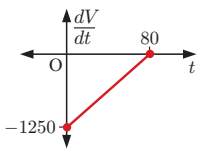
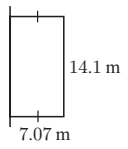
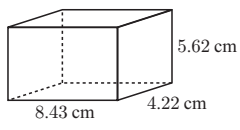


- f**  $\left. \frac{dP}{dt} \right|_{t=4} = 4$  Profit is increasing at \$4000 per year after 4 years.  
 $\left. \frac{dP}{dt} \right|_{t=10} = 28$  Profit is increasing at \$28 000 per year after 10 years.  
 $\left. \frac{dP}{dt} \right|_{t=25} = 88$  Profit is increasing at \$88 000 per year after 25 years.
- 2 a** **i**  $Q(0) = 100$  **ii**  $Q(25) = 50$  **iii**  $Q(100) = 0$   
**b** **i** decr. 1 unit per year **ii** decr.  $\frac{1}{\sqrt{2}}$  units per year  
**c**  $Q'(t) = -\frac{5}{\sqrt{t}} < 0$
- 3 a** 0.5 m  
**b**  $t = 4$ : 9.17 m,  $t = 8$ : 12.5 m,  $t = 12$ : 14.3 m  
**c**  $t = 0$ : 3.9 m year<sup>-1</sup>,  $t = 5$ : 0.975 m year<sup>-1</sup>,  
 $t = 10$ : 0.433 m year<sup>-1</sup>  
**d** As  $\frac{dH}{dt} = \frac{97.5}{(t+5)^2} > 0$  for all  $t \geq 0$ , the tree is always growing.
- 4 a**  $C'(x) = 0.0009x^2 + 0.04x + 4$  dollars per pair  
**b**  $C'(220) = \$56.36$  per pair. This estimates the additional cost of making one more pair of jeans if 220 pairs are currently being made.  
**c** \$56.58 This is the actual increase in cost to make an extra pair of jeans (221 rather than 220).  
**d**  $C''(x) = 0.0018x + 0.04$   
 $C''(x) = 0$  when  $x = -22.2$ . This is where the rate of change is a minimum, however it is out of the bounds of the model (you cannot make  $< 0$  jeans!).
- 5 a** **i** €4500 **ii** €4000  
**b** **i** decrease of €210.22 per km h<sup>-1</sup>  
**ii** increase of €11.31 per km h<sup>-1</sup>  
**c**  $\frac{dC}{dv} = 0$  at  $v = \sqrt[3]{500\,000} \approx 79.4$  km h<sup>-1</sup>
- 6 a**  $\frac{dV}{dt} = -1250 \left(1 - \frac{t}{80}\right)$  L min<sup>-1</sup>  
**b** at  $t = 0$  when the tap was first opened  
**c**  $\frac{d^2V}{dt^2} = \frac{125}{8}$  L min<sup>-2</sup>
- 
- This shows that the rate of change of  $V$  is constantly increasing, so the outflow is decreasing at a constant rate.
- 7 a** The near part of the lake is 2 km from the sea, the furthest part is 3 km.  
**b**  $\frac{dy}{dx} = \frac{3}{10}x^2 - x + \frac{3}{5}$   
 $\left. \frac{dy}{dx} \right|_{x=\frac{1}{2}} = 0.175$ , height of hill is increasing as gradient is positive.  
 $\left. \frac{dy}{dx} \right|_{x=1\frac{1}{2}} = -0.225$ , height of hill is decreasing as gradient is negative.  
 $\therefore$  top of the hill is between  $x = \frac{1}{2}$  and  $x = 1\frac{1}{2}$ .  
**c** 2.55 km from the sea, 63.1 m deep
- 8 a**  $k = \frac{1}{50} \ln 2 \approx 0.0139$   
**b** **i** 20 grams **ii** 14.3 grams **iii** 1.95 grams  
**c** 9 days and 6 minutes (216 hours)  
**d** **i**  $-0.0693$  g h<sup>-1</sup> **ii**  $-2.64 \times 10^{-7}$  g h<sup>-1</sup>  
**e** Hint: You should find  $\frac{dW}{dt} = -\frac{1}{50} \ln 2 \times 20e^{-\frac{1}{50} \ln 2t}$

- 9 a**  $k = \frac{1}{15} \ln \left(\frac{19}{3}\right) \approx 0.123$  **b** 100°C  
**c**  $c = -k \approx -0.123$   
**d** **i** decreasing at 11.7°C min<sup>-1</sup>  
**ii** decreasing at 3.42°C min<sup>-1</sup>  
**iii** decreasing at 0.998°C min<sup>-1</sup>
- 10 a** 43.9 cm **b** 10.4 years  
**c** **i** growing at 5.45 cm per year  
**ii** growing at 1.88 cm per year
- 11 a**  $A(0) = 0$   
**b** **i**  $k = \frac{\ln 2}{3}$  ( $\approx 0.231$ )  
**ii** 0.728 litres of alcohol produced per hour
- 12**  $\frac{21}{\sqrt{2}}$  cm<sup>2</sup> per radian
- 13 a** rising at 2.73 m per hour **b** rising
- 14 a** **i** 0 **ii** 1 **iii**  $\approx 1.11$

## EXERCISE 14E

- 1** 250 items
- 2 b**  $L_{\min} \approx 28.3$  m,  $x \approx 7.07$  m **c**
- 
- 3** 10 blankets **4** 14.8 km h<sup>-1</sup> **5** at 4.41 months old
- 6 a** Hint:  $V = 200 = 2x \times x \times h$   
**b** Hint: Show  $h = \frac{100}{x^2}$  and substitute into the surface area equation.  
**c**  $SA_{\min} \approx 213$  cm<sup>2</sup>,  $x \approx 4.22$  cm **d**
- 
- 7** 20 kettles **8**  $C\left(\frac{1}{\sqrt{2}}, e^{-\frac{1}{2}}\right)$
- 9 a** Recall that  $V_{\text{cylinder}} = \pi r^2 h$  and that 1 L = 1000 cm<sup>3</sup>.  
**b** Recall that  $SA_{\text{cylinder}} = 2\pi r^2 + 2\pi r h$ .  
**c** radius  $\approx 5.42$  cm, height  $\approx 10.8$  cm
- 10 b**  $\theta \approx 1.91$ ,  $A \approx 237$  cm<sup>2</sup> **11 b** 6 cm  $\times$  6 cm
- 12 a**  $0 \leq x \leq 63.7$   
**b**  $l = 100$  m,  $x = \frac{100}{\pi} \approx 31.83$  m,  $A = \frac{20\,000}{\pi} \approx 6366$  m<sup>2</sup>
- 13** after 13.8 weeks **14** after 40 minutes
- 15 c**  $\theta = 30^\circ$ ,  $A \approx 130$  cm<sup>2</sup>
- 16 a** Hint: Show that  $AC = \frac{\theta}{360} \times 2\pi \times 10$   
**b** Hint: Show that  $2\pi r = AC$   
**c** Hint: Use the result from **b** and Pythagoras' theorem.  
**d**  $V = \frac{1}{3}\pi \left(\frac{\theta}{36}\right)^2 \sqrt{100 - \left(\frac{\theta}{36}\right)^2}$  **e**  $\theta \approx 294^\circ$
- 17** 1 hour 34 min 53 s when  $\theta \approx 36.9^\circ$  **18** 9.87 m

## EXERCISE 14F

- 1**  $a$  is decreasing at 7.5 units per second  
**2** increasing at 1 cm per minute  
**3 a** 4 $\pi$  m<sup>2</sup> per second **b** 8 $\pi$  m<sup>2</sup> per second  
**4** increasing at 6 $\pi$  m<sup>2</sup> per minute

5 decreasing at  $0.16 \text{ m}^3$  per minute    6  $\frac{20}{3} \text{ cm}$  per minute

7  $\frac{25\sqrt{3}}{6} \approx 7.22 \text{ cm}$  per minute

8 decreasing at  $\frac{250}{13} \approx 19.2 \text{ m s}^{-1}$

9 a  $0.2 \text{ m s}^{-1}$     b  $\frac{4}{45} \text{ m s}^{-1}$

10 decreasing at  $\frac{\sqrt{2}}{100}$  radians per second

11 increasing at  $0.12$  radians per minute

# REVIEW SET 14A

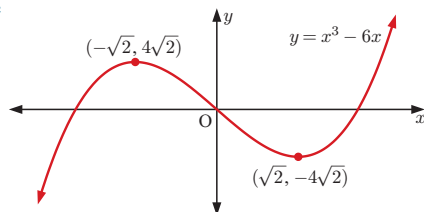
1 a  $y = 4x + 2$     b  $y = 4x + 4 \ln 2 - 4$     c  $y = e^2$

2  $a = \frac{5}{2}$ ,  $b = -\frac{3}{2}$

3 a  $a = -6$

b local max.  $(-\sqrt{2}, 4\sqrt{2})$ , local min.  $(\sqrt{2}, -4\sqrt{2})$

c



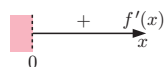
4 a  $y = \frac{1}{5}x - \frac{11}{5}$  (or  $x - 5y = 11$ )    b  $y = -4x + 14$

5  $\frac{3267}{152} \text{ units}^2$     6  $a = 64$     7 P(0, 7.5), Q(3, 0)

9  $3x - 4y = -5$

10 a  $x > 0$

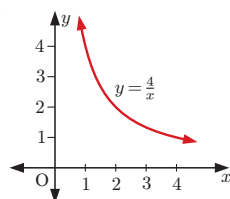
b Sign diagram of  $f'(x)$



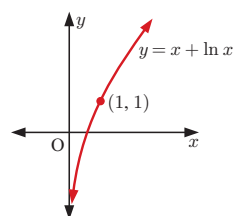
$f(x)$  is increasing for all  $x > 0$ .

d normal is  $x + 2y = 3$

11 a



c



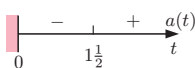
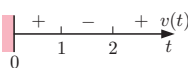
b  $y = -\frac{4}{k^2}x + \frac{8}{k}$

c A(2k, 0), B(0,  $\frac{8}{k}$ )

d Area = 8 units<sup>2</sup>

e  $k = 2$

12 a  $v(t) = (6t^2 - 18t + 12) \text{ cm s}^{-1}$ ,  $a(t) = (12t - 18) \text{ cm s}^{-2}$



b  $s(0) = -5 \text{ cm}$  (5 cm to the left of origin)

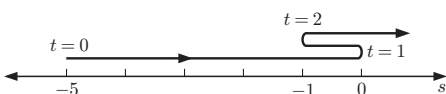
$v(0) = 12 \text{ cm s}^{-1}$  towards origin

$a(0) = -18 \text{ cm s}^{-2}$  (reducing speed)

c At  $t = 2$ , particle is 1 cm to the left of the origin, is stationary and is accelerating towards the origin.

d  $t = 1$ ,  $s = 0$  and  $t = 2$ ,  $s = -1$

e



f  $1 \leq t \leq 1\frac{1}{2}$  and  $t \geq 2$

13 b  $k = 9$

14 a  $x(0) = 3 \text{ cm}$ ,  $x'(0) = 2 \text{ cm s}^{-1}$ ,  $x''(0) = 0 \text{ cm s}^{-2}$

b  $t = \frac{\pi}{4} \text{ s}$  and  $\frac{3\pi}{4} \text{ s}$     c 4 cm

15 6 cm from each end

16 a  $y = \frac{1}{x^2}$ ,  $x > 0$

c base is 1.26 m square, height 0.630 m

17 a  $v(t) = 15 + \frac{120}{(t+1)^3} \text{ cm s}^{-1}$ ,  $a(t) = \frac{-360}{(t+1)^4} \text{ cm s}^{-2}$

b At  $t = 3$ , particle is 41.25 cm to the right of the origin, moving to the right at  $16.88 \text{ cm s}^{-1}$  and decelerating at  $1.41 \text{ cm s}^{-2}$ .

c speed is never increasing

18 A  $(\frac{1}{2}, \frac{1}{e})$     19  $\frac{20\sqrt{10}}{3} \approx 21.1 \text{ m per minute}$

20 a  $V(r) = \frac{8}{9}\pi r^3 \text{ m}^3$

b  $\frac{dr}{dt} = -\frac{8}{375\pi} \approx -0.00679 \text{ m min}^{-1}$

# REVIEW SET 14B

1 a  $x = 1$     b  $ex - 2y = e - \frac{2}{e}$     c  $y = 16x - \frac{127}{2}$

2  $a = -14$ ,  $b = 21$

3 a  $f(3) = 2$ ,  $f'(3) = -1$     b  $f(x) = x^2 - 7x + 14$

4 a  $2x + 3y = \frac{2\pi}{3} + 2\sqrt{3}$     b  $\sqrt{2}y - 4x = 1 - 2\pi$

5  $p = 1$ ,  $q = -8$     6  $(-2, -25)$     7  $a = \frac{1}{2}$

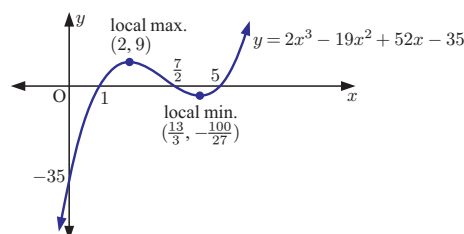
8 a local minimum at (0, 1)    c  $f''(x) = e^x$

9 (0,  $\ln 4 - 1$ )

10 a y-intercept = -35    b  $x = 1, \frac{7}{2}, 5$

c local maximum at (2, 9), local minimum at  $(\frac{13}{3}, -\frac{100}{27})$

d



11 BC =  $\frac{8\sqrt{10}}{3} \text{ units}$

12 a 60 cm    b i 4.24 years    ii 201 years

c i 16 cm per year    ii 1.95 cm per year

13 a  $v(t) = -8e^{-\frac{t}{10}} - 40 \text{ m s}^{-1}$

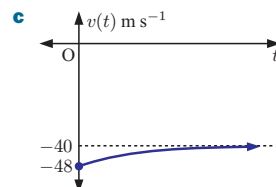
$a(t) = \frac{4}{5}e^{-\frac{t}{10}} \text{ m s}^{-2} \{t \geq 0\}$

b  $s(0) = 80 \text{ m}$

$v(0) = -48 \text{ m s}^{-1}$

$a(0) = 0.8 \text{ m s}^{-2}$

d  $t = 10 \ln 2$  seconds



14 a i \$535    ii \$1385.79

b i  $-\$0.267 \text{ per km h}^{-1}$     ii  $\$2.33 \text{ per km h}^{-1}$

c  $51.3 \text{ km h}^{-1}$

**15 a**  $v(t) = 3 - \frac{1}{2\sqrt{t+1}}$   $a(t) = \frac{1}{4(t+1)^{\frac{3}{2}}}$



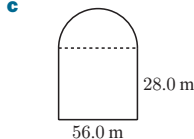
**b**  $x(0) = -1$ ,  $v(0) = 2.5$ ,  $a(0) = 0.25$

Particle is 1 cm to the left of the origin, is travelling to the right at  $2.5 \text{ cm s}^{-1}$ , and accelerating at  $0.25 \text{ cm s}^{-2}$ .

**c** Particle is 21 cm to the right of the origin, is travelling to the right at  $2.83 \text{ cm s}^{-1}$ , and accelerating at  $0.00926 \text{ cm s}^{-2}$ .

**d** never changes direction **e** never decreasing

**16 b**  $A = 200x - 2x^2 - \frac{1}{2}\pi x^2$



**17 a**  $v(0) = 0 \text{ cm s}^{-1}$ ,  $v(\frac{1}{2}) = -\pi \text{ cm s}^{-1}$ ,  $v(1) = 0 \text{ cm s}^{-1}$ ,  
 $v(\frac{3}{2}) = \pi \text{ cm s}^{-1}$ ,  $v(2) = 0 \text{ cm s}^{-1}$

**b**  $0 \leq t \leq 1$ ,  $2 \leq t \leq 3$ ,  $4 \leq t \leq 5$ , etc.

So, for  $2n \leq t \leq 2n+1$ ,  $n \in \{0, 1, 2, 3, \dots\}$

**18**  $x = \frac{k}{2} \left(1 - \frac{1}{\sqrt{3}}\right)$  **19**  $3.60 \text{ m s}^{-1}$

**20** increasing at 0.128 radians per second

**21 a**  $\frac{\sqrt{3}}{2}\pi \text{ cm s}^{-1}$  **b**  $0 \text{ cm s}^{-1}$

**22 a i**  $y = -\frac{a^2}{4b} + \frac{a}{2b}x$  **ii** when  $y = 0$ ,  $x = \frac{a}{2}$

**b i**  $y = -\frac{2b}{a}x + b$  **ii** when  $x = 0$ ,  $y = b$

**iii Hint:** Let  $P''$  be the point on the line  $y = -b$  where the distance to  $P$  is shortest. Show that  $FP = P''P$ .

**c i Hint:** Show that  $\triangle FPP' \cong \triangle P''PP'$ .

**ii Hint:** Show that the tangents meet at  $\left(\frac{a+c}{2}, \frac{ac}{4b}\right)$ .

### EXERCISE 15A.1

**1 a i** 0.6 units<sup>2</sup> **ii** 0.4 units<sup>2</sup> **b** 0.5 units<sup>2</sup>

**2 a** 0.737 units<sup>2</sup> **b** 0.653 units<sup>2</sup>

**3**

$n$	$A_L$	$A_U$
10	2.1850	2.4850
25	2.2736	2.3936
50	2.3034	2.3634
100	2.3184	2.3484
500	2.3303	2.3363

converges to  $\frac{7}{3}$

**4 a i**

$n$	$A_L$	$A_U$
5	0.16000	0.36000
10	0.20250	0.30250
50	0.24010	0.26010
100	0.24503	0.25503
500	0.24900	0.25100
1000	0.24950	0.25050
10000	0.24995	0.25005

**ii**

$n$	$A_L$	$A_U$
5	0.40000	0.60000
10	0.45000	0.55000
50	0.49000	0.51000
100	0.49500	0.50500
500	0.49900	0.50100
1000	0.49950	0.50050
10000	0.49995	0.50005

**iii**

$n$	$A_L$	$A_U$
5	0.54974	0.74974
10	0.61051	0.71051
50	0.65610	0.67610
100	0.66146	0.67146
500	0.66565	0.66765
1000	0.66616	0.66716
10000	0.66662	0.66672

**iv**

$n$	$A_L$	$A_U$
5	0.61867	0.81867
10	0.68740	0.78740
50	0.73851	0.75851
100	0.74441	0.75441
500	0.74893	0.75093
1000	0.74947	0.75047
10000	0.74995	0.75005

**b i**  $\frac{1}{4}$  **ii**  $\frac{1}{2}$  **iii**  $\frac{2}{3}$  **iv**  $\frac{3}{4}$  **c** area =  $\frac{1}{a+1}$

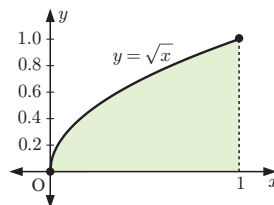
**5 a**

$n$	Rational bounds for $\pi$
10	$2.9045 < \pi < 3.3045$
50	$3.0983 < \pi < 3.1783$
100	$3.1204 < \pi < 3.1604$
200	$3.1312 < \pi < 3.1512$
1000	$3.1396 < \pi < 3.1436$
10000	$3.1414 < \pi < 3.1418$

**b**  $n = 10000$

### EXERCISE 15A.2

**1 a**



**b**

$n$	$A_L$	$A_U$
5	0.5497	0.7497
10	0.6105	0.7105
50	0.6561	0.6761
100	0.6615	0.6715
500	0.6656	0.6676

**c**  $\int_0^1 \sqrt{x} dx \approx 0.67$

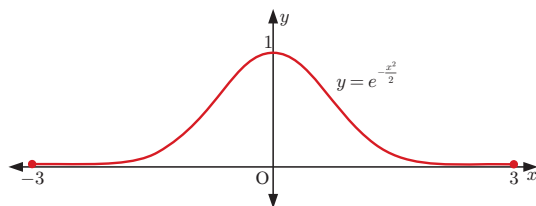
**2 a**  $A_L = \frac{2}{n} \sum_{i=0}^{n-1} \sqrt{1+x_i^3}$ ,  $A_U = \frac{2}{n} \sum_{i=1}^n \sqrt{1+x_i^3}$

**b**

$n$	$A_L$	$A_U$
50	3.2016	3.2816
100	3.2214	3.2614
500	3.2373	3.2453

**c**  $\int_0^2 \sqrt{1+x^3} dx \approx 3.24$

3 a


 b upper  $\approx 1.2506$ , lower  $\approx 1.2493$ 

 c upper  $\approx 1.2506$ , lower  $\approx 1.2493$ 

 d  $\int_{-3}^3 e^{-\frac{x^2}{2}} dx \approx 2.4999$  compared to  $\sqrt{2\pi} \approx 2.5066$ 

4 a 18

b 4.5

 c  $2\pi$ 

## EXERCISE 15B

 1 a i  $\frac{x^2}{2}$  ii  $\frac{x^3}{3}$  iii  $\frac{x^6}{6}$  iv  $-\frac{1}{x}$   
 v  $-\frac{1}{3x^3}$  vi  $\frac{3}{4}x^{\frac{4}{3}}$  vii  $2\sqrt{x}$ 

 b The antiderivative of  $x^n$  is  $\frac{x^{n+1}}{n+1}$  ( $n \neq -1$ ).

 2 a i  $\frac{1}{2}e^{2x}$  ii  $\frac{1}{5}e^{5x}$  iii  $2e^{\frac{1}{2}x}$  iv  $100e^{0.01x}$   
 v  $\frac{1}{\pi}e^{\pi x}$  vi  $3e^{\frac{\pi}{3}}$ 

 b The antiderivative of  $e^{kx}$  is  $\frac{1}{k}e^{kx}$ .

 3 a  $\frac{d}{dx}(x^3 + x^2) = 3x^2 + 2x$   
 $\therefore$  the antiderivative of  $6x^2 + 4x = 2x^3 + 2x^2$ 

 b  $\frac{d}{dx}(e^{3x+1}) = 3e^{3x+1}$   
 $\therefore$  the antiderivative of  $e^{3x+1} = \frac{1}{3}e^{3x+1}$ 

 c  $\frac{d}{dx}(x\sqrt{x}) = \frac{3}{2}\sqrt{x}$   
 $\therefore$  the antiderivative of  $\sqrt{x} = \frac{2}{3}x\sqrt{x}$ 

 d  $\frac{d}{dx}(2x+1)^4 = 8(2x+1)^3$   
 $\therefore$  the antiderivative of  $(2x+1)^3 = \frac{1}{8}(2x+1)^4$ 

## EXERCISE 15C

 1 a  $\frac{1}{4}$  units<sup>2</sup> b  $2\frac{1}{3}$  units<sup>2</sup> c  $\frac{2}{3}$  units<sup>2</sup>  
 3 a  $3\frac{3}{4}$  units<sup>2</sup> b  $24\frac{2}{3}$  units<sup>2</sup> c  $\frac{-2+4\sqrt{2}}{3}$  units<sup>2</sup>  
 d  $\approx 3.48$  units<sup>2</sup> e 2 units<sup>2</sup>

 4 c i  $\int_0^1 (-x^2) dx = -\frac{1}{3}$ , the area between  $y = -x^2$  and the  $x$ -axis from  $x = 0$  to  $x = 1$  is  $\frac{1}{3}$  units<sup>2</sup>.  
 ii  $\int_0^1 (x^2 - x) dx = -\frac{1}{6}$ , the area between  $y = x^2 - x$  and the  $x$ -axis from  $x = 0$  to  $x = 1$  is  $\frac{1}{6}$  units<sup>2</sup>.  
 iii  $\int_{-2}^0 3x dx = -6$ , the area between  $y = 3x$  and the  $x$ -axis from  $x = -2$  to  $x = 0$  is 6 units<sup>2</sup>  
 d  $-\pi$ 

## EXERCISE 15D

 1  $\frac{dy}{dx} = 7x^6$ ,  $\int x^6 dx = \frac{1}{7}x^7 + c$   
 2  $\frac{dy}{dx} = 3x^2 + 2x$ ,  $\int (3x^2 + 2x) dx = x^3 + x^2 + c$ 

 3  $\frac{dy}{dx} = 2e^{2x+1}$ ,  $\int e^{2x+1} dx = \frac{1}{2}e^{2x+1} + c$ 

 4  $\frac{dy}{dx} = 8(2x+1)^3$ ,  $\int (2x+1)^3 dx = \frac{1}{8}(2x+1)^4 + c$ 

 5  $\frac{dy}{dx} = \frac{3}{2}\sqrt{x}$ ,  $\int \sqrt{x} dx = \frac{2}{3}x\sqrt{x} + c$ 

 6  $\frac{dy}{dx} = -\frac{1}{2x\sqrt{x}}$ ,  $\int \frac{1}{x\sqrt{x}} dx = -\frac{2}{\sqrt{x}} + c$ 

 7  $\frac{dy}{dx} = -2\sin 2x$ ,  $\int \sin 2x dx = -\frac{1}{2}\cos 2x + c$ 

 8  $\frac{dy}{dx} = -5\cos(1-5x)$ ,  
 $\int \cos(1-5x) dx = -\frac{1}{5}\sin(1-5x) + c$ 

 9  $\int (2x-1)(x^2-x)^2 dx = \frac{1}{3}(x^2-x)^3 + c$ 

 11  $\frac{dy}{dx} = \frac{-2}{\sqrt{1-4x}}$ ,  $\int \frac{1}{\sqrt{1-4x}} dx = -\frac{1}{2}\sqrt{1-4x} + c$ 

## EXERCISE 15E.1

 1 a  $\frac{x^5}{5} - \frac{x^3}{3} - \frac{x^2}{2} + 2x + c$  b  $x^5 - x^4 - 2x^3 - 7x + c$ 

 c  $\frac{2}{3}x^{\frac{3}{2}} + e^x + c$  d  $3e^x + \frac{1}{3}x^3 + c$ 

 e  $\frac{2}{5}x^{\frac{5}{2}} - 2x + c$  f  $-2x^{-\frac{1}{2}} + 2x^2 + c$ 

 g  $\frac{1}{8}x^4 - \frac{1}{5}x^5 + \frac{3}{4}x^{\frac{4}{3}} + c$  h  $\frac{x^2}{4} + \frac{1}{3}x^3 - e^x + c$ 

 i  $5e^x + \frac{1}{12}x^4 - \frac{2}{3}x^{\frac{3}{2}} + c$ 

 2 a  $-3\cos x - 2x + c$  b  $2x^2 - 2\sin x + c$ 

 c  $-\cos x - 2\sin x + e^x + c$  d  $\frac{2}{7}x^3\sqrt{x} + 10\cos x + c$ 

 e  $\frac{1}{9}x^3 - \frac{1}{6}x^2 + \sin x + c$  f  $\cos x + \frac{4}{3}x\sqrt{x} + c$ 

 3 a  $\frac{1}{3}x^3 + \frac{3}{2}x^2 - 2x + c$  b  $\frac{2}{3}x^{\frac{3}{2}} - 2x^{\frac{1}{2}} + c$ 

 c  $2e^x + \frac{1}{x} + c$  d  $-2x^{-\frac{1}{2}} - 8x^{\frac{1}{2}} + c$ 

 e  $\frac{4}{3}x^3 + 2x^2 + x + c$  f  $\frac{1}{3}x^3 + 2x - \frac{1}{x} + c$ 

 g  $\frac{4}{3}x^{\frac{3}{2}} - 2x^{\frac{1}{2}} + c$  h  $2x^{\frac{1}{2}} + 8x^{-\frac{1}{2}} - \frac{20}{3}x^{-\frac{3}{2}} + c$ 

 i  $\frac{1}{4}x^4 + x^3 + \frac{3}{2}x^2 + x + c$ 

 4 a  $\frac{2}{3}x^{\frac{3}{2}} + \frac{1}{2}\sin x + c$  b  $2e^t + 4\cos t + c$ 

 c  $3\sin t + \cos t + c$ 

 5 a  $y = 6x + c$  b  $y = \frac{4}{3}x^3 + c$ 

 c  $y = \frac{10}{3}x\sqrt{x} - \frac{1}{3}x^3 + c$  d  $y = -\frac{1}{x} + c$ 

 e  $y = 2e^x - 5x + c$  f  $y = x^4 + x^3 + c$ 

 6 a  $f(x) = x - 2x^2 + \frac{4}{3}x^3 + c$ 

 b  $f(x) = \frac{2}{3}x^{\frac{3}{2}} - 4\sqrt{x} + c$  c  $f(x) = x + \frac{5}{x} + c$ 

## EXERCISE 15E.2

 1 a  $f(x) = x^2 - x + 3$  b  $f(x) = x^3 + x^2 - 7$ 

 c  $f(x) = e^x + 2\sqrt{x} - 1 - e$  d  $f(x) = \frac{1}{2}x^2 - 4\sqrt{x} + \frac{11}{2}$ 

 2 a  $f(x) = \frac{x^3}{3} - 4\sin x + 3$ 

 b  $f(x) = 2\sin x + 3\cos x - 2\sqrt{2}$

- 3 a  $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 + x + \frac{1}{3}$   
 b  $f(x) = 4x^{\frac{5}{2}} + 4x^{\frac{3}{2}} - 4x + 5$   
 c  $f(x) = -\cos x - x + 4$  d  $f(x) = \frac{1}{3}x^3 - \frac{16}{3}x + 5$

## EXERCISE 15F

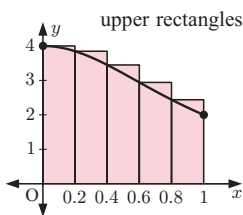
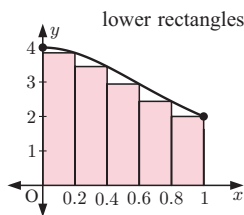
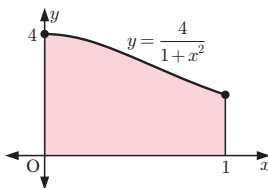
- 1 a  $\frac{1}{8}(2x+5)^4 + c$  b  $\frac{1}{2(3-2x)} + c$   
 c  $\frac{-2}{3(2x-1)^3} + c$  d  $\frac{1}{32}(4x-3)^8 + c$   
 e  $\frac{2}{9}(3x-4)^{\frac{3}{2}} + c$  f  $-4\sqrt{1-5x} + c$   
 g  $-\frac{3}{5}(1-x)^5 + c$  h  $-2\sqrt{3-4x} + c$   
 2 a  $-\frac{1}{3}\cos(3x) + c$  b  $-\frac{1}{2}\sin(-4x) + x + c$   
 c  $6\sin\left(\frac{x}{2}\right) + c$  d  $-\frac{3}{2}\cos(2x) + e^{-x} + c$   
 e  $-\cos\left(2x + \frac{\pi}{6}\right) + c$  f  $3\sin\left(\frac{\pi}{4} - x\right) + c$   
 g  $\frac{1}{2}\sin(2x) - \frac{1}{2}\cos(2x) + c$   
 h  $-\frac{2}{3}\cos(3x) + \frac{5}{4}\sin(4x) + c$   
 i  $\frac{1}{16}\sin(8x) + 3\cos x + c$   
 3  $y = \frac{1}{3}(2x-7)^{\frac{3}{2}} + 2$  4  $(-8, -19)$   
 5 a  $\frac{1}{2}(2x-1)^3 + c$  b  $\frac{1}{5}x^5 - \frac{1}{2}x^4 + \frac{1}{3}x^3 + c$   
 c  $-\frac{1}{12}(1-3x)^4 + c$  d  $x - \frac{2}{3}x^3 + \frac{1}{5}x^5 + c$   
 e  $-\frac{8}{3}(5-x)^{\frac{3}{2}} + c$  f  $\frac{1}{7}x^7 + \frac{3}{5}x^5 + x^3 + x + c$   
 6 a  $2e^x + \frac{5}{2}e^{2x} + c$  b  $\frac{3}{5}e^{5x-2} + c$   
 c  $-\frac{1}{3}e^{7-3x} + c$  d  $\frac{1}{2}e^{2x} + 2x - \frac{1}{2}e^{-2x} + c$   
 e  $-\frac{1}{2}e^{-2x} - 4e^{-x} + 4x + c$  f  $\frac{1}{2}x^2 + 5(1-x)^{-1} + c$   
 7  $y = x - 2e^x + \frac{1}{2}e^{2x} + \frac{11}{2}$   
 8  $p = -\frac{1}{4}$ ,  $f(x) = \frac{1}{2}\cos\left(\frac{1}{2}x\right) + \frac{1}{2}$   
 10  $f(x) = -e^{-2x} + 4$   
 11  $f(x) = \frac{2}{3}x^{\frac{3}{2}} - \frac{1}{8}e^{-4x} + \frac{1}{8}e^{-4} - \frac{2}{3}$

## EXERCISE 15G

- 1 a  $\int_1^4 \sqrt{x} dx = \frac{14}{3}$ ,  $\int_1^4 (-\sqrt{x}) dx = -\frac{14}{3}$   
 b  $\int_0^1 x^7 dx = \frac{1}{8}$ ,  $\int_0^1 (-x^7) dx = -\frac{1}{8}$   
 2 a  $\frac{1}{3}$  b  $\frac{7}{3}$  c  $\frac{8}{3}$  d 1  
 3 a -4 b 6.25 c 2.25 4 a  $\frac{1}{3}$  b  $\frac{2}{3}$  c 1  
 5 a  $\frac{1}{4}$  b  $\frac{2}{3}$  c  $e-1$  ( $\approx 1.72$ ) d  $\frac{1}{2}$   
 e  $1\frac{1}{2}$  f  $6\frac{2}{3}$  g  $\ln 3$  ( $\approx 1.10$ ) h  $\frac{1}{2}$   
 i  $\approx 1.52$  j 2 k  $e-1$  ( $\approx 1.72$ ) l  $\frac{1}{3}$   
 6  $m = -1$  or  $\frac{4}{3}$  7 a  $\frac{\pi}{8} + \frac{1}{4}$  b  $\frac{\pi}{4}$   
 8 a 6.5 b -9 c 0 d -2.5  
 9 a  $2\pi$  b -4 c  $\frac{\pi}{2}$  d  $\frac{5\pi}{2} - 4$   
 10 a  $\int_2^7 f(x) dx$  b  $\int_1^9 g(x) dx$   
 11 a -5 b 4  
 12 a 4 b 0 c -8 d  $k = -\frac{7}{4}$  13 0

## REVIEW SET 15A

1 a



n	$A_L$	$A_U$
5	2.9349	3.3349
50	3.1215	3.1615
100	3.1316	3.1516
500	3.1396	3.1436

c  $\int_0^1 \frac{4}{1+x^2} dx \approx 3.1416$

2 a  $2\pi$ 

b 4

3 a  $8\sqrt{x} + c$  b  $-\frac{1}{4}\cos(4x-5) + c$  c  $-\frac{1}{3}e^{4-3x} + c$ 4 a  $12\frac{4}{9}$  b  $\sqrt{2}$ 

5  $\frac{dy}{dx} = \frac{x}{\sqrt{x^2-4}}$ ,  $\int \frac{x}{\sqrt{x^2-4}} dx = \sqrt{x^2-4} + c$

6  $b = \frac{\pi}{4}, \frac{3\pi}{4}$

7 a  $y = \frac{1}{5}x^5 - \frac{2}{3}x^3 + x + c$  b  $y = 400x + 40e^{-\frac{x}{2}} + c$

8  $f(x) = 3x^3 + 5x^2 + 6x - 1$  9  $a = \ln \sqrt{2}$

10 a  $f(x) = \frac{1}{4}x^4 + \frac{1}{3}x^3 - \frac{10}{3}x + 3$  b  $3x + 26y = 84$

11 a  $e^{3x} + 6e^{2x} + 12e^x + 8$  b  $\frac{1}{3}e^3 + 3e^2 + 12e - 7\frac{1}{3}$

## REVIEW SET 15B

1 a  $A = \frac{17}{4}$ ,  $B = \frac{25}{4}$  b  $\int_0^2 (4-x^2) dx \approx \frac{21}{4}$

2 a  $-2e^{-x} + 3x + c$  b  $\frac{2}{3}x\sqrt{x} - 2\sqrt{x} + c$   
 c  $9x + 3e^{2x-1} + \frac{1}{4}e^{4x-2} + c$

3  $f(x) = \frac{1}{3}x^3 - \frac{3}{2}x^2 + 2x + 2\frac{1}{6}$  4  $\frac{2}{3}(\sqrt{5} - \sqrt{2})$

5  $\frac{d}{dx}(3x^2+x)^3 = 3(3x^2+x)^2(6x+1)$   
 $\int (3x^2+x)^2(6x+1) dx = \frac{1}{3}(3x^2+x)^3 + c$

6 a 6 b 3 7  $f\left(\frac{\pi}{2}\right) = 3 - \frac{\pi}{2}$  8  $e^{-\pi}$

9  $\frac{1}{2(n+1)}(2x+3)^{n+1} + c$ ,  $n \neq -1$

10  $a = \frac{1}{3}$ ,  $f'(x) = 2\sqrt{x} + \frac{1}{3\sqrt{x}}$  is never 0 as  $\sqrt{x} \geq 0$  for all  $x$   
 $\therefore f'(x) > 0$  for all  $x$

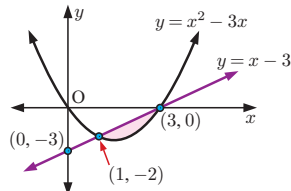
11  $a = 0$  or  $\pm 3$

## EXERCISE 16A

- 1** a 30 units<sup>2</sup>    b  $\frac{9}{2}$  units<sup>2</sup>    c  $\frac{27}{2}$  units<sup>2</sup>    d 2 units<sup>2</sup>  
**2** a  $\frac{1}{3}$  units<sup>2</sup>    b 2 units<sup>2</sup>    c  $63\frac{3}{4}$  units<sup>2</sup>  
      d  $(e-1)$  units<sup>2</sup>    e  $20\frac{5}{6}$  units<sup>2</sup>    f 18 units<sup>2</sup>  
      g  $\frac{1}{2}$  units<sup>2</sup>    h  $4\frac{1}{2}$  units<sup>2</sup>    i  $(2e - \frac{2}{e})$  units<sup>2</sup>  
**3**  $\frac{2}{3}$  units<sup>2</sup>

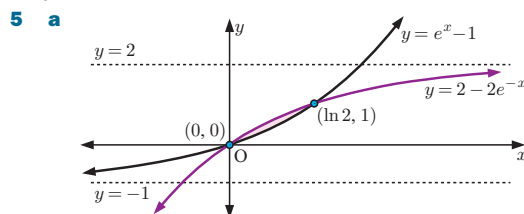
## EXERCISE 16B

- 1** a  $4\frac{1}{2}$  units<sup>2</sup>    b  $(1 + e^{-2})$  units<sup>2</sup>    c  $1\frac{5}{27}$  units<sup>2</sup>  
      d 2 units<sup>2</sup>    e  $2\frac{1}{4}$  units<sup>2</sup>    f  $(\frac{\pi}{2} - 1)$  units<sup>2</sup>  
**2**  $10\frac{2}{3}$  units<sup>2</sup>  
**3** a



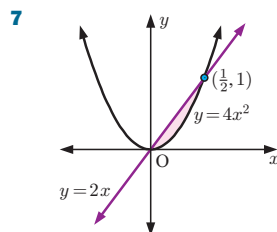
- b (1, -2) and (3, 0)  
 c  $1\frac{1}{3}$  units<sup>2</sup>

- 4**  $\frac{1}{3}$  units<sup>2</sup>

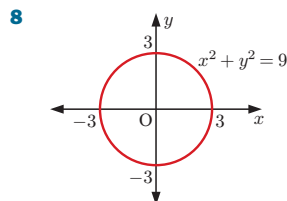


- b (0, 0) and (ln 2, 1)  
 c enclosed area =  $3 \ln 2 - 2$  ( $\approx 0.0794$ ) units<sup>2</sup>

- 6**  $\frac{1}{2}$  units<sup>2</sup>



enclosed area =  $\frac{1}{12}$  units<sup>2</sup>



- 8** a Rearranging  $x^2 + y^2 = 9$  gives  $y = \pm\sqrt{9 - x^2}$ . The upper half has  $y \geq 0$ , so  $y = \sqrt{9 - x^2}$ .  
 b  $\frac{9\pi}{4}$

- 9** a  $40\frac{1}{2}$  units<sup>2</sup>    b 8 units<sup>2</sup>    c 8 units<sup>2</sup>

- 10** a  $C_1$  is  $y = \sin x$ ,  $C_2$  is  $y = 3 \sin x$     b 4 units<sup>2</sup>

- 11** a  $\int_3^5 f(x) dx = -$  (area between  $x = 3$  and  $x = 5$ )

b  $\int_1^3 f(x) dx - \int_3^5 f(x) dx + \int_5^7 f(x) dx$

- 12** a  $C_1$  is  $y = \frac{1}{2} + \frac{1}{2} \cos(2x)$ ,  $C_2$  is  $y = \cos(2x)$

- b A(0, 1), B( $\frac{\pi}{4}$ , 0), C( $\frac{\pi}{2}$ , 0), D( $\frac{3\pi}{4}$ , 0), E( $\pi$ , 1)

c Area =  $\int_0^\pi (\frac{1}{2} + \frac{1}{2} \cos(2x) - \cos(2x)) dx$

- 13** If  $h(x) \geq 0$  on  $a \leq x \leq b$ , the area between  $y = h(x)$  and the  $x$ -axis is  $\int_a^b h(x) dx$ . If  $h(x) < 0$  on  $a \leq x \leq b$ , the area between  $y = h(x)$  and the  $x$ -axis is  $\int_a^b -h(x) dx$ .  
 $\therefore$  the area between  $y = h(x)$  and the  $x$ -axis on  $a \leq x \leq b$  is  $\int_a^b |h(x)| dx$ .

Letting  $h(x) = f(x) - g(x)$ , the area between  $y = f(x) - g(x)$  and the  $x$ -axis  $y = 0$  on  $a \leq x \leq b$  is  $\int_a^b |f(x) - g(x)| dx$ .

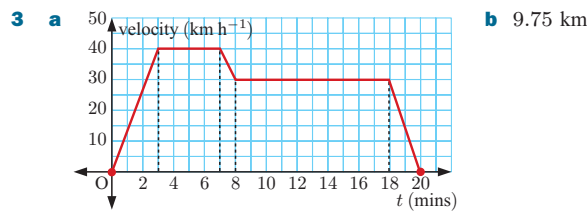
Equivalently, the area between  $y = f(x)$  and  $y = g(x)$  on  $a \leq x \leq b$  is  $\int_a^b |f(x) - g(x)| dx$ .

- 14**  $b \approx 1.3104$

- 15**  $a = \sqrt{3}$

## EXERCISE 16C.1

- 1** 110 m  
**2** a i travelling forwards  
      ii travelling backwards (opposite direction)  
 b 16 km    c 8 km from starting point (on positive side)



## EXERCISE 16C.2

- 1** a  $s(t) = t - t^2 + 2$  cm    b  $\frac{1}{2}$  cm    c 0 cm

- 2** a  $s(t) = \frac{1}{3}t^3 - \frac{1}{2}t^2 - 2t$  cm    b  $5\frac{1}{6}$  cm  
 c  $1\frac{1}{2}$  cm left of its starting point

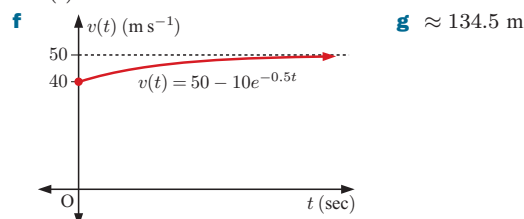
- 3**  $\frac{\sqrt{3}+2}{4}$  m

- 4** a  $s(t) = 32t + 2t^2 + 16$  m  
 b no change of direction  
    so displacement =  $s(t_1) - s(0) = \int_0^{t_1} (32 + 4t) dt$   
 c acceleration =  $4 \text{ m s}^{-2}$

- 5** a 41 units    b 34 units    **6** b 2 m

- 7** a  $40 \text{ m s}^{-1}$     b  $47.8 \text{ m s}^{-1}$     c 1.39 seconds

- d as  $t \rightarrow \infty$ ,  $v(t) \rightarrow 50$  from below  
 e  $a(t) = 5e^{-0.5t}$  and as  $e^x > 0$  for all  $x$ ,  
 $a(t) > 0$  for all  $t$ .



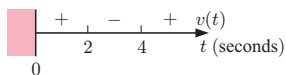
- 8** a  $v(t) = -\frac{1}{(t+1)^2} + 1 \text{ m s}^{-1}$

- b  $s(t) = \frac{1}{t+1} + t - 1 \text{ m}$

- c** The particle is  $\frac{4}{3}$  m to the right of the origin, moving to the right at  $\frac{8}{9} \text{ m s}^{-1}$ , and accelerating at  $\frac{2}{27} \text{ m s}^{-2}$ .
- 9 a**  $v(t) = \frac{t^2}{20} - 3t + 45 \text{ m s}^{-1}$
- b**  $\int_0^{60} v(t) dt = 900$ . The train travels a total of 900 m in the first 60 seconds.
- 10 a** Show that  $v(t) = 100 - 80e^{-\frac{1}{20}t} \text{ m s}^{-1}$  and as  $t \rightarrow \infty$ ,  $v(t) \rightarrow 100 \text{ m s}^{-1}$ .
- b** 370.4 m

**REVIEW SET 16A**

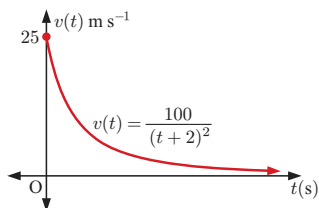
- 1**  $A = \int_a^b [f(x) - g(x)] dx + \int_b^c [g(x) - f(x)] dx$   
 $+ \int_c^d [f(x) - g(x)] dx$
- 2 a**  $2 + \pi$       **b**  $-2$       **c**  $\pi$
- 3** No, total area shaded  $= \int_{-1}^1 f(x) dx - \int_1^3 f(x) dx$ .
- 4**  $k = \sqrt[3]{16}$       **5**  $4.5 \text{ units}^2$
- 6 a**



- b** The particle moves in the positive direction initially, then at  $t = 2$ ,  $6\frac{2}{3}$  m from its starting point, it changes direction. It changes direction again at  $t = 4$ ,  $5\frac{1}{3}$  m from its starting point, and at  $t = 5$ , it is  $6\frac{2}{3}$  m from its starting point again.
- c**  $6\frac{2}{3} \text{ m}$       **d**  $9\frac{1}{3} \text{ m}$
- 7**  $(3 - \ln 4) \text{ units}^2$       **8**  $2.35 \text{ m}$

**REVIEW SET 16B**

- 1 a**  $v(t) = 3t^2 - 30t + 27 \text{ cm s}^{-1}$
- b**  $-162 \text{ cm}$  (162 cm to the left of the origin)
- 2 a**
- c**  $(1 - \frac{\pi}{4}) \text{ units}^2$
- 3**  $a = \ln 3$ ,  $b = \ln 5$
- 4 a**  $a(t) = 2 - 6t \text{ m s}^{-2}$       **b**  $s(t) = t^2 - t^3 + c \text{ m}$
- c**  $-4 \text{ m}$  (4 m to the left)
- 5**  $k = \frac{4}{3}$       **6**  $m = \frac{\pi}{3}$       **7**  $(\frac{4}{\pi} - 1) \text{ units}^2$
- 8 a**  $v(0) = 25 \text{ m s}^{-1}$ ,  $v(3) = 4 \text{ m s}^{-1}$
- b** as  $t \rightarrow \infty$ ,  $v(t) \rightarrow 0$  from above
- c**



- d** 3 seconds      **e**  $a(t) = \frac{-200}{(t+2)^3} \text{ m s}^{-2}$ ,  $t \geq 0$
- f**  $k = \frac{1}{5}$

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