f $\left.\dfrac{dP}{dt}\right|_{t=4} = 4$ Profit is increasing at \$4000 per year after 4 years.

$\left.\dfrac{dP}{dt}\right|_{t=10} = 28$ Profit is increasing at \$28 000 per year after 10 years.

$\left.\dfrac{dP}{dt}\right|_{t=25} = 88$ Profit is increasing at \$88 000 per year after 25 years.

2 **a** **i** $Q(0) = 100$ **ii** $Q(25) = 50$ **iii** $Q(100) = 0$

b **i** decr. 1 unit per year **ii** decr. $\frac{1}{\sqrt{2}}$ units per year

c $Q'(t) = -\dfrac{5}{\sqrt{t}} < 0$

3 **a** 0.5 m

b $t = 4$: 9.17 m, $t = 8$: 12.5 m, $t = 12$: 14.3 m

c $t = 0$: 3.9 m year^{-1}, $t = 5$: 0.975 m year^{-1}, $t = 10$: 0.433 m year^{-1}

d As $\dfrac{dH}{dt} = \dfrac{97.5}{(t+5)^2} > 0$ for all $t \geqslant 0$, the tree is always growing.

4 **a** $C'(x) = 0.0009x^2 + 0.04x + 4$ dollars per pair

b $C'(220) = \$56.36$ per pair. This estimates the additional cost of making one more pair of jeans if 220 pairs are currently being made.

c \$56.58 This is the actual increase in cost to make an extra pair of jeans (221 rather than 220).

d $C''(x) = 0.0018x + 0.04$

$C''(x) = 0$ when $x = -22.2$. This is where the rate of change is a minimum, however it is out of the bounds of the model (you cannot make < 0 jeans!).

5 **a** **i** €4500 **ii** €4000

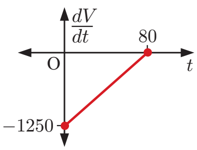
b **i** decrease of €210.22 per km h^{-1}

ii increase of €11.31 per km h^{-1}

c $\dfrac{dC}{dv} = 0$ at $v = \sqrt[3]{500\,000} \approx 79.4$ km h^{-1}

6 **a** $\dfrac{dV}{dt} = -1250\left(1 - \dfrac{t}{80}\right)$ L min^{-1}

b at $t = 0$ when the tap was first opened



c $\dfrac{d^2V}{dt^2} = \dfrac{125}{8}$ L min^{-2}

This shows that the rate of change of V is constantly increasing, so the outflow is decreasing at a constant rate.

7 **a** The near part of the lake is 2 km from the sea, the furthest part is 3 km.

b $\dfrac{dy}{dx} = \frac{3}{10}x^2 - x + \frac{3}{5}$

$\left.\dfrac{dy}{dx}\right|_{x=\frac{1}{2}} = 0.175$, height of hill is increasing as gradient is positive.

$\left.\dfrac{dy}{dx}\right|_{x=1\frac{1}{2}} = -0.225$, height of hill is decreasing as gradient is negative.

$\therefore$ top of the hill is between $x = \frac{1}{2}$ and $x = 1\frac{1}{2}$.

c 2.55 km from the sea, 63.1 m deep

8 **a** $k = \frac{1}{50}\ln 2 \approx 0.0139$

b **i** 20 grams **ii** 14.3 grams **iii** 1.95 grams

c 9 days and 6 minutes (216 hours)

d **i** -0.0693 g h^{-1} **ii** -2.64×10^{-7} g h^{-1}

e **Hint:** You should find $\dfrac{dW}{dt} = -\frac{1}{50}\ln 2 \times 20e^{-\frac{1}{50}\ln 2t}$

9 **a** $k = \frac{1}{15}\ln\left(\frac{19}{3}\right) \approx 0.123$ **b** 100°C

c $c = -k \approx -0.123$

d **i** decreasing at 11.7°C min^{-1}

ii decreasing at 3.42°C min^{-1}

iii decreasing at 0.998°C min^{-1}

10 **a** 43.9 cm **b** 10.4 years

c **i** growing at 5.45 cm per year

ii growing at 1.88 cm per year

11 **a** $A(0) = 0$

b **i** $k = \dfrac{\ln 2}{3}$ (≈ 0.231)

ii 0.728 litres of alcohol produced per hour

12 $\frac{21}{\sqrt{2}}$ cm^2 per radian

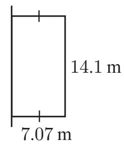
13 **a** rising at 2.73 m per hour **b** rising

14 **b** **i** 0 **ii** 1 **iii** ≈ 1.11

EXERCISE 14E

1 250 items

2 **b** $L_{\min} \approx 28.3$ m, $x \approx 7.07$ m **c**

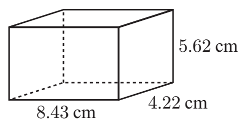


3 10 blankets **4** 14.8 km h^{-1} **5** at 4.41 months old

6 **a** **Hint:** $V = 200 = 2x \times x \times h$

b **Hint:** Show $h = \dfrac{100}{x^2}$ and substitute into the surface area equation.

c $SA_{\min} \approx 213$ cm^2, $x \approx 4.22$ cm **d**



7 20 kettles **8** $\mathrm{C}\left(\frac{1}{\sqrt{2}}, e^{-\frac{1}{2}}\right)$

9 **a** Recall that $V_{\text{cylinder}} = \pi r^2 h$ and that $1\text{ L} = 1000\text{ cm}^3$.

b Recall that $SA_{\text{cylinder}} = 2\pi r^2 + 2\pi rh$.

c radius ≈ 5.42 cm, height ≈ 10.8 cm

10 **b** $\theta \approx 1.91$, $A \approx 237$ cm^2 **11** **b** 6 cm $\times$ 6 cm

12 **a** $0 \leqslant x \leqslant 63.7$

b $l = 100$ m, $x = \frac{100}{\pi} \approx 31.83$ m, $A = \frac{20\,000}{\pi} \approx 6366$ m^2

13 after 13.8 weeks **14** after 40 minutes

15 **c** $\theta = 30°$, $A \approx 130$ cm^2

16 **a** **Hint:** Show that $\mathrm{AC} = \frac{\theta}{360} \times 2\pi \times 10$

b **Hint:** Show that $2\pi r = \mathrm{AC}$

c **Hint:** Use the result from **b** and Pythagoras' theorem.

d $V = \frac{1}{3}\pi\left(\frac{\theta}{36}\right)^2\sqrt{100 - \left(\frac{\theta}{36}\right)^2}$ **e** $\theta \approx 294°$

17 1 hour 34 min 53 s when $\theta \approx 36.9°$ **18** 9.87 m

EXERCISE 14F

1 a is decreasing at 7.5 units per second

2 increasing at 1 cm per minute

3 **a** 4π m^2 per second **b** 8π m^2 per second

4 increasing at 6π m^2 per minute

5 decreasing at $0.16\ \text{m}^3$ per minute

6 $\frac{20}{3}$ cm per minute

7 $\frac{25\sqrt{3}}{6} \approx 7.22$ cm per minute

8 decreasing at $\frac{250}{13} \approx 19.2\ \text{m s}^{-1}$

9 **a** $0.2\ \text{m s}^{-1}$ **b** $\frac{4}{45}\ \text{m s}^{-1}$

10 decreasing at $\frac{\sqrt{2}}{100}$ radians per second

11 increasing at 0.12 radians per minute

REVIEW SET 14A

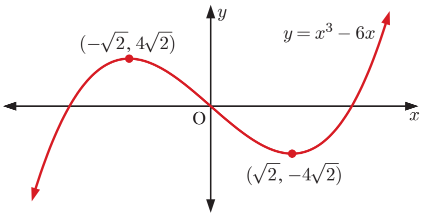
1 **a** $y = 4x + 2$ **b** $y = 4x + 4\ln 2 - 4$ **c** $y = e^2$

2 $a = \frac{5}{2},\ b = -\frac{3}{2}$

3 **a** $a = -6$

b local max. $(-\sqrt{2}, 4\sqrt{2})$, local min. $(\sqrt{2}, -4\sqrt{2})$

c



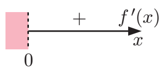
4 **a** $y = \frac{1}{5}x - \frac{11}{5}$ (or $x - 5y = 11$) **b** $y = -4x + 14$

5 $\frac{3267}{152}$ units^2 **6** $a = 64$ **7** P(0, 7.5), Q(3, 0)

9 $3x - 4y = -5$

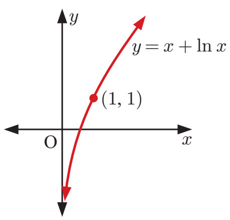
10 **a** $x > 0$

b Sign diagram of $f'(x)$



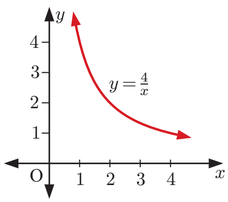
$f(x)$ is increasing for all $x > 0$.

c



d normal is $x + 2y = 3$

11 **a**



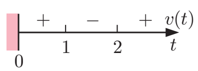
b $y = -\dfrac{4}{k^2}x + \dfrac{8}{k}$

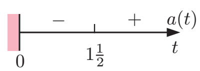
c $\text{A}(2k, 0)$, $\text{B}\left(0, \dfrac{8}{k}\right)$

d Area $= 8$ units^2

e $k = 2$

12 **a** $v(t) = (6t^2 - 18t + 12)\ \text{cm s}^{-1}$, $a(t) = (12t - 18)\ \text{cm s}^{-2}$



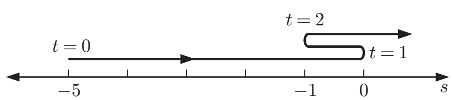


b $s(0) = -5$ cm (5 cm to the left of origin)
$v(0) = 12\ \text{cm s}^{-1}$ towards origin
$a(0) = -18\ \text{cm s}^{-2}$ (reducing speed)

c At $t = 2$, particle is 1 cm to the left of the origin, is stationary and is accelerating towards the origin.

d $t = 1$, $s = 0$ and $t = 2$, $s = -1$

e



f $1 \leqslant t \leqslant 1\frac{1}{2}$ and $t \geqslant 2$

13 **b** $k = 9$

14 **a** $x(0) = 3$ cm, $x'(0) = 2\ \text{cm s}^{-1}$, $x''(0) = 0\ \text{cm s}^{-2}$

b $t = \frac{\pi}{4}$ s and $\frac{3\pi}{4}$ s **c** 4 cm

15 6 cm from each end

16 **a** $y = \dfrac{1}{x^2},\ x > 0$

c base is 1.26 m square, height 0.630 m

17 **a** $v(t) = 15 + \dfrac{120}{(t+1)^3}\ \text{cm s}^{-1}$, $a(t) = \dfrac{-360}{(t+1)^4}\ \text{cm s}^{-2}$

b At $t = 3$, particle is 41.25 cm to the right of the origin, moving to the right at $16.88\ \text{cm s}^{-1}$ and decelerating at $1.41\ \text{cm s}^{-2}$.

c speed is never increasing

18 $\text{A}\left(\dfrac{1}{2}, \dfrac{1}{e}\right)$ **19** $\frac{20\sqrt{10}}{3} \approx 21.1$ m per minute

20 **a** $V(r) = \frac{8}{9}\pi r^3\ \text{m}^3$

b $\dfrac{dr}{dt} = -\dfrac{8}{375\pi} \approx -0.006\,79\ \text{m min}^{-1}$

REVIEW SET 14B

1 **a** $x = 1$ **b** $ex - 2y = e - \dfrac{2}{e}$ **c** $y = 16x - \frac{127}{2}$

2 $a = -14,\ b = 21$

3 **a** $f(3) = 2,\ f'(3) = -1$ **b** $f(x) = x^2 - 7x + 14$

4 **a** $2x + 3y = \frac{2\pi}{3} + 2\sqrt{3}$ **b** $\sqrt{2}y - 4x = 1 - 2\pi$

5 $p = 1,\ q = -8$ **6** $(-2, -25)$ **7** $a = \frac{1}{2}$

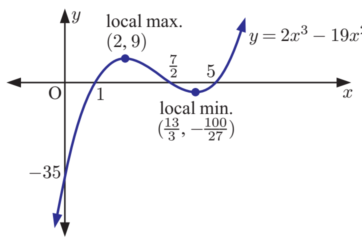
8 **a** local minimum at (0, 1) **c** $f''(x) = e^x$

9 $(0,\ \ln 4 - 1)$

10 **a** y-intercept $= -35$ **b** $x = 1, \frac{7}{2}, 5$

c local maximum at (2, 9), local minimum at $\left(\frac{13}{3}, -\frac{100}{27}\right)$

d



11 $\text{BC} = \frac{8\sqrt{10}}{3}$ units

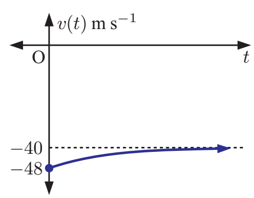
12 **a** 60 cm **b** **i** 4.24 years **ii** 201 years

c **i** 16 cm per year **ii** 1.95 cm per year

13 **a** $v(t) = -8e^{-\frac{t}{10}} - 40\ \text{m s}^{-1}$
$a(t) = \frac{4}{5}e^{-\frac{t}{10}}\ \text{m s}^{-2}$ $\{t \geqslant 0\}$

b $s(0) = 80$ m
$v(0) = -48\ \text{m s}^{-1}$
$a(0) = 0.8\ \text{m s}^{-2}$

c



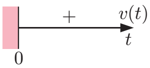
d $t = 10\ln 2$ seconds

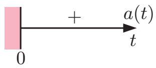
14 **a** **i** \$535 **ii** \$1385.79

b **i** $-\$0.267$ per km h^{-1} **ii** \$2.33 per km h^{-1}

c $51.3\ \text{km h}^{-1}$

15 **a** $v(t) = 3 - \dfrac{1}{2\sqrt{t+1}}$ $\qquad a(t) = \dfrac{1}{4(t+1)^{\frac{3}{2}}}$



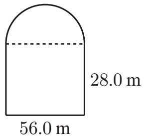


b $x(0) = -1, \quad v(0) = 2.5, \quad a(0) = 0.25$
Particle is 1 cm to the left of the origin, is travelling to the right at 2.5 cm s^{-1}, and accelerating at 0.25 cm s^{-2}.

c Particle is 21 cm to the right of the origin, is travelling to the right at 2.83 cm s^{-1}, and accelerating at 0.009 26 cm s^{-2}.

d never changes direction **e** never decreasing

16 **b** $A = 200x - 2x^2 - \frac{1}{2}\pi x^2$ **c**



17 **a** $v(0) = 0\ \text{cm s}^{-1}, \ v(\frac{1}{2}) = -\pi\ \text{cm s}^{-1}, \ v(1) = 0\ \text{cm s}^{-1},$
$v(\frac{3}{2}) = \pi\ \text{cm s}^{-1}, \ v(2) = 0\ \text{cm s}^{-1}$

b $0 \leqslant t \leqslant 1, \ 2 \leqslant t \leqslant 3, \ 4 \leqslant t \leqslant 5,$ etc.
So, for $2n \leqslant t \leqslant 2n+1, \ n \in \{0, 1, 2, 3, ...\}$

18 $x = \dfrac{k}{2}\left(1 - \dfrac{1}{\sqrt{3}}\right)$ **19** 3.60 m s^{-1}

20 increasing at 0.128 radians per second

21 **a** $\frac{\sqrt{3}}{2}\pi\ \text{cm s}^{-1}$ **b** 0 cm s^{-1}

22 **a** **i** $y = -\dfrac{a^2}{4b} + \dfrac{a}{2b}x$ **ii** when $y = 0, \ x = \dfrac{a}{2}$

b **i** $y = -\dfrac{2b}{a}x + b$ **ii** when $x = 0, \ y = b$

iii **Hint:** Let P$''$ be the point on the line $y = -b$ where the distance to P is shortest.
Show that FP = P$''$P.

c **i** **Hint:** Show that $\triangle$FPP$' \cong \triangle$P$''$PP$'$.

ii **Hint:** Show that the tangents meet at $\left(\dfrac{a+c}{2}, \dfrac{ac}{4b}\right)$.

EXERCISE 15A.1

1 **a** **i** 0.6 units^2 **ii** 0.4 units^2 **b** 0.5 units^2

2 **a** 0.737 units^2 **b** 0.653 units^2

3

n	A_L	A_U
10	2.1850	2.4850
25	2.2736	2.3936
50	2.3034	2.3634
100	2.3184	2.3484
500	2.3303	2.3363

converges to $\frac{7}{3}$

4 **a** **i**

n	A_L	A_U
5	0.160 00	0.360 00
10	0.202 50	0.302 50
50	0.240 10	0.260 10
100	0.245 03	0.255 03
500	0.249 00	0.251 00
1000	0.249 50	0.250 50
10 000	0.249 95	0.250 05

ii

n	A_L	A_U
5	0.400 00	0.600 00
10	0.450 00	0.550 00
50	0.490 00	0.510 00
100	0.495 00	0.505 00
500	0.499 00	0.501 00
1000	0.499 50	0.500 50
10 000	0.499 95	0.500 05

iii

n	A_L	A_U
5	0.549 74	0.749 74
10	0.610 51	0.710 51
50	0.656 10	0.676 10
100	0.661 46	0.671 46
500	0.665 65	0.667 65
1000	0.666 16	0.667 16
10 000	0.666 62	0.666 72

iv

n	A_L	A_U
5	0.618 67	0.818 67
10	0.687 40	0.787 40
50	0.738 51	0.758 51
100	0.744 41	0.754 41
500	0.748 93	0.750 93
1000	0.749 47	0.750 47
10 000	0.749 95	0.750 05

b **i** $\frac{1}{4}$ **ii** $\frac{1}{2}$ **iii** $\frac{2}{3}$ **iv** $\frac{3}{4}$ **c** area $= \dfrac{1}{a+1}$

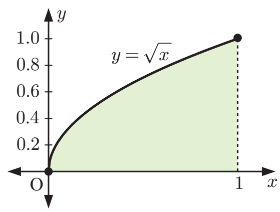
5 **a**

n	Rational bounds for π
10	$2.9045 < \pi < 3.3045$
50	$3.0983 < \pi < 3.1783$
100	$3.1204 < \pi < 3.1604$
200	$3.1312 < \pi < 3.1512$
1000	$3.1396 < \pi < 3.1436$
10 000	$3.1414 < \pi < 3.1418$

b $n = 10\,000$

EXERCISE 15A.2

1 **a**



b

n	A_L	A_U
5	0.5497	0.7497
10	0.6105	0.7105
50	0.6561	0.6761
100	0.6615	0.6715
500	0.6656	0.6676

c $\int_0^1 \sqrt{x}\,dx \approx 0.67$

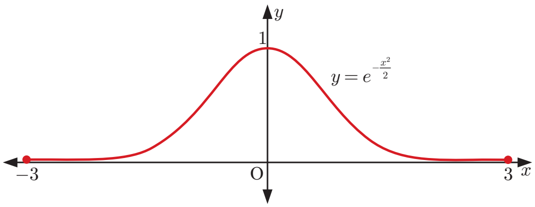
2 **a** $A_L = \dfrac{2}{n}\displaystyle\sum_{i=0}^{n-1}\sqrt{1+{x_i}^3}, \quad A_U = \dfrac{2}{n}\sum_{i=1}^{n}\sqrt{1+{x_i}^3}$

b

n	A_L	A_U
50	3.2016	3.2816
100	3.2214	3.2614
500	3.2373	3.2453

c $\int_0^2 \sqrt{1+x^3}\,dx \approx 3.24$

3 **a**



b upper ≈ 1.2506, lower ≈ 1.2493

c upper ≈ 1.2506, lower ≈ 1.2493

d $\int_{-3}^{3} e^{-\frac{x^2}{2}}\,dx \approx 2.4999$ compared to $\sqrt{2\pi} \approx 2.5066$

4 **a** 18 **b** 4.5 **c** 2π

EXERCISE 15B

1 **a** **i** $\dfrac{x^2}{2}$ **ii** $\dfrac{x^3}{3}$ **iii** $\dfrac{x^6}{6}$ **iv** $-\dfrac{1}{x}$

v $-\dfrac{1}{3x^3}$ **vi** $\frac{3}{4}x^{\frac{4}{3}}$ **vii** $2\sqrt{x}$

b The antiderivative of x^n is $\dfrac{x^{n+1}}{n+1}$ $(n \neq -1)$.

2 **a** **i** $\frac{1}{2}e^{2x}$ **ii** $\frac{1}{5}e^{5x}$ **iii** $2e^{\frac{1}{2}x}$ **iv** $100e^{0.01x}$

v $\frac{1}{\pi}e^{\pi x}$ **vi** $3e^{\frac{x}{3}}$

b The antiderivative of e^{kx} is $\dfrac{1}{k}e^{kx}$.

3 **a** $\dfrac{d}{dx}(x^3+x^2) = 3x^2+2x$

$\therefore$ the antiderivative of $6x^2+4x = 2x^3+2x^2$

b $\dfrac{d}{dx}(e^{3x+1}) = 3e^{3x+1}$

$\therefore$ the antiderivative of $e^{3x+1} = \frac{1}{3}e^{3x+1}$

c $\dfrac{d}{dx}(x\sqrt{x}) = \frac{3}{2}\sqrt{x}$

$\therefore$ the antiderivative of $\sqrt{x} = \frac{2}{3}x\sqrt{x}$

d $\dfrac{d}{dx}(2x+1)^4 = 8(2x+1)^3$

$\therefore$ the antiderivative of $(2x+1)^3 = \frac{1}{8}(2x+1)^4$

EXERCISE 15C

1 **a** $\frac{1}{4}$ units^2 **b** $2\frac{1}{3}$ units^2 **c** $\frac{2}{3}$ units^2

3 **a** $3\frac{3}{4}$ units^2 **b** $24\frac{2}{3}$ units^2 **c** $\frac{-2+4\sqrt{2}}{3}$ units^2

d ≈ 3.48 units^2 **e** 2 units^2

4 **c** **i** $\int_0^1 (-x^2)\,dx = -\frac{1}{3}$, the area between $y=-x^2$ and the x-axis from $x=0$ to $x=1$ is $\frac{1}{3}$ units^2.

ii $\int_0^1 (x^2-x)\,dx = -\frac{1}{6}$, the area between $y=x^2-x$ and the x-axis from $x=0$ to $x=1$ is $\frac{1}{6}$ units^2.

iii $\int_{-2}^{0} 3x\,dx = -6$, the area between $y=3x$ and the x-axis from $x=-2$ to $x=0$ is 6 units^2

d $-\pi$

EXERCISE 15D

1 $\dfrac{dy}{dx} = 7x^6$, $\int x^6\,dx = \frac{1}{7}x^7+c$

2 $\dfrac{dy}{dx} = 3x^2+2x$, $\int (3x^2+2x)\,dx = x^3+x^2+c$

3 $\dfrac{dy}{dx} = 2e^{2x+1}$, $\int e^{2x+1}\,dx = \frac{1}{2}e^{2x+1}+c$

4 $\dfrac{dy}{dx} = 8(2x+1)^3$, $\int (2x+1)^3\,dx = \frac{1}{8}(2x+1)^4+c$

5 $\dfrac{dy}{dx} = \frac{3}{2}\sqrt{x}$, $\int \sqrt{x}\,dx = \frac{2}{3}x\sqrt{x}+c$

6 $\dfrac{dy}{dx} = -\dfrac{1}{2x\sqrt{x}}$, $\displaystyle\int \frac{1}{x\sqrt{x}}\,dx = -\frac{2}{\sqrt{x}}+c$

7 $\dfrac{dy}{dx} = -2\sin 2x$, $\int \sin 2x\,dx = -\frac{1}{2}\cos 2x+c$

8 $\dfrac{dy}{dx} = -5\cos(1-5x)$,

$\int \cos(1-5x)\,dx = -\frac{1}{5}\sin(1-5x)+c$

9 $\int (2x-1)(x^2-x)^2\,dx = \frac{1}{3}(x^2-x)^3+c$

11 $\dfrac{dy}{dx} = \dfrac{-2}{\sqrt{1-4x}}$, $\displaystyle\int \frac{1}{\sqrt{1-4x}}\,dx = -\frac{1}{2}\sqrt{1-4x}+c$

EXERCISE 15E.1

1 **a** $\dfrac{x^5}{5}-\dfrac{x^3}{3}-\dfrac{x^2}{2}+2x+c$ **b** $x^5-x^4-2x^3-7x+c$

c $\frac{2}{3}x^{\frac{3}{2}}+e^x+c$ **d** $3e^x+\frac{1}{3}x^3+c$

e $\frac{2}{5}x^{\frac{5}{2}}-2x+c$ **f** $-2x^{-\frac{1}{2}}+2x^2+c$

g $\frac{1}{8}x^4-\frac{1}{5}x^5+\frac{3}{4}x^{\frac{4}{3}}+c$ **h** $\dfrac{x^2}{4}+\frac{1}{3}x^3-e^x+c$

i $5e^x+\frac{1}{12}x^4-\frac{2}{3}x^{\frac{3}{2}}+c$

2 **a** $-3\cos x-2x+c$ **b** $2x^2-2\sin x+c$

c $-\cos x-2\sin x+e^x+c$ **d** $\frac{2}{7}x^3\sqrt{x}+10\cos x+c$

e $\frac{1}{9}x^3-\frac{1}{6}x^2+\sin x+c$ **f** $\cos x+\frac{4}{3}x\sqrt{x}+c$

3 **a** $\frac{1}{3}x^3+\frac{3}{2}x^2-2x+c$ **b** $\frac{2}{3}x^{\frac{3}{2}}-2x^{\frac{1}{2}}+c$

c $2e^x+\dfrac{1}{x}+c$ **d** $-2x^{-\frac{1}{2}}-8x^{\frac{1}{2}}+c$

e $\frac{4}{3}x^3+2x^2+x+c$ **f** $\frac{1}{3}x^3+2x-\dfrac{1}{x}+c$

g $\frac{4}{3}x^{\frac{3}{2}}-2x^{\frac{1}{2}}+c$ **h** $2x^{\frac{1}{2}}+8x^{-\frac{1}{2}}-\frac{20}{3}x^{-\frac{3}{2}}+c$

i $\frac{1}{4}x^4+x^3+\frac{3}{2}x^2+x+c$

4 **a** $\frac{2}{3}x^{\frac{3}{2}}+\frac{1}{2}\sin x+c$ **b** $2e^t+4\cos t+c$

c $3\sin t+\cos t+c$

5 **a** $y=6x+c$ **b** $y=\frac{4}{3}x^3+c$

c $y=\frac{10}{3}x\sqrt{x}-\frac{1}{3}x^3+c$ **d** $y=-\dfrac{1}{x}+c$

e $y=2e^x-5x+c$ **f** $y=x^4+x^3+c$

6 **a** $f(x)=x-2x^2+\frac{4}{3}x^3+c$

b $f(x)=\frac{2}{3}x^{\frac{3}{2}}-4\sqrt{x}+c$ **c** $f(x)=x+\dfrac{5}{x}+c$

EXERCISE 15E.2

1 **a** $f(x)=x^2-x+3$ **b** $f(x)=x^3+x^2-7$

c $f(x)=e^x+2\sqrt{x}-1-e$ **d** $f(x)=\frac{1}{2}x^2-4\sqrt{x}+\frac{11}{2}$

2 **a** $f(x)=\dfrac{x^3}{3}-4\sin x+3$

b $f(x)=2\sin x+3\cos x-2\sqrt{2}$

3 **a** $f(x)=\frac{1}{3}x^3+\frac{1}{2}x^2+x+\frac{1}{3}$

b $f(x)=4x^{\frac{5}{2}}+4x^{\frac{3}{2}}-4x+5$

c $f(x)=-\cos x-x+4$ **d** $f(x)=\frac{1}{3}x^3-\frac{16}{3}x+5$

EXERCISE 15F

1 **a** $\frac{1}{8}(2x+5)^4+c$ **b** $\dfrac{1}{2(3-2x)}+c$

c $\dfrac{-2}{3(2x-1)^3}+c$ **d** $\frac{1}{32}(4x-3)^8+c$

e $\frac{2}{9}(3x-4)^{\frac{3}{2}}+c$ **f** $-4\sqrt{1-5x}+c$

g $-\frac{3}{5}(1-x)^5+c$ **h** $-2\sqrt{3-4x}+c$

2 **a** $-\frac{1}{3}\cos(3x)+c$ **b** $-\frac{1}{2}\sin(-4x)+x+c$

c $6\sin\left(\frac{x}{2}\right)+c$ **d** $-\frac{3}{2}\cos(2x)+e^{-x}+c$

e $-\cos\left(2x+\frac{\pi}{6}\right)+c$ **f** $3\sin\left(\frac{\pi}{4}-x\right)+c$

g $\frac{1}{2}\sin(2x)-\frac{1}{2}\cos(2x)+c$

h $-\frac{2}{3}\cos(3x)+\frac{5}{4}\sin(4x)+c$

i $\frac{1}{16}\sin(8x)+3\cos x+c$

3 $y=\frac{1}{3}(2x-7)^{\frac{3}{2}}+2$ **4** $(-8,-19)$

5 **a** $\frac{1}{2}(2x-1)^3+c$ **b** $\frac{1}{5}x^5-\frac{1}{2}x^4+\frac{1}{3}x^3+c$

c $-\frac{1}{12}(1-3x)^4+c$ **d** $x-\frac{2}{3}x^3+\frac{1}{5}x^5+c$

e $-\frac{8}{3}(5-x)^{\frac{3}{2}}+c$ **f** $\frac{1}{7}x^7+\frac{3}{5}x^5+x^3+x+c$

6 **a** $2e^x+\frac{5}{2}e^{2x}+c$ **b** $\frac{3}{5}e^{5x-2}+c$

c $-\frac{1}{3}e^{7-3x}+c$ **d** $\frac{1}{2}e^{2x}+2x-\frac{1}{2}e^{-2x}+c$

e $-\frac{1}{2}e^{-2x}-4e^{-x}+4x+c$ **f** $\frac{1}{2}x^2+5(1-x)^{-1}+c$

7 $y=x-2e^x+\frac{1}{2}e^{2x}+\frac{11}{2}$

8 $p=-\frac{1}{4},\ \ f(x)=\frac{1}{2}\cos(\frac{1}{2}x)+\frac{1}{2}$

10 $f(x)=-e^{-2x}+4$

11 $f(x)=\frac{2}{3}x^{\frac{3}{2}}-\frac{1}{8}e^{-4x}+\frac{1}{8}e^{-4}-\frac{2}{3}$

EXERCISE 15G

1 **a** $\int_1^4\sqrt{x}\,dx=\frac{14}{3},\ \ \int_1^4(-\sqrt{x})\,dx=-\frac{14}{3}$

b $\int_0^1 x^7\,dx=\frac{1}{8},\ \ \int_0^1(-x^7)\,dx=-\frac{1}{8}$

2 **a** $\frac{1}{3}$ **b** $\frac{7}{3}$ **c** $\frac{8}{3}$ **d** 1

3 **a** -4 **b** 6.25 **c** 2.25 **4** **a** $\frac{1}{3}$ **b** $\frac{2}{3}$ **c** 1

5 **a** $\frac{1}{4}$ **b** $\frac{2}{3}$ **c** $e-1\ \ (\approx 1.72)$ **d** $\frac{1}{2}$

e $1\frac{1}{2}$ **f** $6\frac{2}{3}$ **g** $\ln 3\ \ (\approx 1.10)$ **h** $\frac{1}{2}$

i ≈ 1.52 **j** 2 **k** $e-1\ \ (\approx 1.72)$ **l** $\frac{1}{3}$

6 $m=-1$ or $\frac{4}{3}$ **7** **a** $\frac{\pi}{8}+\frac{1}{4}$ **b** $\frac{\pi}{4}$

8 **a** 6.5 **b** -9 **c** 0 **d** -2.5

9 **a** 2π **b** -4 **c** $\frac{\pi}{2}$ **d** $\frac{5\pi}{2}-4$

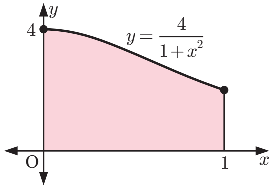
10 **a** $\int_2^7 f(x)\,dx$ **b** $\int_1^9 g(x)\,dx$

11 **a** -5 **b** 4

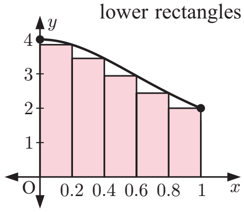
12 **a** 4 **b** 0 **c** -8 **d** $k=-\frac{7}{4}$ **13** 0

REVIEW SET 15A

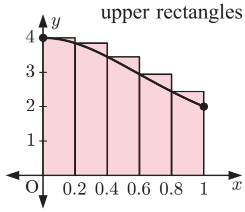
1 **a**



lower rectangles



upper rectangles



b

n	A_L	A_U
5	2.9349	3.3349
50	3.1215	3.1615
100	3.1316	3.1516
500	3.1396	3.1436

c $\displaystyle\int_0^1 \frac{4}{1+x^2}\,dx\approx 3.1416$

2 **a** 2π **b** 4

3 **a** $8\sqrt{x}+c$ **b** $-\frac{1}{4}\cos(4x-5)+c$ **c** $-\frac{1}{3}e^{4-3x}+c$

4 **a** $12\frac{4}{9}$ **b** $\sqrt{2}$

5 $\dfrac{dy}{dx}=\dfrac{x}{\sqrt{x^2-4}},\quad \displaystyle\int\frac{x}{\sqrt{x^2-4}}\,dx=\sqrt{x^2-4}+c$

6 $b=\frac{\pi}{4},\ \frac{3\pi}{4}$

7 **a** $y=\frac{1}{5}x^5-\frac{2}{3}x^3+x+c$ **b** $y=400x+40e^{-\frac{x}{2}}+c$

8 $f(x)=3x^3+5x^2+6x-1$ **9** $a=\ln\sqrt{2}$

10 **a** $f(x)=\frac{1}{4}x^4+\frac{1}{3}x^3-\frac{10}{3}x+3$ **b** $3x+26y=84$

11 **a** $e^{3x}+6e^{2x}+12e^x+8$ **b** $\frac{1}{3}e^3+3e^2+12e-7\frac{1}{3}$

REVIEW SET 15B

1 **a** $A=\frac{17}{4},\ \ B=\frac{25}{4}$ **b** $\int_0^2(4-x^2)\,dx\approx\frac{21}{4}$

2 **a** $-2e^{-x}+3x+c$ **b** $\frac{2}{3}x\sqrt{x}-2\sqrt{x}+c$

c $9x+3e^{2x-1}+\frac{1}{4}e^{4x-2}+c$

3 $f(x)=\frac{1}{3}x^3-\frac{3}{2}x^2+2x+2\frac{1}{6}$ **4** $\frac{2}{3}(\sqrt{5}-\sqrt{2})$

5 $\dfrac{d}{dx}(3x^2+x)^3=3(3x^2+x)^2(6x+1)$

$\int(3x^2+x)^2(6x+1)\,dx=\frac{1}{3}(3x^2+x)^3+c$

6 **a** 6 **b** 3 **7** $f\left(\frac{\pi}{2}\right)=3-\frac{\pi}{2}$ **8** $e^{-\pi}$

9 $\dfrac{1}{2(n+1)}(2x+3)^{n+1}+c,\ \ n\neq -1$

10 $a=\frac{1}{3},\ \ f'(x)=2\sqrt{x}+\dfrac{1}{3\sqrt{x}}$ is never 0 as $\sqrt{x}\geqslant 0$ for all x

$\therefore\ \ f'(x)>0$ for all x

11 $a=0$ or ± 3

EXERCISE 16A

1 **a** 30 units^2 **b** $\frac{9}{2}$ units^2 **c** $\frac{27}{2}$ units^2 **d** 2 units^2

2 **a** $\frac{1}{3}$ units^2 **b** 2 units^2 **c** $63\frac{3}{4}$ units^2
d $(e-1)$ units^2 **e** $20\frac{5}{6}$ units^2 **f** 18 units^2
g $\frac{1}{2}$ units^2 **h** $4\frac{1}{2}$ units^2 **i** $(2e-\frac{2}{e})$ units^2

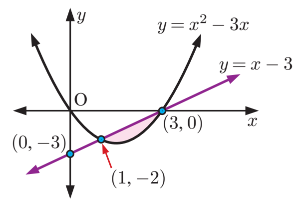
3 $\frac{2}{3}$ units^2

EXERCISE 16B

1 **a** $4\frac{1}{2}$ units^2 **b** $(1+e^{-2})$ units^2 **c** $1\frac{5}{27}$ units^2
d 2 units^2 **e** $2\frac{1}{4}$ units^2 **f** $(\frac{\pi}{2}-1)$ units^2

2 $10\frac{2}{3}$ units^2

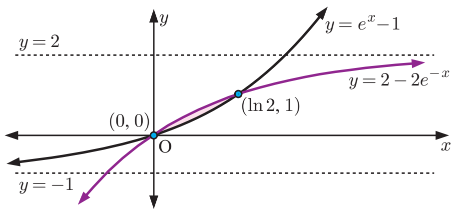
3 **a**



b $(1,-2)$ and $(3,0)$ **c** $1\frac{1}{3}$ units^2

4 $\frac{1}{3}$ units^2

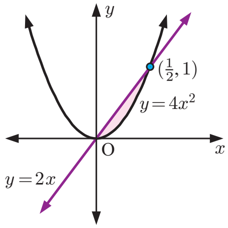
5 **a**



b $(0,0)$ and $(\ln 2, 1)$
c enclosed area $= 3\ln 2 - 2$ (≈ 0.0794) units^2

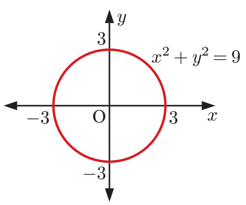
6 $\frac{1}{2}$ units^2

7



enclosed area $= \frac{1}{12}$ units^2

8



a Rearranging $x^2+y^2=9$ gives $y=\pm\sqrt{9-x^2}$. The upper half has $y\geqslant 0$, so $y=\sqrt{9-x^2}$.
b $\frac{9\pi}{4}$

9 **a** $40\frac{1}{2}$ units^2 **b** 8 units^2 **c** 8 units^2

10 **a** C_1 is $y=\sin x$, C_2 is $y=3\sin x$ **b** 4 units^2

11 **a** $\int_3^5 f(x)\,dx = -$ (area between $x=3$ and $x=5$)
b $\int_1^3 f(x)\,dx - \int_3^5 f(x)\,dx + \int_5^7 f(x)\,dx$

12 **a** C_1 is $y=\frac{1}{2}+\frac{1}{2}\cos(2x)$, C_2 is $y=\cos(2x)$
b $A(0,1)$, $B(\frac{\pi}{4},0)$, $C(\frac{\pi}{2},0)$, $D(\frac{3\pi}{4},0)$, $E(\pi,1)$
c Area $= \int_0^{\pi}(\frac{1}{2}+\frac{1}{2}\cos(2x)-\cos(2x))\,dx$

13 If $h(x)\geqslant 0$ on $a\leqslant x\leqslant b$, the area between $y=h(x)$ and the x-axis is $\int_a^b h(x)\,dx$. If $h(x)<0$ on $a\leqslant x\leqslant b$, the area between $y=h(x)$ and the x-axis is $\int_a^b -h(x)\,dx$.
$\therefore$ the area between $y=h(x)$ and the x-axis on $a\leqslant x\leqslant b$ is $\int_a^b |h(x)|\,dx$.
Letting $h(x)=f(x)-g(x)$, the area between $y=f(x)-g(x)$ and the x-axis $y=0$ on $a\leqslant x\leqslant b$ is $\int_a^b |f(x)-g(x)|\,dx$.
Equivalently, the area between $y=f(x)$ and $y=g(x)$ on $a\leqslant x\leqslant b$ is $\int_a^b |f(x)-g(x)|\,dx$.

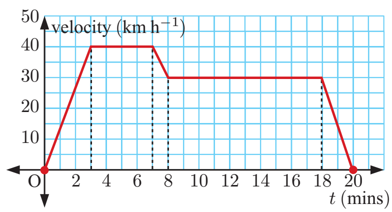
14 $b\approx 1.3104$ **15** $a=\sqrt{3}$

EXERCISE 16C.1

1 110 m

2 **a** **i** travelling forwards
ii travelling backwards (opposite direction)
b 16 km **c** 8 km from starting point (on positive side)

3 **a**



b 9.75 km

EXERCISE 16C.2

1 **a** $s(t)=t-t^2+2$ cm **b** $\frac{1}{2}$ cm **c** 0 cm

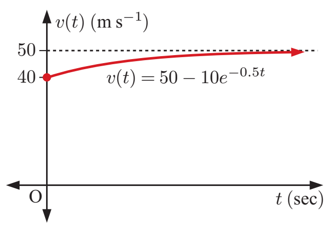
2 **a** $s(t)=\frac{1}{3}t^3-\frac{1}{2}t^2-2t$ cm **b** $5\frac{1}{6}$ cm
c $1\frac{1}{2}$ cm left of its starting point

3 $\frac{\sqrt{3}+2}{4}$ m

4 **a** $s(t)=32t+2t^2+16$ m
b no change of direction
so displacement $= s(t_1)-s(0)=\int_0^{t_1}(32+4t)\,dt$
c acceleration $= 4$ m s^{-2}

5 **a** 41 units **b** 34 units **6** **b** 2 m

7 **a** 40 m s^{-1} **b** 47.8 m s^{-1} **c** 1.39 seconds
d as $t\to\infty$, $v(t)\to 50$ from below
e $a(t)=5e^{-0.5t}$ and as $e^x>0$ for all x, $a(t)>0$ for all t.
f



g ≈ 134.5 m

8 **a** $v(t)=-\dfrac{1}{(t+1)^2}+1$ m s^{-1}
b $s(t)=\dfrac{1}{t+1}+t-1$ m

c The particle is $\frac{4}{3}$ m to the right of the origin, moving to the right at $\frac{8}{9}$ m s^{-1}, and accelerating at $\frac{2}{27}$ m s^{-2}.

9 **a** $v(t) = \dfrac{t^2}{20} - 3t + 45$ m s^{-1}

b $\int_0^{60} v(t)\,dt = 900$. The train travels a total of 900 m in the first 60 seconds.

10 **a** Show that $v(t) = 100 - 80e^{-\frac{1}{20}t}$ m s^{-1} and as $t \to \infty$, $v(t) \to 100$ m s^{-1}.

b 370.4 m

REVIEW SET 16A

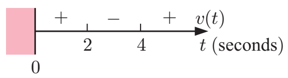
1 $A = \int_a^b [f(x) - g(x)]\,dx + \int_b^c [g(x) - f(x)]\,dx + \int_c^d [f(x) - g(x)]\,dx$

2 **a** $2 + \pi$ **b** -2 **c** π

3 No, total area shaded $= \int_{-1}^{1} f(x)\,dx - \int_1^3 f(x)\,dx$.

4 $k = \sqrt[3]{16}$ **5** 4.5 units^2

6 **a**



b The particle moves in the positive direction initially, then at $t = 2$, $6\frac{2}{3}$ m from its starting point, it changes direction. It changes direction again at $t = 4$, $5\frac{1}{3}$ m from its starting point, and at $t = 5$, it is $6\frac{2}{3}$ m from its starting point again.

c $6\frac{2}{3}$ m **d** $9\frac{1}{3}$ m

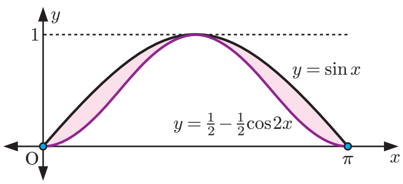
7 $(3 - \ln 4)$ units^2 **8** 2.35 m

REVIEW SET 16B

1 **a** $v(t) = 3t^2 - 30t + 27$ cm s^{-1}

b -162 cm (162 cm to the left of the origin)

2 **a**



c $(1 - \frac{\pi}{4})$ units^2

3 $a = \ln 3$, $b = \ln 5$

4 **a** $a(t) = 2 - 6t$ m s^{-2} **b** $s(t) = t^2 - t^3 + c$ m

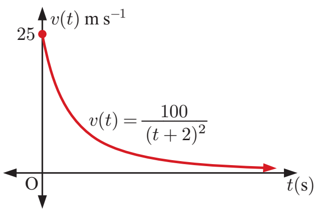
c -4 m (4 m to the left)

5 $k = \frac{4}{3}$ **6** $m = \frac{\pi}{3}$ **7** $(\frac{4}{\pi} - 1)$ units^2

8 **a** $v(0) = 25$ m s^{-1}, $v(3) = 4$ m s^{-1}

b as $t \to \infty$, $v(t) \to 0$ from above

c



d 3 seconds **e** $a(t) = \dfrac{-200}{(t+2)^3}$ m s^{-2}, $t \geqslant 0$

f $k = \frac{1}{5}$

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