b $\overrightarrow{BC}$ and $\overrightarrow{CA}$. Each vector has components of the same magnitude, but differing signs (which do not affect the length of $\overrightarrow{BC}$ or $\overrightarrow{CA}$).

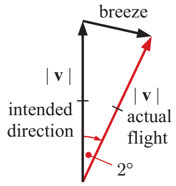
c $\overrightarrow{AC}$, $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$, $2\mathbf{i}-4\mathbf{j}$

2 **a** $\sqrt{13}$ units **b** $\sqrt{10}$ units **c** $\sqrt{109}$ units

3 **a** $k=\pm\frac{12}{13}$ **b** $k=\pm\frac{1}{\sqrt{2}}$ **4** $\begin{pmatrix} 1 \\ 4 \end{pmatrix}$

5 **a** M$(-2, 4)$ **b** 5 units **6** $m=5$

7 **a**



b **i** isosceles triangle $\therefore$ 2 remaining angles $= 89°$ each, breeze makes angle of $180-89=91°$ to intended direction of the arrow.

ii bisect angle $2°$ and use $\sin 1° = \dfrac{\frac{1}{2}\text{ speed}}{|\mathbf{v}|}$

$\therefore$ speed $= 2\,|\mathbf{v}|\sin 1°$

8 $\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} 0 \\ 8 \end{pmatrix}+t\begin{pmatrix} 5 \\ 4 \end{pmatrix}$, $t\in\mathbb{R}$

9 **a** **i** $-6\mathbf{i}+10\mathbf{j}$ **ii** $-5\mathbf{i}-15\mathbf{j}$

iii $(-6-5t)\mathbf{i}+(10-15t)\mathbf{j}$, $t\geqslant 0$

b $t=40$ min ($\frac{2}{3}$ h), yacht is ≈ 9.33 km away from the beacon.

10 **a** **i** $\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} 2 \\ -3 \end{pmatrix}+t\begin{pmatrix} 4 \\ -1 \end{pmatrix}$, $t\in\mathbb{R}$

ii $x=2+4t$, $y=-3-t$, $t\in\mathbb{R}$

b **i** $\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} -1 \\ 6 \end{pmatrix}+t\begin{pmatrix} 3 \\ -4 \end{pmatrix}$, $t\in\mathbb{R}$

ii $x=-1+3t$, $y=6-4t$, $t\in\mathbb{R}$

11 **a** $x_1(t)=2+t$, $y_1(t)=4-3t$, $t\geqslant 0$

b $x_2(t)=13-t$, $y_2(t)=[3-2a]+at$, $t\geqslant 2$

c interception occurred at 2:22:30 pm

d bearing $\approx 12.7°$ west of due south, at ≈ 4.54 km min^{-1}

EXERCISE 12A

1 **a** 1×4 **b** 2×1 **c** 2×2 **d** 3×3

2 **a** $(2 \quad 1 \quad 6 \quad 1)$ **b** $\begin{pmatrix} 1.95 \\ 2.35 \\ 0.45 \\ 2.95 \end{pmatrix}$

c total cost of groceries

3 $\begin{pmatrix} 1000 & 1500 & 1250 \\ 1500 & 1000 & 1000 \\ 800 & 2300 & 1300 \\ 1200 & 1200 & 1200 \end{pmatrix}$ **4** $\begin{pmatrix} 40 & 50 & 55 & 40 \\ 25 & 65 & 44 & 30 \\ 35 & 40 & 40 & 35 \\ 35 & 40 & 35 & 50 \end{pmatrix}$

EXERCISE 12B.1

1 **a** $\begin{pmatrix} 9 & 1 \\ 3 & 3 \end{pmatrix}$ **b** $\begin{pmatrix} 6 & 8 \\ -1 & 1 \end{pmatrix}$

c $\begin{pmatrix} 3 & 4 \\ -6 & -1 \end{pmatrix}$ **d** $\begin{pmatrix} 0 & 0 \\ -11 & -3 \end{pmatrix}$

2 **a** $\begin{pmatrix} 20 & 1 & -8 \\ 8 & 10 & -2 \\ 1 & -5 & 18 \end{pmatrix}$ **b** $\begin{pmatrix} -14 & 9 & -14 \\ 12 & -6 & 14 \\ -5 & 3 & -4 \end{pmatrix}$

c $\begin{pmatrix} 14 & -9 & 14 \\ -12 & 6 & -14 \\ 5 & -3 & 4 \end{pmatrix}$

3 **a** $\begin{array}{c} \text{Friday} \\ \begin{pmatrix} 85 \\ 92 \\ 52 \end{pmatrix} \end{array} \quad \begin{array}{c} \text{Saturday} \\ \begin{pmatrix} 102 \\ 137 \\ 49 \end{pmatrix} \end{array}$ **b** $\begin{pmatrix} 187 \\ 229 \\ 101 \end{pmatrix}$

4 **a** **i** $\begin{pmatrix} 1.72 \\ 27.85 \\ 0.92 \\ 2.53 \\ 3.56 \end{pmatrix}$ **ii** $\begin{pmatrix} 1.79 \\ 28.75 \\ 1.33 \\ 2.25 \\ 3.51 \end{pmatrix}$

b subtract cost price from selling price **c** $\begin{pmatrix} 0.07 \\ 0.90 \\ 0.41 \\ -0.28 \\ -0.05 \end{pmatrix}$

5 **a** $\begin{array}{cc} \begin{array}{cc} \text{L} & \text{R} \end{array} & \\ \begin{pmatrix} 23 & 19 \\ 17 & 29 \\ 31 & 24 \end{pmatrix} & \begin{array}{c} \text{fr} \\ \text{st} \\ \text{mi} \end{array} \end{array}$ **b** $\begin{array}{cc} \begin{array}{cc} \text{L} & \text{R} \end{array} & \\ \begin{pmatrix} 18 & 25 \\ 7 & 13 \\ 36 & 19 \end{pmatrix} & \begin{array}{c} \text{fr} \\ \text{st} \\ \text{mi} \end{array} \end{array}$

c $\begin{array}{cc} \begin{array}{cc} \text{L} & \text{R} \end{array} & \\ \begin{pmatrix} 41 & 44 \\ 24 & 42 \\ 67 & 43 \end{pmatrix} & \begin{array}{c} \text{fr} \\ \text{st} \\ \text{mi} \end{array} \end{array}$

6 **a** $x=-2$, $y=-2$ **b** $x=0$, $y=0$

7 **a** $\mathbf{A}+\mathbf{B}=\begin{pmatrix} 1 & 3 \\ 5 & 2 \end{pmatrix}$, $\mathbf{B}+\mathbf{A}=\begin{pmatrix} 1 & 3 \\ 5 & 2 \end{pmatrix}$

8 **a** $(\mathbf{A}+\mathbf{B})+\mathbf{C}=\begin{pmatrix} 6 & 3 \\ -1 & 6 \end{pmatrix}$, $\mathbf{A}+(\mathbf{B}+\mathbf{C})=\begin{pmatrix} 6 & 3 \\ -1 & 6 \end{pmatrix}$

EXERCISE 12B.2

1 **a** $\begin{pmatrix} 12 & 24 \\ 48 & 12 \end{pmatrix}$ **b** $\begin{pmatrix} 2 & 4 \\ 8 & 2 \end{pmatrix}$

c $\begin{pmatrix} \frac{1}{2} & 1 \\ 2 & \frac{1}{2} \end{pmatrix}$ **d** $\begin{pmatrix} -3 & -6 \\ -12 & -3 \end{pmatrix}$

2 **a** $\begin{pmatrix} 3 & 5 & 6 \\ 2 & 8 & 7 \end{pmatrix}$ **b** $\begin{pmatrix} 1 & 1 & 4 \\ 0 & 4 & 1 \end{pmatrix}$

c $\begin{pmatrix} 5 & 8 & 11 \\ 3 & 14 & 11 \end{pmatrix}$ **d** $\begin{pmatrix} 5 & 7 & 14 \\ 2 & 16 & 9 \end{pmatrix}$

3 $12\mathbf{F}=\begin{pmatrix} 12 \\ 48 \\ 24 \\ 12 \end{pmatrix}$

4 **a** $\mathbf{A}=\begin{pmatrix} 75 \\ 27 \\ 102 \end{pmatrix}\begin{array}{l} \leftarrow \text{DVD} \\ \leftarrow \text{Blu-ray} \\ \leftarrow \text{games} \end{array}$ $\mathbf{B}=\begin{pmatrix} 136 \\ 43 \\ 129 \end{pmatrix}\begin{array}{l} \leftarrow \text{DVD} \\ \leftarrow \text{Blu-ray} \\ \leftarrow \text{games} \end{array}$

b $5\mathbf{A}+2\mathbf{B}=\begin{pmatrix} 647 \\ 221 \\ 768 \end{pmatrix}\begin{array}{l} \leftarrow \text{DVD} \\ \leftarrow \text{Blu-ray} \\ \leftarrow \text{games} \end{array}$

c total weekly average hirings

5 **a** $\begin{array}{c} \begin{array}{cccc} \text{A} & \text{B} & \text{C} & \text{D} \end{array} \\ \begin{pmatrix} 35 & 46 & 46 & 69 \\ 58 & 46 & 35 & 86 \\ 46 & 46 & 58 & 58 \\ 12 & 23 & 23 & 17 \end{pmatrix} \end{array}$ **b** $\begin{array}{c} \begin{array}{cccc} \text{A} & \text{B} & \text{C} & \text{D} \end{array} \\ \begin{pmatrix} 26 & 34 & 34 & 51 \\ 43 & 34 & 26 & 64 \\ 34 & 34 & 43 & 43 \\ 9 & 17 & 17 & 13 \end{pmatrix} \end{array}$

EXERCISE 12B.3

1 **a** $3A$ **b** O **c** $-C$ **d** O **e** $2A+2B$ **f** $-A-B$ **g** $-2A+C$ **h** $4A-B$ **i** $3B$

2 **a** $X=A-B$ **b** $X=C-B$ **c** $X=2C-4B$ **d** $X=\frac{1}{2}A$ **e** $X=\frac{1}{3}B$ **f** $X=A-B$ **g** $X=2C$ **h** $X=\frac{1}{2}B-A$ **i** $X=\frac{1}{4}(A-C)$

3 **a** $X=\begin{pmatrix}3 & 6\\ 9 & 18\end{pmatrix}$ **b** $X=\begin{pmatrix}\frac{1}{2} & -\frac{1}{4}\\ \frac{3}{4} & \frac{5}{4}\end{pmatrix}$ **c** $X=\begin{pmatrix}-1 & -6\\ 1 & -\frac{1}{2}\end{pmatrix}$

EXERCISE 12C.1

1 **a** (11) **b** (22) **c** (16) **2** **b** $\begin{pmatrix}w & x & y & z\end{pmatrix}\begin{pmatrix}\frac{1}{4}\\ \frac{1}{4}\\ \frac{1}{4}\\ \frac{1}{4}\end{pmatrix}$

3 **a** $P=\begin{pmatrix}27 & 35 & 39\end{pmatrix}$, $Q=\begin{pmatrix}4\\ 3\\ 2\end{pmatrix}$

b total cost $=\begin{pmatrix}27 & 35 & 39\end{pmatrix}\begin{pmatrix}4\\ 3\\ 2\end{pmatrix}=\$291$

4 **a** $P=\begin{pmatrix}10 & 6 & 3 & 1\end{pmatrix}$, $N=\begin{pmatrix}3\\ 2\\ 4\\ 2\end{pmatrix}$

b total points $=\begin{pmatrix}10 & 6 & 3 & 1\end{pmatrix}\begin{pmatrix}3\\ 2\\ 4\\ 2\end{pmatrix}=56$ points

EXERCISE 12C.2

1 Number of columns in **A** does not equal number of rows in **B**.

2 **a** $m=n$ **b** 2×3 **c** **B** has 3 columns, **A** has 2 rows

3 **a** does not exist **b** $\begin{pmatrix}28 & 29\end{pmatrix}$

4 **a** (8) **b** $\begin{pmatrix}2 & 0 & 3\\ 8 & 0 & 12\\ 4 & 0 & 6\end{pmatrix}$

5 **a** $\begin{pmatrix}3 & 5 & 3\end{pmatrix}$ **b** $\begin{pmatrix}-2\\ 1\\ 1\end{pmatrix}$

6 **a** $Q=\begin{pmatrix}32 & 24\\ 25 & 16\\ 13 & 9\end{pmatrix}$ **b** $P=\begin{pmatrix}1.19\\ 1.55\end{pmatrix}$

c $QP=\begin{pmatrix}32 & 24\\ 25 & 16\\ 13 & 9\end{pmatrix}\begin{pmatrix}1.19\\ 1.55\end{pmatrix}=\begin{pmatrix}75.28\\ 54.55\\ 29.42\end{pmatrix}$

It represents the total value of sales for each pen colour.

d $\$75.28+\$54.55+\$29.42=\$159.25$

7 **a** $C=\begin{pmatrix}12.5\\ 9.5\end{pmatrix}$ $N=\begin{pmatrix}2375 & 5156\\ 2502 & 3612\end{pmatrix}$

b $\begin{pmatrix}78\,669.5\\ 65\,589\end{pmatrix}$ income from day 1, income from day 2 **c** $\$144\,258.50$

8 **a** $R=\begin{pmatrix}1 & 1\\ 1 & 2\\ 2 & 3\end{pmatrix}$ **b** $P=\begin{pmatrix}7 & 3 & 19\\ 6 & 2 & 22\end{pmatrix}$

c $\begin{pmatrix}48 & 70\\ 52 & 76\end{pmatrix}$ **d** **i** $48 **ii** $76

e The elements of **PR** tell us that, if all the items are to be bought at one store, it is cheapest to do so at store A for both you and your friend. However, the cheapest way is to buy paint from store A, and hammers and screwdrivers from store B.

EXERCISE 12C.3

1 **a** A^2+A **b** B^2+2B **c** A^3-2A^2+A **d** A^3+A^2-2A **e** $AC+AD+BC+BD$ **f** $A^2+AB+BA+B^2$ **g** $A^2-AB+BA-B^2$ **h** A^2+2A+I **i** $9I-6B+B^2$

2 **a** $A^3=3A-2I$, $A^4=4A-3I$
b $B^3=3B-2I$, $B^4=6I-5B$, $B^5=11B-10I$
c $C^3=13C-12I$, $C^5=121C-120I$

3 **a** **i** $I+2A$ **ii** $2I-2A$ **iii** $10A+6I$
b A^2+A+2I **c** **i** $-3A$ **ii** $-2A$ **iii** A

4 **a** $A^2=\begin{pmatrix}\frac{1}{2} & \frac{1}{2}\\ \frac{1}{2} & \frac{1}{2}\end{pmatrix}$

b false as $A(A-I)=O$ does not imply that $A=O$ or $A-I=O$

c $\begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix}$, $\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$, $\begin{pmatrix}a & b\\ \frac{a-a^2}{b} & 1-a\end{pmatrix}$, $b\neq 0$

5 For example, $A=\begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}$ gives $A^2=\begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix}$.

6 **a** $a=3,\ b=-4$ **b** $a=1,\ b=8$

7 **a** $p=-2,\ q=1$ **b** $A^3=5A-2I$ **c** $A^4=-12A+5I$

EXERCISE 12D.1

1 **a** $\begin{pmatrix}3 & 0\\ 0 & 3\end{pmatrix}=3I$, $\begin{pmatrix}1 & -2\\ -\frac{2}{3} & \frac{5}{3}\end{pmatrix}$

b $\begin{pmatrix}10 & 0\\ 0 & 10\end{pmatrix}=10I$, $\begin{pmatrix}0.2 & 0.4\\ -0.1 & 0.3\end{pmatrix}$

2 **a** -2 **b** -1 **c** 0 **d** 1

3 **a** 26 **b** 6 **c** -1 **d** a^2+a

4 **a** -3 **b** -3 **c** -12 **5** **Hint:** Let $A=\begin{pmatrix}a & b\\ c & d\end{pmatrix}$

6 **a** **i** $\det A=ad-bc$ **ii** $\det B=wz-xy$
iii $AB=\begin{pmatrix}aw+by & ax+bz\\ cw+dy & cx+dz\end{pmatrix}$
iv $\det AB=(ad-bc)(wz-xy)$

7 **a** $\det A=-2$, $\det B=-1$
b **i** $\det(2A)=-8$ **ii** $\det(-A)=-2$ **iii** $\det(-3B)=-9$ **iv** $\det(AB)=2$

8 **a** $\frac{1}{14}\begin{pmatrix}5 & -4\\ 1 & 2\end{pmatrix}$ **b** $\begin{pmatrix}1 & 0\\ 1 & -1\end{pmatrix}$ **c** does not exist
d $\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$ **e** $\frac{1}{10}\begin{pmatrix}2 & 0\\ 1 & 5\end{pmatrix}$ **f** does not exist
g $-\frac{1}{15}\begin{pmatrix}7 & -2\\ -4 & -1\end{pmatrix}$ **h** $\frac{1}{10}\begin{pmatrix}2 & -4\\ 1 & 3\end{pmatrix}$ **i** $\begin{pmatrix}-3 & -1\\ 2 & 1\end{pmatrix}$

9 **a** $\frac{1}{2k+6}\begin{pmatrix}2 & -1\\ 6 & k\end{pmatrix}$, $k\neq -3$ **b** $\frac{1}{3k}\begin{pmatrix}k & 1\\ 0 & 3\end{pmatrix}$, $k\neq 0$

c $\dfrac{1}{(k+2)(k-1)}\begin{pmatrix} k & -2 \\ -1 & k+1 \end{pmatrix}$, $k \neq -2$ or 1

d $\dfrac{1}{k(k+1)}\begin{pmatrix} k & -k \\ 3 & k-2 \end{pmatrix}$, $k \neq 0$ or -1

e $\dfrac{1}{k(2-k)}\begin{pmatrix} 1 & 1-k \\ -2k & k^2 \end{pmatrix}$, $k \neq 0$ or 2

f $\dfrac{1}{(k+4)(k-1)}\begin{pmatrix} 3k & -2 \\ -k^2-2 & k+1 \end{pmatrix}$, $k \neq -4$ or 1

EXERCISE 12D.2

1 $\mathbf{X} = \begin{pmatrix} \frac{1}{4} & \frac{3}{4} \\ 1 & 0 \end{pmatrix}$ **2** **a** $\mathbf{X} = \mathbf{ABZ}$ **b** $\mathbf{Z} = \mathbf{B}^{-1}\mathbf{A}^{-1}\mathbf{X}$

3 $\mathbf{A}^2 = 2\mathbf{A} - \mathbf{I}$, $\mathbf{A}^{-1} = 2\mathbf{I} - \mathbf{A}$

4 **a** $\mathbf{A}^{-1} = 4\mathbf{I} - \mathbf{A}$ **b** $\mathbf{A}^{-1} = 5\mathbf{I} + \mathbf{A}$
c $\mathbf{A}^{-1} = \frac{3}{2}\mathbf{A} - 2\mathbf{I}$

6 If $\mathbf{A}^{-1}$ exists, that is, $\det \mathbf{A} \neq 0$.

EXERCISE 12E

1 **a** $\begin{pmatrix} 3 & -1 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 8 \\ 6 \end{pmatrix}$
b $\begin{pmatrix} 4 & -3 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 11 \\ -5 \end{pmatrix}$
c $\begin{pmatrix} 3 & -1 \\ 2 & 7 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 6 \\ -4 \end{pmatrix}$

2 **a** $x = \frac{32}{7}$, $y = \frac{22}{7}$ **b** $x = -\frac{37}{23}$, $y = -\frac{75}{23}$
c $x = \frac{17}{13}$, $y = -\frac{37}{13}$ **d** $x = \frac{59}{13}$, $y = -\frac{25}{13}$
e $x = -40$, $y = -24$ **f** $x = \frac{1}{34}$, $y = \frac{55}{34}$

3 **b** **i** $\mathbf{X} = \begin{pmatrix} -\frac{4}{3} & \frac{13}{9} \\ -1 & \frac{4}{3} \end{pmatrix}$ **ii** $\mathbf{X} = \begin{pmatrix} -1 & 3 \\ 2 & 4 \end{pmatrix}$
iii $\mathbf{X} = \begin{pmatrix} \frac{13}{7} & \frac{3}{7} \\ -\frac{2}{7} & -\frac{8}{7} \end{pmatrix}$ **iv** $\mathbf{X} = \begin{pmatrix} \frac{19}{7} & \frac{6}{7} \\ \frac{20}{7} & -\frac{25}{7} \end{pmatrix}$

4 **a** **i** $\begin{pmatrix} 2 & -3 \\ 4 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 8 \\ 11 \end{pmatrix}$, $\det \mathbf{A} = 10$
ii Yes, $x = 2.5$, $y = -1$
b **i** $\begin{pmatrix} 2 & k \\ 4 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 8 \\ 11 \end{pmatrix}$, $\det \mathbf{A} = -2 - 4k$
ii $k \neq -\frac{1}{2}$, $x = \dfrac{8 + 11k}{2 + 4k}$, $y = \dfrac{5}{1 + 2k}$
iii $k = -\frac{1}{2}$, no solutions

REVIEW SET 12A

1 **a** $\begin{pmatrix} 4 & 2 \\ -2 & 3 \end{pmatrix}$ **b** $\begin{pmatrix} 9 & 6 \\ 0 & -3 \end{pmatrix}$ **c** $\begin{pmatrix} -2 & 0 \\ 4 & -8 \end{pmatrix}$
d $\begin{pmatrix} 2 & 2 \\ 2 & -5 \end{pmatrix}$ **e** $\begin{pmatrix} -5 & -4 \\ -2 & 6 \end{pmatrix}$ **f** $\begin{pmatrix} 7 & 6 \\ 4 & -11 \end{pmatrix}$
g $\begin{pmatrix} -1 & 8 \\ 2 & -4 \end{pmatrix}$ **h** $\begin{pmatrix} 3 & 2 \\ -6 & -8 \end{pmatrix}$ **i** $\begin{pmatrix} \frac{1}{3} & \frac{2}{3} \\ 0 & -1 \end{pmatrix}$
j $\begin{pmatrix} 9 & 4 \\ 0 & 1 \end{pmatrix}$ **k** $\begin{pmatrix} -3 & -10 \\ 6 & 8 \end{pmatrix}$ **l** $\begin{pmatrix} \frac{1}{3} & \frac{2}{3} \\ \frac{1}{6} & \frac{1}{12} \end{pmatrix}$

2 **a** $a = 0$, $b = 5$, $c = 1$, $d = -4$
b $a = 2$, $b = -1$, $c = 3$, $d = 8$

3 **a** $\mathbf{Y} = \mathbf{B} - \mathbf{A}$ **b** $\mathbf{Y} = \frac{1}{2}(\mathbf{A} - \mathbf{C})$ **c** $\mathbf{Y} = \mathbf{A}^{-1}\mathbf{B}$
d $\mathbf{Y} = \mathbf{C}\mathbf{B}^{-1}$ **e** $\mathbf{Y} = \mathbf{A}^{-1}(\mathbf{C} - \mathbf{B})$ **f** $\mathbf{Y} = \mathbf{B}^{-1}\mathbf{A}$

4 **a** $4\mathbf{L}$ **b** $-2\mathbf{L}$

5 **a** $\begin{pmatrix} 10 & -12 \\ -10 & 4 \end{pmatrix}$ **b** $\begin{pmatrix} 2 & 6 & -3 \\ -4 & -2 & 11 \end{pmatrix}$ **c** not possible

6 **a** $\mathbf{A} - \mathbf{A}^2$ **b** $\mathbf{AB} + \mathbf{A}^2 - \mathbf{B}^2 - \mathbf{BA}$
c $4\mathbf{A}^2 - 4\mathbf{A} + \mathbf{I}$

7 $\mathbf{A}^3 = 27\mathbf{A} + 10\mathbf{I}$, $\mathbf{A}^4 = 145\mathbf{A} + 54\mathbf{I}$

8 $a = 4$, $b = -7$

9 **a** $\begin{pmatrix} \frac{7}{2} & -4 \\ -\frac{5}{2} & 3 \end{pmatrix}$ **b** does not exist **c** $\begin{pmatrix} 1 & \frac{5}{3} \\ -2 & -\frac{11}{3} \end{pmatrix}$

10 Unique solution if $k \neq \frac{3}{4}$.

11 **a** $x = 0$, $y = -\frac{1}{2}$ **b** $x = \frac{12}{7}$, $y = \frac{13}{7}$

12 **b** $(\mathbf{A} - \mathbf{I})(\mathbf{A} + 3\mathbf{I}) = 2\mathbf{A} - \mathbf{I}$

REVIEW SET 12B

1 **a** $\begin{pmatrix} 4 & 2 \\ 2 & 4 \\ 3 & 4 \end{pmatrix}$ **b** $\begin{pmatrix} 2 & -2 \\ 0 & 4 \\ -1 & -2 \end{pmatrix}$ **c** $\begin{pmatrix} -\frac{3}{2} & 3 \\ \frac{1}{2} & -4 \\ 2 & \frac{7}{2} \end{pmatrix}$

2 **a** $\mathbf{A} - \mathbf{B} = \begin{pmatrix} 2 & 2 \\ 4 & 1 \\ 3 & 1 \end{pmatrix}$ **b** **i** Book 2 (hard cover)
ii \$101

4 **a** $\mathbf{X} = \frac{1}{2}(\mathbf{B} - \mathbf{A})$ **b** $\mathbf{X} = \frac{1}{3}(2\mathbf{B} - 3\mathbf{A})$
c $\mathbf{X} = \frac{1}{4}(\mathbf{B} - \mathbf{A})$

5 $\mathbf{X} = \begin{pmatrix} -\frac{1}{2} & -\frac{3}{2} \\ 0 & \frac{3}{2} \end{pmatrix}$

6 **a** $\begin{pmatrix} 4 & 8 \\ 0 & 2 \\ 6 & 4 \end{pmatrix}$ **b** $\begin{pmatrix} 1 & 2 \\ 0 & \frac{1}{2} \\ \frac{3}{2} & 1 \end{pmatrix}$ **c** $\begin{pmatrix} 11 & 12 \end{pmatrix}$
d $\mathbf{BA}$ does not exist

7 **a** $\det \mathbf{B} \neq 0$ **b** $\mathbf{AB} = \mathbf{BA}$

8 **a** $\det \mathbf{A} = 5$ **b** $\det(-2\mathbf{A}) = 20$ **c** $\det(\mathbf{A}^2) = 25$

9 **a** $x = \frac{14}{3}$, $y = \frac{1}{3}$ **b** $x = -1$, $y = 3$

10 $k \in \mathbb{R}$, $k \neq 3, -2, 2$

11 Unique solution for $k \neq -3$ or 1. $x = \dfrac{-6}{k-1}$, $y = \dfrac{2}{k-1}$

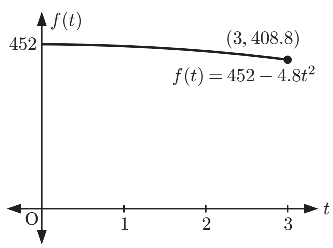
12 $\mathbf{A}(\frac{5}{3}\mathbf{A} - 2\mathbf{I}) = \mathbf{I}$, $\mathbf{A}^{-1} = \frac{5}{3}\mathbf{A} - 2\mathbf{I}$

EXERCISE 13A

1 **a** 7 **b** 7 **c** 11 **d** 16 **e** 0 **f** 5
2 **a** 5 **b** 7 **c** c
3 **a** -2 **b** 7 **c** -1 **d** 1
4 **a** -3 **b** 5 **c** -1 **d** 6 **e** -4 **f** -8
g 1 **h** 2 **i** 5

EXERCISE 13B

1 **a**



b no
c **i** $0\ \text{m s}^{-1}$
ii $9.6\ \text{m s}^{-1}$
iii $19.2\ \text{m s}^{-1}$
iv $28.8\ \text{m s}^{-1}$

2 **a**

x	Point B	Gradient of AB
0	(0, 0)	2
1	(1, 1)	3
1.5	(1.5, 2.25)	3.5
1.9	(1.9, 3.61)	3.9
1.99	(1.99, 3.9601)	3.99
1.999	(1.999, 3.996 001)	3.999

x	Point B	Gradient of AB
5	(5, 25)	7
3	(3, 9)	5
2.5	(2.5, 6.25)	4.5
2.1	(2.1, 4.41)	4.1
2.01	(2.01, 4.0401)	4.01
2.001	(2.001, 4.004 001)	4.001

b $\lim\limits_{x\to 2} \dfrac{x^2-4}{x-2} = 4$

The gradient of the tangent to $y = x^2$ at the point (2, 4) is 4.

EXERCISE 13C

1 **a** $f(2) = 3$ **b** $f'(2) = 0$

2 **a** $f(0) = 4$ **b** $f'(0) = -1$ **3** $f(2) = 3,\ f'(2) = 1$

EXERCISE 13D

1 **a** $f'(x) = 1$ **b** $f'(x) = 0$ **c** $f'(x) = 2$

2 **a** $\dfrac{dy}{dx} = -1$ **b** $\dfrac{dy}{dx} = 2x - 3$ **c** $\dfrac{dy}{dx} = 4x + 1$

3 **a** 3 **b** -12 **c** 9 **d** 10

EXERCISE 13E

1 **a** $f'(x) = 3x^2$ **b** $f'(x) = 6x^2$

c $f'(x) = 14x$ **d** $f'(x) = \dfrac{3}{\sqrt{x}}$

e $f'(x) = \dfrac{1}{\sqrt[3]{x^2}}$ **f** $f'(x) = 2x + 1$

g $f'(x) = -4x$ **h** $f'(x) = 2x + 3$

i $f'(x) = 2x^3 - 12x$ **j** $f'(x) = \dfrac{6}{x^2}$

k $f'(x) = -\dfrac{2}{x^2} + \dfrac{6}{x^3}$ **l** $f'(x) = 2x - \dfrac{5}{x^2}$

m $f'(x) = 2x + \dfrac{3}{x^2}$ **n** $f'(x) = -\dfrac{1}{2x\sqrt{x}}$

o $f'(x) = 8x - 4$ **p** $f'(x) = 3x^2 + 12x + 12$

2 **a** $\dfrac{dy}{dx} = 7.5x^2 - 2.8x$ **b** $\dfrac{dy}{dx} = 2\pi x$

c $\dfrac{dy}{dx} = -\dfrac{2}{5x^3}$ **d** $\dfrac{dy}{dx} = 100$

e $\dfrac{dy}{dx} = 10$ **f** $\dfrac{dy}{dx} = 12\pi x^2$

3 **a** 6 **b** $\dfrac{3\sqrt{x}}{2}$ **c** $2x - 10$ **d** $2 - 9x^2$ **e** $2x - 1$

f $-\dfrac{2}{x^3} + \dfrac{3}{\sqrt{x}}$ **g** $4 + \dfrac{1}{4x^2}$ **h** $6x^2 - 6x - 5$

4 **a** 4 **b** $-\frac{16}{729}$ **c** -7 **d** $\frac{13}{4}$ **e** $\frac{1}{8}$ **f** -11

5 $b = 3,\ c = -4$

6 **a** $f'(x) = \dfrac{2}{\sqrt{x}} + 1$ **b** $f'(x) = \dfrac{1}{3\sqrt[3]{x^2}}$

c $f'(x) = \dfrac{1}{x\sqrt{x}}$ **d** $f'(x) = 2 - \dfrac{1}{2\sqrt{x}}$

e $f'(x) = -\dfrac{2}{x\sqrt{x}}$ **f** $f'(x) = 6x - \frac{3}{2}\sqrt{x}$

g $f'(x) = \dfrac{-25}{2x^3\sqrt{x}}$ **h** $f'(x) = 2 + \dfrac{9}{2x^2\sqrt{x}}$

7 **a** $\dfrac{dy}{dx} = 4 + \dfrac{3}{x^2}$, $\dfrac{dy}{dx}$ is the gradient function of $y = 4x - \dfrac{3}{x}$ from which the gradient at any point can be found.

b $\dfrac{dS}{dt} = 4t + 4 \ \text{m s}^{-1}$, $\dfrac{dS}{dt}$ is the instantaneous rate of change in position at the time t, or the velocity function.

c $\dfrac{dC}{dx} = 3 + 0.004x$ \$ per toaster, $\dfrac{dC}{dx}$ is the instantaneous rate of change in cost as the number of toasters changes.

EXERCISE 13F.1

1 **a** $gf(x) = (2x+7)^2$ **b** $gf(x) = 2x^2 + 7$

c $gf(x) = \sqrt{3-4x}$ **d** $gf(x) = 3 - 4\sqrt{x}$

e $gf(x) = \dfrac{2}{x^2+3}$ **f** $gf(x) = \dfrac{4}{x^2} + 3$

2 **Note:** There may be other answers.

a $g(x) = x^3,\quad f(x) = 3x + 10$

b $g(x) = \dfrac{1}{x},\quad f(x) = 2x + 4$

c $g(x) = \sqrt{x},\quad f(x) = x^2 - 3x$

d $g(x) = \dfrac{10}{x^3},\quad f(x) = 3x - x^2$

EXERCISE 13F.2

1 **a** $u^{-2},\quad u = 2x - 1$ **b** $u^{\frac{1}{2}},\quad u = x^2 - 3x$

c $2u^{-\frac{1}{2}},\quad u = 2 - x^2$ **d** $u^{\frac{1}{3}},\quad u = x^3 - x^2$

e $4u^{-3},\quad u = 3 - x$ **f** $10u^{-1},\quad u = x^2 - 3$

2 **a** $\dfrac{dy}{dx} = 8(4x-5)$ **b** $\dfrac{dy}{dx} = 2(5-2x)^{-2}$

c $\dfrac{dy}{dx} = \frac{1}{2}(3x - x^2)^{-\frac{1}{2}} \times (3 - 2x)$

d $\dfrac{dy}{dx} = -12(1-3x)^3$ **e** $\dfrac{dy}{dx} = -18(5-x)^2$

f $\dfrac{dy}{dx} = \frac{1}{3}(2x^3 - x^2)^{-\frac{2}{3}} \times (6x^2 - 2x)$

g $\dfrac{dy}{dx} = -60(5x-4)^{-3}$

h $\dfrac{dy}{dx} = -4(3x - x^2)^{-2} \times (3 - 2x)$

i $\dfrac{dy}{dx} = 6\left(x^2 - \dfrac{2}{x}\right)^2 \times \left(2x + \dfrac{2}{x^2}\right)$

3 **a** $-\frac{1}{\sqrt{3}}$ **b** -18 **c** -8 **d** -4 **e** $-\frac{3}{32}$ **f** 0

4 $a = 3,\ b = 1$ **5** $a = 2,\ b = 1$

6 **a** $\dfrac{dy}{dx} = 3x^2,\quad \dfrac{dx}{dy} = \frac{1}{3}y^{-\frac{2}{3}}$ **Hint:** Substitute $y = x^3$

b $\dfrac{dy}{dx} \times \dfrac{dx}{dy} = \dfrac{dy}{dy}$ {chain rule} $= 1$

EXERCISE 13G

1 **a** $f'(x) = 2x - 1$ **b** $f'(x) = 4x + 2$

c $f'(x) = 2x(x+1)^{\frac{1}{2}} + \frac{1}{2}x^2(x+1)^{-\frac{1}{2}}$

2 **a** $\dfrac{dy}{dx} = 2x(2x-1) + 2x^2$

b $\dfrac{dy}{dx} = 4(2x+1)^3 + 24x(2x+1)^2$

c $\dfrac{dy}{dx} = 2x(3-x)^{\frac{1}{2}} - \frac{1}{2}x^2(3-x)^{-\frac{1}{2}}$

d $\dfrac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}}(x-3)^2 + 2\sqrt{x}(x-3)$

e $\dfrac{dy}{dx} = 10x(3x^2-1)^2 + 60x^3(3x^2-1)$

f $\dfrac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}}(x-x^2)^3 + 3\sqrt{x}(x-x^2)^2(1-2x)$

3 **a** -48 **b** $406\frac{1}{4}$ **c** $\frac{13}{3}$ **d** $\frac{11}{2}$

4 **b** $x = 3$ or $\frac{3}{5}$ **c** $x \leqslant 0$ **5** $x = -1$ and $x = -\frac{5}{3}$

EXERCISE 13H

1 **a** $\dfrac{dy}{dx} = \dfrac{7}{(2-x)^2}$ **b** $\dfrac{dy}{dx} = \dfrac{2x(2x+1) - 2x^2}{(2x+1)^2}$

c $\dfrac{dy}{dx} = \dfrac{(x^2-3) - 2x^2}{(x^2-3)^2}$

d $\dfrac{dy}{dx} = \dfrac{\frac{1}{2}x^{-\frac{1}{2}}(1-2x) + 2\sqrt{x}}{(1-2x)^2}$

e $\dfrac{dy}{dx} = \dfrac{2x(3x-x^2) - (x^2-3)(3-2x)}{(3x-x^2)^2}$

f $\dfrac{dy}{dx} = \dfrac{(1-3x)^{\frac{1}{2}} + \frac{3}{2}x(1-3x)^{-\frac{1}{2}}}{1-3x}$

2 **a** 1 **b** 1 **c** $-\frac{7}{324}$ **d** $-\frac{28}{27}$

3 **b** **i** never $\left\{\dfrac{dy}{dx}\right.$ is undefined at $\left. x = -1\right\}$

ii $x \leqslant 0$ and $x = 1$

4 **b** **i** $x = -2 \pm \sqrt{11}$ **ii** $x = -2$

EXERCISE 13I

1 **a** $f'(x) = 4e^{4x}$ **b** $f'(x) = e^x$

c $f'(x) = -2e^{-2x}$ **d** $f'(x) = \frac{1}{2}e^{\frac{x}{2}}$

e $f'(x) = -e^{-\frac{x}{2}}$ **f** $f'(x) = 2e^{-x}$

g $f'(x) = 2e^{\frac{x}{2}} + 3e^{-x}$ **h** $f'(x) = \dfrac{e^x - e^{-x}}{2}$

i $f'(x) = -2xe^{-x^2}$ **j** $f'(x) = e^{\frac{1}{x}} \times \dfrac{-1}{x^2}$

k $f'(x) = 20e^{2x}$ **l** $f'(x) = 40e^{-2x}$

m $f'(x) = 2e^{2x+1}$ **n** $f'(x) = \frac{1}{4}e^{\frac{x}{4}}$

o $f'(x) = -4xe^{1-2x^2}$ **p** $f'(x) = -0.02e^{-0.02x}$

2 **a** $e^x + xe^x$ **b** $3x^2e^{-x} - x^3e^{-x}$

c $\dfrac{xe^x - e^x}{x^2}$ **d** $\dfrac{1-x}{e^x}$

e $2xe^{3x} + 3x^2e^{3x}$ **f** $\dfrac{xe^x - \frac{1}{2}e^x}{x\sqrt{x}}$

g $\frac{1}{2}x^{-\frac{1}{2}}e^{-x} - x^{\frac{1}{2}}e^{-x}$ **h** $\dfrac{e^x + 2 + 2e^{-x}}{(e^{-x}+1)^2}$

3 **a** 108 **b** -1 **c** $\frac{9}{\sqrt{19}}$ **4** $k = -9$

5 **a** $\dfrac{dy}{dx} = 2^x \ln 2$ **6** P $= (0, 0)$ or $\left(2, \dfrac{4}{e^2}\right)$

EXERCISE 13J

1 **a** $\dfrac{dy}{dx} = \dfrac{1}{x}$ **b** $\dfrac{dy}{dx} = \dfrac{2}{2x+1}$ **c** $\dfrac{dy}{dx} = \dfrac{1-2x}{x-x^2}$

d $\dfrac{dy}{dx} = -\dfrac{2}{x}$ **e** $\dfrac{dy}{dx} = 2x\ln x + x$

f $\dfrac{dy}{dx} = \dfrac{1-\ln x}{2x^2}$ **g** $\dfrac{dy}{dx} = e^x \ln x + \dfrac{e^x}{x}$

h $\dfrac{dy}{dx} = \dfrac{2\ln x}{x}$ **i** $\dfrac{dy}{dx} = \dfrac{1}{2x\sqrt{\ln x}}$

j $\dfrac{dy}{dx} = \dfrac{e^{-x}}{x} - e^{-x}\ln x$ **k** $\dfrac{dy}{dx} = \dfrac{\ln(2x)}{2\sqrt{x}} + \dfrac{1}{\sqrt{x}}$

l $\dfrac{dy}{dx} = \dfrac{\ln x - 2}{\sqrt{x}(\ln x)^2}$ **m** $\dfrac{dy}{dx} = \dfrac{4}{1-x}$

n $\dfrac{dy}{dx} = \ln(x^2+1) + \dfrac{2x^2}{x^2+1}$

2 **a** $\dfrac{dy}{dx} = \ln 5$ **b** $\dfrac{dy}{dx} = \dfrac{3}{x}$ **c** $\dfrac{dy}{dx} = \dfrac{4x^3+1}{x^4+x}$

d $\dfrac{dy}{dx} = \dfrac{1}{x-2}$ **e** $\dfrac{dy}{dx} = \dfrac{6}{2x+1}[\ln(2x+1)]^2$

f $\dfrac{dy}{dx} = \dfrac{1-\ln(4x)}{x^2}$ **g** $\dfrac{dy}{dx} = -\dfrac{1}{x}$

h $\dfrac{dy}{dx} = \dfrac{1}{x\ln x}$ **i** $\dfrac{dy}{dx} = \dfrac{-1}{x(\ln x)^2}$

3 **a** $\dfrac{dy}{dx} = \dfrac{-1}{1-2x}$ **b** $\dfrac{dy}{dx} = \dfrac{-2}{2x+3}$ **c** $\dfrac{dy}{dx} = 1 + \dfrac{1}{2x}$

d $\dfrac{dy}{dx} = \dfrac{1}{x} - \dfrac{1}{2(2-x)}$ **e** $\dfrac{dy}{dx} = \dfrac{1}{x+3} - \dfrac{1}{x-1}$

f $\dfrac{dy}{dx} = \dfrac{2}{x} + \dfrac{1}{3-x}$ **g** $f'(x) = \dfrac{9}{3x-4}$

h $f'(x) = \dfrac{1}{x} + \dfrac{2x}{x^2+1}$ **i** $f'(x) = \dfrac{2x+2}{x^2+2x} - \dfrac{1}{x-5}$

4 **a** 2 **b** $-\frac{5}{3}$ **5** $a = 3,\ b = -e$

EXERCISE 13K

1 **a** $\dfrac{dy}{dx} = 2\cos(2x)$ **b** $\dfrac{dy}{dx} = \cos x - \sin x$

c $\dfrac{dy}{dx} = -3\sin(3x) - \cos x$ **d** $\dfrac{dy}{dx} = \cos(x+1)$

e $\dfrac{dy}{dx} = 2\sin(3-2x)$ **f** $\dfrac{dy}{dx} = \dfrac{5}{\cos^2(5x)}$

g $\dfrac{dy}{dx} = \frac{1}{2}\cos\left(\frac{x}{2}\right) + 3\sin x$ **h** $\dfrac{dy}{dx} = \dfrac{3\pi}{\cos^2(\pi x)}$

i $\dfrac{dy}{dx} = 4\cos x + 2\sin(2x)$

2 **a** $2x - \sin x$ **b** $\dfrac{1}{\cos^2 x} - 3\cos x$

c $e^x\cos x - e^x\sin x$ **d** $-e^{-x}\sin x + e^{-x}\cos x$

e $\dfrac{\cos x}{\sin x}$ **f** $2e^{2x}\tan x + \dfrac{e^{2x}}{\cos^2 x}$ **g** $3\cos(3x)$

h $-\frac{1}{2}\sin\left(\frac{x}{2}\right)$ **i** $\dfrac{6}{\cos^2(2x)}$ **j** $\cos x - x\sin x$

k $\dfrac{x\cos x - \sin x}{x^2}$ **l** $\tan x + \dfrac{x}{\cos^2 x}$

3 **a** $2x\cos(x^2)$ **b** $-\frac{1}{2\sqrt{x}}\sin(\sqrt{x})$ **c** $-\frac{\sin x}{2\sqrt{\cos x}}$

d $2\sin x\cos x$ **e** $-3\sin x\cos^2 x$

f $-\sin x\sin(2x)+2\cos x\cos(2x)$

g $\sin x\sin(\cos x)$ **h** $-12\sin(4x)\cos^2(4x)$

i $-\frac{\cos x}{\sin^2 x}$ **j** $\frac{2\sin(2x)}{\cos^2(2x)}$

k $-\frac{8\cos(2x)}{\sin^3(2x)}$ **l** $\frac{-12}{\cos^2(\frac{x}{2})\tan^4(\frac{x}{2})}$

4 **a** $-\frac{9}{8}$ **b** 0

EXERCISE 13L

1 **a** $f''(x)=6$ **b** $f''(x)=\frac{3}{2x^{\frac{5}{2}}}$

c $f''(x)=12x-6$ **d** $f''(x)=\frac{12-6x}{x^4}$

e $f''(x)=24-48x$ **f** $f''(x)=\frac{20}{(2x-1)^3}$

2 **a** $\frac{d^2y}{dx^2}=-6x$ **b** $\frac{d^2y}{dx^2}=2-\frac{30}{x^4}$

c $\frac{d^2y}{dx^2}=-\frac{9}{4}x^{-\frac{5}{2}}$ **d** $\frac{d^2y}{dx^2}=\frac{8}{x^3}$

e $\frac{d^2y}{dx^2}=6(x^2-3x)(5x^2-15x+9)$

f $\frac{d^2y}{dx^2}=2+\frac{2}{(1-x)^3}$

3 **a** $f(2)=9$ **b** $f'(2)=10$ **c** $f''(2)=12$

5 **a** $x=1$

b $x=0, \pm\sqrt{6}$

6

x	-1	0	1
$f(x)$	$-$	0	$+$
$f'(x)$	$+$	$-$	$+$
$f''(x)$	$-$	0	$+$

7 **b** $f''(x)=3\sin x\cos 2x+6\cos x\sin 2x$

8 **a** $\frac{d^2y}{dx^2}=\frac{1}{x^2}$ **b** $\frac{d^2y}{dx^2}=\frac{1}{x}$

c $\frac{d^2y}{dx^2}=\frac{2}{x^2}(1-\ln x)$

9 **a** $f(1)=0$ **b** $f'(1)=3$ **c** $f''(1)=0$

10 **Hint:** Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ and substitute into the equation.

REVIEW SET 13A

1 **a** -1 **b** -1 **c** 8

2 **a** $f'(x)=2x+2$ **b** $\frac{dy}{dx}=-6x$

3 **a** $f'(t)=-9.6t\ \text{m s}^{-1}$

b $f'(2)=-19.2\ \text{m s}^{-1}$

(the negative sign indicates travelling downwards)

4 **a** $f(3)=-17$ **b** $f'(3)=-17$ **c** $f''(3)=-6$

5 **a** $\frac{dy}{dx}=6x-4x^3$ **b** $\frac{dy}{dx}=1+\frac{1}{x^2}$

6 $(0, 0)$ **7** **a** $\frac{dy}{dx}=3x^2e^{x^3+2}$ **b** $\frac{dy}{dx}=\frac{1}{x+3}-\frac{2}{x}$

9 **a** $5+3x^{-2}$ **b** $4(3x^2+x)^3(6x+1)$

c $2x(1-x^2)^3-6x(x^2+1)(1-x^2)^2$

10 $(-2, 19)$ and $(1, -2)$

11 **a** $\frac{dy}{dx}=-2(5-4x)^{-\frac{1}{2}}$ **b** $\frac{d^2y}{dx^2}=-4(5-4x)^{-\frac{3}{2}}$

12 **a** $5\cos(5x)\ln x+\frac{\sin(5x)}{x}$

b $\cos x\cos(2x)-2\sin x\sin(2x)$

c $-2e^{-2x}\tan x+\frac{e^{-2x}}{\cos^2 x}$

13 $\frac{\sqrt{3}}{2}$

14 **a** $f'(x)=8x(x^2+3)^3$

b $g'(x)=\frac{\frac{1}{2}x(x+5)^{-\frac{1}{2}}-2(x+5)^{\frac{1}{2}}}{x^3}$

15 **a** $f''(2)=\frac{23}{4}$ **b** $f''(2)=-\frac{1}{8\sqrt{2}}$

16 **a** $10-10\cos(10x)$ **b** $\tan x$

c $5\cos(5x)\ln(2x)+\frac{\sin(5x)}{x}$

REVIEW SET 13B

1 **a** -3 **b** 3 **c** -1 **2** $f'(1)=3$

3 **a** $\frac{dy}{dx}=4x$ **b** when $x=4$, gradient $=16$

c when gradient $=-12$, $x=-3$

4 **a** $\frac{dy}{dx}=3x^2(1-x^2)^{\frac{1}{2}}-x^4(1-x^2)^{-\frac{1}{2}}$

b $\frac{dy}{dx}=\frac{(2x-3)(x+1)^{\frac{1}{2}}-\frac{1}{2}(x^2-3x)(x+1)^{-\frac{1}{2}}}{x+1}$

5 **a** $\frac{d^2y}{dx^2}=36x^2-\frac{4}{x^3}$ **b** $\frac{d^2y}{dx^2}=6x+\frac{3}{4}x^{-\frac{5}{2}}$

6 $(1, e)$ **7** **a** $f'(x)=\frac{e^x}{e^x+3}$ **b** $f'(x)=\frac{3}{x+2}-\frac{1}{x}$

8 When $x=1$, $\frac{dy}{dx}=0$.

9 **a** $\frac{dy}{dx}=\frac{3x^2-3}{x^3-3x}$ **b** $\frac{dy}{dx}=\frac{e^x(x-2)}{x^3}$

10 $x=-\frac{1}{2}, \frac{3}{2}$

11 **a** $f(\pi)=\pi+1$ **b** $f'(\frac{\pi}{2})=2$ **c** $f''(\frac{3\pi}{4})=-\frac{\sqrt{2}}{2}$

12 **a** $f'(x)=\frac{1}{2}x^{-\frac{1}{2}}\cos(4x)-4x^{\frac{1}{2}}\sin(4x)$,

$f''(x)=-\frac{1}{4}x^{-\frac{3}{2}}\cos(4x)-4x^{-\frac{1}{2}}\sin(4x)$

$-16x^{\frac{1}{2}}\cos(4x)$

b $f'(\frac{\pi}{16})\approx-0.455$, $f''(\frac{\pi}{8})\approx-6.38$

14 **a** $x=-6\pm\sqrt{33}$ **b** $x=\pm\sqrt{3}$ **c** $x=0, \pm3$

15 **a** $f(x)=-5\sin 4x$

b $f'(x)=0$ when $x=\frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}$, $0\leqslant x\leqslant\pi$

16 $\frac{dy}{dx}=3b\cos(bx)+2a\sin(2x)$, $a=2$, $b=\pm1$

EXERCISE 14A

1 **a** $y=-7x+11$ **b** $x-4y=-8$ **c** $y=-2x-2$

d $y=-2x+6$ **e** $y=-5x-9$ **f** $y=-5x-1$

2 **a** $x+6y=57$ **b** $x+7y=26$ **c** $x-3y=-11$

d $x+6y=43$

3 $y = 21$ and $y = -6$

4 $(\frac{1}{2}, 2\sqrt{2})$

5 $k = -5$

6 $y = -3x + 1$

7 $a = -4,\ b = 7$

8 $a = 2,\ b = 4$

10 **a** $x - 3y = -5$ **b** $x - 9y = -4$
c $x - 16y = 3$ **d** $y = -4$

11 **a** $y = 2x - \frac{7}{4}$ **b** $y = -27x - \frac{242}{3}$
c $4x + 57y = 1042$ **d** $x - 2y = -1$

12 $a = 4,\ b = 3$

13 **a** $x + ey = 2$ **b** $x + 3y = 3\ln 3 - 1$
c $2x + e^2 y = \dfrac{2}{e^2} - e^2$

15 **a** $y = x$ **b** $y = x$ **c** $2x - y = \frac{\pi}{3} - \frac{\sqrt{3}}{2}$ **d** $x = \frac{\pi}{4}$

16 **a** $(-4, -64)$ **b** $(4, -31)$

17 **a** $f'(x) = 2x - \dfrac{8}{x^3}$ **b** $x = \pm\sqrt{2}$ **c** tangent is $y = 4$

18 A is $(\frac{2}{3}, 0)$, B is $(0, -2e)$

19 **a** $y = (2a-1)x - a^2 + 9$
b $y = 5x$, contact at $(3, 15)$, $y = -7x$, contact at $(-3, 21)$

20 $y = 0$, $y = 27x + 54$

21 $y = -\sqrt{14}x + 4\sqrt{14}$

22 $y = e^a x + e^a(1-a)$ so $y = ex$ is the tangent to $y = e^x$ from the origin.

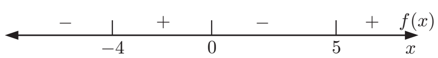
23 **a** **Hint:** They must have the same y-coordinate at $x = b$ and the same gradient.
c $a = \dfrac{1}{2e}$ **d** $y = e^{-\frac{1}{2}}x - \frac{1}{2}$

24 $\approx 63.43°$

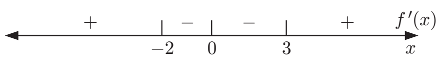
25 **a** **Hint:** $y = f(a) + f'(a)(x-a)$
b **Hint:** Expand $f(x) = 4 - 8(x+1) - (x+1)^2 + 2(x+1)^3$
c Notice the first 2 terms in **b** are the same as the tangent line found in part **a**.

EXERCISE 14B

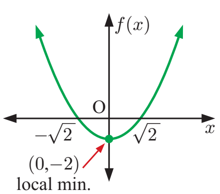
1 **a** A - local max, B - stationary inflection, C - local min.
b **i**



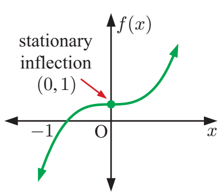
ii



2 **a**



b



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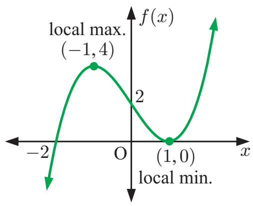

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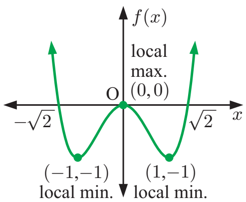

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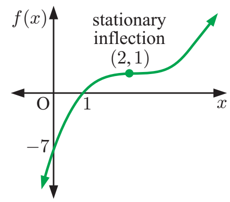

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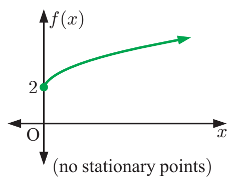
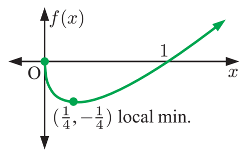
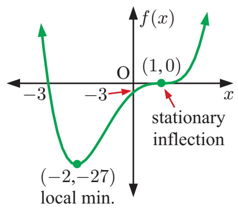


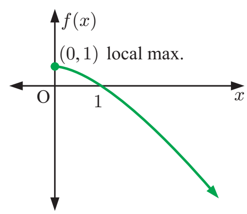
g



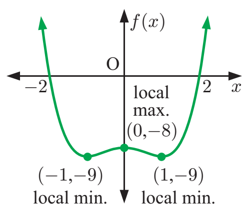
h



i



j



3 $x = -\dfrac{b}{2a}$, local min if $a > 0$, local max if $a < 0$

4 $a = 9$

5 **a** $a = -12,\ b = -13$
b $(-2, 3)$ local max., $(2, -29)$ local min.

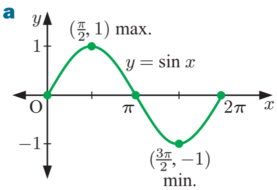
6 **a** local maximum at $(1, e^{-1})$
b local maximum at $(-2, 4e^{-2})$, local minimum at $(0, 0)$
c local minimum at $(1, e)$
d local maximum at $(-1, e)$

7 **a** $x > 0$

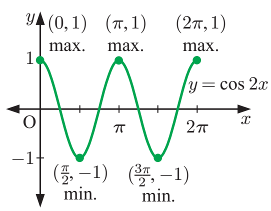
8 **a** Greatest value is 63 when $x = 5$,
least value is -18 when $x = 2$.
b Greatest value is 4 when $x = 3$ and $x = 0$,
least value is -16 when $x = -2$.

9 $P(x) = -9x^3 - 9x^2 + 9x + 2$

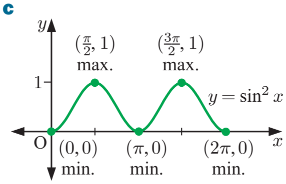
10 **a**



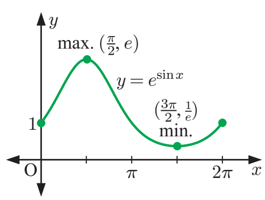
b



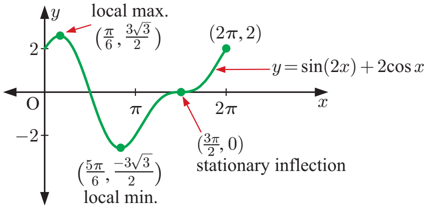
c



d



e



11 **Hint:** Find $\frac{dy}{dx}$, then determine the nature of the stationary points.

12 **Hint:** Show that as $x \to 0$, $f(x) \to -\infty$, and as $x \to \infty$, $f(x) \to 0$.

13 **a** **Hint:** Find $f'(x)$, then determine the nature of the stationary points.
b **Hint:** Show that $f(x) \geqslant 1$ for all $x > 0$.

EXERCISE 14C.1

1 **a** $7\ \text{m s}^{-1}$ **b** $(h+5)\ \text{m s}^{-1}$
c $5\ \text{m s}^{-1} = s'(1)$ is the instantaneous velocity at $t = 1$ s
d average velocity $= (2t + h + 3)\ \text{m s}^{-1}$,
$\lim\limits_{h \to 0} (2t + h + 3) = 2t + 3\ \text{m s}^{-1}$ is the instantaneous velocity at time t seconds.

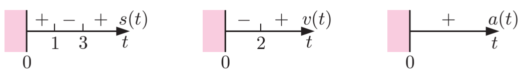
2 **a** $-14\ \text{cm s}^{-1}$ **b** $(-8 - 2h)\ \text{cm s}^{-1}$
c $-8\ \text{cm s}^{-1} = s'(2)$
$\therefore$ instantaneous velocity $= -8\ \text{cm s}^{-1}$ at $t = 2$
d $-4t = s'(t) = v(t)$ is the instantaneous velocity at time t seconds.

3 **a** $\frac{2}{3}\ \text{cm s}^{-2}$ **b** $\dfrac{2\sqrt{1+h} - 2}{h}\ \text{cm s}^{-2}$
c $1\ \text{cm s}^{-2} = v'(1)$ is the instantaneous accn. at $t = 1$ s
d $\dfrac{1}{\sqrt{t}}\ \text{cm s}^{-2} = v'(t)$, the instantaneous accn. at time t

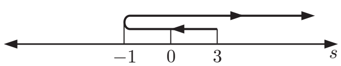
4 **a** velocity at $t = 4$ **b** acceleration at $t = 4$

EXERCISE 14C.2

1 **a** $v(t) = 2t - 4\ \text{cm s}^{-1}$, $a(t) = 2\ \text{cm s}^{-2}$

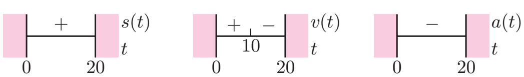


b $s(0) = 3$ cm, $v(0) = -4\ \text{cm s}^{-1}$, $a(0) = 2\ \text{cm s}^{-2}$
The object is initially 3 cm to the right of the origin and is moving to the left at $4\ \text{cm s}^{-1}$. It is accelerating at $2\ \text{cm s}^{-2}$ to the right.
c $s(2) = -1$ cm, $v(2) = 0\ \text{cm s}^{-1}$, $a(2) = 2\ \text{cm s}^{-2}$
The object is instantaneously stationary, 1 cm to the left of the origin and is accelerating to the right at $2\ \text{cm s}^{-2}$.
d At $t = 2$, $s(2) = 1$ cm to the left of the origin.
e



f $0 \leqslant t \leqslant 2$

2 **a** $v(t) = 98 - 9.8t\ \text{m s}^{-1}$, $a(t) = -9.8\ \text{m s}^{-2}$

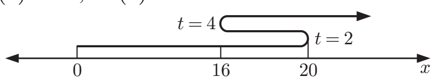


b $s(0) = 0$ m above the ground, $v(0) = 98\ \text{m s}^{-1}$ skyward
c $t = 5$ s Stone is 367.5 m above the ground and moving skyward at $49\ \text{m s}^{-1}$. Its speed is decreasing.
$t = 12$ s Stone is 470.4 m above the ground and moving groundward at $19.6\ \text{m s}^{-1}$. Its speed is increasing.
d 490 m **e** 20 seconds

3 **a** 1.2 m
b $s'(t) = 28.1 - 9.8t$ represents the instantaneous velocity of the ball.
c $t = 2.87$ s. The ball has reached its maximum height and is instantaneously at rest.
d 41.5 m
e **i** $28.1\ \text{m s}^{-1}$ **ii** $8.5\ \text{m s}^{-1}$ **iii** $20.9\ \text{m s}^{-1}$
$s'(t) \geqslant 0$ when the ball is travelling upwards.
$s'(t) \leqslant 0$ when the ball is travelling downwards.
f 5.78 s
g $s''(t)$ is the rate of change of $s'(t)$, or the instantaneous acceleration.

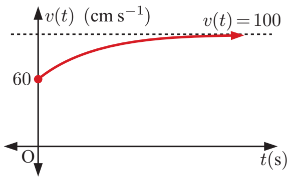
4 **a** $v(t) = 3t^2 - 18t + 24\ \text{m s}^{-1}$ $a(t) = 6t - 18\ \text{m s}^{-2}$

b $x(2) = 20$, $x(4) = 16$



c **i** $0 \leqslant t \leqslant 2$ and $3 \leqslant t \leqslant 4$ **ii** $0 \leqslant t \leqslant 3$
d 28 m

5 **a** $v(t) = 100 - 40e^{-\frac{t}{5}}\ \text{cm s}^{-1}$, $a(t) = 8e^{-\frac{t}{5}}\ \text{cm s}^{-2}$
b $s(0) = 200$ cm on positive side of origin
$v(0) = 60\ \text{cm s}^{-1}$, $a(0) = 8\ \text{cm s}^{-2}$
c



d after 3.47 s

6 **a** $x(0) = -1$ cm, $v(0) = 0\ \text{cm s}^{-1}$, $a(0) = 2\ \text{cm s}^{-2}$
b At $t = \frac{\pi}{4}$ seconds, the particle is $(\sqrt{2} - 1)$ cm left of the origin, moving right at $\sqrt{2}\ \text{cm s}^{-1}$, with increasing speed.
c changes direction when $t = \pi$, $x(\pi) = 3$ cm
d $0 \leqslant t \leqslant \frac{\pi}{2}$ and $\pi \leqslant t \leqslant \frac{3\pi}{2}$

7 **Hint:** Assume that $s(t) = at^2 + bt + c$
$s'(t) = v(t)$ and $s''(t) = a(t) = g$
Show that $a = \frac{1}{2}g$, $b = v(0)$, $c = 0$.

8 **a** 0.675 s
b **i** $S'(t) = u + at\ \text{m s}^{-1}$ **ii** $t = -\dfrac{u}{a}$ s
iii $a = -\frac{640}{99} \approx -6.46\ \text{m s}^{-2}$
iv **Hint:** Substitute $t = -\dfrac{u}{a}$ into $S(t)$.
v If the speed u is doubled, then the braking distance is quadrupled ($2^2 = 4$ times).

EXERCISE 14D

1 **a** \$118 000 **b** $\dfrac{dP}{dt} = 4t - 12$, \$1000s per year
c $\dfrac{dP}{dt}$ is the rate of change in profit with time
d **i** $0 \leqslant t \leqslant 3$ years **ii** $t > 3$ years
e minimum profit is \$100 000 when $t = 3$