SYMBOLS AND NOTATION USED IN THIS BOOK

$\mathbb{N}$	the set of natural numbers, $\{1, 2, 3, ...\}$
$\mathbb{Z}$	the set of integers, $\{0, \pm 1, \pm 2, \pm 3, ...\}$
$\mathbb{Z}^+$	the set of positive integers, $\{1, 2, 3, ...\}$
$\mathbb{Q}$	the set of rational numbers
$\mathbb{Q}^+$	the set of positive rational numbers, $\{x \in \mathbb{Q},\ x > 0\}$
$\mathbb{R}$	the set of real numbers
$\mathbb{R}^+$	the set of positive real numbers, $\{x \in \mathbb{R},\ x > 0\}$
$[a, b]$	the closed interval $\{x \in \mathbb{R} : a \leqslant x \leqslant b\}$
$[a, b)$	the interval $\{x \in \mathbb{R} : a \leqslant x < b\}$
$(a, b]$	the interval $\{x \in \mathbb{R} : a < x \leqslant b\}$
(a, b)	the open interval $\{x \in \mathbb{R} : a < x < b\}$
$\{x_1, x_2, ...\}$	the set with elements $x_1, x_2, ...$
$n(A)$	the number of elements in the finite set A
$\{x : ....$	the set of all x such that
$\in$	is an element of
$\notin$	is not an element of
$\varnothing$ or $\{\ \}$	the empty set
$\mathscr{E}$	the universal set
$\cup$	union
$\cap$	intersection
$\subseteq$	is a subset of
$\subset$	is a proper subset of
$\nsubseteq$	is not a subset of
$\not\subset$	is not a proper subset of
A'	the complement of the set A
$a^{\frac{1}{n}},\ \sqrt[n]{a}$	a to the power of $\dfrac{1}{n}$, nth root of a (if $a \geqslant 0$ then $\sqrt[n]{a} \geqslant 0$)
$a^{\frac{1}{2}},\ \sqrt{a}$	a to the power $\frac{1}{2}$, square root of a (if $a \geqslant 0$ then $\sqrt{a} \geqslant 0$)
$\lvert x \rvert$	the modulus or absolute value of x, that is $\begin{cases} x \text{ for } x \geqslant 0, & x \in \mathbb{R} \\ -x \text{ for } x < 0, & x \in \mathbb{R} \end{cases}$
$\equiv$	identity or is equivalent to
$\approx$	is approximately equal to
$n!$	n factorial for $n \in \mathbb{N}$ $(0! = 1)$
$\binom{n}{r}$	the binomial coefficient $\dfrac{n!}{r!(n-r)!}$ for $n, r \in \mathbb{N},\ 0 \leqslant r \leqslant n$
$>$	is greater than
$\geq$ or $\geqslant$	is greater than or equal to
$<$	is less than
$\leq$ or $\leqslant$	is less than or equal to

$\sum_{i=1}^{n} a_i$	$a_1 + a_2 + ... + a_n$
f	function f
$f : x \mapsto y$	f is a function under which x is mapped to y
$f(x)$	the image of x under the function f
f^{-1}	the inverse function of the function f
$g \circ f$, gf	the composite function of f and g
$\lim_{x \to a} f(x)$	the limit of $f(x)$ as x tends to a
$\frac{dy}{dx}$	the derivative of y with respect to x
$\frac{d^2y}{dx^2}$	the second derivative of y with respect to x
$f'(x)$	the derivative of $f(x)$ with respect to x
$f''(x)$	the second derivative of $f(x)$ with respect to x
$\int y\, dx$	the indefinite integral of y with respect to x
$\int_a^b y\, dx$	the definite integral of y with respect to x for values of x between a and b
e	base of natural logarithms
e^x	exponential function of x
$\lg x$	logarithm of x to base 10
$\ln x$	natural logarithm of x
$\log_a x$	logarithm to the base a of x
sin, cos, tan, cosec, sec, cot	the circular functions
A(x, y)	the point A in the plane with Cartesian coordinates x and y
AB	the line segment with endpoints A and B the distance from A to B the line containing points A and B
$\widehat{\mathrm{A}}$	the angle at A
$\mathrm{C\widehat{A}B}$	the angle between CA and AB
$\triangle\mathrm{ABC}$	the triangle whose vertices are A, B, and C
$\mathbf{a}$	the vector $\mathbf{a}$
$\overrightarrow{\mathrm{AB}}$	the vector represented in magnitude and direction by the directed line segment from A to B
$\lvert\mathbf{a}\rvert$	the magnitude of vector $\mathbf{a}$
$\lvert\overrightarrow{\mathrm{AB}}\rvert$	the magnitude of $\overrightarrow{\mathrm{AB}}$
$\mathbf{i}$, $\mathbf{j}$	unit vectors in the directions of the Cartesian coordinate axes
$\mathbf{M}$	a matrix $\mathbf{M}$
$\mathbf{M}^{-1}$	the inverse of the square matrix $\mathbf{M}$
$\det \mathbf{M}$	the determinant of the square matrix $\mathbf{M}$

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Sets and Venn diagrams

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Opening problem

A city has three football teams in the national league: A, B, and C.

In the last season, 20% of the city's population saw team A play, 24% saw team B, and 28% saw team C. Of these, 4% saw both A and B, 5% saw both A and C, and 6% saw both B and C. 1% saw all three teams play.

Things to think about:

a Writing out all of this information in sentences is very complicated. How can we represent this information more simply on a diagram?

b What percentage of the population:

i saw only team A play

ii saw team A *or* team B play but not team C

iii did not see any of the teams play?

A SETS

SET NOTATION

A **set** is a collection of numbers or objects.

For example:

- the set of digits which we use to write numbers is $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$
- if V is the set of all vowels, then $V = \{a, e, i, o, u\}$.

The numbers or objects in a set are called the **elements** or **members** of the set.

We use the symbol $\in$ to mean *is an element of* and $\notin$ to mean *is not an element of*.

So, for the set $A = \{1, 2, 3, 4, 5, 6, 7\}$ we can say $4 \in A$ but $9 \notin A$.

The set $\{ \}$ or $\varnothing$ is called the **empty set** and contains no elements.

SPECIAL NUMBER SETS

The following is a list of some special number sets you should be familiar with:

- $\mathbb{N} = \{1, 2, 3, 4, 5, 6, 7, ...\}$ is the set of all **natural** or **counting numbers**.
- $\mathbb{Z} = \{0, \pm 1, \pm 2, \pm 3, \pm 4, ...\}$ is the set of all **integers**.
- $\mathbb{Z}^+ = \{1, 2, 3, 4, 5, 6, 7, ...\}$ is the set of all **positive integers**.
- $\mathbb{Z}^- = \{-1, -2, -3, -4, -5, ...\}$ is the set of all **negative integers**.
- $\mathbb{Q}$ is the set of all **rational numbers**, or numbers which can be written in the form $\frac{p}{q}$ where p and q are integers and $q \neq 0$.
- $\mathbb{R}$ is the set of all **real numbers**, which are all numbers which can be placed on the number line.

The set of natural numbers $\mathbb{N}$ is often defined to include 0.

COUNTING ELEMENTS OF SETS

The number of elements in set A is written $n(A)$.

For example, the set $A = \{2, 3, 5, 8, 13, 21\}$ has 6 elements, so we write $n(A) = 6$.

A set which has a finite number of elements is called a **finite set**.

For example: $A = \{2, 3, 5, 8, 13, 21\}$ is a finite set.
$\varnothing$ is also a finite set, since $n(\varnothing) = 0$.

Infinite sets are sets which have infinitely many elements.

For example, the set of positive integers $\{1, 2, 3, 4, ...\}$ does not have a largest element, but rather keeps on going forever. It is therefore an infinite set.

In fact, the sets $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Z}^+$, $\mathbb{Z}^-$, $\mathbb{Q}$, and $\mathbb{R}$ are all infinite sets.

SUBSETS

Suppose A and B are two sets. A is a **subset** of B if every element of A is also an element of B. We write $A \subseteq B$.

For example, $\{2, 3, 5\} \subseteq \{1, 2, 3, 4, 5, 6\}$ as every element in the first set is also in the second set.

A is a **proper subset** of B if A is a subset of B but is *not equal to* B.
We write $A \subset B$.

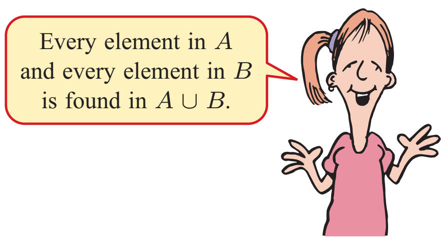
For example, $\mathbb{Z} \subset \mathbb{Q}$ since any integer $n = \frac{n}{1} \in \mathbb{Q}$. However, $\frac{1}{2} \in \mathbb{Q}$ but $\frac{1}{2} \notin \mathbb{Z}$.

We use $A \not\subseteq B$ to indicate that A is *not* a subset of B
and $A \not\subset B$ to indicate that A is *not* a proper subset of B.

UNION AND INTERSECTION

If A and B are two sets, then

- $A \cap B$ is the **intersection** of A and B, and consists of all elements which are in **both** A **and** B
- $A \cup B$ is the **union** of A and B, and consists of all elements which are in A **or** B.



For example:

- If $A = \{1, 3, 4\}$ and $B = \{2, 3, 5\}$ then $A \cap B = \{3\}$ and $A \cup B = \{1, 2, 3, 4, 5\}$.
- The set of integers is made up of the set of negative integers, zero, and the set of positive integers: $\mathbb{Z} = (\mathbb{Z}^- \cup \{0\} \cup \mathbb{Z}^+)$

DEMO

DISJOINT SETS

Two sets are **disjoint** or **mutually exclusive** if they have no elements in common.

If A and B are disjoint then $A \cap B = \varnothing$.

Example 1 — Self Tutor

$M = \{2, 3, 5, 7, 8, 9\}$ and $N = \{3, 4, 6, 9, 10\}$

a True or false? **i** $4 \in M$ **ii** $6 \notin M$

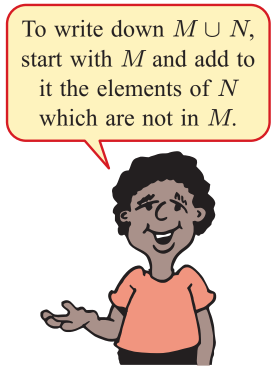
b List the sets: **i** $M \cap N$ **ii** $M \cup N$

c Is **i** $M \subseteq N$ **ii** $\{9, 6, 3\} \subseteq N$?

a **i** 4 is not an element of M, so $4 \in M$ is false.
ii 6 is not an element of M, so $6 \notin M$ is true.

b **i** $M \cap N = \{3, 9\}$ since 3 and 9 are elements of both sets.
ii Every element which is in either M or N is in the union of M and N.
$\therefore$ $M \cup N = \{2, 3, 4, 5, 6, 7, 8, 9, 10\}$

c **i** No. Not every element of M is an element of N.
ii Yes, as 9, 6, and 3 are also in N.



EXERCISE 1A

1 Write using set notation:

a 5 is an element of set D **b** 6 is not an element of set G

c d is not an element of the set of all English vowels

d $\{2, 5\}$ is a subset of $\{1, 2, 3, 4, 5, 6\}$ **e** $\{3, 8, 6\}$ is not a subset of $\{1, 2, 3, 4, 5, 6\}$.

2 Find **i** $A \cap B$ **ii** $A \cup B$ for:

a $A = \{6, 7, 9, 11, 12\}$ and $B = \{5, 8, 10, 13, 9\}$

b $A = \{1, 2, 3, 4\}$ and $B = \{5, 6, 7, 8\}$

c $A = \{1, 3, 5, 7\}$ and $B = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

3 Suppose $A = \{0, 3, 5, 8, 14\}$ and $B = \{1, 4, 5, 8, 11, 13\}$. Write down the number of elements in:

a A **b** B **c** $A \cap B$ **d** $A \cup B$

4 True or false?

a $\mathbb{Z}^+ \subseteq \mathbb{N}$ **b** $\mathbb{N} \subset \mathbb{Z}$ **c** $\mathbb{N} = \mathbb{Z}^+$ **d** $\mathbb{Z}^- \subseteq \mathbb{Z}$

e $\mathbb{Q} \subset \mathbb{Z}$ **f** $\{0\} \subseteq \mathbb{Z}$ **g** $\mathbb{Z} \subseteq \mathbb{Q}$ **h** $\mathbb{Z}^+ \cup \mathbb{Z}^- = \mathbb{Z}$

5 Describe the following sets as either finite or infinite:

a the set of counting numbers between 10 and 20

b the set of counting numbers greater than 5

c the set of all rational numbers $\mathbb{Q}$

d the set of all rational numbers between 0 and 1.

6 True or false?

a $127 \in \mathbb{N}$ **b** $\frac{138}{279} \in \mathbb{Q}$ **c** $3\frac{1}{7} \notin \mathbb{Q}$ **d** $-\frac{4}{11} \in \mathbb{Q}$

7 Which of these pairs of sets are disjoint?

a $A = \{3, 5, 7, 9\}$ and $B = \{2, 4, 6, 8\}$ **b** $P = \{3, 5, 6, 7, 8, 10\}$ and $Q = \{4, 9, 10\}$

8 True or false? If R and S are two non-empty sets and $R \cap S = \varnothing$, then R and S are disjoint.

9 **a** How many proper subsets does the set $\{a, b, c, d\}$ have?

b Copy and complete: "If a set has n elements then it has ...... proper subsets."

B INTERVAL NOTATION

To avoid having to list all members of a set, we often use a general description of its members. We often describe a set of all values of x with a particular property.

The notation $\{x : ......\}$ or $\{x \mid ......\}$ is used to describe "the set of all x such that ......".

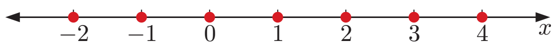
For example:

- $A = \{x \in \mathbb{Z} : -2 \leqslant x \leqslant 4\}$ reads "the set of all integers x such that x is between -2 and 4, including -2 and 4."

  such that

  the set of all

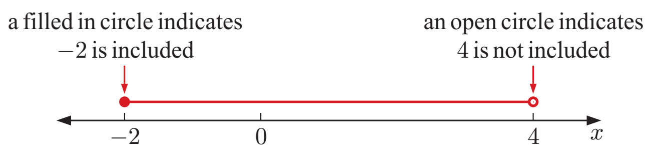
  We can represent A on a number line as:



  A is a finite set, and $n(A) = 7$.

- $B = \{x \in \mathbb{R} : -2 \leqslant x < 4\}$ reads "the set of all real x such that x is greater than or equal to -2 and less than 4."

  We represent B on a number line as:



  B is an infinite set, and $n(B) = \infty$.

  We could also write $B = \{x : -2 \leqslant x < 4\}$, in which case we would *assume* that $x \in \mathbb{R}$.

Example 2 Self Tutor

Suppose $A = \{x \in \mathbb{Z} : 3 < x \leqslant 10\}$.

a Write down the meaning of the interval notation.

b List the elements of set A. **c** Find $n(A)$.

a The set of all integers x such that x is between 3 and 10, including 10.

b $A = \{4, 5, 6, 7, 8, 9, 10\}$ **c** There are 7 elements, so $n(A) = 7$.

CLOSED AND OPEN INTERVALS

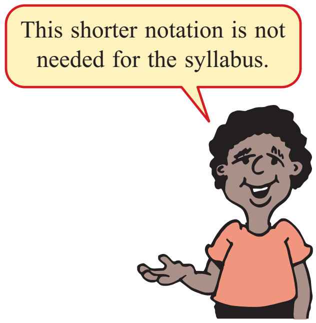
An **interval** is a connected subset of the number line $\mathbb{R}$.

An interval is **closed** if *both* of its endpoints are included.

An interval is **open** if *both* of its endpoints are *not* included.

For $x \in \mathbb{R}$, we commonly use the following notation to concisely write intervals:

$[a,\, b]$	represents the closed interval	$\{x \in \mathbb{R} \;:\; a \leqslant x \leqslant b\}$
$[a,\, b)$	represents the interval	$\{x \in \mathbb{R} \;:\; a \leqslant x < b\}$
$(a,\, b]$	represents the interval	$\{x \in \mathbb{R} \;:\; a < x \leqslant b\}$
$(a,\, b)$	represents the open interval	$\{x \in \mathbb{R} \;:\; a < x < b\}$

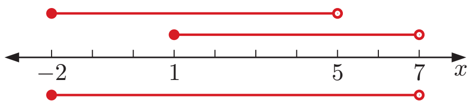


An interval which extends to infinity has no defined endpoint.
So, for $\{x \in \mathbb{R} : x \geqslant a\}$ we write $[a,\, \infty)$.

INTERVALS WHICH OVERLAP

When two intervals overlap, we consolidate them into a single interval.

For example: $[-2,\, 5) \cup [1,\, 7) = [-2,\, 7)$



EXERCISE 1B

1 Explain whether the following sets are finite or infinite:

- **a** $\{x \in \mathbb{Z} : -2 \leqslant x \leqslant 1\}$
- **b** $\{x \in \mathbb{R} : -2 \leqslant x \leqslant 1\}$
- **c** $\{x \in \mathbb{Z} : x \geqslant 5\}$
- **d** $\{x \in \mathbb{Q} : 0 \leqslant x \leqslant 1\}$
- **e** $(2,\, 4)$
- **f** $[-3,\, 7]$
- **g** $(-\infty,\, 0)$

2 For the following sets:

- **i** Write down the meaning of the interval notation.
- **ii** If possible, list the elements of A.
- **iii** Find $n(A)$.
- **iv** If possible, sketch A on a number line.

- **a** $A = \{x \in \mathbb{Z} : -1 \leqslant x \leqslant 7\}$
- **b** $A = \{x \in \mathbb{N} : -2 < x < 8\}$
- **c** $A = \{x \in \mathbb{R} : 0 \leqslant x \leqslant 1\}$
- **d** $A = \{x \in \mathbb{Q} : 5 \leqslant x \leqslant 6\}$
- **e** $A = [-1,\, 5)$
- **f** $A = \{x \in \mathbb{R} : 3 < x \leqslant 5 \;\cup\; x > 7\}$
- **g** $A = (-\infty,\, 1] \cup (2,\, \infty)$
- **h** $A = (-\infty,\, 2) \cup [1,\, \infty)$

3 Write in interval notation:

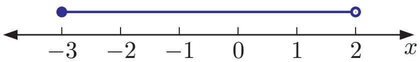
- **a** the set of all integers between -100 and 100
- **b** the set of all real numbers greater than 1000
- **c** the set of all rational numbers between 2 and 3, including 2 and 3.

4 Write using interval notation:

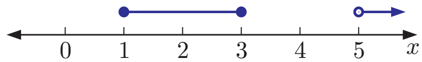
a $\{-2, -1, 0, 1, 2, 3\}$

b $\{...., -6, -5, -4, -3\}$

c



d



5 State whether $A \subseteq B$:

a $A = \varnothing$, $B = \{2, 5, 7, 9\}$

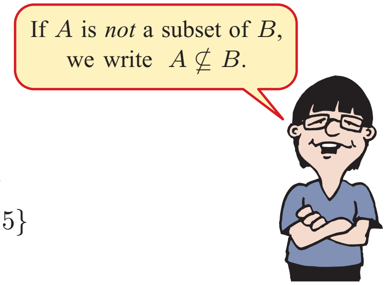
b $A = \{2, 5, 8, 9\}$, $B = \{8, 9\}$

c $A = \{x \in \mathbb{R} : 2 \leqslant x \leqslant 3\}$, $B = \{x \in \mathbb{R}\}$

d $A = \{x \in \mathbb{Q} : 3 \leqslant x \leqslant 9\}$, $B = \{x \in \mathbb{R} : 0 \leqslant x \leqslant 10\}$

e $A = \{x \in \mathbb{Z} : -10 \leqslant x \leqslant 10\}$, $B = \{z \in \mathbb{Z} : 0 \leqslant z \leqslant 5\}$

f $A = \{x \in \mathbb{Q} : 0 \leqslant x \leqslant 1\}$, $B = \{y \in \mathbb{Q} : 0 < y \leqslant 2\}$



6 For each of the following sets, determine whether the interval described is closed, open, or neither:

a $[2, 5)$ **b** $(-1, 3)$ **c** $(-\infty, -4]$ **d** $(4, \infty)$ **e** $[-2, 2]$ **f** $[0, 11)$

7 Given that $\mathbb{Q}$ is the set of rational numbers, we can define

$\mathbb{Q}^+$ as the set of positive rational numbers, $\{x \in \mathbb{Q} : x > 0\}$

and $\mathbb{Q}_0^+$ as the set of positive rational numbers and zero, $\{x \in \mathbb{Q} : x \geqslant 0\}$.

a Explain why the set $\mathbb{Q}$ cannot be illustrated on a number line.

b Describe in words, in interval notation, and using a number line, what would be meant by the set:

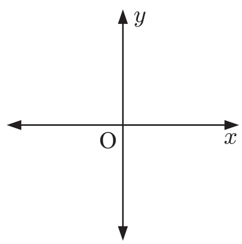
i $\mathbb{R}^+$ **ii** $\mathbb{R}_0^+$

C RELATIONS

A **relation** is any set of points which connect two variables.

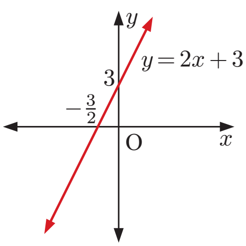
You should be familiar with points (x, y) in the Cartesian plane.

Any set of these points is a relation.

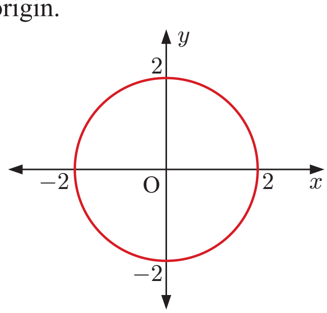


For example:

- $\{(x, y) : y = 2x + 3\}$ is the set of points which form a straight line with gradient 2 and y-intercept 3.



- $\{(x, y) : x^2 + y^2 = 4\}$ is the set of points which form a circle with radius 2 units centred at the origin.



EXERCISE 1C

GRAPHING PACKAGE

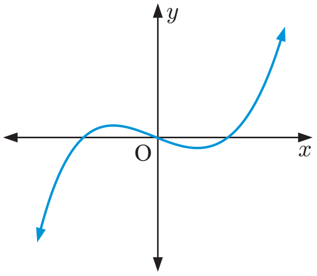
1 Illustrate the following sets in the Cartesian plane. In each case state whether the set is finite or infinite.

a $\{(x, y) : y = x\}$ **b** $\{(x, y) : x + y = 1\}$

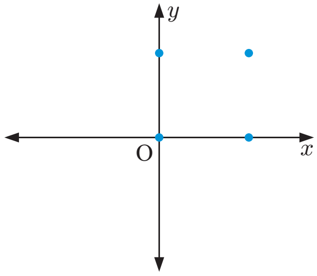
c $\{(x, y) : x > 0, \ y > 0\}$ **d** $\{(x, y) : x + y > 1\}$

2 Let A be the set of points in each graph below. State whether A is finite or infinite.

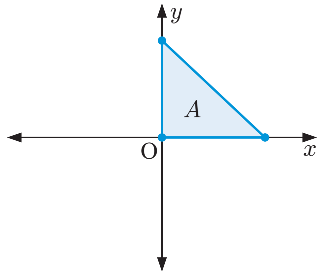
a



b



c



3 Suppose A is the set of points which define a straight line and B is the set of points which define a circle.

a Describe in words the meaning of: **i** $A \cap B$ **ii** $A \cup B$

b Describe, with illustration, what it means if $n(A \cap B)$ equals: **i** 2 **ii** 1 **iii** 0

D COMPLEMENTS OF SETS

UNIVERSAL SETS

Suppose we are only interested in the natural numbers from 1 to 20, and we want to consider subsets of this set. We say the set $\mathscr{E} = \{x \in \mathbb{N} : 1 \leqslant x \leqslant 20\}$ is the *universal set* in this situation.

The symbol $\mathscr{E}$ is used to represent the **universal set** under consideration.

COMPLEMENTARY SETS

The **complement** of A, denoted A', is the set of all elements of $\mathscr{E}$ which are *not* in A.

$$A' = \{x \in \mathscr{E} : x \notin A\}$$

For example, if the universal set $\mathscr{E} = \{1, 2, 3, 4, 5, 6, 7, 8\}$, and the set $A = \{1, 3, 5, 7, 8\}$, then the complement of A is $A' = \{2, 4, 6\}$.

Three obvious relationships are observed connecting A and A'. These are:

- $A \cap A' = \varnothing$ as A' and A have no common members.
- $A \cup A' = \mathscr{E}$ as all elements of A and A' combined make up $\mathscr{E}$.
- $n(A) + n(A') = n(\mathscr{E})$

For example, $\mathbb{Q} \cap \mathbb{Q}' = \varnothing$ and $\mathbb{Q} \cup \mathbb{Q}' = \mathbb{R}$.

Example 3 — Self Tutor

Find C' given that:

a $\mathscr{E} = \{\text{all positive integers}\}$ and $C = \{\text{all even integers}\}$

b $C = \{x \in \mathbb{Z} : x \geqslant 2\}$ and $\mathscr{E} = \mathbb{Z}$

a $C' = \{\text{all odd integers}\}$ **b** $C' = \{x \in \mathbb{Z} : x \leqslant 1\}$

Example 4 — Self Tutor

Suppose $\mathscr{E} = \{x \in \mathbb{Z} : -5 \leqslant x \leqslant 5\}$, $A = \{x \in \mathbb{Z} : 1 \leqslant x \leqslant 4\}$, and $B = \{x \in \mathbb{Z} : -3 \leqslant x < 2\}$. List the elements of:

a A **b** B **c** A' **d** B'
e $A \cap B$ **f** $A \cup B$ **g** $A' \cap B$ **h** $A' \cup B'$

a $A = \{1, 2, 3, 4\}$
b $B = \{-3, -2, -1, 0, 1\}$
c $A' = \{-5, -4, -3, -2, -1, 0, 5\}$
d $B' = \{-5, -4, 2, 3, 4, 5\}$
e $A \cap B = \{1\}$
f $A \cup B = \{-3, -2, -1, 0, 1, 2, 3, 4\}$
g $A' \cap B = \{-3, -2, -1, 0\}$
h $A' \cup B' = \{-5, -4, -3, -2, -1, 0, 2, 3, 4, 5\}$

EXERCISE 1D

1 Find the complement of C given that:

a $\mathscr{E} = \{\text{letters of the English alphabet}\}$ and $C = \{\text{vowels}\}$

b $\mathscr{E} = \{\text{integers}\}$ and $C = \{\text{negative integers}\}$

c $\mathscr{E} = \mathbb{Z}$ and $C = \{x \in \mathbb{Z} : x \leqslant -5\}$

d $\mathscr{E} = \mathbb{Q}$ and $C = \{x \in \mathbb{Q} : x \leqslant 2 \cup x \geqslant 8\}$

2 Suppose $\mathscr{E} = \{x \in \mathbb{Z} : 0 \leqslant x \leqslant 8\}$, $A = \{x \in \mathbb{Z} : 2 \leqslant x \leqslant 7\}$, and $B = \{x \in \mathbb{Z} : 5 \leqslant x \leqslant 8\}$. List the elements of:

a A **b** A' **c** B **d** B'
e $A \cap B$ **f** $A \cup B$ **g** $A \cap B'$ **h** $A' \cup B'$

3 Suppose P and Q' are subsets of $\mathscr{E}$. $n(\mathscr{E}) = 15$, $n(P) = 6$, and $n(Q') = 4$. Find:

a $n(P')$ **b** $n(Q)$

4 True or false?

a If $n(\mathscr{E}) = a$ and $n(A) = b$ where $A \subseteq \mathscr{E}$, then $n(A') = b - a$.

b If Q is a subset of $\mathscr{E}$ then $Q' = \{x \in \mathscr{E} : x \notin Q\}$.

5 Suppose $\mathscr{E} = \{x \in \mathbb{Z} : 0 < x \leqslant 12\}$, $A = \{x \in \mathbb{Z} : 2 \leqslant x \leqslant 7\}$,
$B = \{x \in \mathbb{Z} : 3 \leqslant x \leqslant 9\}$, and $C = \{x \in \mathbb{Z} : 5 \leqslant x \leqslant 11\}$.

List the elements of:

a B' **b** C' **c** A' **d** $A \cap B$
e $(A \cap B)'$ **f** $A' \cap C$ **g** $B' \cup C$ **h** $(A \cup C) \cap B'$

6 Consider the set of real numbers $\mathbb{R}$. Write down the complement of:

a $(-\infty, 0)$ **b** $[1, \infty)$ **c** $[-3, 2)$ **d** $(-5, 7]$

e $(-\infty, 1) \cup [3, \infty)$ **f** $[-5, 0) \cup (1, \infty)$

E PROPERTIES OF UNION AND INTERSECTION

In this section we will explore the number of elements in unions and intersections of sets.

Example 5 Self Tutor

Suppose $\mathscr{E} = \{\text{positive integers}\}$, $P = \{\text{multiples of 4 less than 50}\}$, and $Q = \{\text{multiples of 6 less than 50}\}$.

a List P and Q. **b** Find $P \cap Q$. **c** Find $P \cup Q$.

d Verify that $n(P \cup Q) = n(P) + n(Q) - n(P \cap Q)$.

a $P = \{4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48\}$
$Q = \{6, 12, 18, 24, 30, 36, 42, 48\}$

b $P \cap Q = \{12, 24, 36, 48\}$

c $P \cup Q = \{4, 6, 8, 12, 16, 18, 20, 24, 28, 30, 32, 36, 40, 42, 44, 48\}$

d $n(P \cup Q) = 16$ and $n(P) + n(Q) - n(P \cap Q) = 12 + 8 - 4 = 16$
So, $n(P \cup Q) = n(P) + n(Q) - n(P \cap Q)$ is verified.

EXERCISE 1E

1 Suppose $\mathscr{E} = \mathbb{Z}^+$, $P = \{\text{prime numbers} < 25\}$, and $Q = \{2, 4, 5, 11, 12, 15\}$.

a List P. **b** Find $P \cap Q$. **c** Find $P \cup Q$.

d Verify that $n(P \cup Q) = n(P) + n(Q) - n(P \cap Q)$.

2 Suppose $\mathscr{E} = \mathbb{Z}^+$, $P = \{\text{factors of 28}\}$, and $Q = \{\text{factors of 40}\}$.

a List P and Q. **b** Find $P \cap Q$. **c** Find $P \cup Q$.

d Verify that $n(P \cup Q) = n(P) + n(Q) - n(P \cap Q)$.

3 Suppose $\mathscr{E} = \mathbb{Z}^+$, $M = \{\text{multiples of 4 between 30 and 60}\}$, and $N = \{\text{multiples of 6 between 30 and 60}\}$.

a List M and N. **b** Find $M \cap N$. **c** Find $M \cup N$.

d Verify that $n(M \cup N) = n(M) + n(N) - n(M \cap N)$.

4 Suppose $\mathscr{E} = \mathbb{Z}$, $R = \{x \in \mathbb{Z} : -2 \leqslant x \leqslant 4\}$, and $S = \{x \in \mathbb{Z} : 0 \leqslant x < 7\}$.

a List R and S. **b** Find $R \cap S$. **c** Find $R \cup S$.

d Verify that $n(R \cup S) = n(R) + n(S) - n(R \cap S)$.

5 Suppose $\mathscr{E} = \mathbb{Z}$, $C = \{y \in \mathbb{Z} : -4 \leqslant y \leqslant -1\}$, and $D = \{y \in \mathbb{Z} : -7 \leqslant y < 0\}$.

a List C and D. **b** Find $C \cap D$. **c** Find $C \cup D$.

d Verify that $n(C \cup D) = n(C) + n(D) - n(C \cap D)$.

6 Suppose $\mathscr{E} = \mathbb{Z}^+$, $P = \{\text{factors of } 12\}$, $Q = \{\text{factors of } 18\}$, and $R = \{\text{factors of } 27\}$.

a List the sets P, Q, and R.

b Find: **i** $P \cap Q$ **ii** $P \cap R$ **iii** $Q \cap R$ **iv** $P \cup Q$ **v** $P \cup R$ **vi** $Q \cup R$

c Find: **i** $P \cap Q \cap R$ **ii** $P \cup Q \cup R$

7 Suppose $\mathscr{E} = \mathbb{Z}^+$, $A = \{\text{multiples of 4 less than } 40\}$, $B = \{\text{multiples of 6 less than } 40\}$, and $C = \{\text{multiples of 12 less than } 40\}$.

a List the sets A, B, and C.

b Find: **i** $A \cap B$ **ii** $B \cap C$ **iii** $A \cap C$ **iv** $A \cap B \cap C$ **v** $A \cup B \cup C$

c Verify that
$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$.

8 Suppose $\mathscr{E} = \mathbb{Z}^+$, $A = \{\text{multiples of 6 less than } 31\}$, $B = \{\text{factors of } 30\}$, and $C = \{\text{primes } < 30\}$.

a List the sets A, B, and C.

b Find: **i** $A \cap B$ **ii** $B \cap C$ **iii** $A \cap C$ **iv** $A \cap B \cap C$ **v** $A \cup B \cup C$

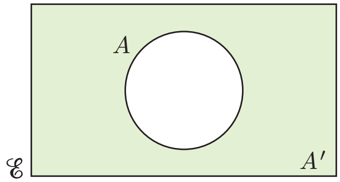
c Verify that
$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$.

F VENN DIAGRAMS

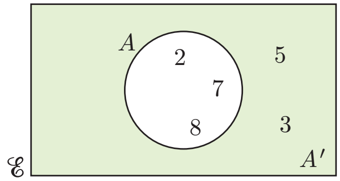
A **Venn diagram** consists of a universal set $\mathscr{E}$ represented by a rectangle. Sets within the universal set are usually represented by circles.

For example:

- This Venn diagram shows set A within the universal set $\mathscr{E}$. A', the complement of A, is the shaded region outside the circle.



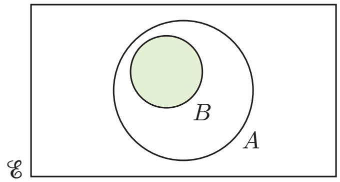
- The sets $\mathscr{E} = \{2, 3, 5, 7, 8\}$, $A = \{2, 7, 8\}$, and $A' = \{3, 5\}$ are represented by:



SUBSETS

If $B \subseteq A$ then every element of B is also in A.

The circle representing B is placed within the circle representing A.

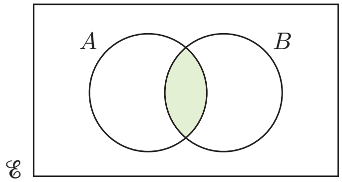


$B \subseteq A$

INTERSECTION

$A \cap B$ consists of all elements common to both A and B.

It is the shaded region where the circles representing A and B overlap.

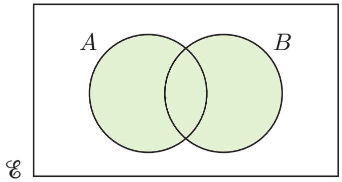


$A \cap B$

UNION

$A \cup B$ consists of all elements in A or B or both.

It is the shaded region which includes both circles.



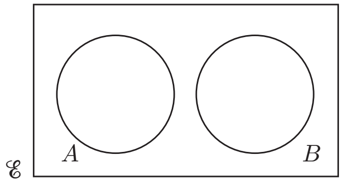
$A \cup B$

DISJOINT OR MUTUALLY EXCLUSIVE SETS

Disjoint sets do not have common elements.

They are represented by non-overlapping circles.

For example, if $A = \{2, 3, 8\}$ and $B = \{4, 5, 9\}$
then $A \cap B = \varnothing$.



$A \cap B = \varnothing$

Example 6 — Self Tutor

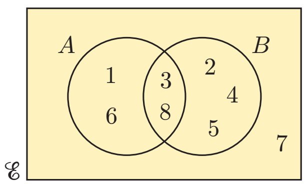
Suppose $\mathscr{E} = \{1, 2, 3, 4, 5, 6, 7, 8\}$. Illustrate on a Venn diagram the sets:

a $A = \{1, 3, 6, 8\}$ and $B = \{2, 3, 4, 5, 8\}$

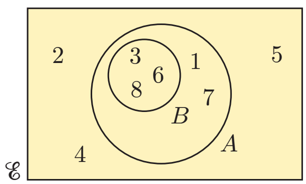
b $A = \{1, 3, 6, 7, 8\}$ and $B = \{3, 6, 8\}$

c $A = \{2, 4, 8\}$ and $B = \{3, 6, 7\}$.

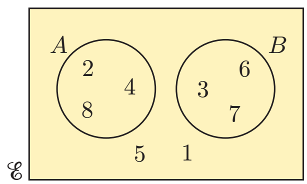
a $A \cap B = \{3, 8\}$



b $A \cap B = \{3, 6, 8\}$,
$B \subseteq A$



c $A \cap B = \varnothing$



EXERCISE 1F.1

1 Represent sets A and B on a Venn diagram, given:

a $\mathscr{E} = \{1, 2, 3, 4, 5, 6, 7\}$, $A = \{2, 4, 5, 6\}$, and $B = \{1, 4, 6, 7\}$

b $\mathscr{E} = \{2, 3, 4, 5, 6, 7\}$, $A = \{2, 4, 6\}$, and $B = \{5, 7\}$

c $\mathscr{E} = \{2, 3, 4, 5, 6, 7\}$, $A = \{2, 4, 6\}$, and $B = \{3, 5, 7\}$

d $\mathscr{E} = \{3, 4, 5, 7\}$, $A = \{3, 4, 5, 7\}$, and $B = \{3, 5\}$

2 Suppose $\mathscr{E} = \{x \in \mathbb{Z} : 1 \leqslant x \leqslant 10\}$, $A = \{\text{odd numbers} < 10\}$, and $B = \{\text{primes} < 10\}$.

a List sets A and B. **b** Find $A \cap B$ and $A \cup B$.

c Represent the sets A and B on a Venn diagram.

3 Suppose $\mathscr{E} = \{x \in \mathbb{Z} : 1 \leqslant x \leqslant 9\}$, $A = \{\text{factors of } 6\}$, and $B = \{\text{factors of } 9\}$.

a List sets A and B. **b** Find $A \cap B$ and $A \cup B$.

c Represent the sets A and B on a Venn diagram.

4 Suppose $\mathscr{E} = \{\text{even numbers between 0 and } 30\}$,
$P = \{\text{multiples of 4 less than } 30\}$, and
$Q = \{\text{multiples of 6 less than } 30\}$.

a List sets P and Q. **b** Find $P \cap Q$ and $P \cup Q$.

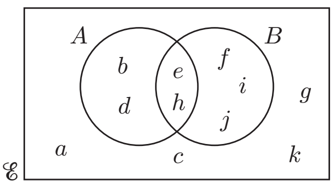
c Represent the sets P and Q on a Venn diagram.

5 Suppose $\mathscr{E} = \{x \in \mathbb{Z}^+ : x \leqslant 30\}$, $R = \{\text{primes less than } 30\}$, and
$S = \{\text{composites less than } 30\}$.

a List sets R and S. **b** Find $R \cap S$ and $R \cup S$.

c Represent the sets R and S on a Venn diagram.

6



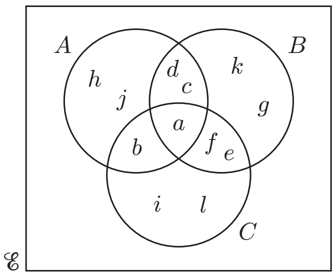
List the elements of set:

a A **b** B **c** A'

d B' **e** $A \cap B$ **f** $A \cup B$

g $(A \cup B)'$ **h** $A' \cup B'$

7



This Venn diagram consists of three overlapping circles A, B, and C.

a List the letters in set:

i A **ii** B

iii C **iv** $A \cap B$

v $A \cup B$ **vi** $B \cap C$

vii $A \cap B \cap C$ **viii** $A \cup B \cup C$

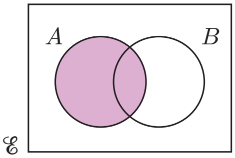
b Find: **i** $n(A \cup B \cup C)$

ii $n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$

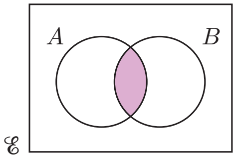
c What do you notice about your answers in **b**?

USING VENN DIAGRAMS TO ILLUSTRATE REGIONS

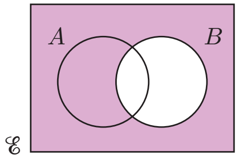
We can use shading to show various sets on a Venn diagram.
For example, for two intersecting sets A and B:



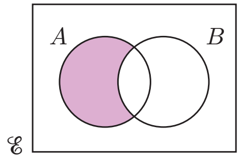
A is shaded



$A \cap B$ is shaded



B' is shaded



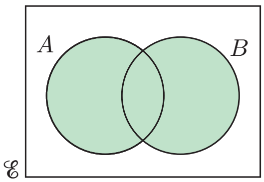
$A \cap B'$ is shaded

Example 7 — Self Tutor

Shade the following regions for two intersecting sets A and B:

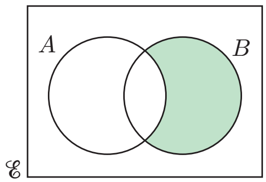
a $A \cup B$ **b** $A' \cap B$ **c** $(A \cap B)'$

a



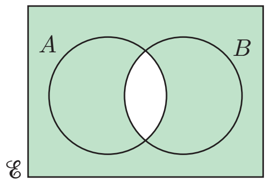
(in A, B, or both)

b



(outside A, intersected with B)

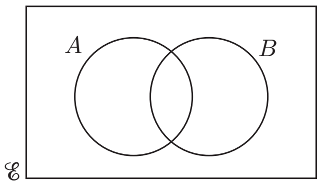
c



(outside $A \cap B$)

EXERCISE 1F.2

1

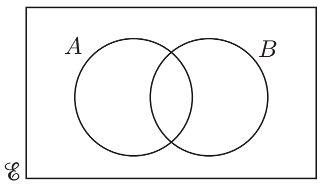


On separate Venn diagrams, shade regions for:

a $A \cap B$ **b** $A \cap B'$
c $A' \cup B$ **d** $A \cup B'$
e $A' \cap B$ **f** $A' \cap B'$

PRINTABLE VENN DIAGRAMS (OVERLAPPING)

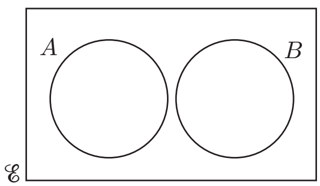
2



On separate Venn diagrams, shade regions for:

a $A \cup B$ **b** $(A \cup B)'$
c $(A \cap B)'$ **d** $A' \cup B'$
e $(A' \cup B')'$ **f** $(A \cup B')'$

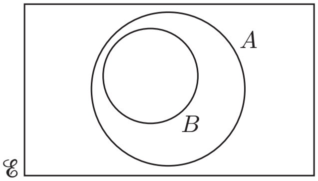
3



Suppose A and B are two disjoint sets. Shade on separate Venn diagrams:

a A **b** B
c A' **d** B'
e $A \cap B$ **f** $A \cup B$
g $A' \cap B$ **h** $A \cup B'$
i $(A \cap B)'$

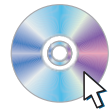
PRINTABLE VENN DIAGRAMS (DISJOINT)

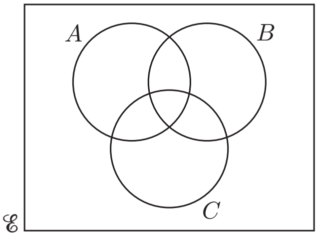
4 

Suppose $B \subseteq A$, as shown in the given Venn diagram. Shade on separate Venn diagrams:

- **a** A
- **b** B
- **c** A'
- **d** B'
- **e** $A \cap B$
- **f** $A \cup B$
- **g** $A' \cap B$
- **h** $A \cup B'$
- **i** $(A \cap B)'$

PRINTABLE VENN DIAGRAMS (SUBSET)

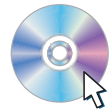


5 

This Venn diagram consists of three intersecting sets. Shade on separate Venn diagrams:

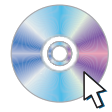
- **a** A
- **b** B'
- **c** $B \cap C$
- **d** $A \cup B$
- **e** $A \cap B \cap C$
- **f** $A \cup B \cup C$
- **g** $(A \cap B \cap C)'$
- **h** $(B \cap C) \cup A$
- **i** $(A \cup B) \cap C$
- **j** $(A \cap C) \cup (B \cap C)$
- **k** $(A \cap B) \cup C$
- **l** $(A \cup C) \cap (B \cup C)$

PRINTABLE VENN DIAGRAMS (3 SETS)



Click on the icon to practise shading regions representing various subsets. You can practise with both two and three intersecting sets.

VENN DIAGRAMS



Discovery — *The algebra of sets*

For the set of real numbers $\mathbb{R}$, we can write laws for the operations $+$ and $\times$:

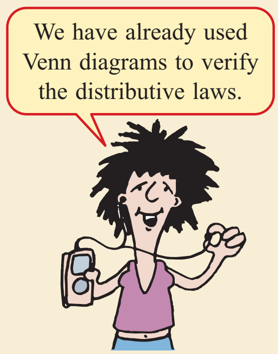
For any real numbers a, b, and c:

- **commutative** $a + b = b + a$ and $ab = ba$
- **identity** Identity elements 0 and 1 exist such that $a + 0 = 0 + a = a$ and $a \times 1 = 1 \times a = a$.
- **associativity** $(a + b) + c = a + (b + c)$ and $(ab)c = a(bc)$
- **distributive** $a(b + c) = ab + ac$

The following are the **laws for the algebra of sets** under the operations $\cup$ and $\cap$:

For any subsets A, B, and C of the universal set $\mathscr{E}$:

- **commutative** $A \cap B = B \cap A$ and $A \cup B = B \cup A$
- **associativity** $A \cap (B \cap C) = (A \cap B) \cap C$ and $A \cup (B \cup C) = (A \cup B) \cup C$
- **distributive** $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ and $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
- **identity** $A \cup \varnothing = A$ and $A \cap \mathscr{E} = A$
- **complement** $A \cup A' = \mathscr{E}$ and $A \cap A' = \varnothing$
- **domination** $A \cup \mathscr{E} = \mathscr{E}$ and $A \cap \varnothing = \varnothing$
- **idempotent** $A \cap A = A$ and $A \cup A = A$
- **DeMorgan's** $(A \cap B)' = A' \cup B'$ and $(A \cup B)' = A' \cap B'$
- **involution** $(A')' = A$



What to do:

1 With the aid of Venn diagrams, explain why the following laws are valid:

a the *complement* law $(A')' = A$

b the *commutative* laws $A \cap B = B \cap A$ and $A \cup B = B \cup A$

c the *idempotent* laws $A \cap A = A$ and $A \cup A = A$

d the *associative* laws $A \cap (B \cap C) = (A \cap B) \cap C$ and $A \cup (B \cup C) = (A \cup B) \cup C$

e the *distributive* laws $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ and $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

2 Use the laws for the algebra of sets to show that:

a $A \cup (B \cup A') = \mathscr{E}$

b $A \cap (B \cap A') = \varnothing$

c $A \cup (B \cap A') = A \cup B$

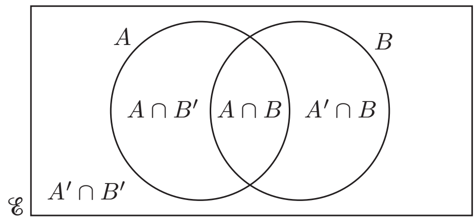
d $(A' \cup B')' = A \cap B$

e $(A \cup B) \cap (A' \cap B') = \varnothing$

f $(A \cup B) \cap (C \cup D) = (A \cap C) \cup (A \cap D) \cup (B \cap C) \cup (B \cap D)$.

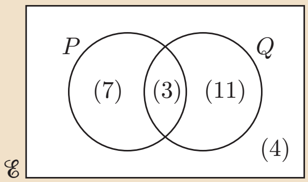
G NUMBERS IN REGIONS

We have seen that there are four regions on a Venn diagram which contains two overlapping sets A and B.



There are many situations where we are only interested in the **number of elements** of $\mathscr{E}$ that are in each region. We do not need to show all the elements on the diagram, so instead we write the number of elements in each region in brackets.

Example 8 — Self Tutor



In the Venn diagram given, (3) means that there are 3 elements in the set $P \cap Q$.

How many elements are there in:

a P

b Q'

c $P \cup Q$

d P, but not Q

e Q, but not P

f neither P nor Q?

a $n(P) = 7 + 3 = 10$

b $n(Q') = 7 + 4 = 11$

c $n(P \cup Q) = 7 + 3 + 11 = 21$

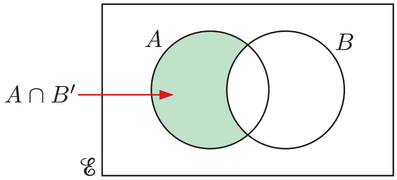
d $n(P$, but not $Q) = 7$

e $n(Q$, but not $P) = 11$

f $n(\text{neither } P \text{ nor } Q) = 4$

Venn diagrams allow us to easily visualise identities such as

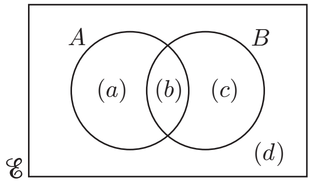
$$n(A \cap B') = n(A) - n(A \cap B)$$
$$n(A' \cap B) = n(B) - n(A \cap B)$$



Example 9 — Self Tutor

Given $n(\mathscr{E}) = 30$, $n(A) = 14$, $n(B) = 17$, and $n(A \cap B) = 6$, find:

a $n(A \cup B)$ **b** $n(A, \text{ but not } B)$



We see that
$$\begin{aligned} b &= 6 && \{\text{as } n(A \cap B) = 6\} \\ a + b &= 14 && \{\text{as } n(A) = 14\} \\ b + c &= 17 && \{\text{as } n(B) = 17\} \\ a + b + c + d &= 30 && \{\text{as } n(\mathscr{E}) = 30\} \end{aligned}$$

$\therefore$ $b = 6$, $a = 8$, and $c = 11$

$\therefore$ $8 + 6 + 11 + d = 30$

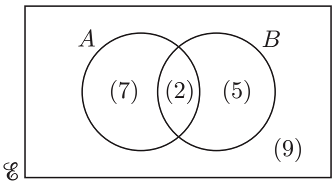
$\therefore$ $d = 5$

a $n(A \cup B) = a + b + c = 25$

b $n(A, \text{ but not } B) = a = 8$

EXERCISE 1G

1

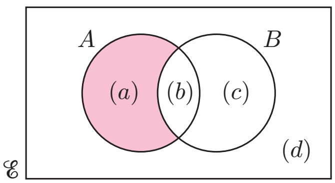


In the Venn diagram given, (2) means that there are 2 elements in the set $A \cap B$.

How many elements are there in:

a B
b A'
c $A \cup B$
d A, but not B
e B, but not A
f neither A nor B?

2

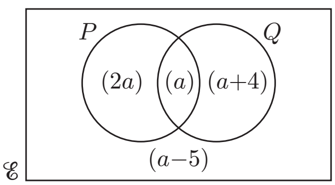


In the Venn diagram given, (a) means that there are a elements in the shaded region.

Notice that $n(A) = a + b$. Find:

a $n(B)$
b $n(A')$
c $n(A \cap B)$
d $n(A \cup B)$
e $n((A \cap B)')$
f $n((A \cup B)')$

3



The Venn diagram shows that $n(P \cap Q) = a$ and $n(P) = 3a$.

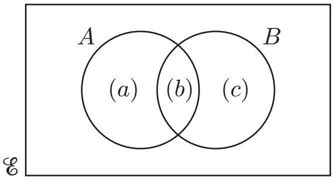
a Find:

i $n(Q)$ **ii** $n(P \cup Q)$ **iii** $n(Q')$ **iv** $n(\mathscr{E})$

b Find a if:

i $n(\mathscr{E}) = 29$ **ii** $n(\mathscr{E}) = 31$

Comment on your results.

4 

Use the Venn diagram to show that:
$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

5 Given $n(\mathscr{E}) = 26$, $n(A) = 11$, $n(B) = 12$, and $n(A \cap B) = 8$, find:

a $n(A \cup B)$ **b** $n(B$, but not $A)$

6 Given $n(\mathscr{E}) = 32$, $n(M) = 13$, $n(M \cap N) = 5$, and $n(M \cup N) = 26$, find:

a $n(N)$ **b** $n((M \cup N)')$

7 Given $n(\mathscr{E}) = 50$, $n(S) = 30$, $n(R) = 25$, and $n(R \cup S) = 48$, find:

a $n(R \cap S)$ **b** $n(S$, but not $R)$

H PROBLEM SOLVING WITH VENN DIAGRAMS

In this section we use Venn diagrams to illustrate real world situations. We can solve problems by considering the number of elements in each region.

Example 10 Self Tutor

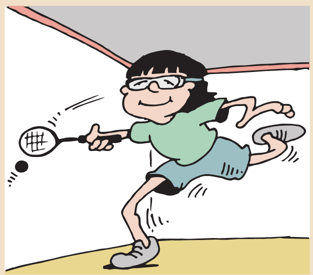
A squash club has 27 members. 19 have black hair, 14 have brown eyes, and 11 have both black hair and brown eyes.

a Place this information on a Venn diagram.

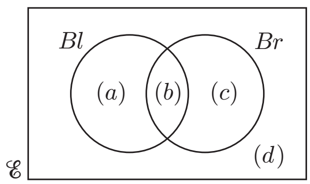
b Hence find the number of members with:

i black hair or brown eyes

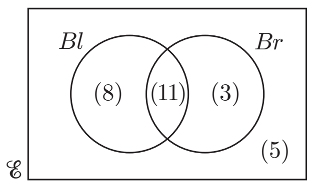
ii black hair, but not brown eyes.



a Let Bl represent the black hair set and Br represent the brown eyes set.



$a + b + c + d = 27$ {total members}
$a + b = 19$ {black hair}
$b + c = 14$ {brown eyes}
$b = 11$ {black hair and brown eyes}
$\therefore\ a = 8,\ c = 3,\ d = 5$



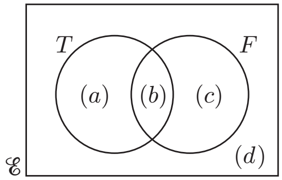
b **i** $n(Bl \cup Br) = 8 + 11 + 3 = 22$
22 members have black hair or brown eyes.

ii $n(Bl \cap Br') = 8$
8 members have black hair, but not brown eyes.

Example 11 — Self Tutor

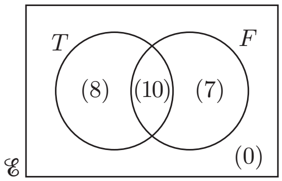
A platform diving squad of 25 has 18 members who dive from 10 m and 17 who dive from 5 m. How many dive from both platforms?

Let T represent those who dive from 10 m and F represent those who dive from 5 m.



$d = 0$ {as all divers in the squad must dive from at least one of the platforms}

$$\begin{aligned} a+b &= 18 \\ b+c &= 17 \\ a+b+c &= 25 \end{aligned} \qquad \therefore\ a=8,\ b=10,\ c=7$$

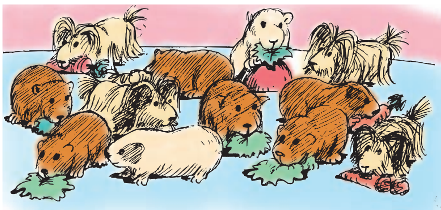


$$\begin{aligned} n(\text{both } T \text{ and } F) &= n(T \cap F) \\ &= 10 \end{aligned}$$

10 members dive from both platforms.

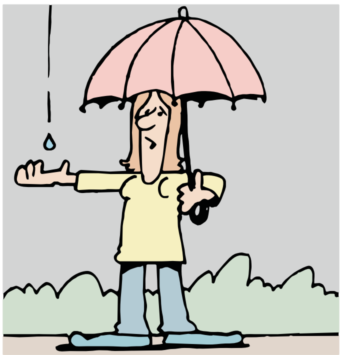
EXERCISE 1H

1 Pelé has 14 cavies as pets. Five have long hair and 8 are brown. Two are both brown and have long hair.



- **a** Place this information on a Venn diagram.
- **b** Hence find the number of cavies that:
  - **i** do not have long hair
  - **ii** have long hair and are not brown
  - **iii** are neither long-haired nor brown.

2



During a 2 week period, Murielle took her umbrella with her on 8 days. It rained on 9 days, and Murielle took her umbrella on five of the days when it rained.

- **a** Display this information on a Venn diagram.
- **b** Hence find the number of days that:
  - **i** Murielle did not take her umbrella and it rained
  - **ii** Murielle did not take her umbrella and it did not rain.

3 A badminton club has 31 playing members. 28 play singles and 16 play doubles. How many play both singles and doubles?

4 In a factory, 56 people work on the assembly line. 47 work day shifts and 29 work night shifts. How many work both day shifts and night shifts?

Example 12

Self Tutor

Consider the **Opening Problem** on page **12**:

A city has three football teams in the national league: A, B, and C.

In the last season, 20% of the city's population saw team A play, 24% saw team B, and 28% saw team C. Of these, 4% saw both A and B, 5% saw both A and C, and 6% saw both B and C. 1% saw all three teams play.

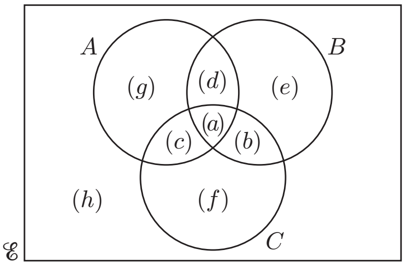
Using a Venn diagram, find the percentage of the city's population which:

a saw only team A play

b saw team A or team B play but not team C

c did not see any of the teams play.

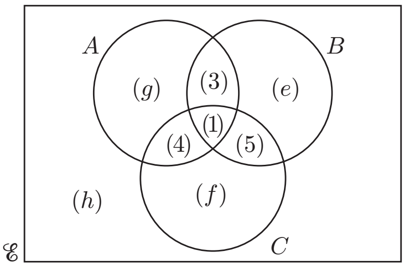
We construct the Venn diagram in terms of percentages.



Using the given information,

$$\begin{aligned} a &= 1 && \{1\% \text{ saw all three teams play}\} \\ a+d &= 4 && \{4\% \text{ saw } A \text{ and } B\} \\ a+b &= 6 && \{6\% \text{ saw } B \text{ and } C\} \\ a+c &= 5 && \{5\% \text{ saw } A \text{ and } C\} \end{aligned}$$

$\therefore \quad d = 3, \quad b = 5, \quad$ and $\quad c = 4$

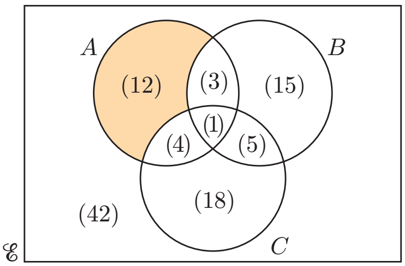


In total, 20% saw team A play,
so $\quad g + 1 + 4 + 3 = 20 \quad \therefore \quad g = 12$

24% saw team B play,
so $\quad e + 1 + 5 + 3 = 24 \quad \therefore \quad e = 15$

28% saw team C play,
so $\quad f + 1 + 5 + 4 = 28 \quad \therefore \quad f = 18$

In total we cover 100% of the population, so $h = 42$.



a $n(\text{saw } A \text{ only}) = 12\%$ {shaded}

b $\quad n(A \text{ or } B, \text{ but not } C)$
$= 12\% + 3\% + 15\%$
$= 30\%$

c $n(\text{saw none of the teams}) = 42\%$

5 In a year group of 63 students, 22 study Biology, 26 study Chemistry, and 25 study Physics. 18 study both Physics and Chemistry, 4 study both Biology and Chemistry, and 3 study both Physics and Biology. 1 studies all three subjects.

a Display this information on a Venn diagram.

b How many students study:

i Biology only

ii Physics or Chemistry

iii none of Biology, Physics, or Chemistry

iv Physics but not Chemistry?

6 36 students participated in the mid-year adventure trip. 19 students went paragliding, 21 went abseiling, and 16 went white water rafting. 7 went abseiling and rafting, 8 went paragliding and rafting, and 11 went paragliding and abseiling. 5 students did all three activities.
Find the number of students who:

- **a** went paragliding or abseiling
- **b** only went white water rafting
- **c** did not participate in any of the activities mentioned
- **d** did exactly two of the activities mentioned.

7

There are 32 students in the woodwind section of the school orchestra. 11 students can play the flute, 15 can play the clarinet, and 12 can play the saxophone. 2 can play the flute and the saxophone, 2 can play the flute and the clarinet, and 6 can play the clarinet and the saxophone. 1 student can play all three instruments. Find the number of students who can play:

- **a** none of the instruments mentioned
- **b** only the saxophone
- **c** the saxophone and the clarinet, but not the flute
- **d** only one of the clarinet, saxophone, or flute.

8 In a particular region, most farms have livestock and crops. A survey of 21 farms showed that 15 grow crops, 9 have cattle, and 11 have sheep. 4 have sheep and cattle, 7 have cattle and crops, and 8 have sheep and crops. 3 have cattle, sheep, and crops. Find the number of farms with:

- **a** only crops
- **b** only animals
- **c** exactly one type of animal, and crops.

Review set 1A

1 Suppose $S = \{x \in \mathbb{Z} : 2 < x \leqslant 7\}$.

- **a** List the elements of S.
- **b** Find $n(S)$.
- **c** How many proper subsets does S have?

2 Determine whether $A \subseteq B$ for the following sets:

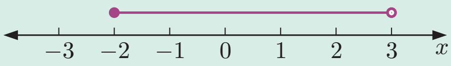
- **a** $A = \{2, 4, 6, 8\}$ and $B = \{x \in \mathbb{Z} : 0 < x < 10\}$
- **b** $A = \varnothing$ and $B = \{x \in (2, 3)\}$
- **c** $A = \{x \in \mathbb{Q} : 2 < x \leqslant 4\}$ and $B = \{x \in \mathbb{R} : 0 \leqslant x < 4\}$
- **d** $A = \{x \in (-\infty, 3)\}$ and $B = \{x \in (-\infty, 4]\}$

3 Find the complement of X given that:

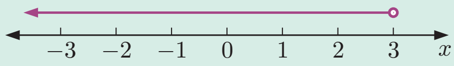
- **a** $\mathscr{E} = \{\text{the 7 colours of the rainbow}\}$ and $X = \{\text{red, indigo, violet}\}$
- **b** $\mathscr{E} = \{x \in \mathbb{Z} : -5 \leqslant x \leqslant 5\}$ and $X = \{-4, -1, 3, 4\}$
- **c** $\mathscr{E} = \{x \in \mathbb{Q}\}$ and $X = \{x \in \mathbb{Q} : x < -8\}$
- **d** $\mathscr{E} = \{x \in \mathbb{R}\}$ and $X = \{x \in [-3, 1) \cup (4, \infty)\}$

4 Write using interval notation, and state whether the interval is closed, open, or neither:

a



b

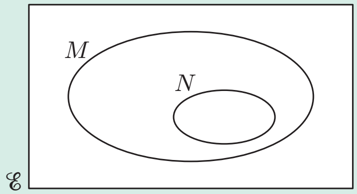


5 Illustrate in the Cartesian plane:

a $\{(x, y) : y = -2x\}$ **b** $\{(x, y) : x \leqslant y\}$

6 On separate Venn diagrams like the one alongside, shade:

a N' **b** $M \cap N$ **c** $M \cap N'$



7 Let $\mathscr{E} = \{$the letters in the English alphabet$\}$, $A = \{$the letters in "springbok"$\}$, and $B = \{$the letters in "waterbuck"$\}$.

a Find:

i $A \cup B$ **ii** $A \cap B$ **iii** $A \cap B'$

b Write a description for each of the sets in **a**.

c Show $\mathscr{E}$, A, and B on a Venn diagram.

8 Let $\mathscr{E} = \{x \in \mathbb{Z}^+ : x \leqslant 30\}$, $P = \{$factors of 24$\}$, and $Q = \{$factors of 30$\}$.

a List the elements of:

i P **ii** Q **iii** $P \cap Q$ **iv** $P \cup Q$

b Illustrate the sets P and Q on a Venn diagram.

9 A school has 564 students. During Term 1, 229 of them were absent for at least one day due to sickness, and 111 students missed some school because of family holidays. 296 students attended every day of Term 1.

a Display this information on a Venn diagram.

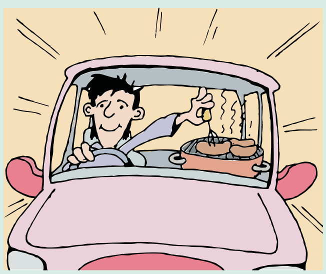
b Find the number of students who were away:

i for both sickness and holidays **ii** for holidays but not sickness

iii during Term 1 for any reason.

10 The main courses at a restaurant all contain rice or onion. Of the 23 choices, 17 contain onion and 14 contain rice. How many dishes contain both rice and onion?

11 38 students were asked what life skills they had. 15 could swim, 12 could drive, and 23 could cook. 9 could cook and swim, 5 could swim and drive, and 6 could drive and cook. There was 1 student who could do all three. Find the number of students who:



a could only cook

b could not do any of these things

c had exactly two life skills.

12 Consider the sets $\mathscr{E} = \{x \in \mathbb{Z}^+ : x \leqslant 10\}$, $P = \{\text{odd numbers less than } 10\}$, and $Q = \{\text{even numbers less than } 11\}$.

- **a** List the sets P and Q.
- **b** What can be said about sets P and Q?
- **c** Illustrate sets P and Q on a Venn diagram.

Review set 1B

1 True or false?

- **a** $\mathbb{N} \subset \mathbb{Q}$
- **b** $0 \in \mathbb{Z}^+$
- **c** $0 \in \mathbb{Q}$
- **d** $\mathbb{R} \subseteq \mathbb{Q}$
- **e** $\mathbb{Z}^+ \cap \mathbb{Z}^- = \{0\}$

2 **a** Write using interval notation:

- **i** the real numbers between 5 and 12
- **ii** the integers between -4 and 7, including -4
- **iii** the natural numbers greater than 45.

b Which sets in **a** are finite and which are infinite?

3 List the subsets of $\{1, 3, 5\}$.

4 Let $\mathscr{E} = \{x \in \mathbb{Z} : 0 < x < 10\}$, $A = \{\text{the even integers between 0 and 9}\}$, and $B = \{\text{the factors of } 8\}$.

a List the elements of:

- **i** A
- **ii** $A \cap B$
- **iii** $(A \cup B)'$

b Represent this information on a Venn diagram.

5 S and T are disjoint sets. $n(S) = s$ and $n(T) = t$. Find:

- **a** $S \cap T$
- **b** $n(S \cup T)$

6 For each of the following sets, determine whether the interval described is closed, open, or neither:

- **a** $x \in (-4, 3]$
- **b** $x \in [-2, 2]$
- **c** $x \in \mathbb{R}$

7 Suppose A and B are each sets of points which define straight lines.

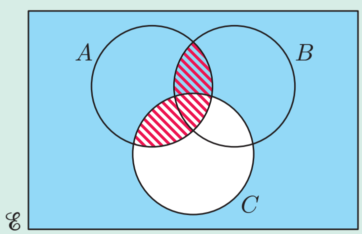
a Describe in words the meaning of:

- **i** $A \cap B$
- **ii** $A \cup B$

b Is $A \cap B$ necessarily finite? Explain your answer.

c If $A \cap B$ is finite, what possible values can $n(A \cap B)$ have?

8



Give an expression for the region shaded in:

- **a** blue
- **b** red.