EXERCISE 6A.3

1 **a** quotient is $x+1$, remainder is $-x-4$
b quotient is 3, remainder is $-x+3$
c quotient is $3x$, remainder is $-2x-1$
d quotient is 0, remainder is $x-4$

2 **a** $1-\dfrac{2x}{x^2+x+1}$, $x^2-x+1=1(x^2+x+1)-2x$
b $x-\dfrac{2x}{x^2+2}$, $x^3=x(x^2+2)-2x$
c $x^2+x+3+\dfrac{3x-4}{x^2-x+1}$,
$x^4+3x^2+x-1=(x^2+x+3)(x^2-x+1)+3x-4$
d $2x+4+\dfrac{5x+2}{(x-1)^2}$,
$2x^3-x+6=(2x+4)(x-1)^2+5x+2$
e $x^2-2x+3-\dfrac{4x+3}{(x+1)^2}$,
$x^4=(x^2-2x+3)(x+1)^2-4x-3$
f $x^2-3x+5+\dfrac{15-10x}{(x-1)(x+2)}$,
$x^4-2x^3+x+5=(x^2-3x+5)(x-1)(x+2)+15-10x$

3 quotient is x^2+2x+3, remainder is 7
4 quotient is x^2-3x+5, remainder is $15-10x$

EXERCISE 6B.1

1 **a** $4, -\frac{3}{2}$ **b** $-3\pm\sqrt{10}$ **c** $5\pm\sqrt{19}$
d $0, \pm 2$ **e** $0, \pm\sqrt{11}$ **f** $\pm 2, \pm\sqrt{2}$
2 **a** $1, -\frac{2}{5}$ **b** $-\frac{1}{2}, \pm\sqrt{3}$ **c** $-3, \frac{1}{3}, 2$
d $0, 1\pm\sqrt{3}$ **e** $0, \pm\sqrt{7}$ **f** $\pm\sqrt{2}, \pm\sqrt{5}$
3 **a** $(2x+3)(x-5)$ **b** $x(x-7)(x-4)$
c $(x-3-\sqrt{6})(x-3+\sqrt{6})$
d $x(x+1+\sqrt{5})(x+1-\sqrt{5})$ **e** $x(3x-2)(2x+1)$
f $(x+1)(x-1)(x+\sqrt{5})(x-\sqrt{5})$
4 $P(\alpha)=0$, $P(\beta)=0$, $P(\gamma)=0$
5 **a** $P(x)=a(x+3)(x-4)(x-5)$, $a\neq 0$
b $P(x)=a(x+2)(x-2)(x-3)$, $a\neq 0$
c $P(x)=a(x-3)(x^2-2x-4)$, $a\neq 0$
d $P(x)=a(x+1)(x^2+4x+2)$, $a\neq 0$
6 **a** $P(x)=a(x^2-1)(x^2-2)$, $a\neq 0$
b $P(x)=a(x-2)(5x+1)(x^2-3)$, $a\neq 0$
c $P(x)=a(x+3)(4x-1)(x^2-2x-1)$, $a\neq 0$
d $P(x)=a(x^2-4x-1)(x^2+4x-3)$, $a\neq 0$

EXERCISE 6B.2

1 **a** $a=2$, $b=5$, $c=5$ **b** $a=3$, $b=4$, $c=3$
c $a=2$, $b=-5$, $c=4$
2 **a** $a=2$, $b=-2$ or $a=-2$, $b=2$
b $a=3$, $b=-1$
3 **a** $a=1$, $b=6$, $c=-7$ **b** $(x+3)(x+7)(x-1)$
4 **a** $p=2$, $q=7$, $r=5$ **b** $x=\frac{1}{2}, -1, -\frac{5}{2}$
5 **a** $a=3$, $b=-2$, $c=1$
b $3x^3+10x^2-7x+4=(x+4)(3x^2-2x+1)$
Δ of $3x^2-2x+1$ is -8,
$\therefore$ the only real zero is -4.
6 **a** $a=1$, $b=-2$, $c=-1$, $k=-4$
b $-\frac{2}{3}, 1\pm\sqrt{2}$
7 **a** $a=-2$, $b=2$ **b** $-1\pm\sqrt{3}$
8 $a=-11$, zeros are $\frac{3}{2}$, $\dfrac{-3\pm\sqrt{13}}{2}$
9 **a** $a=-9$, $b=-1$
b $P(x)=0$ when $x=-1, -\frac{1}{2}, 2, 4$
10 **Hint:** Let $x^3+3x^2-9x+c=(x+a)^2(x+b)$
When $c=5$, the cubic is $(x-1)^2(x+5)$.
When $c=-27$, the cubic is $(x+3)^2(x-3)$.

EXERCISE 6C

1 **a** $P(x)=Q(x)(x-2)+7$, $P(x)$ divided by $x-2$ leaves a remainder of 7.
b $P(-3)=-8$, $P(x)$ divided by $x+3$ leaves a remainder of -8.
c $P(5)=11$, $P(x)=Q(x)(x-5)+11$
2 **a** 4 **b** -19 **c** 1 **3** 4
4 **a** $a=3$ **b** $a=2$ **5** $a=-5$, $b=6$
6 $a=-5$, $b=6$ **7** -7
8 **a** $P(x)=Q(x)(2x-1)+R$
$$\begin{aligned}P(\tfrac{1}{2})&=Q(\tfrac{1}{2})(2\times\tfrac{1}{2}-1)+R\\&=Q(\tfrac{1}{2})\times 0+R\\&=R\end{aligned}$$
b **i** -3 **ii** 7 **iii** -7
9 $a=3$, $b=10$ **10** **a** -3 **b** 1

EXERCISE 6D

1 **a** factor **b** not a factor **c** factor **d** not a factor
2 **a** $c=2$ **b** $c=-2$ **c** $b=3$
3 $k=-8$, $P(x)=(x+2)(x-2)(2x+1)$
4 **a** $k=-8$ **b** $P(x)=(x-3)(3x^2+x-2)$
c $x=-1, \frac{2}{3}, 3$
5 $a=7$, $b=-14$ **6** $a=3$, $b=2$
7 **a** $a=7$, $b=-6$ **b** 60
c $P(x)=(x+3)(2x^2+3x-2)$ **d** $-3, -2, \frac{1}{2}$
8 **a** $a=7$, $b=2$ **b** $x=-2\pm\sqrt{6}$
9 **a** **i** $P(a)=0$, $x-a$ is a factor
ii $(x-a)(x^2+ax+a^2)$
b **i** $P(-a)=0$, $x+a$ is a factor
ii $(x+a)(x^2-ax+a^2)$
10 $a=2$

EXERCISE 6E

1 **a** $x=1, 2, 3$ **b** $x=-1, 2$ {2 is a double root}
c $x=1, -1, -2$ **d** $x=-1, 3, 4$ **e** $x=-5, -4, 4$
f $x=-3, -5$ {-5 is a double root}
2 **a** $x=-2, 2, 3$ **b** $x=-3, -2, 6$ **c** $x=-3, 4, 7$

REVIEW SET 6A

1 **a** $8x^2+6x+3$ **b** $7x^2-9x+9$
c $15x^4+32x^3+29x-4$
2 **a** quotient $=2x+5$, remainder $=3$
b quotient $=x^2-4x+2$, remainder $=-5$
3 **a** $\frac{4}{3}, -2$ **b** $-4\pm\sqrt{5}$

4 **a** $a = 1,\ b = -2,\ c = 3$
b Δ of $x^2 - 2x + 3$ is -8
$\therefore$ the only real root is $x = -3$.

5 **a** 1 **b** -53 **6** **a** not a factor **b** factor

7 $k = 6$ **8** $a = 4,\ b = -1$ **9** $c = 3$

10 **a** $a = -19,\ b = -20$ **b** $-5, -1, 4$

11 $x = -3, -1, 5$

REVIEW SET 6B

1 **a** $12x^4 - 9x^3 + 8x^2 - 26x + 15$
b $4x^4 - 4x^3 + 13x^2 - 6x + 9$

2 **a** $x^2 - 2x + 4 - \dfrac{8}{x+2}$ **b** $x - 5 + \dfrac{19x + 30}{(x+2)(x+3)}$

3 $P(x) = a(4x - 1)(x^2 - 2x - 4),\quad a \neq 0$

4 For $k = 3,\ b = 27,\ x = 3$ or -3.
For $k = -1,\ b = -5,\ x = -1$ or 5.

5 **a** -3 **b** -7 **6** **a** $a = 5$ **b** -12

7 **b** $(x - 2)(x^2 + 2x - 9)$ **c** $2, -1 \pm \sqrt{10}$

8 $a = \frac{8}{7},\ b = \frac{174}{7}$

9 $k = 8$, the zeros are $-1, -2$ {-2 is a double root}

10 **a** $a = -20,\ b = 12$ **b** $f(x) = (2x - 1)(x - 6)(x + 2)$

11 $x = -4, 2, 3$

EXERCISE 7A.1

1 **a** gradient $= 3$, y-intercept is 5
b gradient $= 4$, y-intercept is -2
c gradient $= \frac{1}{5}$, y-intercept is $\frac{3}{5}$
d gradient $= -7$, y-intercept is -3
e gradient $= \frac{1}{6}$, y-intercept is $\frac{1}{3}$
f gradient $= -\frac{5}{3}$, y-intercept is $\frac{8}{3}$

2 **a** $y = x - 2$ **b** $y = -x + 4$ **c** $y = 2x$
d $y = -\frac{1}{2}x + 3$

3 **a** $y = 4x - 13$ **b** $y = -3x - 5$ **c** $y = -5x + 32$
d $y = \frac{1}{2}x + \frac{7}{2}$ **e** $y = -\frac{1}{3}x + \frac{8}{3}$ **f** $y = 6$

4 **a** $2x - 3y = -11$ **b** $3x - 5y = -23$ **c** $x + 3y = 5$
d $2x + 7y = -2$ **e** $4x - y = -11$ **f** $2x + y = 7$
g $7x + 2y = 18$ **h** $6x - y = -40$

5 **a** $y = \frac{5}{2}x - 2$ **b** $y = -2x + 3$ **c** $y = -2$
d $y = -\frac{1}{5}x + \frac{2}{5}$ **e** $y = \frac{1}{6}x - \frac{11}{6}$ **f** $y = -\frac{2}{3}x - \frac{11}{3}$

6 **a** $x - 3y = -3$ **b** $5x - y = 1$ **c** $x - y = 3$
d $4x - 5y = 10$ **e** $x - 2y = -1$ **f** $2x + 3y = -5$

7 **a** $\sqrt{45}$ units **b** $(-1, \frac{7}{2})$ **c** $\frac{1}{2}$ **d** $y = \frac{1}{2}x + 4$

8 **a** $y = \frac{4}{3}x - 1$ **b** $2x - 3y = -13$ **c** $y = x + 1$
d $2x + y = -2$ **e** $y = -\frac{2}{3}x + 2$ **f** $3x + 7y = -9$

9 **a** $M = \frac{1}{3}p + 2$ **b** $R = -\frac{5}{4}n + 2$ **c** $T = \frac{1}{2}x - 1$
d $F = \frac{1}{10}x + 1$ **e** $H = -\frac{1}{2}z + 2$ **f** $W = -\frac{1}{6}t - 2$

10 **a** $x + 2y = 13$ **b** $(13, 0)$

11 **a** $3x + 5y = 10$ **b** $(0, 2)$ **12** 54 units^2

EXERCISE 7A.2

1 **a** $\sqrt{160}$ units **b** $(-1, 1)$ **c** -3 **d** $x - 3y = -4$

2 **a** $y = x - 4$ **b** $y = 2x + 6$ **c** $y = \frac{6}{5}x + \frac{7}{2}$ **d** $y = 1$

3 15 units^2

EXERCISE 7B

1 **a** $(1, 3)$ **b** $(6, -3)$ **c** $(-5, 3)$ **d** $(-1, -2)$

2 **a** $3x + 5y = 9$ **b** $(-2, 3)$ **3** $(4, 2)$

4 **a** $x - 3y = -8$ **b** $y = -3x - 4$ **c** $(-2, 2)$

5 **a** $(0, -1)$ **b** 25 units^2

6 **a** $(-1, 0)$ **b** 26 units^2 **7** 30 units^2

8 **a** **i** $(5, 0)$ **ii** $(7, -4)$ **iii** $(6, -2)$
b **Hint:** Find the gradients of MN and AC.
c **i** 15 units^2 **ii** 20 units^2

EXERCISE 7C

1 $(-1, -2)$ and $(\frac{11}{5}, -\frac{2}{5})$ **2** $\sqrt{18}$ units

3 $x - 2y = 0$ **4** $(-\frac{4}{3}, -\frac{8}{3})$ and $(2, -1)$

5 $\sqrt{125}$ units **6** $x - 3y = -13$

7 $(3, -\frac{3}{2})$ and $(4, -1)$ **8** $(\frac{7}{3}, \frac{5}{2})$

EXERCISE 7D

1 **a** $y = \frac{1}{2}x^3 + 2$ **b** $y = 3\sqrt{x} - 1,\quad x \geqslant 0$
c $y = 3 - x^4$ **d** $y = \frac{1}{3} \times 2^x$
e $y = \dfrac{2}{x} + 1$ **f** $y = -\frac{3}{2} \times 3^x + 11$

2 **a** **i** $y = x^2 + 3x$ **ii** $y = 18$
b **i** $y = -\frac{1}{2}\sqrt{x} + \dfrac{10}{\sqrt{x}},\quad x > 0$ **ii** $y = \frac{17\sqrt{3}}{6}$
c **i** $y = \dfrac{5}{3x} \times 2^x$ **ii** $y = \frac{40}{9}$
d **i** $y = 2x^3 - 9x$ **ii** $y = 27$
e **i** $y = \dfrac{1}{x^2} - \dfrac{12}{x} + 36$ **ii** $y = 32\frac{1}{9}$
f **i** $y = (x + 2)^2 + 3$ **ii** $y = 28$

3 **a** $\lg y = 2x - 1$ **b** $y = \frac{1}{10} \times 10^{2x}$

4 $y = 1000 \times 10^{-\frac{3}{2}x}$

5 **a** $y = \frac{1}{10\,000} \times 10^x$ **b** $y = 10\,000 \times (\frac{1}{10})^x$
c $y = 5 \times 4^x$

6 **a** $y = 10 \times 10^{\frac{1}{3}x}$ **b** $y = 1000$

7 **a** $\lg y = -\frac{1}{2}\lg x + 2$ **b** $y = \dfrac{100}{\sqrt{x}}$

8 **a** $y = x^{\frac{1}{4}}$ **b** $y = \dfrac{1000}{x}$ **c** $y = x^2\sqrt{1000}$

9 **a** $K = 7\sqrt{t}$ **b** $K = 21$ **10** **a** 3 **b** $\lg 4$

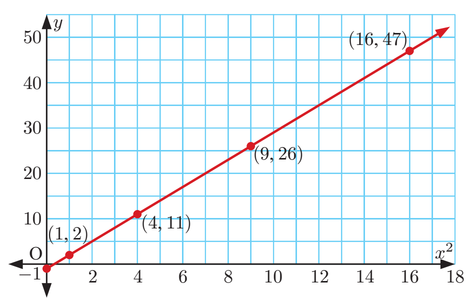
EXERCISE 7E

1 **a**

x^2	1	4	9	16
y	2	11	26	47

c $y = 3x^2 - 1$

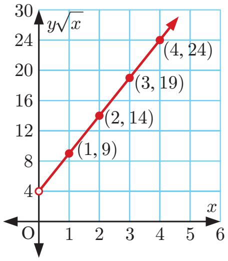
b



2 **a**

x	1	2	3	4
$y\sqrt{x}$	9	14	19	24

b



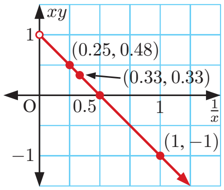
c $y = 5\sqrt{x} + \dfrac{4}{\sqrt{x}}, \quad x > 0$

d $y = 21$

3 **a**

$\dfrac{1}{x}$	1	0.5	0.33	0.25
xy	-1	0	0.33	0.48

b



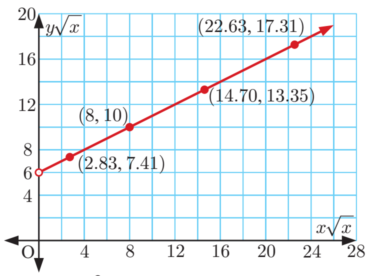
c $a = 1, \quad b = -2$

d $y = 0.08$

4 **a**

$x\sqrt{x}$	2.83	8	14.70	22.63
$y\sqrt{x}$	7.41	10	13.35	17.31

b



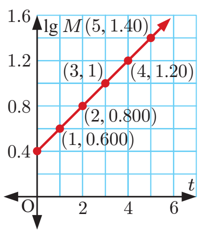
c $y = \frac{1}{2}x + \dfrac{6}{\sqrt{x}}, \quad x > 0$

d $y = 6.5$

5 **a**

t	1	2	3	4	5
$\lg M$	0.600	0.800	1	1.20	1.40

b

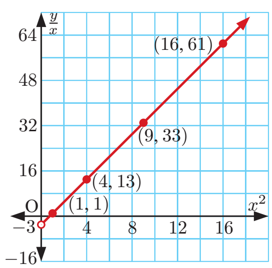


c $M \approx 2.51 \times 1.58^t$

d ≈ 2.51 g

6 **a** x^2

b



c $a = 4,$ $b = -3$

d $y = 485$

7 Plot xy against $\sqrt{x}$. $\quad y = \dfrac{8}{x} - \dfrac{4}{\sqrt{x}} \quad \{a = 8, \ b = -4\}$

8 **a** $a \approx 4.90, \ b \approx 2.00$ **b** ≈ 44.1 m **c** ≈ 4.04 seconds

REVIEW SET 7A

1 **a** $\sqrt{40}$ units **b** $(2, 5)$ **c** $x + 3y = 17$

2 $y = -2x + 6$

3 The gradient of a vertical line is undefined.

4 **a** $x + 2y = 7$ **b** $(7, 0)$ **5** $(3, -1)$

6 **a** $(-1, 4)$ **b** $32\frac{1}{2}$ units^2

7 $(-\frac{7}{5}, \frac{26}{5})$ and $(2, -5)$ **8** $y = x - 5$

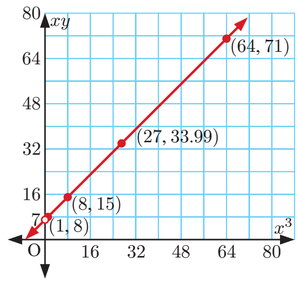
10 **a** $y = \dfrac{3}{\sqrt{x}} - \dfrac{2}{x}, \quad x > 0$ **b** $y = 1$

11 **a** $\lg y = \frac{1}{2}\lg x + 1$ **b** $y = 10\sqrt{x}$

12 **a**

x^3	1	8	27	64
xy	8	15	33.99	71

b



c $y = x^2 + \dfrac{7}{x}$

d $y = 50$

REVIEW SET 7B

1 $y = -3x + 7$ **2** $\sqrt{80}$ units

3 **a** $y = 5x - \dfrac{2}{x}$ **b** $39\frac{3}{4}$

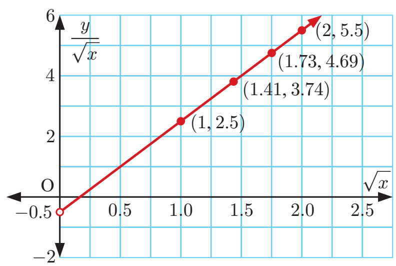
4 **a** $r = \frac{5}{7}a + 2$ **b** $K = \frac{3}{5}s + 3$

5 $(3, 2)$ **6** $5x - 8y = 31$

7 **a**

$\sqrt{x}$	1	1.41	1.73	2
$\dfrac{y}{\sqrt{x}}$	2.5	3.74	4.69	5.5

b



c $y = 3x - \dfrac{\sqrt{x}}{2}$

8 **a** $bx + ay = ab$

b **Hint:** $\cos\theta = \dfrac{a}{\sqrt{a^2 + b^2}}, \quad \sin\theta = \dfrac{b}{\sqrt{a^2 + b^2}}$

9 **a** **i** $(2, 12)$ **ii** $(11, 0)$ **b** 75 units^2

10 $(\frac{7}{8}, \frac{1}{2})$

11 **a** Plot $\lg y$ against x.

$y = 100 \times (10^{-\frac{1}{3}})^x \quad \{a = 100, \ b = 10^{-\frac{1}{3}}\}$

b $y \approx 46.4$

EXERCISE 8A

1 **a** $\frac{\pi}{2}^c$ **b** $\frac{\pi}{3}^c$ **c** $\frac{\pi}{6}^c$ **d** $\frac{\pi}{10}^c$ **e** $\frac{\pi}{20}^c$
f $\frac{3\pi}{4}^c$ **g** $\frac{5\pi}{4}^c$ **h** $\frac{3\pi}{2}^c$ **i** $2\pi^c$ **j** $4\pi^c$
k $\frac{7\pi}{4}^c$ **l** $3\pi^c$ **m** $\frac{\pi}{5}^c$ **n** $\frac{4\pi}{9}^c$ **o** $\frac{23\pi}{18}^c$

2 **a** 0.641^c **b** 2.39^c **c** 5.55^c **d** 3.83^c **e** 6.92^c

3 **a** $36°$ **b** $108°$ **c** $135°$ **d** $10°$ **e** $20°$
f $140°$ **g** $18°$ **h** $27°$ **i** $210°$ **j** $22.5°$

4 **a** $114.59°$ **b** $87.66°$ **c** $49.68°$ **d** $182.14°$
e $301.78°$

5 **a**

Degrees	0	45	90	135	180	225	270	315	360
Radians	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π

b

Deg.	0	30	60	90	120	150	180	210	240	270	300	330	360
Rad.	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	2π

EXERCISE 8B

1 **a** 49.5 cm, 223 cm^2 **b** 23.0 cm, 56.8 cm^2

2 **a** 3.14 m **b** 9.30 m^2 **3** **a** 5.91 cm **b** 18.9 cm

4 **a** 0.686^c **b** 0.6^c

5 **a** $\theta = 0.75^c$, area $= 24$ cm^2
b $\theta = 1.68^c$, area $= 21$ cm^2
c $\theta \approx 2.32^c$, area $= 126.8$ cm^2

6 10 cm, 25 cm^2

8 **a** 11.7 cm **b** $r \approx 11.7$ **c** 37.7 cm **d** 3.23^c

9 **a** $\alpha \approx 18.43$ **b** $\theta \approx 143.1$ **c** 387 m^2

10 25.9 cm **11** **b** 2 h 49 min **12** 227 m^2

EXERCISE 8C

1 **a** **i** A$(\cos 26°, \sin 26°)$, B$(\cos 146°, \sin 146°)$, C$(\cos 199°, \sin 199°)$
ii A$(0.899, 0.438)$, B$(-0.829, 0.559)$, C$(-0.946, -0.326)$
b **i** A$(\cos 123°, \sin 123°)$, B$(\cos 251°, \sin 251°)$, C$(\cos(-35°), \sin(-35°))$
ii A$(-0.545, 0.839)$, B$(-0.326, -0.946)$, C$(0.819, -0.574)$

2

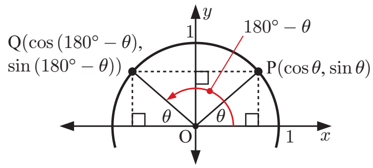
θ (degrees)	$0°$	$90°$	$180°$	$270°$	$360°$	$450°$
θ (radians)	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π	$\frac{5\pi}{2}$
sine	0	1	0	-1	0	1
cosine	1	0	-1	0	1	0
tangent	0	undef	0	undef	0	undef

3 **a** **i** $\frac{1}{\sqrt{2}} \approx 0.707$ **ii** $\frac{\sqrt{3}}{2} \approx 0.866$

b

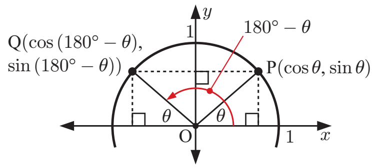
θ (degrees)	$30°$	$45°$	$60°$	$135°$	$150°$	$240°$	$315°$
θ (radians)	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\frac{4\pi}{3}$	$\frac{7\pi}{4}$
sine	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{2}}$
cosine	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\frac{1}{\sqrt{2}}$
tangent	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	$\sqrt{3}$	-1

4 **a** **i** 0.985 **ii** 0.985 **iii** 0.866 **iv** 0.866
v 0.5 **vi** 0.5 **vii** 0.707 **viii** 0.707
b $\sin(180° - \theta) = \sin\theta$
c $\sin\theta$ and $\sin(180° - \theta)$ have the same value, as P and Q have the same y-coordinate.



d **i** $135°$ **ii** $129°$ **iii** $\frac{2\pi}{3}$ **iv** $\frac{5\pi}{6}$

5 **a** **i** 0.342 **ii** -0.342 **iii** 0.5 **iv** -0.5
v 0.906 **vi** -0.906 **vii** 0.174 **viii** -0.174
b $\cos(180° - \theta) = -\cos\theta$
c $\cos(180° - \theta) = -\cos\theta$, as the x-coordinates of P and Q are negatives of each other.



d **i** $140°$ **ii** $161°$ **iii** $\frac{4\pi}{5}$ **iv** $\frac{3\pi}{5}$

6 **a** ≈ 0.6820 **b** ≈ 0.8572 **c** ≈ -0.7986
d ≈ 0.9135 **e** ≈ 0.9063 **f** ≈ -0.6691

7 **a**

Quadrant	*Degree measure*	*Radian measure*	$\cos\theta$	$\sin\theta$	$\tan\theta$
1	$0° < \theta < 90°$	$0 < \theta < \frac{\pi}{2}$	+ve	+ve	+ve
2	$90° < \theta < 180°$	$\frac{\pi}{2} < \theta < \pi$	$-$ve	+ve	$-$ve
3	$180° < \theta < 270°$	$\pi < \theta < \frac{3\pi}{2}$	$-$ve	$-$ve	+ve
4	$270° < \theta < 360°$	$\frac{3\pi}{2} < \theta < 2\pi$	+ve	$-$ve	$-$ve

b **i** 1 and 4 **ii** 2 and 3 **iii** 3 **iv** 2

8 **a** $\widehat{AOQ} = 180° - \theta$ or $\pi - \theta$ radians
b [OQ] is a reflection of [OP] in the y-axis and so Q has coordinates $(-\cos\theta, \sin\theta)$.
c $\cos(180° - \theta) = -\cos\theta$, $\sin(180° - \theta) = \sin\theta$

9 **a**

θ^c	$\sin\theta$	$\sin(-\theta)$	$\cos\theta$	$\cos(-\theta)$
0.75	0.682	-0.682	0.732	0.732
1.772	0.980	-0.980	-0.200	-0.200
3.414	-0.269	0.269	-0.963	-0.963
6.25	-0.0332	0.0332	0.999	0.999
-1.17	-0.921	0.921	0.390	0.390

b $\sin(-\theta) = -\sin\theta$, $\cos(-\theta) = \cos\theta$

EXERCISE 8D.1

1 **a** $\cos\theta = \pm\frac{\sqrt{3}}{2}$ **b** $\cos\theta = \pm\frac{2\sqrt{2}}{3}$ **c** $\cos\theta = \pm 1$
d $\cos\theta = 0$

2 **a** $\sin\theta = \pm\frac{3}{5}$ **b** $\sin\theta = \pm\frac{\sqrt{7}}{4}$ **c** $\sin\theta = 0$
d $\sin\theta = \pm 1$

3 **a** $\sin\theta = \frac{\sqrt{5}}{3}$ **b** $\cos\theta = -\frac{\sqrt{21}}{5}$ **c** $\cos\theta = \frac{4}{5}$
d $\sin\theta = -\frac{12}{13}$

4 **a** $\tan\theta = -\frac{1}{2\sqrt{2}}$ **b** $\tan\theta = -2\sqrt{6}$ **c** $\tan\theta = \frac{1}{\sqrt{2}}$
d $\tan\theta = -\frac{\sqrt{7}}{3}$

5 **a** $\sin x = \frac{2}{\sqrt{13}}$, $\cos x = \frac{3}{\sqrt{13}}$
b $\sin x = \frac{4}{5}$, $\cos x = -\frac{3}{5}$
c $\sin x = -\sqrt{\frac{5}{14}}$, $\cos x = -\frac{3}{\sqrt{14}}$
d $\sin x = -\frac{12}{13}$, $\cos x = \frac{5}{13}$

6 $\sin\theta = \dfrac{-k}{\sqrt{k^2+1}}$, $\cos\theta = \dfrac{-1}{\sqrt{k^2+1}}$

EXERCISE 8D.2

1 **a** $\theta \approx 1.33$ or 4.47 **b** $\theta \approx 0.592$ or 5.69
c $\theta \approx 0.644$ or 2.50 **d** $\theta = \frac{\pi}{2}$ or $\frac{3\pi}{2}$
e $\theta \approx 0.876$ or 4.02 **f** $\theta \approx 0.674$ or 5.61
g $\theta \approx 0.0910$ or 3.05 **h** $\theta \approx 1.52$ or 4.66
i $\theta \approx 1.35$ or 1.79

2 **a** $\theta \approx 1.82$ or 4.46 **b** $\theta = 0, \pi$, or 2π
c $\theta \approx 1.88$ or 5.02 **d** $\theta \approx 3.58$ or 5.85
e $\theta \approx 1.72$ or 4.86 **f** $\theta \approx 1.69$ or 4.59
g $\theta \approx 1.99$ or 5.13 **h** $\theta \approx 2.19$ or 4.10
i $\theta \approx 3.83$ or 5.60

EXERCISE 8E

1

	a	b	c	d	e
$\sin\theta$	$\frac{1}{\sqrt{2}}$	1	$-\frac{1}{\sqrt{2}}$	0	$-\frac{1}{\sqrt{2}}$
$\cos\theta$	$\frac{1}{\sqrt{2}}$	0	$\frac{1}{\sqrt{2}}$	-1	$-\frac{1}{\sqrt{2}}$
$\tan\theta$	1	undef	-1	0	1

2

	a	b	c	d	e
$\sin\beta$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$
$\cos\beta$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$\tan\beta$	$\frac{1}{\sqrt{3}}$	$-\sqrt{3}$	$\frac{1}{\sqrt{3}}$	$-\sqrt{3}$	$-\frac{1}{\sqrt{3}}$

3 **a** $\cos 120° = -\frac{1}{2}$, $\sin 120° = \frac{\sqrt{3}}{2}$, $\tan 120° = -\sqrt{3}$
b $\cos(-45°) = \frac{1}{\sqrt{2}}$, $\sin(-45°) = -\frac{1}{\sqrt{2}}$, $\tan(-45°) = -1$

4 **a** $\cos 270° = 0$, $\sin 270° = -1$
b $\tan 270°$ is undefined

5 **a** $\frac{3}{4}$ **b** $\frac{1}{4}$ **c** 3 **d** $\frac{1}{4}$ **e** $-\frac{1}{4}$ **f** 1
g $\sqrt{2}$ **h** $\frac{1}{2}$ **i** $\frac{1}{2}$ **j** 2 **k** -1 **l** $-\sqrt{3}$

6 **a** $30°, 150°$ **b** $60°, 120°$ **c** $45°, 315°$
d $120°, 240°$ **e** $135°, 225°$ **f** $240°, 300°$

7 **a** $\frac{\pi}{4}, \frac{5\pi}{4}$ **b** $\frac{3\pi}{4}, \frac{7\pi}{4}$ **c** $\frac{\pi}{3}, \frac{4\pi}{3}$
d $0, \pi, 2\pi$ **e** $\frac{\pi}{6}, \frac{7\pi}{6}$ **f** $\frac{2\pi}{3}, \frac{5\pi}{3}$

8 **a** $\frac{\pi}{6}, \frac{11\pi}{6}, \frac{13\pi}{6}, \frac{23\pi}{6}$ **b** $\frac{7\pi}{6}, \frac{11\pi}{6}, \frac{19\pi}{6}, \frac{23\pi}{6}$ **c** $\frac{3\pi}{2}, \frac{7\pi}{2}$

9 **a** $\theta = \frac{\pi}{3}, \frac{5\pi}{3}$ **b** $\theta = \frac{\pi}{3}, \frac{2\pi}{3}$ **c** $\theta = \pi$
d $\theta = \frac{\pi}{2}$ **e** $\theta = \frac{3\pi}{4}, \frac{5\pi}{4}$ **f** $\theta = \frac{\pi}{2}, \frac{3\pi}{2}$
g $\theta = 0, \pi, 2\pi$ **h** $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$
i $\theta = \frac{5\pi}{6}, \frac{11\pi}{6}$ **j** $\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

10 **a** $\theta = k\pi,\ k \in \mathbb{Z}$ **b** $\theta = \frac{\pi}{2} + k\pi,\ k \in \mathbb{Z}$

EXERCISE 8F

1 **a** $\frac{2}{\sqrt{3}}$ **b** $-\frac{1}{\sqrt{3}}$ **c** $-\frac{2}{\sqrt{3}}$ **d** undefined
e $-\frac{2}{\sqrt{3}}$ **f** $\sqrt{2}$

2 **a** $\operatorname{cosec} x = \frac{5}{3}$, $\sec x = \frac{5}{4}$, $\cot x = \frac{4}{3}$
b $\operatorname{cosec} x = -\frac{3}{\sqrt{5}}$, $\sec x = \frac{3}{2}$, $\cot x = -\frac{2}{\sqrt{5}}$

3 **a** $\sin\theta = -\frac{\sqrt{7}}{4}$, $\tan\theta = -\frac{\sqrt{7}}{3}$, $\operatorname{cosec}\theta = -\frac{4}{\sqrt{7}}$,
$\sec\theta = \frac{4}{3}$, $\cot\theta = -\frac{3}{\sqrt{7}}$
b $\cos x = -\frac{\sqrt{5}}{3}$, $\tan x = \frac{2}{\sqrt{5}}$, $\operatorname{cosec} x = -\frac{3}{2}$,
$\sec x = -\frac{3}{\sqrt{5}}$, $\cot x = \frac{\sqrt{5}}{2}$
c $\sin x = \frac{\sqrt{21}}{5}$, $\cos x = \frac{2}{5}$, $\tan x = \frac{\sqrt{21}}{2}$,
$\operatorname{cosec} x = \frac{5}{\sqrt{21}}$, $\cot x = \frac{2}{\sqrt{21}}$
d $\sin\theta = \frac{1}{2}$, $\cos\theta = -\frac{\sqrt{3}}{2}$, $\tan\theta = -\frac{1}{\sqrt{3}}$,
$\sec\theta = -\frac{2}{\sqrt{3}}$, $\cot\theta = -\sqrt{3}$
e $\sin\beta = -\frac{1}{\sqrt{5}}$, $\cos\beta = -\frac{2}{\sqrt{5}}$, $\operatorname{cosec}\beta = -\sqrt{5}$,
$\sec\beta = -\frac{\sqrt{5}}{2}$, $\cot\beta = 2$
f $\sin\theta = -\frac{3}{5}$, $\cos\theta = -\frac{4}{5}$, $\tan\theta = \frac{3}{4}$,
$\operatorname{cosec}\theta = -\frac{5}{3}$, $\sec\theta = -\frac{5}{4}$

4 **a** $\theta = k\pi,\ k \in \mathbb{Z}$ **b** $\theta = \frac{\pi}{2} + k\pi,\ k \in \mathbb{Z}$
c $\theta = \frac{\pi}{2} + k\pi,\ k \in \mathbb{Z}$ **d** $\theta = k\pi,\ k \in \mathbb{Z}$

REVIEW SET 8A

1 **a** $\frac{2\pi}{3}$ **b** $\frac{5\pi}{4}$ **c** $\frac{5\pi}{6}$ **d** 3π

2 **a** $\frac{\pi}{3}$ **b** $15°$ **c** $84°$

3 **a** 0.358 **b** -0.035 **c** 0.259 **d** -0.731

4 111 cm^2 **5** $\theta = \frac{3\pi}{4}, \frac{7\pi}{4}$

6

	a	b	c	d
$\sin\theta$	0	$\frac{\sqrt{3}}{2}$	0	$\frac{\sqrt{3}}{2}$
$\cos\theta$	1	$-\frac{1}{2}$	-1	$-\frac{1}{2}$
$\tan\theta$	0	$-\sqrt{3}$	0	$-\sqrt{3}$

7 $\sin\theta = \pm\frac{\sqrt{7}}{4}$ **8** **a** $\frac{\sqrt{3}}{2}$ **b** 0 **c** $\frac{1}{2}$

9 **a** $\frac{2}{\sqrt{13}}$ **b** $-\frac{3}{\sqrt{13}}$

10 perimeter $= 12$ units, area $= 8\text{ units}^2$ **11** $\frac{\sqrt{6}}{\sqrt{11}}$

12 **a** $150°, 210°$ **b** $45°, 315°$ **c** $120°, 300°$

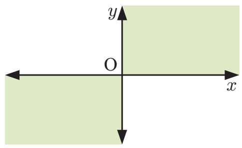
13 **a** $\theta = \pi$ **b** $\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

14 $\cos x = -\frac{\sqrt{15}}{4}$, $\tan x = \frac{1}{\sqrt{15}}$, $\sec x = -\frac{4}{\sqrt{15}}$,
$\operatorname{cosec} x = -4$, $\cot x = \sqrt{15}$

REVIEW SET 8B

1 **a** 72° **b** 225° **c** 140° **d** 330°

2



3 **a** $\cos(\frac{3\pi}{2}) = 0, \quad \sin(\frac{3\pi}{2}) = -1$

b $\cos(-\frac{\pi}{2}) = 0, \quad \sin(-\frac{\pi}{2}) = -1$

4 **a** $\sin(\pi - p) = m$ **b** $\sin(p + 2\pi) = m$

c $\cos p = \sqrt{1 - m^2}$ **d** $\tan p = \dfrac{m}{\sqrt{1 - m^2}}$

5 **a** **i** 60° **ii** $\frac{\pi}{3}$ **b** $\frac{\pi}{3}$ units **c** $\frac{\pi}{6}$ units^2

7 $\sin\theta = \frac{\sqrt{21}}{5}, \quad \tan\theta = -\frac{\sqrt{21}}{2}, \quad \sec\theta = -\frac{5}{2},$

$\operatorname{cosec}\theta = \frac{5}{\sqrt{21}}, \quad \cot\theta = -\frac{2}{\sqrt{21}}$

8 **a** $2\frac{1}{2}$ **b** $1\frac{1}{2}$ **c** $-\frac{1}{2}$

9 **a** $\theta \approx 0.841$ or 5.44 **b** $\theta \approx 3.39$ or 6.03

c $\theta \approx 1.25$ or 4.39

10 perimeter ≈ 34.1 cm, area ≈ 66.5 cm^2

11 $r \approx 8.79$ cm, area ≈ 81.0 cm^2 **12** **a** 0 **b** $\sin\theta$

13 $\sin\alpha = \frac{\sqrt{91}}{10}, \quad \cos\alpha = -\frac{3}{10}, \quad \tan\alpha = -\frac{\sqrt{91}}{3},$

$\operatorname{cosec}\alpha = \frac{10}{\sqrt{91}}, \quad \cot\alpha = -\frac{3}{\sqrt{91}}$

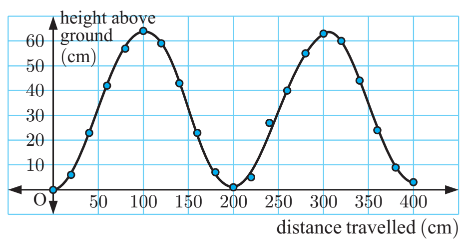
EXERCISE 9A

1 **a** periodic **b** periodic **c** periodic

d not periodic **e** periodic **f** periodic

g not periodic **h** not periodic

2 **a**



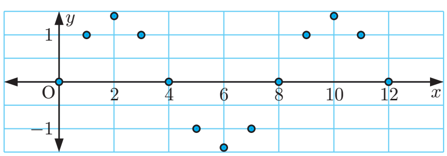
b A curve can be fitted to the data.

c The data is periodic.

i $y = 32$ (approx.) **ii** ≈ 64 cm

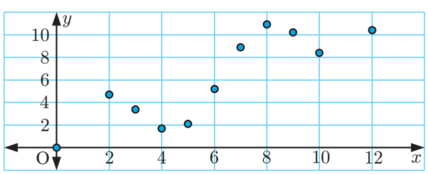
iii ≈ 200 cm **iv** ≈ 32 cm

3 **a**



Data exhibits periodic behaviour.

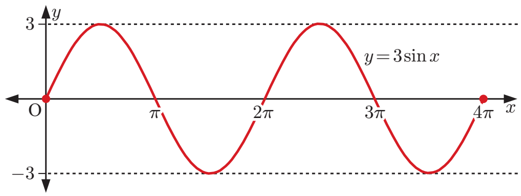
b



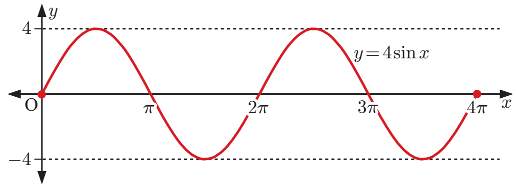
Not enough information to say data is periodic.

EXERCISE 9B

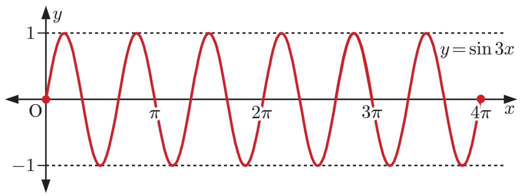
1 **a**



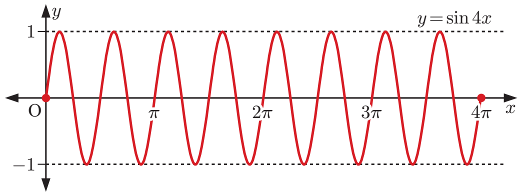
b



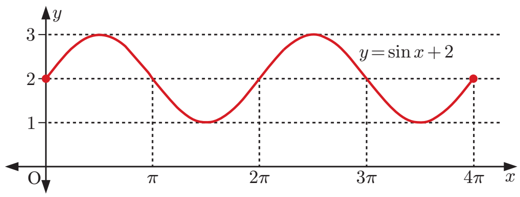
c



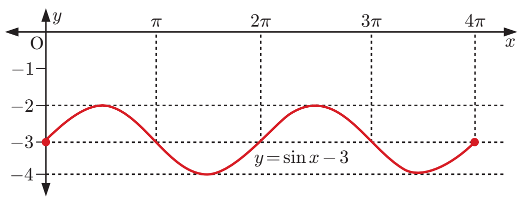
d



e



f

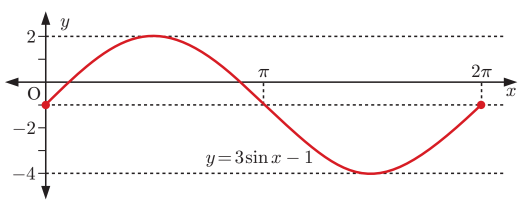


2 **a** $a = 2$ **b** $a = 5$ **c** $a = 11$

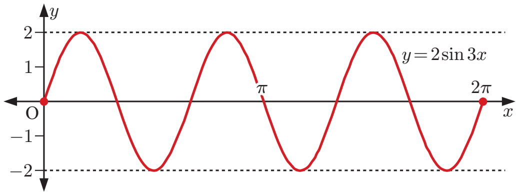
3 **a** $b = 3$ **b** $b = 5$ **c** $b = 6$ **d** $b = 4$

4 **a** $c = 3$ **b** $c = -1$ **c** $c = 5$

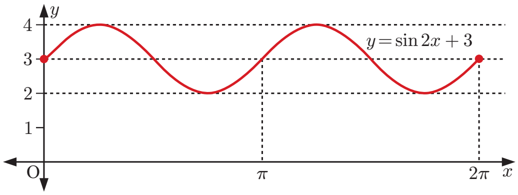
5 **a**



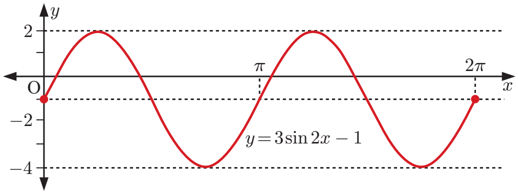
b



c



d



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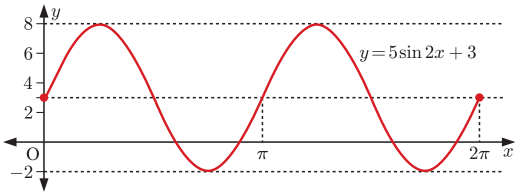
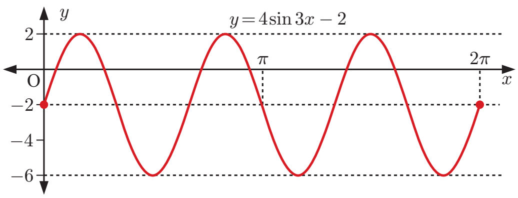


f

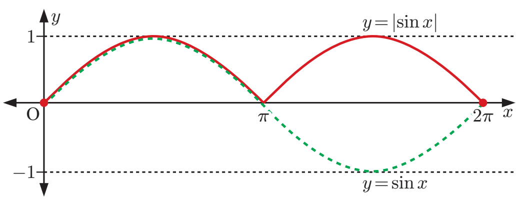


6 **a** $a = 3, \ b = 1, \ c = 0$ **b** $a = 2, \ b = 5, \ c = 6$

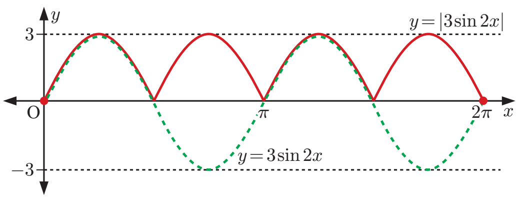
c $a = 5, \ b = 3, \ c = -2$

7 $m = 2, \ n = -3$

8 **a**

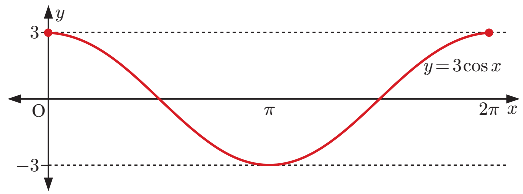


b

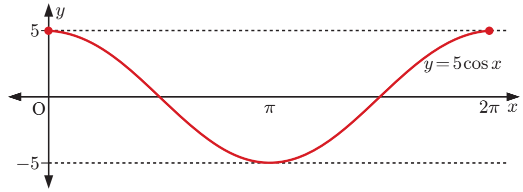


EXERCISE 9C

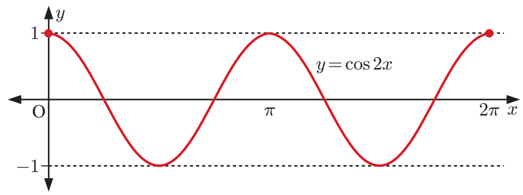
1 **a**



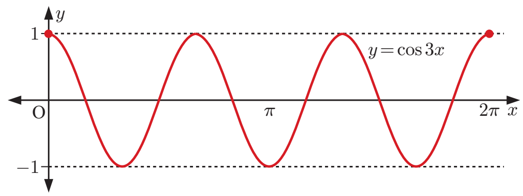
b



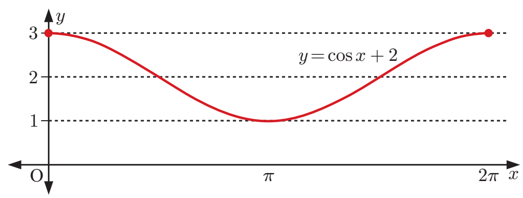
c



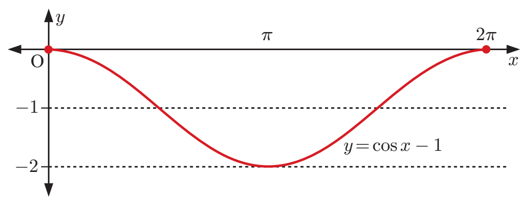
d



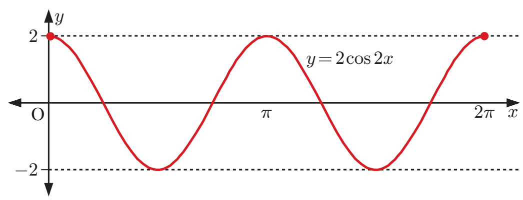
e



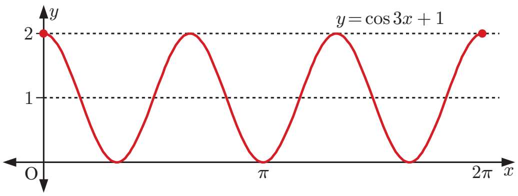
f



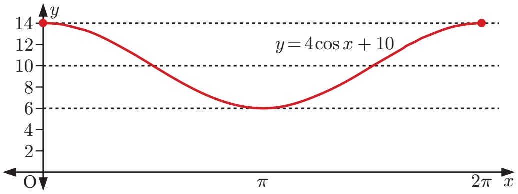
g



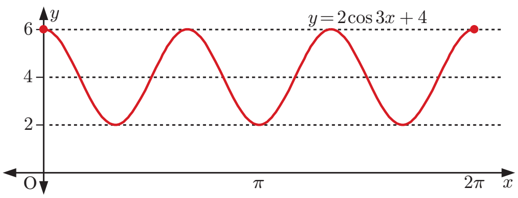
h



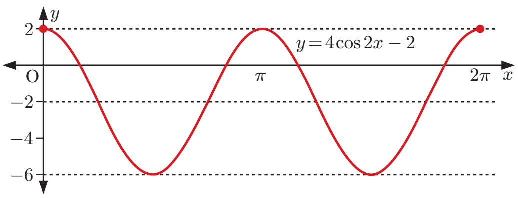
i



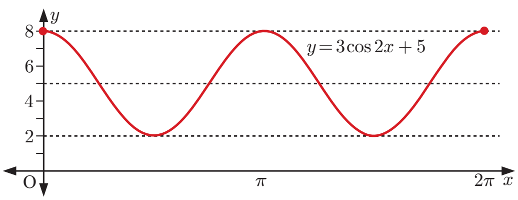
j



k



l

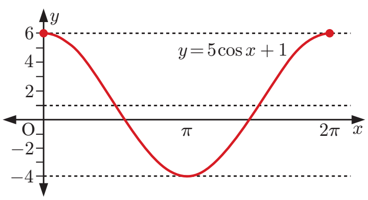


2 **a** $a = 4, \ b = 3, \ c = -1$ **b** $a = 3, \ b = 5, \ c = 3$

3 **a** $y = 2\cos 2x$ **b** $y = \cos\left(\frac{x}{2}\right) + 2$

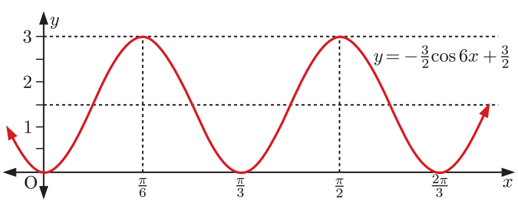
4 **a** $a = 5,$
$b = 1,$
$c = 1$

b



5 **a** $a = \frac{3}{2}, \ b = 6, \ c = -\frac{3}{2}$

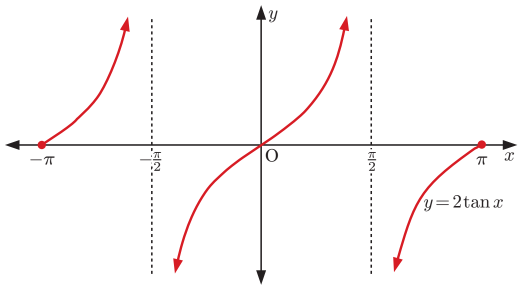
b



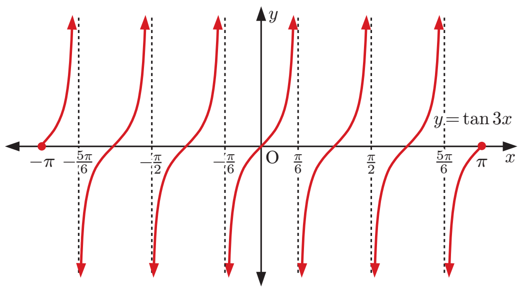
c $y = -\frac{3}{2}\cos 6x + \frac{3}{2}$

EXERCISE 9D

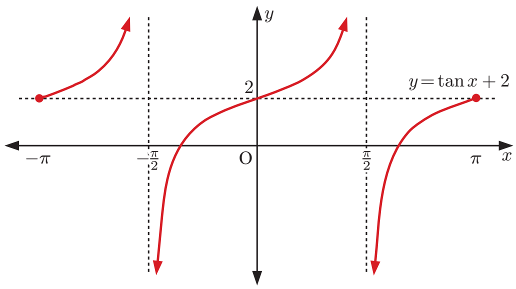
1 **a**



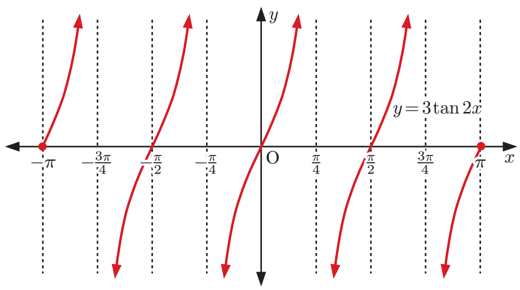
b



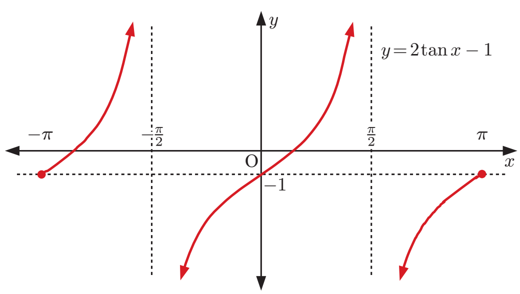
c



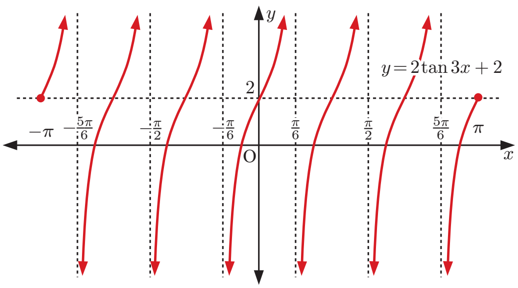
d



e



f



2 **a** $b = \frac{3}{2}$, $c = 2$ **b** $b = 2$, $c = -3$

3 $p = \frac{1}{2}$, $q = 1$

EXERCISE 9E.1

1 **a** $x \approx 0.3, 2.8, 6.6, 9.1, 12.9$ **b** $x \approx 5.9, 9.8, 12.2$

2 **a** $x \approx 0.9, 5.4, 7.2$ **b** $x \approx 4.4, 8.2, 10.7$

3 **a** $x \approx 0.4, 1.2, 3.5, 4.3, 6.7, 7.5, 9.8, 10.6, 13.0, 13.7$
b $x \approx 1.7, 3.0, 4.9, 6.1, 8.0, 9.3, 11.1, 12.4, 14.3, 15.6$

4 **a** **i** ≈ 1.6 **ii** ≈ -1.1
b **i** $x \approx 1.1, 4.2, 7.4$ **ii** $x \approx 2.2, 5.3$

5 **a** $x \approx 0.446, 2.70, 6.73, 8.98$
b $x \approx 2.52, 3.76, 8.80, 10.0$
c $x \approx 0.588, 3.73, 6.87, 10.0$

6 **a** $x \approx -0.644, 0.644$
b $x \approx -4.56, -1.42, 1.72, 4.87$
c $x \approx -2.76, -0.384, 3.53$

7 **a** $x \approx 1.57$ **b** $m = -2$ or $m = 1$
c $-1 \leqslant \sin x \leqslant 1$, so $m = \sin x = -2$ is not a valid solution.

EXERCISE 9E.2

1 **a** $x = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}$ **b** $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{9\pi}{4}, \frac{11\pi}{4}$
c $x = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}$

2 **a** $x = -\frac{5\pi}{3}, -\frac{4\pi}{3}, \frac{\pi}{3}, \frac{2\pi}{3}$ **b** $x = -\frac{5\pi}{4}, -\frac{3\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}$
c $x = -\frac{5\pi}{4}, -\frac{\pi}{4}, \frac{3\pi}{4}, \frac{7\pi}{4}$

3 **a** $x = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}$ **b** $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$

4 **a** $x = \frac{5\pi}{4}, \frac{7\pi}{4}$ **b** $x = \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{13\pi}{12}, \frac{5\pi}{4}, \frac{7\pi}{4}, \frac{23\pi}{12}$

5 **a** $x = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}, \frac{14\pi}{3}$
b $x = -330°, -210°, 30°, 150°$
c $x = \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{17\pi}{6}$ **d** $x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$
e $x = -\frac{8\pi}{9}, -\frac{4\pi}{9}, -\frac{2\pi}{9}, \frac{2\pi}{9}, \frac{4\pi}{9}, \frac{8\pi}{9}$

6 $x = \frac{\pi}{3}, \frac{4\pi}{3}$
a $x = \frac{\pi}{12}, \frac{\pi}{3}, \frac{7\pi}{12}, \frac{5\pi}{6}, \frac{13\pi}{12}, \frac{4\pi}{3}, \frac{19\pi}{12}, \frac{11\pi}{6}$
b $x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

7 $x = \frac{\pi}{18}, \frac{7\pi}{18}, \frac{13\pi}{18}$

8 **a** $x = -\frac{2\pi}{3}, \frac{2\pi}{3}$ **b** $x = -\frac{5\pi}{6}, -\frac{2\pi}{3}, \frac{\pi}{6}, \frac{\pi}{3}$
c $x = -\frac{\pi}{2}, \frac{\pi}{2}$

9 **a** $x = \frac{\pi}{4}, \frac{5\pi}{4}$ **b** $x = \frac{3\pi}{4}, \frac{7\pi}{4}$
c $x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{3\pi}{4}, \frac{13\pi}{12}, \frac{17\pi}{12}, \frac{7\pi}{4}$
d $x = \frac{\pi}{6}, \frac{2\pi}{3}, \frac{7\pi}{6}, \frac{5\pi}{3}$

10 $x = \frac{\pi}{2}$

EXERCISE 9F.1

1 **a** $2\sin\theta$ **b** $3\cos\theta$ **c** $2\sin\theta$ **d** $\sin\theta$
e $-2\tan\theta$ **f** $-3\cos^2\theta$

2 **a** 3 **b** -2 **c** -1 **d** $3\cos^2\theta$
e $4\sin^2\theta$ **f** $\cos\theta$ **g** $-\sin^2\theta$ **h** $-\cos^2\theta$
i $-2\sin^2\theta$ **j** 1 **k** $\sin\theta$ **l** $\sin\theta$

3 **a** $2\tan x$ **b** $\tan^2 x$ **c** $\sin x$ **d** $\cos x$
e $5\sin x$ **f** $2\sec x$ **g** 1 **h** 1
i $\operatorname{cosec} x$ **j** $\cos x$ **k** $\cos x$ **l** $5\sin x$

4 **a** $1 + 2\sin\theta + \sin^2\theta$ **b** $\sin^2\alpha - 4\sin\alpha + 4$
c $\tan^2\alpha - 2\tan\alpha + 1$ **d** $1 + 2\sin\alpha\cos\alpha$
e $1 - 2\sin\beta\cos\beta$ **f** $-4 + 4\cos\alpha - \cos^2\alpha$

5 **a** $-\tan^2\beta$ **b** 1 **c** $\sin^2\alpha$
d $\sin^2 x - \tan^2 x$ **e** 13 **f** $\cos^2\theta$ **g** 0

EXERCISE 9F.2

1 **a** $(1 - \sin\theta)(1 + \sin\theta)$
b $(\sin\alpha + \cos\alpha)(\sin\alpha - \cos\alpha)$
c $(\tan\alpha + 1)(\tan\alpha - 1)$ **d** $\sin\beta(2\sin\beta - 1)$
e $\cos\phi(2 + 3\cos\phi)$ **f** $3\sin\theta(\sin\theta - 2)$
g $(\tan\theta + 3)(\tan\theta + 2)$ **h** $(2\cos\theta + 1)(\cos\theta + 3)$
i $(3\cos\alpha + 1)(2\cos\alpha - 1)$ **j** $\tan\alpha(3\tan\alpha - 2)$
k $(\sec\beta + \operatorname{cosec}\beta)(\sec\beta - \operatorname{cosec}\beta)$
l $(2\cot x - 1)(\cot x - 1)$
m $(2\sin x + \cos x)(\sin x + 3\cos x)$

2 **a** $1 + \sin\alpha$ **b** $\tan\beta - 1$ **c** $\cos\phi - \sin\phi$
d $\cos\phi + \sin\phi$ **e** $\dfrac{1}{\sin\alpha - \cos\alpha}$ **f** $\dfrac{\cos\theta}{2}$
g $\sin\theta$ **h** $\cos\theta$ **i** $\sec\theta + 1$

EXERCISE 9G

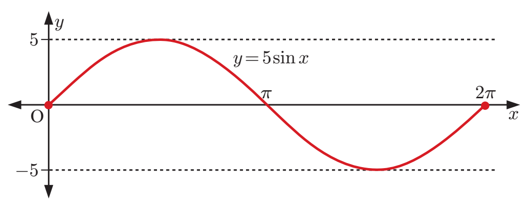
1 **a** $x = 0, \pi, \frac{7\pi}{6}, \frac{11\pi}{6}, 2\pi$ **b** $x = \frac{\pi}{3}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{3}$
c $x = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$ **d** $x = \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$
e no solutions **f** $x = 0, 2\pi$

2 **a** $x = \pi$ **b** $x = \frac{\pi}{6}, \frac{5\pi}{6}$

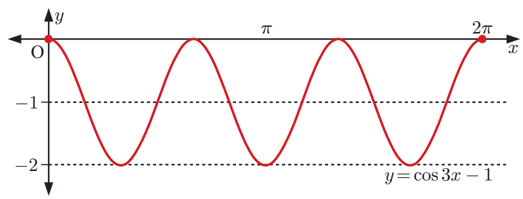
REVIEW SET 9A

1 **a** no **b** yes

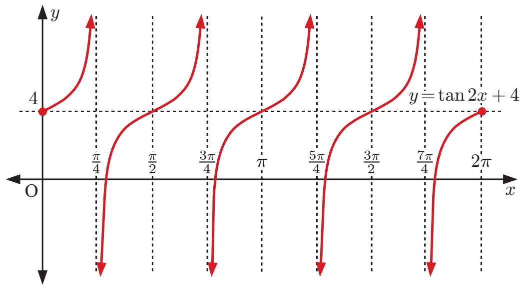
2 **a**



b



c



3 **a** minimum $= 0$, maximum $= 2$
b minimum $= -2$, maximum $= 2$
c minimum $= -3$, maximum $= 3$
d minimum $= -2$, maximum $= 0$

4 **a** 2π **b** $\frac{\pi}{2}$ **c** π **d** $\frac{\pi}{3}$

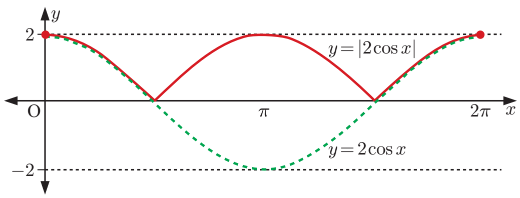
5

Function	*Period*	*Amplitude*
$y = 3\sin 2x + 1$	π	3
$y = \tan 2x$	$\frac{\pi}{2}$	undefined
$y = 2\cos 3x - 3$	$\frac{2\pi}{3}$	2

Function	*Domain*	*Range*
$y = 3\sin 2x + 1$	$x \in \mathbb{R}$	$-2 \leqslant y \leqslant 4$
$y = \tan 2x$	$x \neq \pm\frac{\pi}{4}, \pm\frac{3\pi}{4}, \ldots$	$y \in \mathbb{R}$
$y = 2\cos 3x - 3$	$x \in \mathbb{R}$	$-5 \leqslant y \leqslant -1$

6 $y = 4\cos 2x$

7



8 **a** $x \approx 115°, 245°, 475°, 605°$ **b** $x \approx 25°, 335°, 385°$

9 **a** $x = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{19\pi}{6}, \frac{23\pi}{6}$ **b** $x = -\frac{7\pi}{4}, -\frac{5\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}$
c $x = \frac{4\pi}{9}, \frac{5\pi}{9}, \frac{10\pi}{9}, \frac{11\pi}{9}, \frac{16\pi}{9}, \frac{17\pi}{9}$
d $x = \frac{\pi}{4}, \frac{7\pi}{4}, \frac{9\pi}{4}, \frac{15\pi}{4}$

10 **a** $1 - \cos\theta$ **b** $\dfrac{1}{\sin\alpha + \cos\alpha}$ **c** $-\dfrac{\cos\alpha}{2}$
d $\operatorname{cosec}\theta + 1$

12 **a** $x = -\frac{2\pi}{3}, -\frac{\pi}{6}, \frac{\pi}{3}, \frac{5\pi}{6}$ **b** $x = -\frac{2\pi}{3}, -\frac{\pi}{3}, \frac{\pi}{3}, \frac{2\pi}{3}$

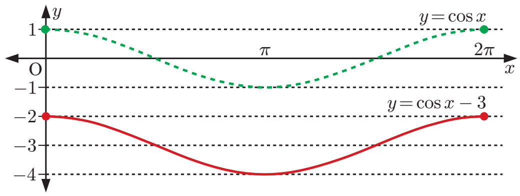
REVIEW SET 9B

1 **a** The function repeats itself over and over in a horizontal direction, in intervals of length 8 units.
b **i** 8 **ii** 5 **iii** -1

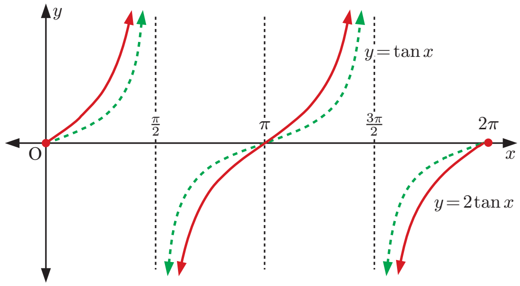
2 **a** $b = 6$ **b** $b = 24$

3 **a** minimum $= -8$, maximum $= 2$
b minimum $= -2$, maximum $= 4$
c minimum $= 5$, maximum $= 13$

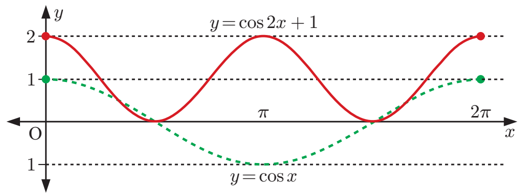
4 **a**



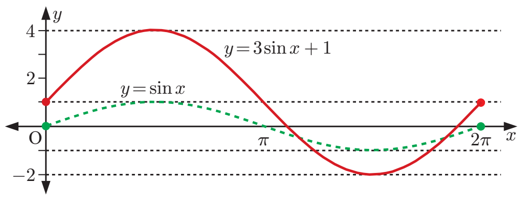
b



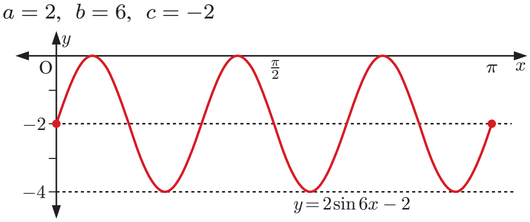
c



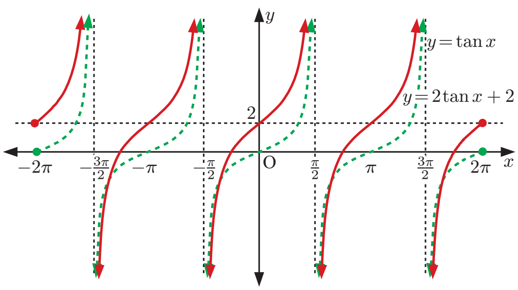
d



5 **a** $a = 2$, $b = 6$, $c = -2$
b



6 **a** $y = 2\tan x + 2$
b



7 **a** $x \approx -6.1, -3.4$ **b** $x \approx 0.8$

8 $m = 3$, $n = -1$

9 **a** $x = \frac{3\pi}{2}$ **b** $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

10 **a** $\cos\theta$ **b** $-\sin\theta$ **c** $5\cos^2\theta$ **d** $-\cos\theta$

11 **a** $4\sin^2\alpha - 4\sin\alpha + 1$ **b** $1 - 2\sin\alpha\cos\alpha$

EXERCISE 10A

1 18 **2** **a** 4 **b** 8 **c** 24 **3** 6

4 42 **5** 1680 **6** **a** 125 **b** 60

7 17 576 000 **8** **a** 4 **b** 9 **c** 81

EXERCISE 10B

1 **a** 13 **b** 20 **c** 19 **d** 32 **2** 13

EXERCISE 10C.1

1 1, 1, 2, 6, 24, 120, 720, 5040, 40 320, 362 880, 3 628 800

2 **a** 6 **b** 30 **c** $\frac{1}{7}$ **d** $\frac{1}{30}$ **e** 100 **f** 21

3 **a** $n,\ n \geqslant 1$ **b** $(n+2)(n+1),\ n \geqslant 0$
c $(n+1)n,\ n \geqslant 1$

4 **a** $\dfrac{7!}{4!}$ **b** $\dfrac{10!}{8!}$ **c** $\dfrac{11!}{6!}$ **d** $\dfrac{13!}{10!3!}$ **e** $\dfrac{3!}{6!}$ **f** $\dfrac{4!16!}{20!}$

5 **a** $6\times 4!$ **b** $10\times 10!$ **c** $73\times 7!$ **d** $131\times 10!$
e $81\times 7!$ **f** $62\times 6!$ **g** $10\times 11!$ **h** $32\times 8!$

6 **a** $11!$ **b** $9!$ **c** $8!$ **d** 9
e 34 **f** $n+1$ **g** $(n-1)!$ **h** $(n+1)!$

EXERCISE 10C.2

1 **a** 3 **b** 6 **c** 35 **d** 210

2 **a** **i** 28 **ii** 28 **3** $k = 3$ or 6

EXERCISE 10D

1 **a** W, X, Y, Z
b WX, WY, WZ, XW, XY, XZ, YW, YX, YZ, ZW, ZX, ZY
c WXY, WXZ, WYX, WYZ, WZX, WZY, XWY, XWZ, XYW, XYZ, XZW, XZY, YWX, YWZ, YXW, YXZ, YZX, YZW, ZWX, ZWY, ZXW, ZXY, ZYW, ZYX

2 **a** AB, AC, AD, AE, BA, BC, BD, BE, CA, CB, CD, CE, DA, DB, DC, DE, EA, EB, EC, ED
b ABC, ABD, ABE, ACB, ACD, ACE, ADB, ADC, ADE, AEB, AEC, AED, BAC, BAD, BAE, BCA, BCD, BCE, BDA, BDC, BDE, BEA, BEC, BED, CAB, CAD, CAE, CBA, CBD, CBE, CDA, CDB, CDE, CEA, CEB, CED, DAB, DAC, DAE, DBA, DBC, DBE, DCA, DCB, DCE, DEA, DEB, DEC, EAB, EAC, EAD, EBA, EBC, EBD, ECA, ECB, ECD, EDA, EDB, EDC

3 **a** 120 **b** 336 **c** 5040 **4** 110

5 **a** 12 **b** 24 **c** 36 **6** **a** 15 120 **b** 720

7 **a** 720 **b** **i** 24 **ii** 24 **iii** 48

8 **a** 343 **b** 210 **c** 120

9 **a** 648 **b** 64 **c** 72 **d** 136

10 **a** 6720 **b** 240 **c** 4200

11 **a** 120 **b** 48 **c** 72

12 **a** 3 628 800 **b** 241 920

13 **a** 720 **b** 144 **c** 72 **d** 144

14 **a** 48 **b** 24 **c** 15 **15** **a** 360 **b** 336 **c** 288

16 **a** 3 628 800 **b** **i** 151 200 **ii** 33 600

EXERCISE 10E

1 **a** permutation **b** combination
c permutation **d** combination

2 ABCD, ABCE, ABCF, ABDE, ABDF, ABEF, ACDE, ACDF, ACEF, ADEF, BCDE, BCDF, BCEF, BDEF, CDEF, $\binom{6}{4} = 15$

3 $\binom{17}{11} = 12\,376$ **4** **a** $\binom{9}{5} = 126$ **b** $\binom{1}{1}\binom{8}{4} = 70$

5 **a** $\binom{13}{3} = 286$ **b** $\binom{1}{1}\binom{12}{2} = 66$

6 **a** $\binom{12}{5} = 792$
b **i** $\binom{2}{2}\binom{10}{3} = 120$ **ii** $\binom{2}{1}\binom{10}{4} = 420$

7 $\binom{3}{3}\binom{1}{0}\binom{11}{6} = 462$

8 **a** $\binom{1}{1}\binom{9}{3} = 84$ **b** $\binom{2}{0}\binom{8}{4} = 70$
c $\binom{2}{0}\binom{1}{1}\binom{7}{3} = 35$

9 **a** $\binom{16}{5} = 4368$ **b** $\binom{10}{3}\binom{6}{2} = 1800$
c $\binom{10}{5}\binom{6}{0} = 252$
d $\binom{10}{3}\binom{6}{2} + \binom{10}{4}\binom{6}{1} + \binom{10}{5}\binom{6}{0} = 3312$
e $\binom{16}{5} - \binom{10}{5}\binom{6}{0} - \binom{10}{0}\binom{6}{5} = 4110$

10 **a** 6435 **b** 2520 **c** 36 **d** 4005 **11** 1050

12 **a** $\binom{6}{2}\binom{3}{1}\binom{7}{2} = 945$ **b** $\binom{6}{2}\binom{10}{3} = 1800$
c $\binom{16}{5} - \binom{9}{0}\binom{7}{5} = 4347$

13 $\binom{20}{2} - 20 = 170$

14 **a** **i** $\binom{12}{2} = 66$ **ii** $\binom{11}{1} = 11$
b **i** $\binom{12}{3} = 220$ **ii** $\binom{11}{2} = 55$

15 $\binom{9}{4} = 126$

16 **a** Selecting the different committees of 4 from 5 men and 6 women in all possible ways.
b $\binom{m+n}{r}$

17 **a** $\dfrac{\binom{12}{6}}{2} = 462$ **b** $\dfrac{\binom{12}{4}\binom{8}{4}\binom{4}{4}}{3!} = 5775$

18 **a** 45, yes **b** 37 128 **c** 3 628 800

EXERCISE 10F

1 **a** $p^3 + 3p^2q + 3pq^2 + q^3$ **b** $x^3 + 3x^2 + 3x + 1$
c $x^3 - 9x^2 + 27x - 27$ **d** $8 + 12x + 6x^2 + x^3$
e $27x^3 - 27x^2 + 9x - 1$ **f** $8x^3 + 60x^2 + 150x + 125$
g $8a^3 - 12a^2b + 6ab^2 - b^3$ **h** $27x^3 - 9x^2 + x - \frac{1}{27}$
i $8x^3 + 12x + \dfrac{6}{x} + \dfrac{1}{x^3}$

2 **a** $1 + 4x + 6x^2 + 4x^3 + x^4$
b $p^4 - 4p^3q + 6p^2q^2 - 4pq^3 + q^4$
c $x^4 - 8x^3 + 24x^2 - 32x + 16$
d $81 - 108x + 54x^2 - 12x^3 + x^4$
e $1 + 8x + 24x^2 + 32x^3 + 16x^4$
f $16x^4 - 96x^3 + 216x^2 - 216x + 81$
g $16x^4 + 32x^3b + 24x^2b^2 + 8xb^3 + b^4$
h $x^4 + 4x^2 + 6 + \dfrac{4}{x^2} + \dfrac{1}{x^4}$
i $16x^4 - 32x^2 + 24 - \dfrac{8}{x^2} + \dfrac{1}{x^4}$

3 **a** $x^5 + 10x^4 + 40x^3 + 80x^2 + 80x + 32$
b $x^5 - 10x^4y + 40x^3y^2 - 80x^2y^3 + 80xy^4 - 32y^5$
c $1 + 10x + 40x^2 + 80x^3 + 80x^4 + 32x^5$
d $x^5 - 5x^3 + 10x - \dfrac{10}{x} + \dfrac{5}{x^3} - \dfrac{1}{x^5}$

4 $64 + 160x^2 + 20x^4$

5 **a** 1 6 15 20 15 6 1

b **i** $x^6 + 12x^5 + 60x^4 + 160x^3 + 240x^2 + 192x + 64$

ii $64x^6 - 192x^5 + 240x^4 - 160x^3 + 60x^2 - 12x + 1$

iii $x^6 + 6x^4 + 15x^2 + 20 + \dfrac{15}{x^2} + \dfrac{6}{x^4} + \dfrac{1}{x^6}$

6 **a** $7 + 5\sqrt{2}$ **b** $161 + 72\sqrt{5}$ **c** $232 - 164\sqrt{2}$

7 $\dfrac{59 + 34\sqrt{3}}{13}$

8 **a** $64 + 192x + 240x^2 + 160x^3 + 60x^4 + 12x^5 + x^6$

b $65.944\,160\,601\,201$

9 **a** $a = 2$ and $b = e^x$ **b** $T_3 = 6e^{2x}$ and $T_4 = e^{3x}$

10 $2x^5 + 11x^4 + 24x^3 + 26x^2 + 14x + 3$

11 **a** 270 **b** 4320

EXERCISE 10G

1 **a** $1^{11} + \binom{11}{1}(2x)^1 + \binom{11}{2}(2x)^2 + \ldots. + \binom{11}{10}(2x)^{10} + (2x)^{11}$

b $(3x)^{15} + \binom{15}{1}(3x)^{14}\left(\frac{2}{x}\right)^1 + \binom{15}{2}(3x)^{13}\left(\frac{2}{x}\right)^2 + \ldots.$

$\ldots. + \binom{15}{14}(3x)^1\left(\frac{2}{x}\right)^{14} + \left(\frac{2}{x}\right)^{15}$

c $(2x)^{20} + \binom{20}{1}(2x)^{19}\left(-\frac{3}{x}\right)^1 + \binom{20}{2}(2x)^{18}\left(-\frac{3}{x}\right)^2 + \ldots.$

$\ldots. + \binom{20}{19}(2x)^1\left(-\frac{3}{x}\right)^{19} + \left(-\frac{3}{x}\right)^{20}$

2 **a** $T_6 = \binom{15}{5}(2x)^{10}5^5$ **b** $T_4 = \binom{9}{3}(x^2)^6y^3$

c $T_{10} = \binom{17}{9}x^8\left(-\frac{2}{x}\right)^9$ **d** $T_9 = \binom{21}{8}(2x^2)^{13}\left(-\frac{1}{x}\right)^8$

3 **a** $\binom{12}{4}2^83^4 = 10\,264\,320$ **b** $\binom{12}{7}2^53^7 = 55\,427\,328$

4 **a** $\binom{10}{3}1^7(-3)^3 = -3240$ **b** $\binom{10}{7}1^3(-3)^7 = -262\,440$

5 **a** 144 **b** 5376 **c** 2304

6 **a** $T_{r+1} = \binom{7}{r}x^{7-r}b^r$ **b** $b = -2$

7 **a** $\binom{15}{5}2^5 = 96\,096$ **b** $\binom{9}{3}(-3)^3 = -2268$

8 **a** $\binom{10}{5}3^52^5 = 1\,959\,552$ **b** $\binom{6}{3}2^3(-3)^3 = -4320$

c $\binom{6}{3}2^3(-3)^3 = -4320$ **d** $\binom{12}{4}2^8(-1)^4 = 126\,720$

9 $k = 5$ **10** $a = 3$ **11** **b** $a = 5,\ b = 2$

12 $\binom{8}{6} = 28$ **13** $2\binom{9}{3}3^6x^6 - \binom{9}{4}3^5x^6 = 91\,854x^6$

14 **a** $\binom{7}{4}3^3(-2)^4 = 15\,120$

b $\binom{7}{4}3^3(-2)^4 + 3\binom{7}{3}3^4(-2)^3 = -52\,920$

15 **a** $\binom{8}{3}2^5(-5)^3 - 3\binom{8}{1}2^7(-5)^1 = -208\,640$

b $\binom{6}{3}2^3 - \binom{6}{4}2^4 = -80$

16 $a = 3,\ b = -2,\ c = 57$ **17** $n = 8$

18 $n = 6$ **19** $84x^3$ **20** $k = -2,\ n = 6$

REVIEW SET 10A

1 **a** $n(n-1),\ n \geqslant 2$ **b** $n + 2$ **2** 28

3 **a** 24 **b** 6 **4** **a** 900 **b** 180

5 **a** $a = e^x$ and $b = -e^{-x}$

b $(e^x - e^{-x})^4 = e^{4x} - 4e^{2x} + 6 - 4e^{-2x} + e^{-4x}$

6 $362 + 209\sqrt{3}$ **7** It does not have one. **8** $c = 3$

9 **a** 720 **b** 72 **c** 504 **10** 2500

11 **a** 252 **b** 246 **12** $\binom{12}{6}2^6(-3)^6 = 43\,110\,144$

13 $8\binom{6}{2} - 6\binom{6}{1} = 84$ **14** $a = \pm 4$ **15** $k = 0$ or ± 2

REVIEW SET 10B

1 **a** $26^2 \times 10^4 = 6\,760\,000$

b $5 \times 26 \times 10^4 = 1\,300\,000$

c $26 \times 25 \times 10 \times 9 \times 8 \times 7 = 3\,276\,000$

2 **a** 3003 **b** 980 **c** 2982

3 **a** $x^3 - 6x^2y + 12xy^2 - 8y^3$

b $81x^4 + 216x^3 + 216x^2 + 96x + 16$

4 20 000 **5** 60 **6** $-103 + 74\sqrt{2}$ **7** 4200

8 $\binom{5}{2}2^3 - 3\binom{5}{1}2^4 = -160$

9 **a** 3024 **b** 840 **c** 42

10 $q = 0$ or $\pm\sqrt{\frac{3}{35}}$ **11** 4320 **12** $k = 180$

13 **a** 43 758 teams **b** 11 550 teams **c** 41 283 teams

d 3861 teams

14 $n = 7$ **15** $k = -\frac{1}{4},\ n = 16$

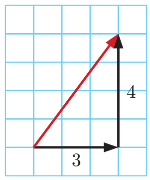
EXERCISE 11A

1 **a** $\begin{pmatrix} 7 \\ 3 \end{pmatrix}$, $7\mathbf{i} + 3\mathbf{j}$ **b** $\begin{pmatrix} -6 \\ 0 \end{pmatrix}$, $-6\mathbf{i}$

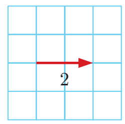
c $\begin{pmatrix} 2 \\ -5 \end{pmatrix}$, $2\mathbf{i} - 5\mathbf{j}$ **d** $\begin{pmatrix} 0 \\ 6 \end{pmatrix}$, $6\mathbf{j}$

e $\begin{pmatrix} -6 \\ 3 \end{pmatrix}$, $-6\mathbf{i} + 3\mathbf{j}$ **f** $\begin{pmatrix} -5 \\ -5 \end{pmatrix}$, $-5\mathbf{i} - 5\mathbf{j}$

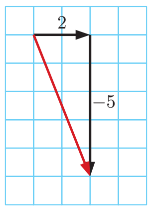
2 **a** $3\mathbf{i} + 4\mathbf{j}$



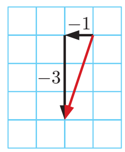
b $2\mathbf{i}$



c $2\mathbf{i} - 5\mathbf{j}$



d $-\mathbf{i} - 3\mathbf{j}$



3 **a** **i** $\begin{pmatrix} 4 \\ 1 \end{pmatrix}$, $4\mathbf{i} + \mathbf{j}$ **ii** $\begin{pmatrix} -4 \\ -1 \end{pmatrix}$, $-4\mathbf{i} - \mathbf{j}$

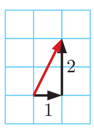
iii $\begin{pmatrix} -1 \\ -5 \end{pmatrix}$, $-\mathbf{i} - 5\mathbf{j}$ **iv** $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$, $2\mathbf{i}$

v $\begin{pmatrix} 3 \\ -4 \end{pmatrix}$, $3\mathbf{i} - 4\mathbf{j}$ **vi** $\begin{pmatrix} 4 \\ 1 \end{pmatrix}$, $4\mathbf{i} + \mathbf{j}$

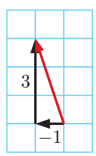
b $\overrightarrow{AB}$ and $\overrightarrow{DE}$. They have the same magnitude and direction.

c $\overrightarrow{BA}$ is the negative of both $\overrightarrow{AB}$ and $\overrightarrow{DE}$. They have the same magnitude but opposite direction.

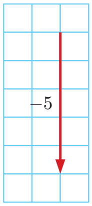
4 **a** $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$



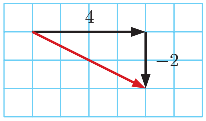
b $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$



c $\begin{pmatrix}0\\-5\end{pmatrix}$



d $\begin{pmatrix}4\\-2\end{pmatrix}$



5 **a** $\begin{pmatrix}-1\\-4\end{pmatrix}$ **b** $\begin{pmatrix}2\\-3\end{pmatrix}$ **c** $\begin{pmatrix}-5\\2\end{pmatrix}$ **d** $\begin{pmatrix}0\\6\end{pmatrix}$

EXERCISE 11B

1 **a** 5 units **b** 5 units **c** 2 units
d $\sqrt{8}$ units **e** 3 units

2 **a** $\sqrt{2}$ units **b** 13 units **c** $\sqrt{17}$ units
d 3 units **e** $|k|$ units

3 **a** unit vector **b** unit vector **c** not a unit vector
d unit vector **e** not a unit vector

4 **a** $k = \pm 1$ **b** $k = \pm 1$ **c** $k = 0$
d $k = \pm\frac{1}{\sqrt{2}}$ **e** $k = \pm\frac{\sqrt{3}}{2}$

5 $p = \pm 3$

EXERCISE 11C

1 **a** $\begin{pmatrix}-2\\6\end{pmatrix}$ **b** $\begin{pmatrix}-2\\6\end{pmatrix}$ **c** $\begin{pmatrix}-1\\-1\end{pmatrix}$ **d** $\begin{pmatrix}-1\\-1\end{pmatrix}$
e $\begin{pmatrix}-5\\-3\end{pmatrix}$ **f** $\begin{pmatrix}-5\\-3\end{pmatrix}$ **g** $\begin{pmatrix}-6\\4\end{pmatrix}$ **h** $\begin{pmatrix}-4\\1\end{pmatrix}$

2 **a** $\begin{pmatrix}-3\\7\end{pmatrix}$ **b** $\begin{pmatrix}-4\\-3\end{pmatrix}$ **c** $\begin{pmatrix}-8\\-1\end{pmatrix}$ **d** $\begin{pmatrix}-6\\9\end{pmatrix}$
e $\begin{pmatrix}0\\-5\end{pmatrix}$ **f** $\begin{pmatrix}6\\-9\end{pmatrix}$

3 **a** $\mathbf{a} + \mathbf{0} = \begin{pmatrix}a_1\\a_2\end{pmatrix} + \begin{pmatrix}0\\0\end{pmatrix} = \begin{pmatrix}a_1+0\\a_2+0\end{pmatrix} = \begin{pmatrix}a_1\\a_2\end{pmatrix} = \mathbf{a}$

b $\mathbf{a} - \mathbf{a} = \begin{pmatrix}a_1\\a_2\end{pmatrix} - \begin{pmatrix}a_1\\a_2\end{pmatrix} = \begin{pmatrix}a_1-a_1\\a_2-a_2\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix} = \mathbf{0}$

4 **a** $\begin{pmatrix}-3\\-15\end{pmatrix}$ **b** $\begin{pmatrix}-1\\2\end{pmatrix}$ **c** $\begin{pmatrix}0\\14\end{pmatrix}$ **d** $\begin{pmatrix}5\\-3\end{pmatrix}$
e $\begin{pmatrix}\frac{5}{2}\\\frac{11}{2}\end{pmatrix}$ **f** $\begin{pmatrix}-7\\7\end{pmatrix}$ **g** $\begin{pmatrix}5\\11\end{pmatrix}$ **h** $\begin{pmatrix}3\\\frac{17}{3}\end{pmatrix}$

5 **a** $\begin{pmatrix}8\\-1\end{pmatrix}$ **b** $\begin{pmatrix}8\\-1\end{pmatrix}$ **c** $\begin{pmatrix}8\\-1\end{pmatrix}$

In each case, the result is $2\mathbf{p} + 3\mathbf{q} = \begin{pmatrix}8\\-1\end{pmatrix}$.

6 **a** $\sqrt{13}$ units **b** $\sqrt{17}$ units **c** $5\sqrt{2}$ units
d $\sqrt{10}$ units **e** $\sqrt{29}$ units

7 **a** $\sqrt{10}$ units **b** $2\sqrt{10}$ units **c** $2\sqrt{10}$ units
d $3\sqrt{10}$ units **e** $3\sqrt{10}$ units **f** $2\sqrt{5}$ units
g $8\sqrt{5}$ units **h** $8\sqrt{5}$ units **i** $\sqrt{5}$ units
j $\sqrt{5}$ units

8 **a** $3\mathbf{i} + 2\mathbf{j}$ **b** $-\mathbf{i} + 9\mathbf{j}$ **c** $6\mathbf{i} - \mathbf{j}$ **d** $7\mathbf{j}$
e 2 units **f** $2\sqrt{10}$ units

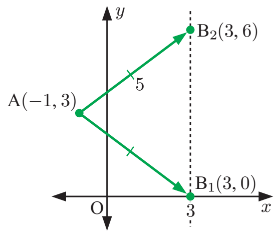
EXERCISE 11D

1 **a** $\begin{pmatrix}2\\4\end{pmatrix}$ **b** $\begin{pmatrix}-2\\5\end{pmatrix}$ **c** $\begin{pmatrix}3\\-3\end{pmatrix}$ **d** $\begin{pmatrix}1\\-5\end{pmatrix}$
e $\begin{pmatrix}6\\-5\end{pmatrix}$ **f** $\begin{pmatrix}1\\3\end{pmatrix}$

2 **a** B(4, 2) **b** C(2, 2) **3** **a** $\begin{pmatrix}2\\1\end{pmatrix}$ **b** Q(3, 3)

4 **a** $\begin{pmatrix}5\\1\end{pmatrix}$ **b** $\begin{pmatrix}-5\\-1\end{pmatrix}$ **c** D(−1, −2)

5 **a** $\overrightarrow{AB} = \begin{pmatrix}4\\k-3\end{pmatrix}$, $|\overrightarrow{AB}| = \sqrt{16 + (k-3)^2} = 5$ units
b $k = 0$ or 6 **c**



6 **a** $\overrightarrow{AB} = \begin{pmatrix}2\\3\end{pmatrix}$, $\overrightarrow{AC} = \begin{pmatrix}3\\-3\end{pmatrix}$
b $\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC} = -\overrightarrow{AB} + \overrightarrow{AC}$ **c** $\overrightarrow{BC} = \begin{pmatrix}1\\-6\end{pmatrix}$

7 **a** $\begin{pmatrix}-5\\4\end{pmatrix}$ **b** $\begin{pmatrix}1\\2\end{pmatrix}$ **c** $\begin{pmatrix}6\\-5\end{pmatrix}$

8 **a** M(1, 4) **b** $\overrightarrow{CA} = \begin{pmatrix}7\\5\end{pmatrix}$, $\overrightarrow{CM} = \begin{pmatrix}5\\3\end{pmatrix}$, $\overrightarrow{CB} = \begin{pmatrix}3\\1\end{pmatrix}$

EXERCISE 11E

1 $r = 3$ **2** $a = -6$

3 **a** $\overrightarrow{AB}$ is parallel and in the same direction as $\overrightarrow{CD}$, and 3 times its length.
b $\overrightarrow{RS}$ is parallel and in the opposite direction to $\overrightarrow{KL}$, and half its length.
c A, B, and C are collinear. $\overrightarrow{AB}$ is parallel and in the same direction as $\overrightarrow{BC}$, and twice its length.

4 **a** $\begin{pmatrix}4\\8\end{pmatrix}$ **b** $\begin{pmatrix}-1\\-2\end{pmatrix}$

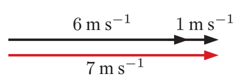
5 **a** $\frac{1}{\sqrt{5}}\mathbf{i} + \frac{2}{\sqrt{5}}\mathbf{j}$ **b** $\frac{1}{\sqrt{10}}\mathbf{i} - \frac{3}{\sqrt{10}}\mathbf{j}$ **c** $\frac{2}{\sqrt{5}}\mathbf{i} - \frac{1}{\sqrt{5}}\mathbf{j}$

6 **a** $\mathbf{v} = \frac{3}{\sqrt{5}}\begin{pmatrix}2\\-1\end{pmatrix}$ **b** $\mathbf{v} = \frac{2}{\sqrt{17}}\begin{pmatrix}1\\4\end{pmatrix}$

7 **a** $\overrightarrow{AB} = \begin{pmatrix}2\sqrt{2}\\-2\sqrt{2}\end{pmatrix}$ **b** $\overrightarrow{OB} = \begin{pmatrix}3+2\sqrt{2}\\2-2\sqrt{2}\end{pmatrix}$
c $B(3 + 2\sqrt{2},\ 2 - 2\sqrt{2})$

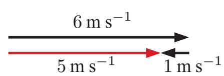
EXERCISE 11F

1 **a**



$\therefore$ 7 m s^{-1}

b



$\therefore$ 5 m s^{-1}

2 **a** 1.34 m s^{-1} in the direction 26.6° to the right of intended line
b **i** 30° to the left of Q **ii** 1.04 m s^{-1}

3 **a** $24.6\ \text{km h}^{-1}$ **b** $\approx 9.93°$ east of south

4 **a** 82.5 m **b** $23.3°$ to the left of directly across **c** 48.4 s

5 **a** The plane's speed in still air would be $\approx 437\ \text{km h}^{-1}$. The wind slows the plane down to $400\ \text{km h}^{-1}$.
b $4.64°$ north of due east

EXERCISE 11G

1 **a** **i** $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ -4 \end{pmatrix} + t\begin{pmatrix} 1 \\ 4 \end{pmatrix},\ t \in \mathbb{R}$
ii $x = 3 + t,\ y = -4 + 4t,\ t \in \mathbb{R}$ **iii** $4x - y = 16$

b **i** $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -6 \\ 0 \end{pmatrix} + t\begin{pmatrix} 3 \\ 7 \end{pmatrix},\ t \in \mathbb{R}$
ii $x = -6 + 3t,\ y = 7t,\ t \in \mathbb{R}$ **iii** $7x - 3y = -42$

c **i** $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 \\ 11 \end{pmatrix} + t\begin{pmatrix} -2 \\ 1 \end{pmatrix},\ t \in \mathbb{R}$
ii $x = -1 - 2t,\ y = 11 + t,\ t \in \mathbb{R}$ **iii** $x + 2y = 21$

2 **a** $x = -1 + 2t,\ y = 4 - t,\ t \in \mathbb{R}$

b

t	0	1	3	-1	-4
Point	$(-1, 4)$	$(1, 3)$	$(5, 1)$	$(-3, 5)$	$(-9, 8)$

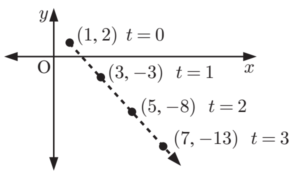
3 **a** When $t = 1,\ x = 3,\ y = -2,\ \therefore$ yes **b** $k = -5$

4 **a** $(0, 8)$ **b** It is a non-zero scalar multiple of $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$.

c $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 8 \end{pmatrix} + s\begin{pmatrix} 1 \\ -3 \end{pmatrix},\ s \in \mathbb{R}$

EXERCISE 11H

1 **a** $(1, 2)$ **b**



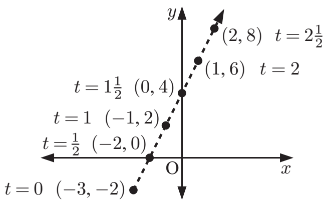
c $\begin{pmatrix} 2 \\ -5 \end{pmatrix}$

d $\sqrt{29}\ \text{cm s}^{-1}$

2 **a** $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} + t\begin{pmatrix} 4 \\ -5 \end{pmatrix},\ t \geqslant 0$ **b** $(8, -4.5)$

c 45 minutes

3 **a** $\begin{pmatrix} -3 + 2t \\ -2 + 4t \end{pmatrix}$ **d**



b $\begin{pmatrix} 2 \\ 8 \end{pmatrix}$

c **i** $t = 1.5$ s
ii $t = 0.5$ s

4 **a** **i** $(-4, 3)$ **ii** $\begin{pmatrix} 12 \\ 5 \end{pmatrix}$ **iii** $13\ \text{m s}^{-1}$

b **i** $(3, 0)$ **ii** $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$ **iii** $\sqrt{5}\ \text{m s}^{-1}$

5 **a** $\begin{pmatrix} 120 \\ -90 \end{pmatrix}$ **b** $\begin{pmatrix} 20\sqrt{5} \\ 10\sqrt{5} \end{pmatrix}$ **6** $\begin{pmatrix} 35 \\ -84 \end{pmatrix}$

7 **a** A is at $(4, 5)$, B is at $(1, -8)$

b For A it is $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$. For B it is $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$.

c For A, speed is $\sqrt{5}\ \text{km h}^{-1}$. For B, speed is $\sqrt{5}\ \text{km h}^{-1}$.

d Yacht A: $2x + y = 13$, Yacht B: $x - 2y = 17$

e Yacht A moves with gradient -2; Yacht B with gradient $\frac{1}{2}$. So, their paths are perpendicular.

f no

8 **a** $\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} -5 \\ 4 \end{pmatrix} + t\begin{pmatrix} 3 \\ -1 \end{pmatrix},\ t \geqslant 0$

$\therefore\ x_1(t) = -5 + 3t,\ y_1(t) = 4 - t,\ t \geqslant 0$

b speed $= \sqrt{10}\ \text{km min}^{-1}$

c a minutes later, $(t - a)$ min have elapsed.

$\therefore\ \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} 15 \\ 7 \end{pmatrix} + (t - a)\begin{pmatrix} -4 \\ -3 \end{pmatrix},\ t \geqslant 0$

$\therefore\ x_2(t) = 15 - 4(t - a),\ y_2(t) = 7 - 3(t - a),\ t \geqslant 0$

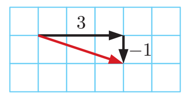
d Torpedo is fired at 1:35:28 pm and the explosion occurs at 1:37:42 pm.

REVIEW SET 11A

1 **a** $\mathbf{x} = \begin{pmatrix} 5 \\ 1 \end{pmatrix} = 5\mathbf{i} + \mathbf{j},\ \mathbf{y} = \begin{pmatrix} 1 \\ -2 \end{pmatrix} = \mathbf{i} - 2\mathbf{j}$

b **i** $6\mathbf{i} - \mathbf{j}$ **ii** $-9\mathbf{i} - 4\mathbf{j}$

2 **a** $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$ **b**



c $\begin{pmatrix} -3 \\ 1 \end{pmatrix}$

d $\sqrt{10}$ units

3 **a** $k = \pm\frac{1}{\sqrt{2}}$ **b** $\begin{pmatrix} -2\sqrt{5} \\ \sqrt{5} \end{pmatrix}$

4 **a** $\begin{pmatrix} 3 \\ -3 \end{pmatrix}$ **b** $\begin{pmatrix} 7 \\ -3 \end{pmatrix}$ **c** 5 units

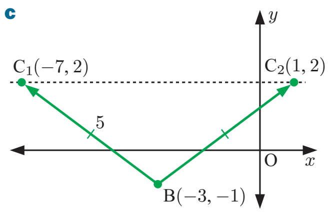
5 **a** $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$ **b** $\begin{pmatrix} -3 \\ -6 \end{pmatrix}$ **c** $\sqrt{34}$ units

6 **a** $\overrightarrow{BC} = \begin{pmatrix} k + 3 \\ 3 \end{pmatrix}$

$|\overrightarrow{BC}| = 5$

b $k = -7$ or 1

c



7 **a** $11.5°$ east of due north **b** $\approx 343\ \text{km h}^{-1}$

8 **a** $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -6 \\ 3 \end{pmatrix} + t\begin{pmatrix} 4 \\ -3 \end{pmatrix},\ t \in \mathbb{R}$

b $x = -6 + 4t,\ y = 3 - 3t,\ t \in \mathbb{R}$ **c** $3x + 4y = -6$

9 $m = 10$ **10** $\begin{pmatrix} 6\sqrt{10} \\ -2\sqrt{10} \end{pmatrix}$

11 **a** $(5, 2)$ **b** $\begin{pmatrix} 4 \\ 10 \end{pmatrix}$ is a non-zero scalar multiple of $\begin{pmatrix} 2 \\ 5 \end{pmatrix}$

c $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \end{pmatrix} + s\begin{pmatrix} 4 \\ 10 \end{pmatrix},\ s \in \mathbb{R}$

12 **a** $(-4, 3)$ **b** $(28, 27)$ **c** $\begin{pmatrix} 8 \\ 6 \end{pmatrix}$ **d** $10\ \text{m s}^{-1}$

REVIEW SET 11B

1 **a** **i** $\overrightarrow{AB} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} = 4\mathbf{i}$ **ii** $\overrightarrow{BC} = \begin{pmatrix} -2 \\ -4 \end{pmatrix} = -2\mathbf{i} - 4\mathbf{j}$

iii $\overrightarrow{CA} = \begin{pmatrix} -2 \\ 4 \end{pmatrix} = -2\mathbf{i} + 4\mathbf{j}$

b $\overrightarrow{BC}$ and $\overrightarrow{CA}$. Each vector has components of the same magnitude, but differing signs (which do not affect the length of $\overrightarrow{BC}$ or $\overrightarrow{CA}$).

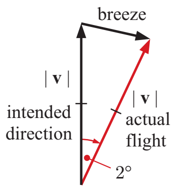
c $\overrightarrow{AC}$, $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$, $2\mathbf{i} - 4\mathbf{j}$

2 **a** $\sqrt{13}$ units **b** $\sqrt{10}$ units **c** $\sqrt{109}$ units

3 **a** $k = \pm\frac{12}{13}$ **b** $k = \pm\frac{1}{\sqrt{2}}$ **4** $\begin{pmatrix} 1 \\ 4 \end{pmatrix}$

5 **a** M$(-2, 4)$ **b** 5 units **6** $m = 5$

7 **a**



b **i** isosceles triangle $\therefore$ 2 remaining angles $= 89°$ each, breeze makes angle of $180 - 89 = 91°$ to intended direction of the arrow.

ii bisect angle $2°$ and use $\sin 1° = \dfrac{\frac{1}{2}\text{ speed}}{|\mathbf{v}|}$

$\therefore$ speed $= 2\,|\mathbf{v}|\sin 1°$

8 $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 8 \end{pmatrix} + t\begin{pmatrix} 5 \\ 4 \end{pmatrix}$, $t \in \mathbb{R}$

9 **a** **i** $-6\mathbf{i} + 10\mathbf{j}$ **ii** $-5\mathbf{i} - 15\mathbf{j}$

iii $(-6 - 5t)\mathbf{i} + (10 - 15t)\mathbf{j}$, $t \geqslant 0$

b $t = 40$ min ($\frac{2}{3}$ h), yacht is ≈ 9.33 km away from the beacon.

10 **a** **i** $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \end{pmatrix} + t\begin{pmatrix} 4 \\ -1 \end{pmatrix}$, $t \in \mathbb{R}$

ii $x = 2 + 4t$, $y = -3 - t$, $t \in \mathbb{R}$

b **i** $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 \\ 6 \end{pmatrix} + t\begin{pmatrix} 3 \\ -4 \end{pmatrix}$, $t \in \mathbb{R}$

ii $x = -1 + 3t$, $y = 6 - 4t$, $t \in \mathbb{R}$

11 **a** $x_1(t) = 2 + t$, $y_1(t) = 4 - 3t$, $t \geqslant 0$

b $x_2(t) = 13 - t$, $y_2(t) = [3 - 2a] + at$, $t \geqslant 2$

c interception occurred at 2:22:30 pm

d bearing $\approx 12.7°$ west of due south, at ≈ 4.54 km min^{-1}

EXERCISE 12A

1 **a** 1×4 **b** 2×1 **c** 2×2 **d** 3×3

2 **a** $(2 \quad 1 \quad 6 \quad 1)$ **b** $\begin{pmatrix} 1.95 \\ 2.35 \\ 0.45 \\ 2.95 \end{pmatrix}$

c total cost of groceries

3 $\begin{pmatrix} 1000 & 1500 & 1250 \\ 1500 & 1000 & 1000 \\ 800 & 2300 & 1300 \\ 1200 & 1200 & 1200 \end{pmatrix}$ **4** $\begin{pmatrix} 40 & 50 & 55 & 40 \\ 25 & 65 & 44 & 30 \\ 35 & 40 & 40 & 35 \\ 35 & 40 & 35 & 50 \end{pmatrix}$

EXERCISE 12B.1

1 **a** $\begin{pmatrix} 9 & 1 \\ 3 & 3 \end{pmatrix}$ **b** $\begin{pmatrix} 6 & 8 \\ -1 & 1 \end{pmatrix}$

c $\begin{pmatrix} 3 & 4 \\ -6 & -1 \end{pmatrix}$ **d** $\begin{pmatrix} 0 & 0 \\ -11 & -3 \end{pmatrix}$

2 **a** $\begin{pmatrix} 20 & 1 & -8 \\ 8 & 10 & -2 \\ 1 & -5 & 18 \end{pmatrix}$ **b** $\begin{pmatrix} -14 & 9 & -14 \\ 12 & -6 & 14 \\ -5 & 3 & -4 \end{pmatrix}$

c $\begin{pmatrix} 14 & -9 & 14 \\ -12 & 6 & -14 \\ 5 & -3 & 4 \end{pmatrix}$

3 **a** $\begin{array}{cc} \text{Friday} & \text{Saturday} \\ \begin{pmatrix} 85 \\ 92 \\ 52 \end{pmatrix} & \begin{pmatrix} 102 \\ 137 \\ 49 \end{pmatrix} \end{array}$ **b** $\begin{pmatrix} 187 \\ 229 \\ 101 \end{pmatrix}$

4 **a** **i** $\begin{pmatrix} 1.72 \\ 27.85 \\ 0.92 \\ 2.53 \\ 3.56 \end{pmatrix}$ **ii** $\begin{pmatrix} 1.79 \\ 28.75 \\ 1.33 \\ 2.25 \\ 3.51 \end{pmatrix}$

b subtract cost price from selling price **c** $\begin{pmatrix} 0.07 \\ 0.90 \\ 0.41 \\ -0.28 \\ -0.05 \end{pmatrix}$

5 **a** $\begin{array}{cc} \begin{matrix} \text{L} & \text{R} \end{matrix} & \\ \begin{pmatrix} 23 & 19 \\ 17 & 29 \\ 31 & 24 \end{pmatrix} & \begin{matrix} \text{fr} \\ \text{st} \\ \text{mi} \end{matrix} \end{array}$ **b** $\begin{array}{cc} \begin{matrix} \text{L} & \text{R} \end{matrix} & \\ \begin{pmatrix} 18 & 25 \\ 7 & 13 \\ 36 & 19 \end{pmatrix} & \begin{matrix} \text{fr} \\ \text{st} \\ \text{mi} \end{matrix} \end{array}$

c $\begin{array}{cc} \begin{matrix} \text{L} & \text{R} \end{matrix} & \\ \begin{pmatrix} 41 & 44 \\ 24 & 42 \\ 67 & 43 \end{pmatrix} & \begin{matrix} \text{fr} \\ \text{st} \\ \text{mi} \end{matrix} \end{array}$

6 **a** $x = -2$, $y = -2$ **b** $x = 0$, $y = 0$

7 **a** $\mathbf{A} + \mathbf{B} = \begin{pmatrix} 1 & 3 \\ 5 & 2 \end{pmatrix}$, $\mathbf{B} + \mathbf{A} = \begin{pmatrix} 1 & 3 \\ 5 & 2 \end{pmatrix}$

8 **a** $(\mathbf{A} + \mathbf{B}) + \mathbf{C} = \begin{pmatrix} 6 & 3 \\ -1 & 6 \end{pmatrix}$, $\mathbf{A} + (\mathbf{B} + \mathbf{C}) = \begin{pmatrix} 6 & 3 \\ -1 & 6 \end{pmatrix}$

EXERCISE 12B.2

1 **a** $\begin{pmatrix} 12 & 24 \\ 48 & 12 \end{pmatrix}$ **b** $\begin{pmatrix} 2 & 4 \\ 8 & 2 \end{pmatrix}$

c $\begin{pmatrix} \frac{1}{2} & 1 \\ 2 & \frac{1}{2} \end{pmatrix}$ **d** $\begin{pmatrix} -3 & -6 \\ -12 & -3 \end{pmatrix}$

2 **a** $\begin{pmatrix} 3 & 5 & 6 \\ 2 & 8 & 7 \end{pmatrix}$ **b** $\begin{pmatrix} 1 & 1 & 4 \\ 0 & 4 & 1 \end{pmatrix}$

c $\begin{pmatrix} 5 & 8 & 11 \\ 3 & 14 & 11 \end{pmatrix}$ **d** $\begin{pmatrix} 5 & 7 & 14 \\ 2 & 16 & 9 \end{pmatrix}$

3 $12\mathbf{F} = \begin{pmatrix} 12 \\ 48 \\ 24 \\ 12 \end{pmatrix}$

4 **a** $\mathbf{A} = \begin{pmatrix} 75 \\ 27 \\ 102 \end{pmatrix} \begin{matrix} \leftarrow \text{DVD} \\ \leftarrow \text{Blu-ray} \\ \leftarrow \text{games} \end{matrix}$ $\mathbf{B} = \begin{pmatrix} 136 \\ 43 \\ 129 \end{pmatrix} \begin{matrix} \leftarrow \text{DVD} \\ \leftarrow \text{Blu-ray} \\ \leftarrow \text{games} \end{matrix}$

b $5\mathbf{A} + 2\mathbf{B} = \begin{pmatrix} 647 \\ 221 \\ 768 \end{pmatrix} \begin{matrix} \leftarrow \text{DVD} \\ \leftarrow \text{Blu-ray} \\ \leftarrow \text{games} \end{matrix}$

c total weekly average hirings

5 **a** $\begin{array}{c} \begin{matrix} \text{A} & \text{B} & \text{C} & \text{D} \end{matrix} \\ \begin{pmatrix} 35 & 46 & 46 & 69 \\ 58 & 46 & 35 & 86 \\ 46 & 46 & 58 & 58 \\ 12 & 23 & 23 & 17 \end{pmatrix} \end{array}$ **b** $\begin{array}{c} \begin{matrix} \text{A} & \text{B} & \text{C} & \text{D} \end{matrix} \\ \begin{pmatrix} 26 & 34 & 34 & 51 \\ 43 & 34 & 26 & 64 \\ 34 & 34 & 43 & 43 \\ 9 & 17 & 17 & 13 \end{pmatrix} \end{array}$

EXERCISE 12B.3

1 **a** $3A$ **b** O **c** $-C$ **d** O **e** $2A + 2B$ **f** $-A - B$ **g** $-2A + C$ **h** $4A - B$ **i** $3B$

2 **a** $X = A - B$ **b** $X = C - B$ **c** $X = 2C - 4B$ **d** $X = \frac{1}{2}A$ **e** $X = \frac{1}{3}B$ **f** $X = A - B$ **g** $X = 2C$ **h** $X = \frac{1}{2}B - A$ **i** $X = \frac{1}{4}(A - C)$

3 **a** $X = \begin{pmatrix} 3 & 6 \\ 9 & 18 \end{pmatrix}$ **b** $X = \begin{pmatrix} \frac{1}{2} & -\frac{1}{4} \\ \frac{3}{4} & \frac{5}{4} \end{pmatrix}$

c $X = \begin{pmatrix} -1 & -6 \\ 1 & -\frac{1}{2} \end{pmatrix}$

EXERCISE 12C.1

1 **a** (11) **b** (22) **c** (16) **2** **b** $\begin{pmatrix} w & x & y & z \end{pmatrix}\begin{pmatrix} \frac{1}{4} \\ \frac{1}{4} \\ \frac{1}{4} \\ \frac{1}{4} \end{pmatrix}$

3 **a** $P = \begin{pmatrix} 27 & 35 & 39 \end{pmatrix}$, $Q = \begin{pmatrix} 4 \\ 3 \\ 2 \end{pmatrix}$

b total cost $= \begin{pmatrix} 27 & 35 & 39 \end{pmatrix}\begin{pmatrix} 4 \\ 3 \\ 2 \end{pmatrix} = \$291$

4 **a** $P = \begin{pmatrix} 10 & 6 & 3 & 1 \end{pmatrix}$, $N = \begin{pmatrix} 3 \\ 2 \\ 4 \\ 2 \end{pmatrix}$

b total points $= \begin{pmatrix} 10 & 6 & 3 & 1 \end{pmatrix}\begin{pmatrix} 3 \\ 2 \\ 4 \\ 2 \end{pmatrix} = 56$ points

EXERCISE 12C.2

1 Number of columns in **A** does not equal number of rows in **B**.

2 **a** $m = n$ **b** 2×3 **c** **B** has 3 columns, **A** has 2 rows

3 **a** does not exist **b** $\begin{pmatrix} 28 & 29 \end{pmatrix}$

4 **a** (8) **b** $\begin{pmatrix} 2 & 0 & 3 \\ 8 & 0 & 12 \\ 4 & 0 & 6 \end{pmatrix}$

5 **a** $\begin{pmatrix} 3 & 5 & 3 \end{pmatrix}$ **b** $\begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$

6 **a** $Q = \begin{pmatrix} 32 & 24 \\ 25 & 16 \\ 13 & 9 \end{pmatrix}$ **b** $P = \begin{pmatrix} 1.19 \\ 1.55 \end{pmatrix}$

c $QP = \begin{pmatrix} 32 & 24 \\ 25 & 16 \\ 13 & 9 \end{pmatrix}\begin{pmatrix} 1.19 \\ 1.55 \end{pmatrix} = \begin{pmatrix} 75.28 \\ 54.55 \\ 29.42 \end{pmatrix}$

It represents the total value of sales for each pen colour.

d $\$75.28 + \$54.55 + \$29.42 = \$159.25$

7 **a** $C = \begin{pmatrix} 12.5 \\ 9.5 \end{pmatrix}$ $N = \begin{pmatrix} 2375 & 5156 \\ 2502 & 3612 \end{pmatrix}$

b $\begin{pmatrix} 78\,669.5 \\ 65\,589 \end{pmatrix}$ income from day 1, income from day 2 **c** $\$144\,258.50$

8 **a** $R = \begin{pmatrix} 1 & 1 \\ 1 & 2 \\ 2 & 3 \end{pmatrix}$ **b** $P = \begin{pmatrix} 7 & 3 & 19 \\ 6 & 2 & 22 \end{pmatrix}$

c $\begin{pmatrix} 48 & 70 \\ 52 & 76 \end{pmatrix}$ **d** **i** $48 **ii** $76

e The elements of **PR** tell us that, if all the items are to be bought at one store, it is cheapest to do so at store A for both you and your friend. However, the cheapest way is to buy paint from store A, and hammers and screwdrivers from store B.

EXERCISE 12C.3

1 **a** $A^2 + A$ **b** $B^2 + 2B$ **c** $A^3 - 2A^2 + A$ **d** $A^3 + A^2 - 2A$ **e** $AC + AD + BC + BD$ **f** $A^2 + AB + BA + B^2$ **g** $A^2 - AB + BA - B^2$ **h** $A^2 + 2A + I$ **i** $9I - 6B + B^2$

2 **a** $A^3 = 3A - 2I$, $A^4 = 4A - 3I$
b $B^3 = 3B - 2I$, $B^4 = 6I - 5B$, $B^5 = 11B - 10I$
c $C^3 = 13C - 12I$, $C^5 = 121C - 120I$

3 **a** **i** $I + 2A$ **ii** $2I - 2A$ **iii** $10A + 6I$
b $A^2 + A + 2I$ **c** **i** $-3A$ **ii** $-2A$ **iii** A

4 **a** $A^2 = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

b false as $A(A - I) = O$ does not imply that $A = O$ or $A - I = O$

c $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$, $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $\begin{pmatrix} a & b \\ \dfrac{a - a^2}{b} & 1 - a \end{pmatrix}$, $b \neq 0$

5 For example, $A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ gives $A^2 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$.

6 **a** $a = 3,\ b = -4$ **b** $a = 1,\ b = 8$

7 **a** $p = -2,\ q = 1$ **b** $A^3 = 5A - 2I$ **c** $A^4 = -12A + 5I$

EXERCISE 12D.1

1 **a** $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix} = 3I$, $\begin{pmatrix} 1 & -2 \\ -\frac{2}{3} & \frac{5}{3} \end{pmatrix}$

b $\begin{pmatrix} 10 & 0 \\ 0 & 10 \end{pmatrix} = 10I$, $\begin{pmatrix} 0.2 & 0.4 \\ -0.1 & 0.3 \end{pmatrix}$

2 **a** -2 **b** -1 **c** 0 **d** 1

3 **a** 26 **b** 6 **c** -1 **d** $a^2 + a$

4 **a** -3 **b** -3 **c** -12 **5** **Hint:** Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

6 **a** **i** $\det A = ad - bc$ **ii** $\det B = wz - xy$
iii $AB = \begin{pmatrix} aw + by & ax + bz \\ cw + dy & cx + dz \end{pmatrix}$
iv $\det AB = (ad - bc)(wz - xy)$

7 **a** $\det A = -2$, $\det B = -1$
b **i** $\det(2A) = -8$ **ii** $\det(-A) = -2$ **iii** $\det(-3B) = -9$ **iv** $\det(AB) = 2$

8 **a** $\frac{1}{14}\begin{pmatrix} 5 & -4 \\ 1 & 2 \end{pmatrix}$ **b** $\begin{pmatrix} 1 & 0 \\ 1 & -1 \end{pmatrix}$ **c** does not exist
d $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ **e** $\frac{1}{10}\begin{pmatrix} 2 & 0 \\ 1 & 5 \end{pmatrix}$ **f** does not exist
g $-\frac{1}{15}\begin{pmatrix} 7 & -2 \\ -4 & -1 \end{pmatrix}$ **h** $\frac{1}{10}\begin{pmatrix} 2 & -4 \\ 1 & 3 \end{pmatrix}$ **i** $\begin{pmatrix} -3 & -1 \\ 2 & 1 \end{pmatrix}$

9 **a** $\dfrac{1}{2k + 6}\begin{pmatrix} 2 & -1 \\ 6 & k \end{pmatrix}$, $k \neq -3$ **b** $\dfrac{1}{3k}\begin{pmatrix} k & 1 \\ 0 & 3 \end{pmatrix}$, $k \neq 0$

c $\frac{1}{(k+2)(k-1)}\begin{pmatrix} k & -2 \\ -1 & k+1 \end{pmatrix}$, $k \neq -2$ or 1

d $\frac{1}{k(k+1)}\begin{pmatrix} k & -k \\ 3 & k-2 \end{pmatrix}$, $k \neq 0$ or -1

e $\frac{1}{k(2-k)}\begin{pmatrix} 1 & 1-k \\ -2k & k^2 \end{pmatrix}$, $k \neq 0$ or 2

f $\frac{1}{(k+4)(k-1)}\begin{pmatrix} 3k & -2 \\ -k^2-2 & k+1 \end{pmatrix}$, $k \neq -4$ or 1

EXERCISE 12D.2

1 $\mathbf{X} = \begin{pmatrix} \frac{1}{4} & \frac{3}{4} \\ 1 & 0 \end{pmatrix}$ **2** **a** $\mathbf{X} = \mathbf{ABZ}$ **b** $\mathbf{Z} = \mathbf{B}^{-1}\mathbf{A}^{-1}\mathbf{X}$

3 $\mathbf{A}^2 = 2\mathbf{A} - \mathbf{I}$, $\mathbf{A}^{-1} = 2\mathbf{I} - \mathbf{A}$

4 **a** $\mathbf{A}^{-1} = 4\mathbf{I} - \mathbf{A}$ **b** $\mathbf{A}^{-1} = 5\mathbf{I} + \mathbf{A}$
c $\mathbf{A}^{-1} = \frac{3}{2}\mathbf{A} - 2\mathbf{I}$

6 If $\mathbf{A}^{-1}$ exists, that is, $\det \mathbf{A} \neq 0$.

EXERCISE 12E

1 **a** $\begin{pmatrix} 3 & -1 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 8 \\ 6 \end{pmatrix}$
b $\begin{pmatrix} 4 & -3 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 11 \\ -5 \end{pmatrix}$
c $\begin{pmatrix} 3 & -1 \\ 2 & 7 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 6 \\ -4 \end{pmatrix}$

2 **a** $x = \frac{32}{7}$, $y = \frac{22}{7}$ **b** $x = -\frac{37}{23}$, $y = -\frac{75}{23}$
c $x = \frac{17}{13}$, $y = -\frac{37}{13}$ **d** $x = \frac{59}{13}$, $y = -\frac{25}{13}$
e $x = -40$, $y = -24$ **f** $x = \frac{1}{34}$, $y = \frac{55}{34}$

3 **b** **i** $\mathbf{X} = \begin{pmatrix} -\frac{4}{3} & \frac{13}{9} \\ -1 & \frac{4}{3} \end{pmatrix}$ **ii** $\mathbf{X} = \begin{pmatrix} -1 & 3 \\ 2 & 4 \end{pmatrix}$
iii $\mathbf{X} = \begin{pmatrix} \frac{13}{7} & \frac{3}{7} \\ -\frac{2}{7} & -\frac{8}{7} \end{pmatrix}$ **iv** $\mathbf{X} = \begin{pmatrix} \frac{19}{7} & \frac{6}{7} \\ \frac{20}{7} & -\frac{25}{7} \end{pmatrix}$

4 **a** **i** $\begin{pmatrix} 2 & -3 \\ 4 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 8 \\ 11 \end{pmatrix}$, $\det \mathbf{A} = 10$
ii Yes, $x = 2.5$, $y = -1$
b **i** $\begin{pmatrix} 2 & k \\ 4 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 8 \\ 11 \end{pmatrix}$, $\det \mathbf{A} = -2 - 4k$
ii $k \neq -\frac{1}{2}$, $x = \frac{8+11k}{2+4k}$, $y = \frac{5}{1+2k}$
iii $k = -\frac{1}{2}$, no solutions

REVIEW SET 12A

1 **a** $\begin{pmatrix} 4 & 2 \\ -2 & 3 \end{pmatrix}$ **b** $\begin{pmatrix} 9 & 6 \\ 0 & -3 \end{pmatrix}$ **c** $\begin{pmatrix} -2 & 0 \\ 4 & -8 \end{pmatrix}$
d $\begin{pmatrix} 2 & 2 \\ 2 & -5 \end{pmatrix}$ **e** $\begin{pmatrix} -5 & -4 \\ -2 & 6 \end{pmatrix}$ **f** $\begin{pmatrix} 7 & 6 \\ 4 & -11 \end{pmatrix}$
g $\begin{pmatrix} -1 & 8 \\ 2 & -4 \end{pmatrix}$ **h** $\begin{pmatrix} 3 & 2 \\ -6 & -8 \end{pmatrix}$ **i** $\begin{pmatrix} \frac{1}{3} & \frac{2}{3} \\ 0 & -1 \end{pmatrix}$
j $\begin{pmatrix} 9 & 4 \\ 0 & 1 \end{pmatrix}$ **k** $\begin{pmatrix} -3 & -10 \\ 6 & 8 \end{pmatrix}$ **l** $\begin{pmatrix} \frac{1}{3} & \frac{2}{3} \\ \frac{1}{6} & \frac{1}{12} \end{pmatrix}$

2 **a** $a = 0$, $b = 5$, $c = 1$, $d = -4$
b $a = 2$, $b = -1$, $c = 3$, $d = 8$

3 **a** $\mathbf{Y} = \mathbf{B} - \mathbf{A}$ **b** $\mathbf{Y} = \frac{1}{2}(\mathbf{A} - \mathbf{C})$ **c** $\mathbf{Y} = \mathbf{A}^{-1}\mathbf{B}$
d $\mathbf{Y} = \mathbf{CB}^{-1}$ **e** $\mathbf{Y} = \mathbf{A}^{-1}(\mathbf{C} - \mathbf{B})$ **f** $\mathbf{Y} = \mathbf{B}^{-1}\mathbf{A}$

4 **a** $4\mathbf{L}$ **b** $-2\mathbf{L}$

5 **a** $\begin{pmatrix} 10 & -12 \\ -10 & 4 \end{pmatrix}$ **b** $\begin{pmatrix} 2 & 6 & -3 \\ -4 & -2 & 11 \end{pmatrix}$ **c** not possible

6 **a** $\mathbf{A} - \mathbf{A}^2$ **b** $\mathbf{AB} + \mathbf{A}^2 - \mathbf{B}^2 - \mathbf{BA}$
c $4\mathbf{A}^2 - 4\mathbf{A} + \mathbf{I}$

7 $\mathbf{A}^3 = 27\mathbf{A} + 10\mathbf{I}$, $\mathbf{A}^4 = 145\mathbf{A} + 54\mathbf{I}$

8 $a = 4$, $b = -7$

9 **a** $\begin{pmatrix} \frac{7}{2} & -4 \\ -\frac{5}{2} & 3 \end{pmatrix}$ **b** does not exist **c** $\begin{pmatrix} 1 & \frac{5}{3} \\ -2 & -\frac{11}{3} \end{pmatrix}$

10 Unique solution if $k \neq \frac{3}{4}$.

11 **a** $x = 0$, $y = -\frac{1}{2}$ **b** $x = \frac{12}{7}$, $y = \frac{13}{7}$

12 **b** $(\mathbf{A} - \mathbf{I})(\mathbf{A} + 3\mathbf{I}) = 2\mathbf{A} - \mathbf{I}$

REVIEW SET 12B

1 **a** $\begin{pmatrix} 4 & 2 \\ 2 & 4 \\ 3 & 4 \end{pmatrix}$ **b** $\begin{pmatrix} 2 & -2 \\ 0 & 4 \\ -1 & -2 \end{pmatrix}$ **c** $\begin{pmatrix} -\frac{3}{2} & 3 \\ \frac{1}{2} & -4 \\ 2 & \frac{7}{2} \end{pmatrix}$

2 **a** $\mathbf{A} - \mathbf{B} = \begin{pmatrix} 2 & 2 \\ 4 & 1 \\ 3 & 1 \end{pmatrix}$ **b** **i** Book 2 (hard cover)
ii \$101

4 **a** $\mathbf{X} = \frac{1}{2}(\mathbf{B} - \mathbf{A})$ **b** $\mathbf{X} = \frac{1}{3}(2\mathbf{B} - 3\mathbf{A})$
c $\mathbf{X} = \frac{1}{4}(\mathbf{B} - \mathbf{A})$

5 $\mathbf{X} = \begin{pmatrix} -\frac{1}{2} & -\frac{3}{2} \\ 0 & \frac{3}{2} \end{pmatrix}$

6 **a** $\begin{pmatrix} 4 & 8 \\ 0 & 2 \\ 6 & 4 \end{pmatrix}$ **b** $\begin{pmatrix} 1 & 2 \\ 0 & \frac{1}{2} \\ \frac{3}{2} & 1 \end{pmatrix}$ **c** $\begin{pmatrix} 11 & 12 \end{pmatrix}$
d $\mathbf{BA}$ does not exist

7 **a** $\det \mathbf{B} \neq 0$ **b** $\mathbf{AB} = \mathbf{BA}$

8 **a** $\det \mathbf{A} = 5$ **b** $\det(-2\mathbf{A}) = 20$ **c** $\det(\mathbf{A}^2) = 25$

9 **a** $x = \frac{14}{3}$, $y = \frac{1}{3}$ **b** $x = -1$, $y = 3$

10 $k \in \mathbb{R}$, $k \neq 3, -2, 2$

11 Unique solution for $k \neq -3$ or 1. $x = \frac{-6}{k-1}$, $y = \frac{2}{k-1}$

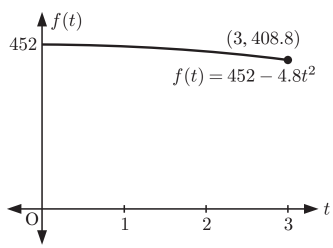
12 $\mathbf{A}(\frac{5}{3}\mathbf{A} - 2\mathbf{I}) = \mathbf{I}$, $\mathbf{A}^{-1} = \frac{5}{3}\mathbf{A} - 2\mathbf{I}$

EXERCISE 13A

1 **a** 7 **b** 7 **c** 11 **d** 16 **e** 0 **f** 5
2 **a** 5 **b** 7 **c** c
3 **a** -2 **b** 7 **c** -1 **d** 1
4 **a** -3 **b** 5 **c** -1 **d** 6 **e** -4 **f** -8
g 1 **h** 2 **i** 5

EXERCISE 13B

1 **a**



b no
c **i** 0 m s^{-1}
ii 9.6 m s^{-1}
iii 19.2 m s^{-1}
iv 28.8 m s^{-1}

2 **a**

x	Point B	Gradient of AB
0	(0, 0)	2
1	(1, 1)	3
1.5	(1.5, 2.25)	3.5
1.9	(1.9, 3.61)	3.9
1.99	(1.99, 3.9601)	3.99
1.999	(1.999, 3.996 001)	3.999

x	Point B	Gradient of AB
5	(5, 25)	7
3	(3, 9)	5
2.5	(2.5, 6.25)	4.5
2.1	(2.1, 4.41)	4.1
2.01	(2.01, 4.0401)	4.01
2.001	(2.001, 4.004 001)	4.001

b $\lim\limits_{x\to 2} \dfrac{x^2-4}{x-2} = 4$

The gradient of the tangent to $y = x^2$ at the point (2, 4) is 4.

EXERCISE 13C

1 **a** $f(2) = 3$ **b** $f'(2) = 0$

2 **a** $f(0) = 4$ **b** $f'(0) = -1$ **3** $f(2) = 3,\ f'(2) = 1$

EXERCISE 13D

1 **a** $f'(x) = 1$ **b** $f'(x) = 0$ **c** $f'(x) = 2$

2 **a** $\dfrac{dy}{dx} = -1$ **b** $\dfrac{dy}{dx} = 2x-3$ **c** $\dfrac{dy}{dx} = 4x+1$

3 **a** 3 **b** -12 **c** 9 **d** 10

EXERCISE 13E

1 **a** $f'(x) = 3x^2$ **b** $f'(x) = 6x^2$

c $f'(x) = 14x$ **d** $f'(x) = \dfrac{3}{\sqrt{x}}$

e $f'(x) = \dfrac{1}{\sqrt[3]{x^2}}$ **f** $f'(x) = 2x+1$

g $f'(x) = -4x$ **h** $f'(x) = 2x+3$

i $f'(x) = 2x^3 - 12x$ **j** $f'(x) = \dfrac{6}{x^2}$

k $f'(x) = -\dfrac{2}{x^2} + \dfrac{6}{x^3}$ **l** $f'(x) = 2x - \dfrac{5}{x^2}$

m $f'(x) = 2x + \dfrac{3}{x^2}$ **n** $f'(x) = -\dfrac{1}{2x\sqrt{x}}$

o $f'(x) = 8x-4$ **p** $f'(x) = 3x^2+12x+12$

2 **a** $\dfrac{dy}{dx} = 7.5x^2 - 2.8x$ **b** $\dfrac{dy}{dx} = 2\pi x$

c $\dfrac{dy}{dx} = -\dfrac{2}{5x^3}$ **d** $\dfrac{dy}{dx} = 100$

e $\dfrac{dy}{dx} = 10$ **f** $\dfrac{dy}{dx} = 12\pi x^2$

3 **a** 6 **b** $\dfrac{3\sqrt{x}}{2}$ **c** $2x-10$ **d** $2-9x^2$ **e** $2x-1$

f $-\dfrac{2}{x^3} + \dfrac{3}{\sqrt{x}}$ **g** $4 + \dfrac{1}{4x^2}$ **h** $6x^2-6x-5$

4 **a** 4 **b** $-\frac{16}{729}$ **c** -7 **d** $\frac{13}{4}$ **e** $\frac{1}{8}$ **f** -11

5 $b = 3,\ c = -4$

6 **a** $f'(x) = \dfrac{2}{\sqrt{x}} + 1$ **b** $f'(x) = \dfrac{1}{3\sqrt[3]{x^2}}$

c $f'(x) = \dfrac{1}{x\sqrt{x}}$ **d** $f'(x) = 2 - \dfrac{1}{2\sqrt{x}}$

e $f'(x) = -\dfrac{2}{x\sqrt{x}}$ **f** $f'(x) = 6x - \frac{3}{2}\sqrt{x}$

g $f'(x) = \dfrac{-25}{2x^3\sqrt{x}}$ **h** $f'(x) = 2 + \dfrac{9}{2x^2\sqrt{x}}$

7 **a** $\dfrac{dy}{dx} = 4 + \dfrac{3}{x^2}$, $\dfrac{dy}{dx}$ is the gradient function of $y = 4x - \dfrac{3}{x}$ from which the gradient at any point can be found.

b $\dfrac{dS}{dt} = 4t+4$ m s^{-1}, $\dfrac{dS}{dt}$ is the instantaneous rate of change in position at the time t, or the velocity function.

c $\dfrac{dC}{dx} = 3 + 0.004x$ \$ per toaster, $\dfrac{dC}{dx}$ is the instantaneous rate of change in cost as the number of toasters changes.

EXERCISE 13F.1

1 **a** $gf(x) = (2x+7)^2$ **b** $gf(x) = 2x^2+7$

c $gf(x) = \sqrt{3-4x}$ **d** $gf(x) = 3 - 4\sqrt{x}$

e $gf(x) = \dfrac{2}{x^2+3}$ **f** $gf(x) = \dfrac{4}{x^2} + 3$

2 **Note:** There may be other answers.

a $g(x) = x^3,\quad f(x) = 3x+10$

b $g(x) = \dfrac{1}{x},\quad f(x) = 2x+4$

c $g(x) = \sqrt{x},\quad f(x) = x^2-3x$

d $g(x) = \dfrac{10}{x^3},\quad f(x) = 3x - x^2$

EXERCISE 13F.2

1 **a** $u^{-2},\ u = 2x-1$ **b** $u^{\frac{1}{2}},\ u = x^2-3x$

c $2u^{-\frac{1}{2}},\ u = 2-x^2$ **d** $u^{\frac{1}{3}},\ u = x^3-x^2$

e $4u^{-3},\ u = 3-x$ **f** $10u^{-1},\ u = x^2-3$

2 **a** $\dfrac{dy}{dx} = 8(4x-5)$ **b** $\dfrac{dy}{dx} = 2(5-2x)^{-2}$

c $\dfrac{dy}{dx} = \frac{1}{2}(3x-x^2)^{-\frac{1}{2}} \times (3-2x)$

d $\dfrac{dy}{dx} = -12(1-3x)^3$ **e** $\dfrac{dy}{dx} = -18(5-x)^2$

f $\dfrac{dy}{dx} = \frac{1}{3}(2x^3-x^2)^{-\frac{2}{3}} \times (6x^2-2x)$

g $\dfrac{dy}{dx} = -60(5x-4)^{-3}$

h $\dfrac{dy}{dx} = -4(3x-x^2)^{-2} \times (3-2x)$

i $\dfrac{dy}{dx} = 6\left(x^2 - \dfrac{2}{x}\right)^2 \times \left(2x + \dfrac{2}{x^2}\right)$

3 **a** $-\frac{1}{\sqrt{3}}$ **b** -18 **c** -8 **d** -4 **e** $-\frac{3}{32}$ **f** 0

4 $a = 3,\ b = 1$ **5** $a = 2,\ b = 1$

6 **a** $\dfrac{dy}{dx} = 3x^2$, $\dfrac{dx}{dy} = \frac{1}{3}y^{-\frac{2}{3}}$ **Hint:** Substitute $y = x^3$

b $\dfrac{dy}{dx} \times \dfrac{dx}{dy} = \dfrac{dy}{dy}$ {chain rule} $= 1$

EXERCISE 13G

1 **a** $f'(x) = 2x - 1$ **b** $f'(x) = 4x + 2$

c $f'(x) = 2x(x+1)^{\frac{1}{2}} + \frac{1}{2}x^2(x+1)^{-\frac{1}{2}}$

2 **a** $\dfrac{dy}{dx} = 2x(2x-1) + 2x^2$

b $\dfrac{dy}{dx} = 4(2x+1)^3 + 24x(2x+1)^2$

c $\dfrac{dy}{dx} = 2x(3-x)^{\frac{1}{2}} - \frac{1}{2}x^2(3-x)^{-\frac{1}{2}}$

d $\dfrac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}}(x-3)^2 + 2\sqrt{x}(x-3)$

e $\dfrac{dy}{dx} = 10x(3x^2-1)^2 + 60x^3(3x^2-1)$

f $\dfrac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}}(x-x^2)^3 + 3\sqrt{x}(x-x^2)^2(1-2x)$

3 **a** -48 **b** $406\frac{1}{4}$ **c** $\frac{13}{3}$ **d** $\frac{11}{2}$

4 **b** $x = 3$ or $\frac{3}{5}$ **c** $x \leqslant 0$ **5** $x = -1$ and $x = -\frac{5}{3}$

EXERCISE 13H

1 **a** $\dfrac{dy}{dx} = \dfrac{7}{(2-x)^2}$ **b** $\dfrac{dy}{dx} = \dfrac{2x(2x+1) - 2x^2}{(2x+1)^2}$

c $\dfrac{dy}{dx} = \dfrac{(x^2-3) - 2x^2}{(x^2-3)^2}$

d $\dfrac{dy}{dx} = \dfrac{\frac{1}{2}x^{-\frac{1}{2}}(1-2x) + 2\sqrt{x}}{(1-2x)^2}$

e $\dfrac{dy}{dx} = \dfrac{2x(3x-x^2) - (x^2-3)(3-2x)}{(3x-x^2)^2}$

f $\dfrac{dy}{dx} = \dfrac{(1-3x)^{\frac{1}{2}} + \frac{3}{2}x(1-3x)^{-\frac{1}{2}}}{1-3x}$

2 **a** 1 **b** 1 **c** $-\frac{7}{324}$ **d** $-\frac{28}{27}$

3 **b** **i** never $\{\dfrac{dy}{dx}$ is undefined at $x = -1\}$

ii $x \leqslant 0$ and $x = 1$

4 **b** **i** $x = -2 \pm \sqrt{11}$ **ii** $x = -2$

EXERCISE 13I

1 **a** $f'(x) = 4e^{4x}$ **b** $f'(x) = e^x$

c $f'(x) = -2e^{-2x}$ **d** $f'(x) = \frac{1}{2}e^{\frac{x}{2}}$

e $f'(x) = -e^{-\frac{x}{2}}$ **f** $f'(x) = 2e^{-x}$

g $f'(x) = 2e^{\frac{x}{2}} + 3e^{-x}$ **h** $f'(x) = \dfrac{e^x - e^{-x}}{2}$

i $f'(x) = -2xe^{-x^2}$ **j** $f'(x) = e^{\frac{1}{x}} \times \dfrac{-1}{x^2}$

k $f'(x) = 20e^{2x}$ **l** $f'(x) = 40e^{-2x}$

m $f'(x) = 2e^{2x+1}$ **n** $f'(x) = \frac{1}{4}e^{\frac{x}{4}}$

o $f'(x) = -4xe^{1-2x^2}$ **p** $f'(x) = -0.02e^{-0.02x}$

2 **a** $e^x + xe^x$ **b** $3x^2e^{-x} - x^3e^{-x}$

c $\dfrac{xe^x - e^x}{x^2}$ **d** $\dfrac{1-x}{e^x}$

e $2xe^{3x} + 3x^2e^{3x}$ **f** $\dfrac{xe^x - \frac{1}{2}e^x}{x\sqrt{x}}$

g $\frac{1}{2}x^{-\frac{1}{2}}e^{-x} - x^{\frac{1}{2}}e^{-x}$ **h** $\dfrac{e^x + 2 + 2e^{-x}}{(e^{-x}+1)^2}$

3 **a** 108 **b** -1 **c** $\frac{9}{\sqrt{19}}$ **4** $k = -9$

5 **a** $\dfrac{dy}{dx} = 2^x \ln 2$ **6** P $= (0, 0)$ or $(2, \dfrac{4}{e^2})$

EXERCISE 13J

1 **a** $\dfrac{dy}{dx} = \dfrac{1}{x}$ **b** $\dfrac{dy}{dx} = \dfrac{2}{2x+1}$ **c** $\dfrac{dy}{dx} = \dfrac{1-2x}{x-x^2}$

d $\dfrac{dy}{dx} = -\dfrac{2}{x}$ **e** $\dfrac{dy}{dx} = 2x\ln x + x$

f $\dfrac{dy}{dx} = \dfrac{1-\ln x}{2x^2}$ **g** $\dfrac{dy}{dx} = e^x \ln x + \dfrac{e^x}{x}$

h $\dfrac{dy}{dx} = \dfrac{2\ln x}{x}$ **i** $\dfrac{dy}{dx} = \dfrac{1}{2x\sqrt{\ln x}}$

j $\dfrac{dy}{dx} = \dfrac{e^{-x}}{x} - e^{-x}\ln x$ **k** $\dfrac{dy}{dx} = \dfrac{\ln(2x)}{2\sqrt{x}} + \dfrac{1}{\sqrt{x}}$

l $\dfrac{dy}{dx} = \dfrac{\ln x - 2}{\sqrt{x}(\ln x)^2}$ **m** $\dfrac{dy}{dx} = \dfrac{4}{1-x}$

n $\dfrac{dy}{dx} = \ln(x^2+1) + \dfrac{2x^2}{x^2+1}$

2 **a** $\dfrac{dy}{dx} = \ln 5$ **b** $\dfrac{dy}{dx} = \dfrac{3}{x}$ **c** $\dfrac{dy}{dx} = \dfrac{4x^3+1}{x^4+x}$

d $\dfrac{dy}{dx} = \dfrac{1}{x-2}$ **e** $\dfrac{dy}{dx} = \dfrac{6}{2x+1}[\ln(2x+1)]^2$

f $\dfrac{dy}{dx} = \dfrac{1-\ln(4x)}{x^2}$ **g** $\dfrac{dy}{dx} = -\dfrac{1}{x}$

h $\dfrac{dy}{dx} = \dfrac{1}{x\ln x}$ **i** $\dfrac{dy}{dx} = \dfrac{-1}{x(\ln x)^2}$

3 **a** $\dfrac{dy}{dx} = \dfrac{-1}{1-2x}$ **b** $\dfrac{dy}{dx} = \dfrac{-2}{2x+3}$ **c** $\dfrac{dy}{dx} = 1 + \dfrac{1}{2x}$

d $\dfrac{dy}{dx} = \dfrac{1}{x} - \dfrac{1}{2(2-x)}$ **e** $\dfrac{dy}{dx} = \dfrac{1}{x+3} - \dfrac{1}{x-1}$

f $\dfrac{dy}{dx} = \dfrac{2}{x} + \dfrac{1}{3-x}$ **g** $f'(x) = \dfrac{9}{3x-4}$

h $f'(x) = \dfrac{1}{x} + \dfrac{2x}{x^2+1}$ **i** $f'(x) = \dfrac{2x+2}{x^2+2x} - \dfrac{1}{x-5}$

4 **a** 2 **b** $-\frac{5}{3}$ **5** $a = 3,\ b = -e$

EXERCISE 13K

1 **a** $\dfrac{dy}{dx} = 2\cos(2x)$ **b** $\dfrac{dy}{dx} = \cos x - \sin x$

c $\dfrac{dy}{dx} = -3\sin(3x) - \cos x$ **d** $\dfrac{dy}{dx} = \cos(x+1)$

e $\dfrac{dy}{dx} = 2\sin(3-2x)$ **f** $\dfrac{dy}{dx} = \dfrac{5}{\cos^2(5x)}$

g $\dfrac{dy}{dx} = \frac{1}{2}\cos\left(\frac{x}{2}\right) + 3\sin x$ **h** $\dfrac{dy}{dx} = \dfrac{3\pi}{\cos^2(\pi x)}$

i $\dfrac{dy}{dx} = 4\cos x + 2\sin(2x)$

2 **a** $2x - \sin x$ **b** $\dfrac{1}{\cos^2 x} - 3\cos x$

c $e^x\cos x - e^x \sin x$ **d** $-e^{-x}\sin x + e^{-x}\cos x$

e $\dfrac{\cos x}{\sin x}$ **f** $2e^{2x}\tan x + \dfrac{e^{2x}}{\cos^2 x}$ **g** $3\cos(3x)$

h $-\frac{1}{2}\sin\left(\frac{x}{2}\right)$ **i** $\dfrac{6}{\cos^2(2x)}$ **j** $\cos x - x\sin x$

k $\dfrac{x\cos x - \sin x}{x^2}$ **l** $\tan x + \dfrac{x}{\cos^2 x}$

3 **a** $2x\cos(x^2)$ **b** $-\dfrac{1}{2\sqrt{x}}\sin(\sqrt{x})$ **c** $-\dfrac{\sin x}{2\sqrt{\cos x}}$

d $2\sin x\cos x$ **e** $-3\sin x\cos^2 x$

f $-\sin x\sin(2x)+2\cos x\cos(2x)$

g $\sin x\sin(\cos x)$ **h** $-12\sin(4x)\cos^2(4x)$

i $-\dfrac{\cos x}{\sin^2 x}$ **j** $\dfrac{2\sin(2x)}{\cos^2(2x)}$

k $-\dfrac{8\cos(2x)}{\sin^3(2x)}$ **l** $\dfrac{-12}{\cos^2(\frac{x}{2})\tan^4(\frac{x}{2})}$

4 **a** $-\frac{9}{8}$ **b** 0

EXERCISE 13L

1 **a** $f''(x)=6$ **b** $f''(x)=\dfrac{3}{2x^{\frac{5}{2}}}$

c $f''(x)=12x-6$ **d** $f''(x)=\dfrac{12-6x}{x^4}$

e $f''(x)=24-48x$ **f** $f''(x)=\dfrac{20}{(2x-1)^3}$

2 **a** $\dfrac{d^2y}{dx^2}=-6x$ **b** $\dfrac{d^2y}{dx^2}=2-\dfrac{30}{x^4}$

c $\dfrac{d^2y}{dx^2}=-\frac{9}{4}x^{-\frac{5}{2}}$ **d** $\dfrac{d^2y}{dx^2}=\dfrac{8}{x^3}$

e $\dfrac{d^2y}{dx^2}=6(x^2-3x)(5x^2-15x+9)$

f $\dfrac{d^2y}{dx^2}=2+\dfrac{2}{(1-x)^3}$

3 **a** $f(2)=9$ **b** $f'(2)=10$ **c** $f''(2)=12$

5 **a** $x=1$

b $x=0,\ \pm\sqrt{6}$

6

x	-1	0	1
$f(x)$	$-$	0	$+$
$f'(x)$	$+$	$-$	$+$
$f''(x)$	$-$	0	$+$

7 **b** $f''(x)=3\sin x\cos 2x+6\cos x\sin 2x$

8 **a** $\dfrac{d^2y}{dx^2}=\dfrac{1}{x^2}$ **b** $\dfrac{d^2y}{dx^2}=\dfrac{1}{x}$

c $\dfrac{d^2y}{dx^2}=\dfrac{2}{x^2}(1-\ln x)$

9 **a** $f(1)=0$ **b** $f'(1)=3$ **c** $f''(1)=0$

10 **Hint:** Find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$ and substitute into the equation.

REVIEW SET 13A

1 **a** -1 **b** -1 **c** 8

2 **a** $f'(x)=2x+2$ **b** $\dfrac{dy}{dx}=-6x$

3 **a** $f'(t)=-9.6t\ \text{m s}^{-1}$

b $f'(2)=-19.2\ \text{m s}^{-1}$

(the negative sign indicates travelling downwards)

4 **a** $f(3)=-17$ **b** $f'(3)=-17$ **c** $f''(3)=-6$

5 **a** $\dfrac{dy}{dx}=6x-4x^3$ **b** $\dfrac{dy}{dx}=1+\dfrac{1}{x^2}$

6 $(0,0)$ **7** **a** $\dfrac{dy}{dx}=3x^2e^{x^3+2}$ **b** $\dfrac{dy}{dx}=\dfrac{1}{x+3}-\dfrac{2}{x}$

9 **a** $5+3x^{-2}$ **b** $4(3x^2+x)^3(6x+1)$

c $2x(1-x^2)^3-6x(x^2+1)(1-x^2)^2$

10 $(-2,19)$ and $(1,-2)$

11 **a** $\dfrac{dy}{dx}=-2(5-4x)^{-\frac{1}{2}}$ **b** $\dfrac{d^2y}{dx^2}=-4(5-4x)^{-\frac{3}{2}}$

12 **a** $5\cos(5x)\ln x+\dfrac{\sin(5x)}{x}$

b $\cos x\cos(2x)-2\sin x\sin(2x)$

c $-2e^{-2x}\tan x+\dfrac{e^{-2x}}{\cos^2 x}$

13 $\frac{\sqrt{3}}{2}$

14 **a** $f'(x)=8x(x^2+3)^3$

b $g'(x)=\dfrac{\frac{1}{2}x(x+5)^{-\frac{1}{2}}-2(x+5)^{\frac{1}{2}}}{x^3}$

15 **a** $f''(2)=\frac{23}{4}$ **b** $f''(2)=-\frac{1}{8\sqrt{2}}$

16 **a** $10-10\cos(10x)$ **b** $\tan x$

c $5\cos(5x)\ln(2x)+\dfrac{\sin(5x)}{x}$

REVIEW SET 13B

1 **a** -3 **b** 3 **c** -1 **2** $f'(1)=3$

3 **a** $\dfrac{dy}{dx}=4x$ **b** when $x=4$, gradient $=16$

c when gradient $=-12$, $x=-3$

4 **a** $\dfrac{dy}{dx}=3x^2(1-x^2)^{\frac{1}{2}}-x^4(1-x^2)^{-\frac{1}{2}}$

b $\dfrac{dy}{dx}=\dfrac{(2x-3)(x+1)^{\frac{1}{2}}-\frac{1}{2}(x^2-3x)(x+1)^{-\frac{1}{2}}}{x+1}$

5 **a** $\dfrac{d^2y}{dx^2}=36x^2-\dfrac{4}{x^3}$ **b** $\dfrac{d^2y}{dx^2}=6x+\frac{3}{4}x^{-\frac{5}{2}}$

6 $(1,e)$ **7** **a** $f'(x)=\dfrac{e^x}{e^x+3}$ **b** $f'(x)=\dfrac{3}{x+2}-\dfrac{1}{x}$

8 When $x=1$, $\dfrac{dy}{dx}=0$.

9 **a** $\dfrac{dy}{dx}=\dfrac{3x^2-3}{x^3-3x}$ **b** $\dfrac{dy}{dx}=\dfrac{e^x(x-2)}{x^3}$

10 $x=-\frac{1}{2},\ \frac{3}{2}$

11 **a** $f(\pi)=\pi+1$ **b** $f'(\frac{\pi}{2})=2$ **c** $f''(\frac{3\pi}{4})=-\frac{\sqrt{2}}{2}$

12 **a** $f'(x)=\frac{1}{2}x^{-\frac{1}{2}}\cos(4x)-4x^{\frac{1}{2}}\sin(4x)$,

$f''(x)=-\frac{1}{4}x^{-\frac{3}{2}}\cos(4x)-4x^{-\frac{1}{2}}\sin(4x)$

$-16x^{\frac{1}{2}}\cos(4x)$

b $f'(\frac{\pi}{16})\approx-0.455$, $f''(\frac{\pi}{8})\approx-6.38$

14 **a** $x=-6\pm\sqrt{33}$ **b** $x=\pm\sqrt{3}$ **c** $x=0,\ \pm3$

15 **a** $f(x)=-5\sin 4x$

b $f'(x)=0$ when $x=\frac{\pi}{8},\ \frac{3\pi}{8},\ \frac{5\pi}{8},\ \frac{7\pi}{8}$, $0\leqslant x\leqslant\pi$

16 $\dfrac{dy}{dx}=3b\cos(bx)+2a\sin(2x)$, $a=2$, $b=\pm1$

EXERCISE 14A

1 **a** $y=-7x+11$ **b** $x-4y=-8$ **c** $y=-2x-2$

d $y=-2x+6$ **e** $y=-5x-9$ **f** $y=-5x-1$

2 **a** $x+6y=57$ **b** $x+7y=26$ **c** $x-3y=-11$

d $x+6y=43$

3 $y = 21$ and $y = -6$ **4** $(\frac{1}{2}, 2\sqrt{2})$

5 $k = -5$ **6** $y = -3x + 1$

7 $a = -4,\ b = 7$ **8** $a = 2,\ b = 4$

10 **a** $x - 3y = -5$ **b** $x - 9y = -4$
c $x - 16y = 3$ **d** $y = -4$

11 **a** $y = 2x - \frac{7}{4}$ **b** $y = -27x - \frac{242}{3}$
c $4x + 57y = 1042$ **d** $x - 2y = -1$

12 $a = 4,\ b = 3$

13 **a** $x + ey = 2$ **b** $x + 3y = 3\ln 3 - 1$
c $2x + e^2 y = \dfrac{2}{e^2} - e^2$

15 **a** $y = x$ **b** $y = x$ **c** $2x - y = \frac{\pi}{3} - \frac{\sqrt{3}}{2}$ **d** $x = \frac{\pi}{4}$

16 **a** $(-4, -64)$ **b** $(4, -31)$

17 **a** $f'(x) = 2x - \dfrac{8}{x^3}$ **b** $x = \pm\sqrt{2}$ **c** tangent is $y = 4$

18 A is $(\frac{2}{3}, 0)$, B is $(0, -2e)$

19 **a** $y = (2a - 1)x - a^2 + 9$
b $y = 5x$, contact at $(3, 15)$, $y = -7x$, contact at $(-3, 21)$

20 $y = 0,\ y = 27x + 54$ **21** $y = -\sqrt{14}x + 4\sqrt{14}$

22 $y = e^a x + e^a(1 - a)$ so $y = ex$ is the tangent to $y = e^x$ from the origin.

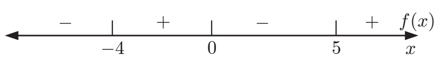
23 **a** **Hint:** They must have the same y-coordinate at $x = b$ and the same gradient.
c $a = \dfrac{1}{2e}$ **d** $y = e^{-\frac{1}{2}}x - \frac{1}{2}$

24 $\approx 63.43°$

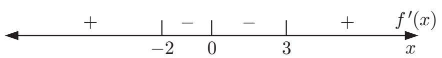
25 **a** **Hint:** $y = f(a) + f'(a)(x - a)$
b **Hint:** Expand $f(x) = 4 - 8(x+1) - (x+1)^2 + 2(x+1)^3$
c Notice the first 2 terms in **b** are the same as the tangent line found in part **a**.

EXERCISE 14B

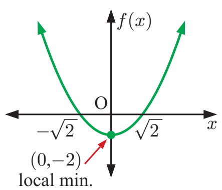
1 **a** A - local max, B - stationary inflection, C - local min.
b **i**



ii



2 **a**



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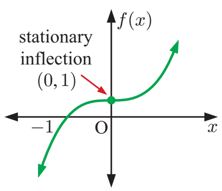
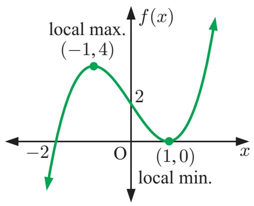
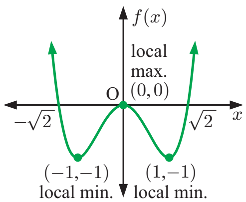


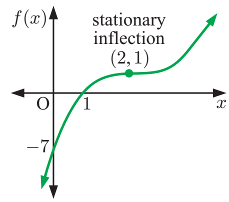
c



d



e



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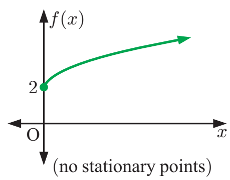

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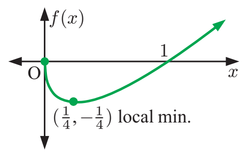

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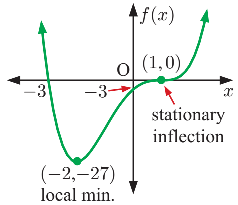

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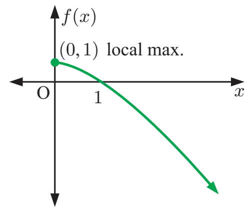

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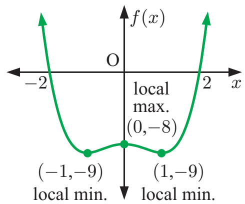


3 $x = -\dfrac{b}{2a}$, local min if $a > 0$, local max if $a < 0$

4 $a = 9$

5 **a** $a = -12,\ b = -13$
b $(-2, 3)$ local max., $(2, -29)$ local min.

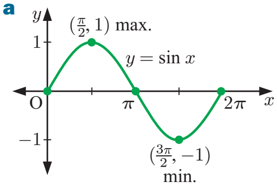
6 **a** local maximum at $(1, e^{-1})$
b local maximum at $(-2, 4e^{-2})$, local minimum at $(0, 0)$
c local minimum at $(1, e)$
d local maximum at $(-1, e)$

7 **a** $x > 0$

8 **a** Greatest value is 63 when $x = 5$, least value is -18 when $x = 2$.
b Greatest value is 4 when $x = 3$ and $x = 0$, least value is -16 when $x = -2$.

9 $P(x) = -9x^3 - 9x^2 + 9x + 2$

10 **a**



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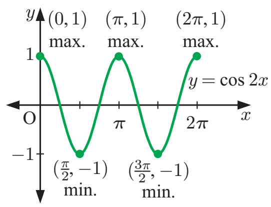

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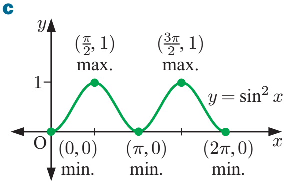

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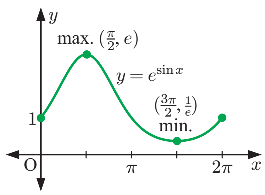

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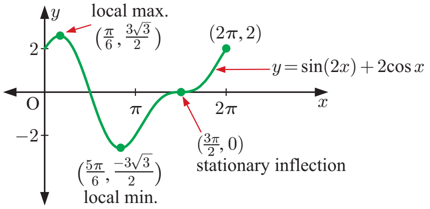


11 **Hint:** Find $\frac{dy}{dx}$, then determine the nature of the stationary points.

12 **Hint:** Show that as $x \to 0$, $f(x) \to -\infty$, and as $x \to \infty$, $f(x) \to 0$.

13 **a** **Hint:** Find $f'(x)$, then determine the nature of the stationary points.

b **Hint:** Show that $f(x) \geqslant 1$ for all $x > 0$.

EXERCISE 14C.1

1 **a** 7 m s^{-1} **b** $(h+5)\text{ m s}^{-1}$

c $5\text{ m s}^{-1} = s'(1)$ is the instantaneous velocity at $t = 1$ s

d average velocity $= (2t + h + 3)\text{ m s}^{-1}$,
$\lim\limits_{h \to 0}(2t + h + 3) = 2t + 3\text{ m s}^{-1}$ is the instantaneous velocity at time t seconds.

2 **a** -14 cm s^{-1} **b** $(-8 - 2h)\text{ cm s}^{-1}$

c $-8\text{ cm s}^{-1} = s'(2)$
$\therefore$ instantaneous velocity $= -8\text{ cm s}^{-1}$ at $t = 2$

d $-4t = s'(t) = v(t)$ is the instantaneous velocity at time t seconds.

3 **a** $\frac{2}{3}\text{ cm s}^{-2}$ **b** $\dfrac{2\sqrt{1+h}-2}{h}\text{ cm s}^{-2}$

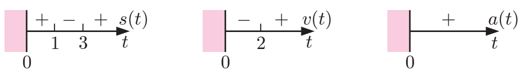
c $1\text{ cm s}^{-2} = v'(1)$ is the instantaneous accn. at $t = 1$ s

d $\dfrac{1}{\sqrt{t}}\text{ cm s}^{-2} = v'(t)$, the instantaneous accn. at time t

4 **a** velocity at $t = 4$ **b** acceleration at $t = 4$

EXERCISE 14C.2

1 **a** $v(t) = 2t - 4\text{ cm s}^{-1}$, $a(t) = 2\text{ cm s}^{-2}$

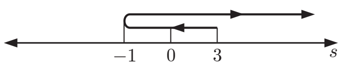


b $s(0) = 3$ cm, $v(0) = -4\text{ cm s}^{-1}$, $a(0) = 2\text{ cm s}^{-2}$
The object is initially 3 cm to the right of the origin and is moving to the left at 4 cm s^{-1}. It is accelerating at 2 cm s^{-2} to the right.

c $s(2) = -1$ cm, $v(2) = 0\text{ cm s}^{-1}$, $a(2) = 2\text{ cm s}^{-2}$
The object is instantaneously stationary, 1 cm to the left of the origin and is accelerating to the right at 2 cm s^{-2}.

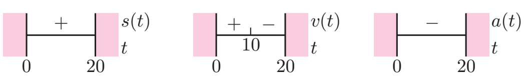
d At $t = 2$, $s(2) = 1$ cm to the left of the origin.

e



f $0 \leqslant t \leqslant 2$

2 **a** $v(t) = 98 - 9.8t\text{ m s}^{-1}$, $a(t) = -9.8\text{ m s}^{-2}$



b $s(0) = 0$ m above the ground, $v(0) = 98\text{ m s}^{-1}$ skyward

c $t = 5$ s Stone is 367.5 m above the ground and moving skyward at 49 m s^{-1}. Its speed is decreasing.
$t = 12$ s Stone is 470.4 m above the ground and moving groundward at 19.6 m s^{-1}. Its speed is increasing.

d 490 m **e** 20 seconds

3 **a** 1.2 m

b $s'(t) = 28.1 - 9.8t$ represents the instantaneous velocity of the ball.

c $t = 2.87$ s. The ball has reached its maximum height and is instantaneously at rest.

d 41.5 m

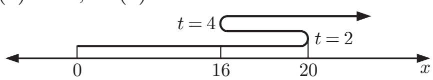
e **i** 28.1 m s^{-1} **ii** 8.5 m s^{-1} **iii** 20.9 m s^{-1}
$s'(t) \geqslant 0$ when the ball is travelling upwards.
$s'(t) \leqslant 0$ when the ball is travelling downwards.

f 5.78 s

g $s''(t)$ is the rate of change of $s'(t)$, or the instantaneous acceleration.

4 **a** $v(t) = 3t^2 - 18t + 24\text{ m s}^{-1}$ $a(t) = 6t - 18\text{ m s}^{-2}$

b $x(2) = 20$, $x(4) = 16$



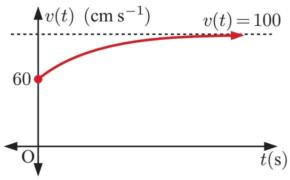
c **i** $0 \leqslant t \leqslant 2$ and $3 \leqslant t \leqslant 4$ **ii** $0 \leqslant t \leqslant 3$

d 28 m

5 **a** $v(t) = 100 - 40e^{-\frac{t}{5}}\text{ cm s}^{-1}$, $a(t) = 8e^{-\frac{t}{5}}\text{ cm s}^{-2}$

b $s(0) = 200$ cm on positive side of origin
$v(0) = 60\text{ cm s}^{-1}$, $a(0) = 8\text{ cm s}^{-2}$

c



d after 3.47 s

6 **a** $x(0) = -1$ cm, $v(0) = 0\text{ cm s}^{-1}$, $a(0) = 2\text{ cm s}^{-2}$

b At $t = \frac{\pi}{4}$ seconds, the particle is $(\sqrt{2} - 1)$ cm left of the origin, moving right at $\sqrt{2}\text{ cm s}^{-1}$, with increasing speed.

c changes direction when $t = \pi$, $x(\pi) = 3$ cm

d $0 \leqslant t \leqslant \frac{\pi}{2}$ and $\pi \leqslant t \leqslant \frac{3\pi}{2}$

7 **Hint:** Assume that $s(t) = at^2 + bt + c$
$s'(t) = v(t)$ and $s''(t) = a(t) = g$
Show that $a = \frac{1}{2}g$, $b = v(0)$, $c = 0$.

8 **a** 0.675 s

b **i** $S'(t) = u + at\text{ m s}^{-1}$ **ii** $t = -\dfrac{u}{a}$ s

iii $a = -\frac{640}{99} \approx -6.46\text{ m s}^{-2}$

iv **Hint:** Substitute $t = -\dfrac{u}{a}$ into $S(t)$.

v If the speed u is doubled, then the braking distance is quadrupled ($2^2 = 4$ times).

EXERCISE 14D

1 **a** \$118 000 **b** $\dfrac{dP}{dt} = 4t - 12$, \$1000s per year

c $\dfrac{dP}{dt}$ is the rate of change in profit with time

d **i** $0 \leqslant t \leqslant 3$ years **ii** $t > 3$ years

e minimum profit is \$100 000 when $t = 3$

f $\left.\dfrac{dP}{dt}\right|_{t=4} = 4$ Profit is increasing at \$4000 per year after 4 years.

$\left.\dfrac{dP}{dt}\right|_{t=10} = 28$ Profit is increasing at \$28 000 per year after 10 years.

$\left.\dfrac{dP}{dt}\right|_{t=25} = 88$ Profit is increasing at \$88 000 per year after 25 years.

2 **a** **i** $Q(0) = 100$ **ii** $Q(25) = 50$ **iii** $Q(100) = 0$

b **i** decr. 1 unit per year **ii** decr. $\frac{1}{\sqrt{2}}$ units per year

c $Q'(t) = -\dfrac{5}{\sqrt{t}} < 0$

3 **a** 0.5 m

b $t = 4$: 9.17 m, $t = 8$: 12.5 m, $t = 12$: 14.3 m

c $t = 0$: 3.9 m year^{-1}, $t = 5$: 0.975 m year^{-1}, $t = 10$: 0.433 m year^{-1}

d As $\dfrac{dH}{dt} = \dfrac{97.5}{(t+5)^2} > 0$ for all $t \geqslant 0$, the tree is always growing.

4 **a** $C'(x) = 0.0009x^2 + 0.04x + 4$ dollars per pair

b $C'(220) = \$56.36$ per pair. This estimates the additional cost of making one more pair of jeans if 220 pairs are currently being made.

c \$56.58 This is the actual increase in cost to make an extra pair of jeans (221 rather than 220).

d $C''(x) = 0.0018x + 0.04$
$C''(x) = 0$ when $x = -22.2$. This is where the rate of change is a minimum, however it is out of the bounds of the model (you cannot make < 0 jeans!).

5 **a** **i** €4500 **ii** €4000

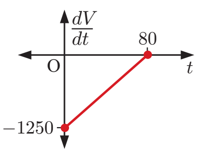
b **i** decrease of €210.22 per km h^{-1}

ii increase of €11.31 per km h^{-1}

c $\dfrac{dC}{dv} = 0$ at $v = \sqrt[3]{500\,000} \approx 79.4$ km h^{-1}

6 **a** $\dfrac{dV}{dt} = -1250\left(1 - \dfrac{t}{80}\right)$ L min^{-1}

b at $t = 0$ when the tap was first opened



c $\dfrac{d^2V}{dt^2} = \dfrac{125}{8}$ L min^{-2}

This shows that the rate of change of V is constantly increasing, so the outflow is decreasing at a constant rate.

7 **a** The near part of the lake is 2 km from the sea, the furthest part is 3 km.

b $\dfrac{dy}{dx} = \frac{3}{10}x^2 - x + \frac{3}{5}$

$\left.\dfrac{dy}{dx}\right|_{x=\frac{1}{2}} = 0.175$, height of hill is increasing as gradient is positive.

$\left.\dfrac{dy}{dx}\right|_{x=1\frac{1}{2}} = -0.225$, height of hill is decreasing as gradient is negative.

$\therefore$ top of the hill is between $x = \frac{1}{2}$ and $x = 1\frac{1}{2}$.

c 2.55 km from the sea, 63.1 m deep

8 **a** $k = \frac{1}{50}\ln 2 \approx 0.0139$

b **i** 20 grams **ii** 14.3 grams **iii** 1.95 grams

c 9 days and 6 minutes (216 hours)

d **i** -0.0693 g h^{-1} **ii** -2.64×10^{-7} g h^{-1}

e **Hint:** You should find $\dfrac{dW}{dt} = -\frac{1}{50}\ln 2 \times 20e^{-\frac{1}{50}\ln 2t}$

9 **a** $k = \frac{1}{15}\ln\left(\frac{19}{3}\right) \approx 0.123$ **b** 100°C

c $c = -k \approx -0.123$

d **i** decreasing at 11.7°C min^{-1}

ii decreasing at 3.42°C min^{-1}

iii decreasing at 0.998°C min^{-1}

10 **a** 43.9 cm **b** 10.4 years

c **i** growing at 5.45 cm per year

ii growing at 1.88 cm per year

11 **a** $A(0) = 0$

b **i** $k = \dfrac{\ln 2}{3}$ (≈ 0.231)

ii 0.728 litres of alcohol produced per hour

12 $\frac{21}{\sqrt{2}}$ cm^2 per radian

13 **a** rising at 2.73 m per hour **b** rising

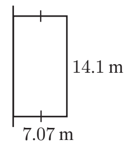
14 **b** **i** 0 **ii** 1 **iii** ≈ 1.11

EXERCISE 14E

1 250 items

2 **b** $L_{\min} \approx 28.3$ m, $x \approx 7.07$ m

c



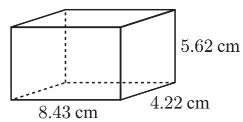
3 10 blankets **4** 14.8 km h^{-1} **5** at 4.41 months old

6 **a** **Hint:** $V = 200 = 2x \times x \times h$

b **Hint:** Show $h = \dfrac{100}{x^2}$ and substitute into the surface area equation.

c $SA_{\min} \approx 213$ cm^2, $x \approx 4.22$ cm

d



7 20 kettles **8** $\mathrm{C}\left(\frac{1}{\sqrt{2}}, e^{-\frac{1}{2}}\right)$

9 **a** Recall that $V_{\text{cylinder}} = \pi r^2 h$ and that $1\text{ L} = 1000\text{ cm}^3$.

b Recall that $SA_{\text{cylinder}} = 2\pi r^2 + 2\pi r h$.

c radius ≈ 5.42 cm, height ≈ 10.8 cm

10 **b** $\theta \approx 1.91$, $A \approx 237$ cm^2 **11** **b** 6 cm × 6 cm

12 **a** $0 \leqslant x \leqslant 63.7$

b $l = 100$ m, $x = \frac{100}{\pi} \approx 31.83$ m, $A = \frac{20\,000}{\pi} \approx 6366$ m^2

13 after 13.8 weeks **14** after 40 minutes

15 **c** $\theta = 30°$, $A \approx 130$ cm^2

16 **a** **Hint:** Show that $\text{AC} = \frac{\theta}{360} \times 2\pi \times 10$

b **Hint:** Show that $2\pi r = \text{AC}$

c **Hint:** Use the result from **b** and Pythagoras' theorem.

d $V = \frac{1}{3}\pi\left(\frac{\theta}{36}\right)^2\sqrt{100 - \left(\frac{\theta}{36}\right)^2}$ **e** $\theta \approx 294°$

17 1 hour 34 min 53 s when $\theta \approx 36.9°$ **18** 9.87 m

EXERCISE 14F

1 a is decreasing at 7.5 units per second

2 increasing at 1 cm per minute

3 **a** 4π m^2 per second **b** 8π m^2 per second

4 increasing at 6π m^2 per minute

5 decreasing at $0.16\ \text{m}^3$ per minute

6 $\frac{20}{3}$ cm per minute

7 $\frac{25\sqrt{3}}{6} \approx 7.22$ cm per minute

8 decreasing at $\frac{250}{13} \approx 19.2\ \text{m s}^{-1}$

9 **a** $0.2\ \text{m s}^{-1}$ **b** $\frac{4}{45}\ \text{m s}^{-1}$

10 decreasing at $\frac{\sqrt{2}}{100}$ radians per second

11 increasing at 0.12 radians per minute

REVIEW SET 14A

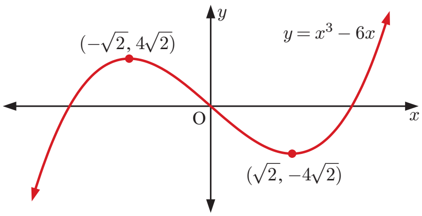
1 **a** $y = 4x + 2$ **b** $y = 4x + 4\ln 2 - 4$ **c** $y = e^2$

2 $a = \frac{5}{2},\ b = -\frac{3}{2}$

3 **a** $a = -6$

b local max. $(-\sqrt{2}, 4\sqrt{2})$, local min. $(\sqrt{2}, -4\sqrt{2})$

c



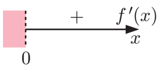
4 **a** $y = \frac{1}{5}x - \frac{11}{5}$ (or $x - 5y = 11$) **b** $y = -4x + 14$

5 $\frac{3267}{152}$ units^2 **6** $a = 64$ **7** P(0, 7.5), Q(3, 0)

9 $3x - 4y = -5$

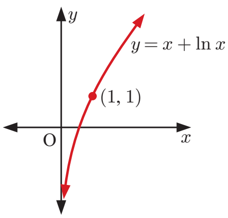
10 **a** $x > 0$

b Sign diagram of $f'(x)$



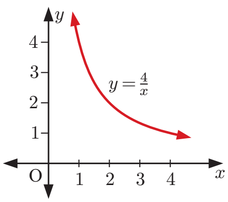
$f(x)$ is increasing for all $x > 0$.

c



d normal is $x + 2y = 3$

11 **a**



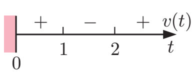
b $y = -\dfrac{4}{k^2}x + \dfrac{8}{k}$

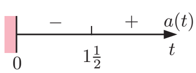
c $A(2k, 0)$, $B\left(0, \dfrac{8}{k}\right)$

d Area $= 8$ units^2

e $k = 2$

12 **a** $v(t) = (6t^2 - 18t + 12)\ \text{cm s}^{-1}$, $a(t) = (12t - 18)\ \text{cm s}^{-2}$



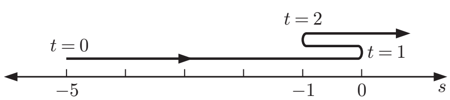


b $s(0) = -5$ cm (5 cm to the left of origin)
$v(0) = 12\ \text{cm s}^{-1}$ towards origin
$a(0) = -18\ \text{cm s}^{-2}$ (reducing speed)

c At $t = 2$, particle is 1 cm to the left of the origin, is stationary and is accelerating towards the origin.

d $t = 1,\ s = 0$ and $t = 2,\ s = -1$

e



f $1 \leqslant t \leqslant 1\frac{1}{2}$ and $t \geqslant 2$

13 **b** $k = 9$

14 **a** $x(0) = 3$ cm, $x'(0) = 2\ \text{cm s}^{-1}$, $x''(0) = 0\ \text{cm s}^{-2}$

b $t = \frac{\pi}{4}$ s and $\frac{3\pi}{4}$ s **c** 4 cm

15 6 cm from each end

16 **a** $y = \dfrac{1}{x^2},\ x > 0$

c base is 1.26 m square, height 0.630 m

17 **a** $v(t) = 15 + \dfrac{120}{(t+1)^3}\ \text{cm s}^{-1}$, $a(t) = \dfrac{-360}{(t+1)^4}\ \text{cm s}^{-2}$

b At $t = 3$, particle is 41.25 cm to the right of the origin, moving to the right at $16.88\ \text{cm s}^{-1}$ and decelerating at $1.41\ \text{cm s}^{-2}$.

c speed is never increasing

18 $A\left(\dfrac{1}{2}, \dfrac{1}{e}\right)$ **19** $\frac{20\sqrt{10}}{3} \approx 21.1$ m per minute

20 **a** $V(r) = \frac{8}{9}\pi r^3\ \text{m}^3$

b $\dfrac{dr}{dt} = -\dfrac{8}{375\pi} \approx -0.006\,79\ \text{m min}^{-1}$

REVIEW SET 14B

1 **a** $x = 1$ **b** $ex - 2y = e - \dfrac{2}{e}$ **c** $y = 16x - \frac{127}{2}$

2 $a = -14,\ b = 21$

3 **a** $f(3) = 2,\ f'(3) = -1$ **b** $f(x) = x^2 - 7x + 14$

4 **a** $2x + 3y = \frac{2\pi}{3} + 2\sqrt{3}$ **b** $\sqrt{2}y - 4x = 1 - 2\pi$

5 $p = 1,\ q = -8$ **6** $(-2, -25)$ **7** $a = \frac{1}{2}$

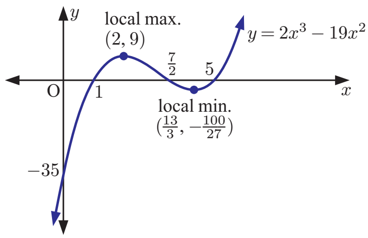
8 **a** local minimum at (0, 1) **c** $f''(x) = e^x$

9 $(0, \ln 4 - 1)$

10 **a** y-intercept $= -35$ **b** $x = 1, \frac{7}{2}, 5$

c local maximum at (2, 9), local minimum at $(\frac{13}{3}, -\frac{100}{27})$

d



11 $BC = \frac{8\sqrt{10}}{3}$ units

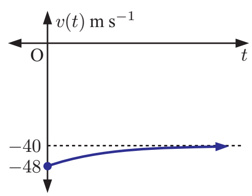
12 **a** 60 cm **b** **i** 4.24 years **ii** 201 years

c **i** 16 cm per year **ii** 1.95 cm per year

13 **a** $v(t) = -8e^{-\frac{t}{10}} - 40\ \text{m s}^{-1}$
$a(t) = \frac{4}{5}e^{-\frac{t}{10}}\ \text{m s}^{-2}$ $\{t \geqslant 0\}$

b $s(0) = 80$ m
$v(0) = -48\ \text{m s}^{-1}$
$a(0) = 0.8\ \text{m s}^{-2}$

c



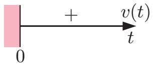
d $t = 10\ln 2$ seconds

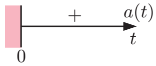
14 **a** **i** \$535 **ii** \$1385.79

b **i** $-\$0.267$ per km h^{-1} **ii** \$2.33 per km h^{-1}

c 51.3 km h^{-1}

15 **a** $v(t) = 3 - \dfrac{1}{2\sqrt{t+1}}$ $\quad$ $a(t) = \dfrac{1}{4(t+1)^{\frac{3}{2}}}$



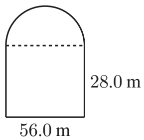


b $x(0) = -1, \quad v(0) = 2.5, \quad a(0) = 0.25$
Particle is 1 cm to the left of the origin, is travelling to the right at 2.5 cm s^{-1}, and accelerating at 0.25 cm s^{-2}.

c Particle is 21 cm to the right of the origin, is travelling to the right at 2.83 cm s^{-1}, and accelerating at 0.009 26 cm s^{-2}.

d never changes direction $\quad$ **e** never decreasing

16 **b** $A = 200x - 2x^2 - \frac{1}{2}\pi x^2$ $\quad$ **c**



17 **a** $v(0) = 0 \text{ cm s}^{-1}, \ v(\frac{1}{2}) = -\pi \text{ cm s}^{-1}, \ v(1) = 0 \text{ cm s}^{-1},$
$v(\frac{3}{2}) = \pi \text{ cm s}^{-1}, \ v(2) = 0 \text{ cm s}^{-1}$

b $0 \leqslant t \leqslant 1, \ 2 \leqslant t \leqslant 3, \ 4 \leqslant t \leqslant 5,$ etc.
So, for $2n \leqslant t \leqslant 2n+1, \ n \in \{0, 1, 2, 3, ...\}$

18 $x = \dfrac{k}{2}\left(1 - \dfrac{1}{\sqrt{3}}\right)$ $\quad$ **19** 3.60 m s^{-1}

20 increasing at 0.128 radians per second

21 **a** $\frac{\sqrt{3}}{2}\pi \text{ cm s}^{-1}$ $\quad$ **b** 0 cm s^{-1}

22 **a** **i** $y = -\dfrac{a^2}{4b} + \dfrac{a}{2b}x$ $\quad$ **ii** when $y = 0, \ x = \dfrac{a}{2}$

b **i** $y = -\dfrac{2b}{a}x + b$ $\quad$ **ii** when $x = 0, \ y = b$

iii **Hint:** Let P$''$ be the point on the line $y = -b$ where the distance to P is shortest.
Show that FP = P$''$P.

c **i** **Hint:** Show that $\triangle$FPP$' \cong \triangle$P$''$PP$'$.

ii **Hint:** Show that the tangents meet at $\left(\dfrac{a+c}{2}, \dfrac{ac}{4b}\right)$.

EXERCISE 15A.1

1 **a** **i** 0.6 units^2 $\quad$ **ii** 0.4 units^2 $\quad$ **b** 0.5 units^2

2 **a** 0.737 units^2 $\quad$ **b** 0.653 units^2

3

n	A_L	A_U
10	2.1850	2.4850
25	2.2736	2.3936
50	2.3034	2.3634
100	2.3184	2.3484
500	2.3303	2.3363

converges to $\frac{7}{3}$

4 **a** **i**

n	A_L	A_U
5	0.160 00	0.360 00
10	0.202 50	0.302 50
50	0.240 10	0.260 10
100	0.245 03	0.255 03
500	0.249 00	0.251 00
1000	0.249 50	0.250 50
10 000	0.249 95	0.250 05

ii

n	A_L	A_U
5	0.400 00	0.600 00
10	0.450 00	0.550 00
50	0.490 00	0.510 00
100	0.495 00	0.505 00
500	0.499 00	0.501 00
1000	0.499 50	0.500 50
10 000	0.499 95	0.500 05

iii

n	A_L	A_U
5	0.549 74	0.749 74
10	0.610 51	0.710 51
50	0.656 10	0.676 10
100	0.661 46	0.671 46
500	0.665 65	0.667 65
1000	0.666 16	0.667 16
10 000	0.666 62	0.666 72

iv

n	A_L	A_U
5	0.618 67	0.818 67
10	0.687 40	0.787 40
50	0.738 51	0.758 51
100	0.744 41	0.754 41
500	0.748 93	0.750 93
1000	0.749 47	0.750 47
10 000	0.749 95	0.750 05

b **i** $\frac{1}{4}$ $\quad$ **ii** $\frac{1}{2}$ $\quad$ **iii** $\frac{2}{3}$ $\quad$ **iv** $\frac{3}{4}$ $\quad$ **c** area $= \dfrac{1}{a+1}$

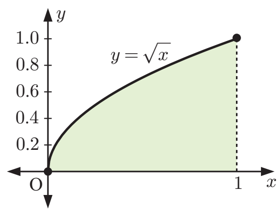
5 **a**

n	Rational bounds for π
10	$2.9045 < \pi < 3.3045$
50	$3.0983 < \pi < 3.1783$
100	$3.1204 < \pi < 3.1604$
200	$3.1312 < \pi < 3.1512$
1000	$3.1396 < \pi < 3.1436$
10 000	$3.1414 < \pi < 3.1418$

b $n = 10\,000$

EXERCISE 15A.2

1 **a**



b

n	A_L	A_U
5	0.5497	0.7497
10	0.6105	0.7105
50	0.6561	0.6761
100	0.6615	0.6715
500	0.6656	0.6676

c $\int_0^1 \sqrt{x}\, dx \approx 0.67$

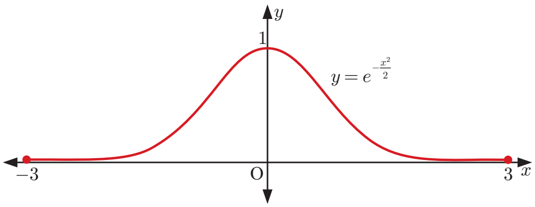
2 **a** $A_L = \dfrac{2}{n}\displaystyle\sum_{i=0}^{n-1}\sqrt{1+x_i^{\,3}}, \quad A_U = \dfrac{2}{n}\sum_{i=1}^{n}\sqrt{1+x_i^{\,3}}$

b

n	A_L	A_U
50	3.2016	3.2816
100	3.2214	3.2614
500	3.2373	3.2453

c $\int_0^2 \sqrt{1+x^3}\, dx \approx 3.24$

3 **a**



b upper ≈ 1.2506, lower ≈ 1.2493

c upper ≈ 1.2506, lower ≈ 1.2493

d $\int_{-3}^{3} e^{-\frac{x^2}{2}}\,dx \approx 2.4999$ compared to $\sqrt{2\pi} \approx 2.5066$

4 **a** 18 **b** 4.5 **c** 2π

EXERCISE 15B

1 **a** **i** $\dfrac{x^2}{2}$ **ii** $\dfrac{x^3}{3}$ **iii** $\dfrac{x^6}{6}$ **iv** $-\dfrac{1}{x}$

v $-\dfrac{1}{3x^3}$ **vi** $\frac{3}{4}x^{\frac{4}{3}}$ **vii** $2\sqrt{x}$

b The antiderivative of x^n is $\dfrac{x^{n+1}}{n+1}$ $(n \neq -1)$.

2 **a** **i** $\frac{1}{2}e^{2x}$ **ii** $\frac{1}{5}e^{5x}$ **iii** $2e^{\frac{1}{2}x}$ **iv** $100e^{0.01x}$

v $\frac{1}{\pi}e^{\pi x}$ **vi** $3e^{\frac{x}{3}}$

b The antiderivative of e^{kx} is $\dfrac{1}{k}e^{kx}$.

3 **a** $\dfrac{d}{dx}(x^3+x^2) = 3x^2+2x$

$\therefore$ the antiderivative of $6x^2+4x = 2x^3+2x^2$

b $\dfrac{d}{dx}(e^{3x+1}) = 3e^{3x+1}$

$\therefore$ the antiderivative of $e^{3x+1} = \frac{1}{3}e^{3x+1}$

c $\dfrac{d}{dx}(x\sqrt{x}) = \frac{3}{2}\sqrt{x}$

$\therefore$ the antiderivative of $\sqrt{x} = \frac{2}{3}x\sqrt{x}$

d $\dfrac{d}{dx}(2x+1)^4 = 8(2x+1)^3$

$\therefore$ the antiderivative of $(2x+1)^3 = \frac{1}{8}(2x+1)^4$

EXERCISE 15C

1 **a** $\frac{1}{4}$ units^2 **b** $2\frac{1}{3}$ units^2 **c** $\frac{2}{3}$ units^2

3 **a** $3\frac{3}{4}$ units^2 **b** $24\frac{2}{3}$ units^2 **c** $\frac{-2+4\sqrt{2}}{3}$ units^2

d ≈ 3.48 units^2 **e** 2 units^2

4 **c** **i** $\int_0^1 (-x^2)\,dx = -\frac{1}{3}$, the area between $y=-x^2$ and the x-axis from $x=0$ to $x=1$ is $\frac{1}{3}$ units^2.

ii $\int_0^1 (x^2-x)\,dx = -\frac{1}{6}$, the area between $y=x^2-x$ and the x-axis from $x=0$ to $x=1$ is $\frac{1}{6}$ units^2.

iii $\int_{-2}^{0} 3x\,dx = -6$, the area between $y=3x$ and the x-axis from $x=-2$ to $x=0$ is 6 units^2

d $-\pi$

EXERCISE 15D

1 $\dfrac{dy}{dx} = 7x^6$, $\int x^6\,dx = \frac{1}{7}x^7+c$

2 $\dfrac{dy}{dx} = 3x^2+2x$, $\int (3x^2+2x)\,dx = x^3+x^2+c$

3 $\dfrac{dy}{dx} = 2e^{2x+1}$, $\int e^{2x+1}\,dx = \frac{1}{2}e^{2x+1}+c$

4 $\dfrac{dy}{dx} = 8(2x+1)^3$, $\int (2x+1)^3\,dx = \frac{1}{8}(2x+1)^4+c$

5 $\dfrac{dy}{dx} = \frac{3}{2}\sqrt{x}$, $\int \sqrt{x}\,dx = \frac{2}{3}x\sqrt{x}+c$

6 $\dfrac{dy}{dx} = -\dfrac{1}{2x\sqrt{x}}$, $\displaystyle\int \frac{1}{x\sqrt{x}}\,dx = -\frac{2}{\sqrt{x}}+c$

7 $\dfrac{dy}{dx} = -2\sin 2x$, $\int \sin 2x\,dx = -\frac{1}{2}\cos 2x+c$

8 $\dfrac{dy}{dx} = -5\cos(1-5x)$,

$\int \cos(1-5x)\,dx = -\frac{1}{5}\sin(1-5x)+c$

9 $\int (2x-1)(x^2-x)^2\,dx = \frac{1}{3}(x^2-x)^3+c$

11 $\dfrac{dy}{dx} = \dfrac{-2}{\sqrt{1-4x}}$, $\displaystyle\int \frac{1}{\sqrt{1-4x}}\,dx = -\frac{1}{2}\sqrt{1-4x}+c$

EXERCISE 15E.1

1 **a** $\dfrac{x^5}{5} - \dfrac{x^3}{3} - \dfrac{x^2}{2} + 2x + c$ **b** $x^5 - x^4 - 2x^3 - 7x + c$

c $\frac{2}{3}x^{\frac{3}{2}} + e^x + c$ **d** $3e^x + \frac{1}{3}x^3 + c$

e $\frac{2}{5}x^{\frac{5}{2}} - 2x + c$ **f** $-2x^{-\frac{1}{2}} + 2x^2 + c$

g $\frac{1}{8}x^4 - \frac{1}{5}x^5 + \frac{3}{4}x^{\frac{4}{3}} + c$ **h** $\dfrac{x^2}{4} + \frac{1}{3}x^3 - e^x + c$

i $5e^x + \frac{1}{12}x^4 - \frac{2}{3}x^{\frac{3}{2}} + c$

2 **a** $-3\cos x - 2x + c$ **b** $2x^2 - 2\sin x + c$

c $-\cos x - 2\sin x + e^x + c$ **d** $\frac{2}{7}x^3\sqrt{x} + 10\cos x + c$

e $\frac{1}{9}x^3 - \frac{1}{6}x^2 + \sin x + c$ **f** $\cos x + \frac{4}{3}x\sqrt{x} + c$

3 **a** $\frac{1}{3}x^3 + \frac{3}{2}x^2 - 2x + c$ **b** $\frac{2}{3}x^{\frac{3}{2}} - 2x^{\frac{1}{2}} + c$

c $2e^x + \dfrac{1}{x} + c$ **d** $-2x^{-\frac{1}{2}} - 8x^{\frac{1}{2}} + c$

e $\frac{4}{3}x^3 + 2x^2 + x + c$ **f** $\frac{1}{3}x^3 + 2x - \dfrac{1}{x} + c$

g $\frac{4}{3}x^{\frac{3}{2}} - 2x^{\frac{1}{2}} + c$ **h** $2x^{\frac{1}{2}} + 8x^{-\frac{1}{2}} - \frac{20}{3}x^{-\frac{3}{2}} + c$

i $\frac{1}{4}x^4 + x^3 + \frac{3}{2}x^2 + x + c$

4 **a** $\frac{2}{3}x^{\frac{3}{2}} + \frac{1}{2}\sin x + c$ **b** $2e^t + 4\cos t + c$

c $3\sin t + \cos t + c$

5 **a** $y = 6x + c$ **b** $y = \frac{4}{3}x^3 + c$

c $y = \frac{10}{3}x\sqrt{x} - \frac{1}{3}x^3 + c$ **d** $y = -\dfrac{1}{x} + c$

e $y = 2e^x - 5x + c$ **f** $y = x^4 + x^3 + c$

6 **a** $f(x) = x - 2x^2 + \frac{4}{3}x^3 + c$

b $f(x) = \frac{2}{3}x^{\frac{3}{2}} - 4\sqrt{x} + c$ **c** $f(x) = x + \dfrac{5}{x} + c$

EXERCISE 15E.2

1 **a** $f(x) = x^2 - x + 3$ **b** $f(x) = x^3 + x^2 - 7$

c $f(x) = e^x + 2\sqrt{x} - 1 - e$ **d** $f(x) = \frac{1}{2}x^2 - 4\sqrt{x} + \frac{11}{2}$

2 **a** $f(x) = \dfrac{x^3}{3} - 4\sin x + 3$

b $f(x) = 2\sin x + 3\cos x - 2\sqrt{2}$

3 **a** $f(x)=\frac{1}{3}x^3+\frac{1}{2}x^2+x+\frac{1}{3}$

b $f(x)=4x^{\frac{5}{2}}+4x^{\frac{3}{2}}-4x+5$

c $f(x)=-\cos x-x+4$ **d** $f(x)=\frac{1}{3}x^3-\frac{16}{3}x+5$

EXERCISE 15F

1 **a** $\frac{1}{8}(2x+5)^4+c$ **b** $\dfrac{1}{2(3-2x)}+c$

c $\dfrac{-2}{3(2x-1)^3}+c$ **d** $\frac{1}{32}(4x-3)^8+c$

e $\frac{2}{9}(3x-4)^{\frac{3}{2}}+c$ **f** $-4\sqrt{1-5x}+c$

g $-\frac{3}{5}(1-x)^5+c$ **h** $-2\sqrt{3-4x}+c$

2 **a** $-\frac{1}{3}\cos(3x)+c$ **b** $-\frac{1}{2}\sin(-4x)+x+c$

c $6\sin\left(\frac{x}{2}\right)+c$ **d** $-\frac{3}{2}\cos(2x)+e^{-x}+c$

e $-\cos\left(2x+\frac{\pi}{6}\right)+c$ **f** $3\sin\left(\frac{\pi}{4}-x\right)+c$

g $\frac{1}{2}\sin(2x)-\frac{1}{2}\cos(2x)+c$

h $-\frac{2}{3}\cos(3x)+\frac{5}{4}\sin(4x)+c$

i $\frac{1}{16}\sin(8x)+3\cos x+c$

3 $y=\frac{1}{3}(2x-7)^{\frac{3}{2}}+2$ **4** $(-8,\,-19)$

5 **a** $\frac{1}{2}(2x-1)^3+c$ **b** $\frac{1}{5}x^5-\frac{1}{2}x^4+\frac{1}{3}x^3+c$

c $-\frac{1}{12}(1-3x)^4+c$ **d** $x-\frac{2}{3}x^3+\frac{1}{5}x^5+c$

e $-\frac{8}{3}(5-x)^{\frac{3}{2}}+c$ **f** $\frac{1}{7}x^7+\frac{3}{5}x^5+x^3+x+c$

6 **a** $2e^x+\frac{5}{2}e^{2x}+c$ **b** $\frac{3}{5}e^{5x-2}+c$

c $-\frac{1}{3}e^{7-3x}+c$ **d** $\frac{1}{2}e^{2x}+2x-\frac{1}{2}e^{-2x}+c$

e $-\frac{1}{2}e^{-2x}-4e^{-x}+4x+c$ **f** $\frac{1}{2}x^2+5(1-x)^{-1}+c$

7 $y=x-2e^x+\frac{1}{2}e^{2x}+\frac{11}{2}$

8 $p=-\frac{1}{4},\ f(x)=\frac{1}{2}\cos(\frac{1}{2}x)+\frac{1}{2}$

10 $f(x)=-e^{-2x}+4$

11 $f(x)=\frac{2}{3}x^{\frac{3}{2}}-\frac{1}{8}e^{-4x}+\frac{1}{8}e^{-4}-\frac{2}{3}$

EXERCISE 15G

1 **a** $\int_1^4 \sqrt{x}\,dx=\frac{14}{3},\ \int_1^4(-\sqrt{x})\,dx=-\frac{14}{3}$

b $\int_0^1 x^7\,dx=\frac{1}{8},\ \int_0^1(-x^7)\,dx=-\frac{1}{8}$

2 **a** $\frac{1}{3}$ **b** $\frac{7}{3}$ **c** $\frac{8}{3}$ **d** 1

3 **a** -4 **b** 6.25 **c** 2.25 **4** **a** $\frac{1}{3}$ **b** $\frac{2}{3}$ **c** 1

5 **a** $\frac{1}{4}$ **b** $\frac{2}{3}$ **c** $e-1$ (≈ 1.72) **d** $\frac{1}{2}$

e $1\frac{1}{2}$ **f** $6\frac{2}{3}$ **g** $\ln 3$ (≈ 1.10) **h** $\frac{1}{2}$

i ≈ 1.52 **j** 2 **k** $e-1$ (≈ 1.72) **l** $\frac{1}{3}$

6 $m=-1$ or $\frac{4}{3}$ **7** **a** $\frac{\pi}{8}+\frac{1}{4}$ **b** $\frac{\pi}{4}$

8 **a** 6.5 **b** -9 **c** 0 **d** -2.5

9 **a** 2π **b** -4 **c** $\frac{\pi}{2}$ **d** $\frac{5\pi}{2}-4$

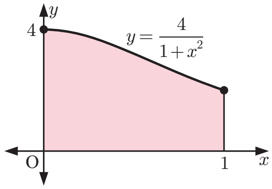
10 **a** $\int_2^7 f(x)\,dx$ **b** $\int_1^9 g(x)\,dx$

11 **a** -5 **b** 4

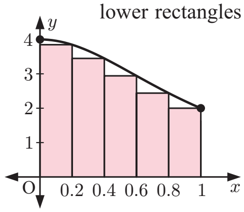
12 **a** 4 **b** 0 **c** -8 **d** $k=-\frac{7}{4}$ **13** 0

REVIEW SET 15A

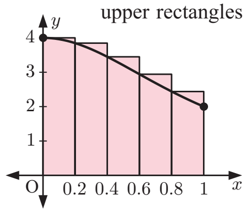
1 **a**



lower rectangles



upper rectangles



b

n	A_L	A_U
5	2.9349	3.3349
50	3.1215	3.1615
100	3.1316	3.1516
500	3.1396	3.1436

c $\displaystyle\int_0^1 \frac{4}{1+x^2}\,dx\approx 3.1416$

2 **a** 2π **b** 4

3 **a** $8\sqrt{x}+c$ **b** $-\frac{1}{4}\cos(4x-5)+c$ **c** $-\frac{1}{3}e^{4-3x}+c$

4 **a** $12\frac{4}{9}$ **b** $\sqrt{2}$

5 $\dfrac{dy}{dx}=\dfrac{x}{\sqrt{x^2-4}},\quad \displaystyle\int\frac{x}{\sqrt{x^2-4}}\,dx=\sqrt{x^2-4}+c$

6 $b=\frac{\pi}{4},\ \frac{3\pi}{4}$

7 **a** $y=\frac{1}{5}x^5-\frac{2}{3}x^3+x+c$ **b** $y=400x+40e^{-\frac{x}{2}}+c$

8 $f(x)=3x^3+5x^2+6x-1$ **9** $a=\ln\sqrt{2}$

10 **a** $f(x)=\frac{1}{4}x^4+\frac{1}{3}x^3-\frac{10}{3}x+3$ **b** $3x+26y=84$

11 **a** $e^{3x}+6e^{2x}+12e^x+8$ **b** $\frac{1}{3}e^3+3e^2+12e-7\frac{1}{3}$

REVIEW SET 15B

1 **a** $A=\frac{17}{4},\ B=\frac{25}{4}$ **b** $\int_0^2(4-x^2)\,dx\approx\frac{21}{4}$

2 **a** $-2e^{-x}+3x+c$ **b** $\frac{2}{3}x\sqrt{x}-2\sqrt{x}+c$

c $9x+3e^{2x-1}+\frac{1}{4}e^{4x-2}+c$

3 $f(x)=\frac{1}{3}x^3-\frac{3}{2}x^2+2x+2\frac{1}{6}$ **4** $\frac{2}{3}(\sqrt{5}-\sqrt{2})$

5 $\dfrac{d}{dx}(3x^2+x)^3=3(3x^2+x)^2(6x+1)$

$\int(3x^2+x)^2(6x+1)\,dx=\frac{1}{3}(3x^2+x)^3+c$

6 **a** 6 **b** 3 **7** $f\left(\frac{\pi}{2}\right)=3-\frac{\pi}{2}$ **8** $e^{-\pi}$

9 $\dfrac{1}{2(n+1)}(2x+3)^{n+1}+c,\quad n\neq -1$

10 $a=\frac{1}{3},\ f'(x)=2\sqrt{x}+\dfrac{1}{3\sqrt{x}}$ is never 0 as $\sqrt{x}\geqslant 0$ for all x

$\therefore\ f'(x)>0$ for all x

11 $a=0$ or ± 3

EXERCISE 16A

1 **a** 30 units^2 **b** $\frac{9}{2}$ units^2 **c** $\frac{27}{2}$ units^2 **d** 2 units^2

2 **a** $\frac{1}{3}$ units^2 **b** 2 units^2 **c** $63\frac{3}{4}$ units^2
d $(e-1)$ units^2 **e** $20\frac{5}{6}$ units^2 **f** 18 units^2
g $\frac{1}{2}$ units^2 **h** $4\frac{1}{2}$ units^2 **i** $(2e-\frac{2}{e})$ units^2

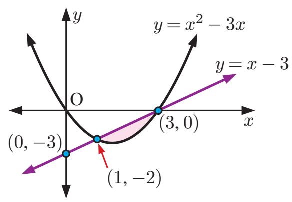
3 $\frac{2}{3}$ units^2

EXERCISE 16B

1 **a** $4\frac{1}{2}$ units^2 **b** $(1+e^{-2})$ units^2 **c** $1\frac{5}{27}$ units^2
d 2 units^2 **e** $2\frac{1}{4}$ units^2 **f** $(\frac{\pi}{2}-1)$ units^2

2 $10\frac{2}{3}$ units^2

3 **a**

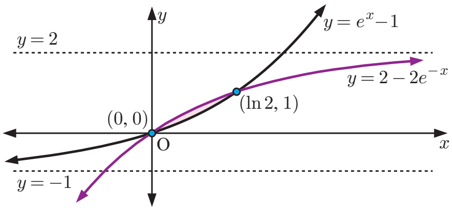


b $(1,\ -2)$ and $(3,\ 0)$

c $1\frac{1}{3}$ units^2

4 $\frac{1}{3}$ units^2

5 **a**

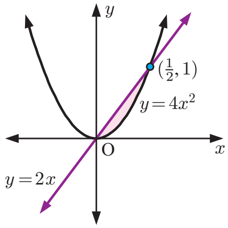


b $(0,\ 0)$ and $(\ln 2,\ 1)$

c enclosed area $= 3\ln 2 - 2\ \ (\approx 0.0794)$ units^2

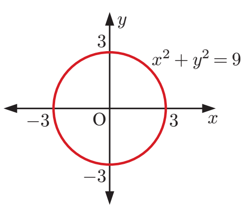
6 $\frac{1}{2}$ units^2

7



enclosed area $= \frac{1}{12}$ units^2

8



a Rearranging $x^2+y^2=9$ gives $y=\pm\sqrt{9-x^2}$. The upper half has $y \geqslant 0$, so $y=\sqrt{9-x^2}$.

b $\frac{9\pi}{4}$

9 **a** $40\frac{1}{2}$ units^2 **b** 8 units^2 **c** 8 units^2

10 **a** C_1 is $y=\sin x$, C_2 is $y=3\sin x$ **b** 4 units^2

11 **a** $\int_3^5 f(x)\,dx = -$ (area between $x=3$ and $x=5$)
b $\int_1^3 f(x)\,dx - \int_3^5 f(x)\,dx + \int_5^7 f(x)\,dx$

12 **a** C_1 is $y=\frac{1}{2}+\frac{1}{2}\cos(2x)$, C_2 is $y=\cos(2x)$
b $A(0,\ 1)$, $B(\frac{\pi}{4},\ 0)$, $C(\frac{\pi}{2},\ 0)$, $D(\frac{3\pi}{4},\ 0)$, $E(\pi,\ 1)$
c Area $= \int_0^\pi \left(\frac{1}{2}+\frac{1}{2}\cos(2x)-\cos(2x)\right) dx$

13 If $h(x) \geqslant 0$ on $a \leqslant x \leqslant b$, the area between $y=h(x)$ and the x-axis is $\int_a^b h(x)\,dx$. If $h(x)<0$ on $a \leqslant x \leqslant b$, the area between $y=h(x)$ and the x-axis is $\int_a^b -h(x)\,dx$.
$\therefore$ the area between $y=h(x)$ and the x-axis on $a \leqslant x \leqslant b$ is $\int_a^b |h(x)|\,dx$.
Letting $h(x)=f(x)-g(x)$, the area between $y=f(x)-g(x)$ and the x-axis $y=0$ on $a \leqslant x \leqslant b$ is $\int_a^b |f(x)-g(x)|\,dx$.
Equivalently, the area between $y=f(x)$ and $y=g(x)$ on $a \leqslant x \leqslant b$ is $\int_a^b |f(x)-g(x)|\,dx$.

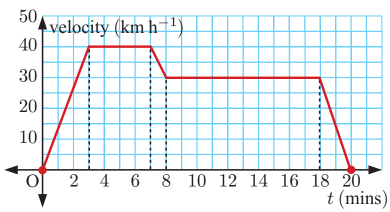
14 $b \approx 1.3104$ **15** $a=\sqrt{3}$

EXERCISE 16C.1

1 110 m

2 **a** **i** travelling forwards
ii travelling backwards (opposite direction)
b 16 km **c** 8 km from starting point (on positive side)

3 **a**



b 9.75 km

EXERCISE 16C.2

1 **a** $s(t)=t-t^2+2$ cm **b** $\frac{1}{2}$ cm **c** 0 cm

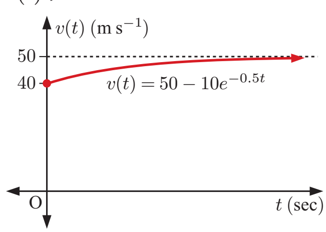
2 **a** $s(t)=\frac{1}{3}t^3-\frac{1}{2}t^2-2t$ cm **b** $5\frac{1}{6}$ cm
c $1\frac{1}{2}$ cm left of its starting point

3 $\frac{\sqrt{3}+2}{4}$ m

4 **a** $s(t)=32t+2t^2+16$ m
b no change of direction
so displacement $= s(t_1)-s(0) = \int_0^{t_1} (32+4t)\,dt$
c acceleration $= 4\ \text{m s}^{-2}$

5 **a** 41 units **b** 34 units **6** **b** 2 m

7 **a** $40\ \text{m s}^{-1}$ **b** $47.8\ \text{m s}^{-1}$ **c** 1.39 seconds
d as $t \to \infty$, $v(t) \to 50$ from below
e $a(t)=5e^{-0.5t}$ and as $e^x>0$ for all x, $a(t)>0$ for all t.
f



g ≈ 134.5 m

8 **a** $v(t) = -\dfrac{1}{(t+1)^2}+1\ \text{m s}^{-1}$
b $s(t) = \dfrac{1}{t+1}+t-1$ m

c The particle is $\frac{4}{3}$ m to the right of the origin, moving to the right at $\frac{8}{9}$ m s^{-1}, and accelerating at $\frac{2}{27}$ m s^{-2}.

9 **a** $v(t) = \dfrac{t^2}{20} - 3t + 45$ m s^{-1}

b $\int_0^{60} v(t)\,dt = 900$. The train travels a total of 900 m in the first 60 seconds.

10 **a** Show that $v(t) = 100 - 80e^{-\frac{1}{20}t}$ m s^{-1} and as $t \to \infty$, $v(t) \to 100$ m s^{-1}.

b 370.4 m

REVIEW SET 16A

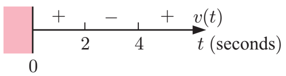
1 $A = \int_a^b [f(x) - g(x)]\,dx + \int_b^c [g(x) - f(x)]\,dx + \int_c^d [f(x) - g(x)]\,dx$

2 **a** $2 + \pi$ **b** -2 **c** π

3 No, total area shaded $= \int_{-1}^{1} f(x)\,dx - \int_1^3 f(x)\,dx$.

4 $k = \sqrt[3]{16}$ **5** 4.5 units^2

6 **a**



b The particle moves in the positive direction initially, then at $t = 2$, $6\frac{2}{3}$ m from its starting point, it changes direction. It changes direction again at $t = 4$, $5\frac{1}{3}$ m from its starting point, and at $t = 5$, it is $6\frac{2}{3}$ m from its starting point again.

c $6\frac{2}{3}$ m **d** $9\frac{1}{3}$ m

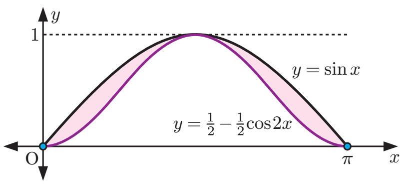
7 $(3 - \ln 4)$ units^2 **8** 2.35 m

REVIEW SET 16B

1 **a** $v(t) = 3t^2 - 30t + 27$ cm s^{-1}

b -162 cm (162 cm to the left of the origin)

2 **a**



c $(1 - \frac{\pi}{4})$ units^2

3 $a = \ln 3$, $b = \ln 5$

4 **a** $a(t) = 2 - 6t$ m s^{-2} **b** $s(t) = t^2 - t^3 + c$ m

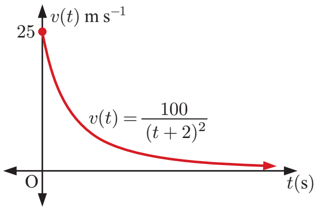
c -4 m (4 m to the left)

5 $k = \frac{4}{3}$ **6** $m = \frac{\pi}{3}$ **7** $(\frac{4}{\pi} - 1)$ units^2

8 **a** $v(0) = 25$ m s^{-1}, $v(3) = 4$ m s^{-1}

b as $t \to \infty$, $v(t) \to 0$ from above

c



d 3 seconds **e** $a(t) = \dfrac{-200}{(t+2)^3}$ m s^{-2}, $t \geqslant 0$

f $k = \frac{1}{5}$

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