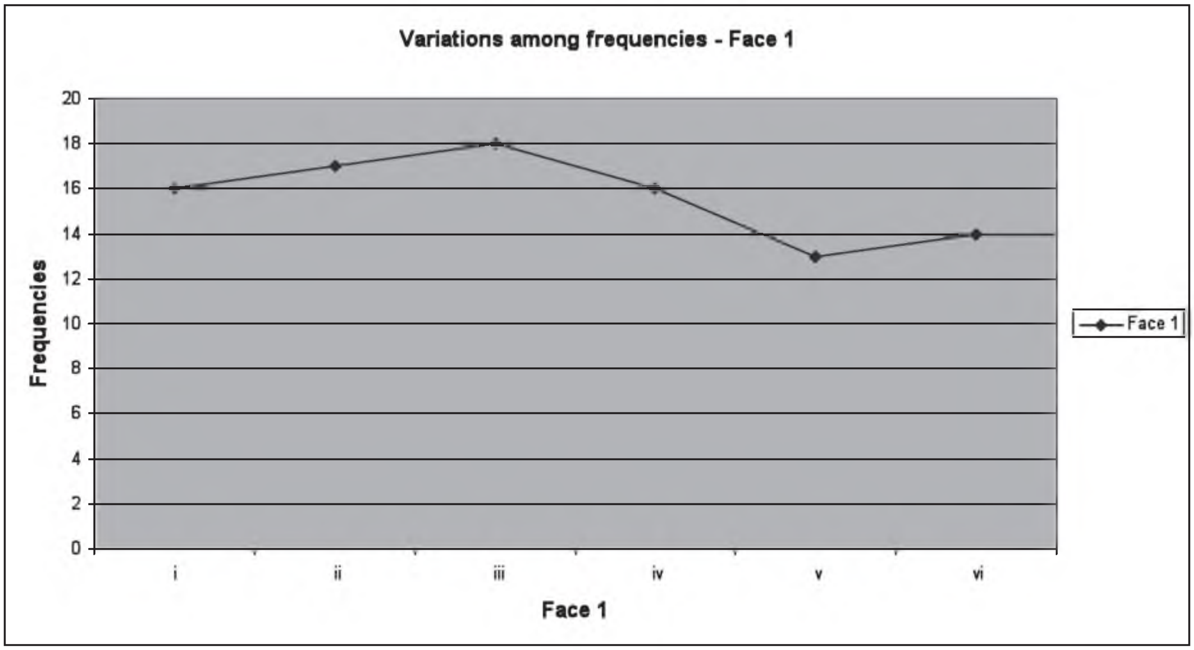
The following line graph illustrates the variations among the frequencies for face 1.



Graphs for each of faces 2, 3, 4, 5, and 6 may be shown.

Mathematical Knowledge/Analysis of Data

Choosing to use the different methods for carrying out these experiments, as described in Data Collection, took into account that different conditions of the throws of the die may have significant influences in the outcomes of these throws. The size of the cups chosen may have a particular influence on these outcomes. The inside surface of the two types of cups chosen are also factors that may influence these outcomes. The number of times the die is tossed around in the palm of the hand and/or the number of times it is tossed around in the cups may influence these outcomes. The different coverings of the surface of the table top may also influence these outcomes.

In the absence of more in-depth and elaborate statistical techniques, these simple experiments were intended to give some idea of the theory of classical probability. The limiting relative frequency of an event over a long series of trials is the conceptual foundation of the frequency interpretation of probability. In this framework, it is assumed that as the length of the series increases without bound, the fraction of the experiments in which we observe the event will stabilize.

120 throws under each of the conditions selected should allow for simple comparison of the observed and theoretical frequencies.

Using the principle of relative probability, the following table shows the probability distribution for Experiment (i) and the theoretical probability of obtaining any of the faces numbered 1, 2, 3, 4, 5, 6 landing uppermost.

# on face	1	2	3	4	5	6
Relative probability	$\frac{2}{15}$ = 0.13	$\frac{7}{60}$ = 0.12	$\frac{9}{40}$ = 0.23	$\frac{1}{8}$ = 0.13	$\frac{5}{24}$ = 0.21	$\frac{23}{120}$ = 0.19
Theoretical probability	$\frac{1}{6}$ = 0.17	$\frac{1}{6}$ = 0.17	$\frac{1}{6}$ = 0.17	$\frac{1}{6}$ = 0.17	$\frac{1}{6}$ = 0.17	$\frac{1}{6}$ = 0.17

Comparisons of the differences of the observed and theoretical frequencies for 120 throws of the die under the conditions described should be considered as sufficient for an explanation of any significant variation in determining whether the die was biased in favour of any particular face. Barring any significant variation among the relative frequencies, it may be reasoned that the die is not biased.

The relative probabilities can also be calculated for Experiments (ii) through (vi)

Furthermore we can combine the results of all six experiments to arrive at an overall probability for each face as shown in the table below:

# on face	1	2	3	4	5	6
Relative frequency	$\frac{96}{720}$ = 0.13	$\frac{124}{720}$ = 0.17	$\frac{130}{720}$ = 0.18	$\frac{128}{720}$ = 0.18	$\frac{123}{720}$ = 0.17	$\frac{121}{720}$ = 0.17

The above table clearly shows that the relative frequency of each face is close to the true probability (0.17) when the number of trials (720) is large. This is strong evidence to claim that the die is unbiased even though there were differences among the observed frequencies for the six experiments.

Further analysis must be taken in light of any limitations that the project may have. Considering the mean and standard deviation of each of these experiments, account may be taken of the size of the variations of the observed and theoretical values. This aspect may explain any significant variation from the expected mean and variance of these outcomes

The standard deviations for the frequencies of faces 1 through 6 for Experiments (i), (ii), (iii), (iv), (v) and (vi) are 1.67, 1.71, 1.62, 1.63, 1.63 and 1.60 respectively.

Except for Face #2 and to a lesser extent (Face #1), the variances among the outcomes do not appear to suggest significant differences in the results.

Conclusions

These experiments can be considered simplistic but reasonably effective for the purpose of determining bias in an ordinary gaming die. The number of throws, 120, may be considered sufficient for obtaining relative frequencies and relative probability for the experiments. Increasing the number of throws should result in observed frequencies very close to the theoretical frequencies.

Further statistical analyses can explain variations between the observed and theoretical results. These experiments may be refined by using other methods of throwing the die. Results can be compared for similarity among these results and for a reasonable conclusion about fairness of the die.

Procedures for Reporting and Submitting School Based Assessment

(i) Teachers are required to record the mark awarded to each candidate under the appropriate profile dimension on the mark sheet provided by CXC. The completed mark sheets should be submitted to CXC no later than April 30 of the year of the examination.

Note: The school is advised to keep a copy of the project for each candidate as well as copies of the mark sheets.

(ii) Teachers will be required to submit to CXC copies of the projects of a sample of candidates as indicated by CXC. This sample will be re-marked by CXC for moderation purposes.

Moderation of School Based Assessment

The candidate's performance on the project will be moderated. The standard and range of marks awarded by the teacher will be adjusted where appropriate. However, the rank order assigned by the teacher will be adjusted only in special circumstances and then only after consideration of the data provided by the sample of marked projects submitted by the teacher and re-marked by CXC.

♦ RESOURCES

The following is a list of books and other printed material that might be used for Additional Mathematics. The list is by no means exhaustive. Each student should have access to at least one text.

Talbert, J. F. And Heng, H. H. *Additional Mathematics – Pure and Applied*, Singapore: Longman Publishers, 1991.

Website:
http://www.saskschools.ca/curr_content/physics30kindy n/ for kinematics

♦ GLOSSARY

KEY TO ABBREVIATIONS

K - Knowledge
C - Comprehension
R - Reasoning

WORD	**DEFINITION**	**NOTES**
analyse	examine in detail	
annotate	add a brief note to a label	Simple phrase or a few words only.
apply	use knowledge/principles to solve problems	Make inferences/conclusions.
assess	present reasons for the importance of particular structures, relationships or processes	Compare the advantages and disadvantages or the merits and demerits of a particular structure, relationship or process.
calculate	arrive at the solution to a numerical problem	Steps should be shown; units must be included.
classify	divide into groups according to observable characteristics	
comment	state opinion or view with supporting reasons	
compare	state similarities and differences	An explanation of the significance of each similarity and difference stated may be required for comparisons which are other than structural.
construct	use a specific format to make and/or draw a graph, histogram, pie chart or other representation using data or material provided or drawn from practical investigations, build (for example, a model), draw scale diagram	Such representations should normally bear a title, appropriate headings and legend.

WORD	DEFINITION	NOTES
deduce	make a logical connection between two or more pieces of information; use data to arrive at a conclusion	
define	state concisely the meaning of a word or term	This should include the defining equation/formula where relevant.
demonstrate	show; direct attention to...	
derive	to deduce, determine or extract from data by a set of logical steps some relationship, formula or result	This relationship may be general or specific.
describe	provide detailed factual information of the appearance or arrangement of a specific structure or a sequence of a specific process	Description may be in words, drawings or diagrams or any appropriate combination. Drawings or diagrams should be annotated to show appropriate detail where necessary.
determine	find the value of a physical quantity	
design	plan and present with appropriate practical detail	Where hypotheses are stated or when tests are to be conducted, possible outcomes should be clearly stated and/or the way in which data will be analyzed and presented.
develop	expand or elaborate an idea or argument with supporting reasons	
diagram	simplified representation showing the relationship between components	
differentiate/distinguish (between/among)	state or explain briefly those differences between or among items which can be used to define the items or place them into separate categories	
discuss	present reasoned argument; consider points both for and against; explain the	

WORD	DEFINITION	NOTES
	relative merits of a case	
draw	make a line representation from specimens or apparatus which shows an accurate relation between the parts	In the case of drawings from specimens, the magnification must always be stated.
estimate	make an approximate quantitative judgement	
evaluate	weigh evidence and make judgements based on given criteria	The use of logical supporting reasons for a particular point of view is more important than the view held; usually both sides of an argument should be considered.
explain	give reasons based on recall; account for	
find	locate a feature or obtain as from a graph	
formulate	devise a hypothesis	
identify	name or point out specific components or features	
illustrate	show clearly by using appropriate examples or diagrams, sketches	
interpret	explain the meaning of	
investigate	use simple systematic procedures to observe, record data and draw logical conclusions	
justify	explain the correctness of	
label	add names to identify structures or parts indicated by pointers	
list	itemize without detail	
measure	take accurate quantitative readings using appropriate instruments	

WORD	DEFINITION	NOTES
name	give only the name of	No additional information is required.
note	write down observations	
observe	pay attention to details which characterize a specimen, reaction or change taking place; to examine and note scientifically	Observations may involve all the senses and/or extensions of them but would normally exclude the sense of taste.
outline	give basic steps only	
plan	prepare to conduct an investigation	
predict	use information provided to arrive at a likely conclusion or suggest a possible outcome	
record	write an accurate description of the full range of observations made during a given procedure	This includes the values for any variable being investigated; where appropriate, recorded data may be depicted in graphs, histograms or tables.
relate	show connections between; explain how one set of facts or data depend on others or are determined by them	
sketch	make a simple freehand diagram showing relevant proportions and any important details	
state	provide factual information in concise terms outlining explanations	
suggest	offer an explanation deduced from information provided or previous knowledge. (... a hypothesis; provide a generalization which offers a likely explanation for a set of data or observations.)	No correct or incorrect solution is presumed but suggestions must be acceptable within the limits of scientific knowledge.
use	apply knowledge/principles to solve problems	Make inferences/conclusions.

<u>Western Zone Office</u>
3 May 2010

CARIBBEAN EXAMINATIONS COUNCIL

Caribbean Advanced Proficiency Examination
CAPE®

ADDITIONAL MATHEMATICS

Specimen Papers and Mark Schemes/Keys

Specimen Papers:	-	Unit 1, Paper 01 (Mark Scheme included) Unit 1, Paper 02 Unit 1, Paper 03/2
Mark Schemes and Keys:	-	Unit 1, Paper 02 Unit 1, Paper 03/2

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

SECONDARY EDUCATION CERTIFICATE EXAMINATION

ADDITIONAL MATHEMATICS

SPECIMEN PAPER

Paper 01 – General Proficiency

90 minutes

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This paper consists of **45** items. You will have 90 minutes to answer them.

2. In addition to this test booklet, you should have an answer sheet.

3. Each item in this test has four suggested answers, lettered (A), (B), (C) and (D). Read each item you are about to answer and decide which choice is best.

4. On your answer sheet, find the number which corresponds to your item and shade the space having the same letter as the answer you have chosen. Look at the sample item below.

Sample Item

Evaluate $(4^{-2})^2 \div (1/16)^2$

(A) 4^{-2}
(B) 4^{-1}
(C) 4^{0}
(D) 4^{2}

Sample Answer

A B ● D

The best answer to this item is "   ", so answer space (C) has been shaded.

5. If you want to change your answer, erase it completely before you fill in your new choice.

6. When you are told to begin, turn the page and work as quickly and as carefully as you can. If you cannot answer an item, omit it and go on to the one. You can return later to the item omitted. You score will be the number of correct answers produced.

7. You may do any rough work in the booklet.

8. You may use a silent non-programmable calculator to answer questions.

1. Given that $f(x) = x^3 + 2x^2 - 5x + k$, and that $x - 2$ is a factor of $f(x)$ then k is equal to

(A) -6
(B) -2
(C) 2
(D) 6

2. $a(b + c) - b(a + c)$ is equal to

(A) $a(c - b)$
(B) $a(b - c)$
(C) $c(a - b)$
(D) $c(b - a)$

3. The value of $\sum_{r=1}^{20}(3r - 1)$ is

(A) 590
(B) 610
(C) 650
(D) 1220

4. A teacher illustrates AP's by cutting a length of string into 10 pieces so that the lengths of the pieces are in arithmetic progression and the entire length of the string is used up exactly. If the first piece measures 30 cm and the fourth piece measures 24 cm, the total length of the string is

(A) 60 cm
(B) 210 cm
(C) 240 cm
(D) 390 cm

5. The first term of a GP is 16 and the fifth term is 81. Given that the common ratio is positive, the value of the 4th term is

(A) $\frac{81}{16}$
(B) 24
(C) 54
(D) 64

6. The first four terms of a convergent GP is given by 81, 27, 9, 3. The sum to infinity of this GP is

(A) 54
(B) 120.5
(C) 121.5
(D) 243

7. Given that $2 \times 4^{x+1} = 16^{2x}$, the value of x is

(A) -1
(B) $\frac{1}{4}$
(C) $\frac{1}{3}$
(D) $\frac{1}{2}$

8. $\sqrt[n]{2 \times 4^m}$ is equal to

(A) $\sqrt[n]{8^m}$
(B) 2^{n+2m}
(C) 2^{n+mn}
(D) $2^{\frac{2m+1}{n}}$

9. Given that $\log_2 x + \log_2(6x + 1) = 1$, the value of x is

(A) $-\frac{2}{3}$
(B)
(C) $\frac{2}{3}$
(D) $\frac{3}{2}$

10. The value of

$\log_4 (8) - \log_4 (2) + \log_4 \left(\frac{1}{16}\right)$ is

(A) -1

(B) $\frac{1}{2}$

(C) 3

(D) 4

11. The expression $\frac{1+\sqrt{3}}{\sqrt{3}-1}$ when simplified is equal to

(A) -1

(B) 1

(C) $\frac{\sqrt{3}+2}{2}$

(D) $\sqrt{3}+2$

12. $f(x) = -5 - 8x - 2x^2$. By completing the square f (x) can be expressed as

(A) $2(x+2)^2 - 4$

(B) $4 - 2(x-2)^2$

(C) $3 - 2(x+2)^2$

(D) $3 - 2(x-2)^2$

13. The roots of the equation $2x^2 - x + 1 = 0$ are

(A) real and equal
(B) real and distinct
(C) not real and equal
(D) not real and distinct

14. For $-2 \leq x \leq 2$, the maximum value of $4-(x+1)^2$, and the value of x for which $4-(x+1)^2$ is maximum are respectively

(A) 5 and 1
(B) 2 and -1
(C) 4 and -1
(D) 4 and 1

15. $f(x) = x(x+5)+6$. Given that f (x) is one-to-one for $x \geq k$, the value of k is

(A) $-\frac{5}{2}$

(B) $-\frac{2}{5}$

(C) $\frac{2}{5}$

(D) $\frac{5}{2}$

16. If a function f is defined by $f : x \rightarrow \frac{x+3}{x-1}$, $x \neq 1$, then $f^{-1}(-4)$ is equal to

(A) -1

(B) $\frac{1}{5}$

(C) 1

(D) 5

17. A function g is defined by $g : x \rightarrow 3x - 1$. Expressed in terms of a, g $(3a - 1)$ is

(A) $9a - 1$
(B) $3a - 4$
(C) $9a - 2$
(D) $9a - 4$

18. Functions f and g are defined by
$f : x \rightarrow 3x - 2$ and
$g : x \rightarrow \frac{12}{x} - 4,\ x \neq 0$.

The composite function fg is defined by

(A) $\text{fg} : x \rightarrow \frac{36}{x} - 4,\ x \neq 0$

(B) $\text{fg} : x \rightarrow \frac{12}{x} - 12,\ x \neq 0$

(C) $\text{fg} : x \rightarrow \frac{12}{x} - 6,\ x \neq 0$

(D) $\text{fg} : x \rightarrow \frac{36}{x} - 14,\ x \neq 0$

19. The range of values for which $2x^2 < 5x + 3$ is

(A) $-\frac{1}{2} < x < 3$

(B) $\frac{1}{2} < x < 3$

(C) $x < -\frac{1}{2}$ and $x < 3$

(D) $x > -\frac{1}{2}$ and $x > 3$

20. The values of x which satisfy the inequality $\frac{2x - 3}{x + 1} > 0$ are

(A) $x > -1$ and $x > \frac{3}{2}$

(B) $x > \frac{3}{2}$

(C) $x < -1$ or $x > \frac{3}{2}$

(D) $x > -1$

21. The coordinates of the points A and B are (2, -3) and (-10, -5) respectively. The perpendicular bisector to the line AB is given by the equation

(A) $x - 6y + 20 = 0$

(B) $6x + y + 28 = 0$

(C) $x + 6y - 20 = 0$

(D) $6x + y - 28 = 0$

22. The lines $2y - 3x - 13 = 0$ and $y + x + 1 = 0$ intersect at the point P, where the coordinates of P are

(A) (3, 2)
(B) (3, -2)
(C) (-3, -2)
(D) (-3, 2)

23. The radius, r, and the coordinates of the centre, C, of the circle with equation $x^2 + y^2 - 6x + 4y - 12 = 0$ are

(A) $r = 5$, C (-2, 3),

(B) $r = 25$, C (2, -3),

(C) $r = 12$, C (-3, 2),

(D) $r = 5$, C (3, -2),

24. If the length of the vector $\mathbf{p} = 2\mathbf{i} - k\mathbf{j}$ is $\sqrt{13}$ and k is real, then

I. $k = 3$
II. $k = -3$
III. $k = \sqrt{17}$
IV. $k = -\sqrt{17}$

(A) I or II only
(B) I or III only
(C) I or IV only
(D) II or IV only

25. The value of the real number t for which the two vectors $\mathbf{a} = 4\mathbf{i} + t\mathbf{j}$ and $\mathbf{b} = 2\mathbf{i} - 3\mathbf{j}$ are parallel is

(A) -6

(B) $-\frac{3}{4}$

(C) $\frac{4}{3}$

(D) 6

26. The position vectors of A and B relative to an origin O are $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 7 \\ 4 \end{pmatrix}$ respectively. The acute angle AOB is given by

(A) $\cos^{-1}\left(\frac{2}{\sqrt{65}}\right)$

(B) $\cos^{-1}\left(\frac{\sqrt{26}}{13 \times 65}\right)$

(C) $\cos^{-1}\left(\frac{\sqrt{2}}{\sqrt{65}}\right)$

(D) $\cos^{-1}\left(\frac{26}{\sqrt{13}\sqrt{65}}\right)$

27. The trigonometrical expression $\frac{1 + \sin x}{\cos x} + \frac{\cos x}{1 + \sin x}$ is identical to

(A) 1

(B) $\frac{2}{\cos x}$

(C) $\frac{1 + \sin x + \cos x}{\cos x(1 + \sin x)}$

(D) $\frac{2}{\cos x(1 + \sin x)}$

28. $\cos(A - B) - \cos(A + B) \equiv$

(A) $2 \sin A \sin B$
(B) $-2 \sin A \cos B$
(C) $2 \cos A \sin B$
(D) $2 \cos A \cos B$

29. If $\sin\theta = \frac{15}{17}$ and θ is obtuse, then $\cos\theta$ is equal to

(A) $-\frac{8}{15}$

(B) $-\frac{8}{17}$

B. $\frac{8}{15}$

(D) $\frac{8}{17}$

30. The smallest positive angle for which the equation $\sin\theta + \cos\theta = 0$ is

(A) $\frac{\pi}{4}$

(B) $\frac{3\pi}{4}$

(C) $\frac{5\pi}{4}$

(D) $\frac{7\pi}{4}$

31. For $0 \le \theta \le 2\pi$, solutions for the equation $4\sin^2\theta - 1 = 0$ exist in quadrants

(A) 1, 2 and 3
(B) 1, 3 and 4
(C) 2, 3 and 4
(D) 1, 2, 3 and 4

32. $2\sin\left(x - \frac{\pi}{2}\right)$ is equal to

(A) 2 sin x - 2

(B) - 2 cos x

(C) $2\cos\left(x + \frac{\pi}{2}\right)$

(D) 2 sin x - π

33. For which of the following ranges of values is f (x) = 2 + cos 3x valid?

(A) $1 \leq f(x) \leq 3$

(B) $-1 \leq f(x) \leq 1$

(C) $-2 \leq f(x) \leq 2$

(D) $0 \leq f(x) \leq 2$

34. For $0 \leq x \leq 2\pi$, the values of x which satisfy the equation $2\cos^2 x + 3\sin x = 0$ are

(A) $x = \frac{\pi}{6}, x = \frac{5\pi}{6}$

(B) $x = -\frac{\pi}{6}, x = -\frac{5\pi}{6}$

(C) $x = \frac{7\pi}{6}, x = \frac{11\pi}{6}$

(D) $x = \frac{5\pi}{6}, x = \frac{7\pi}{6}$

35. Given that $y = (3x-2)^3$, then $\frac{dy}{dx} =$

(A) $3(3x - 2)^2$

(B) $3(3x)^2$

(C) $3(3x - 2)^3$

(D) $9(3x - 2)^2$

36. Given that $y = \frac{3x+5}{2x-11}$, then $\frac{dy}{dx} =$

(A) $\frac{(3x+5)(2) + (2x-11)(3)}{(2x-11)^2}$

(B) $\frac{(2x-11)(3) + (3x+5)(2)}{(2x-11)^2}$

(C) $\frac{(2x-11)(3) - (3x+5)(2)}{(2x-11)^2}$

(D) $\frac{(3x+5)(2) - (2x-11)(3)}{(2x-11)^2}$

37. The curve C is given by the equation $y = 3\sin x + 2$. The value of $\frac{dy}{dx}$ at the point where $x = \frac{\pi}{3}$ is

(A) $\frac{1}{2}$

(B) $\frac{3}{2}$

(C) $\frac{7}{2}$

(C) 3

38. The point P (2, 2) lies on the curve with equation $y = x(x-3)^2$. The equation of the normal to the curve at the point P is given by

(A) $y - 2 = 3(x - 2)$

(B) $y - 2 = -3(x - 2)$

(C) $y - 2 = \frac{1}{3}(x - 2)$

(D) $y - 2 = \frac{-1}{3}(x - 2)$

39. The curve C is given by the equation $y = 4x + \frac{9}{x}$. The second derivative, $\frac{d^2y}{dx^2}$, is given by

(A) $4 + \frac{9}{x^3}$

(B) $\frac{18}{x^3}$

C. $4 - \frac{9}{x^3}$

(D) $-\frac{9}{2x^3}$

40. The positive value of z for which $\int_0^z x^2 \, dx = 9$ is

(A) 3
(B) 4.5
(C) 9
(D) 27

41. The gradient of the tangent to a curve C at (x, y) is given by $\frac{dy}{dx} = \frac{1}{(3x+4)^2}$. The curve passes through the point $P\left(-1, \frac{2}{3}\right)$. The equation of the curve C is given by

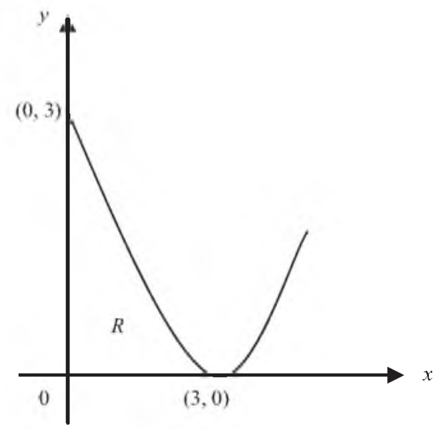
(A) $y = \frac{2}{(3x+4)} + 1$

(B) $y = \frac{-6}{(3x+4)^3}$

(C) $y = \frac{-2}{3(3x+4)} + 4$

(D) $y = \frac{-1}{3(3x+4)} + 1$

Item **42** refers to the figure below.



42. The finite region R is bounded by the y-axis, the x-axis, and the arc of the curve $y = (x-3)^2$ as shown in the figure above. The area of R in square units is

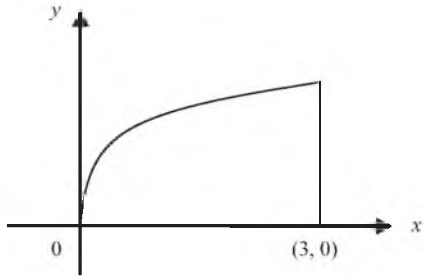
(A) 1

(B) 3

(C) 9

(D) 27

<u>Item 43</u> refers to the figure below.



43. The finite region enclosed by the curve $y = \sqrt{x}$, $x \geq 0$, the x-axis and the line $x = 3$, as shown in the figure above, is rotated completely about the x-axis. The volume of the solid of revolution formed is given by

(A) $\int_0^3 \sqrt{x}\, dx$

(B) $\pi \int_0^3 x\, dx$

(C) $\pi \int_0^3 \sqrt{x}\, dx$

(D) $\pi \int_0^3 x^2\, dx$

44. $\int (2x + 3)^5\, dx =$

(A) $\left[\frac{1}{6}(2x+3)^6\right] + C$

(B) $\left[\frac{1}{2}(2x+3)^6\right] + C$

(C) $\left[\frac{1}{12}(2x+3)^6\right] + C$

(D) $\left[\frac{1}{3}(2x+3)^6\right] + C$

45. Given $\frac{dy}{dx} = 3 \sin x - 2 \cos x$, the indefinite integral is given by

(A) $y = 3 \cos x - 2 \sin x + C$

(B) $y = -3 \cos x + 2 \sin x + C$

(C) $y = -3 \cos x - 2 \sin x + C$

(D) $y = 3 \cos x + 2 \sin x + C$

END OF TEST

TEST CODE 01254020/SPEC

FORM TP 2011037/SPEC

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

SECONDARY EDUCATION CERTIFICATE EXAMINATION

ADDITIONAL MATHEMATICS

SPECIMEN PAPER

Paper 02 – General Proficiency

2 hours and 40 minutes

INSTRUCTIONS TO CANDIDATES

1. DO NOT open this examination paper until instructed to do so.

2. This paper consists of FOUR sections. Answer ALL questions in Section 1, Section 2 and Section 3.

3. Answer ONE question in Section 4.

4. Write your solutions with full working in the booklet provided.

Required Examination Materials

Electronic calculator (non programmable)
Geometry Set
Mathematical Tables (provided)
Graph paper (provided)

SECTION 1

Answer BOTH questions.

All working must be clearly shown.

1. (a) A function f : R→ R is defined on domain R and co-domain R, where R is the set of real numbers, by the rule

$$f(x) = x^2$$

(i) State with reason, whether f is many-to-one or one-to-one.

[1 mark]

(ii) If instead, the domain of f is the set of non-negative real numbers,

a) Determine a function g such that $g\left[f(x)\right] = x$ for all values of x in this domain. **[1 mark]**

b) On the same pair of axes, sketch the graphs of f and g. **[2 mark]**

(b) Use the remainder theorem, or otherwise, to find the reminder when $x^3 - 2x^2 + 4x - 21$ is divided by $x - 3$.

[2 marks]

(c) A student collects laboratory data for two quantities q and p as shown in Table 1.

Table 1

q	1	2	3	4
p	0.50	0.63	0.72	0.80

The student reasons a relationship of the form $p = aq^n$

(i) Use logarithms to reduce this relation to a linear form.

[2 marks]

(ii) Using the graph paper provided and a scale of 1 cm to represent 0.1 units on the horizontal axis and a scale of 2 cm to represent 0.1 units on the vertical axis, plot a suitable straight line graph and hence estimate the constants a and n.

[6 marks]

Total 14 marks

2. (a) Let $f(x) = 3x^2 + 12x - 18$.

(i) Express $f(x)$ in the form $a(x + b)^2 + c$. **[3 marks]**

(ii) State the minimum value of $f(x)$. **[1 mark]**

(iii) Determine the value of x for which $f(x)$ is a minimum. **[1 mark]**

(b) Find the set of values of x for which $2x^2 + 2 > 5x$. **[4 marks]**

(c) Given the series $\frac{1}{2} + \frac{1}{2^4} + \frac{1}{2^7} + \frac{1}{2^{10}} + \ldots$

(i) show that this series is geometric, **[3 marks]**

(ii) hence, find the sum to infinity of this series. **[2 marks]**

Total 14 marks

SECTION 2

Answer BOTH Questions

3. (i) Write the equation of the circle C, with centre $(-1, 2)$ and radius $\sqrt{13}$ units.

[1 mark]

(ii) Find the equation of the tangent to the circle C at the point P(2,4).

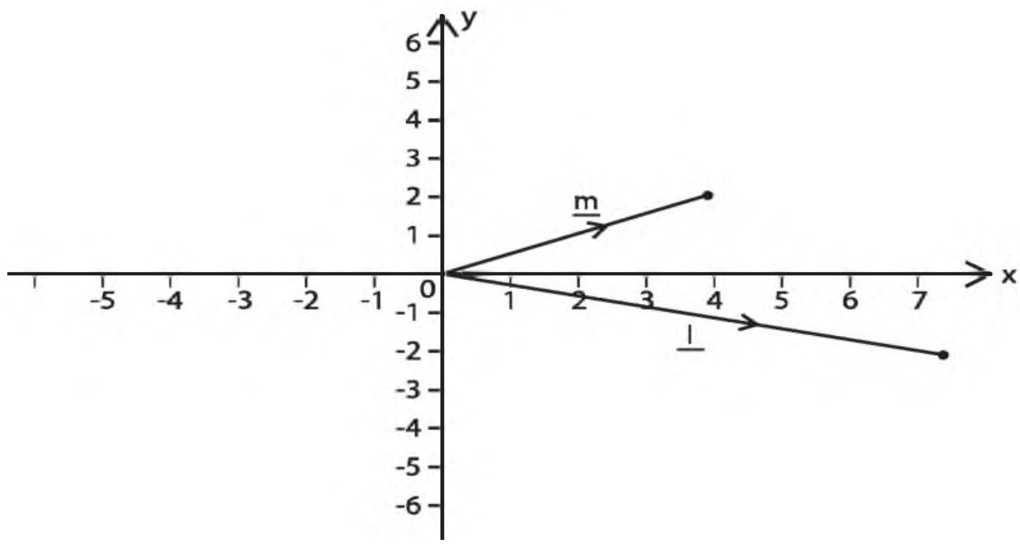
[4 marks]

(b) The position vector of two points, A and B, relative to a fixed origin O, are $3t\mathbf{i} + 2t\mathbf{j}$ and $4\mathbf{i} - 2t\mathbf{j}$ respectively, where $t > 0$.

Find the value of t such that $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are perpendicular.

[4 marks]

(c) The points L and M referred to a fixed origin O are represented by the vectors $\mathbf{l} = 7\mathbf{i} - 2\mathbf{j}$ and $\mathbf{m} = 4t\mathbf{i} + 2\mathbf{j}$ respectively, as shown in the diagram below.

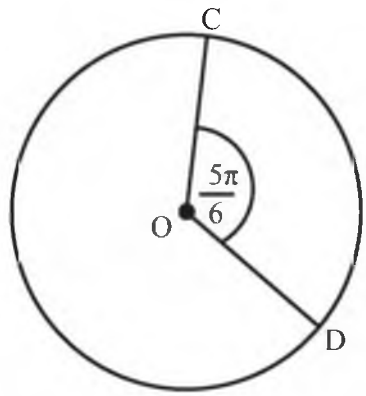


Find the unit vector in the direction of $\overrightarrow{LM}$.

[3 marks]

Total 12 marks

4. (a) The diagram below shows a circle of centre O and radius 6 cm. The sector COD subtends the angle $\frac{5\pi}{6}$ at the centre.



Working in **radians**, calculate, giving your answers in terms of π,

(i) the length of the minor arc CD

[1 mark]

(ii) the area of the minor sector OCD

[2 marks]

(b) (i) Given that $\sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$, where x is acute, show that

$$\sin\left(x - \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}(\sin x - \cos x).$$

[2 marks]

(ii) Using the fact that $\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$ and $\cos \frac{\pi}{3} = \frac{1}{2}$, find the exact value of $\sin \frac{\pi}{12}$ showing ALL steps in your working.

[3 marks]

(c) Prove the identity $\left(\tan\theta - \frac{1}{\cos\theta}\right)^2 \equiv -\frac{\sin\theta - 1}{\sin\theta + 1}$.

[4 marks]

Total 12 marks

SECTION 3

Answer BOTH Questions

5. (a) Differentiate the following with respect to x, simplifying your result as far as possible

$$(5-2x)(1+x)^4$$

[4 marks]

(b) The point P lies on the curve $y = x^2$. The value of x at P is -2.

Find the equation of the tangent to the curve at P.

[4 marks]

(c) Find the stationary points on the curve $f(x) = 2x^3 - 9x^2 + 12x$ and distinguish their nature.

[6 marks]

Total 14 marks

6. (a) Evaluate $\int_1^2 (3x-1)^2 \, dx$. **[4 marks]**

(b) Evaluate $\int_0^{\frac{\pi}{2}} (5 \sin x - 3 \cos x) \, dx$. **[4 marks]**

(c) A curve passes through the point P $\left(0, \frac{7}{2}\right)$ and is such that $\frac{dy}{dx} = 2 - x$.

(i) Find the equation of the curve. **[3 marks]**

(ii) Find the area of the finite region bounded by the curve, the x-axis, the y-axis and the line $x=5$.

[3 marks]

Total 14 marks

SECTION 4

Answer Only **ONE** Question

7. (a) In a Lower Sixth Form there are 43 students who are studying either Statistics or Physics or both Statistics and Physics. 28 students study Statistics and 19 students study Physics. If a student is selected at random, what is the probability that he/she is studying

(i) both Statistics and Physics, **[3 marks]**

(ii) Physics only. **[2 marks]**

(b) A tetrahedral die has four faces. Two fair tetrahedral dice, one red and one blue, have faces numbered 0, 1, 2, and 3 respectively. The dice are rolled and the numbers face down on the two dice are recorded. *R* is the score on the red die and *B* is the score on the blue die.

(i) Find $P(R = 3 \text{ and } B = 0)$.

[2 marks]

The random variable *T* is *R* multiplied by *B*.

(ii) Complete the diagram below to represent the sample space that shows all the possible values of *T*

3				
2		2		
1	0			
0				
B \ R	**0**	**1**	**2**	**3**

Sample space diagram of T

[3 marks]

The table below gives the probability of each possible value of *t*.

t	0	1	2	3	4	6	9
$P(T = t)$	a	$\frac{1}{16}$	$\frac{1}{8}$	b	c		

(iii) Find the values of a, *b* and *c*.

[3 marks]

7. (c) The number of cars parked on a local beachfront on each night in August last year is summarized in the following stem and leaf diagram.

1	0 5	
2	1 2 4 8	
3	0 3 3 3 4 7 8 8	Key: 1|0 means 10
4	1 1 3 5 8 8 8 9 9	
5	2 3 6 6 7	
6	2 3 4	

(i) Find the median and quartiles for these data.

[3 marks]

(ii) Construct a box-and-whisker plot to illustrate these data and comment on the shape of the distribution.

[4 marks]

Total 20 marks

8. (a) A car moves along a horizontal straight road, passing two points A and B. The speed of the car at A is 15m s^{-1}. When the driver passes A, he sees a warning sign W ahead of him, 120 m away. He immediately applies the brakes and the car decelerates uniformly, reaching W at a speed of 5 ms^{-1}. At W, the driver sees that the road is clear. He then immediately accelerates the car with uniform acceleration for 16 secs to reach a speed of V ms^{-1} where $V > 15$. He then maintains a constant speed of V ms^{-1} for 22 secs, passing B.

(i) Sketch, on the graph paper provided a velocity-time graph to illustrate the motion of the car as it moves from A to B.

[3 marks]

(ii) Find the time taken for the car to move from A to B.

[3 marks]

The distance from A to B is 1 km.

(iii) Find the value of V.

[5 marks]

(b) A particle moves in a straight line so that t seconds after passing through a fixed point O, its velocity, v m/s is given by

$$v = 3t^2 - 30t + 72.$$

Calculate the:

(i) values of t when the particle is at instantaneous rest,

[3 marks]

(ii) distance moved by the particle during the interval between the two values of t found in b (i).

[3 marks]

(iii) total distance moved by the particle between t = 0 and t =7.

[3 marks]

Total 20 marks

END OF TEST

TEST CODE 01254030/SPEC

FORM TP 2011038/SPEC

CARIBBEAN EXAMINATIONS COUNCIL

HEADQUARTERS

SECONDARY EDUCATION CERTIFICATE EXAMINATION

ADDITIONAL MATHEMATICS

PAPER 03/2

ALTERNATIVE

90 minutes

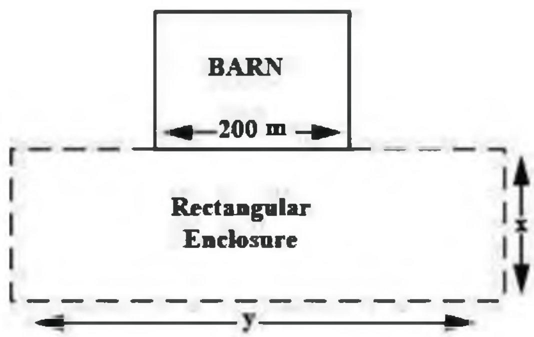
Answer the given questions

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO

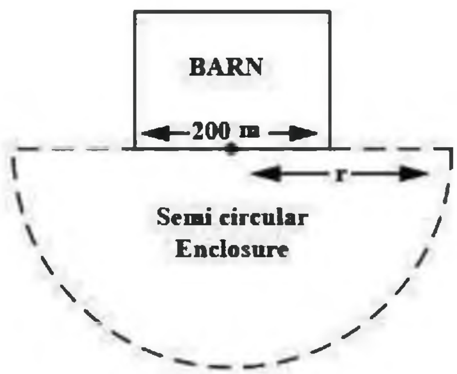
A farmer plans to construct an enclosure for his goats making use of one side of a barn that is 200 m in length. He has 800 m of fencing material available and wants to maximize the enclosed area.

The farmer must decide whether to make a rectangular or semicircular enclosure as shown in the diagrams below.

You are given that the radius of the semi circular enclosure is r, the length of the rectangular enclosure is x and the width is y. .



Option I



Option II

(i) Formulate the given real world problem **mathematically.**

(7 marks)

(ii) Show that for Option I a square enclosure maximizes the area and determine the MAXIMUM possible area.

(7 marks)

(iii) Determine the MAXIMUM area of the enclosure in Option II.

(4 marks)

(iv) Make a recommendation to the farmer on the MOST appropriate enclosure, giving a reason.

(2 marks)

Total 20 marks

CSEC ADDITIONAL MATHEMATICS SPECIMEN PAPER 01

Item	Key	Specific Objective
1	A	1A4
2	C	1A1
3	B	1F4
4	B	1F9
5	C	1F7
6	C	1F11
7	D	1E3
8	D	1E2
9	B	1E6
10	A	1E5
11	D	1E1
12	C	1B1
13	D	1B4
14	C	1B2
15	A	1D3
16	B	1D4
17	D	1D7
18	D	1D7
19	A	1C1
20	C	1C2
21	B	2A2
22	D	2A3
23	D	2A5
24	A	2B7
25	A	2B10
26	D	2B9
27	B	2C10
28	A	2C10
29	B	2C4
30	B	2C11
31	D	2C11
32	B	2C8
33	A	2C6
34	C	2C11
35	D	3A8
36	C	3A8
37	B	3A5
38	C	3A17
39	B	3A14
40	A	3B8
41	D	3B9
42	C	3B10(i)
43	B	3B10(ii)
44	C	3B5
45	C	3B7

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L
HEADQUARTERS

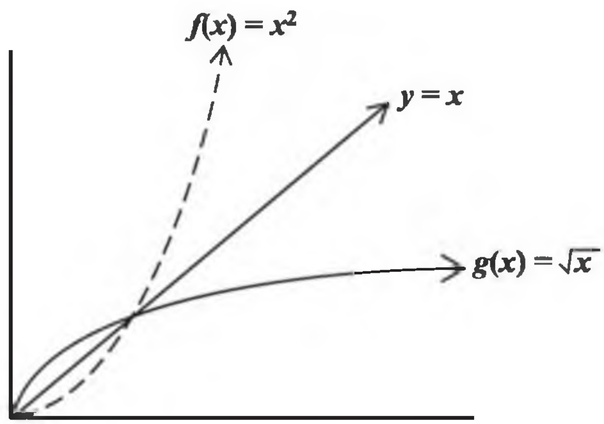
SECONDARY EDUCATION CERTIFICATE EXAMINATION

ADDITIONAL MATHEMATICS

PAPER 02

SPECIMEN PAPER
KEY AND MARK SCHEME

ADDITIONAL MATHEMATICS
PAPER 02
KEY AND MARK SCHEME

Question 1	CK	AK	R
(a) (i) *f* cannot be 1-1 since for example $f(-2) = 4$ and $f(2) = 4$. Two objects can have the same images. Thus *f* is many-to-one (accept any reasonable explanation or horizontal line test)			1
(ii)			
a) If $g[f(x)] = x$ then *g* must be the inverse of *f*. Let $y = x^2$ where *x* is a non-negative real number. Then $x = \sqrt{y} \Rightarrow g(x) = \sqrt{x}$ or $= \sqrt{(x^2)} = x$, for all $x \geq 0$.		1	
b) 	1	1	
(b) Using f (3) = R			1
$f(3) = 3^3 - 2(3)^2 + 4(3) - 21$		1	
$f(3) = 0$			
(c) (i) $p = aq^n \Rightarrow \log p = \log a + n \log q$	1		1

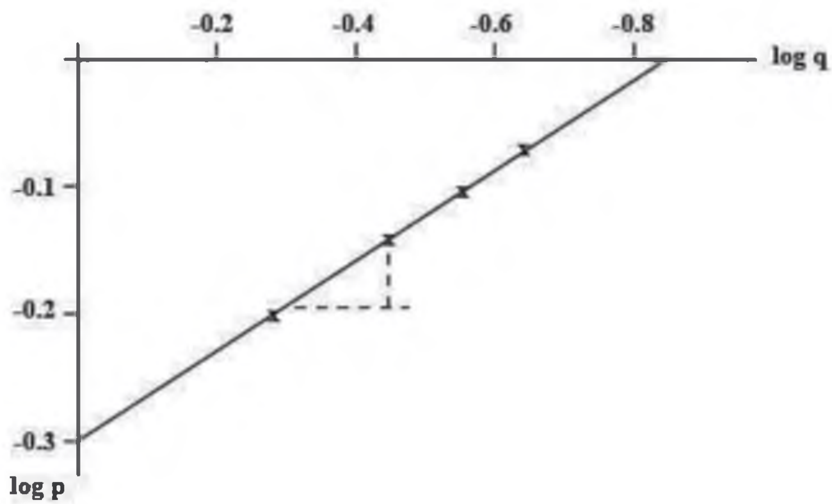
ADDITIONAL MATHEMATICS
PAPER 02
KEY AND MARK SCHEME

(ii)

Plot $\log p$ against $\log q$: First find the table of values

$\log q$	0	0.3	0.48	0.6
$\log p$	-0.30	-0.20	-0.14	-0.10

On the graph paper provided plot the points and draw a straight line through the points



The y intercept $c = -0.3 = \log a$

$\therefore\ a = 10^{-0.3} = 0.5$

The slope of the line is $\dfrac{y_2 - y_1}{x_2 - x_1} = m = 0.33$ (by measurement)

$\therefore\ n = 0.33$

(graphical measurements of slope may vary slightly above or below this value)

(Specific Objectives-Sect 1: A3, D3-5, D7, E7-8)

CK	AK	R
	1	
1		
		1
	1	
	1	
		1
3	**6**	**5**

ADDITIONAL MATHEMATICS
PAPER 02
KEY AND MARK SCHEME

Question 2

			CK	AK	R
(a)	(i)	$3x^2 + 12x - 18$			
		$3\,[x^2 + 4x] - 18$	1		
		$3[(x + 2)^2 - 4] - 18$		1	
		$3(x + 2)^2 - 12 - 18$		1	
		$3\,(x + 2)^2 - 30$			
	(ii)	Minimum value is $y = -30$			1
	(iii)	Value of x at minimum point is $x = -2$			1
(b)		$2x^2 + 2 > 5x$			
		$2x^2 - 5x + 2 > 0$		1	
		$(2x - 1)(x - 2) > 0$		1	
		$x < \frac{1}{2}$			1
		or $x > 2$			1
(c)	(i)	Given the series $\frac{1}{2} + \frac{1}{2^4} + \frac{1}{2^7} + \frac{1}{2^{10}} + \ldots$			
		S is a geometric series if $\frac{T_n}{T_1} = \frac{T_n}{T_{n-1}} = r$	1		
		i.e. S has a common ratio r.			
		$r = \dfrac{\frac{1}{2^7}}{\frac{1}{2^4}} = \dfrac{\frac{1}{2^4}}{\frac{1}{2}}$			1
		$r = \frac{1}{2^3} = \frac{1}{8}$		1	

ADDITIONAL MATHEMATICS
PAPER 02
KEY AND MARK SCHEME

Question 2

(cont'd)

(c) (ii)

$$S_\infty = \frac{a}{1-r} = \frac{\frac{1}{2}}{1-\frac{1}{8}}$$

$$S_\infty = \frac{\frac{1}{2}}{\frac{7}{8}} = \frac{4}{7}$$

(Specific Objectives - Sect 1: B1, B2, C1, F6, F11)

CK	AK	R
1		
	1	
3	**6**	**5**

ADDITIONAL MATHEMATICS
PAPER 02
KEY AND MARK SCHEME

Question 3

			CK	AK	R
(a)	(i)	$(x+1)^2 + (y-2)^2 = \left(\sqrt{13}\right)^2$			
		$(x+1)^2 + (y-2)^2 = 13$	1		
	(ii)	The gradient, m, of the radius through P (2, 4)			
		is $m = \dfrac{4-2}{2+1}$	1		
		$= \dfrac{2}{3}$			
		The gradient of the tangent to circle P is $-\dfrac{3}{2}$			1
		The equation of the tangent at P is given by			
		$\dfrac{y-4}{x-2} = -\dfrac{3}{2}$		1	
		$2y - 8 = -3x + 6$		1	
		$2y + 3x = 14$			
(b)		$(3t\mathbf{i} + 2t\mathbf{j}) \cdot (4\mathbf{i} - 2t\mathbf{j}) = 0$			1
		$12t - 4t^2 = 0$		1	
		$4t(3 - t) = 0$		1	
		$t = 0$ or $t = 3$			
		$t = 3$, since $t > 0$			1

ADDITIONAL MATHEMATICS
PAPER 02
KEY AND MARK SCHEME

Question 3 (cont'd)

		CK	AK	R
(c)	$\overrightarrow{LM} = \overrightarrow{LO} + \overrightarrow{OM}$			
	$= -\begin{pmatrix} 7 \\ -2 \end{pmatrix} + \begin{pmatrix} 4 \\ 2 \end{pmatrix}$	1		
	$= \begin{pmatrix} -3 \\ 4 \end{pmatrix}$			
	$= -3\mathbf{i} + 4\mathbf{j}$			
	The unit vector in the direction of $\overrightarrow{LM}$ is			
	$\frac{1}{\sqrt{(-3)^2 + (4)^2}}(-3\mathbf{i} + 4\mathbf{j})$	1		
	$= \frac{1}{5}(-3\mathbf{i} + 4\mathbf{j})$		1	
	(Specific Objectives-Sect 2: A4, A6, B5, B10)	**4**	**5**	**3**

ADDITIONAL MATHEMATICS
PAPER 02
KEY AND MARK SCHEME

Question 4

		Working	CK	AK	R
(a)	(i)	$l_{\text{CD}} = r\theta$			
		$= 6 \times \dfrac{5\pi}{6} = 5\pi \text{ cm}$		1	
	(ii)	$\text{Area}_{\text{sector COD}} = \dfrac{1}{2}r^2\theta$			
		$= \dfrac{1}{2}(6)^2\left(\dfrac{5\pi}{6}\right) = 15\pi \text{ cm}^2$	1		1
(b)	(i)	$\sin\left(x - \dfrac{\pi}{4}\right) = \sin x \cos\dfrac{\pi}{4} - \cos x \sin\dfrac{\pi}{4}$			1
		$= \dfrac{\sqrt{2}}{2}\sin x - \dfrac{\sqrt{2}}{2}\cos x$			
		$= \dfrac{\sqrt{2}}{2}(\sin x - \cos x)$		1	
	(ii)	$\sin\left(\dfrac{\pi}{12}\right) = \sin\left(\dfrac{\pi}{3} - \dfrac{\pi}{4}\right)$	1		1
		$= \dfrac{\sqrt{2}}{2}\left(\sin\dfrac{\pi}{3} - \cos\dfrac{\pi}{3}\right)$			
		$= \dfrac{\sqrt{2}}{2}\left(\dfrac{\sqrt{3}}{2} - \dfrac{1}{2}\right)$		1	
		$= \dfrac{\sqrt{2}}{4}\left(\sqrt{3} - 1\right)$			

ADDITIONAL MATHEMATICS
PAPER 02
KEY AND MARK SCHEME

Question 4
(cont'd)

			CK	AK	R
4	(c)	$\text{LHS} \equiv \left(\dfrac{\sin\theta}{\cos\theta} - \dfrac{1}{\cos\theta}\right)^2$			1
		$\equiv \left(\dfrac{\sin\theta - 1}{\cos\theta}\right)^2$			
		$\equiv \dfrac{(\sin\theta - 1)^2}{1 - \sin^2\theta}$		1	
		$\equiv \dfrac{(\sin\theta - 1)^2}{(1 - \sin\theta)(1 + \sin\theta)}$		1	
		$\equiv -\dfrac{\sin\theta - 1}{\sin\theta + 1}$			1
		(Specific objectives-Sect 2: C3, C5, C8, C10)	**2**	**5**	**5**

ADDITIONAL MATHEMATICS
PAPER 02
KEY AND MARK SCHEME

Question 5

		CK	AK	R
(a)	$\frac{\mathrm{d}}{\mathrm{d}x}(5-2x)(1+x)^4 = (1+x)^4\frac{\mathrm{d}}{\mathrm{d}x}(5-2x) + (5-2x)\frac{\mathrm{d}}{\mathrm{d}x}(1+x)^4$	1		
	$= (1+x)^4(-2) + (5-2x)\left[4(1+x)^3\ (1)\right]$		1	
	$= 2(1+x)^3(-1-x+10-4x\ )$			1
	$= 2(1+x)^3(9-5x\ )$		1	
(b)	$\frac{\mathrm{d}y}{\mathrm{d}x} = 2x$	1		
	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_{(-2,\,4)} = -4$			1
	tangent$_{\mathrm{p}}$: $y-4=-4(x+2)$		1	
	$4x+y+4=0$		1	
(c)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(2x^3-9x^2+12x\right) = 6x^2-18x+12$		1	
	$6x^2-18x+12=0 \Rightarrow (x-1)(x-2)=0$			1
	$x=1,\ y=5 \quad x=2,\ y=4$		1	
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 12x-18$			1
	$\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)_1 < 0$ (1, 5) is a maximum			1
	$\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)_2 > 0$ (2, 4) is a minimum			1
	(Specific objectives: Sect 3: A4, A7-9, A11-17)	**2**	**6**	**6**

ADDITIONAL MATHEMATICS
PAPER 02
KEY AND MARK SCHEME

Question 6

	Working	CK	AK	R
(a)	$\int_1^2 (3x-1)^2 \, dx = \int_1^2 \left(9x^2 \ 6x + 1\right) dx$			1
	$= \left[3x^3 - 3x^2 + x\right]_1^2 = (24 - 12 + 2) - (3 - 3 + 1)$	1	1	
	$= 13$		1	
(b)	$\int_0^{\frac{\pi}{2}} (5 \sin x - 3 \cos x) \, dx = \left[-5 \cos x - 3 \sin x\right]_0^{\frac{\pi}{2}}$	1	1	
	$= [-5(0) - 3(1)] - [-5(1) - 3(0)]$			1
	$= 2$		1	
(c)	(i)			
	$y = \int (2 - x) \, dx = 2x - \dfrac{x^2}{2} + C$	1		
	At $\left(0, \dfrac{7}{2}\right)$ $C = \dfrac{7}{2}$		1	1
	$y = 2x - \dfrac{x^2}{2} + \dfrac{7}{2}$			
	(ii) Area $= \int_0^5 \left(2x - \dfrac{x^2}{2} + \dfrac{7}{2}\right) dx = \left[x^2 - \dfrac{x^3}{6} + \dfrac{7x}{2}\right]_0^5$	1		1
	Area $= \left(25 - \dfrac{125}{6} + \dfrac{35}{2}\right) = \dfrac{65}{3}$ units^2		1	
	(Specific Objectives- Sect 3: B4-5, B7--10)	4	6	4

ADDITIONAL MATHEMATICS
PAPER 02
KEY AND MARK SCHEME

Question 7

(a) (i) P (Statistics) = $\frac{28}{43}$ P (Physics) = $\frac{19}{43}$

P (Statistics and Physics) = $\frac{28}{43} + \frac{19}{43} - \frac{43}{43} = \frac{4}{43}$

(ii) P (Physics only) = $\frac{19}{43} - \frac{4}{43} = \frac{15}{43}$

(b) (i) P (Red = 3 and Blue = 0) = $\frac{1}{16}$

(ii)

3	0	3	6	9
2	0	2	4	6
1	0	1	2	3
0	0	0	0	0
B \ R	**0**	**1**	**2**	**3**

(iii)

t	0	1	2	3	4	6	9
P (t)	$\frac{7}{16}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{16}$

$a = \frac{7}{16}$, $b = \frac{1}{8}$ and $c = \frac{1}{16}$

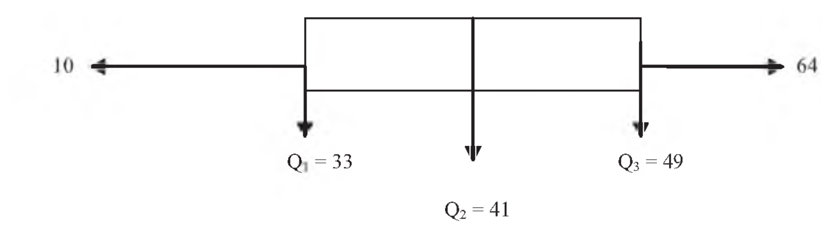
	CK	**AK**	**R**
(a)(i)	1	1	1
(a)(ii)		1	1
(b)(i)	1	1	
(b)(ii)	1	1	1
(b)(iii)	1	1	1

ADDITIONAL MATHEMATICS
PAPER 02
KEY AND MARK SCHEME

Question 7 cont'd

(c) (i) $Q_1 = 33$ $Q_2 = 41$ $Q_3 = 49$

(ii)



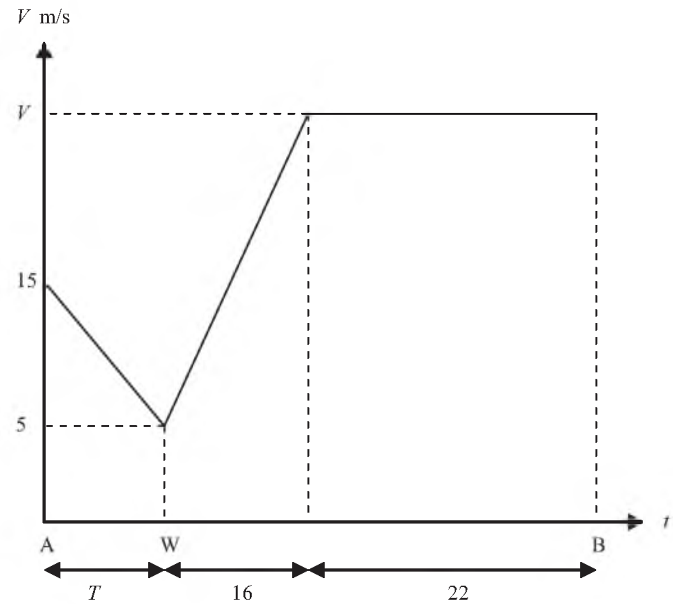
The shape is symmetrical about the median

(Specific Objectives- Sect 4: A 4-6, B2-4, B9).

CK	AK	R
1	1	1
1	1	1
	1	
6	**8**	**6**

ADDITIONAL MATHEMATICS
PAPER 02
KEY AND MARK SCHEME

Question 8

	CK	AK	R
(a) (i) 	1	1	1
(ii) Using area under the curve $\frac{1}{2}(15+5)(T) = 120$	1		
$T = 12$		1	
Total time = 12 + 16 + 22 = 50 secs			1
(iii) Using area under the curve			1
$120 + \frac{1}{2}(V+5)\times 16 + 22V = 1000$	1	1	1
$V = 28$		1	

ADDITIONAL MATHEMATICS
PAPER 02
KEY AND MARK SCHEME

Question 8
(cont'd)

	CK	AK	R
(b) (i) $v = 0 \Rightarrow 3t^2 - 30t + 72 = 0$			1
$(t-6)(t-4) = 0$		1	
$t = 4$ secs, 6 secs	1		
(ii) Distance = $\left\lvert \int_4^6 \left(3t^2 - 30t + 72\right) \mathrm{d}t \right\rvert$ (below the x-axis)			1
$= \left\lvert \left[t^3 - 15t^2 + 72t\right]_4^6 \right\rvert$		1	
$= \left\lvert (216 - 540 + 432) - (64 - 240 + 288) \right\rvert$			
= 4 metres	1		
(iii) Distance in first 4 secs = $\int_0^4 \left(3t^2 - 30t + 72\right) \mathrm{d}t$			
= 112 metres	1		
Distance between 6 secs and 7 secs			
$= \int_6^7 \left(3t^2 - 30t + 72\right) \mathrm{d}t$			
= 4 metres		1	
Total distance = 112 + 4 + 4 = 120 metres		1	
(Specific Objectives- Sect 4: C2 – 4)	**6**	**8**	**6**

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L
HEADQUARTERS

SECONDARY EDUCATION CERTIFICATE EXAMINATION

ADDITIONAL MATHEMATICS

PAPER 03/2

SPECIMEN PAPER

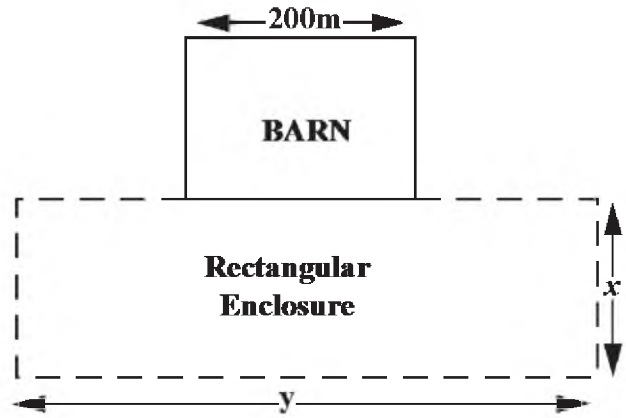
KEY AND MARK SCHEME

MATHEMATICS
PAPER 03
KEY AND MARK SCHEME

Question 1

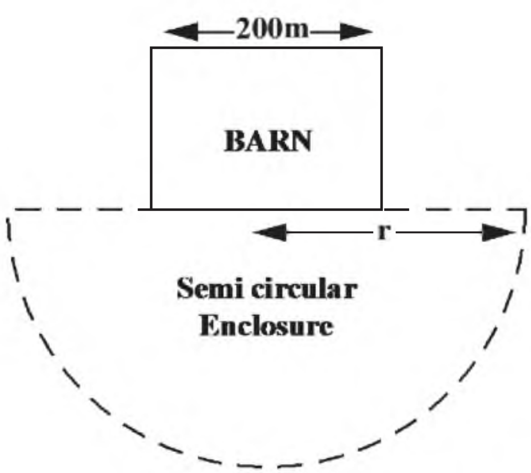
(i) **Mathematical Formulation of the problem**

Option I



	CK	AK	R
Let the rectangular enclosure have length x *m* and width y m.			1
Since one side of the barn of length 200 m is used in making the enclosure and an additional 800m of fencing is available then the perimeter of the enclosure is 200 + 800 = 1000 m			1
The perimeter of the rectangle is $(2x + 2y)$ m and	1		
Area of the rectangle enclosure is xy m^2			
Thus the problem can be formulated mathematically as			
Maximise $A_1 = xy$			
Subject to $2x + 2y = 1000$	1		

Option II



MATHEMATICS
PAPER 03
KEY AND MARK SCHEME

	CK	AK	R
Let the semicircular enclosure have radius r			1
Since half the circumference of a circle is πr and diameter is 2r, the problem is to find the Area A_2 of the semicircle when πr + 2r = 1000m	1		
The solution lies in comparing the two areas Maximum A_1 and A_2 to see which is larger and recommend the larger option to the farmer.			1
(ii) **To show that for Option I a square maximizes the area:**			
Maximise $A_1 = xy$ (area) Subject to $2x + 2y = 1000$ (perimeter)			
<u>Solution</u>			
$2x+2y = 1000 \longrightarrow y = \frac{1000-2x}{2} = 500 - x$		1	
Substituting in A_1 we obtain			
$A_1 = xy = x\,(500-x) = 500x - x^2$			
To maximize A_1 with respect to x we first find the stationary points:			1
$\frac{dA}{dx} = 500 - 2x$			
$500 - 2x = 0 \longrightarrow 2x = 500$		1	
$\therefore$ x = 250 at maximum		1	
(we need not find the 2nd derivative as there is only one solution but			
$d^2A/dx^2 = -2 \longrightarrow$ stationary point is a maximum)			
$\therefore y = 500 - x = 250$		1	
Since x = y the rectangle is a square			1
Maximum Area is $A_1 = 250^2 = 62\,500$ m^2		1	
(iii) **Option II Area**			
Perimeter of enclosure = $\pi r + 2r = 1000\ m$			
$\therefore$ r = $\frac{1000}{\pi+2}$ = 194.49 m (radius of enclosure)			1
Area of semicircle is $A_2 = \frac{\pi}{2}\left(\frac{1000}{\pi+2}\right)^2$	1		1
This is the maximum area of the semicircle since r is fixed			
Area of semicircle is $A_2 = 59\,419$ m^2		1	

MATHEMATICS
PAPER 03
KEY AND MARK SCHEME

		CK	AK	R
	Recommendation			
(iv)	Since 62500 > 59419, the square enclosure has greater area than the semicircular. I would therefore recommend Option I to the farmer,			**1**
	i.e. build a square enclosure of side 250 m			**1**
		4	**6**	**10**

C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

REPORT ON CANDIDATES' WORK IN THE CARIBBEAN SECONDARY EDUCATION CERTIFICATE® EXAMINATION

MAY/JUNE 2012

ADDITIONAL MATHEMATICS GENERAL PROFICIENCY EXAMINATION

GENERAL COMMENTS

Additional Mathematics, the newest CXC subject offering at the CSEC level, was tested for the first time in the May/June 2012 examinations. The subject is intended to bridge a perceived gap between the CSEC General Mathematics and the CAPE Unit1 Pure Mathematics. The intent is to offer CSEC Mathematics students a more seamless transition to the advanced thinking and skills needed for CAPE Unit 1 Mathematics courses, although a good CSEC Mathematics student should still be able to meet the skills and thinking demands of the CAPE Unit 1 Mathematics course.

The examinations consist of three papers:
- Paper 01 — a 45 – item multiple choice paper;
- Paper 02 — a structured, 'essay-type' paper consisting of 8 questions;
- Paper 031 or Paper 032 — Paper 03 represents an School-Based Assessment (SBA) project component for candidates in schools, and Paper 032 an alternative to the SBA for out-of-school candidates.

The Additional Mathematics syllabus (CXC 37/G/SYLL 10) tests content in four main topic areas divided as follows: Section 1 — Algebra and Functions, Section 2 — Coordinate Geometry, Vectors and Trigonometry, Section 3 — Introductory Calculus, and Section 4 — Basic Mathematical Applications.

Paper 01 tests content from Sections 1, 2 and 3 of the syllabus. This paper carries 45 items which are weighted up to 60 for the overall examination. Paper 02 tests content from all four sections of the syllabus. This year the paper consisted of four sections, divided as described previously for the outline of the syllabus. Each section contained two problem-solving type questions. The questions in Sections 1, 2 and 3 were all compulsory. The two questions in these sections were worth a total of 28, 24 and 28 marks respectively. Section 4 also contained two questions, one on Data Representation and Probability and the other on Kinematics worth 20 marks each; candidates were to choose one question from this section. Paper 03 represents the SBA component of the examination. Candidates can do a project chosen from two project types, a mathematical modelling project (Project A) and a data handling/statistical analysis project (Project B). The SBA component is worth 20 marks. Alternatively, candidates can sit an alternative to the SBA paper, Paper 032, which consists of a more in-depth extended question from Sections 1, 2 and/or 3 of the syllabus. This paper carries 20 marks.

DETAILED COMMENTS

Paper 01 – Structured Essay Questions

This was a 45 - item paper covering Sections 1, 2 and 3 of the syllabus. A total of 1720 candidates sat this paper of the examination. Candidates scored a mean of 23.22, with standard deviation 8.77.

Paper 02 – Structured Essay Questions

No formula sheet was provided for this sitting of the examination. However, in cases where it was suspected that candidates might have been affected by the absence of a formula sheet the mark scheme was adjusted so that candidates were awarded follow through marks if they used an incorrect reasonable formula, and were not penalized for this wrong formula. For future examinations, a formula sheet will be provided for papers 01 and 02.

Section 1: Algebra and Functions

Question 1

This question tested candidates' ability to:

- determine a composite function, its range and inverse;
- make use of the Factor Theorem;
- use the laws of indices to solve exponential equations in one unknown; and
- apply logarithms to reduce a relationship to linear form.

There were 1746 responses to this question. The mean mark was 5.72 with standard deviation 3.26. Thirty-nine candidates obtained full marks.

Candidates performed best on Parts (a) (i), (iii) and (b) which required them to determine the composite function $g(f(x))$ of given functions $f(x)$ and $g(x)$, and the inverse of the composite function. In Part (a) (iii) some candidates experienced difficulty at the interchange step, that is,

1) (i) $g(f(x)) = x^3 + 6$; Let $y = g(f(x)$ (ii) $\left[*y^3 = x+6*\right]$ commonly seen error

$$y = \sqrt[3]{x+6}$$

$$\therefore \left[gf(x)\right]^{-1} = \sqrt[3]{x+6}$$

Part (b) was generally well done by candidates. However, some of them did have difficulty here. The following common errors were seen:

- Substituting $x = 2$ into $f(x)$ rather than $x = -2$;
- Some candidates who did correctly substitute $x = -2$ then made errors in their manipulation of the directed numbers;
- A few candidates attempted long division approach to solving this problem; however they very often encountered some difficulty with this.

Parts (a) (ii), (c) and (d) presented the most difficulty to candidates. In Part (a) (ii), which required candidates to state the range of the composite function, generally candidates were unable to do this, or stated discrete values for the range, that is, {6, 7, 14, 33}, not recognizing it as the infinite set of Real Numbers, $6 \leq g(f(x)) \leq 33$.

In Part (c), many candidates attempted to solve the given equation $3^{2x} - 9(3^{-2x}) = 8$ without (appropriate) substitution. Having not recognized a suitable substitution to use in order to transform the equation into polynomial form, candidates incorrectly took logarithms of each term in the equation. Some candidates were able to recognize that $y = 3^{2x}$ or $y = 3^x$ were suitable substitutions and quite often were able to derive an appropriate polynomial equation. Those who derived the quadratic equation were able to easily solve the indices equation. However, those who derived the quartic equation at times found it challenging to solve. A number of candidates who got to the solution stage of the problem did not recognize (or know) that the logarithm of a negative number did not exist.

In Part (d), although many candidates knew how to transform $x^3 = 10^{x-3}$ to logarithmic form, they did not use brackets and so wrote $3 \log_{10} x = x - 3 \log_{10} 10$, instead of the expected $3 \log_{10} x = (x - 3) \log_{10} 10$. Whilst candidates did not lose marks this time given the nature of the specific case here (base is the same as the number so that $\log_{10} 10$ is 1), teachers are asked to encourage their students to use brackets to avoid possible errors in the future. Another common error made was having reached to the stage of 3 $\log_{10} x = x - 3$, a number of candidates then went on to give the gradient as 1, seemingly not recognizing that they had to write the equation in terms of log x, that is. to divide through by 3 so that the coefficient of log x was 1, i.e. $\log_{10} x = \frac{1}{3}x - 1$, so that the gradient of the linear form was $\frac{1}{3}$. Other common errors observed were:

- $\text{Log}_{10} x^3 = \log_{10} 10^{x-3}$
  $\text{Log}_{10} 3x = \log_{10} x - \log_{10} 3$

- $\text{Log}_{10} x^3 = \log_{10} 10x - \log_{10} 30$
  $3\log_{10} x = 10x - 30$

Solutions

(a)	(i) $g(f(x)) = x^3 + 6$	(ii) $6 \leq g(f(x)) \leq 33$	(iii) $[gf(x)]^{-1} = \sqrt[3]{x-6}$
(b)	$a = 20$		
(c)	$x = 1$		
(d)	(i) $\log_{10} x = \frac{1}{3}x - 1$	(ii) gradient $= \frac{1}{3}$	

Question 2

This question tested candidates' ability to:

- make use of the relationship between the sums and products of the roots of quadratic equations;
- solve an inequality of the form $\frac{ax+b}{cx+d} \geq 0$;
- identify an AP, obtain an expression for its general term, find its sum to a finite term.

There were 1733 responses to this question. The mean mark was 5.64 with standard deviation 4.27. One hundred and four candidates obtained full marks

Candidates performed best on Part (c) where they had to recognize the problem as an Arithmetic Progression (AP) and find its first term and common difference.

Parts (a) and (b) presented the most difficulty to candidates. In Part (a), many candidates attempted to solve the quadratic equation whether by factorizing or using the formula, showing a seeming lack of knowledge about the sum and product of roots of quadratic equations. In Part (b), although some candidates were able to determine the boundary values for the inequality, they had difficulty with the direction of the inequality for these boundary values which would make the overall inequality 'true'. Although more rarely seen, there was generally more success among candidates who attempted a graphical solution to this question.

In Part (c), a common error observed related to candidates finding the 24th term of the AP (T_{24}) rather than the sum of the first 24 terms (S_{24}). Another common error observed was that some candidates could not correctly write the S_n formula, commonly writing it as $S_n = \frac{1}{2}[a + (n - 1)d]$, instead of the expected $S_n = \frac{n}{2}[2a + (n - 1)d]$.

Solutions

(a) $\alpha^2 + \beta^2 = 4$

(b) $x > \frac{5}{2}$ and $x < -\frac{1}{3}$

(c) Total paid = \$2100

Section 2: Coordinate Geometry, Vectors and Trigonometry

Question 3

This question tested candidates' ability to:

- find the centre of a given circle and showing that a given line passes through this centre;
- find the equation of a tangent to a circle at a given point;
- add and subtract vectors, find displacement vectors and find the magnitude of a vector.

There were 1715 responses to this question. One hundred and sixteen candidates obtained full marks.

Candidates performed best on Part (a) (ii) which required them to calculate the gradient of the radius of the circle (from two points), and hence determine the gradient and equation of the tangent to the circle's radius.

Candidates had difficulty with Part (a) (i) which required them to show that the line $x + y + 1 = 0$ passed through a given circle's centre. Although many could find the coordinates of the circle's centre, some candidates had difficulty in showing (that is, proving) that the given line passed through this centre, indicating a seeming lack of knowledge of what constituted a simple proof in this case. Candidates also had difficulty with Part (b), writing $\overrightarrow{BP}$ in terms of its position vectors, or correctly adding vectors to get $\overrightarrow{BP}$. A common error seen here was that $\overrightarrow{OP} = \overrightarrow{BP}$.

Solutions

(a) (ii) $y = \frac{4}{3}x + 11$

(b) (i) $\vec{BP} = \frac{3}{2}\,\mathbf{a} - \mathbf{b}$ (ii) $|\vec{BP}| = \frac{1}{2}$