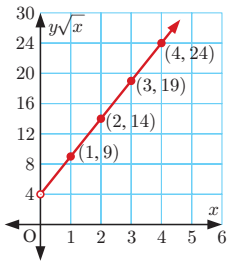
2 **a**

x	1	2	3	4
$y\sqrt{x}$	9	14	19	24

b



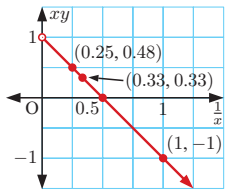
c $y = 5\sqrt{x} + \dfrac{4}{\sqrt{x}}, \quad x > 0$

d $y = 21$

3 **a**

$\frac{1}{x}$	1	0.5	0.33	0.25
xy	-1	0	0.33	0.48

b



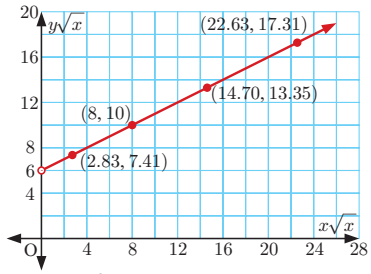
c $a = 1, \quad b = -2$

d $y = 0.08$

4 **a**

$x\sqrt{x}$	2.83	8	14.70	22.63
$y\sqrt{x}$	7.41	10	13.35	17.31

b

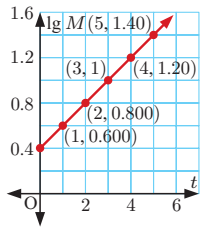


c $y = \frac{1}{2}x + \dfrac{6}{\sqrt{x}}, \quad x > 0$ **d** $y = 6.5$

5 **a**

t	1	2	3	4	5
$\lg M$	0.600	0.800	1	1.20	1.40

b



c $M \approx 2.51 \times 1.58^t$

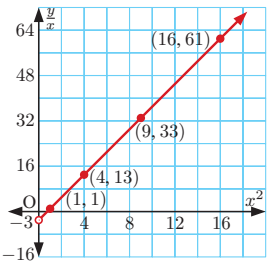
d ≈ 2.51 g

6 **a** x^2

c $a = 4, \quad b = -3$

d $y = 485$

b



7 Plot xy against $\sqrt{x}$. $\quad y = \dfrac{8}{x} - \dfrac{4}{\sqrt{x}} \quad \{a = 8, \ b = -4\}$

8 **a** $a \approx 4.90, \ b \approx 2.00$ **b** ≈ 44.1 m **c** ≈ 4.04 seconds

REVIEW SET 7A

1 **a** $\sqrt{40}$ units **b** $(2, 5)$ **c** $x + 3y = 17$

2 $y = -2x + 6$

3 The gradient of a vertical line is undefined.

4 **a** $x + 2y = 7$ **b** $(7, 0)$ **5** $(3, -1)$

6 **a** $(-1, 4)$ **b** $32\frac{1}{2}$ units^2

7 $(-\frac{7}{5}, \frac{26}{5})$ and $(2, -5)$ **8** $y = x - 5$

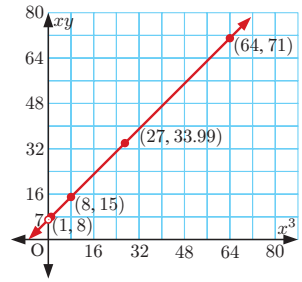
10 **a** $y = \dfrac{3}{\sqrt{x}} - \dfrac{2}{x}, \quad x > 0$ **b** $y = 1$

11 **a** $\lg y = \frac{1}{2}\lg x + 1$ **b** $y = 10\sqrt{x}$

12 **a**

x^3	1	8	27	64
xy	8	15	33.99	71

b



c $y = x^2 + \dfrac{7}{x}$

d $y = 50$

REVIEW SET 7B

1 $y = -3x + 7$ **2** $\sqrt{80}$ units

3 **a** $y = 5x - \dfrac{2}{x}$ **b** $39\frac{3}{4}$

4 **a** $r = \frac{5}{7}a + 2$ **b** $K = \frac{3}{5}s + 3$

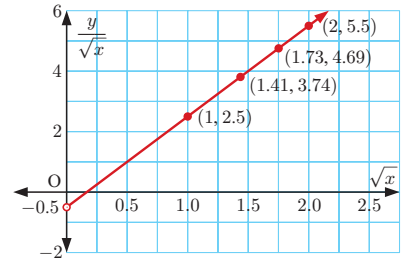
5 $(3, 2)$ **6** $5x - 8y = 31$

7 **a**

$\sqrt{x}$	1	1.41	1.73	2
$\frac{y}{\sqrt{x}}$	2.5	3.74	4.69	5.5

c $y = 3x - \dfrac{\sqrt{x}}{2}$

b



8 **a** $bx + ay = ab$

b **Hint:** $\cos\theta = \dfrac{a}{\sqrt{a^2 + b^2}}, \quad \sin\theta = \dfrac{b}{\sqrt{a^2 + b^2}}$

9 **a** **i** $(2, 12)$ **ii** $(11, 0)$ **b** 75 units^2

10 $(\frac{7}{8}, \frac{1}{2})$

11 **a** Plot $\lg y$ against x.

$y = 100 \times (10^{-\frac{1}{3}})^x \quad \{a = 100, \ b = 10^{-\frac{1}{3}}\}$

b $y \approx 46.4$

EXERCISE 8A

1 **a** $\frac{\pi}{2}^c$ **b** $\frac{\pi}{3}^c$ **c** $\frac{\pi}{6}^c$ **d** $\frac{\pi}{10}^c$ **e** $\frac{\pi}{20}^c$
f $\frac{3\pi}{4}^c$ **g** $\frac{5\pi}{4}^c$ **h** $\frac{3\pi}{2}^c$ **i** $2\pi^c$ **j** $4\pi^c$
k $\frac{7\pi}{4}^c$ **l** $3\pi^c$ **m** $\frac{\pi}{5}^c$ **n** $\frac{4\pi}{9}^c$ **o** $\frac{23\pi}{18}^c$

2 **a** 0.641^c **b** 2.39^c **c** 5.55^c **d** 3.83^c **e** 6.92^c

3 **a** $36°$ **b** $108°$ **c** $135°$ **d** $10°$ **e** $20°$
f $140°$ **g** $18°$ **h** $27°$ **i** $210°$ **j** $22.5°$

4 **a** $114.59°$ **b** $87.66°$ **c** $49.68°$ **d** $182.14°$
e $301.78°$

5 **a**

Degrees	0	45	90	135	180	225	270	315	360
Radians	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π

b

Deg.	0	30	60	90	120	150	180	210	240	270	300	330	360
Rad.	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	2π

EXERCISE 8B

1 **a** 49.5 cm, $223\,\text{cm}^2$ **b** 23.0 cm, $56.8\,\text{cm}^2$

2 **a** 3.14 m **b** $9.30\,\text{m}^2$ **3** **a** 5.91 cm **b** 18.9 cm

4 **a** 0.686^c **b** 0.6^c

5 **a** $\theta = 0.75^c$, area $= 24\,\text{cm}^2$
b $\theta = 1.68^c$, area $= 21\,\text{cm}^2$
c $\theta \approx 2.32^c$, area $= 126.8\,\text{cm}^2$

6 10 cm, $25\,\text{cm}^2$

8 **a** 11.7 cm **b** $r \approx 11.7$ **c** 37.7 cm **d** 3.23^c

9 **a** $\alpha \approx 18.43$ **b** $\theta \approx 143.1$ **c** $387\,\text{m}^2$

10 25.9 cm **11** **b** 2 h 49 min **12** $227\,\text{m}^2$

EXERCISE 8C

1 **a** **i** A$(\cos 26°, \sin 26°)$, B$(\cos 146°, \sin 146°)$, C$(\cos 199°, \sin 199°)$
ii A$(0.899, 0.438)$, B$(-0.829, 0.559)$, C$(-0.946, -0.326)$
b **i** A$(\cos 123°, \sin 123°)$, B$(\cos 251°, \sin 251°)$, C$(\cos(-35°), \sin(-35°))$
ii A$(-0.545, 0.839)$, B$(-0.326, -0.946)$, C$(0.819, -0.574)$

2

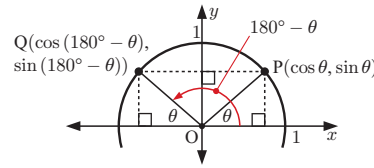
θ (degrees)	0°	90°	180°	270°	360°	450°
θ (radians)	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π	$\frac{5\pi}{2}$
sine	0	1	0	-1	0	1
cosine	1	0	-1	0	1	0
tangent	0	undef	0	undef	0	undef

3 **a** **i** $\frac{1}{\sqrt{2}} \approx 0.707$ **ii** $\frac{\sqrt{3}}{2} \approx 0.866$

b

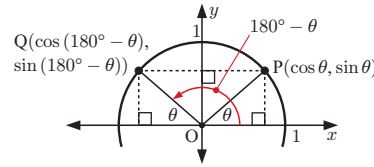
θ (degrees)	30°	45°	60°	135°	150°	240°	315°
θ (radians)	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\frac{4\pi}{3}$	$\frac{7\pi}{4}$
sine	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{2}}$
cosine	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\frac{1}{\sqrt{2}}$
tangent	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	$\sqrt{3}$	-1

4 **a** **i** 0.985 **ii** 0.985 **iii** 0.866 **iv** 0.866
v 0.5 **vi** 0.5 **vii** 0.707 **viii** 0.707
b $\sin(180° - \theta) = \sin\theta$
c $\sin\theta$ and $\sin(180° - \theta)$ have the same value, as P and Q have the same y-coordinate.



d **i** $135°$ **ii** $129°$ **iii** $\frac{2\pi}{3}$ **iv** $\frac{5\pi}{6}$

5 **a** **i** 0.342 **ii** -0.342 **iii** 0.5 **iv** -0.5
v 0.906 **vi** -0.906 **vii** 0.174 **viii** -0.174
b $\cos(180° - \theta) = -\cos\theta$
c $\cos(180° - \theta) = -\cos\theta$, as the x-coordinates of P and Q are negatives of each other.



d **i** $140°$ **ii** $161°$ **iii** $\frac{4\pi}{5}$ **iv** $\frac{3\pi}{5}$

6 **a** ≈ 0.6820 **b** ≈ 0.8572 **c** ≈ -0.7986
d ≈ 0.9135 **e** ≈ 0.9063 **f** ≈ -0.6691

7 **a**

Quadrant	*Degree measure*	*Radian measure*	$\cos\theta$	$\sin\theta$	$\tan\theta$
1	$0° < \theta < 90°$	$0 < \theta < \frac{\pi}{2}$	+ve	+ve	+ve
2	$90° < \theta < 180°$	$\frac{\pi}{2} < \theta < \pi$	−ve	+ve	−ve
3	$180° < \theta < 270°$	$\pi < \theta < \frac{3\pi}{2}$	−ve	−ve	+ve
4	$270° < \theta < 360°$	$\frac{3\pi}{2} < \theta < 2\pi$	+ve	−ve	−ve

b **i** 1 and 4 **ii** 2 and 3 **iii** 3 **iv** 2

8 **a** $\widehat{AOQ} = 180° - \theta$ or $\pi - \theta$ radians
b [OQ] is a reflection of [OP] in the y-axis and so Q has coordinates $(-\cos\theta, \sin\theta)$.
c $\cos(180° - \theta) = -\cos\theta$, $\sin(180° - \theta) = \sin\theta$

9 **a**

θ^c	$\sin\theta$	$\sin(-\theta)$	$\cos\theta$	$\cos(-\theta)$
0.75	0.682	-0.682	0.732	0.732
1.772	0.980	-0.980	-0.200	-0.200
3.414	-0.269	0.269	-0.963	-0.963
6.25	-0.0332	0.0332	0.999	0.999
-1.17	-0.921	0.921	0.390	0.390

b $\sin(-\theta) = -\sin\theta$, $\cos(-\theta) = \cos\theta$

EXERCISE 8D.1

1 **a** $\cos\theta = \pm\frac{\sqrt{3}}{2}$ **b** $\cos\theta = \pm\frac{2\sqrt{2}}{3}$ **c** $\cos\theta = \pm 1$
d $\cos\theta = 0$

2 **a** $\sin\theta = \pm\frac{3}{5}$ **b** $\sin\theta = \pm\frac{\sqrt{7}}{4}$ **c** $\sin\theta = 0$
d $\sin\theta = \pm 1$

3 **a** $\sin\theta = \frac{\sqrt{5}}{3}$ **b** $\cos\theta = -\frac{\sqrt{21}}{5}$ **c** $\cos\theta = \frac{4}{5}$
d $\sin\theta = -\frac{12}{13}$

4 **a** $\tan\theta = -\frac{1}{2\sqrt{2}}$ **b** $\tan\theta = -2\sqrt{6}$ **c** $\tan\theta = \frac{1}{\sqrt{2}}$
d $\tan\theta = -\frac{\sqrt{7}}{3}$

5 **a** $\sin x = \frac{2}{\sqrt{13}}$, $\cos x = \frac{3}{\sqrt{13}}$
b $\sin x = \frac{4}{5}$, $\cos x = -\frac{3}{5}$
c $\sin x = -\sqrt{\frac{5}{14}}$, $\cos x = -\frac{3}{\sqrt{14}}$
d $\sin x = -\frac{12}{13}$, $\cos x = \frac{5}{13}$

6 $\sin\theta = \dfrac{-k}{\sqrt{k^2+1}}$, $\cos\theta = \dfrac{-1}{\sqrt{k^2+1}}$

EXERCISE 8D.2

1 **a** $\theta \approx 1.33$ or 4.47 **b** $\theta \approx 0.592$ or 5.69
c $\theta \approx 0.644$ or 2.50 **d** $\theta = \frac{\pi}{2}$ or $\frac{3\pi}{2}$
e $\theta \approx 0.876$ or 4.02 **f** $\theta \approx 0.674$ or 5.61
g $\theta \approx 0.0910$ or 3.05 **h** $\theta \approx 1.52$ or 4.66
i $\theta \approx 1.35$ or 1.79

2 **a** $\theta \approx 1.82$ or 4.46 **b** $\theta = 0$, π, or 2π
c $\theta \approx 1.88$ or 5.02 **d** $\theta \approx 3.58$ or 5.85
e $\theta \approx 1.72$ or 4.86 **f** $\theta \approx 1.69$ or 4.59
g $\theta \approx 1.99$ or 5.13 **h** $\theta \approx 2.19$ or 4.10
i $\theta \approx 3.83$ or 5.60

EXERCISE 8E

1

	a	b	c	d	e
$\sin\theta$	$\frac{1}{\sqrt{2}}$	1	$-\frac{1}{\sqrt{2}}$	0	$-\frac{1}{\sqrt{2}}$
$\cos\theta$	$\frac{1}{\sqrt{2}}$	0	$\frac{1}{\sqrt{2}}$	-1	$-\frac{1}{\sqrt{2}}$
$\tan\theta$	1	undef	-1	0	1

2

	a	b	c	d	e
$\sin\beta$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$
$\cos\beta$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$\tan\beta$	$\frac{1}{\sqrt{3}}$	$-\sqrt{3}$	$\frac{1}{\sqrt{3}}$	$-\sqrt{3}$	$-\frac{1}{\sqrt{3}}$

3 **a** $\cos 120° = -\frac{1}{2}$, $\sin 120° = \frac{\sqrt{3}}{2}$, $\tan 120° = -\sqrt{3}$
b $\cos(-45°) = \frac{1}{\sqrt{2}}$, $\sin(-45°) = -\frac{1}{\sqrt{2}}$, $\tan(-45°) = -1$

4 **a** $\cos 270° = 0$, $\sin 270° = -1$
b $\tan 270°$ is undefined

5 **a** $\frac{3}{4}$ **b** $\frac{1}{4}$ **c** 3 **d** $\frac{1}{4}$ **e** $-\frac{1}{4}$ **f** 1
g $\sqrt{2}$ **h** $\frac{1}{2}$ **i** $\frac{1}{2}$ **j** 2 **k** -1 **l** $-\sqrt{3}$

6 **a** 30°, 150° **b** 60°, 120° **c** 45°, 315°
d 120°, 240° **e** 135°, 225° **f** 240°, 300°

7 **a** $\frac{\pi}{4}, \frac{5\pi}{4}$ **b** $\frac{3\pi}{4}, \frac{7\pi}{4}$ **c** $\frac{\pi}{3}, \frac{4\pi}{3}$
d 0, π, 2π **e** $\frac{\pi}{6}, \frac{7\pi}{6}$ **f** $\frac{2\pi}{3}, \frac{5\pi}{3}$

8 **a** $\frac{\pi}{6}, \frac{11\pi}{6}, \frac{13\pi}{6}, \frac{23\pi}{6}$ **b** $\frac{7\pi}{6}, \frac{11\pi}{6}, \frac{19\pi}{6}, \frac{23\pi}{6}$ **c** $\frac{3\pi}{2}, \frac{7\pi}{2}$

9 **a** $\theta = \frac{\pi}{3}, \frac{5\pi}{3}$ **b** $\theta = \frac{\pi}{3}, \frac{2\pi}{3}$ **c** $\theta = \pi$
d $\theta = \frac{\pi}{2}$ **e** $\theta = \frac{3\pi}{4}, \frac{5\pi}{4}$ **f** $\theta = \frac{\pi}{2}, \frac{3\pi}{2}$
g $\theta = 0, \pi, 2\pi$ **h** $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$
i $\theta = \frac{5\pi}{6}, \frac{11\pi}{6}$ **j** $\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

10 **a** $\theta = k\pi,\ k \in \mathbb{Z}$ **b** $\theta = \frac{\pi}{2} + k\pi,\ k \in \mathbb{Z}$

EXERCISE 8F

1 **a** $\frac{2}{\sqrt{3}}$ **b** $-\frac{1}{\sqrt{3}}$ **c** $-\frac{2}{\sqrt{3}}$ **d** undefined
e $-\frac{2}{\sqrt{3}}$ **f** $\sqrt{2}$

2 **a** $\operatorname{cosec} x = \frac{5}{3}$, $\sec x = \frac{5}{4}$, $\cot x = \frac{4}{3}$
b $\operatorname{cosec} x = -\frac{3}{\sqrt{5}}$, $\sec x = \frac{3}{2}$, $\cot x = -\frac{2}{\sqrt{5}}$

3 **a** $\sin\theta = -\frac{\sqrt{7}}{4}$, $\tan\theta = -\frac{\sqrt{7}}{3}$, $\operatorname{cosec}\theta = -\frac{4}{\sqrt{7}}$, $\sec\theta = \frac{4}{3}$, $\cot\theta = -\frac{3}{\sqrt{7}}$
b $\cos x = -\frac{\sqrt{5}}{3}$, $\tan x = \frac{2}{\sqrt{5}}$, $\operatorname{cosec} x = -\frac{3}{2}$, $\sec x = -\frac{3}{\sqrt{5}}$, $\cot x = \frac{\sqrt{5}}{2}$
c $\sin x = \frac{\sqrt{21}}{5}$, $\cos x = \frac{2}{5}$, $\tan x = \frac{\sqrt{21}}{2}$, $\operatorname{cosec} x = \frac{5}{\sqrt{21}}$, $\cot x = \frac{2}{\sqrt{21}}$
d $\sin\theta = \frac{1}{2}$, $\cos\theta = -\frac{\sqrt{3}}{2}$, $\tan\theta = -\frac{1}{\sqrt{3}}$, $\sec\theta = -\frac{2}{\sqrt{3}}$, $\cot\theta = -\sqrt{3}$
e $\sin\beta = -\frac{1}{\sqrt{5}}$, $\cos\beta = -\frac{2}{\sqrt{5}}$, $\operatorname{cosec}\beta = -\sqrt{5}$, $\sec\beta = -\frac{\sqrt{5}}{2}$, $\cot\beta = 2$
f $\sin\theta = -\frac{3}{5}$, $\cos\theta = -\frac{4}{5}$, $\tan\theta = \frac{3}{4}$, $\operatorname{cosec}\theta = -\frac{5}{3}$, $\sec\theta = -\frac{5}{4}$

4 **a** $\theta = k\pi,\ k \in \mathbb{Z}$ **b** $\theta = \frac{\pi}{2} + k\pi,\ k \in \mathbb{Z}$
c $\theta = \frac{\pi}{2} + k\pi,\ k \in \mathbb{Z}$ **d** $\theta = k\pi,\ k \in \mathbb{Z}$

REVIEW SET 8A

1 **a** $\frac{2\pi}{3}$ **b** $\frac{5\pi}{4}$ **c** $\frac{5\pi}{6}$ **d** 3π

2 **a** $\frac{\pi}{3}$ **b** 15° **c** 84°

3 **a** 0.358 **b** -0.035 **c** 0.259 **d** -0.731

4 111 cm^2 **5** $\theta = \frac{3\pi}{4}, \frac{7\pi}{4}$

6

	a	b	c	d
$\sin\theta$	0	$\frac{\sqrt{3}}{2}$	0	$\frac{\sqrt{3}}{2}$
$\cos\theta$	1	$-\frac{1}{2}$	-1	$-\frac{1}{2}$
$\tan\theta$	0	$-\sqrt{3}$	0	$-\sqrt{3}$

7 $\sin\theta = \pm\frac{\sqrt{7}}{4}$ **8** **a** $\frac{\sqrt{3}}{2}$ **b** 0 **c** $\frac{1}{2}$

9 **a** $\frac{2}{\sqrt{13}}$ **b** $-\frac{3}{\sqrt{13}}$

10 perimeter = 12 units, area = 8 units^2 **11** $\frac{\sqrt{6}}{\sqrt{11}}$

12 **a** 150°, 210° **b** 45°, 315° **c** 120°, 300°

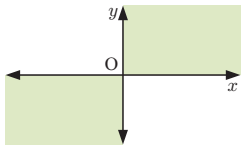
13 **a** $\theta = \pi$ **b** $\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

14 $\cos x = -\frac{\sqrt{15}}{4}$, $\tan x = \frac{1}{\sqrt{15}}$, $\sec x = -\frac{4}{\sqrt{15}}$, $\operatorname{cosec} x = -4$, $\cot x = \sqrt{15}$

REVIEW SET 8B

1 **a** $72°$ **b** $225°$ **c** $140°$ **d** $330°$

2



3 **a** $\cos(\frac{3\pi}{2}) = 0, \quad \sin(\frac{3\pi}{2}) = -1$

b $\cos(-\frac{\pi}{2}) = 0, \quad \sin(-\frac{\pi}{2}) = -1$

4 **a** $\sin(\pi - p) = m$ **b** $\sin(p + 2\pi) = m$

c $\cos p = \sqrt{1 - m^2}$ **d** $\tan p = \dfrac{m}{\sqrt{1 - m^2}}$

5 **a** **i** $60°$ **ii** $\frac{\pi}{3}$ **b** $\frac{\pi}{3}$ units **c** $\frac{\pi}{6}$ units^2

7 $\sin\theta = \frac{\sqrt{21}}{5}, \quad \tan\theta = -\frac{\sqrt{21}}{2}, \quad \sec\theta = -\frac{5}{2},$

$\operatorname{cosec}\theta = \frac{5}{\sqrt{21}}, \quad \cot\theta = -\frac{2}{\sqrt{21}}$

8 **a** $2\frac{1}{2}$ **b** $1\frac{1}{2}$ **c** $-\frac{1}{2}$

9 **a** $\theta \approx 0.841$ or 5.44 **b** $\theta \approx 3.39$ or 6.03

c $\theta \approx 1.25$ or 4.39

10 perimeter ≈ 34.1 cm, area ≈ 66.5 cm^2

11 $r \approx 8.79$ cm, area ≈ 81.0 cm^2 **12** **a** 0 **b** $\sin\theta$

13 $\sin\alpha = \frac{\sqrt{91}}{10}, \quad \cos\alpha = -\frac{3}{10}, \quad \tan\alpha = -\frac{\sqrt{91}}{3},$

$\operatorname{cosec}\alpha = \frac{10}{\sqrt{91}}, \quad \cot\alpha = -\frac{3}{\sqrt{91}}$

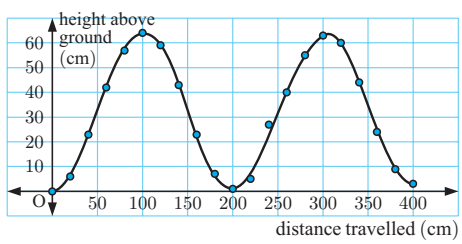
EXERCISE 9A

1 **a** periodic **b** periodic **c** periodic

d not periodic **e** periodic **f** periodic

g not periodic **h** not periodic

2 **a**



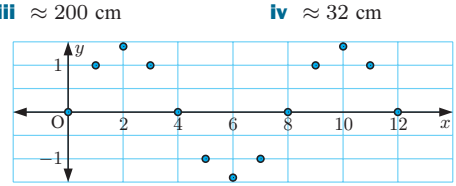
b A curve can be fitted to the data.

c The data is periodic.

i $y = 32$ (approx.) **ii** ≈ 64 cm

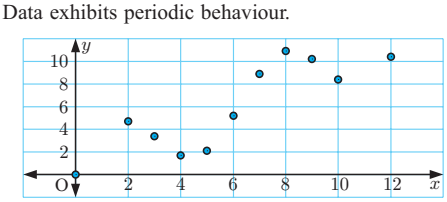
iii ≈ 200 cm **iv** ≈ 32 cm

3 **a**



Data exhibits periodic behaviour.

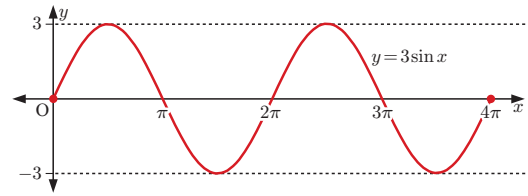
b



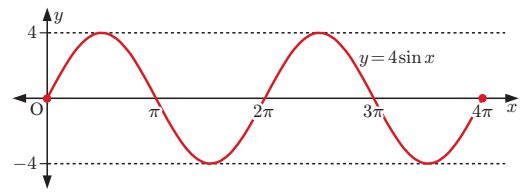
Not enough information to say data is periodic.

EXERCISE 9B

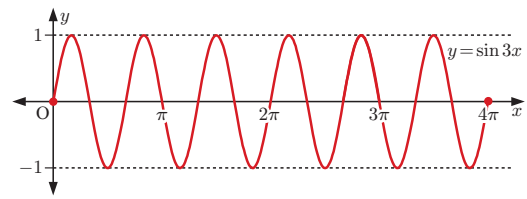
1 **a**



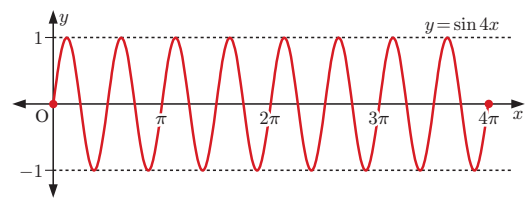
b



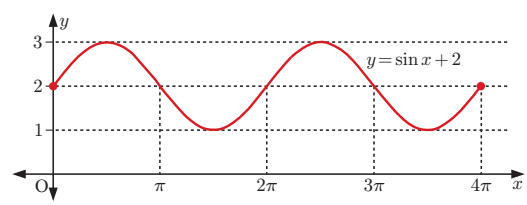
c



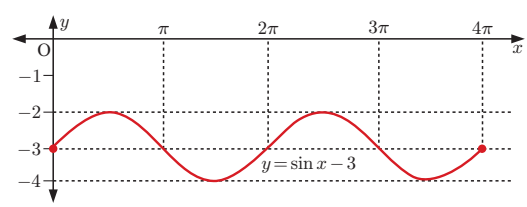
d



e



f

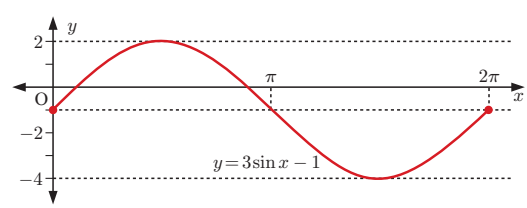


2 **a** $a = 2$ **b** $a = 5$ **c** $a = 11$

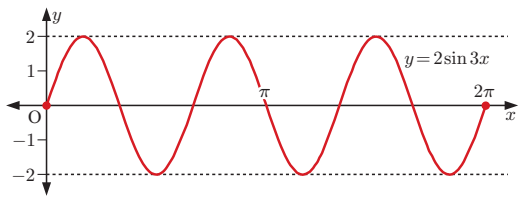
3 **a** $b = 3$ **b** $b = 5$ **c** $b = 6$ **d** $b = 4$

4 **a** $c = 3$ **b** $c = -1$ **c** $c = 5$

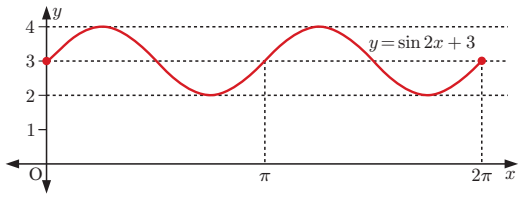
5 **a**



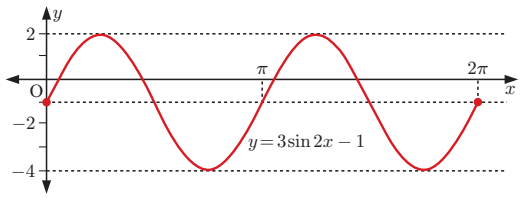
b



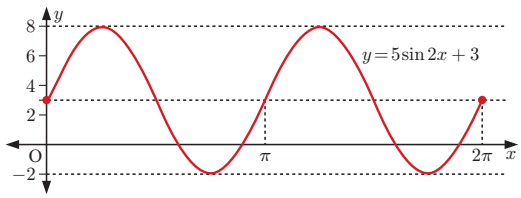
c



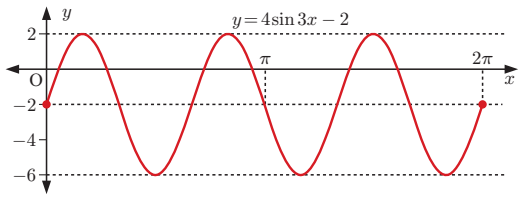
d



e



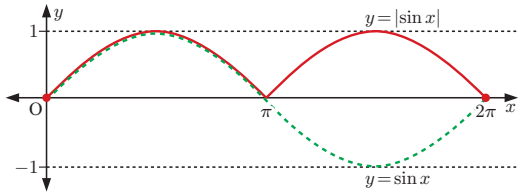
f



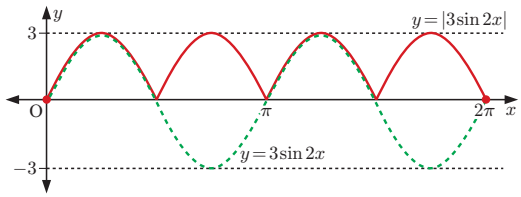
6 **a** $a = 3,\ b = 1,\ c = 0$ **b** $a = 2,\ b = 5,\ c = 6$
c $a = 5,\ b = 3,\ c = -2$

7 $m = 2,\ n = -3$

8 **a**

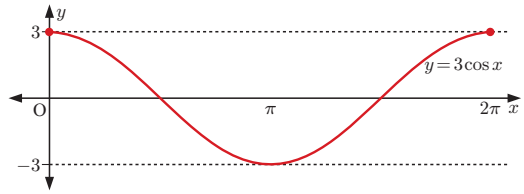


b

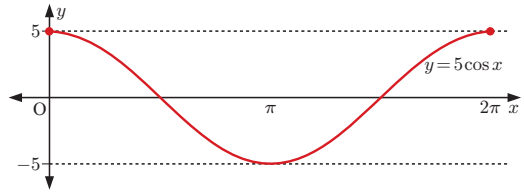


EXERCISE 9C

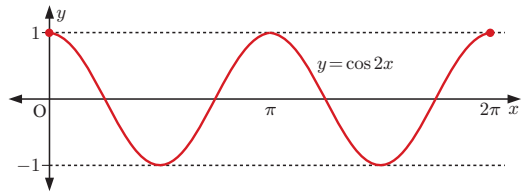
1 **a**



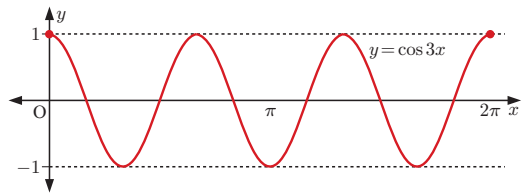
b



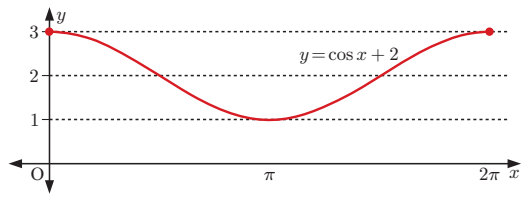
c



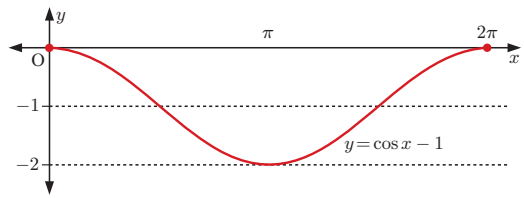
d



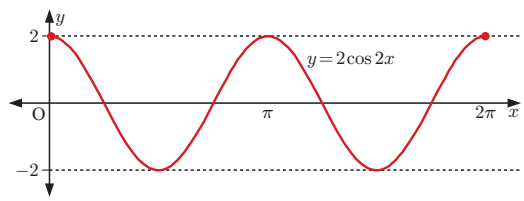
e



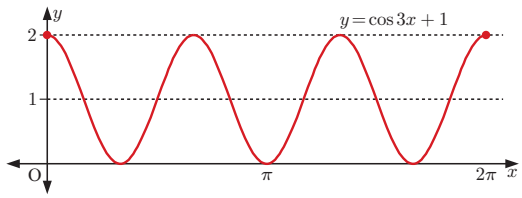
f



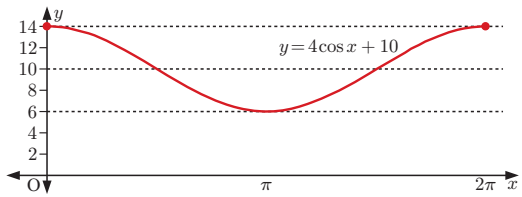
g



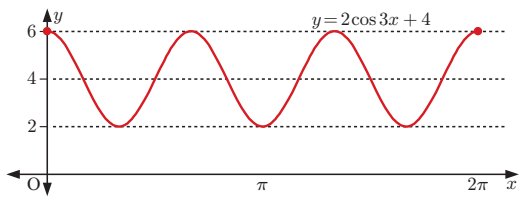
h



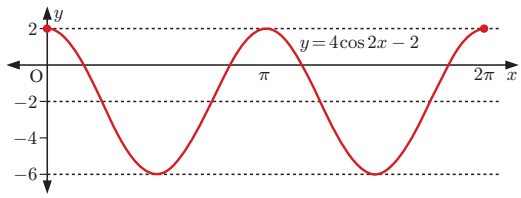
i



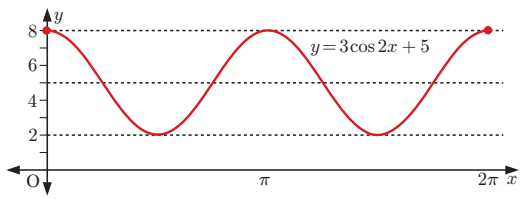
j



k



l

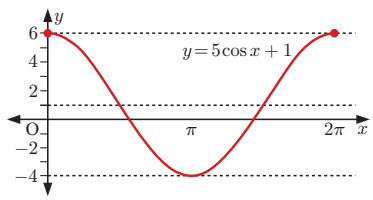


2 **a** $a = 4,\ b = 3,\ c = -1$ **b** $a = 3,\ b = 5,\ c = 3$

3 **a** $y = 2\cos 2x$ **b** $y = \cos\left(\frac{x}{2}\right) + 2$

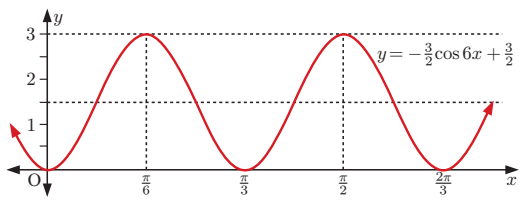
4 **a** $a = 5,\ b = 1,\ c = 1$

b



5 **a** $a = \frac{3}{2},\ b = 6,\ c = -\frac{3}{2}$

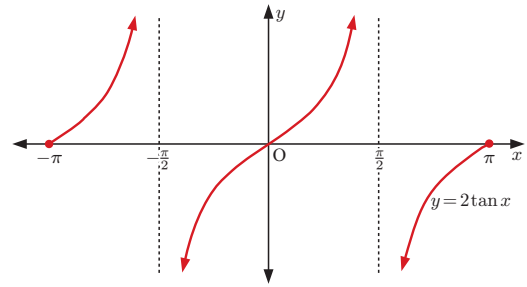
b



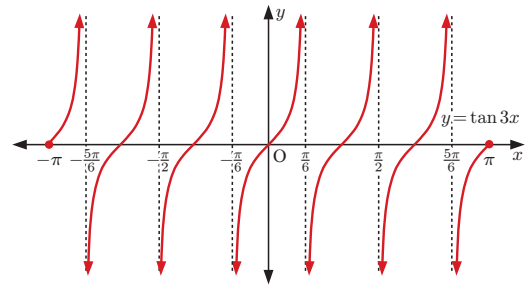
c $y = -\frac{3}{2}\cos 6x + \frac{3}{2}$

EXERCISE 9D

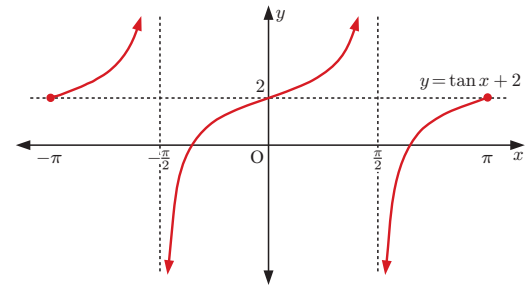
1 **a**



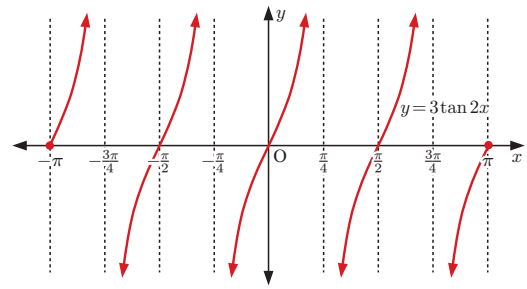
b



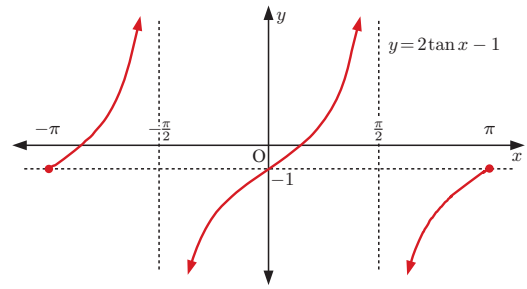
c



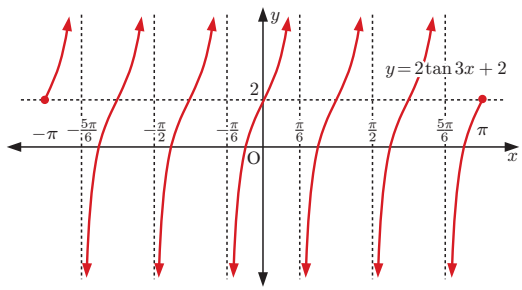
d



e



f



2 **a** $b = \frac{3}{2},\ c = 2$ **b** $b = 2,\ c = -3$

3 $p = \frac{1}{2},\ q = 1$

EXERCISE 9E.1

1 **a** $x \approx 0.3, 2.8, 6.6, 9.1, 12.9$ **b** $x \approx 5.9, 9.8, 12.2$

2 **a** $x \approx 0.9, 5.4, 7.2$ **b** $x \approx 4.4, 8.2, 10.7$

3 **a** $x \approx 0.4, 1.2, 3.5, 4.3, 6.7, 7.5, 9.8, 10.6, 13.0, 13.7$
b $x \approx 1.7, 3.0, 4.9, 6.1, 8.0, 9.3, 11.1, 12.4, 14.3, 15.6$

4 **a** **i** ≈ 1.6 **ii** ≈ -1.1
b **i** $x \approx 1.1, 4.2, 7.4$ **ii** $x \approx 2.2, 5.3$

5 **a** $x \approx 0.446, 2.70, 6.73, 8.98$
b $x \approx 2.52, 3.76, 8.80, 10.0$
c $x \approx 0.588, 3.73, 6.87, 10.0$

6 **a** $x \approx -0.644, 0.644$
b $x \approx -4.56, -1.42, 1.72, 4.87$
c $x \approx -2.76, -0.384, 3.53$

7 **a** $x \approx 1.57$ **b** $m = -2$ or $m = 1$
c $-1 \leqslant \sin x \leqslant 1$, so $m = \sin x = -2$ is not a valid solution.

EXERCISE 9E.2

1 **a** $x = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}$ **b** $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{9\pi}{4}, \frac{11\pi}{4}$
c $x = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}$

2 **a** $x = -\frac{5\pi}{3}, -\frac{4\pi}{3}, \frac{\pi}{3}, \frac{2\pi}{3}$ **b** $x = -\frac{5\pi}{4}, -\frac{3\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}$
c $x = -\frac{5\pi}{4}, -\frac{\pi}{4}, \frac{3\pi}{4}, \frac{7\pi}{4}$

3 **a** $x = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}$ **b** $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$

4 **a** $x = \frac{5\pi}{4}, \frac{7\pi}{4}$ **b** $x = \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{13\pi}{12}, \frac{5\pi}{4}, \frac{7\pi}{4}, \frac{23\pi}{12}$

5 **a** $x = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}, \frac{14\pi}{3}$
b $x = -330°, -210°, 30°, 150°$
c $x = \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{17\pi}{6}$ **d** $x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$
e $x = -\frac{8\pi}{9}, -\frac{4\pi}{9}, -\frac{2\pi}{9}, \frac{2\pi}{9}, \frac{4\pi}{9}, \frac{8\pi}{9}$

6 $x = \frac{\pi}{3}, \frac{4\pi}{3}$
a $x = \frac{\pi}{12}, \frac{\pi}{3}, \frac{7\pi}{12}, \frac{5\pi}{6}, \frac{13\pi}{12}, \frac{4\pi}{3}, \frac{19\pi}{12}, \frac{11\pi}{6}$
b $x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

7 $x = \frac{\pi}{18}, \frac{7\pi}{18}, \frac{13\pi}{18}$

8 **a** $x = -\frac{2\pi}{3}, \frac{2\pi}{3}$ **b** $x = -\frac{5\pi}{6}, -\frac{2\pi}{3}, \frac{\pi}{6}, \frac{\pi}{3}$
c $x = -\frac{\pi}{2}, \frac{\pi}{2}$

9 **a** $x = \frac{\pi}{4}, \frac{5\pi}{4}$ **b** $x = \frac{3\pi}{4}, \frac{7\pi}{4}$
c $x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{3\pi}{4}, \frac{13\pi}{12}, \frac{17\pi}{12}, \frac{7\pi}{4}$
d $x = \frac{\pi}{6}, \frac{2\pi}{3}, \frac{7\pi}{6}, \frac{5\pi}{3}$

10 $x = \frac{\pi}{2}$

EXERCISE 9F.1

1 **a** $2\sin\theta$ **b** $3\cos\theta$ **c** $2\sin\theta$ **d** $\sin\theta$
e $-2\tan\theta$ **f** $-3\cos^2\theta$

2 **a** 3 **b** -2 **c** -1 **d** $3\cos^2\theta$
e $4\sin^2\theta$ **f** $\cos\theta$ **g** $-\sin^2\theta$ **h** $-\cos^2\theta$
i $-2\sin^2\theta$ **j** 1 **k** $\sin\theta$ **l** $\sin\theta$

3 **a** $2\tan x$ **b** $\tan^2 x$ **c** $\sin x$ **d** $\cos x$
e $5\sin x$ **f** $2\sec x$ **g** 1 **h** 1
i $\operatorname{cosec} x$ **j** $\cos x$ **k** $\cos x$ **l** $5\sin x$

4 **a** $1 + 2\sin\theta + \sin^2\theta$ **b** $\sin^2\alpha - 4\sin\alpha + 4$
c $\tan^2\alpha - 2\tan\alpha + 1$ **d** $1 + 2\sin\alpha\cos\alpha$
e $1 - 2\sin\beta\cos\beta$ **f** $-4 + 4\cos\alpha - \cos^2\alpha$

5 **a** $-\tan^2\beta$ **b** 1 **c** $\sin^2\alpha$
d $\sin^2 x - \tan^2 x$ **e** 13 **f** $\cos^2\theta$ **g** 0

EXERCISE 9F.2

1 **a** $(1 - \sin\theta)(1 + \sin\theta)$
b $(\sin\alpha + \cos\alpha)(\sin\alpha - \cos\alpha)$
c $(\tan\alpha + 1)(\tan\alpha - 1)$ **d** $\sin\beta(2\sin\beta - 1)$
e $\cos\phi(2 + 3\cos\phi)$ **f** $3\sin\theta(\sin\theta - 2)$
g $(\tan\theta + 3)(\tan\theta + 2)$ **h** $(2\cos\theta + 1)(\cos\theta + 3)$
i $(3\cos\alpha + 1)(2\cos\alpha - 1)$ **j** $\tan\alpha(3\tan\alpha - 2)$
k $(\sec\beta + \operatorname{cosec}\beta)(\sec\beta - \operatorname{cosec}\beta)$
l $(2\cot x - 1)(\cot x - 1)$
m $(2\sin x + \cos x)(\sin x + 3\cos x)$

2 **a** $1 + \sin\alpha$ **b** $\tan\beta - 1$ **c** $\cos\phi - \sin\phi$
d $\cos\phi + \sin\phi$ **e** $\dfrac{1}{\sin\alpha - \cos\alpha}$ **f** $\dfrac{\cos\theta}{2}$
g $\sin\theta$ **h** $\cos\theta$ **i** $\sec\theta + 1$

EXERCISE 9G

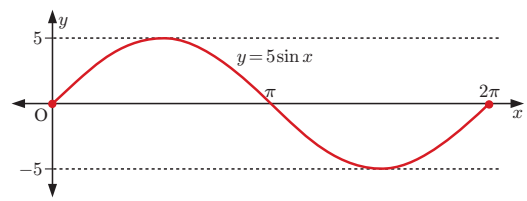
1 **a** $x = 0, \pi, \frac{7\pi}{6}, \frac{11\pi}{6}, 2\pi$ **b** $x = \frac{\pi}{3}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{3}$
c $x = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$ **d** $x = \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$
e no solutions **f** $x = 0, 2\pi$

2 **a** $x = \pi$ **b** $x = \frac{\pi}{6}, \frac{5\pi}{6}$

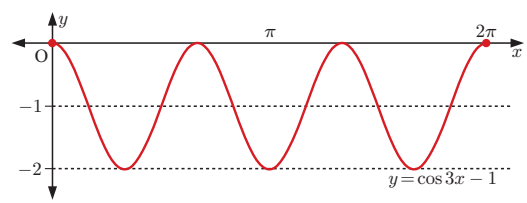
REVIEW SET 9A

1 **a** no **b** yes

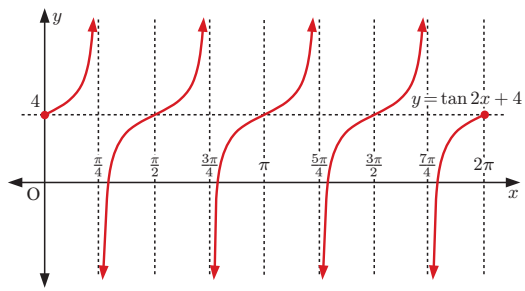
2 **a**



b



c



3 **a** minimum $= 0$, maximum $= 2$
b minimum $= -2$, maximum $= 2$
c minimum $= -3$, maximum $= 3$
d minimum $= -2$, maximum $= 0$

4 **a** 2π **b** $\frac{\pi}{2}$ **c** π **d** $\frac{\pi}{3}$

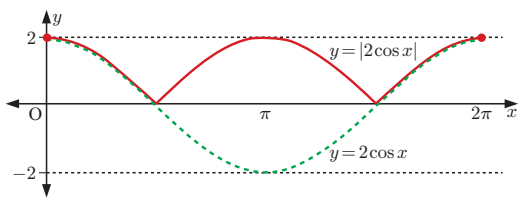
5

Function	*Period*	*Amplitude*
$y = 3\sin 2x + 1$	π	3
$y = \tan 2x$	$\frac{\pi}{2}$	undefined
$y = 2\cos 3x - 3$	$\frac{2\pi}{3}$	2

Function	*Domain*	*Range*
$y = 3\sin 2x + 1$	$x \in \mathbb{R}$	$-2 \leqslant y \leqslant 4$
$y = \tan 2x$	$x \neq \pm\frac{\pi}{4}, \pm\frac{3\pi}{4}, \ldots.$	$y \in \mathbb{R}$
$y = 2\cos 3x - 3$	$x \in \mathbb{R}$	$-5 \leqslant y \leqslant -1$

6 $y = 4\cos 2x$

7



8 **a** $x \approx 115°, 245°, 475°, 605°$ **b** $x \approx 25°, 335°, 385°$

9 **a** $x = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{19\pi}{6}, \frac{23\pi}{6}$ **b** $x = -\frac{7\pi}{4}, -\frac{5\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}$
c $x = \frac{4\pi}{9}, \frac{5\pi}{9}, \frac{10\pi}{9}, \frac{11\pi}{9}, \frac{16\pi}{9}, \frac{17\pi}{9}$
d $x = \frac{\pi}{4}, \frac{7\pi}{4}, \frac{9\pi}{4}, \frac{15\pi}{4}$

10 **a** $1 - \cos\theta$ **b** $\dfrac{1}{\sin\alpha + \cos\alpha}$ **c** $-\dfrac{\cos\alpha}{2}$
d $\operatorname{cosec}\theta + 1$

12 **a** $x = -\frac{2\pi}{3}, -\frac{\pi}{6}, \frac{\pi}{3}, \frac{5\pi}{6}$ **b** $x = -\frac{2\pi}{3}, -\frac{\pi}{3}, \frac{\pi}{3}, \frac{2\pi}{3}$

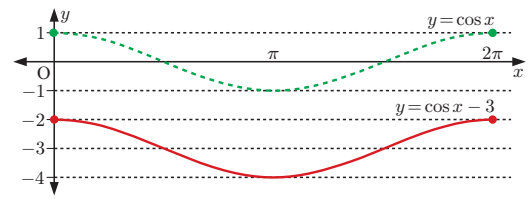
REVIEW SET 9B

1 **a** The function repeats itself over and over in a horizontal direction, in intervals of length 8 units.
b **i** 8 **ii** 5 **iii** -1

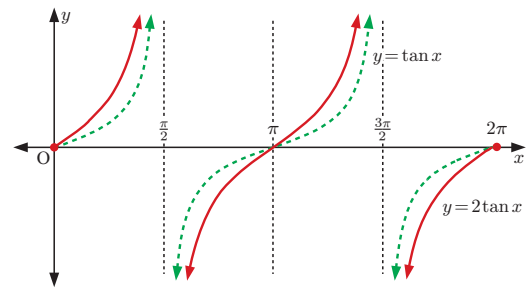
2 **a** $b = 6$ **b** $b = 24$

3 **a** minimum $= -8$, maximum $= 2$
b minimum $= -2$, maximum $= 4$
c minimum $= 5$, maximum $= 13$

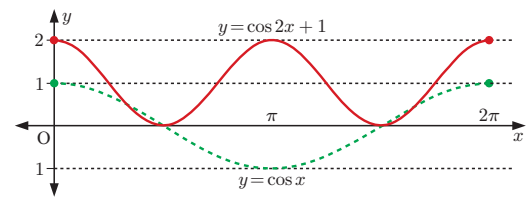
4 **a**



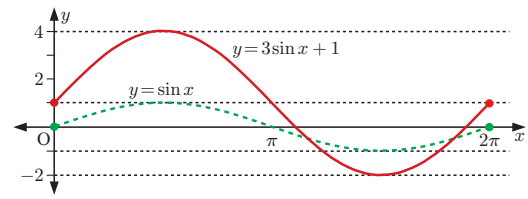
b



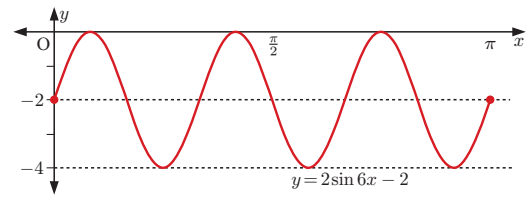
c



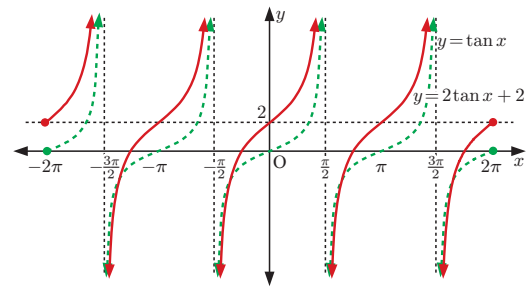
d



5 **a** $a = 2$, $b = 6$, $c = -2$
b



6 **a** $y = 2\tan x + 2$
b



7 **a** $x \approx -6.1, -3.4$ **b** $x \approx 0.8$

8 $m = 3$, $n = -1$

9 **a** $x = \frac{3\pi}{2}$ **b** $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

10 **a** $\cos\theta$ **b** $-\sin\theta$ **c** $5\cos^2\theta$ **d** $-\cos\theta$

11 **a** $4\sin^2\alpha - 4\sin\alpha + 1$ **b** $1 - 2\sin\alpha\cos\alpha$

EXERCISE 10A

1 18 **2** **a** 4 **b** 8 **c** 24 **3** 6

4 42 **5** 1680 **6** **a** 125 **b** 60

7 17 576 000 **8** **a** 4 **b** 9 **c** 81

EXERCISE 10B

1 **a** 13 **b** 20 **c** 19 **d** 32 **2** 13

EXERCISE 10C.1

1 1, 1, 2, 6, 24, 120, 720, 5040, 40 320, 362 880, 3 628 800

2 **a** 6 **b** 30 **c** $\frac{1}{7}$ **d** $\frac{1}{30}$ **e** 100 **f** 21

3 **a** $n,\ n \geqslant 1$ **b** $(n+2)(n+1),\ n \geqslant 0$

c $(n+1)n,\ n \geqslant 1$

4 **a** $\frac{7!}{4!}$ **b** $\frac{10!}{8!}$ **c** $\frac{11!}{6!}$ **d** $\frac{13!}{10!3!}$ **e** $\frac{3!}{6!}$ **f** $\frac{4!16!}{20!}$

5 **a** $6 \times 4!$ **b** $10 \times 10!$ **c** $73 \times 7!$ **d** $131 \times 10!$

e $81 \times 7!$ **f** $62 \times 6!$ **g** $10 \times 11!$ **h** $32 \times 8!$

6 **a** 11! **b** 9! **c** 8! **d** 9

e 34 **f** $n+1$ **g** $(n-1)!$ **h** $(n+1)!$

EXERCISE 10C.2

1 **a** 3 **b** 6 **c** 35 **d** 210

2 **a** **i** 28 **ii** 28 **3** $k = 3$ or 6

EXERCISE 10D

1 **a** W, X, Y, Z

b WX, WY, WZ, XW, XY, XZ, YW, YX, YZ, ZW, ZX, ZY

c WXY, WXZ, WYX, WYZ, WZX, WZY, XWY, XWZ, XYW, XYZ, XZW, XZY, YWX, YWZ, YXW, YXZ, YZX, YZW, ZWX, ZWY, ZXW, ZXY, ZYW, ZYX

2 **a** AB, AC, AD, AE, BA, BC, BD, BE, CA, CB, CD, CE, DA, DB, DC, DE, EA, EB, EC, ED

b ABC, ABD, ABE, ACB, ACD, ACE, ADB, ADC, ADE, AEB, AEC, AED, BAC, BAD, BAE, BCA, BCD, BCE, BDA, BDC, BDE, BEA, BEC, BED, CAB, CAD, CAE, CBA, CBD, CBE, CDA, CDB, CDE, CEA, CEB, CED, DAB, DAC, DAE, DBA, DBC, DBE, DCA, DCB, DCE, DEA, DEB, DEC, EAB, EAC, EAD, EBA, EBC, EBD, ECA, ECB, ECD, EDA, EDB, EDC

3 **a** 120 **b** 336 **c** 5040 **4** 110

5 **a** 12 **b** 24 **c** 36 **6** **a** 15 120 **b** 720

7 **a** 720 **b** **i** 24 **ii** 24 **iii** 48

8 **a** 343 **b** 210 **c** 120

9 **a** 648 **b** 64 **c** 72 **d** 136

10 **a** 6720 **b** 240 **c** 4200

11 **a** 120 **b** 48 **c** 72

12 **a** 3 628 800 **b** 241 920

13 **a** 720 **b** 144 **c** 72 **d** 144

14 **a** 48 **b** 24 **c** 15 **15** **a** 360 **b** 336 **c** 288

16 **a** 3 628 800 **b** **i** 151 200 **ii** 33 600

EXERCISE 10E

1 **a** permutation **b** combination

c permutation **d** combination

2 ABCD, ABCE, ABCF, ABDE, ABDF, ABEF, ACDE, ACDF, ACEF, ADEF, BCDE, BCDF, BCEF, BDEF, CDEF, $\binom{6}{4} = 15$

3 $\binom{17}{11} = 12\,376$ **4** **a** $\binom{9}{5} = 126$ **b** $\binom{1}{1}\binom{8}{4} = 70$

5 **a** $\binom{13}{3} = 286$ **b** $\binom{1}{1}\binom{12}{2} = 66$

6 **a** $\binom{12}{5} = 792$

b **i** $\binom{2}{2}\binom{10}{3} = 120$ **ii** $\binom{2}{1}\binom{10}{4} = 420$

7 $\binom{3}{3}\binom{1}{0}\binom{11}{6} = 462$

8 **a** $\binom{1}{1}\binom{9}{3} = 84$ **b** $\binom{2}{0}\binom{8}{4} = 70$

c $\binom{2}{0}\binom{1}{1}\binom{7}{3} = 35$

9 **a** $\binom{16}{5} = 4368$ **b** $\binom{10}{3}\binom{6}{2} = 1800$

c $\binom{10}{5}\binom{6}{0} = 252$

d $\binom{10}{3}\binom{6}{2} + \binom{10}{4}\binom{6}{1} + \binom{10}{5}\binom{6}{0} = 3312$

e $\binom{16}{5} - \binom{10}{5}\binom{6}{0} - \binom{10}{0}\binom{6}{5} = 4110$

10 **a** 6435 **b** 2520 **c** 36 **d** 4005 **11** 1050

12 **a** $\binom{6}{2}\binom{3}{1}\binom{7}{2} = 945$ **b** $\binom{6}{2}\binom{10}{3} = 1800$

c $\binom{16}{5} - \binom{9}{0}\binom{7}{5} = 4347$

13 $\binom{20}{2} - 20 = 170$

14 **a** **i** $\binom{12}{2} = 66$ **ii** $\binom{11}{1} = 11$

b **i** $\binom{12}{3} = 220$ **ii** $\binom{11}{2} = 55$

15 $\binom{9}{4} = 126$

16 **a** Selecting the different committees of 4 from 5 men and 6 women in all possible ways.

b $\binom{m+n}{r}$

17 **a** $\dfrac{\binom{12}{6}}{2} = 462$ **b** $\dfrac{\binom{12}{4}\binom{8}{4}\binom{4}{4}}{3!} = 5775$

18 **a** 45, yes **b** 37 128 **c** 3 628 800

EXERCISE 10F

1 **a** $p^3 + 3p^2q + 3pq^2 + q^3$ **b** $x^3 + 3x^2 + 3x + 1$

c $x^3 - 9x^2 + 27x - 27$ **d** $8 + 12x + 6x^2 + x^3$

e $27x^3 - 27x^2 + 9x - 1$ **f** $8x^3 + 60x^2 + 150x + 125$

g $8a^3 - 12a^2b + 6ab^2 - b^3$ **h** $27x^3 - 9x^2 + x - \frac{1}{27}$

i $8x^3 + 12x + \dfrac{6}{x} + \dfrac{1}{x^3}$

2 **a** $1 + 4x + 6x^2 + 4x^3 + x^4$

b $p^4 - 4p^3q + 6p^2q^2 - 4pq^3 + q^4$

c $x^4 - 8x^3 + 24x^2 - 32x + 16$

d $81 - 108x + 54x^2 - 12x^3 + x^4$

e $1 + 8x + 24x^2 + 32x^3 + 16x^4$

f $16x^4 - 96x^3 + 216x^2 - 216x + 81$

g $16x^4 + 32x^3b + 24x^2b^2 + 8xb^3 + b^4$

h $x^4 + 4x^2 + 6 + \dfrac{4}{x^2} + \dfrac{1}{x^4}$

i $16x^4 - 32x^2 + 24 - \dfrac{8}{x^2} + \dfrac{1}{x^4}$

3 **a** $x^5 + 10x^4 + 40x^3 + 80x^2 + 80x + 32$

b $x^5 - 10x^4y + 40x^3y^2 - 80x^2y^3 + 80xy^4 - 32y^5$

c $1 + 10x + 40x^2 + 80x^3 + 80x^4 + 32x^5$

d $x^5 - 5x^3 + 10x - \dfrac{10}{x} + \dfrac{5}{x^3} - \dfrac{1}{x^5}$

4 $64 + 160x^2 + 20x^4$

5 **a** 1 6 15 20 15 6 1

b **i** $x^6 + 12x^5 + 60x^4 + 160x^3 + 240x^2 + 192x + 64$

ii $64x^6 - 192x^5 + 240x^4 - 160x^3 + 60x^2 - 12x + 1$

iii $x^6 + 6x^4 + 15x^2 + 20 + \frac{15}{x^2} + \frac{6}{x^4} + \frac{1}{x^6}$

6 **a** $7 + 5\sqrt{2}$ **b** $161 + 72\sqrt{5}$ **c** $232 - 164\sqrt{2}$

7 $\frac{59 + 34\sqrt{3}}{13}$

8 **a** $64 + 192x + 240x^2 + 160x^3 + 60x^4 + 12x^5 + x^6$

b 65.944 160 601 201

9 **a** $a = 2$ and $b = e^x$ **b** $T_3 = 6e^{2x}$ and $T_4 = e^{3x}$

10 $2x^5 + 11x^4 + 24x^3 + 26x^2 + 14x + 3$

11 **a** 270 **b** 4320

EXERCISE 10G

1 **a** $1^{11} + \binom{11}{1}(2x)^1 + \binom{11}{2}(2x)^2 + ... + \binom{11}{10}(2x)^{10} + (2x)^{11}$

b $(3x)^{15} + \binom{15}{1}(3x)^{14}\left(\frac{2}{x}\right)^1 + \binom{15}{2}(3x)^{13}\left(\frac{2}{x}\right)^2 + ...$
$.... + \binom{15}{14}(3x)^1\left(\frac{2}{x}\right)^{14} + \left(\frac{2}{x}\right)^{15}$

c $(2x)^{20} + \binom{20}{1}(2x)^{19}\left(-\frac{3}{x}\right)^1 + \binom{20}{2}(2x)^{18}\left(-\frac{3}{x}\right)^2 + ...$
$.... + \binom{20}{19}(2x)^1\left(-\frac{3}{x}\right)^{19} + \left(-\frac{3}{x}\right)^{20}$

2 **a** $T_6 = \binom{15}{5}(2x)^{10}5^5$ **b** $T_4 = \binom{9}{3}(x^2)^6y^3$

c $T_{10} = \binom{17}{9}x^8\left(-\frac{2}{x}\right)^9$ **d** $T_9 = \binom{21}{8}(2x^2)^{13}\left(-\frac{1}{x}\right)^8$

3 **a** $\binom{12}{4}2^8 3^4 = 10\,264\,320$ **b** $\binom{12}{7}2^5 3^7 = 55\,427\,328$

4 **a** $\binom{10}{3}1^7(-3)^3 = -3240$ **b** $\binom{10}{7}1^3(-3)^7 = -262\,440$

5 **a** 144 **b** 5376 **c** 2304

6 **a** $T_{r+1} = \binom{7}{r}x^{7-r}b^r$ **b** $b = -2$

7 **a** $\binom{15}{5}2^5 = 96\,096$ **b** $\binom{9}{3}(-3)^3 = -2268$

8 **a** $\binom{10}{5}3^5 2^5 = 1\,959\,552$ **b** $\binom{6}{3}2^3(-3)^3 = -4320$

c $\binom{6}{3}2^3(-3)^3 = -4320$ **d** $\binom{12}{4}2^8(-1)^4 = 126\,720$

9 $k = 5$ **10** $a = 3$ **11** **b** $a = 5,\ b = 2$

12 $\binom{8}{6} = 28$ **13** $2\binom{9}{3}3^6x^6 - \binom{9}{4}3^5x^6 = 91\,854x^6$

14 **a** $\binom{7}{4}3^3(-2)^4 = 15\,120$

b $\binom{7}{4}3^3(-2)^4 + 3\binom{7}{3}3^4(-2)^3 = -52\,920$

15 **a** $\binom{8}{3}2^5(-5)^3 - 3\binom{8}{1}2^7(-5)^1 = -208\,640$

b $\binom{6}{3}2^3 - \binom{6}{4}2^4 = -80$

16 $a = 3,\ b = -2,\ c = 57$ **17** $n = 8$

18 $n = 6$ **19** $84x^3$ **20** $k = -2,\ n = 6$

REVIEW SET 10A

1 **a** $n(n-1),\ n \geqslant 2$ **b** $n + 2$ **2** 28

3 **a** 24 **b** 6 **4** **a** 900 **b** 180

5 **a** $a = e^x$ and $b = -e^{-x}$

b $(e^x - e^{-x})^4 = e^{4x} - 4e^{2x} + 6 - 4e^{-2x} + e^{-4x}$

6 $362 + 209\sqrt{3}$ **7** It does not have one. **8** $c = 3$

9 **a** 720 **b** 72 **c** 504 **10** 2500

11 **a** 252 **b** 246 **12** $\binom{12}{6}2^6(-3)^6 = 43\,110\,144$

13 $8\binom{6}{2} - 6\binom{6}{1} = 84$ **14** $a = \pm 4$ **15** $k = 0$ or ± 2

REVIEW SET 10B

1 **a** $26^2 \times 10^4 = 6\,760\,000$

b $5 \times 26 \times 10^4 = 1\,300\,000$

c $26 \times 25 \times 10 \times 9 \times 8 \times 7 = 3\,276\,000$

2 **a** 3003 **b** 980 **c** 2982

3 **a** $x^3 - 6x^2y + 12xy^2 - 8y^3$

b $81x^4 + 216x^3 + 216x^2 + 96x + 16$

4 20 000 **5** 60 **6** $-103 + 74\sqrt{2}$ **7** 4200

8 $\binom{5}{2}2^3 - 3\binom{5}{1}2^4 = -160$

9 **a** 3024 **b** 840 **c** 42

10 $q = 0$ or $\pm\sqrt{\frac{3}{35}}$ **11** 4320 **12** $k = 180$

13 **a** 43 758 teams **b** 11 550 teams **c** 41 283 teams

d 3861 teams

14 $n = 7$ **15** $k = -\frac{1}{4},\ n = 16$

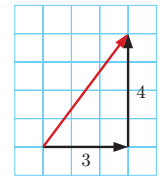
EXERCISE 11A

1 **a** $\begin{pmatrix} 7 \\ 3 \end{pmatrix}$, $7\mathbf{i} + 3\mathbf{j}$ **b** $\begin{pmatrix} -6 \\ 0 \end{pmatrix}$, $-6\mathbf{i}$

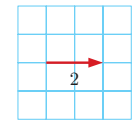
c $\begin{pmatrix} 2 \\ -5 \end{pmatrix}$, $2\mathbf{i} - 5\mathbf{j}$ **d** $\begin{pmatrix} 0 \\ 6 \end{pmatrix}$, $6\mathbf{j}$

e $\begin{pmatrix} -6 \\ 3 \end{pmatrix}$, $-6\mathbf{i} + 3\mathbf{j}$ **f** $\begin{pmatrix} -5 \\ -5 \end{pmatrix}$, $-5\mathbf{i} - 5\mathbf{j}$

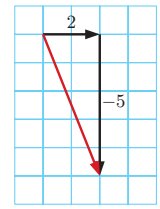
2 **a** $3\mathbf{i} + 4\mathbf{j}$



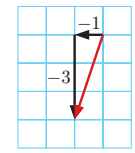
b $2\mathbf{i}$



c $2\mathbf{i} - 5\mathbf{j}$



d $-\mathbf{i} - 3\mathbf{j}$



3 **a** **i** $\begin{pmatrix} 4 \\ 1 \end{pmatrix}$, $4\mathbf{i} + \mathbf{j}$ **ii** $\begin{pmatrix} -4 \\ -1 \end{pmatrix}$, $-4\mathbf{i} - \mathbf{j}$

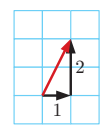
iii $\begin{pmatrix} -1 \\ -5 \end{pmatrix}$, $-\mathbf{i} - 5\mathbf{j}$ **iv** $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$, $2\mathbf{i}$

v $\begin{pmatrix} 3 \\ -4 \end{pmatrix}$, $3\mathbf{i} - 4\mathbf{j}$ **vi** $\begin{pmatrix} 4 \\ 1 \end{pmatrix}$, $4\mathbf{i} + \mathbf{j}$

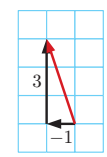
b $\overrightarrow{AB}$ and $\overrightarrow{DE}$. They have the same magnitude and direction.

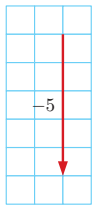
c $\overrightarrow{BA}$ is the negative of both $\overrightarrow{AB}$ and $\overrightarrow{DE}$. They have the same magnitude but opposite direction.

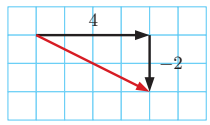
4 **a** $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$



b $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$



c $\begin{pmatrix}0\\-5\end{pmatrix}$ 

d $\begin{pmatrix}4\\-2\end{pmatrix}$ 

5 **a** $\begin{pmatrix}-1\\-4\end{pmatrix}$ **b** $\begin{pmatrix}2\\-3\end{pmatrix}$ **c** $\begin{pmatrix}-5\\2\end{pmatrix}$ **d** $\begin{pmatrix}0\\6\end{pmatrix}$

EXERCISE 11B

1 **a** 5 units **b** 5 units **c** 2 units
d $\sqrt{8}$ units **e** 3 units

2 **a** $\sqrt{2}$ units **b** 13 units **c** $\sqrt{17}$ units
d 3 units **e** $|k|$ units

3 **a** unit vector **b** unit vector **c** not a unit vector
d unit vector **e** not a unit vector

4 **a** $k = \pm 1$ **b** $k = \pm 1$ **c** $k = 0$
d $k = \pm\frac{1}{\sqrt{2}}$ **e** $k = \pm\frac{\sqrt{3}}{2}$

5 $p = \pm 3$

EXERCISE 11C

1 **a** $\begin{pmatrix}-2\\6\end{pmatrix}$ **b** $\begin{pmatrix}-2\\6\end{pmatrix}$ **c** $\begin{pmatrix}-1\\-1\end{pmatrix}$ **d** $\begin{pmatrix}-1\\-1\end{pmatrix}$
e $\begin{pmatrix}-5\\-3\end{pmatrix}$ **f** $\begin{pmatrix}-5\\-3\end{pmatrix}$ **g** $\begin{pmatrix}-6\\4\end{pmatrix}$ **h** $\begin{pmatrix}-4\\1\end{pmatrix}$

2 **a** $\begin{pmatrix}-3\\7\end{pmatrix}$ **b** $\begin{pmatrix}-4\\-3\end{pmatrix}$ **c** $\begin{pmatrix}-8\\-1\end{pmatrix}$ **d** $\begin{pmatrix}-6\\9\end{pmatrix}$
e $\begin{pmatrix}0\\-5\end{pmatrix}$ **f** $\begin{pmatrix}6\\-9\end{pmatrix}$

3 **a** $\mathbf{a} + \mathbf{0} = \begin{pmatrix}a_1\\a_2\end{pmatrix} + \begin{pmatrix}0\\0\end{pmatrix} = \begin{pmatrix}a_1+0\\a_2+0\end{pmatrix} = \begin{pmatrix}a_1\\a_2\end{pmatrix} = \mathbf{a}$

b $\mathbf{a} - \mathbf{a} = \begin{pmatrix}a_1\\a_2\end{pmatrix} - \begin{pmatrix}a_1\\a_2\end{pmatrix} = \begin{pmatrix}a_1-a_1\\a_2-a_2\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix} = \mathbf{0}$

4 **a** $\begin{pmatrix}-3\\-15\end{pmatrix}$ **b** $\begin{pmatrix}-1\\2\end{pmatrix}$ **c** $\begin{pmatrix}0\\14\end{pmatrix}$ **d** $\begin{pmatrix}5\\-3\end{pmatrix}$
e $\begin{pmatrix}\frac{5}{2}\\\frac{11}{2}\end{pmatrix}$ **f** $\begin{pmatrix}-7\\7\end{pmatrix}$ **g** $\begin{pmatrix}5\\11\end{pmatrix}$ **h** $\begin{pmatrix}3\\\frac{17}{3}\end{pmatrix}$

5 **a** $\begin{pmatrix}8\\-1\end{pmatrix}$ **b** $\begin{pmatrix}8\\-1\end{pmatrix}$ **c** $\begin{pmatrix}8\\-1\end{pmatrix}$

In each case, the result is $2\mathbf{p} + 3\mathbf{q} = \begin{pmatrix}8\\-1\end{pmatrix}$.

6 **a** $\sqrt{13}$ units **b** $\sqrt{17}$ units **c** $5\sqrt{2}$ units
d $\sqrt{10}$ units **e** $\sqrt{29}$ units

7 **a** $\sqrt{10}$ units **b** $2\sqrt{10}$ units **c** $2\sqrt{10}$ units
d $3\sqrt{10}$ units **e** $3\sqrt{10}$ units **f** $2\sqrt{5}$ units
g $8\sqrt{5}$ units **h** $8\sqrt{5}$ units **i** $\sqrt{5}$ units
j $\sqrt{5}$ units

8 **a** $3\mathbf{i} + 2\mathbf{j}$ **b** $-\mathbf{i} + 9\mathbf{j}$ **c** $6\mathbf{i} - \mathbf{j}$ **d** $7\mathbf{j}$
e 2 units **f** $2\sqrt{10}$ units

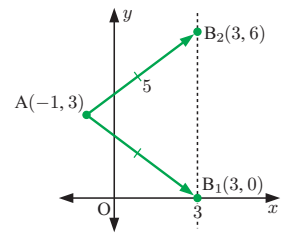
EXERCISE 11D

1 **a** $\begin{pmatrix}2\\4\end{pmatrix}$ **b** $\begin{pmatrix}-2\\5\end{pmatrix}$ **c** $\begin{pmatrix}3\\-3\end{pmatrix}$ **d** $\begin{pmatrix}1\\-5\end{pmatrix}$
e $\begin{pmatrix}6\\-5\end{pmatrix}$ **f** $\begin{pmatrix}1\\3\end{pmatrix}$

2 **a** B(4, 2) **b** C(2, 2) **3** **a** $\begin{pmatrix}2\\1\end{pmatrix}$ **b** Q(3, 3)

4 **a** $\begin{pmatrix}5\\1\end{pmatrix}$ **b** $\begin{pmatrix}-5\\-1\end{pmatrix}$ **c** D(−1, −2)

5 **a** $\overrightarrow{AB} = \begin{pmatrix}4\\k-3\end{pmatrix}$, $|\overrightarrow{AB}| = \sqrt{16 + (k-3)^2} = 5$ units

b $k = 0$ or 6 **c** 

6 **a** $\overrightarrow{AB} = \begin{pmatrix}2\\3\end{pmatrix}$, $\overrightarrow{AC} = \begin{pmatrix}3\\-3\end{pmatrix}$

b $\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC} = -\overrightarrow{AB} + \overrightarrow{AC}$ **c** $\overrightarrow{BC} = \begin{pmatrix}1\\-6\end{pmatrix}$

7 **a** $\begin{pmatrix}-5\\4\end{pmatrix}$ **b** $\begin{pmatrix}1\\2\end{pmatrix}$ **c** $\begin{pmatrix}6\\-5\end{pmatrix}$

8 **a** M(1, 4) **b** $\overrightarrow{CA} = \begin{pmatrix}7\\5\end{pmatrix}$, $\overrightarrow{CM} = \begin{pmatrix}5\\3\end{pmatrix}$, $\overrightarrow{CB} = \begin{pmatrix}3\\1\end{pmatrix}$

EXERCISE 11E

1 $r = 3$ **2** $a = -6$

3 **a** $\overrightarrow{AB}$ is parallel and in the same direction as $\overrightarrow{CD}$, and 3 times its length.

b $\overrightarrow{RS}$ is parallel and in the opposite direction to $\overrightarrow{KL}$, and half its length.

c A, B, and C are collinear. $\overrightarrow{AB}$ is parallel and in the same direction as $\overrightarrow{BC}$, and twice its length.

4 **a** $\begin{pmatrix}4\\8\end{pmatrix}$ **b** $\begin{pmatrix}-1\\-2\end{pmatrix}$

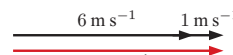
5 **a** $\frac{1}{\sqrt{5}}\mathbf{i} + \frac{2}{\sqrt{5}}\mathbf{j}$ **b** $\frac{1}{\sqrt{10}}\mathbf{i} - \frac{3}{\sqrt{10}}\mathbf{j}$ **c** $\frac{2}{\sqrt{5}}\mathbf{i} - \frac{1}{\sqrt{5}}\mathbf{j}$

6 **a** $\mathbf{v} = \frac{3}{\sqrt{5}}\begin{pmatrix}2\\-1\end{pmatrix}$ **b** $\mathbf{v} = \frac{2}{\sqrt{17}}\begin{pmatrix}1\\4\end{pmatrix}$

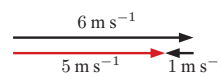
7 **a** $\overrightarrow{AB} = \begin{pmatrix}2\sqrt{2}\\-2\sqrt{2}\end{pmatrix}$ **b** $\overrightarrow{OB} = \begin{pmatrix}3+2\sqrt{2}\\2-2\sqrt{2}\end{pmatrix}$

c $B(3 + 2\sqrt{2},\ 2 - 2\sqrt{2})$

EXERCISE 11F

1 **a** 

$\therefore$ 7 m s^{-1}

b 

$\therefore$ 5 m s^{-1}

2 **a** 1.34 m s^{-1} in the direction 26.6° to the right of intended line

b **i** 30° to the left of Q **ii** 1.04 m s^{-1}

3 **a** 24.6 km h^{-1} **b** $\approx 9.93^\circ$ east of south

4 **a** 82.5 m **b** 23.3° to the left of directly across **c** 48.4 s

5 **a** The plane's speed in still air would be $\approx 437\text{ km h}^{-1}$. The wind slows the plane down to 400 km h^{-1}.

b 4.64° north of due east

EXERCISE 11G

1 **a** **i** $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ -4 \end{pmatrix} + t\begin{pmatrix} 1 \\ 4 \end{pmatrix},\ t \in \mathbb{R}$

ii $x = 3 + t,\ y = -4 + 4t,\ t \in \mathbb{R}$ **iii** $4x - y = 16$

b **i** $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -6 \\ 0 \end{pmatrix} + t\begin{pmatrix} 3 \\ 7 \end{pmatrix},\ t \in \mathbb{R}$

ii $x = -6 + 3t,\ y = 7t,\ t \in \mathbb{R}$ **iii** $7x - 3y = -42$

c **i** $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 \\ 11 \end{pmatrix} + t\begin{pmatrix} -2 \\ 1 \end{pmatrix},\ t \in \mathbb{R}$

ii $x = -1 - 2t,\ y = 11 + t,\ t \in \mathbb{R}$ **iii** $x + 2y = 21$

2 **a** $x = -1 + 2t,\ y = 4 - t,\ t \in \mathbb{R}$

b

t	0	1	3	-1	-4
Point	$(-1, 4)$	$(1, 3)$	$(5, 1)$	$(-3, 5)$	$(-9, 8)$

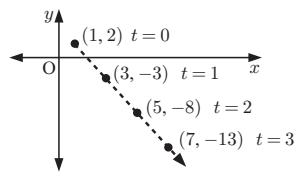
3 **a** When $t = 1,\ x = 3,\ y = -2,\ \therefore$ yes **b** $k = -5$

4 **a** $(0, 8)$ **b** It is a non-zero scalar multiple of $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$.

c $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 8 \end{pmatrix} + s\begin{pmatrix} 1 \\ -3 \end{pmatrix},\ s \in \mathbb{R}$

EXERCISE 11H

1 **a** $(1, 2)$ **b**



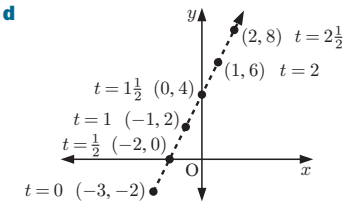
c $\begin{pmatrix} 2 \\ -5 \end{pmatrix}$

d $\sqrt{29}\text{ cm s}^{-1}$

2 **a** $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} + t\begin{pmatrix} 4 \\ -5 \end{pmatrix},\ t \geqslant 0$ **b** $(8, -4.5)$

c 45 minutes

3 **a** $\begin{pmatrix} -3 + 2t \\ -2 + 4t \end{pmatrix}$ **d**



b $\begin{pmatrix} 2 \\ 8 \end{pmatrix}$

c **i** $t = 1.5$ s **ii** $t = 0.5$ s

4 **a** **i** $(-4, 3)$ **ii** $\begin{pmatrix} 12 \\ 5 \end{pmatrix}$ **iii** 13 m s^{-1}

b **i** $(3, 0)$ **ii** $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$ **iii** $\sqrt{5}\text{ m s}^{-1}$

5 **a** $\begin{pmatrix} 120 \\ -90 \end{pmatrix}$ **b** $\begin{pmatrix} 20\sqrt{5} \\ 10\sqrt{5} \end{pmatrix}$ **6** $\begin{pmatrix} 35 \\ -84 \end{pmatrix}$

7 **a** A is at $(4, 5)$, B is at $(1, -8)$

b For A it is $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$. For B it is $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$.

c For A, speed is $\sqrt{5}\text{ km h}^{-1}$. For B, speed is $\sqrt{5}\text{ km h}^{-1}$.

d Yacht A: $2x + y = 13$, Yacht B: $x - 2y = 17$

e Yacht A moves with gradient -2; Yacht B with gradient $\frac{1}{2}$. So, their paths are perpendicular.

f no

8 **a** $\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} -5 \\ 4 \end{pmatrix} + t\begin{pmatrix} 3 \\ -1 \end{pmatrix},\ t \geqslant 0$

$\therefore\ x_1(t) = -5 + 3t,\ y_1(t) = 4 - t,\ t \geqslant 0$

b speed $= \sqrt{10}\text{ km min}^{-1}$

c a minutes later, $(t - a)$ min have elapsed.

$\therefore\ \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} 15 \\ 7 \end{pmatrix} + (t - a)\begin{pmatrix} -4 \\ -3 \end{pmatrix},\ t \geqslant 0$

$\therefore\ x_2(t) = 15 - 4(t - a),\ y_2(t) = 7 - 3(t - a),\ t \geqslant 0$

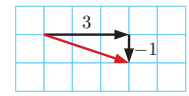
d Torpedo is fired at 1:35:28 pm and the explosion occurs at 1:37:42 pm.

REVIEW SET 11A

1 **a** $\mathbf{x} = \begin{pmatrix} 5 \\ 1 \end{pmatrix} = 5\mathbf{i} + \mathbf{j},\ \mathbf{y} = \begin{pmatrix} 1 \\ -2 \end{pmatrix} = \mathbf{i} - 2\mathbf{j}$

b **i** $6\mathbf{i} - \mathbf{j}$ **ii** $-9\mathbf{i} - 4\mathbf{j}$

2 **a** $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$ **b**



c $\begin{pmatrix} -3 \\ 1 \end{pmatrix}$

d $\sqrt{10}$ units

3 **a** $k = \pm\frac{1}{\sqrt{2}}$ **b** $\begin{pmatrix} -2\sqrt{5} \\ \sqrt{5} \end{pmatrix}$

4 **a** $\begin{pmatrix} 3 \\ -3 \end{pmatrix}$ **b** $\begin{pmatrix} 7 \\ -3 \end{pmatrix}$ **c** 5 units

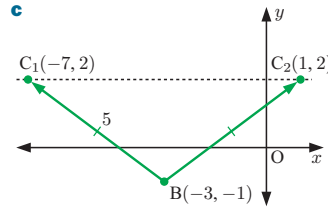
5 **a** $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$ **b** $\begin{pmatrix} -3 \\ -6 \end{pmatrix}$ **c** $\sqrt{34}$ units

6 **a** $\overrightarrow{BC} = \begin{pmatrix} k + 3 \\ 3 \end{pmatrix}$

$|\overrightarrow{BC}| = 5$

b $k = -7$ or 1

c



7 **a** 11.5° east of due north **b** $\approx 343\text{ km h}^{-1}$

8 **a** $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -6 \\ 3 \end{pmatrix} + t\begin{pmatrix} 4 \\ -3 \end{pmatrix},\ t \in \mathbb{R}$

b $x = -6 + 4t,\ y = 3 - 3t,\ t \in \mathbb{R}$ **c** $3x + 4y = -6$

9 $m = 10$ **10** $\begin{pmatrix} 6\sqrt{10} \\ -2\sqrt{10} \end{pmatrix}$

11 **a** $(5, 2)$ **b** $\begin{pmatrix} 4 \\ 10 \end{pmatrix}$ is a non-zero scalar multiple of $\begin{pmatrix} 2 \\ 5 \end{pmatrix}$

c $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \end{pmatrix} + s\begin{pmatrix} 4 \\ 10 \end{pmatrix},\ s \in \mathbb{R}$

12 **a** $(-4, 3)$ **b** $(28, 27)$ **c** $\begin{pmatrix} 8 \\ 6 \end{pmatrix}$ **d** 10 m s^{-1}

REVIEW SET 11B

1 **a** **i** $\overrightarrow{AB} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} = 4\mathbf{i}$ **ii** $\overrightarrow{BC} = \begin{pmatrix} -2 \\ -4 \end{pmatrix} = -2\mathbf{i} - 4\mathbf{j}$

iii $\overrightarrow{CA} = \begin{pmatrix} -2 \\ 4 \end{pmatrix} = -2\mathbf{i} + 4\mathbf{j}$