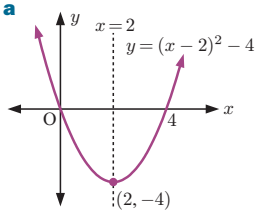
2 **a** $x = 0$ or 4 **b** $x = -\frac{5}{3}$ or 2 **c** $x = 15$ or -4

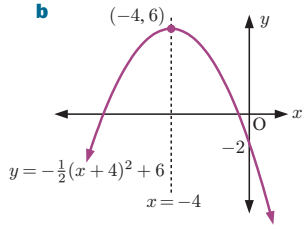
3 **a** $x = -\frac{5}{2} \pm \frac{\sqrt{13}}{2}$ **b** $x = -\frac{11}{6} \pm \frac{\sqrt{145}}{6}$

4 **a** $-3 < x < 7$ **b** $x \leqslant -\frac{1}{3}$ or $x \geqslant 2$

5 **a**



b



6 **a** $y = 3x^2 - 24x + 48$ **b** $y = \frac{2}{5}x^2 + \frac{16}{5}x + \frac{37}{5}$

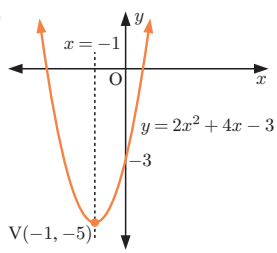
7 $a = -2$ which is < 0 $\therefore$  a max.
max. $= 5$ when $x = 1$

8 $(4, 4)$ and $(-3, 18)$ **9** $k < -3\frac{1}{8}$

10 **a** $m = \frac{9}{8}$ **b** $m < \frac{9}{8}$ **c** $m > \frac{9}{8}$ **11** $\frac{6}{5}$ or $\frac{5}{6}$

12 **Hint:** Let the line have equation $y = mx + 10$.

13 **a** $y = 2(x + 1)^2 - 5$ **b**



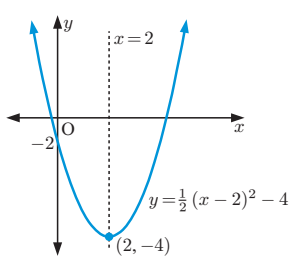
14 **a** $y = \frac{20}{9}(x - 2)^2 - 20$ **b** $y = -\frac{2}{7}(x - 1)(x - 7)$
c $y = \frac{2}{9}(x + 3)^2$

15 $\{y : -13 \leqslant y \leqslant 12\}$ **16** 21 m

REVIEW SET 3B

1 **a** $x = 2$
b $(2, -4)$
c -2
e $\{y : y \geqslant -4\}$

d

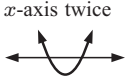


2 **a** $x = \frac{5}{2} \pm \frac{\sqrt{37}}{2}$ **b** $x = \frac{7}{4} \pm \frac{\sqrt{73}}{4}$

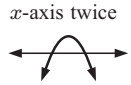
3 **a** $-7 \leqslant x \leqslant 2$ **b** $x < -4$ or $x > \frac{3}{2}$

4 $x = \frac{4}{3}$, $V(\frac{4}{3}, 12\frac{1}{3})$

5 **a** graph cuts x-axis twice



b graph cuts x-axis twice

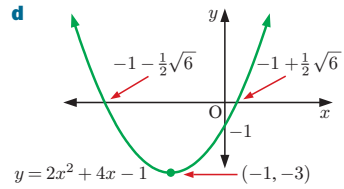


6 **a** $a < 0$, $\Delta > 0$, neither
b $a > 0$, $\Delta < 0$, positive definite

7 $y = -6(x - 2)^2 + 25$ **8** $m < -5$ or $m > 19$

9 **a** $x = -1$
b $(-1, -3)$
c y-intercept -1,
x-ints. $-1 \pm \frac{1}{2}\sqrt{6}$

d



10 $\{y : -55 \leqslant y \leqslant \frac{11}{2}\}$

11 **a** $k = -8$ **b** $k < -8$ or $k > 0$ **c** $-8 < k < 0$

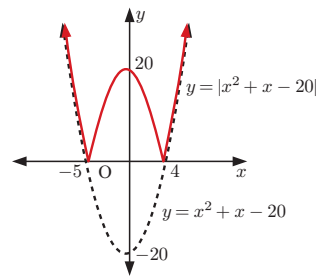
12 **a** $c > -6$
b example: $c = -2$, $(-1, -5)$ and $(3, 7)$

13 **a** $y = -\frac{2}{5}(x + 5)(x - 1)$ **b** $(-2, 3\frac{3}{5})$

14 **a** min. $= 5\frac{2}{3}$ when $x = -\frac{2}{3}$
b max. $= 5\frac{1}{8}$ when $x = -\frac{5}{4}$

15 **b** $37\frac{1}{2}$ m by $33\frac{1}{3}$ m **c** 1250 m^2

16



EXERCISE 4A.1

1 **a** 11 **b** $\sqrt{15}$ **c** 3 **d** $\sqrt{30}$
e 4 **f** 12 **g** 42 **h** 45
i $\sqrt{6}$ **j** $\sqrt{6}$ **k** 2 **l** $\sqrt{5}$

2 **a** $2\sqrt{2}$ **b** $2\sqrt{3}$ **c** $2\sqrt{5}$ **d** $4\sqrt{2}$
e $3\sqrt{3}$ **f** $3\sqrt{5}$ **g** $4\sqrt{3}$ **h** $3\sqrt{6}$
i $5\sqrt{2}$ **j** $4\sqrt{5}$ **k** $4\sqrt{6}$ **l** $6\sqrt{3}$

EXERCISE 4A.2

1 **a** $5\sqrt{2}$ **b** $-\sqrt{2}$ **c** $2\sqrt{5}$ **d** $8\sqrt{5}$
e $-2\sqrt{5}$ **f** $9\sqrt{3}$ **g** $-3\sqrt{6}$ **h** $3\sqrt{2}$

2 **a** $3\sqrt{2} - 2$ **b** $5 + \sqrt{5}$ **c** $3\sqrt{10} + 20$
d $21 - 4\sqrt{7}$ **e** $-5\sqrt{3} - 3$ **f** $12 - 14\sqrt{6}$
g $-8 + 5\sqrt{8}$ **h** $-12\sqrt{2} + 36$

3 **a** $22 + 9\sqrt{2}$ **b** $34 + 15\sqrt{3}$ **c** $22 + 14\sqrt{7}$
d $-7 - \sqrt{3}$ **e** $34 - 15\sqrt{8}$ **f** $-47 + 30\sqrt{5}$

4 **a** $11 + 6\sqrt{2}$ **b** $39 - 12\sqrt{3}$ **c** $6 + 2\sqrt{5}$
d $17 - 6\sqrt{8}$ **e** $28 + 16\sqrt{3}$ **f** $46 + 6\sqrt{5}$
g $89 - 28\sqrt{10}$ **h** $166 - 40\sqrt{6}$

5 **a** 2 **b** -23 **c** 13 **d** 7
e -56 **f** 218

EXERCISE 4A.3

1 **a** $\frac{\sqrt{3}}{3}$ **b** $\sqrt{3}$ **c** $3\sqrt{3}$ **d** $\frac{11\sqrt{3}}{3}$ **e** $\frac{\sqrt{6}}{9}$
f $\sqrt{2}$ **g** $3\sqrt{2}$ **h** $6\sqrt{2}$ **i** $\frac{\sqrt{6}}{2}$ **j** $\frac{\sqrt{2}}{8}$

2 **a** $\sqrt{5}$ **b** $3\sqrt{5}$ **c** $-\frac{3\sqrt{5}}{5}$ **d** $40\sqrt{5}$ **e** $\frac{\sqrt{5}}{15}$
f $\sqrt{7}$ **g** $3\sqrt{7}$ **h** $\frac{2\sqrt{11}}{11}$ **i** $2\sqrt{13}$ **j** $\frac{\sqrt{3}}{9}$

3 **a** $\frac{3-\sqrt{2}}{7}$ **b** $\frac{6+2\sqrt{2}}{7}$ **c** $-2+\sqrt{5}$
d $1+\sqrt{2}$ **e** $2+2\sqrt{6}$ **f** $\frac{\sqrt{21}-2\sqrt{3}}{3}$
g $-3-2\sqrt{2}$ **h** $\frac{3+4\sqrt{3}}{13}$ **i** $4+2\sqrt{2}$
j $-7-3\sqrt{5}$ **k** $\frac{5+3\sqrt{3}}{2}$ **l** $\frac{-38+11\sqrt{10}}{6}$

4 **a** $-\frac{9}{7}-\frac{3}{7}\sqrt{2}$ **b** $4-2\sqrt{2}$ **c** $-\frac{2}{23}-\frac{5}{23}\sqrt{2}$
d $-4+2\sqrt{2}$

5 **a** $-2-2\sqrt{3}$ **b** $12-6\sqrt{3}$ **c** $3+2\sqrt{3}$ **d** $-\frac{1}{2}+\frac{5}{6}\sqrt{3}$

6 **a** $(a+b\sqrt{c})(a-b\sqrt{c})=a^2-b^2c$
which is an integer as a, b, and c are integers.
b **i** $\frac{-1+2\sqrt{3}}{11}$ **ii** $\frac{-6-5\sqrt{2}}{7}$ **iii** $1+\sqrt{2}$

7 **a** $(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b})=a-b$
which is an integer as a and b are integers.
b **i** $\sqrt{3}-\sqrt{2}$ **ii** $\frac{-3-\sqrt{15}}{2}$ **iii** $\frac{2\sqrt{154}-25}{3}$

8 $x=-7+5\sqrt{3}$ **9** $x=\frac{10}{19}+\frac{1}{19}\sqrt{5}$

EXERCISE 4B

1 **a** $2^1=2,\ 2^2=4,\ 2^3=8,\ 2^4=16,\ 2^5=32,\ 2^6=64$
b $3^1=3,\ 3^2=9,\ 3^3=27,\ 3^4=81,\ 3^5=243,\ 3^6=729$
c $4^1=4,\ 4^2=16,\ 4^3=64,\ 4^4=256,\ 4^5=1024,\ 4^6=4096$

2 **a** $5^1=5,\ 5^2=25,\ 5^3=125,\ 5^4=625$
b $6^1=6,\ 6^2=36,\ 6^3=216,\ 6^4=1296$
c $7^1=7,\ 7^2=49,\ 7^3=343,\ 7^4=2401$

3 **a** -1 **b** 1 **c** 1 **d** -1 **e** 1
f -1 **g** -1 **h** -32 **i** -32 **j** -64
k 625 **l** -625

4 **a** $16\,384$ **b** 2401 **c** -3125
d -3125 **e** $262\,144$ **f** $262\,144$
g $-262\,144$ **h** $902.436\,039\,6$ **i** $-902.436\,039\,6$
j $-902.436\,039\,6$

5 **a** $0.\overline{1}$ **b** $0.\overline{1}$ **c** $0.02\overline{7}$ **d** $0.02\overline{7}$
e $0.012\,345\,679$ **f** $0.012\,345\,679$ **g** 1 **h** 1
Notice that $a^{-n}=\frac{1}{a^n}$

6 3 **7** 7

EXERCISE 4C

1 **a** 5^{11} **b** d^8 **c** k^5 **d** $\frac{1}{7}$ **e** x^{10} **f** 3^{16}
g p^{-4} **h** n^{12} **i** 5^{3t} **j** 7^{x+2} **k** 10^{3-q} **l** c^{4m}

2 **a** 2^2 **b** 2^{-2} **c** 2^3 **d** 2^{-3} **e** 2^5 **f** 2^{-5}
g 2^1 **h** 2^{-1} **i** 2^6 **j** 2^{-6} **k** 2^7 **l** 2^{-7}

3 **a** 3^2 **b** 3^{-2} **c** 3^3 **d** 3^{-3} **e** 3^1 **f** 3^{-1}
g 3^4 **h** 3^{-4} **i** 3^0 **j** 3^5 **k** 3^{-5}

4 **a** 2^{a+1} **b** 2^{b+2} **c** 2^{t+3} **d** 2^{2x+2} **e** 2^{n-1}
f 2^{c-2} **g** 2^{2m} **h** 2^{n+1} **i** 2^1 **j** 2^{3x-1}

5 **a** 3^{p+2} **b** 3^{3a} **c** 3^{2n+1} **d** 3^{d+3} **e** 3^{3t+2}
f 3^{y-1} **g** 3^{1-y} **h** 3^{2-3t} **i** 3^{3a-1} **j** 3^3

6 **a** $4a^2$ **b** $27b^3$ **c** a^4b^4 **d** p^3q^3 **e** $\frac{m^2}{n^2}$
f $\frac{a^3}{27}$ **g** $\frac{b^4}{c^4}$ **h** $1,\ a,\ b\neq 0$ **i** $\frac{m^4}{81n^4}$ **j** $\frac{x^3y^3}{8}$

7 **a** $4a^2$ **b** $36b^4$ **c** $-8a^3$ **d** $-27m^6n^6$
e $16a^4b^{16}$ **f** $\frac{-8a^6}{b^6}$ **g** $\frac{16a^6}{b^2}$ **h** $\frac{9p^4}{q^6}$
i $4x^3y^2$ **j** $32a^5b$ **k** $\frac{5a^{12}}{b^2}$ **l** $\frac{-2x^{18}}{y^3}$

8 **a** $\frac{a}{b^2}$ **b** $\frac{1}{a^2b^2}$ **c** $\frac{4a^2}{b^2}$ **d** $\frac{9b^2}{a^4}$ **e** $\frac{a^2}{bc^2}$
f $\frac{a^2c^2}{b}$ **g** a^3 **h** $\frac{b^3}{a^2}$ **i** $\frac{2}{ad^2}$ **j** $12am^3$

9 **a** a^{-n} **b** b^n **c** 3^{n-2} **d** a^nb^m **e** a^{-2n-2}

10 **a** 1 **b** $\frac{4}{7}$ **c** 6 **d** 27 **e** $\frac{9}{16}$ **f** $\frac{5}{2}$
g $\frac{27}{125}$ **h** $\frac{151}{5}$

11 **a** 3^{-2} **b** 2^{-4} **c** 5^{-3} **d** $3^1\times 5^{-1}$ **e** $2^2\times 3^{-3}$
f $2^{c-3}\times 3^{-2}$ **g** $3^{2k}\times 2^{-1}\times 5^{-1}$ **h** $2^p\times 3^{p-1}\times 5^{-2}$

12 **a** $5^3=21+23+25+27+29$
b $7^3=43+45+47+49+51+53+55$
c $12^3=133+135+137+139+141+143+145+147+149+151+153+155$

EXERCISE 4D

1 **a** $2^{\frac{1}{5}}$ **b** $2^{-\frac{1}{5}}$ **c** $2^{\frac{3}{2}}$ **d** $2^{\frac{5}{2}}$ **e** $2^{-\frac{1}{3}}$
f $2^{\frac{4}{3}}$ **g** $2^{\frac{3}{2}}$ **h** $2^{\frac{3}{2}}$ **i** $2^{-\frac{4}{3}}$ **j** $2^{-\frac{3}{2}}$

2 **a** $3^{\frac{1}{3}}$ **b** $3^{-\frac{1}{3}}$ **c** $3^{\frac{1}{4}}$ **d** $3^{\frac{3}{2}}$ **e** $3^{-\frac{5}{2}}$

3 **a** $7^{\frac{1}{3}}$ **b** $3^{\frac{3}{4}}$ **c** $2^{\frac{4}{5}}$ **d** $2^{\frac{5}{3}}$ **e** $7^{\frac{2}{7}}$
f $7^{-\frac{1}{3}}$ **g** $3^{-\frac{3}{4}}$ **h** $2^{-\frac{4}{5}}$ **i** $2^{-\frac{5}{3}}$ **j** $7^{-\frac{2}{7}}$

4 **a** 2.28 **b** 1.83 **c** 0.794 **d** 0.435 **e** 1.68
f 1.93 **g** 0.523

5 **a** 8 **b** 32 **c** 8 **d** 125 **e** 4
f $\frac{1}{2}$ **g** $\frac{1}{27}$ **h** $\frac{1}{16}$ **i** $\frac{1}{81}$ **j** $\frac{1}{25}$

EXERCISE 4E.1

1 **a** $x^5+2x^4+x^2$ **b** 4^x+2^x **c** $x+1$
d $49^x+2(7^x)$ **e** $2(3^x)-1$ **f** x^2+2x+3
g $1+5(2^{-x})$ **h** 5^x+1 **i** $x^{\frac{3}{2}}+x^{\frac{1}{2}}+1$

2 **a** $4^x+2^{x+1}-3$ **b** $9^x+7(3^x)+10$
c $25^x-6(5^x)+8$ **d** $4^x+6(2^x)+9$
e $9^x-2(3^x)+1$ **f** $16^x+14(4^x)+49$

3 **a** $x-4$ **b** 4^x-9 **c** $x-x^{-1}$ **d** $x^2+4+\frac{4}{x^2}$
e $7^{2x}-2+7^{-2x}$ **f** $25-10(2^{-x})+4^{-x}$
g $x^{\frac{4}{3}}+2x+x^{\frac{2}{3}}$ **h** x^3-2x^2+x **i** $4x-4+x^{-1}$

EXERCISE 4E.2

1 **a** $5^x(5^x+1)$ **b** $10(3^n)$ **c** $7^n(1+7^{2n})$
d $5(5^n-1)$ **e** $6(6^{n+1}-1)$ **f** $16(4^n-1)$

2 **a** $(3^x+2)(3^x-2)$ **b** $(2^x+5)(2^x-5)$
c $(4+3^x)(4-3^x)$ **d** $(5+2^x)(5-2^x)$
e $(3^x+2^x)(3^x-2^x)$ **f** $(2^x+3)^2$
g $(3^x+5)^2$ **h** $(2^x-7)^2$ **i** $(5^x-2)^2$

3 **a** $(2^x+3)(2^x+6)$ **b** $(2^x+4)(2^x-5)$
c $(3^x+2)(3^x+7)$ **d** $(3^x+5)(3^x-1)$
e $(5^x+2)(5^x-1)$ **f** $(7^x-4)(7^x-3)$

4 **a** 2^n **b** 10^a **c** 3^b **d** $\frac{1}{5^n}$ **e** 5^x
f $(\frac{3}{4})^a$ **g** 5 **h** 5^n

5 **a** 3^m+1 **b** $1+6^n$ **c** 4^n+2^n **d** 4^x-1
e 6^n **f** 5^n **g** 4 **h** 2^n-1 **i** $\frac{1}{2}$

6 **a** $n\,2^{n+1}$ **b** -3^{n-1}

EXERCISE 4F

1 **a** $x=3$ **b** $x=2$ **c** $x=4$ **d** $x=0$
e $x=-1$ **f** $x=\frac{1}{2}$ **g** $x=-3$ **h** $x=2$
i $x=-3$ **j** $x=-4$ **k** $x=2$ **l** $x=1$

2 **a** $x=\frac{5}{3}$ **b** $x=-\frac{3}{2}$ **c** $x=-\frac{3}{2}$ **d** $x=-\frac{1}{2}$
e $x=-\frac{2}{3}$ **f** $x=-\frac{5}{4}$ **g** $x=\frac{3}{2}$ **h** $x=\frac{5}{2}$
i $x=\frac{1}{8}$ **j** $x=\frac{9}{2}$ **k** $x=-4$ **l** $x=-4$
m $x=0$ **n** $x=\frac{7}{2}$ **o** $x=-2$ **p** $x=-6$

3 **a** $x=\frac{1}{7}$ **b** has no solutions **c** $x=2\frac{1}{2}$

4 **a** $x=1$ **b** $x=2$ **c** $x=1$
d $x=\frac{5}{4}$ **e** $x=2$ **f** $x=-\frac{9}{7}$

5 **a** $x=3$ **b** $x=2$ **c** $x=2$
d $x=2$ **e** $x=-2$ **f** $x=-2$

6 **a** $x=1$ or 2 **b** $x=1$ **c** $x=1$ or 2
d $x=1$ **e** $x=2$ **f** $x=0$

EXERCISE 4G

1 **a** 1.4 **b** 1.7 **c** 2.8 **d** 0.4

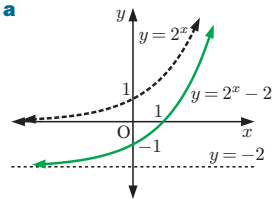
2 **a** $x\approx1.6$ **b** $x\approx-0.7$ **c** $x\approx2.1$ **d** $x\approx-1.7$

3 $y=2^x$ has a horizontal asymptote of $y=0$

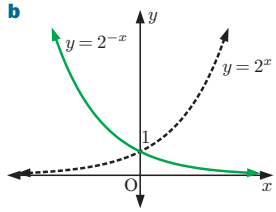
4 **a** 2 **b** 54 **c** $\frac{2}{9}$

5 **a** $g(0)=3,\quad g(-1)=\frac{11}{5}$ **b** $a=2$

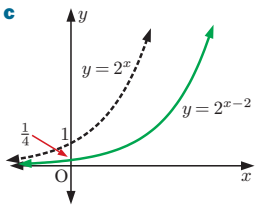
6 **a**



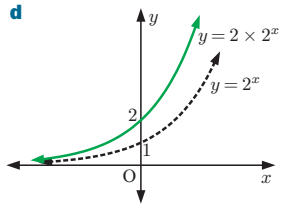
b



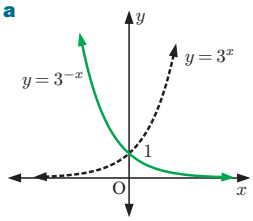
c



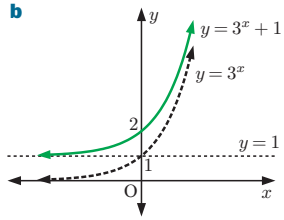
d



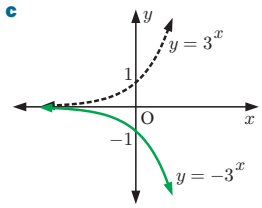
7 **a**



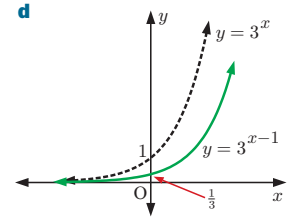
b



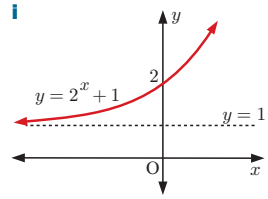
c



d



8 **a** **i**



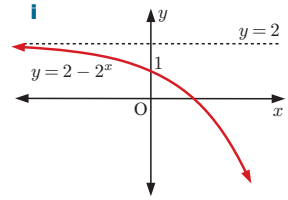
ii Domain: $\{x : x \in \mathbb{R}\}$
Range: $\{y : y > 1\}$

iii $y \approx 3.67$

iv As $x \to \infty,\ y \to \infty$
As $x \to -\infty,\ y \to 1$ from above

v $y = 1$

b **i**



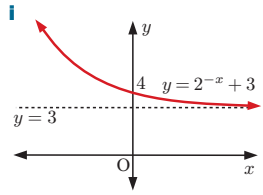
ii Domain: $\{x : x \in \mathbb{R}\}$
Range: $\{y : y < 2\}$

iii $y \approx -0.665$

iv As $x \to \infty,\ y \to -\infty$
As $x \to -\infty,\ y \to 2$ from below

v $y = 2$

c **i**



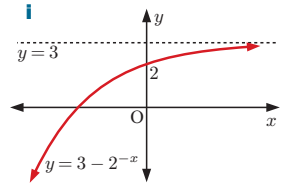
ii Domain: $\{x : x \in \mathbb{R}\}$
Range: $\{y : y > 3\}$

iii $y \approx 3.38$

iv As $x \to \infty,\ y \to 3$ from above
As $x \to -\infty,\ y \to \infty$

v $y = 3$

d **i**



ii Domain: $\{x : x \in \mathbb{R}\}$
Range: $\{y : y < 3\}$

iii $y \approx 2.62$

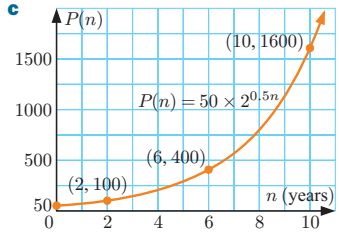
iv As $x \to \infty,\ y \to 3$ from below
As $x \to -\infty,\ y \to -\infty$

v $y = 3$

9 **a** $P_0 = 50$

b **i** 100 possums
ii 400 possums
iii 1600 possums

c



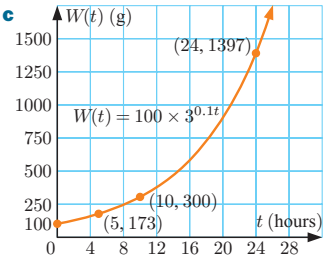
d 8 years

10 **a** 100 g

b **i** ≈ 173 g **ii** 300 g **iii** ≈ 1397 g

d 20 hours

c

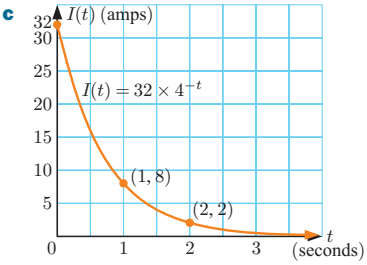


11 **a** 32 amps

b **i** 8 amps **ii** 2 amps

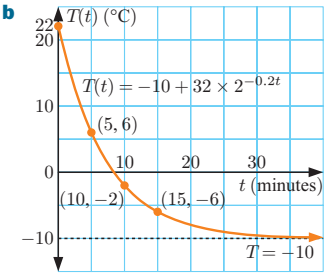
d 3 seconds

c



12 **a** **i** 22°C **ii** 6°C **iii** −2°C **iv** −6°C

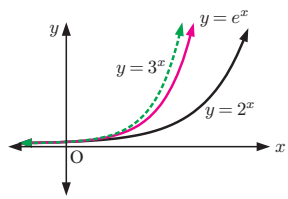
b



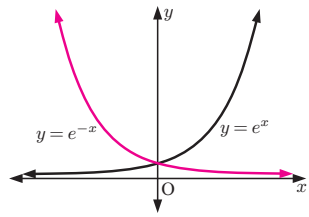
c The temperature will not reach −10°C according to this model, as the model has a horizontal asymptote at $T = -10$.

EXERCISE 4H

1 The graph of $y = e^x$ lies between $y = 2^x$ and $y = 3^x$.



2 One is the other reflected in the y-axis.



3 a

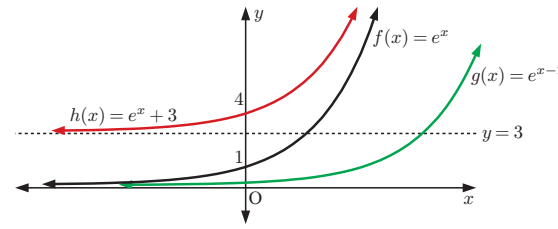
4 **a** $e^x > 0$ for all x

b **i** 0.000 000 004 12 **ii** 970 000 000

5 **a** ≈ 7.39 **b** ≈ 20.1 **c** ≈ 2.01 **d** ≈ 1.65 **e** ≈ 0.368

6 **a** $e^{\frac{1}{2}}$ **b** $e^{-\frac{1}{2}}$ **c** e^{-2} **d** $e^{\frac{3}{2}}$

7

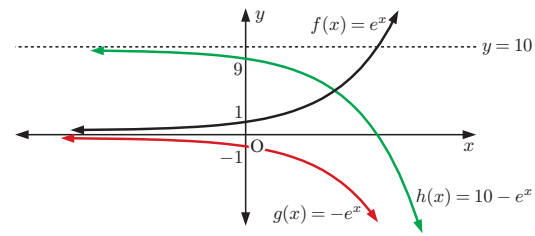


Domain of f, g, and h is $\{x : x \in \mathbb{R}\}$

Range of f is $\{y : y > 0\}$, Range of g is $\{y : y > 0\}$

Range of h is $\{y : y > 3\}$

8



Domain of f, g, and h is $\{x : x \in \mathbb{R}\}$

Range of f is $\{y : y > 0\}$, Range of g is $\{y : y < 0\}$

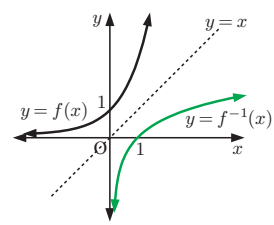
Range of h is $\{y : y < 10\}$

9 **a** $e^{2x} + 2e^x + 1$ **b** $1 - e^{2x}$ **c** $1 - 3e^x$

10 **a** $x = \frac{1}{2}$ **b** $x = -4$

11 **a** $fg(x) = e^{3x+2}$, $gf(x) = 3e^x + 2$ **b** $x = -1$

12 **a**



b Domain of f^{-1} is $\{x : x > 0\}$,

Range of f^{-1} is $\{y : y \in \mathbb{R}\}$

REVIEW SET 4A

1 **a** $-15 + 20\sqrt{3}$ **b** $86 - 60\sqrt{2}$

2 **a** $\frac{2\sqrt{3}}{3}$ **b** $\frac{\sqrt{35}}{5}$ **c** $\frac{\sqrt{7}}{28}$

3 **a** a^6b^7 **b** $\dfrac{2}{3x}$ **c** $\dfrac{y^2}{5}$

4 **a** **i** 81 **ii** $\frac{1}{3}$ **b** $k = 9$

5 **a** $\dfrac{1}{x^5}$ **b** $\dfrac{2}{a^2b^2}$ **c** $\dfrac{2a}{b^2}$

6 **a** 3^{3-2a} **b** $3^{\frac{5}{2}-\frac{9}{2}x}$ **7** **a** 4 **b** $\frac{1}{9}$

8 **a** $\dfrac{m}{n^2}$ **b** $\dfrac{1}{m^3n^3}$ **c** $\dfrac{m^2p^2}{n}$ **d** $\dfrac{16n^2}{m^2}$

9 **a** $9 - 6e^x + e^{2x}$ **b** $x - 4$ **c** $2^x + 1$

10 $x = \frac{9}{34} + \frac{1}{34}\sqrt{13}$

11 **a** $x = -2$ **b** $x = \frac{3}{4}$ **c** $x = -\frac{1}{4}$

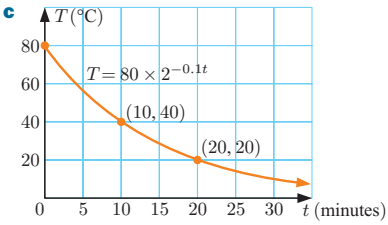
12 **a** **C** **b** **E** **c** **A** **d** **B** **e** **D**

13 **a** 3 **b** 24 **c** $\frac{3}{4}$

14 **a** Range of f is $\{y : y > -3\}$ **b** -2 **c** $x = \frac{1}{2}$

15 **a** 80°C **b** **i** 40°C **ii** 20°C **d** 30 minutes

c



REVIEW SET 4B

1 **a** $17 - 11\sqrt{3}$ **b** 28

2 **a** $\dfrac{5+\sqrt{3}}{22}$ **b** $\dfrac{\sqrt{77}+2\sqrt{11}}{3}$ **c** $\dfrac{26+11\sqrt{2}}{7}$ **d** $\dfrac{-33-14\sqrt{5}}{3}$

3 **a** $x \approx 1.45$ **b** $x \approx -0.6$ **c** $x \approx 1.1$

4 **a** $3-2\sqrt{2}$ **b** $3-2\sqrt{2}$ **c** $3-2\sqrt{2}$ **d** $3-2\sqrt{2}$

5 **a** a^{21} **b** p^4q^6 **c** $\dfrac{4b}{a^3}$

6 **a** 2^{-3} **b** 2^7 **c** 2^{12}

7 **a** $4m^6$ **b** $\dfrac{-a^9}{b^3}$ **c** $3x^3y^2$ **d** $16ab^{\frac{4}{5}}$

8 2^{2x} **9** **a** 5^0 **b** $5^{\frac{3}{2}}$ **c** $5^{-\frac{1}{4}}$ **d** 5^{2a+6}

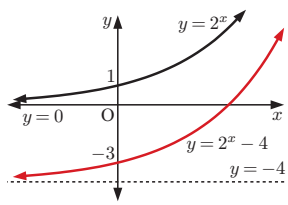
10 **a** $1+e^{2x}$ **b** $2^{2x}+10(2^x)+25$ **c** $x-49$

11 **a** $x=5$ **b** $x=-4$

12 **a** $x=-\frac{2}{5}$ **b** $x=1$ **c** $x=\frac{7}{11}$

13 **a** $\frac{1}{\sqrt{2}}+1 \approx 1.71$ **b** $a=-1$

14

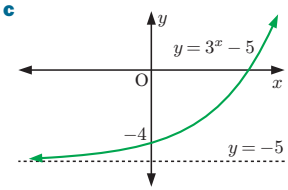


15 **a**

x	-2	-1	0	1	2
y	$-4\frac{8}{9}$	$-4\frac{2}{3}$	-4	-2	4

b as $x \to \infty$, $y \to \infty$;
as $x \to -\infty$, $y \to -5$ (above)

c



d $\{y : y > -5\}$

16 **a** Range of f is $\{y : y > 0\}$ **b** $g(\sqrt{2}) = e^2$ **c** $1+\frac{1}{2}\sqrt{2}$

EXERCISE 5A

1 **a** 4 **b** -3 **c** 1 **d** 0 **e** $\frac{1}{2}$ **f** $\frac{1}{3}$ **g** $-\frac{1}{4}$ **h** $1\frac{1}{2}$ **i** $\frac{2}{3}$ **j** $1\frac{1}{2}$ **k** $1\frac{1}{3}$ **l** $3\frac{1}{2}$

2 **a** n **b** $a+2$ **c** $1-m$ **d** $a-b$

3 **a** $\lg 41 \approx 1.6128$ **b** $41 \approx 10^{1.6128}$

4 **a** $10^{0.7782}$ **b** $10^{1.7782}$ **c** $10^{3.7782}$ **d** $10^{-0.2218}$ **e** $10^{-2.2218}$ **f** $10^{1.1761}$ **g** $10^{3.1761}$ **h** $10^{0.1761}$ **i** $10^{-0.8239}$ **j** $10^{-3.8239}$

5 A negative number cannot be written in the form 10^b where $b \in \mathbb{R}$, so its logarithm cannot be found.

6 **a** **i** 0.477 **ii** 2.477 **b** $\lg 300 = \lg(3 \times 10^2)$

7 **a** **i** 0.699 **ii** -1.301 **b** $\lg 0.05 = \lg(5 \times 10^{-2})$

8 **a** $x=100$ **b** $x=10$ **c** $x=1$ **d** $x=\frac{1}{10}$ **e** $x=10^{\frac{1}{2}}$ **f** $x=10^{-\frac{1}{2}}$ **g** $x=10\,000$ **h** $x=0.000\,01$ **i** $x \approx 6.84$ **j** $x \approx 140$ **k** $x \approx 0.0419$ **l** $x \approx 0.000\,631$

EXERCISE 5B

1 **a** $10^2=100$ **b** $10^4=10\,000$ **c** $10^{-1}=0.1$ **d** $10^{\frac{1}{2}}=\sqrt{10}$ **e** $2^3=8$ **f** $3^2=9$ **g** $2^{-2}=\frac{1}{4}$ **h** $3^{1.5}=\sqrt{27}$ **i** $5^{-\frac{1}{2}}=\frac{1}{\sqrt{5}}$

2 **a** $\log_2 4=2$ **b** $\log_4 64=3$ **c** $\log_5 25=2$ **d** $\log_7 49=2$ **e** $\log_2 64=6$ **f** $\log_2(\frac{1}{8})=-3$ **g** $\log_{10} 0.01=-2$ **h** $\log_2(\frac{1}{2})=-1$ **i** $\log_3(\frac{1}{27})=-3$

3 **a** 5 **b** -2 **c** $\frac{1}{2}$ **d** 3 **e** 6 **f** 7 **g** 2 **h** 3 **i** -3 **j** $\frac{1}{2}$ **k** 2 **l** $\frac{1}{2}$ **m** 5 **n** $\frac{1}{3}$ **o** $n,\ a>0$ **p** $\frac{1}{3}$ **q** $-1,\ t>0$ **r** $\frac{3}{2}$ **s** 0 **t** 1

4 **a** ≈ 2.18 **b** ≈ 1.40 **c** ≈ 1.87 **d** ≈ -0.0969

5 **a** $x=8$ **b** $x=2$ **c** $x=3$ **d** $x=14$

6 **a** 2 **b** 2 **c** -1 **d** $\frac{3}{4}$ **e** $-\frac{1}{2}$ **f** $\frac{5}{2}$ **g** $-\frac{3}{2}$ **h** $-\frac{3}{4}$ **i** $2,\ x>0$ **j** $\frac{1}{2},\ x>0$ **k** $3,\ m>0$ **l** $\frac{3}{2},\ x>0$ **m** $-1,\ n>0$ **n** $-2,\ a>0$ **o** $-\frac{1}{2},\ a>0$ **p** $\frac{5}{2},\ m>0$

EXERCISE 5C

1 **a** $\lg 16$ **b** $\lg 20$ **c** $\lg 8$ **d** $\lg\dfrac{p}{m}$ **e** 1 **f** $\lg 2$ **g** $\lg 24$ **h** $\log_2 6$ **i** $\lg 0.4$ **j** 1 **k** $\lg 200$ **l** $\lg(10^t \times w)$ **m** $\log_m\left(\dfrac{40}{m^2}\right)$ **n** 0 **o** $\lg(0.005)$ **p** $\log_5(\frac{5}{2})$ **q** 2 **r** $\lg 28$

2 **a** $\lg 96$ **b** $\lg 72$ **c** $\lg 8$ **d** $\log_3(\frac{25}{8})$ **e** 1 **f** $\lg\frac{1}{2}$ **g** $\lg 20$ **h** $\lg 25$ **i** $\log_n\left(\dfrac{n^2}{10}\right)$

3 **a** 2 **b** $\frac{3}{2}$ **c** 3 **d** $\frac{1}{2}$ **e** -2 **f** $-\frac{3}{2}$

4 For example, for **a**, $\lg 9 = \lg 3^2 = 2\lg 3$

5 **a** 2 **b** -1 **c** 1

6 **a** $x+z$ **b** $z+2y$ **c** $x+z-y$ **d** $2x+\frac{1}{2}y$ **e** $3y-\frac{1}{2}z$ **f** $2z+\frac{1}{2}y-3x$

7 **a** $p+q$ **b** $2q+r$ **c** $2p+3q$ **d** $r+\frac{1}{2}q-p$ **e** $r-5p$ **f** $p-2q$

8 **a** 0.86 **b** 2.15 **c** 1.075 **9** $\log_b Q = 3$

10 **a** $\log_t A + 3\log_t B = 15,\ 2\log_t A - \log_t B = 9$ **b** $\log_t A = 6,\ \log_t B = 3$ **c** $\log_t(B^5\sqrt{A}) = 18$ **d** $B = t^3$

EXERCISE 5D.1

1 **a** $\lg y = x \lg 2$ **b** $\lg y = 3 \lg x$
c $\lg M = 4 \lg d$ **d** $\lg T = x \lg 5$
e $\lg y = \frac{1}{2} \lg x$ **f** $\lg y = \lg 7 + x \lg 3$
g $\lg S = \lg 9 - \lg t$ **h** $\lg M = 2 + x \lg 7$
i $\lg T = \lg 5 + \frac{1}{2} \lg d$ **j** $\lg F = 3 - \frac{1}{2} \lg n$
k $\lg S = \lg 200 + t \lg 2$ **l** $\lg y = \frac{1}{2} \lg 15 - \frac{1}{2} \lg x$

2 **a** $y = 7^x$ **b** $D = 2x$ **c** $F = \frac{5}{t}$ **d** $y = 6 \times 2^x$
e $P = \sqrt{x}$ **f** $N = \frac{1}{\sqrt[3]{p}}$ **g** $P = 10x^3$ **h** $y = \frac{10^x}{2}$
i $y = \frac{x^2}{10}$ **j** $T = 2k^5$ **k** $P = \frac{n^4}{9}$ **l** $y = 8 \times 16^x$

3 **a** $y = \frac{x^3}{2}$ **b** **i** $y = 4$ **ii** $y = 32$

4 **a** $y = 100(10^{\frac{1}{3}x})$ **b** **i** $y = 100$ **ii** $y = 1000$

5 **a** If there is a *power* relationship between y and x, for example $y = 5x^3$, then there is a *linear* relationship between $\lg y$ and $\lg x$.
b If there is an *exponential* relationship between y and x, for example $y = 4 \times 2^x$, then there is a *linear* relationship between $\lg y$ and x.

EXERCISE 5D.2

1 **a** $x = 25$ **b** $x = 67$ **c** $x = 20$ **d** $x = \frac{125}{64}$
e $x = 5$ **f** no solution **g** $x = \frac{9}{8}$ **h** no solution

2 **a** $x = 5$ **b** $x = 3$ or 6 **c** $x = 2$ or 4 **d** $x = 2$
e $x = 1$ **f** no solution **g** $x = 2$ **h** $x = 4$

3 **a** $x = 8$ **b** $x = 3$ **c** $x = 6$ **d** $x = 4$

EXERCISE 5E.1

1 **a** 2 **b** 3 **c** $\frac{1}{2}$ **d** 0 **e** -1 **f** $\frac{1}{3}$ **g** -2
h $-\frac{1}{2}$

2 **a** 3 **b** 9 **c** $\frac{1}{5}$ **d** $\frac{1}{4}$

3 x does not exist such that $e^x = -2$ or 0

4 **a** a **b** $a + 1$ **c** $a + b$ **d** ab **e** $a - b$

5 **a** $e^{1.7918}$ **b** $e^{4.0943}$ **c** $e^{8.6995}$ **d** $e^{-0.5108}$
e $e^{-5.1160}$ **f** $e^{2.7081}$ **g** $e^{7.3132}$ **h** $e^{0.4055}$
i $e^{-1.8971}$ **j** $e^{-8.8049}$

6 **a** $x \approx 20.1$ **b** $x = e \approx 2.72$ **c** $x = 1$
d $x = \frac{1}{e} \approx 0.368$ **e** $x \approx 0.006\,74$
f $x \approx 2.30$ **g** $x \approx 8.54$ **h** $x \approx 0.0370$

EXERCISE 5E.2

1 **a** $\ln 45$ **b** $\ln 5$ **c** $\ln 4$ **d** $\ln 24$
e $\ln 1 = 0$ **f** $\ln 30$ **g** $\ln(4e)$ **h** $\ln\left(\frac{6}{e}\right)$
i $\ln 20$ **j** $\ln(4e^2)$ **k** $\ln\left(\frac{20}{e^2}\right)$ **l** $\ln 1 = 0$

2 **a** $\ln 972$ **b** $\ln 200$ **c** $\ln 1 = 0$ **d** $\ln 16$ **e** $\ln 6$
f $\ln\left(\frac{1}{3}\right)$ **g** $\ln\left(\frac{1}{2}\right)$ **h** $\ln 2$ **i** $\ln 16$

3 For example, for **a**, $\ln 27 = \ln 3^3 = 3 \ln 3$

4 **Hint:** In **d**, $\ln\left(\frac{e^2}{8}\right) = \ln e^2 - \ln 2^3$

5 **a** $D = ex$ **b** $F = \frac{e^2}{p}$ **c** $P = 5e^{2x}$
d $M = e^3 y^2$ **e** $B = \frac{1}{4}e^{3t}$ **f** $N = \frac{1}{\sqrt[3]{g}}$
g $Q \approx 8.66x^3$ **h** $D \approx 0.518n^{0.4}$ **i** $T \approx \frac{4.85}{e^x}$

EXERCISE 5F

1 **a** $x \approx 3.32$ **b** $x \approx 2.73$ **c** $x \approx 3.32$
d $x = 4$ **e** $x \approx 8.00$ **f** $x = -5$

2 **a** $x \approx 1.43$ **b** $x \approx 1.56$ **c** $x \approx 3.44$
d $x \approx 5.82$ **e** $x \approx -1.34$ **f** $x \approx 2.37$
g $x \approx 0.275$ **h** $x \approx 1.81$ **i** $x \approx 9.64$

3 **a** $x = \ln 10$ **b** $x = \ln 1000$ **c** $x = \ln 0.15$
d $x = 2 \ln 5$ **e** $x = \frac{1}{2} \ln 18$ **f** $x = 0$

4 **a** $x = \frac{1}{2} \ln 300$ **b** $x \approx 2.85$

5 **a** $x = -\frac{\lg(0.03)}{\lg 2}$ **b** $x = \frac{10 \lg\left(\frac{10}{3}\right)}{\lg 5}$ **c** $x = \frac{-4 \lg\left(\frac{1}{8}\right)}{\lg 3}$

6 **a** 3.90 hours **b** 15.5 hours **7** **b** $t \approx 6.93$ hours

8 **a** 50 g **b** $\approx 13\,200$ years

9 **a** $x = \ln 2$ **b** $x = 0$ **c** $x = \ln 2$ or $\ln 3$ **d** $x = 0$
e $x = \ln 4$ **f** $x = \ln\left(\frac{3+\sqrt{5}}{2}\right)$ or $\ln\left(\frac{3-\sqrt{5}}{2}\right)$

10 **a** $(\ln 3, 3)$ **b** $(\ln 2, 5)$ **c** $(0, 2)$ and $(\ln 5, -2)$

EXERCISE 5G

1 **a** ≈ 2.26 **b** ≈ -10.3 **c** ≈ -2.46 **d** ≈ 5.42

2 **a** $x \approx -4.29$ **b** $x \approx 3.87$ **c** $x \approx 0.139$

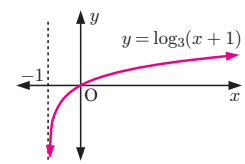
3 **a** $\log_9 26 = \frac{1}{2} \log_3 26$ **b** $\log_2 11 = 2 \log_4 11$
c $\frac{6}{\log_7 25} = 3 \log_5 7$

4 **a** $x = \sqrt[3]{50}$ **b** $x = \sqrt{13}$ **c** $x = 49$
d $x = 5$ **e** $x = 8$ **f** $x = 16$

5 **b** **i** $x = \frac{1}{9}$ or 9 **ii** $x = \frac{1}{2}$ or 32 **iii** $x = 2$ or 64

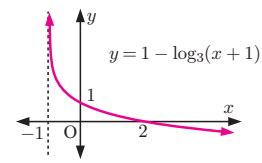
EXERCISE 5H

1 **a** **i** Domain is $\{x : x > -1\}$, Range is $\{y : y \in \mathbb{R}\}$
ii VA is $x = -1$, x and y-intercepts 0
iii



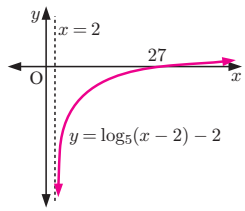
iv $x = -\frac{2}{3}$
v $f^{-1}(x) = 3^x - 1$

b **i** Domain is $\{x : x > -1\}$, Range is $\{y : y \in \mathbb{R}\}$
ii VA is $x = -1$, x-intercept 2, y-intercept 1
iii

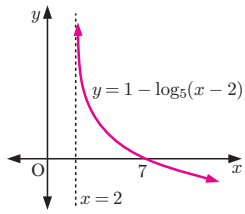


iv $x = 8$
v $f^{-1}(x) = 3^{1-x} - 1$

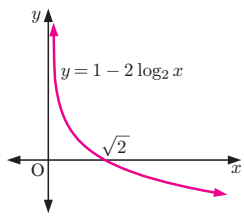
c **i** Domain is $\{x : x > 2\}$, Range is $\{y : y \in \mathbb{R}\}$
ii VA is $x = 2$, x-intercept 27, no y-intercept
iv $x = 7$
v $f^{-1}(x) = 5^{2+x} + 2$
iii



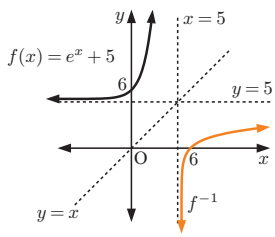
d **i** Domain is $\{x : x > 2\}$, Range is $\{y : y \in \mathbb{R}\}$
ii VA is $x = 2$, x-intercept 7, no y-intercept
iv $x = 27$
v $f^{-1}(x) = 5^{1-x} + 2$
iii



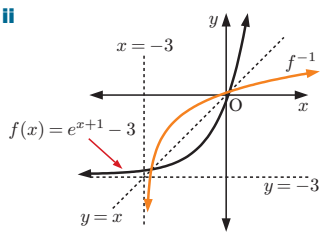
e **i** Domain is $\{x : x > 0\}$, Range is $\{y : y \in \mathbb{R}\}$
ii VA is $x = 0$, x-intercept $\sqrt{2}$, no y-intercept
iv $x = 2$
v $f^{-1}(x) = 2^{\frac{1-x}{2}}$
iii



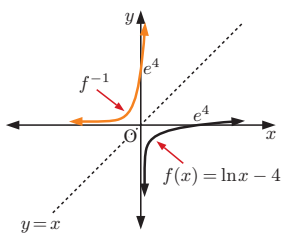
2 **a** **i** $f^{-1}(x) = \ln(x - 5)$
iii Domain of f is $\{x : x \in \mathbb{R}\}$, Range is $\{y : y > 5\}$
Domain of f^{-1} is $\{x : x > 5\}$, Range is $\{y : y \in \mathbb{R}\}$
iv f has a HA $y = 5$, f has y-int 6
f^{-1} has a VA $x = 5$, f^{-1} has x-int 6
ii



b **i** $f^{-1}(x) = \ln(x + 3) - 1$
iii Domain of f is $\{x : x \in \mathbb{R}\}$, Range is $\{y : y > -3\}$
Domain of f^{-1} is $\{x : x > -3\}$, Range is $\{y : y \in \mathbb{R}\}$
iv f has a HA $y = -3$, x-int $\ln 3 - 1$, y-int $e - 3$
f^{-1} has a VA $x = -3$, x-int $e - 3$, y-int $\ln 3 - 1$
ii

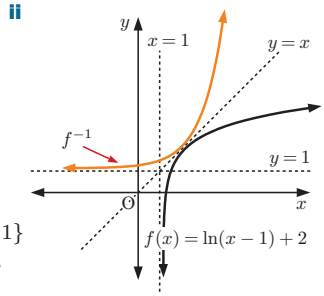


c **i** $f^{-1}(x) = e^{x+4}$
iii Domain of f is $\{x : x > 0\}$, Range of f is $\{y : y \in \mathbb{R}\}$
Domain of f^{-1} is $\{x : x \in \mathbb{R}\}$, Range is $\{y : y > 0\}$
ii

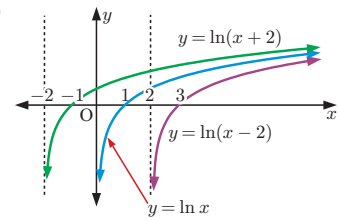


iv f has a VA $x = 0$, x-int e^4
f^{-1} has a HA $y = 0$, y-int e^4

d **i** $f^{-1}(x) = 1 + e^{x-2}$
iii Domain of f is $\{x : x > 1\}$, Range is $\{y : y \in \mathbb{R}\}$
Domain of f^{-1} is $\{x : x \in \mathbb{R}\}$, Range is $\{y : y > 1\}$
iv f has a VA $x = 1$, x-int $1 + e^{-2}$
f^{-1} has a HA $y = 1$, y-int $1 + e^{-2}$
ii



3 **a** A is $y = \ln x$ as its x-intercept is 1
c $y = \ln x$ has VA $x = 0$
$y = \ln(x - 2)$ has VA $x = 2$
$y = \ln(x + 2)$ has VA $x = -2$
b



4 $y = \ln(x^2) = 2\ln x$, so she is correct.
This is because the y-values are twice as large for $y = \ln(x^2)$ as they are for $y = \ln x$.

5 **a** $f^{-1} : x \mapsto \ln(x - 2) - 3$
b **i** $x < -5.30$ **ii** $x < -7.61$ **iii** $x < -9.91$
iv $x < -12.2$ Conjecture HA is $y = 2$
c as $x \to \infty$, $f(x) \to \infty$,
as $x \to -\infty$, $e^{x+3} \to 0$ and $f(x) \to 2$
$\therefore$ HA is $y = 2$
d VA of f^{-1} is $x = 2$, Domain of f^{-1} is $\{x : x > 2\}$

6 **a** **i** $f(5) = 3$ **ii** $f(x^2) = \log_2(x^2 + 3)$
iii $f(2x - 1) = 1 + \log_2(x + 1)$
b Domain of $f(x)$ is $\{x : x > -3\}$ **c** $x = \pm 5$

7 **a** Range is $\{y : y > 1\}$ **b** $f^{-1}(x) = \frac{1}{3}\ln(x - 1)$
c $f^{-1}(10) = \frac{1}{3}\ln 9$
d Domain of $f^{-1}(x)$ is $\{x : x > 1\}$
e $(f \circ f^{-1})(x) = (f^{-1} \circ f)(x) = x$

8 **a** $f^{-1}(x) = \frac{1}{2}\ln x$
i $(f^{-1} \circ g)(x) = \frac{1}{2}\ln(2x - 1)$
ii $(g \circ f)^{-1}(x) = \frac{1}{2}\ln\left(\frac{x+1}{2}\right)$
b $x = 13$

9 **a** $f(1) = \frac{10}{e}$, $g(6) = \ln 3$ **b** x-intercept of $g(x)$ is 4
c $fg(x) = \frac{10}{x - 3}$ **d** $x = \ln 2$

10 **a** Domain of $f(x)$ is $\{x : x > -6\}$
b $f^{-1}(x) = e^x - 6$
c x-intercept is -5, y-intercept is $\ln 6$ **d** $x = -\frac{8}{3}$ or 3

REVIEW SET 5A

1 **a** 3 **b** 8 **c** -2 **d** $\frac{1}{2}$ **e** 0
f $\frac{1}{4}$ **g** -1 **h** $\frac{1}{2}$, $k > 0$

2 **a** $\frac{1}{2}$ **b** $-\frac{1}{3}$ **c** $a+b+1$

3 **a** $\ln 144$ **b** $\ln\left(\frac{3}{2}\right)$ **c** $\ln\left(\frac{25}{e}\right)$ **d** $\ln 3$

4 **a** $\frac{3}{2}$ **b** -3 **c** $2x$ **d** $1-x$

5 **a** $\lg 144$ **b** $\log_2\left(\frac{16}{9}\right)$ **c** $\log_4 80$

6 **a** $\lg P = \lg 3 + x\lg 7$ **b** $\lg m = 3\lg n - \lg 5$

7 **a** $x=3$ **b** $x=5$

8 **Hint:** Use change of base rule.

9 **a** $T = \dfrac{x^2}{5}$ **b** $K = 3\times 2^x$

10 **a** $5\ln 2$ **b** $3\ln 5$ **c** $6\ln 3$

11

Function	$y=\log_2 x$	$y=\ln(x+5)$
Domain	$x>0$	$x>-5$
Range	$y\in\mathbb{R}$	$y\in\mathbb{R}$

12 **a** $2A+2B$ **b** $A+3B$ **c** $3A+\frac{1}{2}B$
d $4B-2A$ **e** $3A-2B$

13 **a** $x=0$ or $\ln\left(\frac{2}{3}\right)$ **b** $x=e^2$

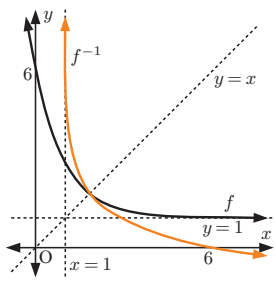
14 **a** $x\approx 2.46$ **b** $x\approx 1.88$ **15** ≈ 6.97 years

16 **a** Range of f is $\{y : y>1\}$

b **i** $f^{-1}(x) = \ln\left(\dfrac{5}{x-1}\right)$ **ii** $f^{-1}(2) = \ln 5$

c Domain of f^{-1} is $\{x : x>1\}$ **d** $x=6$

e



REVIEW SET 5B

1 **a** $\frac{3}{2}$ **b** $\frac{2}{3}$ **c** $a+b$

2 **a** $\approx 10^{1.5051}$ **b** $\approx 10^{-2.8861}$ **c** $\approx 10^{-4.0475}$

3 **a** $x=\frac{1}{8}$ **b** $x\approx 82.7$ **c** $x\approx 0.0316$

4 **a** $k\approx 3.25\times 2^x$ **b** $Q=5P^3$ **c** $A=6\times 2^x$

5 **a** $x=\dfrac{\lg 7}{\lg 5}$ **b** $x=2$

6 -1 **7** $\log_8 30 = \frac{1}{3}\log_2 30$

8 **a** $x=8$ **b** $x=3$ **9** **a** 9 **b** $\ln 5$

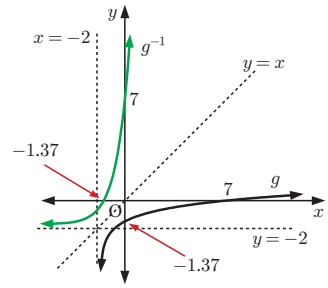
10 **a** $\ln 3$ **b** $\ln 4$ **c** $\ln 125$

11 **a** $\lg M = \lg 5 + x\lg 6$ **b** $\lg T = \lg 5 - \frac{1}{2}\lg l$
c $\lg G = \lg 4 - \lg c$

12 **a** $x=\ln 3$ **b** $x=\ln 3$ or $\ln 4$

13 **a** Domain is $\{x : x>-2\}$, Range is $\{y : y\in\mathbb{R}\}$
b VA is $x=-2$, x-intercept is 7, y-intercept is ≈ -1.37
c $g^{-1}(x) = 3^{x+2}-2$

d



14 13.9 weeks

15 **a** $x=5$ **b** $x=32$ or $\frac{1}{32}$ **c** $x=9$ or 81

16 **a** Domain is $\{x : x>4\}$, Range is $\{y : y\in\mathbb{R}\}$
b x-intercept is 5, no y-intercept **c** $x=4+\sqrt{6}$
d $x=0$

EXERCISE 6A.1

1 **a** $3x^2+6x+9$ **b** $5x^2+7x+9$ **c** $-7x^2-8x-9$
d $4x^4+13x^3+28x^2+27x+18$

2 **a** x^3+x^2-4x+7 **b** x^3-x^2-2x+3
c $3x^3+2x^2-11x+19$ **d** $2x^3-x^2-x+5$
e $x^5-x^4-x^3+8x^2-11x+10$
f $x^4-2x^3+5x^2-4x+4$

3 **a** $2x^3-3x^2+4x+3$ **b** $x^4+x^3-7x^2+7x-2$
c $x^3+6x^2+12x+8$ **d** $4x^4-4x^3+13x^2-6x+9$
e $16x^4-32x^3+24x^2-8x+1$
f $18x^4-87x^3+56x^2+20x-16$

4 **a** $6x^3-11x^2+18x-5$ **b** $8x^3+18x^2-x+10$
c $-2x^3+7x^2+13x+10$ **d** $2x^3-7x^2+4x+4$
e $2x^4-2x^3-9x^2+11x-2$
f $15x^4+x^3-x^2+7x-6$
g $x^4-2x^3+7x^2-6x+9$
h $4x^4+4x^3-15x^2-8x+16$
i $8x^3+60x^2+150x+125$
j $x^6+2x^5+x^4-4x^3-4x^2+4$

EXERCISE 6A.2

1 **a** $Q(x)=x$, $R=-3$, $x^2+2x-3 = x(x+2)-3$
b $Q(x)=x-4$, $R=-3$,
$x^2-5x+1 = (x-4)(x-1)-3$
c $Q(x)=2x^2+10x+16$, $R=35$,
$2x^3+6x^2-4x+3 = (2x^2+10x+16)(x-2)+35$

2 **a** $x^2-3x+6 = (x+1)(x-4)+10$
b $x^2+4x-11 = (x+1)(x+3)-14$
c $2x^2-7x+2 = (2x-3)(x-2)-4$
d $2x^3+3x^2-3x-2 = (x^2+x-2)(2x+1)$
e $3x^3+11x^2+8x+7 = (x^2+4x+4)(3x-1)+11$
f $2x^4-x^3-x^2+7x+4$
$= (x^3-2x^2+\frac{5}{2}x-\frac{1}{4})(2x+3)+\frac{19}{4}$

3 **a** $x+2+\dfrac{9}{x-2}$ **b** $2x+1-\dfrac{1}{x+1}$
c $3x-4+\dfrac{3}{x+2}$ **d** x^2+3x-2
e $2x^2-8x+31-\dfrac{124}{x+4}$ **f** $x^2+3x+6+\dfrac{7}{x-2}$

EXERCISE 6A.3

1 **a** quotient is $x+1$, remainder is $-x-4$
b quotient is 3, remainder is $-x+3$
c quotient is $3x$, remainder is $-2x-1$
d quotient is 0, remainder is $x-4$

2 **a** $1-\dfrac{2x}{x^2+x+1}$, $x^2-x+1=1(x^2+x+1)-2x$

b $x-\dfrac{2x}{x^2+2}$, $x^3=x(x^2+2)-2x$

c $x^2+x+3+\dfrac{3x-4}{x^2-x+1}$,
$x^4+3x^2+x-1=(x^2+x+3)(x^2-x+1)+3x-4$

d $2x+4+\dfrac{5x+2}{(x-1)^2}$,
$2x^3-x+6=(2x+4)(x-1)^2+5x+2$

e $x^2-2x+3-\dfrac{4x+3}{(x+1)^2}$,
$x^4=(x^2-2x+3)(x+1)^2-4x-3$

f $x^2-3x+5+\dfrac{15-10x}{(x-1)(x+2)}$,
$x^4-2x^3+x+5=(x^2-3x+5)(x-1)(x+2)+15-10x$

3 quotient is x^2+2x+3, remainder is 7

4 quotient is x^2-3x+5, remainder is $15-10x$

EXERCISE 6B.1

1 **a** $4, -\frac{3}{2}$ **b** $-3\pm\sqrt{10}$ **c** $5\pm\sqrt{19}$
d $0, \pm2$ **e** $0, \pm\sqrt{11}$ **f** $\pm2, \pm\sqrt{2}$

2 **a** $1, -\frac{2}{5}$ **b** $-\frac{1}{2}, \pm\sqrt{3}$ **c** $-3, \frac{1}{3}, 2$
d $0, 1\pm\sqrt{3}$ **e** $0, \pm\sqrt{7}$ **f** $\pm\sqrt{2}, \pm\sqrt{5}$

3 **a** $(2x+3)(x-5)$ **b** $x(x-7)(x-4)$
c $(x-3-\sqrt{6})(x-3+\sqrt{6})$
d $x(x+1+\sqrt{5})(x+1-\sqrt{5})$ **e** $x(3x-2)(2x+1)$
f $(x+1)(x-1)(x+\sqrt{5})(x-\sqrt{5})$

4 $P(\alpha)=0$, $P(\beta)=0$, $P(\gamma)=0$

5 **a** $P(x)=a(x+3)(x-4)(x-5)$, $a\neq0$
b $P(x)=a(x+2)(x-2)(x-3)$, $a\neq0$
c $P(x)=a(x-3)(x^2-2x-4)$, $a\neq0$
d $P(x)=a(x+1)(x^2+4x+2)$, $a\neq0$

6 **a** $P(x)=a(x^2-1)(x^2-2)$, $a\neq0$
b $P(x)=a(x-2)(5x+1)(x^2-3)$, $a\neq0$
c $P(x)=a(x+3)(4x-1)(x^2-2x-1)$, $a\neq0$
d $P(x)=a(x^2-4x-1)(x^2+4x-3)$, $a\neq0$

EXERCISE 6B.2

1 **a** $a=2$, $b=5$, $c=5$ **b** $a=3$, $b=4$, $c=3$
c $a=2$, $b=-5$, $c=4$

2 **a** $a=2$, $b=-2$ or $a=-2$, $b=2$
b $a=3$, $b=-1$

3 **a** $a=1$, $b=6$, $c=-7$ **b** $(x+3)(x+7)(x-1)$

4 **a** $p=2$, $q=7$, $r=5$ **b** $x=\frac{1}{2}, -1, -\frac{5}{2}$

5 **a** $a=3$, $b=-2$, $c=1$
b $3x^3+10x^2-7x+4=(x+4)(3x^2-2x+1)$
Δ of $3x^2-2x+1$ is -8,
$\therefore$ the only real zero is -4.

6 **a** $a=1$, $b=-2$, $c=-1$, $k=-4$
b $-\frac{2}{3}, 1\pm\sqrt{2}$

7 **a** $a=-2$, $b=2$ **b** $-1\pm\sqrt{3}$

8 $a=-11$, zeros are $\frac{3}{2}$, $\dfrac{-3\pm\sqrt{13}}{2}$

9 **a** $a=-9$, $b=-1$
b $P(x)=0$ when $x=-1, -\frac{1}{2}, 2, 4$

10 **Hint:** Let $x^3+3x^2-9x+c=(x+a)^2(x+b)$
When $c=5$, the cubic is $(x-1)^2(x+5)$.
When $c=-27$, the cubic is $(x+3)^2(x-3)$.

EXERCISE 6C

1 **a** $P(x)=Q(x)(x-2)+7$, $P(x)$ divided by $x-2$ leaves a remainder of 7.
b $P(-3)=-8$, $P(x)$ divided by $x+3$ leaves a remainder of -8.
c $P(5)=11$, $P(x)=Q(x)(x-5)+11$

2 **a** 4 **b** -19 **c** 1 **3** 4

4 **a** $a=3$ **b** $a=2$ **5** $a=-5$, $b=6$

6 $a=-5$, $b=6$ **7** -7

8 **a** $P(x)=Q(x)(2x-1)+R$
$P(\frac{1}{2})=Q(\frac{1}{2})(2\times\frac{1}{2}-1)+R$
$=Q(\frac{1}{2})\times0+R$
$=R$

b **i** -3 **ii** 7 **iii** -7

9 $a=3$, $b=10$ **10** **a** -3 **b** 1

EXERCISE 6D

1 **a** factor **b** not a factor **c** factor **d** not a factor

2 **a** $c=2$ **b** $c=-2$ **c** $b=3$

3 $k=-8$, $P(x)=(x+2)(x-2)(2x+1)$

4 **a** $k=-8$ **b** $P(x)=(x-3)(3x^2+x-2)$
c $x=-1, \frac{2}{3}, 3$

5 $a=7$, $b=-14$ **6** $a=3$, $b=2$

7 **a** $a=7$, $b=-6$ **b** 60
c $P(x)=(x+3)(2x^2+3x-2)$ **d** $-3, -2, \frac{1}{2}$

8 **a** $a=7$, $b=2$ **b** $x=-2\pm\sqrt{6}$

9 **a** **i** $P(a)=0$, $x-a$ is a factor
ii $(x-a)(x^2+ax+a^2)$
b **i** $P(-a)=0$, $x+a$ is a factor
ii $(x+a)(x^2-ax+a^2)$

10 $a=2$

EXERCISE 6E

1 **a** $x=1, 2, 3$ **b** $x=-1, 2$ {2 is a double root}
c $x=1, -1, -2$ **d** $x=-1, 3, 4$ **e** $x=-5, -4, 4$
f $x=-3, -5$ {-5 is a double root}

2 **a** $x=-2, 2, 3$ **b** $x=-3, -2, 6$ **c** $x=-3, 4, 7$

REVIEW SET 6A

1 **a** $8x^2+6x+3$ **b** $7x^2-9x+9$
c $15x^4+32x^3+29x-4$

2 **a** quotient $=2x+5$, remainder $=3$
b quotient $=x^2-4x+2$, remainder $=-5$

3 **a** $\frac{4}{3}, -2$ **b** $-4\pm\sqrt{5}$

4 **a** $a = 1,\ b = -2,\ c = 3$
b Δ of $x^2 - 2x + 3$ is -8
$\therefore$ the only real root is $x = -3$.
5 **a** 1 **b** -53 **6** **a** not a factor **b** factor
7 $k = 6$ **8** $a = 4,\ b = -1$ **9** $c = 3$
10 **a** $a = -19,\ b = -20$ **b** $-5, -1, 4$
11 $x = -3, -1, 5$

REVIEW SET 6B

1 **a** $12x^4 - 9x^3 + 8x^2 - 26x + 15$
b $4x^4 - 4x^3 + 13x^2 - 6x + 9$
2 **a** $x^2 - 2x + 4 - \dfrac{8}{x+2}$ **b** $x - 5 + \dfrac{19x + 30}{(x+2)(x+3)}$
3 $P(x) = a(4x - 1)(x^2 - 2x - 4),\quad a \neq 0$
4 For $k = 3,\ b = 27,\ x = 3$ or -3.
For $k = -1,\ b = -5,\ x = -1$ or 5.
5 **a** -3 **b** -7 **6** **a** $a = 5$ **b** -12
7 **b** $(x - 2)(x^2 + 2x - 9)$ **c** $2, -1 \pm \sqrt{10}$
8 $a = \frac{8}{7},\ b = \frac{174}{7}$
9 $k = 8$, the zeros are $-1, -2$ $\{-2$ is a double root$\}$
10 **a** $a = -20,\ b = 12$ **b** $f(x) = (2x - 1)(x - 6)(x + 2)$
11 $x = -4, 2, 3$

EXERCISE 7A.1

1 **a** gradient $= 3$, y-intercept is 5
b gradient $= 4$, y-intercept is -2
c gradient $= \frac{1}{5}$, y-intercept is $\frac{3}{5}$
d gradient $= -7$, y-intercept is -3
e gradient $= \frac{1}{6}$, y-intercept is $\frac{1}{3}$
f gradient $= -\frac{5}{3}$, y-intercept is $\frac{8}{3}$
2 **a** $y = x - 2$ **b** $y = -x + 4$ **c** $y = 2x$
d $y = -\frac{1}{2}x + 3$
3 **a** $y = 4x - 13$ **b** $y = -3x - 5$ **c** $y = -5x + 32$
d $y = \frac{1}{2}x + \frac{7}{2}$ **e** $y = -\frac{1}{3}x + \frac{8}{3}$ **f** $y = 6$
4 **a** $2x - 3y = -11$ **b** $3x - 5y = -23$ **c** $x + 3y = 5$
d $2x + 7y = -2$ **e** $4x - y = -11$ **f** $2x + y = 7$
g $7x + 2y = 18$ **h** $6x - y = -40$
5 **a** $y = \frac{5}{2}x - 2$ **b** $y = -2x + 3$ **c** $y = -2$
d $y = -\frac{1}{5}x + \frac{2}{5}$ **e** $y = \frac{1}{6}x - \frac{11}{6}$ **f** $y = -\frac{2}{3}x - \frac{11}{3}$
6 **a** $x - 3y = -3$ **b** $5x - y = 1$ **c** $x - y = 3$
d $4x - 5y = 10$ **e** $x - 2y = -1$ **f** $2x + 3y = -5$
7 **a** $\sqrt{45}$ units **b** $(-1, \frac{7}{2})$ **c** $\frac{1}{2}$ **d** $y = \frac{1}{2}x + 4$
8 **a** $y = \frac{4}{3}x - 1$ **b** $2x - 3y = -13$ **c** $y = x + 1$
d $2x + y = -2$ **e** $y = -\frac{2}{3}x + 2$ **f** $3x + 7y = -9$
9 **a** $M = \frac{1}{3}p + 2$ **b** $R = -\frac{5}{4}n + 2$ **c** $T = \frac{1}{2}x - 1$
d $F = \frac{1}{10}x + 1$ **e** $H = -\frac{1}{2}z + 2$ **f** $W = -\frac{1}{6}t - 2$
10 **a** $x + 2y = 13$ **b** $(13, 0)$
11 **a** $3x + 5y = 10$ **b** $(0, 2)$ **12** 54 units^2

EXERCISE 7A.2

1 **a** $\sqrt{160}$ units **b** $(-1, 1)$ **c** -3 **d** $x - 3y = -4$
2 **a** $y = x - 4$ **b** $y = 2x + 6$ **c** $y = \frac{6}{5}x + \frac{7}{2}$ **d** $y = 1$
3 15 units^2

EXERCISE 7B

1 **a** $(1, 3)$ **b** $(6, -3)$ **c** $(-5, 3)$ **d** $(-1, -2)$
2 **a** $3x + 5y = 9$ **b** $(-2, 3)$ **3** $(4, 2)$
4 **a** $x - 3y = -8$ **b** $y = -3x - 4$ **c** $(-2, 2)$
5 **a** $(0, -1)$ **b** 25 units^2
6 **a** $(-1, 0)$ **b** 26 units^2 **7** 30 units^2
8 **a** **i** $(5, 0)$ **ii** $(7, -4)$ **iii** $(6, -2)$
b **Hint:** Find the gradients of MN and AC.
c **i** 15 units^2 **ii** 20 units^2

EXERCISE 7C

1 $(-1, -2)$ and $(\frac{11}{5}, -\frac{2}{5})$ **2** $\sqrt{18}$ units
3 $x - 2y = 0$ **4** $(-\frac{4}{3}, -\frac{8}{3})$ and $(2, -1)$
5 $\sqrt{125}$ units **6** $x - 3y = -13$
7 $(3, -\frac{3}{2})$ and $(4, -1)$ **8** $(\frac{7}{3}, \frac{5}{2})$

EXERCISE 7D

1 **a** $y = \frac{1}{2}x^3 + 2$ **b** $y = 3\sqrt{x} - 1,\quad x \geqslant 0$
c $y = 3 - x^4$ **d** $y = \frac{1}{3} \times 2^x$
e $y = \dfrac{2}{x} + 1$ **f** $y = -\frac{3}{2} \times 3^x + 11$
2 **a** **i** $y = x^2 + 3x$ **ii** $y = 18$
b **i** $y = -\frac{1}{2}\sqrt{x} + \dfrac{10}{\sqrt{x}},\quad x > 0$ **ii** $y = \frac{17\sqrt{3}}{6}$
c **i** $y = \dfrac{5}{3x} \times 2^x$ **ii** $y = \frac{40}{9}$
d **i** $y = 2x^3 - 9x$ **ii** $y = 27$
e **i** $y = \dfrac{1}{x^2} - \dfrac{12}{x} + 36$ **ii** $y = 32\frac{1}{9}$
f **i** $y = (x + 2)^2 + 3$ **ii** $y = 28$
3 **a** $\lg y = 2x - 1$ **b** $y = \frac{1}{10} \times 10^{2x}$
4 $y = 1000 \times 10^{-\frac{3}{2}x}$
5 **a** $y = \frac{1}{10\,000} \times 10^x$ **b** $y = 10\,000 \times (\frac{1}{10})^x$
c $y = 5 \times 4^x$
6 **a** $y = 10 \times 10^{\frac{1}{3}x}$ **b** $y = 1000$
7 **a** $\lg y = -\frac{1}{2}\lg x + 2$ **b** $y = \dfrac{100}{\sqrt{x}}$
8 **a** $y = x^{\frac{1}{4}}$ **b** $y = \dfrac{1000}{x}$ **c** $y = x^2\sqrt{1000}$
9 **a** $K = 7\sqrt{t}$ **b** $K = 21$ **10** **a** 3 **b** $\lg 4$

EXERCISE 7E

1 **a**

x^2	1	4	9	16
y	2	11	26	47

c $y = 3x^2 - 1$

b