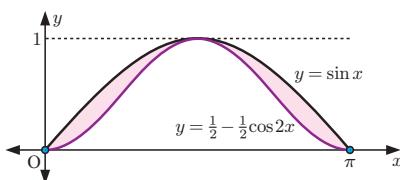
7 $(3 - \ln 4)$ units² **8** 2.35 m

REVIEW SET 16B

1 a $v(t) = 3t^2 - 30t + 27 \text{ cm s}^{-1}$

b -162 cm (162 cm to the left of the origin)

2 a



c $(1 - \frac{\pi}{4})$ units²

3 $a = \ln 3$, $b = \ln 5$

4 a $a(t) = 2 - 6t \text{ m s}^{-2}$ **b** $s(t) = t^2 - t^3 + c \text{ m}$

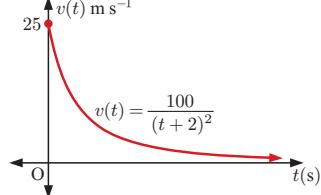
c -4 m (4 m to the left)

5 $k = \frac{4}{3}$ **6** $m = \frac{\pi}{3}$ **7** $(\frac{4}{\pi} - 1)$ units²

8 a $v(0) = 25 \text{ m s}^{-1}$, $v(3) = 4 \text{ m s}^{-1}$

b as $t \rightarrow \infty$, $v(t) \rightarrow 0$ from above

c



d 3 seconds

e $a(t) = \frac{-200}{(t + 2)^3} \text{ m s}^{-2}$, $t \geq 0$

f $k = \frac{1}{5}$

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