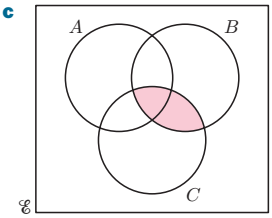
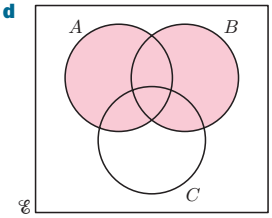
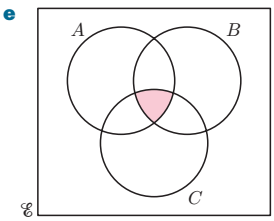
c



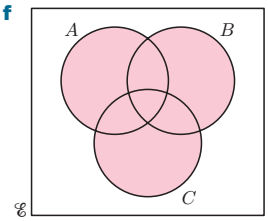
d



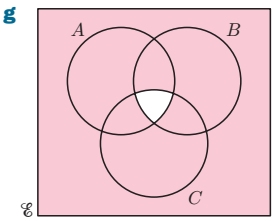
e



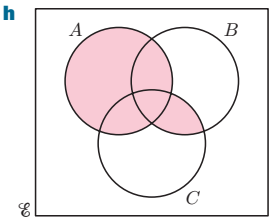
f



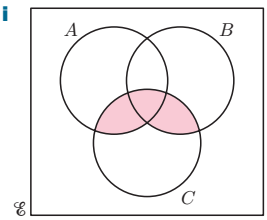
g



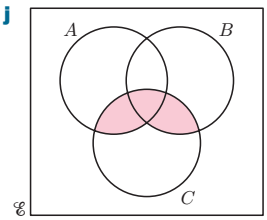
h



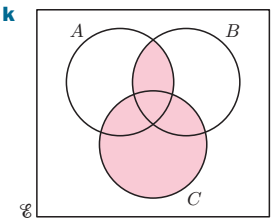
i



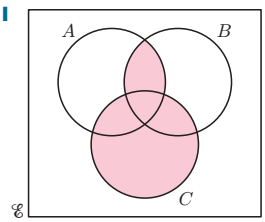
j



k



l



EXERCISE 1G

1 **a** 7 **b** 14 **c** 14 **d** 7 **e** 5 **f** 9

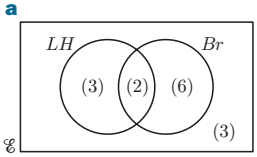
2 **a** $b+c$ **b** $c+d$ **c** b
d $a+b+c$ **e** $a+c+d$ **f** d

3 **a** **i** $2a+4$ **ii** $4a+4$ **iii** $3a-5$ **iv** $5a-1$
b **i** $a=6$ **ii** $a=\frac{32}{5}$
Since $a \in \mathbb{N}$, there cannot be 31 elements in $\mathscr{E}$, but it is possible to have 29 elements.

5 **a** 15 **b** 4 **6** **a** 18 **b** 6 **7** **a** 7 **b** 23

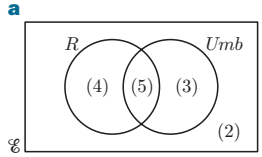
EXERCISE 1H

1 **a**



b **i** 9 **ii** 3 **iii** 3

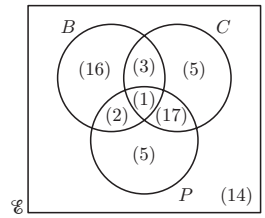
2 **a**



b **i** 4 **ii** 2

3 13 players **4** 20 people

5 **a**



b **i** 16 **ii** 33 **iii** 14 **iv** 7

6 **a** 29 **b** 6 **c** 1 **d** 11

7 **a** 3 **b** 5 **c** 5 **d** 21

8 **a** 3 **b** 4 **c** 9

REVIEW SET 1A

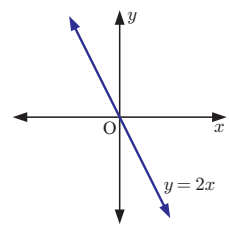
1 **a** $S = \{3, 4, 5, 6, 7\}$ **b** 5 **c** 31

2 **a** yes **b** yes **c** no **d** yes

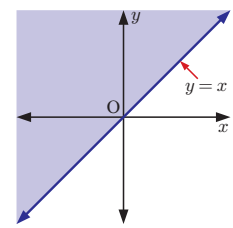
3 **a** $X' = \{\text{orange, yellow, green, blue}\}$
b $X' = \{-5, -3, -2, 0, 1, 2, 5\}$
c $X' = \{x \in \mathbb{Q} : x \geqslant -8\}$
d $X' = \{x \in (-\infty, -3) \cup [1, 4]\}$

4 **a** $\{x \in \mathbb{R} : -2 \leqslant x < 3\}$, neither
b $\{x \in \mathbb{R} : x < 3\}$, open

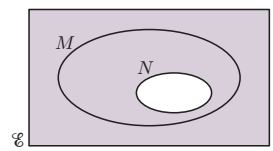
5 **a**



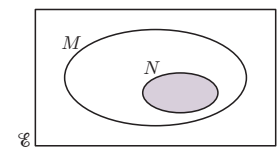
b



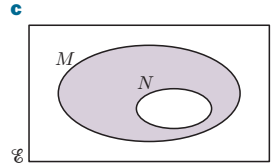
6 **a**



b

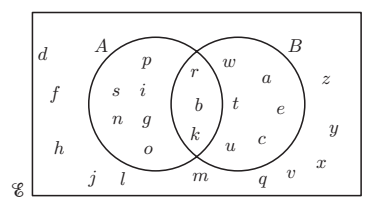


c



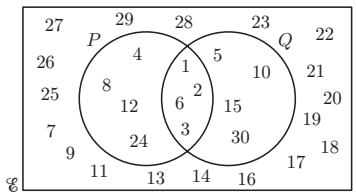
7 **a** **i** $\{s, p, r, i, n, g, b, o, k, w, a, t, e, u, c\}$
ii $\{r, b, k\}$ **iii** $\{g, i, n, o, p, s\}$
b **i** {the letters in 'springbok' or 'waterbuck'}
ii {the letters common to both 'springbok' and 'waterbuck'}
iii {the letters in 'springbok' but not 'waterbuck'}

c

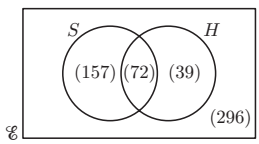


8 **a** **i** $\{1, 2, 3, 4, 6, 8, 12, 24\}$
ii $\{1, 2, 3, 5, 6, 10, 15, 30\}$ **iii** $\{1, 2, 3, 6\}$
iv $\{1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 24, 30\}$

b



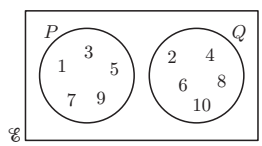
9 **a**



b **i** 72 **ii** 39 **iii** 268

10 8 **11** **a** 9 **b** 7 **c** 17

12 **a** $P = \{1, 3, 5, 7, 9\}$
$Q = \{2, 4, 6, 8, 10\}$
b They are disjoint.
c



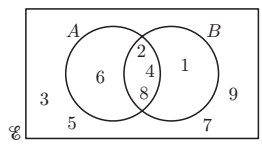
REVIEW SET 1B

1 **a** true **b** false **c** true **d** false **e** false

2 **a** **i** $\{x \in \mathbb{R} : 5 < x < 12\}$ **ii** $\{x \in \mathbb{Z} : -4 \leqslant x < 7\}$
iii $\{x \in \mathbb{N} : x > 45\}$
b **i** infinite **ii** finite **iii** infinite

3 $\varnothing$, $\{1\}$, $\{3\}$, $\{5\}$, $\{1, 3\}$, $\{3, 5\}$, $\{1, 5\}$, $\{1, 3, 5\}$

4 **a** **i** $\{2, 4, 6, 8\}$
ii $\{2, 4, 8\}$
iii $\{3, 5, 7, 9\}$
b



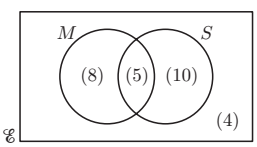
5 **a** $\varnothing$ **b** $s + t$

6 **a** neither **b** closed **c** neither

7 **a** **i** The set of points which lie on both A and B (that is, the point(s) of intersection of line A and line B).
ii The set of points which lie on line A or line B.
b No. If the lines are coincident (so, A and B describe the same line), then $A \cap B$ will be infinite.
c $n(A \cap B) = 0$ or 1

8 **a** C' **b** $(A \cap B) \cup (A \cap C)$ or $A \cap (B \cup C)$

9 **a**

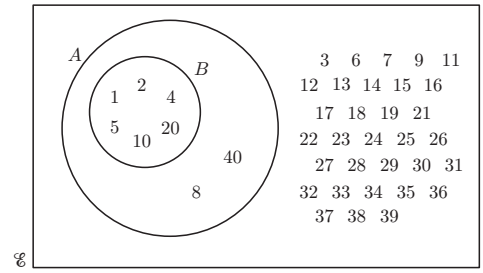


b **i** 27 **ii** 8 **iii** 14

10 4

11 **a** $A = \{1, 2, 4, 5, 8, 10, 20, 40\}$, $B = \{1, 2, 4, 5, 10, 20\}$
b $B \subset A$

c



12 **a** 1 **b** 7 **c** 15

EXERCISE 2A.1

1 **a**, **d**, **e** **2** **a**, **b**, **c**, **e**, **g**, **i**

3 No, for example $(0, 4)$ and $(0, -4)$ satisfy the relation.

EXERCISE 2A.2

1 **a**, **c**, **f**

2 **a** not a function **b** function, one-one
c function, not one-one

3 **a** **i** \$13 **ii** yes **iii** yes
b **i** no **ii** no

EXERCISE 2B

1 **a** 2 **b** 2 **c** -16 **d** -68 **e** $\frac{17}{4}$

2 **a** -3 **b** 3 **c** 3 **d** -3 **e** $\frac{15}{2}$

3 **a** **i** 1 **ii** -1 **b** $x = -4$

4 **a** $7 - 3a$ **b** $7 + 3a$ **c** $-3a - 2$ **d** $10 - 3b$
e $1 - 3x$ **f** $7 - 3x - 3h$

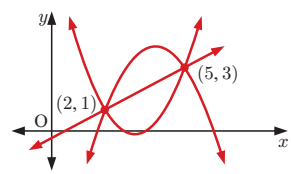
5 **a** $2x^2 + 19x + 43$ **b** $2x^2 - 11x + 13$
c $2x^2 - 3x - 1$ **d** $2x^4 + 3x^2 - 1$
e $2x^4 - x^2 - 2$ **f** $2x^2 + (4h + 3)x + 2h^2 + 3h - 1$

6 **a** **i** $-\frac{7}{2}$ **ii** $-\frac{3}{4}$ **iii** $-\frac{4}{9}$
b $x = 4$ **c** $\dfrac{2x + 7}{x - 2}$ **d** $x = \frac{9}{5}$

7 f is the function which converts x into $f(x)$ whereas $f(x)$ is the value of the function at any value of x.

8 **a** $V(4) = 6210$, the value in dollars after 4 years
b $t = 4.5$, the time in years for the photocopier to reach a value of 5780 dollars.
c 9650 dollars

9



10 $f(x) = -2x + 5$

11 $a = 3$, $b = -2$

12 $a = 3$, $b = -1$, $c = -4$

EXERCISE 2C

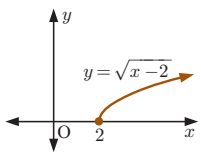
1 **a** Domain $= \{x : x \geqslant -1\}$, Range $= \{y : y \leqslant 3\}$
b Domain $= \{x : -1 < x \leqslant 5\}$, Range $= \{y : 1 < y \leqslant 3\}$
c Domain $= \{x : x \neq 2\}$, Range $= \{y : y \neq -1\}$
d Domain $= \{x : x \in \mathbb{R}\}$, Range $= \{y : 0 < y \leqslant 2\}$
e Domain $= \{x : x \in \mathbb{R}\}$, Range $= \{y : y \geqslant -1\}$
f Domain $= \{x : x \in \mathbb{R}\}$, Range $= \{y : y \leqslant \frac{25}{4}\}$
g Domain $= \{x : x \geqslant -4\}$, Range $= \{y : y \geqslant -3\}$

h Domain $= \{x : x \in \mathbb{R}\}$, Range $= \{y : y > -2\}$
i Domain $= \{x : x \neq \pm 2\}$,
Range $= \{y : y \leqslant -1 \text{ or } y > 0\}$

2 **a** $f(x)$ defined for $x \geqslant -6$, Domain $= \{x : x \geqslant -6\}$
b $f(x)$ defined for $x \neq 0$, Domain $= \{x : x \neq 0\}$
c $f(x)$ defined for $x < \frac{3}{2}$, Domain $= \{x : x < \frac{3}{2}\}$

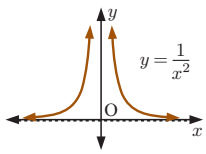
3 **a** Domain $= \{x : x \in \mathbb{R}\}$, Range $= \{y : y \in \mathbb{R}\}$
b Domain $= \{x : x \in \mathbb{R}\}$, Range $= \{3\}$
c Domain $= \{x : x \geqslant 0\}$, Range $= \{y : y \geqslant 0\}$
d Domain $= \{x : x \neq -1\}$, Range $= \{y : y \neq 0\}$
e Domain $= \{x : x > 0\}$, Range $= \{y : y < 0\}$
f Domain $= \{x : x \neq 3\}$, Range $= \{y : y \neq 0\}$

4 **a**



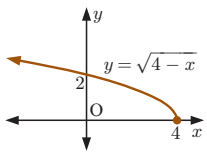
Domain $= \{x : x \geqslant 2\}$
Range $= \{y : y \geqslant 0\}$

b



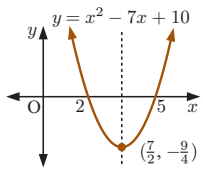
Domain $= \{x : x \neq 0\}$
Range $= \{y : y > 0\}$

c



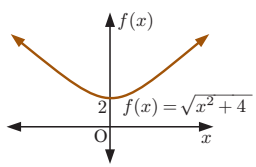
Domain $= \{x : x \leqslant 4\}$
Range $= \{y : y \geqslant 0\}$

d



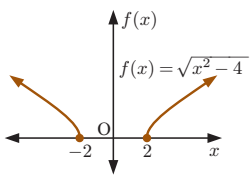
Domain $= \{x : x \in \mathbb{R}\}$
Range $= \{y : y \geqslant -2\frac{1}{4}\}$

e



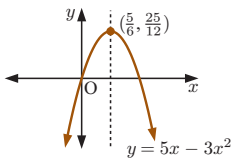
Domain $= \{x : x \in \mathbb{R}\}$
Range $= \{y : y \geqslant 2\}$

f



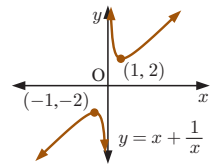
Domain $= \{x : x \leqslant -2$ or $x \geqslant 2\}$
Range $= \{y : y \geqslant 0\}$

g



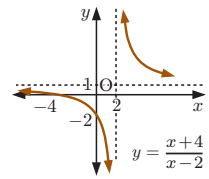
Domain $= \{x : x \in \mathbb{R}\}$
Range $= \{y : y \leqslant \frac{25}{12}\}$

h



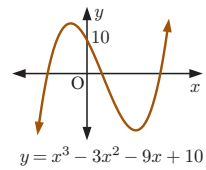
Domain $= \{x : x \neq 0\}$
Range $= \{y : y \leqslant -2$ or $y \geqslant 2\}$

i



Domain $= \{x : x \neq 2\}$
Range $= \{y : y \neq 1\}$

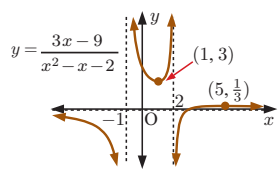
j



$y = x^3 - 3x^2 - 9x + 10$

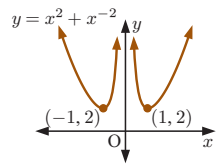
Domain $= \{x : x \in \mathbb{R}\}$
Range $= \{y : y \in \mathbb{R}\}$

k



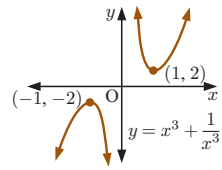
Domain $= \{x : x \neq -1$ and $x \neq 2\}$
Range $= \{y : y \leqslant \frac{1}{3}$ or $y \geqslant 3\}$

l



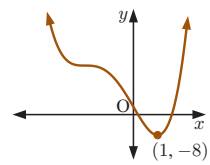
Domain $= \{x : x \neq 0\}$
Range $= \{y : y \geqslant 2\}$

m



Domain $= \{x : x \neq 0\}$
Range $= \{y : y \leqslant -2$ or $y \geqslant 2\}$

n



$y = x^4 + 4x^3 - 16x + 3$

Domain $= \{x : x \in \mathbb{R}\}$
Range $= \{y : y \geqslant -8\}$

EXERCISE 2D.1

1 **a** 5 **b** 5 **c** 4 **d** 4 **e** 6 **f** 0 **g** $\frac{2}{7}$ **h** $\frac{8}{27}$

2 **a** 1 **b** 6 **c** 4 **d** 3

3 **a** 2 **b** -4 **c** -6 **d** -5

EXERCISE 2D.2

1 **a** $x = \pm 3$ **b** no solution **c** $x = 0$
d $x = 4$ or -2 **e** $x = -1$ or 7 **f** no solution

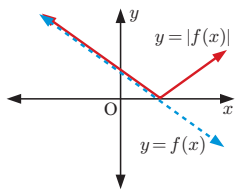
g $x = 1$ or $\frac{1}{3}$ **h** $x = 0$ or 3 **i** $x = -2$ or $\frac{14}{5}$

2 **a** $x = -\frac{1}{4}$ or $\frac{3}{2}$ **b** $x = -6$ or $-\frac{4}{3}$ **c** $x = \frac{1}{2}$

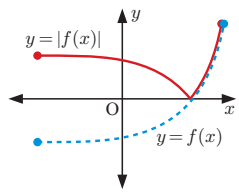
d $x = \frac{5}{2}$ **e** $x = 0$ or $\frac{2}{5}$ **f** $x = -2$ or 0

EXERCISE 2D.3

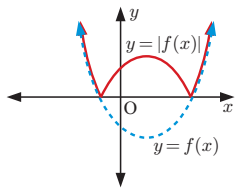
1 **a**



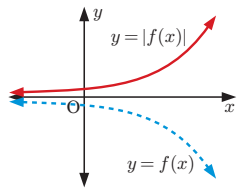
b



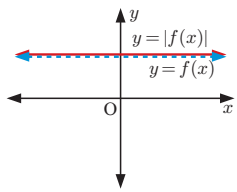
c



d



e



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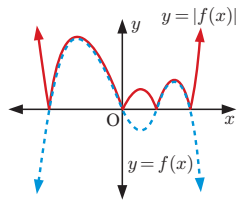
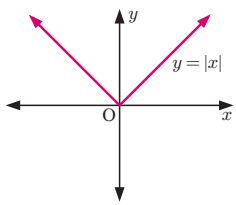


2 function **d** **3** $\{y : 0 \leqslant y \leqslant 6\}$

4 **a** false **b** true **c** true **d** false

5 **a**



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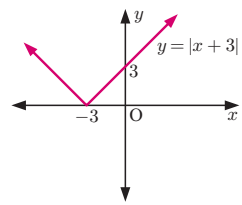

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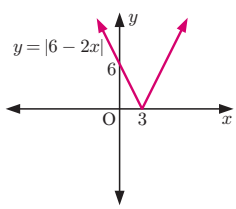

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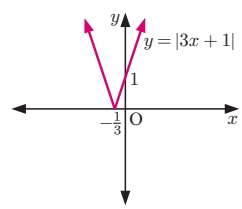
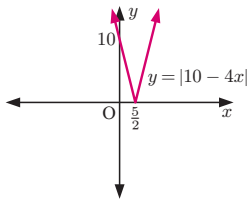
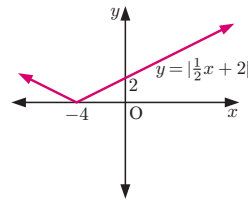


e



f



EXERCISE 2E

1 **a** $5 - 2x$ **b** $-2x - 2$ **c** 11

2 **a** $5 - x$ **b** $1 - x$ **c** $4 + x$

3 **a** $25x - 42$ **b** $\sqrt{8}$ **c** -7

4 $f(g(x)) = (2 - x)^2$, Domain $= \{x : x \in \mathbb{R}\}$, Range $= \{y : y \geqslant 0\}$

$g(f(x)) = 2 - x^2$, Domain $= \{x : x \in \mathbb{R}\}$, Range $= \{y : y \leqslant 2\}$

5 **a** $(f \circ g)(x) = 6x - 4$ **b** $x = -\frac{3}{4}$

6 **a** **i** $x^2 - 6x + 10$ **ii** $2 - x^2$ **b** $x = \pm\frac{1}{\sqrt{2}}$

7 **a** Let $x = 0$, $\therefore\ b = d$ and so

$ax + b = cx + b$

$\therefore\ ax = cx$ for all x

Let $x = 1$, $\therefore\ a = c$

b $(f \circ g)(x) = [2a]x + [2b + 3] = 1x + 0$ for all x

$\therefore\ 2a = 1$ and $2b + 3 = 0$

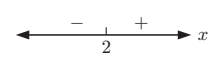
c Yes, $\{(g \circ f)(x) = [2a]x + [3a + b]\}$

8 **a** $(f \circ g)(x) = \sqrt{1 - x^2}$

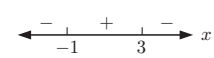
b Domain $= \{x : -1 \leqslant x \leqslant 1\}$, Range $= \{y : 0 \leqslant y \leqslant 1\}$

EXERCISE 2F

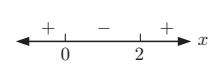
1 **a**



b



c



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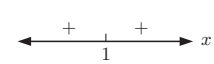

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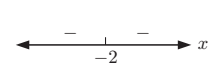
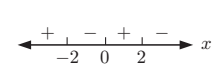


f



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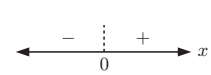

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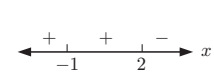
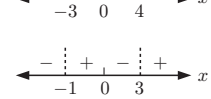


i

j

k



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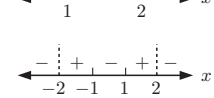
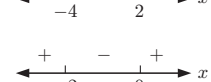


2 **a**

b

c



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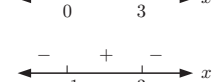

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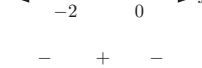

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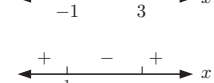

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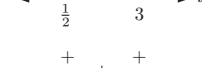

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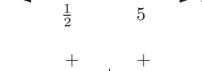

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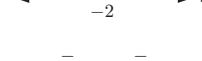
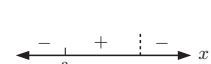


3 **a**

b

c



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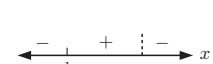

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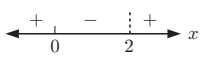

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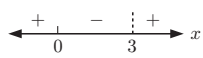

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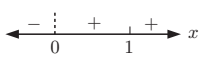

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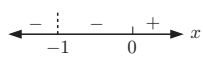
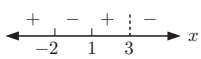
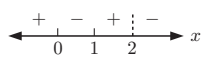


i



j



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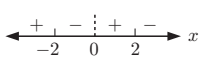

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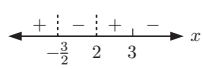
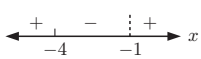


4 a



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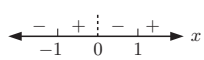

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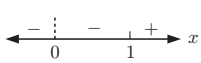
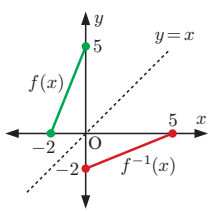


EXERCISE 2G

1 a



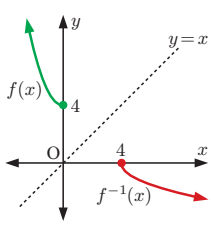
Domain of $f(x)$ is $\{x : -2 \leqslant x \leqslant 0\}$

Range of $f(x)$ is $\{y : 0 \leqslant y \leqslant 5\}$

Domain of $f^{-1}(x)$ is $\{x : 0 \leqslant x \leqslant 5\}$

Range of $f^{-1}(x)$ is $\{y : -2 \leqslant y \leqslant 0\}$

b



Domain of $f(x)$ is $\{x : x \leqslant 0\}$

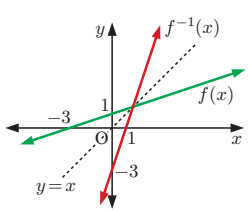
Range of $f(x)$ is $\{y : y \geqslant 4\}$

Domain of $f^{-1}(x)$ is $\{x : x \geqslant 4\}$

Range of $f^{-1}(x)$ is $\{y : y \leqslant 0\}$

c The function does not have an inverse, as it is not one-one.

d



Domain of $f(x)$ is $\{x : x \in \mathbb{R}\}$

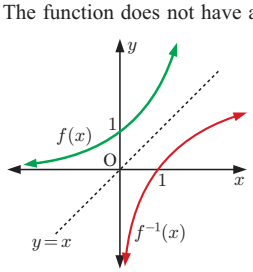
Range of $f(x)$ is $\{y : y \in \mathbb{R}\}$

Domain of $f^{-1}(x)$ is $\{x : x \in \mathbb{R}\}$

Range of $f^{-1}(x)$ is $\{y : y \in \mathbb{R}\}$

e The function does not have an inverse, as it is not one-one.

f



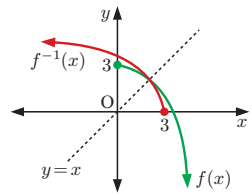
Domain of $f(x)$ is $\{x : x \in \mathbb{R}\}$

Range of $f(x)$ is $\{y : y > 0\}$

Domain of $f^{-1}(x)$ is $\{x : x > 0\}$

Range of $f^{-1}(x)$ is $\{y : y \in \mathbb{R}\}$

g



Domain of $f(x)$ is $\{x : x \geqslant 0\}$

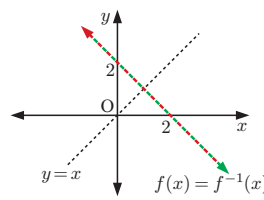
Range of $f(x)$ is $\{y : y \leqslant 3\}$

Domain of $f^{-1}(x)$ is $\{x : x \leqslant 3\}$

Range of $f^{-1}(x)$ is $\{y : y \geqslant 0\}$

h The function does not have an inverse, as it is not one-one.

i



Domain of $f(x)$ is $\{x : x \in \mathbb{R}\}$

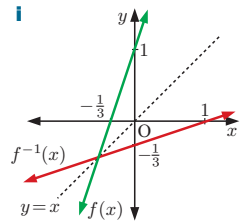
Range of $f(x)$ is $\{y : y \in \mathbb{R}\}$

Domain of $f^{-1}(x)$ is $\{x : x \in \mathbb{R}\}$

Range of $f^{-1}(x)$ is $\{y : y \in \mathbb{R}\}$

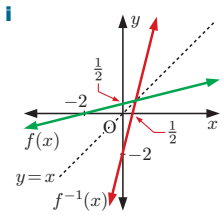
2 function **i** **3** Range $= \{y : -2 \leqslant y < 3\}$

4 a i



ii $f^{-1}(x) = \dfrac{x-1}{3}$

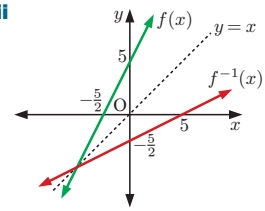
b i



ii $f^{-1}(x) = 4x - 2$

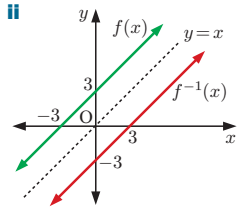
5 a i $f^{-1}(x) = \dfrac{x-5}{2}$

ii



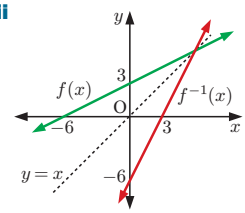
b i $f^{-1}(x) = x - 3$

ii



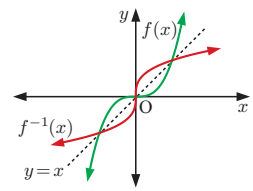
c i $f^{-1}(x) = 2x - 6$

ii



6 $f(x)$ is the same as $(f^{-1})^{-1}(x)$

7



8 $f^{-1}(x) = \frac{1}{x}$ and $f(x) = \frac{1}{x}$ $\therefore$ $f = f^{-1}$

$\therefore$ f is a self-inverse function

9 **a** $f^{-1}(x) = 2x + 2$

b **i** $(f \circ f^{-1})(x) = x$ **ii** $(f^{-1} \circ f)(x) = x$

10 **a** 10

b $f^{-1}(x) = \frac{x-5}{2}$ and $f^{-1}(-3) = -4$

$g^{-1}(x) = 8 - 2x$ and $g^{-1}(6) = -4$

$\therefore$ $f^{-1}(-3) - g^{-1}(6) = 0$

c $x = 3$

11 **a** **i** 25 **ii** 16 **b** $x = 1$

12 **a** Is not **b** Is **c** Is **d** Is **e** Is **f** Is not

13 $(f^{-1} \circ g^{-1})(x) = \frac{x+3}{8}$ and $(g \circ f)^{-1}(x) = \frac{x+3}{8}$

REVIEW SET 2A

1 **a** function **b** function **c** not a function **d** function

2 $a = -6,\ b = 13$

3 **a** $x = -2$ or 12 **b** $x = -5$ or 1

4 **a** 10 **b** $x^2 - x - 2$

5 **a** **i** Domain is $\{x : x \geqslant -3\}$, Range is $\{y : y \geqslant 2\}$

ii function is one-one

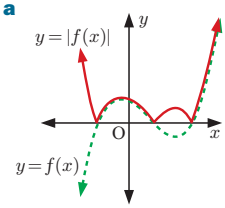
b **i** Domain is $\{x : x \in \mathbb{R}\}$, Range is $\{y : y \geqslant -5\}$

ii function is not one-one

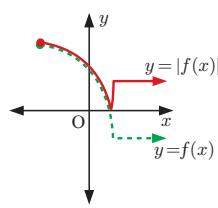
c **i** Domain is $\{x : x \in \mathbb{R}\}$,
Range is $\{y : y = -3$ or $y = 1\}$

ii function is not one-one

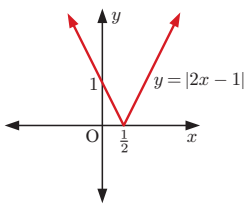
6 **a**



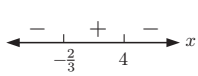
b



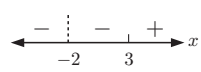
7



8 **a**

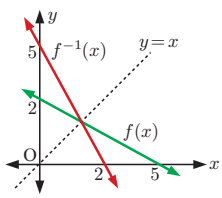


b

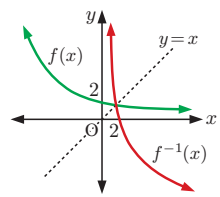


9 **a** $2x^2 + 1$ **b** $4x^2 - 12x + 11$

10 **a**



b



c The function does not have an inverse.

11 **a** $f^{-1}(x) = \frac{x-2}{4}$ **b** $f^{-1}(x) = \frac{3-4x}{5}$

12 **a** $f(-3) = (-3)^2 = 9$ **b** 169 **c** $x = -4$

$g(-\frac{4}{3}) = 1 - 6(-\frac{4}{3}) = 9$

13 $(f^{-1} \circ h^{-1})(x) = (h \circ f)^{-1}(x) = x - 2$

REVIEW SET 2B

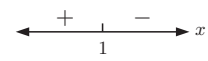
1 **a** not a function **b** function, one-one

c function, not one-one

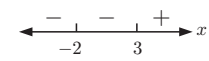
2 **a** 12 **b** $x = \pm 1$

3 **a** $x = -5$ or 6 **b** $x = 1$ or 3

4 **a**



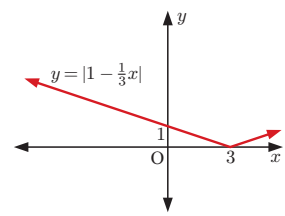
b



5 **a** $10 - 6x$ **b** $9x - 14$ **c** -23

6 $\{y : 3 \leqslant y \leqslant 7\}$

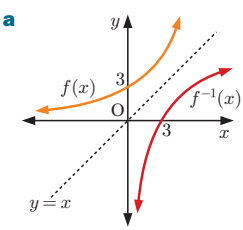
7



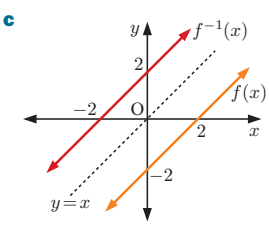
8 **a** **i** $1 - 10x$ **ii** $5 - 10x$ **b** $x = -\frac{3}{5}$

9 $a = 1,\ b = -6,\ c = 5$

10 **a**



c



b The function does not have an inverse.

11 **a** $f^{-1}(x) = \frac{7-x}{4}$ **b** $f^{-1}(x) = \frac{5x-3}{2}$

12 $(f^{-1} \circ h^{-1})(x) = (h \circ f)^{-1}(x) = \frac{4x+6}{15}$ **13** 16

EXERCISE 3A.1

1 **a** $x = 0, -\frac{7}{4}$ **b** $x = 0, -\frac{1}{3}$ **c** $x = 0, \frac{7}{3}$

d $x = 0, \frac{11}{2}$ **e** $x = 0, \frac{8}{3}$ **f** $x = 0, \frac{3}{2}$

g $x = 3, 2$ **h** $x = 4, -2$ **i** $x = 3, 7$

j $x = 3$ **k** $x = -4, 3$ **l** $x = -11, 3$

2 **a** $x = \frac{2}{3}$ **b** $x = -\frac{1}{2}, 7$ **c** $x = -\frac{2}{3}, 6$

d $x = \frac{1}{3}, -2$ **e** $x = \frac{3}{2}, 1$ **f** $x = -\frac{2}{3}, -2$

g $x = -\frac{2}{3}, 4$ **h** $x = \frac{1}{2}, -\frac{3}{2}$ **i** $x = -\frac{1}{4}, 3$

j $x = -\frac{3}{4}, \frac{5}{3}$ **k** $x = \frac{1}{7}, -1$ **l** $x = -2, \frac{28}{15}$

3 **a** $x = 2, 5$ **b** $x = -3, 2$ **c** $x = 0, -\frac{3}{2}$

d $x = 1, 2$ **e** $x = \frac{1}{2}, -1$ **f** $x = 3$

EXERCISE 3A.2

1 **a** $x = -5 \pm \sqrt{2}$ **b** no real solutions **c** $x = 4 \pm 2\sqrt{2}$
d $x = 8 \pm \sqrt{7}$ **e** $x = -3 \pm \sqrt{5}$ **f** $x = 2 \pm \sqrt{6}$
g $x = -1 \pm \sqrt{10}$ **h** $x = -\frac{1}{2} \pm \frac{1}{2}\sqrt{3}$ **i** $x = \frac{1}{3} \pm \frac{\sqrt{7}}{3}$

2 **a** $x = 2 \pm \sqrt{3}$ **b** $x = -3 \pm \sqrt{7}$ **c** $x = 7 \pm \sqrt{3}$
d $x = 2 \pm \sqrt{7}$ **e** $x = -3 \pm \sqrt{2}$ **f** $x = 1 \pm \sqrt{7}$
g $x = -3 \pm \sqrt{11}$ **h** $x = 4 \pm \sqrt{6}$ **i** no real solns.

3 **a** $x = -1 \pm \frac{1}{\sqrt{2}}$ **b** $x = \frac{5}{2} \pm \frac{\sqrt{19}}{2}$ **c** $x = -2 \pm \sqrt{\frac{7}{3}}$
d $x = 1 \pm \sqrt{\frac{7}{3}}$ **e** $x = \frac{3}{2} \pm \sqrt{\frac{37}{20}}$ **f** $x = -\frac{1}{2} \pm \frac{\sqrt{6}}{2}$

4 **a** $x = \frac{2}{3} \pm \frac{\sqrt{10}}{3}$ **b** $x = -\frac{1}{10} \pm \frac{\sqrt{21}}{10}$ **c** $x = -\frac{5}{6} \pm \frac{\sqrt{13}}{6}$

5 $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

EXERCISE 3A.3

1 **a** $x = 2 \pm \sqrt{7}$ **b** $x = -3 \pm \sqrt{2}$ **c** $x = 2 \pm \sqrt{3}$
d $x = -2 \pm \sqrt{5}$ **e** $x = 2 \pm \sqrt{2}$ **f** $x = \frac{1}{2} \pm \frac{1}{2}\sqrt{7}$
g $x = \frac{5}{6} \pm \frac{\sqrt{37}}{6}$ **h** $x = 2 \pm \sqrt{10}$ **i** $x = \frac{7}{4} \pm \frac{\sqrt{33}}{4}$

2 **a** $x = -2 \pm 2\sqrt{2}$ **b** $x = -\frac{5}{8} \pm \frac{\sqrt{57}}{8}$ **c** $x = \frac{5}{2} \pm \frac{\sqrt{13}}{2}$
d $x = -\frac{4}{9} \pm \frac{\sqrt{7}}{9}$ **e** $x = -\frac{7}{4} \pm \frac{\sqrt{97}}{4}$ **f** $x = \frac{1}{8} \pm \frac{\sqrt{145}}{8}$
g $x = \frac{1}{2} \pm \frac{1}{2}\sqrt{7}$ **h** $x = \frac{1}{2} \pm \frac{\sqrt{5}}{2}$ **i** $x = \frac{3}{4} \pm \frac{\sqrt{17}}{4}$

EXERCISE 3B

1 **a** $x \leqslant -3$ or $x \geqslant 2$ **b** $-1 < x < 4$
c $x < -\frac{1}{2}$ or $x > 3$ **d** $x \leqslant 0$ or $x \geqslant 1$
e $x \leqslant 0$ or $x \geqslant 3$ **f** $-\frac{2}{3} < x < 0$
g $-2 < x < 2$ **h** $x \leqslant -3$ or $x \geqslant 3$
i $x \neq -2$ **j** $x < -5$ or $x > 3$
k $4 \leqslant x \leqslant 7$ **l** $-6 < x < -4$
m $x \leqslant -2$ or $x \geqslant 15$ **n** $x \leqslant -1$ or $x \geqslant \frac{3}{2}$
o no solutions **p** $-\frac{3}{2} < x < \frac{1}{3}$
q $x < -\frac{4}{3}$ or $x > 4$ **r** no solutions **s** $\frac{1}{3} \leqslant x \leqslant \frac{1}{2}$
t $x \in \mathbb{R}$ **u** $-\frac{11}{8} - \frac{\sqrt{73}}{8} < x < -\frac{11}{8} + \frac{\sqrt{73}}{8}$

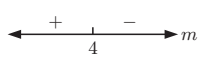
2 **a** $\square = <$ **b** $\square = \leqslant$ **c** $\square = \geqslant$ or $>$

EXERCISE 3C

1 **a** 2 distinct irrational roots **b** 2 distinct rational roots
c 2 distinct rational roots **d** 2 distinct irrational roots
e no real roots **f** a repeated root

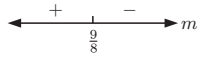
2 **a**, **c**, **d**, **f**

3 **a** $\Delta = 16 - 4m$



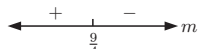
i $m = 4$ **ii** $m < 4$ **iii** $m > 4$

b $\Delta = 9 - 8m$



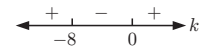
i $m = \frac{9}{8}$ **ii** $m < \frac{9}{8}$, $m \neq 0$ **iii** $m > \frac{9}{8}$

c $\Delta = 9 - 4m$



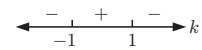
i $m = \frac{9}{4}$ **ii** $m < \frac{9}{4}$, $m \neq 0$ **iii** $m > \frac{9}{4}$

4 **a** $\Delta = k^2 + 8k$



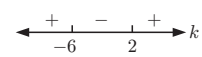
i $k < -8$ or $k > 0$ **ii** $k \leqslant -8$ or $k \geqslant 0$
iii $k = -8$ or 0 **iv** $-8 < k < 0$

b $\Delta = 4 - 4k^2$



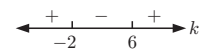
i $-1 < k < 1$, $k \neq 0$ **ii** $-1 \leqslant k \leqslant 1$, $k \neq 0$
iii $k = \pm 1$ **iv** $k < -1$ or $k > 1$

c $\Delta = k^2 + 4k - 12$



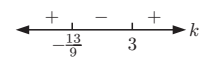
i $k < -6$ or $k > 2$ **ii** $k \leqslant -6$ or $k \geqslant 2$
iii $k = -6$ or 2 **iv** $-6 < k < 2$

d $\Delta = k^2 - 4k - 12$



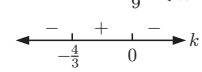
i $k < -2$ or $k > 6$ **ii** $k \leqslant -2$ or $k \geqslant 6$
iii $k = 6$ or -2 **iv** $-2 < k < 6$

e $\Delta = 9k^2 - 14k - 39$



i $k < -\frac{13}{9}$ or $k > 3$ **ii** $k \leqslant -\frac{13}{9}$ or $k \geqslant 3$
iii $k = -\frac{13}{9}$ or 3 **iv** $-\frac{13}{9} < k < 3$

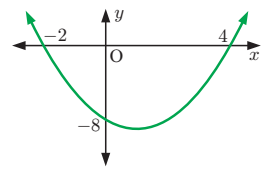
f $\Delta = -3k^2 - 4k$



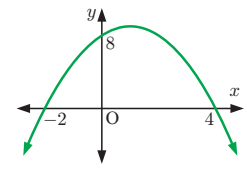
i $-\frac{4}{3} < k < 0$, $k \neq -1$ **ii** $-\frac{4}{3} \leqslant k \leqslant 0$, $k \neq -1$
iii $k = -\frac{4}{3}$ or 0 **iv** $k < -\frac{4}{3}$ or $k > 0$

EXERCISE 3D.1

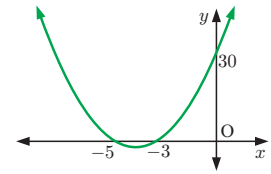
1 **a** $y = (x - 4)(x + 2)$



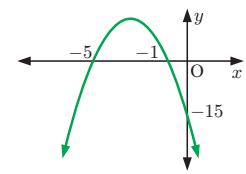
b $f(x) = -(x - 4)(x + 2)$



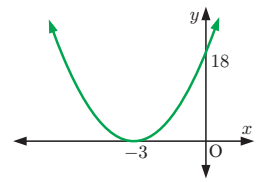
c $y = 2(x + 3)(x + 5)$



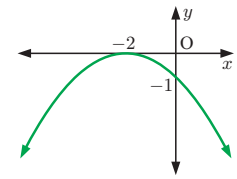
d $f(x) = -3(x + 1)(x + 5)$



e $f(x) = 2(x + 3)^2$



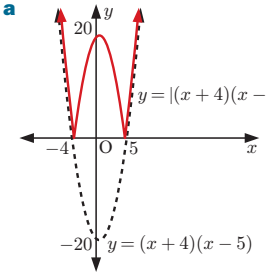
f $y = -\frac{1}{4}(x + 2)^2$



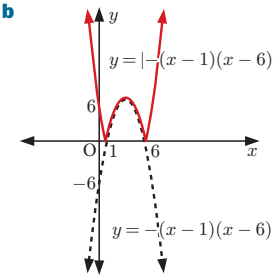
2 **a** $x = 1$ **b** $x = 1$ **c** $x = -4$
d $x = -3$ **e** $x = -3$ **f** $x = -2$

3 **a** C **b** E **c** B **d** F **e** G **f** H **g** A **h** D

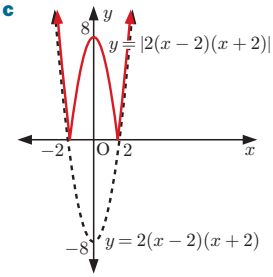
4 **a**



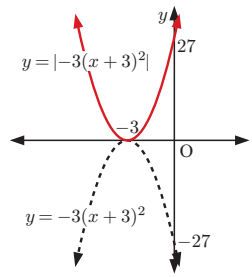
b



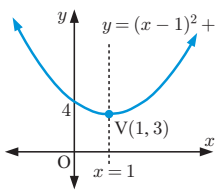
c



d



5 **a**



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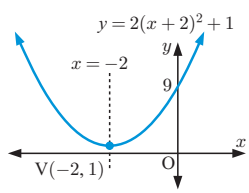

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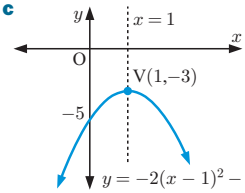

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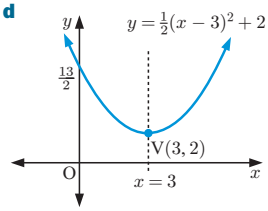

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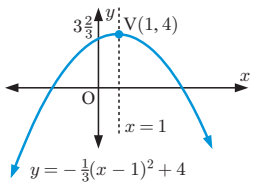

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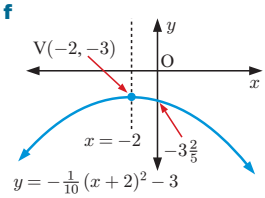
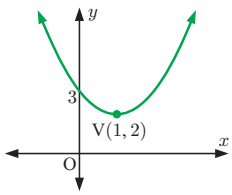


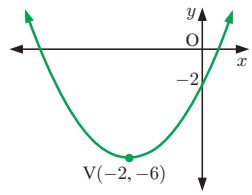
6 **a** G **b** A **c** E **d** B **e** I **f** C **g** D **h** F **i** H

EXERCISE 3D.2

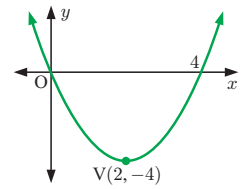
1 **a** $y = (x-1)^2 + 2$



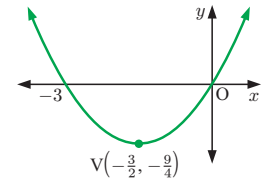
b $y = (x+2)^2 - 6$



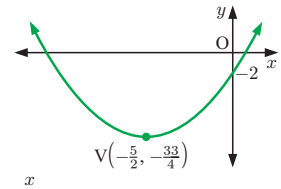
c $y = (x-2)^2 - 4$



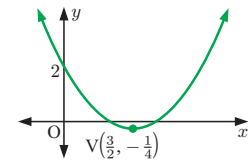
d $y = \left(x + \frac{3}{2}\right)^2 - \frac{9}{4}$



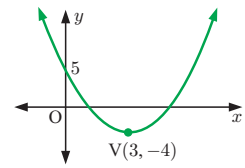
e $y = \left(x + \frac{5}{2}\right)^2 - \frac{33}{4}$



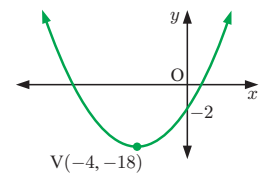
f $y = \left(x - \frac{3}{2}\right)^2 - \frac{1}{4}$



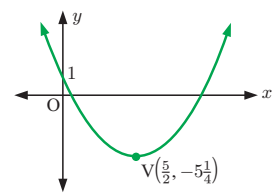
g $y = (x-3)^2 - 4$



h $y = (x+4)^2 - 18$

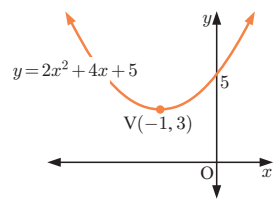


i $y = \left(x - \frac{5}{2}\right)^2 - 5\frac{1}{4}$



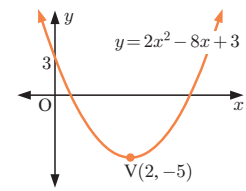
2 **a** **i** $y = 2(x+1)^2 + 3$ **ii** $(-1, 3)$ **iii** 5

iv



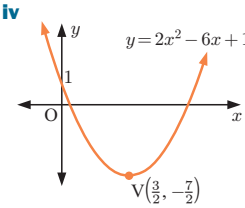
b **i** $y = 2(x-2)^2 - 5$ **ii** $(2, -5)$ **iii** 3

iv



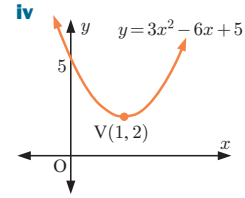
c **i** $y = 2(x - \frac{3}{2})^2 - \frac{7}{2}$ **ii** $(\frac{3}{2}, -\frac{7}{2})$ **iii** 1

iv

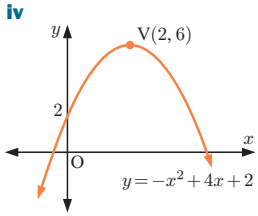


d **i** $y = 3(x-1)^2 + 2$ **ii** $(1, 2)$ **iii** 5

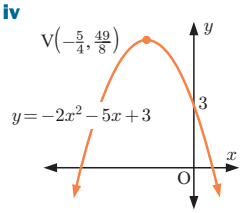
iv



e **i** $y = -(x-2)^2 + 6$
ii $(2, 6)$ **iii** 2
iv



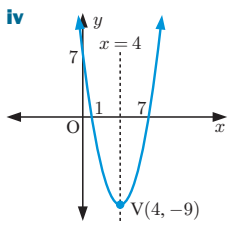
f **i** $y = -2(x + \frac{5}{4})^2 + \frac{49}{8}$
ii $(-\frac{5}{4}, \frac{49}{8})$ **iii** 3
iv



EXERCISE 3D.3

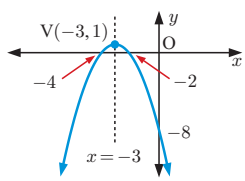
1 **a** $(2, -2)$ **b** $(-1, -4)$ **c** $(0, 4)$
d $(0, 1)$ **e** $(-2, -15)$ **f** $(-2, -5)$
g $(-\frac{3}{2}, -\frac{11}{2})$ **h** $(\frac{5}{2}, -\frac{19}{2})$ **i** $(1, -\frac{9}{2})$

2 **a** **i** $x = 4$
ii $(4, -9)$
iii x-intercepts 1, 7, y-intercept 7
iv



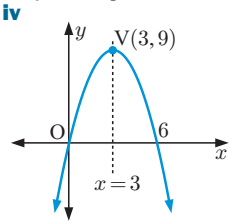
v $\{y : y \geqslant -9\}$

b **i** $x = -3$
ii $(-3, 1)$
iii x-int. -2, -4, y-intercept -8
iv



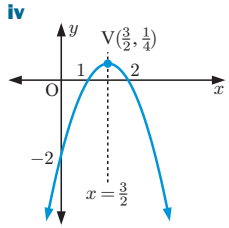
v $\{y : y \leqslant 1\}$

c **i** $x = 3$
ii $(3, 9)$
iii x-intercepts 0, 6, y-intercept 0
iv



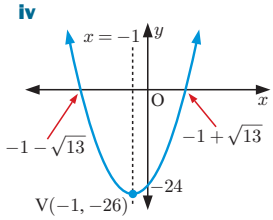
v $\{y : y \leqslant 9\}$

d **i** $x = \frac{3}{2}$
ii $(\frac{3}{2}, \frac{1}{4})$
iii x-intercepts 1, 2, y-intercept -2
iv



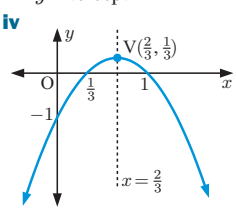
v $\{y : y \leqslant \frac{1}{4}\}$

e **i** $x = -1$
ii $(-1, -26)$
iii x-int. $-1 \pm \sqrt{13}$, y-intercept -24
iv



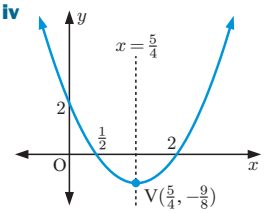
v $\{y : y \geqslant -26\}$

f **i** $x = \frac{2}{3}$
ii $(\frac{2}{3}, \frac{1}{3})$
iii x-intercepts $\frac{1}{3}$, 1, y-intercept -1
iv



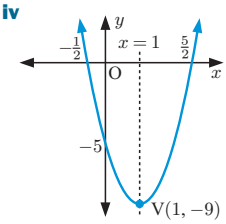
v $\{y : y \leqslant \frac{1}{3}\}$

g **i** $x = \frac{5}{4}$
ii $(\frac{5}{4}, -\frac{9}{8})$
iii x-intercepts $\frac{1}{2}$, 2, y-intercept 2
iv



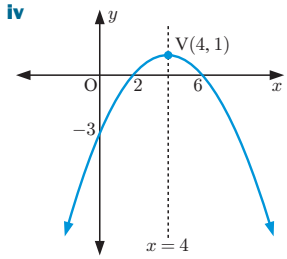
v $\{y : y \geqslant -\frac{9}{8}\}$

h **i** $x = 1$
ii $(1, -9)$
iii x-intercepts $-\frac{1}{2}$, $\frac{5}{2}$, y-intercept -5
iv

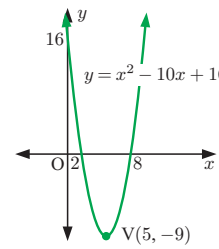


v $\{y : y \geqslant -9\}$

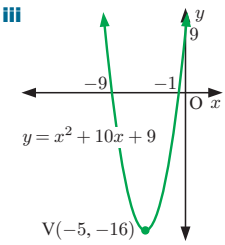
i **i** $x = 4$
ii $(4, 1)$
iii x-intercepts 2, 6, y-intercept -3
v $\{y : y \leqslant 1\}$
iv



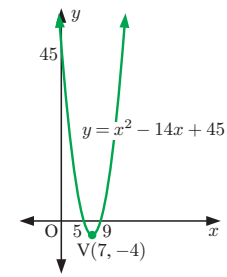
3 **a** **i** $y = (x-2)(x-8)$, roots are 2 and 8
ii $y = (x-5)^2 - 9$, vertex is $(5, -9)$
iii



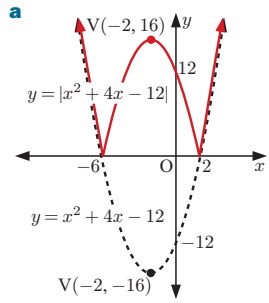
b **i** $y = (x+1)(x+9)$, roots are -1 and -9
ii $y = (x+5)^2 - 16$, vertex is $(-5, -16)$
iii



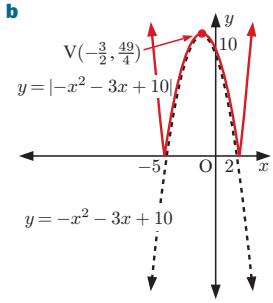
c **i** $y = (x-5)(x-9)$, roots are 5 and 9
ii $y = (x-7)^2 - 4$, vertex is $(7, -4)$
iii



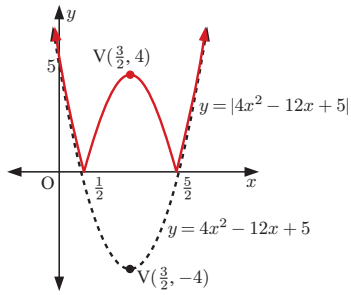
4 **a**



b



c



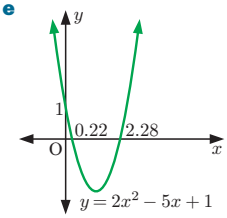
5 **a** $\{y : -10 \leqslant y \leqslant 15\}$ **b** $\{y : 3 \leqslant y \leqslant 19\}$
c $\{y : -13 \leqslant y \leqslant 37\}$ **d** $\{y : -8 \leqslant y \leqslant \frac{49}{4}\}$

EXERCISE 3D.4

1 **a** cuts x-axis twice, concave up
b cuts x-axis twice, concave up
c lies entirely below the x-axis, concave down, negative definite
d cuts x-axis twice, concave up
e touches x-axis, concave up
f cuts x-axis twice, concave down
g cuts x-axis twice, concave up
h cuts x-axis twice, concave down
i touches x-axis, concave up

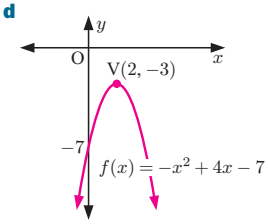
2 **a** concave up
b $\Delta = 17$ which is > 0
c x-intercepts ≈ 0.22 and 2.28
d y-intercept $= 1$

e



3 **a** $\Delta = -12$ which is < 0
b negative definite
c vertex is $(2, -3)$, y-intercept $= -7$

d



4 **a** $a = 1$ which is > 0 and $\Delta = -15$ which is < 0 so is entirely above the x-axis.
b $a = -1$ which is < 0 and $\Delta = -8$ which is < 0 so is entirely below the x-axis.
c $a = 2$ which is > 0 and $\Delta = -40$ which is < 0 so is entirely above the x-axis.
d $a = -2$ which is < 0 and $\Delta = -23$ which is < 0 so is entirely below the x-axis.

5 $a = 3$ which is > 0 and $\Delta = k^2 + 12$ which is always > 0 {as $k^2 \geqslant 0$ for all k} $\therefore$ always cuts x-axis twice.

6 $-4 < k < 4$

EXERCISE 3E

1 **a** $y = 2(x-1)(x-2)$ **b** $y = 2(x-2)^2$
c $y = (x-1)(x-3)$ **d** $y = -(x-3)(x+1)$
e $y = -3(x-1)^2$ **f** $y = -2(x+2)(x-3)$

2 **a** $y = \frac{3}{2}(x-2)(x-4)$ **b** $y = -\frac{1}{2}(x+4)(x-2)$
c $y = -\frac{4}{3}(x+3)^2$

3 **a** $y = 3x^2 - 18x + 15$ **b** $y = -4x^2 + 6x + 4$
c $y = -x^2 + 6x - 9$ **d** $y = 4x^2 + 16x + 16$

4 **a** $y = \frac{3}{2}x^2 - 6x + \frac{9}{2}$ **b** $y = -\frac{1}{3}x^2 + \frac{2}{3}x + 5$

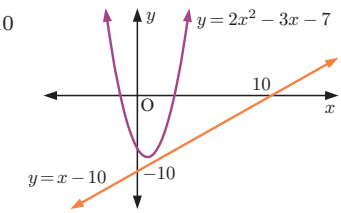
5 **a** $y = -(x-2)^2 + 4$ **b** $y = 2(x-2)^2 - 1$
c $y = -2(x-3)^2 + 8$ **d** $y = \frac{2}{3}(x-4)^2 - 6$
e $y = -2(x-2)^2 + 3$ **f** $y = 2(x - \frac{1}{2})^2 - \frac{3}{2}$

EXERCISE 3F

1 **a** $(1, 7)$ and $(2, 8)$ **b** $(4, 5)$ and $(-3, -9)$
c $(3, 0)$ (touching) **d** graphs do not meet

2 $c = -9$ **3** $m = 0$ or -8 **4** -1 or 11

5 **a** $c < -9$
b example: $c = -10$



6 **a** $c > -2$ **b** $c = -2$ **c** $c < -2$

7 **a** $m < -1$ or $m > 7$ **b** $m = -1$ or $m = 7$
c $-1 < m < 7$

8 **Hint:** A straight line through $(0, 3)$ will have an equation of the form $y = mx + 3$.

EXERCISE 3G

1 7 and -5 or -7 and 5 **2** 5 or $\frac{1}{5}$ **3** 14
4 18 and 20 or -18 and -20 **5** 15 sides
6 3.48 cm **7** **b** 6 cm by 6 cm by 7 cm
8 11.2 cm square **9** no

11 **a** $y = -\frac{8}{9}x^2 + 8$
b No, as the tunnel is only 4.44 m high when it is the same width as the truck.

12 **b** The graph is a parabola. **c** 21.25 m
d $f(x) = -0.05x^2 + 2x + 1.25$ **e** yes

EXERCISE 3H

1 **a** min. -1, when $x = 1$ **b** max. 8, when $x = -1$
c max. $8\frac{1}{3}$, when $x = \frac{1}{3}$ **d** min. $-1\frac{1}{8}$, when $x = -\frac{1}{4}$
e min. $4\frac{15}{16}$, when $x = \frac{1}{8}$ **f** max. $6\frac{1}{8}$, when $x = \frac{7}{4}$

2 **a** 40 refrigerators **b** \$4000

4 500 m by 250 m **5** **c** 100 m by 112.5 m

6 **a** $41\frac{2}{3}$ m by $41\frac{2}{3}$ m **b** 50 m by $31\frac{1}{4}$ m

7 **b** $3\frac{1}{8}$ units **8** **a** $y = 6 - \frac{3}{4}x$ **b** 3 cm by 4 cm

REVIEW SET 3A

1 **a** $-2, 1$
b $x = -\frac{1}{2}$
c 4
d $(-\frac{1}{2}, \frac{9}{2})$
f $\{y : y \leqslant \frac{9}{2}\}$

e