Section 3: Introductory Calculus

Question 5

This question tested candidates' ability to:

- Use the product rule to differentiate the product of a quadratic expression and a trigonometric function of the form sin *ax* where $a \in Z$.
- Determine the stationary points of a polynomial of order 3; determine the nature of the stationary points.
- Use the concept of connected rates to determine the rate of change of the radius of a spherical balloon w.r.t. time, given the rate of change of the volume w.r.t. time and the formula for the volume of a sphere.

There were approximately 2891 responses to this question. The mean mark was 6.35 with standard deviation of 4.06.

Candidates' understanding of the algorithmic procedures in the Introductory Calculus and Coordinate Geometry content in this section of the syllabus was generally good. However, most candidates were challenged by the real-life application of the rate of change in Part (c).

In Part (a), candidates easily obtained the derivative of $2x^2 + 3$ to be $4x$. Their responses were often incorrect for $\frac{d \sin 5x}{dx}$. The coefficient 5 was often omitted and in a few instances -5 or $\frac{1}{5}$ were presented as coefficients of cos 5*x*. In addition, many candidates left out the coefficient, 5, which was part of the argument of the sine function. Candidates correctly applied the product rule but the mark for simplifying their result was rarely obtained.

In Part (b), differentiating the function $f(x) = x^3 - 5x^2 + 3x + 1$ to obtain the first derivative $f'(x) = 3x^2 - 10x + 3$ and equating the resulting quadratic equation to zero to obtain the coordinates of the stationary points are successfully done. Candidates also correctly differentiated $f'(x)$ to obtain the second derivative; $f''(x) = 6x - 10$ which enabled them to correctly determine the nature of each of the stationary points. However, a few of them came to the wrong conclusions based on the signs of the second derivative; at the stationary points. Some of them concluded that a negative value of the second derivative at the stationary point resulted in a minimum point and a positive value resulted in a maximum point.

In Part (c), candidates did not show full understanding of the terminology for the derivatives. Assigning 200 $cm^3\ s^{-1}$ correctly to $\frac{dV}{dt}$ was not commonly seen and correct substitution into derivatives was not often done. In addition, formulating the chain rule was also challenging and hence the algorithmic mark for the correct result was often lost.

Solutions

(a) $(10x^2 + 15)\cos 5x + 4x \sin 5x$

(b) (i) Coordinates stationary points are $\left(\frac{1}{3}, \frac{40}{27}\right)$ and (3, −8)

(ii) Maximum point is (3, −8); Minimum point is $\left(\frac{1}{3}, \frac{40}{27}\right)$

(c) $\dfrac{dr}{dt} = \dfrac{1}{2\pi}$

Question 6

This question tested candidates' ability to:

- Evaluate a definite integral for a trigonometric function with a multiple angle.
- Equate the derivative of a function to a given value to determine the value of an unknown constant.
- Formulate the equation of a curve given its gradient function and a point on the curve.
- Find the volume obtained by rotating a finite region, in the first quadrant bounded by a curve and the x-axis and the lines $x = 0$ and $x = 1$, through 360°.

Part (a) was poorly done, very few of the candidates were able to integrate $\cos 3\theta$ correctly. Those who remembered that the integral of $\cos\theta$ is $\sin\theta$ often gave the integral of $\cos 3\theta$ as $\sin 3\theta$.

In Part (b) (i), approximately 65 per cent of the candidates were able to correctly calculate the value of the unknown constant, k, by substituting $x = 2$ and equating the result to 14. A few of them substituted the value $y = 3$ for $\dfrac{dy}{dx}$. This clearly demonstrates very superficial understanding of the topic. However, the majority of the candidates was able to use their value of k to proceed to Part (b) (ii).

In Part (b) (ii), about 50 per cent of the candidates incorrectly used the equation of a line $y = mx + c$, as the equation of a curve. Candidates who correctly calculated the value of k sometimes had difficulty integrating correctly. In a few cases, the constant of integration was left out, which resulted in incorrect equations.

Part (c) was generally well done, with most of the candidates knowing the volume of revolution formula, as well as how to obtain the limits of integration. However, incorrect expansions of y^2, given $y = x^2 + 1$ was frequently observed, $x^4 + 1$ or $x^4 + 2x^2 + 2$ were some of the expansions seen. Some candidates were very careless when integrating, they either forgot to integrate the constant term or integrated the variable terms incorrectly. Simple addition of fractions such as $\dfrac{1}{5} + \dfrac{2}{3} + 1$ proved challenging for some candidates. Candidates also gave their answers in decimal form which reflected a lack of knowledge of exactness.

Solutions

(a) $-\dfrac{1}{3}$

(b) (i) $k = 7$

(ii) Equation of the curve is $y = \dfrac{7}{3}x^3 - \dfrac{7}{2}x^2 - \dfrac{5}{3}$

(c) Volume generated = $\dfrac{28}{15}\pi$ units^3

Section 4: Basic Mathematical Applications

Question 7

This question tested candidates’ ability to:

- Use the concept of a possibility space to solve problems involving probability.
- Calculate the mean for grouped data.
- Construct a tree diagram and use it to calculate probabilities.

This question was the least popular of the optional questions with about 1530 candidates attempting it. It was poorly done with approximately 70 per cent of the candidates scoring under 10 marks, however, there were 29 candidates scoring full marks.

Candidates performed poorly on Part (a), which required finding the probability involving three independent events. Most candidates were able to identify that they were independent events. A few of the candidates worked with two events instead of the three events that were given because they were unable to correctly define the possibility space for the question.

Candidates performed best on Part (b), many of them knew that they had to calculate the midpoints in order to calculate the mean. However, far too many candidates could not do this part which is also covered in the CSEC General Mathematics syllabus.

In Part (c), the majority of the candidates received most of their marks from Part (c) (i), which required drawing a tree diagram. Some of the candidates did not label the branches properly; for example, they did not show their probabilities on the branches, while others did not label the days. Some of the candidates did not have the percentage sign (%) or the decimal equivalent for their probabilities. Most candidates knew that total probabilities on their branches should add to 1 and the majority of them attempted to use their tree diagram to calculate the probability of rain falling on Wednesday.

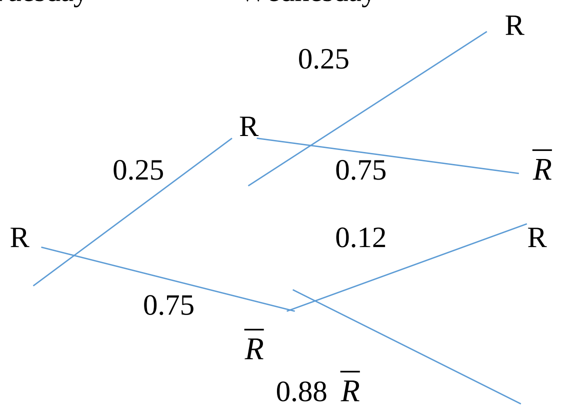
Solutions

(a) (i) Probability motorist has to stop at only one traffic light = 0.42

(ii) Probability motorist has to stop at at least two traffic lights is = 0.5

(b) Mean = $\frac{340}{25} = 13.6$

(c) (i) Monday Tuesday Wednesday



Key: R means it rains; $\bar{R}$ means it does not rain

(ii) Probability it rain on Wednesday = 0.1525

Question 8

This question tested candidates' ability to:

- Draw and make use of a velocity–time graph for rectilinear motion to find acceleration and increase in displacement.
- Apply the result $v = \int a\mathrm{d}t$ and use it to calculate the velocity function, given the velocity at a specified time and use the result $s = \int v\mathrm{d}t$ to determine the displacement of a particle at a given instance of time.

There were approximately 2298 responses to this question. The mean mark was 10.27 with a standard deviation of 5.59. Two hundred sixty-three candidates obtained full marks (20 marks).

Candidates performed best on Parts (a) (i) and (ii). They were able to draw the correct velocity–time graph for the given information, as well as calculate the acceleration of the particle.

They used a variety of methods to determine the displacement in Part (a) (iii); some calculated the displacement by finding the area of a trapezium, others found the area of a rectangle plus the area of a triangle while others used equations of motion formulae or determined the equation of the velocity—time graph and integrated between the limits t = 0 and t = 4. Approximately 25 per cent of the candidates only calculated part of the area under the graph.

The majority of the candidates who attempted Part (a) (ii), knew that the acceleration could be obtained using the formula $\text{.acceleration} = \dfrac{\text{change in velocity}}{\text{change in time}}$. However, instead of using the initial and final velocities and the corresponding times, a few of the candidates used non- exact values which led to incorrect answers.

Parts (b) (i) and (ii) presented the most problems for more than half of the candidates. Many of them used equations of motion, which are only valid when the acceleration is constant, to do these parts. In Part (b), $a = 3t - 1$ was given and candidates were required to obtain an expression for the velocity at the instant when the time was 4, and the displacement from the starting point at $\boldsymbol{t}$ = 3. Those who knew they had to integrate the velocity function to obtain the displacement sometimes forgot to include their constant of integration, which had to be calculated using the given information. This led to incorrect answers for both the velocity and displacement.

Solutions

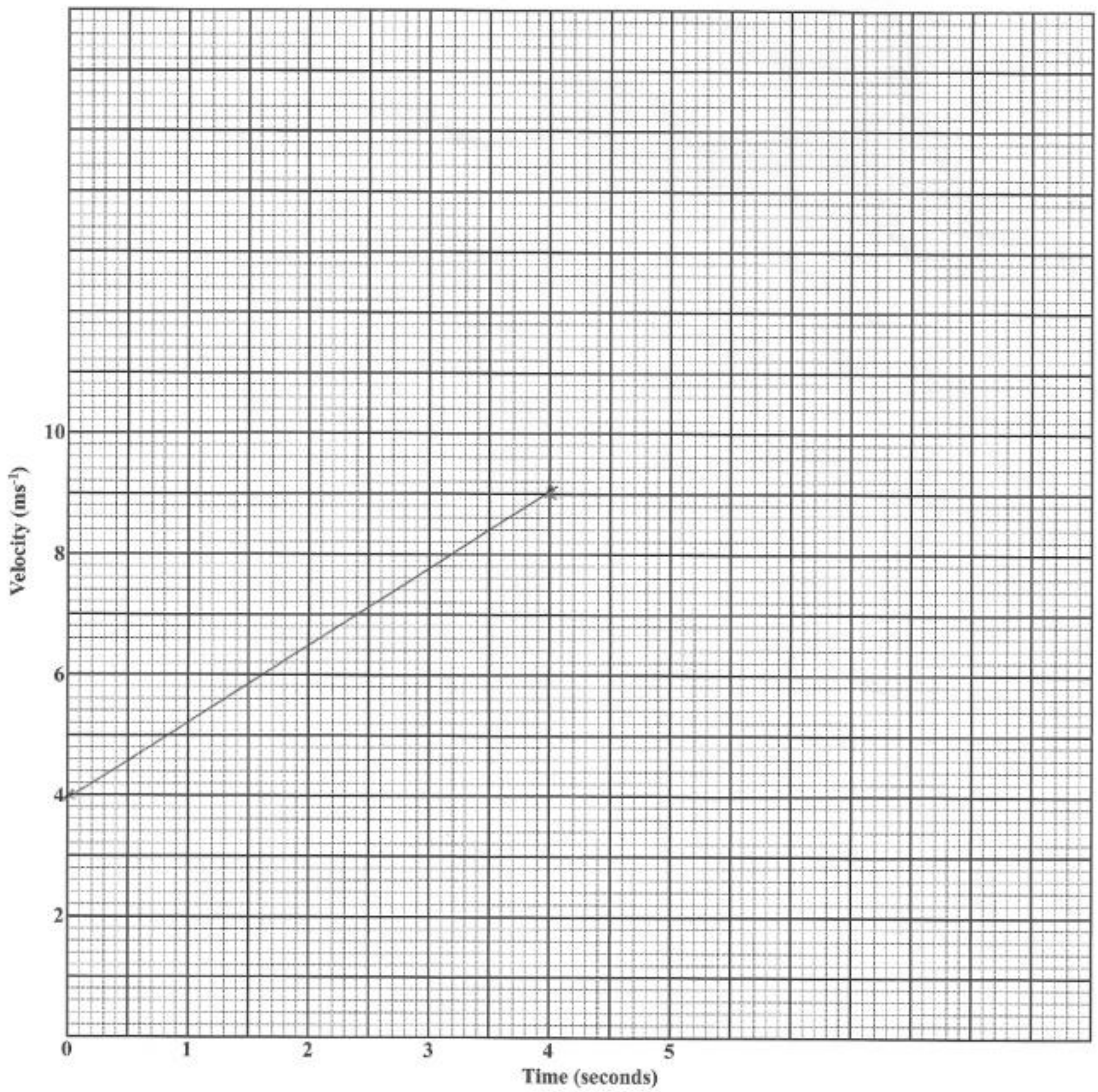
Recommendation for Teachers:

- Students must be aware of the mathematical jargon associated with kinematics, for example, motion in a straight line (rectilinear motion) invariably means that the acceleration is constant.
- Students should be encouraged to use the information given in plotting a straight line graph to obtain the gradient, instead of choosing their own non-exact values.
- Where graph paper with the axes already written is provided, teachers should insist that that graph paper be used instead of any other graph paper.
- Students should be informed that the equations of motion (i) $v = u + at$ (ii) $s = ut + \frac{1}{2}at^2$ and (iii) $v^2 = u^2 + 2as$ should only be used if the acceleration is constant.

- Teachers need to give a greater number of examples and practice questions after completing topics so that their students can get a clearer grasp of the concepts.

Solutions

(a) (i) See graph below
(ii) Acceleration = 1.5 m s^{-2}
(iii) Displacement = 24 m
(b) (i) Velocity = 20 m s^{-1} (ii) displacement = 13 m



Paper 031 — School Based Assessment (SBA)

The marking team moderated many more projects this year from 191 examination centres, a further indication of the increase in the number of new schools as well as studnets pursuing studies in Additional Mathematics. Many of the projects submitted were of a high quality and generally related to real-world situations. The mean mark awarded on the SBAs was 32.08 (out of a possible total of 40 marks), with a standard deviation of 6.33, compared with 33.06 and 6.30 respectively in 2014. Many of the sample submissions received full marks.

Comments specifically for Project A:

<u>Title</u> — *what is the project investigating?*

- Many of the students gave a name for the project rather than a title, these needs to be more specific.
- In most cases, students related to real world problems.

<u>Purpose of Project</u> — *why this project/investigation is being done?*

- In many cases, the purpose was a replica of the title.
- Few students were specific enough about the purpose of the project.

<u>Mathematical Formulation</u> — *how the problem described in the purpose is going to be investigated/solved*

- Generally, most students correctly identified all the important elements of the problem and showed understanding of the relationship between elements.
- In many cases, students did not use appropriate additional mathematical methods/models to solve the problems.
- The content used in some cases reflected Form 3/Grade 8 level mathematics, for example, some projects had only simple calculations with area and volume.

<u>The Problem Solution</u> — *here the problem is solved*

- The assumptions were not clearly stated (must be explicit and not simply implied). However, some students stated limitations of the project.
- Explanations were not sufficient and not placed between every successive step in the problem solution.
- Calculations were precise but solutions were not clearly stated.

<u>Application of Solution</u> — *here the solution derived is applied to the real–world context*

- This section was fairly well done. However, many students were unaware of how to show that the solution or proof of the given problem was valid.
- Validation can be achieved by an alternative method or by comparison to a similar study.

<u>Discussion of Findings/Conclusion</u> — *interprets the solution and relates it to the purpose of the study*

- The discussions were too general and in most cases the findings/solutions were not related to the purpose of the project.
- The conclusions in many instances were found not to be valid. Some students in made many conclusions but not all were related to the purpose of the project.
- Suggestions for future analysis can also include the use of this study in related areas or improvements for further studies.

<u>Overall Presentation</u>

- There was a marked improvement in the correct use of spelling and grammar.
- Students used the correct mathematical jargon and symbols appropriately most of the time.

Areas of Strength

- Many of the submissions were of a high quality and related to real-world situations to which mathematical concepts were applied. A few were unique and interesting and represented the investigation of good ideas.
- There was evidence indicating that students applied a high degree of effort to the task. They understood the focus of their topic selection and conducted the requisite research thoroughly.

Areas of Weakness

- The CSEC Additional Mathematics rubric was absent from the samples submitted by some centres.
- Some SBA project titles and aims were not clear, not concise and not well defined.
- Teachers must be alert to any suspected plagiarism. Plagiarism must not be condoned and projects should be marked accordingly.
- The majority of students did not incorporate or suggest any future analysis in their submissions.
- Some students failed to adequately connect their finding(s)/conclusion(s) to their aim(s), their data collected and their analysis of the data.

Comments Specifically for Project B

<u>Title</u> — *what the project is investigating?*

- Many of the students gave a name for the project rather than a title.
- In most cases, the students related to real–world problems.

<u>Purpose of Project</u> — *why this project/investigation is being done?*

- In many cases, the purpose was a replica of the title.
- Few students were specific enough about the purpose of the project
- The majority of the students was **not** able to identify the variables in the project properly.

<u>Method of data collection</u> — *how the data is being collected*

- Most students were able to describe the data collection method clearly.
- It is expected that students mention the type of data collected (primary, secondary) and/or sampling used for example, random sampling, stratified sampling. This occurred only in a few instances.

Presentation of data

- Most studdnts had a minimum of one table or chart.
- Most students used the graphs, figures and tables appropriately in their presentation.

Mathematical Knowledge (Analysis of Data)

- Mathematical concepts in many cases were not used appropriately from the Additional Mathematics syllabus
- In many cases, students used concepts and representations which reflected Form 2 level Mathematics. For example, some students used only pie charts and bar charts.
- In many instances, the analysis was attempted, but students were not able to use it in their discussions. For example, standard deviation was calculated but was not used in the discussion (there was no mention of spread or skewness and what they mean? in relation to the project).
- Students were not too clear on the two approaches to be used in the analysis.
- In many cases the two approaches used were of the same concept. For example, mean, mode and median are all measures of central tendencies.

Discussion of Findings/Conclusions

- In some instances there was no statement of findings. However, most statements made were linked to the solution of the problems.
- Suggestions for future analysis can also include the use of this study in related areas or improvements for further studies.

Overall Presentation

- There was a marked improvement in the correct use of spelling and grammar.
- Students used the correct mathematical jargon and symbols most of the time.

Paper 032 — Alternative to SBA

This paper tested candidates' ability to:

- Determine the maximum profit for a given profit function which was quadratic.
- Represent real–world situations using quadratic equations and solve for unknown variables.
- Derive an expression, in algebraic form, for the area of a composite figure given its dimensions.
- Determine the range of values of an unknown side for which the area of the shape exists.

There were 83 candidates who wrote this paper, 81 of them attempted the question. The mean mark was 5.26 with a standard deviation of 4.46 compared with 8.69 and 7.75 respectively in 2014. No candidate received full marks (20 marks). The highest mark recorded was 18 out of 20.

This paper was very poorly done. Only about 3.7 per cent of candidates were able to achieve 75 per cent and above. Of the three parts, which comprised this paper, Part (b) was done the best. Candidates need to know the functions of calculus and when it should be used as opposed to using normal algebraic procedures.

In Part (a), candidates were required to use differentiation, or some other suitable method, to determine the maximum daily profit of a company. Many candidates tried to solve for x by factorizing the expression, completing the square or using the quadratic formula. Obviously, candidates did not connect differentiation with this question. After this part, the majority of candidates understood that substitution was necessary to determine the number of hundred dollars needed, even if the value they substituted was obtained erroneously. In Part (a), some of the candidates who solved the given equation obtained the values $x = -30$ and $x = 10$. They then concluded that $x = 10$ would give the maximum profit of $1000.

Common Errors

Candidates solved for the roots of the equation rather than differentiate and equate to 0.

Candidates did not multiply by 100 to get the required dollar value for their answer.

In Part (b), candidates were required to determine dimensions of a square and rectangle given specific conditions. This part was relatively well done compared with the other two parts. Expressions for the dimensions of the square and rectangle were often correctly obtained, thus $A = x^2 + (x+3)(x+6)$ was usually given as the total area although there were many variations presented such as $w = s + 3$ and $l = w + 3$, where s represented the length of the square, w represented the width of the rectangle and l the length of the rectangle. The candidates who represented the total area correctly invariably obtained a quadratic equation and even though some factorized incorrectly, they were able to apply their answers to produce acceptable dimensions for each shape. Some candidates used the trial and error method to arrive at the correct dimensions. This method was not penalized once supporting statements were seen.

In Part (c) (i), candidates were required to obtain an expression for area in terms of x only. This part was poorly done and it was seen that candidates could not apply the formula for the area of a triangle to algebraic processes. Many knew they needed to add the areas of the rectangle and triangle, but were confused by what should be done next. Many candidates did not recognize the need to use Pythagoras' theorem in representing h, the height of the isosceles triangle in terms of its base, x, and its slant side which was 4 cm. In cases where Pythagoras' theorem was used, it was incorrectly applied.

In Part (c) (ii), candidates were required to find the required domain for area. Very few candidates even attempted this part and it was clear that they could not link area in Part (c) (i) to the length of a side, x, even though this was stated in the question. Additionally, candidates could not relate the need for x to be positive in order to represent length. Candidates also did not use the fact that the height, h, of the triangle had to be positive in order to find the required range for x using the result $h = \sqrt{\frac{64 - x^2}{4}} > o$ which would imply that $\sqrt{64 - x^2} > o$.

With respect to candidates' responses on this question, it is recommended that candidates be given more practice questions. This will allow them to apply theory to real situations.

Solutions

(a) Maximum daily profit = $10 000

(b) (i) Dimensions of the square are 5.5 cm by 5.5 cm
(ii) Dimensions of the rectangle are 11.5 cm by 8.5 cm

(c) (i) Area of shaded surface $= 4x + \frac{x\sqrt{64 - x^2}}{4}$

(ii) Domain of x $\{x : 0 < x < 8, x \in \mathbf{R}\}$

Generally, teachers need to ensure that the syllabus is completed with sufficient time to give students time to do practice exercise and past papers.