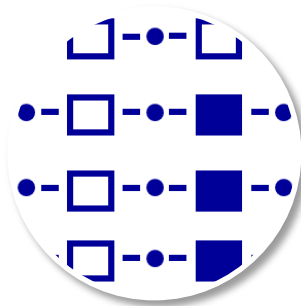


Regulation Analysis using Restricted Boltzmann Machines

iBIOS, 2/2/2012

Patrick Michl
Heidelberg University



Biological Problem

Analysing the regulation of metabolism



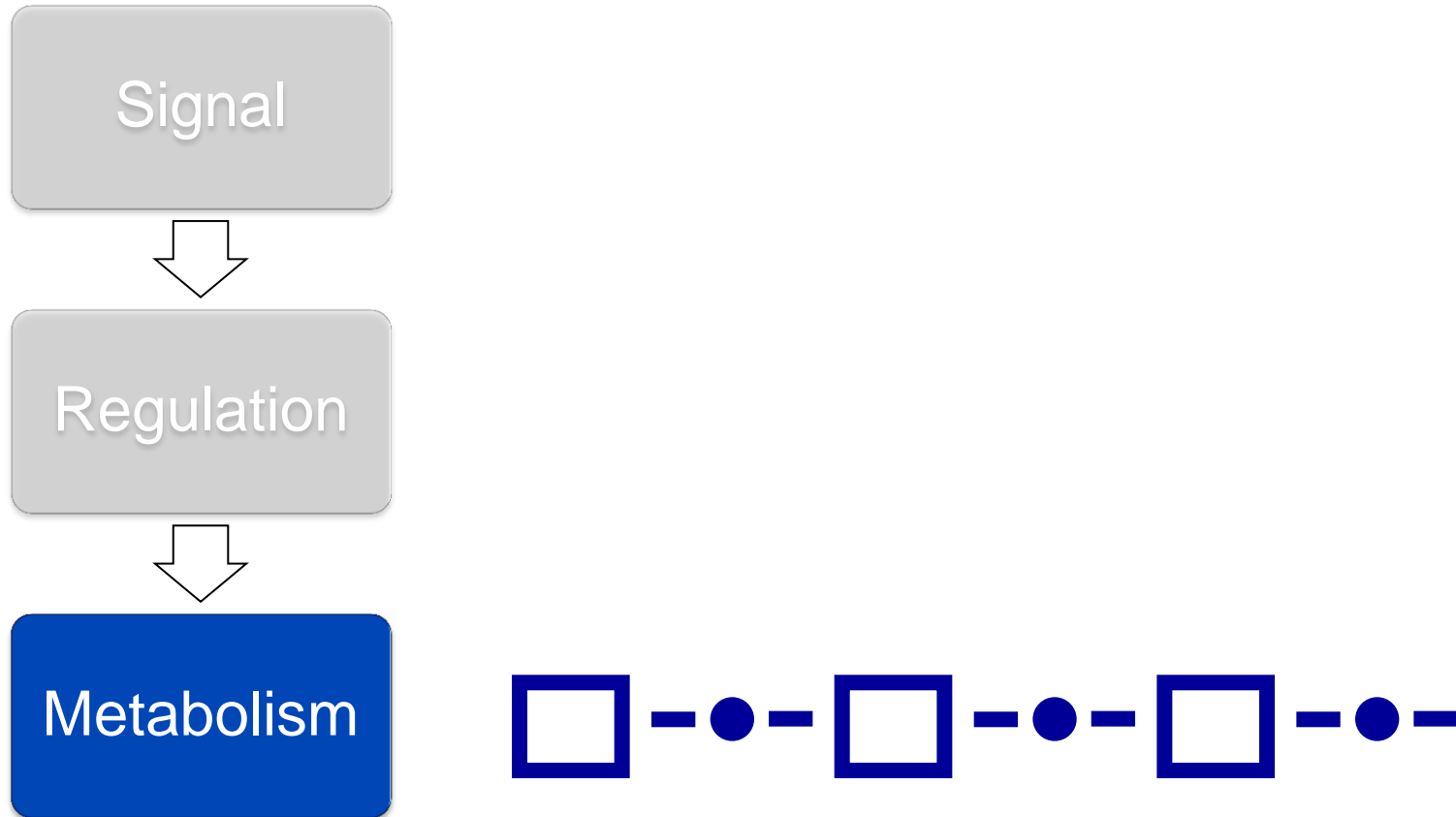
Machine Learning



Implementation

Biological Problem

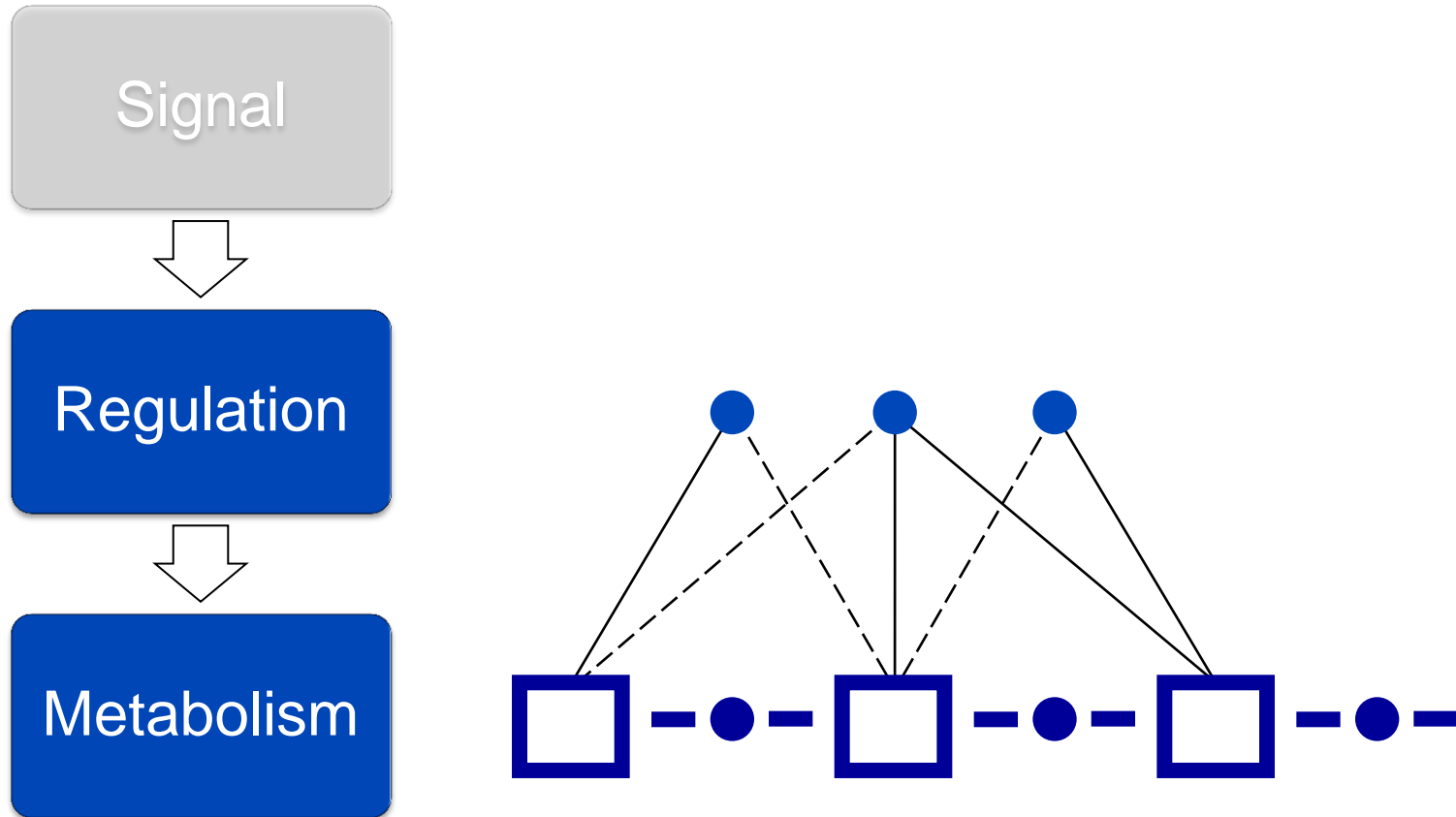
Analysing the regulation of metabolism



A linear metabolic pathway of enzymes (E) ...

Biological Problem

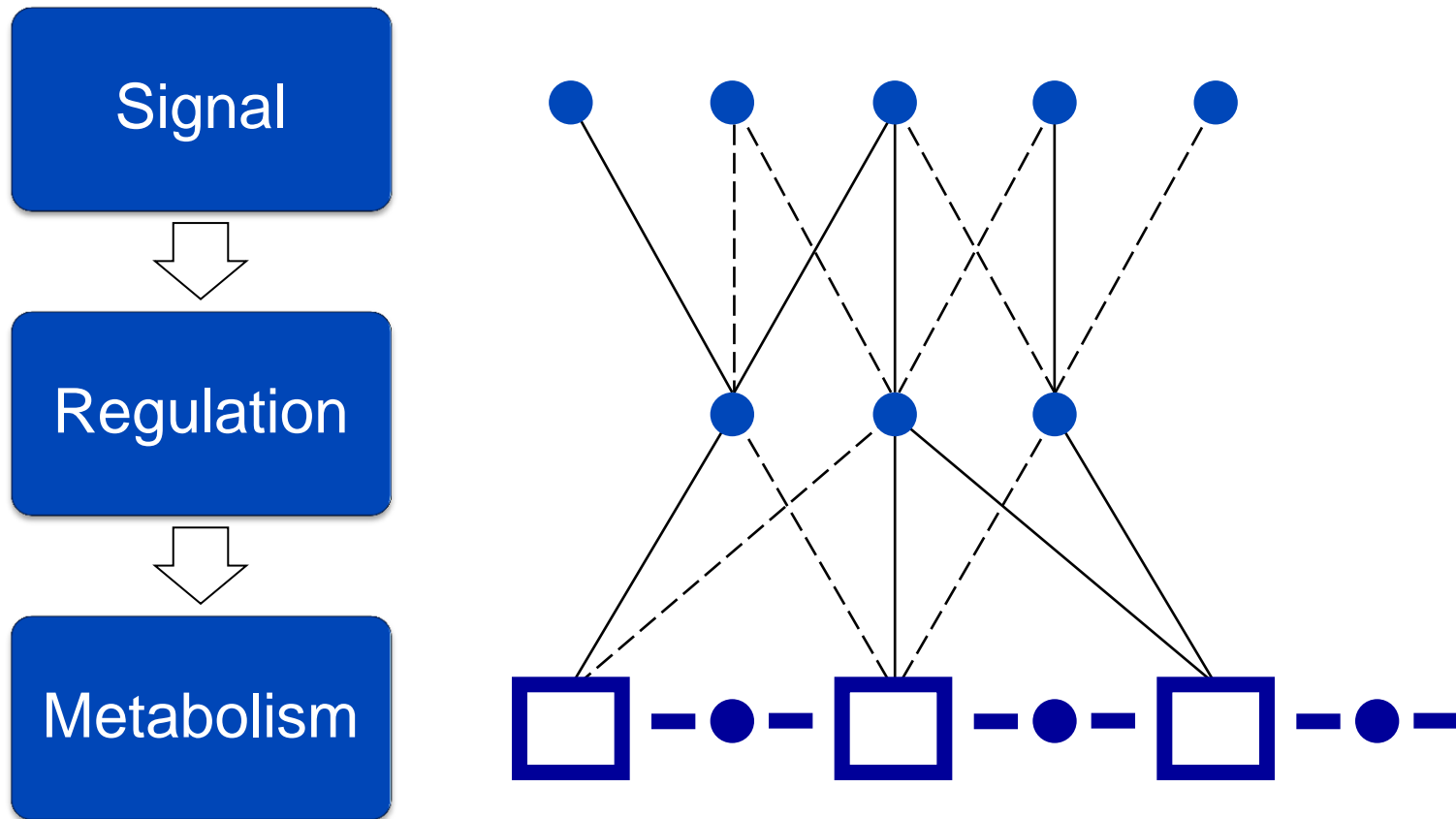
Analysing the regulation of metabolism



... is regulated by **transcription factors (TF)** ...

Biological Problem

Analysing the regulation of metabolism



... which respond to **signals** (S)

Biological Problem

Analysing the regulation of metabolism

P 4

P 3

P 2

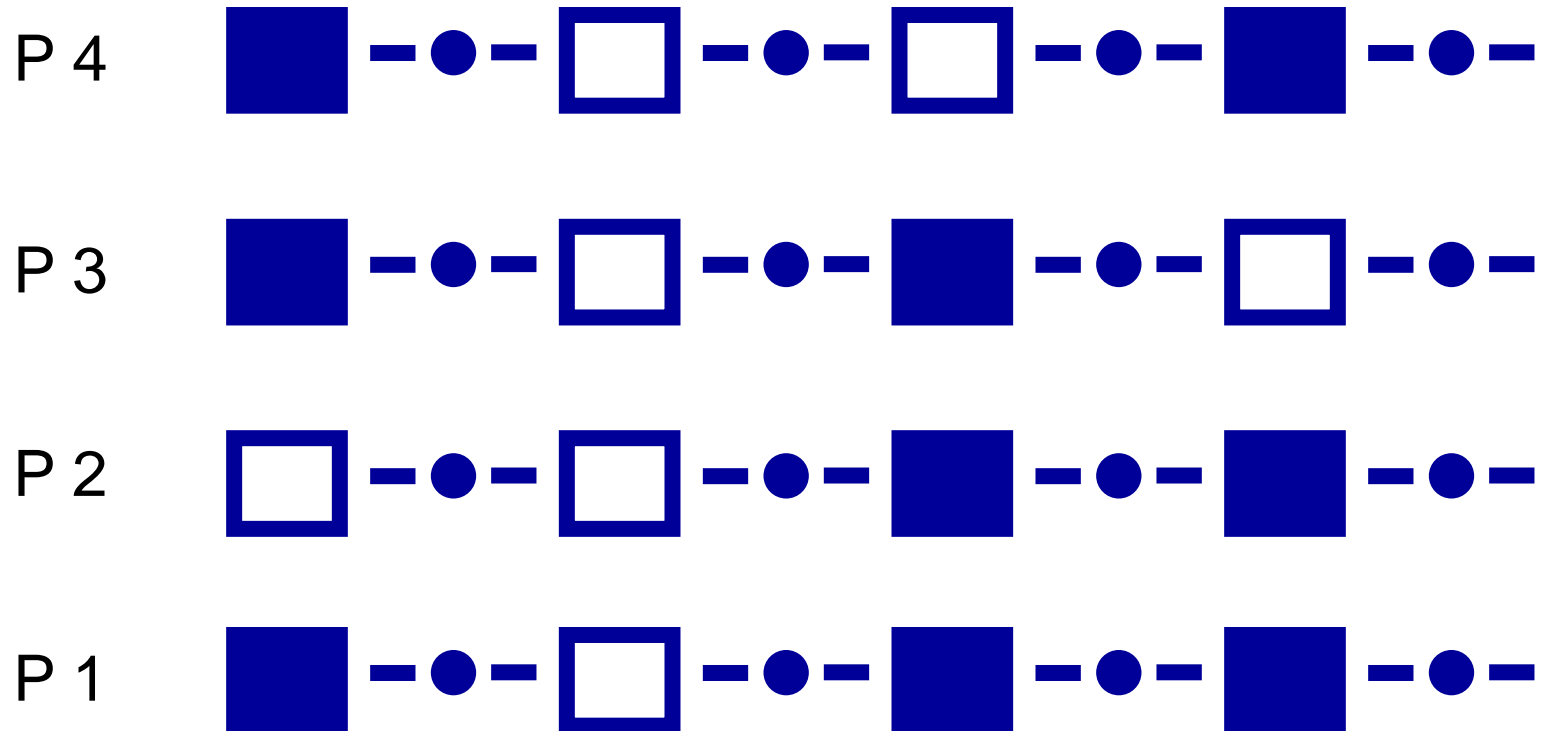
P 1



Upregulated linear pathways ...

Biological Problem

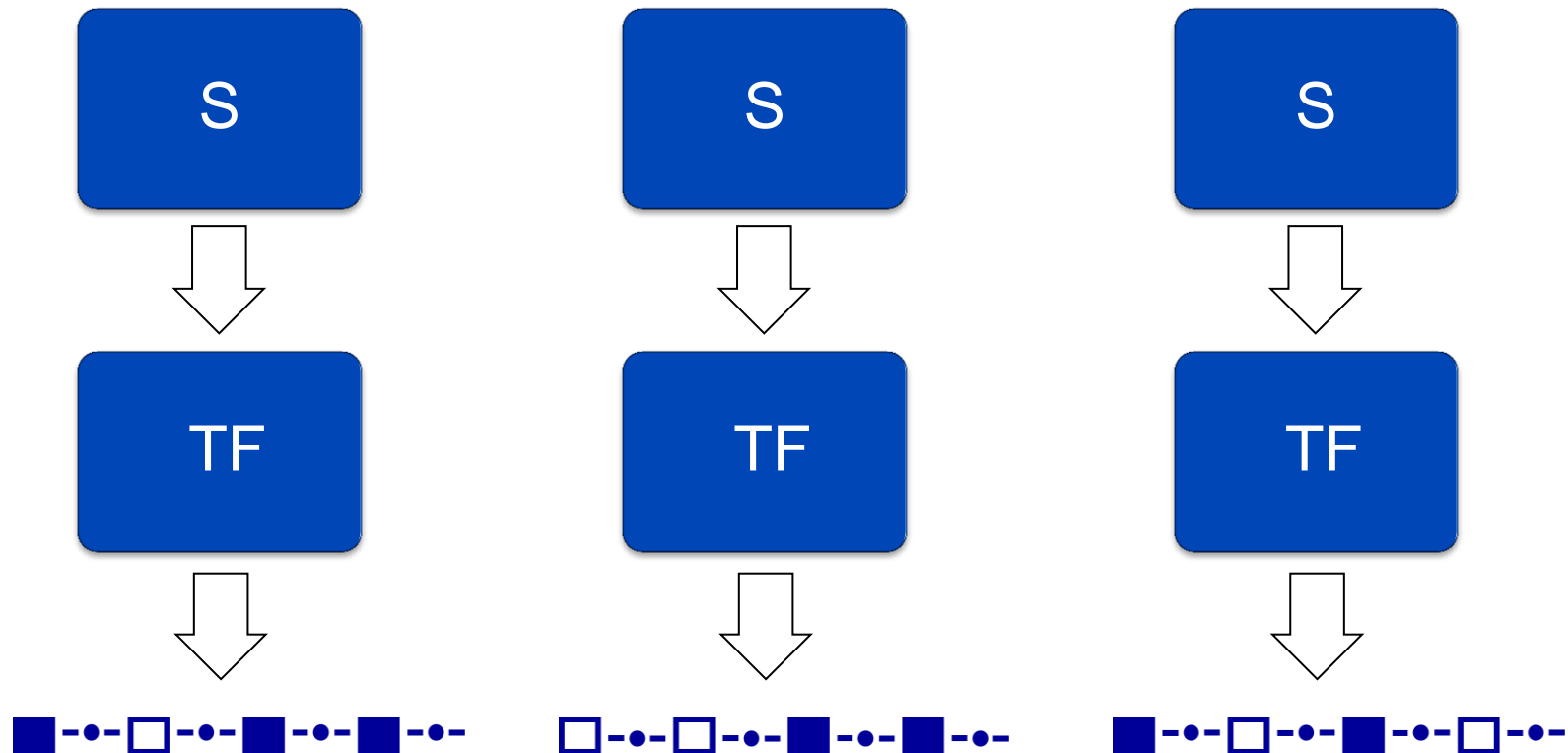
Analysing the regulation of metabolism



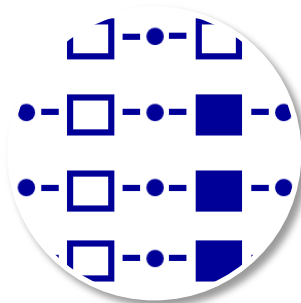
... can appear in **different patterns**

Biological Problem

Analysing the regulation of metabolism

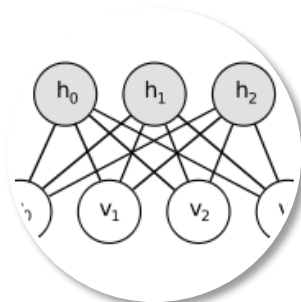


Which transcription factors and signals cause this patterns?



Biological Problem

Analysing the regulation of metabolism

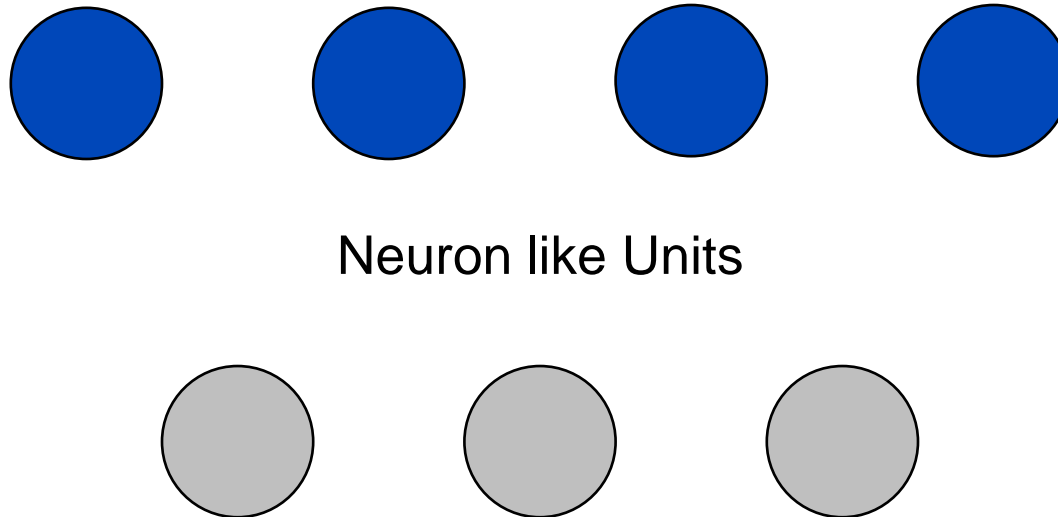


Machine Learning

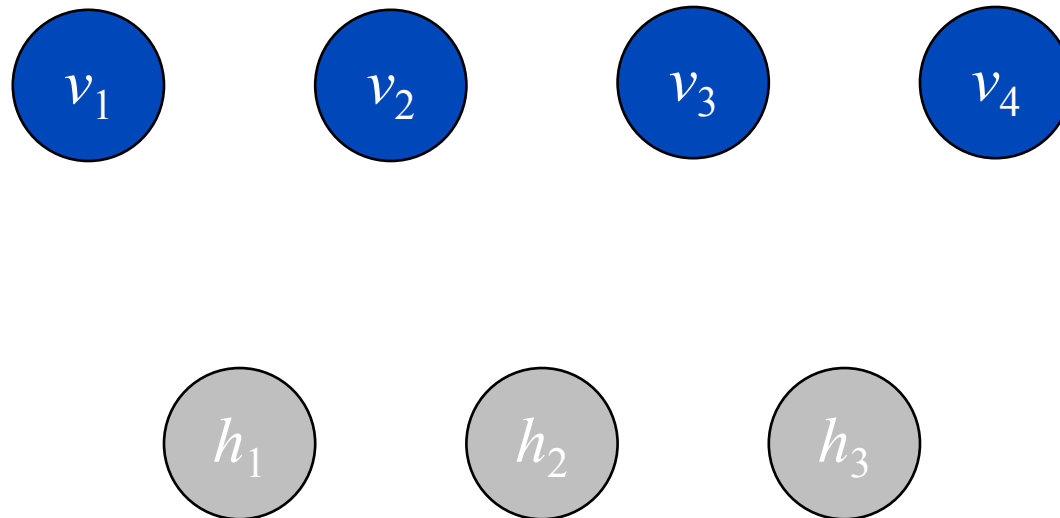
Restricted Boltzmann Machines (RBM)



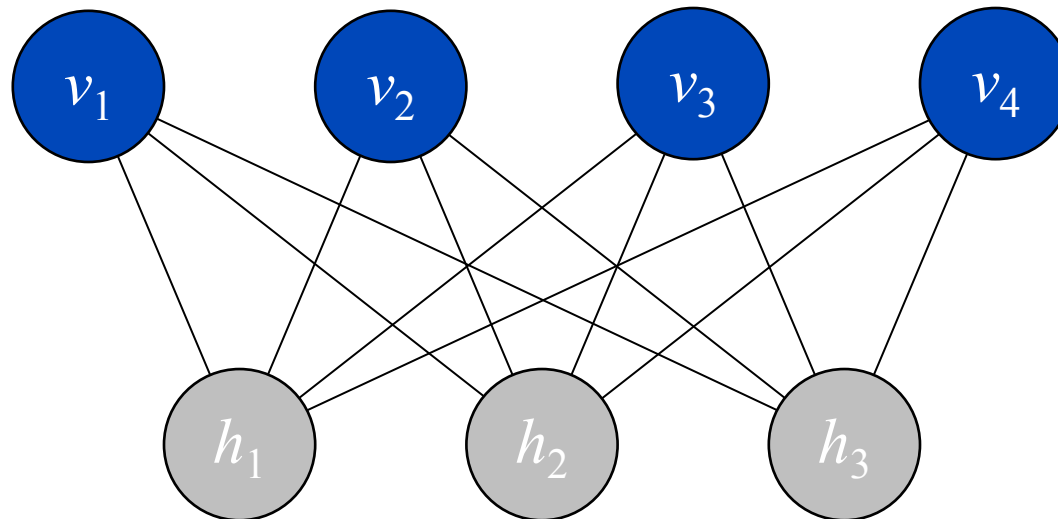
Implementation



A Restricted Boltzmann Machine (RBM) is an **ANN** ...



... with two layers: **visible units** (v) and **hidden units** (h)



Visible units are **strictly** connected with hidden units

V := set of visible units

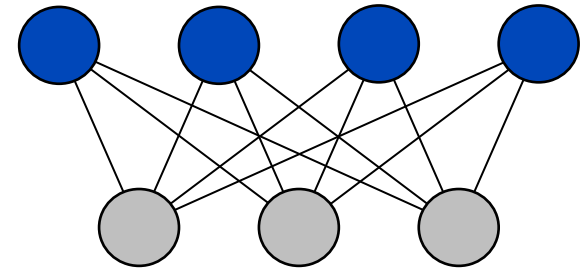
H := set of hidden units

s_v := value of v , $\forall v \in V$

s_h := value of h , $\forall h \in H$

$s_v \in \{0, 1\}$, $\forall v \in V$

$s_h \in \{0, 1\}$, $\forall h \in H$



In the most common model all units have **binary values** ...

V := set of visible units

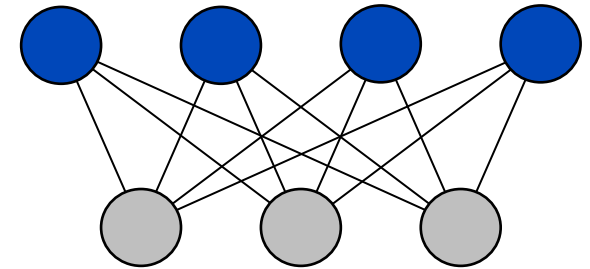
H := set of hidden units

s_v := value of v , $\forall v \in V$

s_h := value of h , $\forall h \in H$

$s_v \in \{0, 1\}$, $\forall v \in V$

$s_h \in \{0, 1\}$, $\forall h \in H$



θ_v := threshold of v , $\forall v \in V$

θ_h := threshold of h , $\forall h \in H$

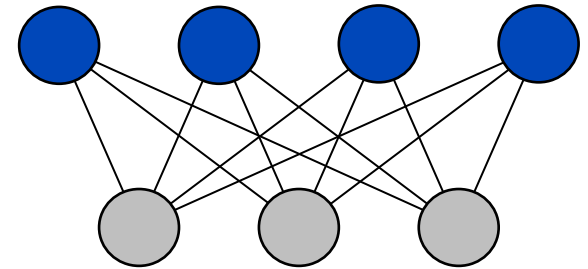
... and **arbitrary thresholds** which allow us ...

$w_{v,h} := \text{weight of } edge(v,h)$

Local Energy

$$E_v := - \sum_h w_{v,h} s_v s_h + \theta_v s_v$$

$$E_h := - \sum_v w_{v,h} s_v s_h + \theta_h s_h$$



... to define **energy** functions: **Local** energies E_v and E_h ...

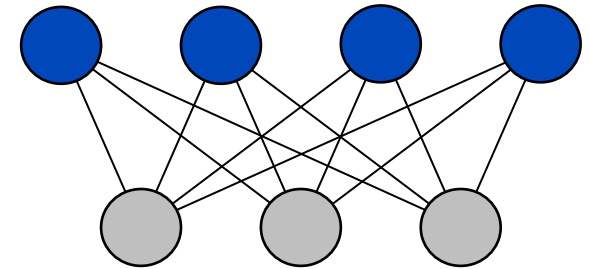
$w_{v,h} := \text{weight of } edge(v,h)$

Local Energy

$$E_v := - \sum_h w_{v,h} s_v s_h + \theta_v s_v$$

1

$$E_h := - \sum_v w_{v,h} s_v s_h + \theta_h s_h$$



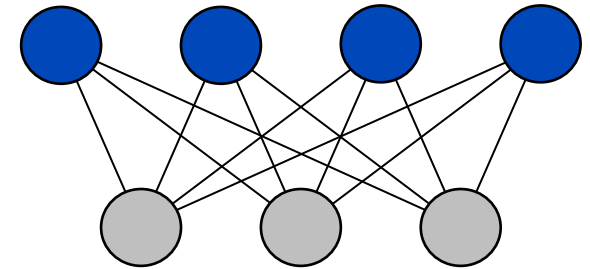
Global Energy

$$E := \sum_v E_v + \sum_h E_h$$

... and the **global** Energy E . We want to minimize E

Energy Delta for visible units

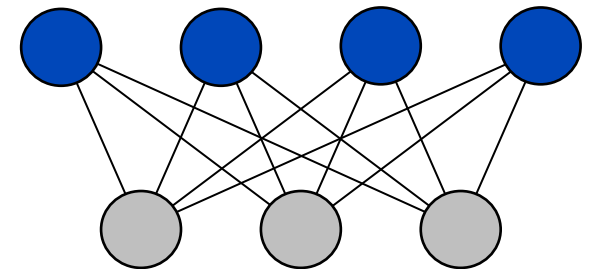
$$\begin{aligned}\Delta E_v &= E_{v, off} - E_{v, on} \\ &= \dots\end{aligned}$$



If we approximate a **Boltzmann Distribution** for $E_v \dots$

Energy Delta for visible units

$$\begin{aligned}\Delta E_v &= E_{v, off} - E_{v, on} \\ &= -k_B T \ln P[v, off] \\ &\quad - (-k_B T \ln P[v, on])\end{aligned}$$

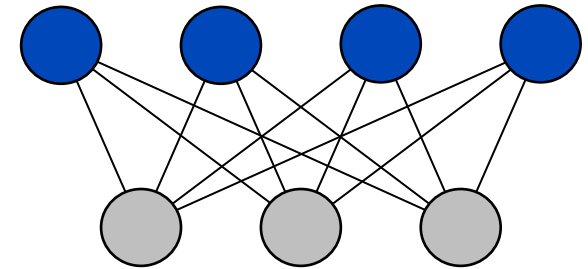


... we can use the **Boltzmann Factor** ...

Energy Delta for visible units

$$\begin{aligned}\Delta E_v &= E_{v, off} - E_{v, on} \\ &= -k_B T \ln P[v, off] \\ &\quad - (-k_B T \ln P[v, on])\end{aligned}$$

$$P[v, off] = 1 - P[v, on]$$



$$P[v, on] = \frac{1}{1 + e^{-\frac{\Delta E_v}{k_B T}}}$$

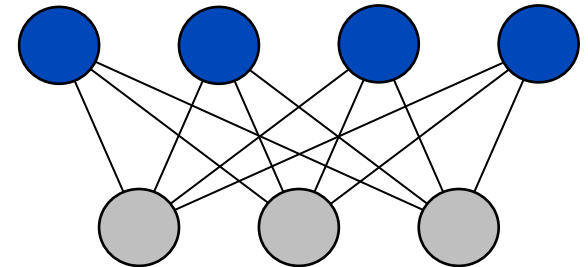
... to get a term for the **probability** [v, on]

Probabilities

2

$$P[v, \text{on}] = \frac{1}{1 + e^{-\frac{\Delta E v}{k_B T}}}$$

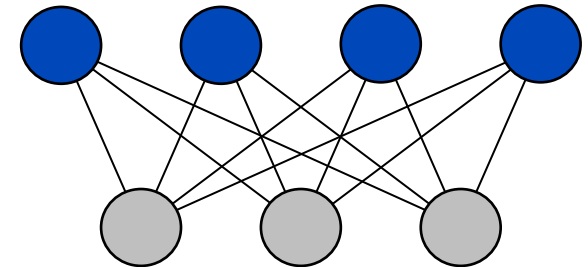
$$P[h, \text{on}] = \frac{1}{1 + e^{-\frac{\Delta E h}{k_B T}}}$$



... and similar for hidden units

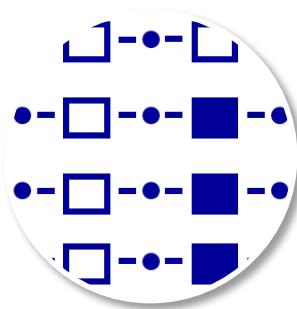
Simulated Annealing

```
set  $T = T_{\text{Max}}$   
  
while ( $T > T_{\text{Min}}$ )  
  forall  $v$   
    if ( $P[v, \text{on}] > \text{rand}(0,1)$ ) set  $s_v = 1$   
  forall  $h$   
    if ( $P[h, \text{on}] > \text{rand}(0,1)$ ) set  $s_h = 1$   
  set  $T$  smaller
```



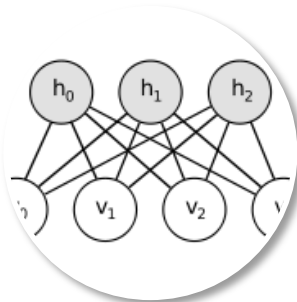
$E \rightarrow \min$

With (1) an (2) we can perform **Simulated Annealing**



Biological Problem

Analysing the regulation of metabolism



Machine Learning

Restricted Boltzmann Machines (RBM)

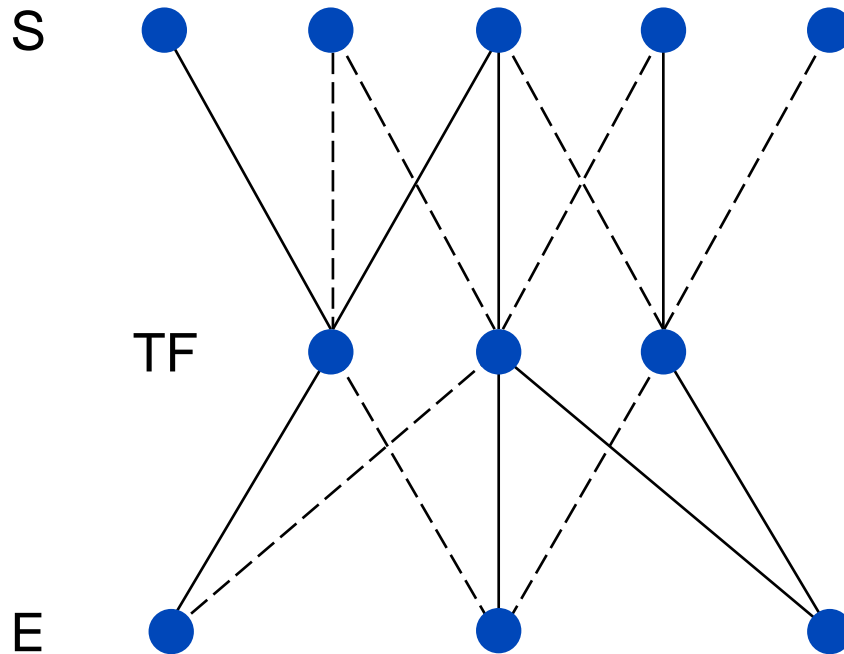
```
class RBM:
    def __init__(self,
                 self.num_hidden = 10,
                 self.num_visible = 100,
                 self.learning_rate = 0.1):

        # Initialize a weight matrix
        # a Gaussian distribution
        self.weights = 0.01 * np.random.randn(
            self.num_hidden, self.num_visible)
        # Insert weights
```

Implementation

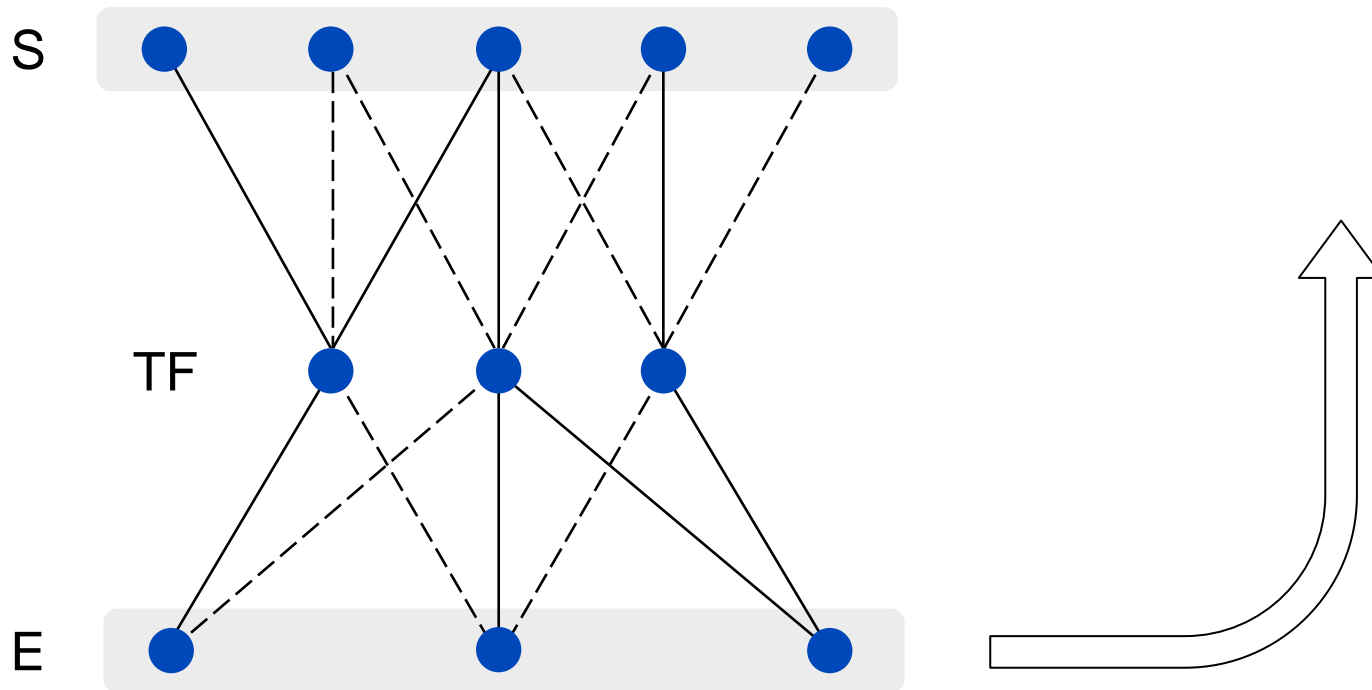
Modeling the Problem / Example

Modeling Regulation as RBM

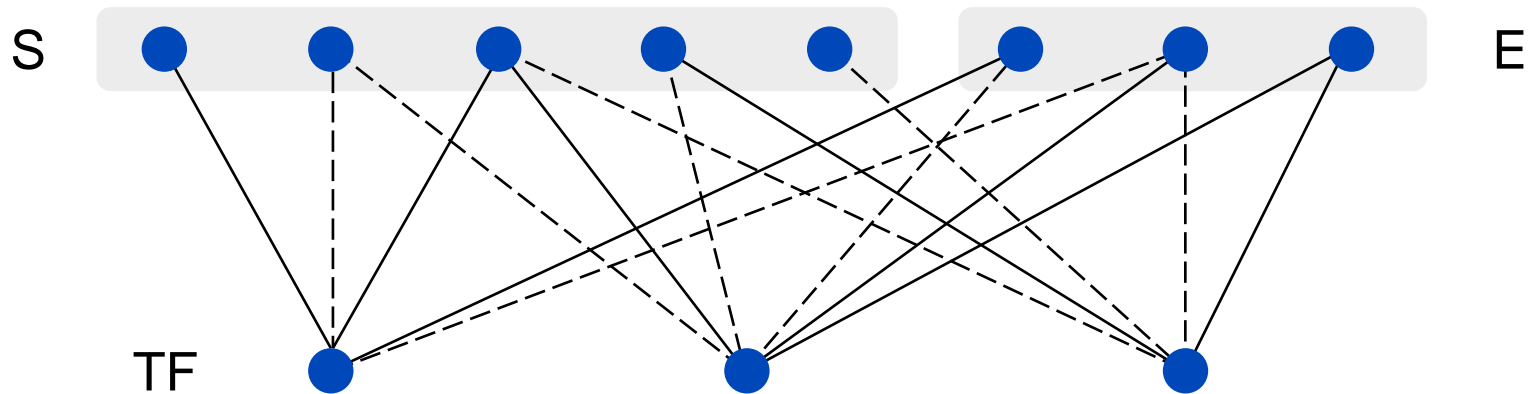


To **model** regulation as an RBM ...

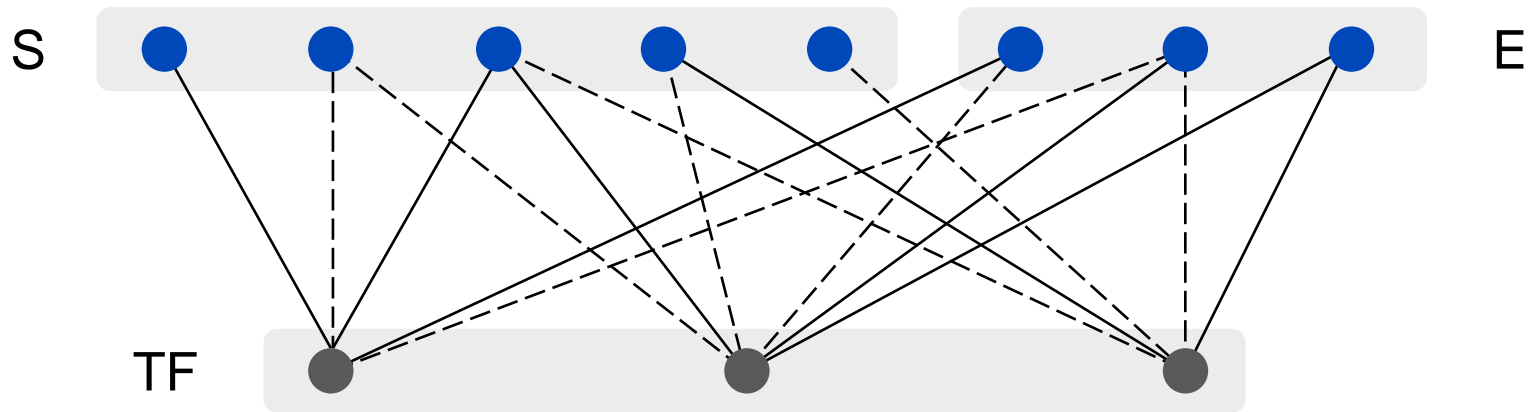
Modeling Regulation as RBM



... we define S and E as **visible Layer** ...

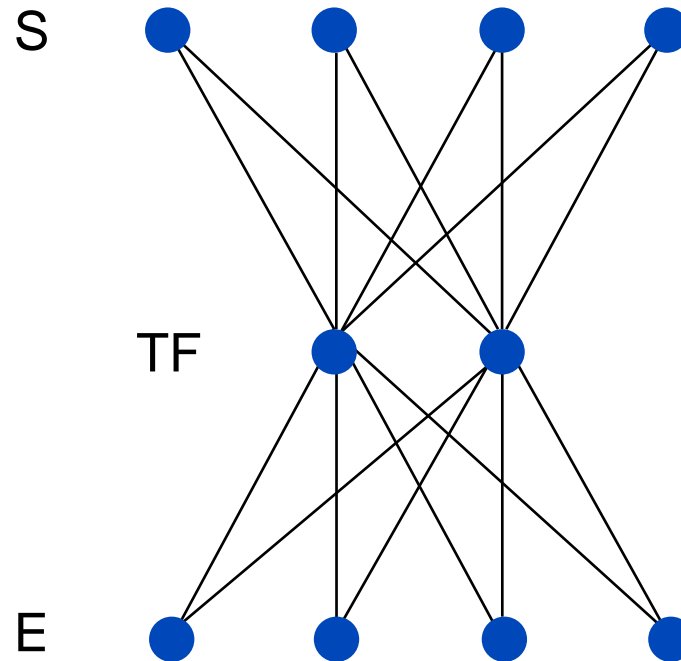


... we define S and E as **visible Layer** ...



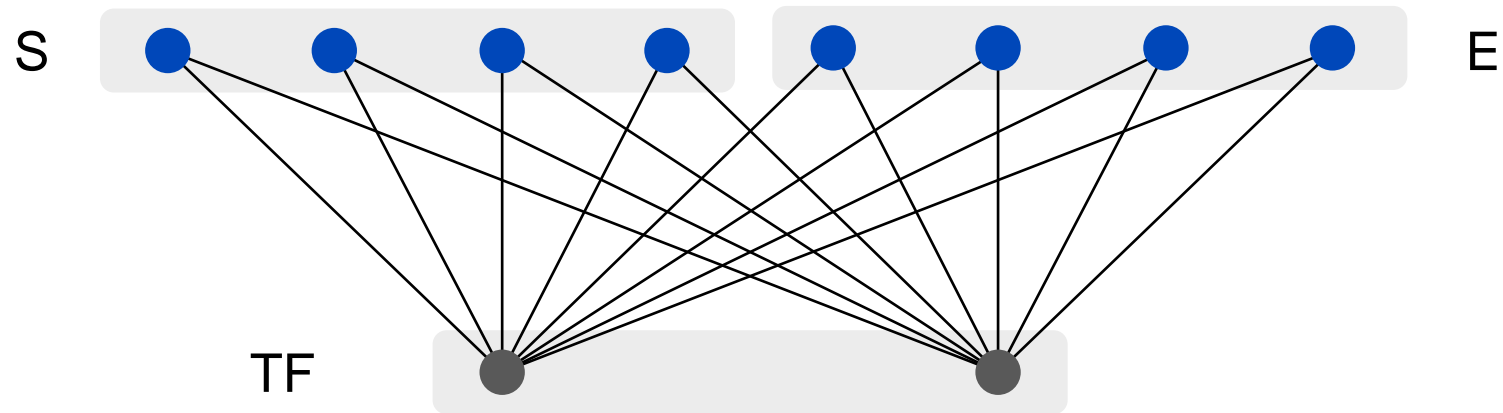
... and TF as **hidden Layer**

Example



Let's try it as simple as possible

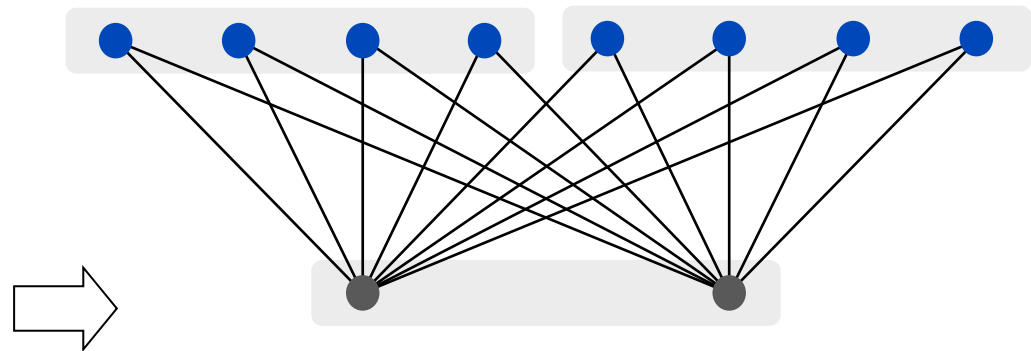
Example



... so we get 8 visible and 2 hidden units, fully connected

Learning samples

S	E
1,0,0,1	1,0,0,0
1,0,0,1	1,1,0,0
1,0,0,1	1,0,1,0
1,0,0,1	1,0,0,1
1,0,1,1	0,0,0,0
1,0,1,1	0,1,0,0
1,0,1,1	0,0,1,0
1,0,1,1	0,0,0,1

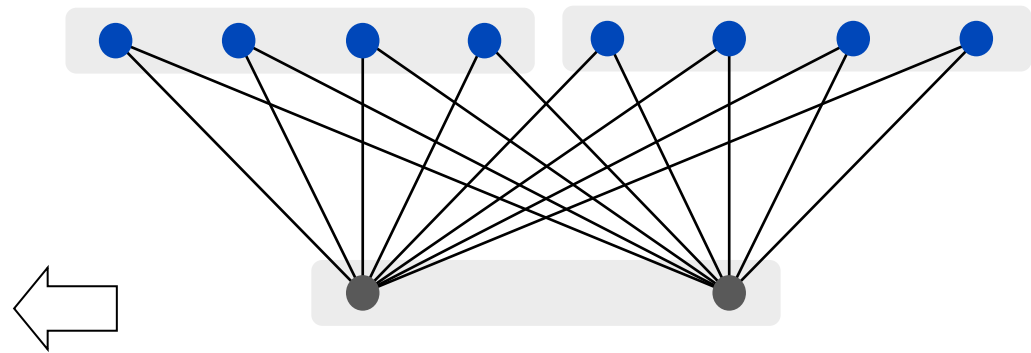


Let's feed the machine with **learning samples** ...

Example

Weight matrix

	TF_1	TF_2
S_1	0,3	0,8
S_2	0,5	0,6
S_3	0,9	0,1
S_4	0,3	0,8
E_1	1,0	0,0
E_2	0,1	0,0
E_3	0,1	0,0
E_4	0,2	0,0

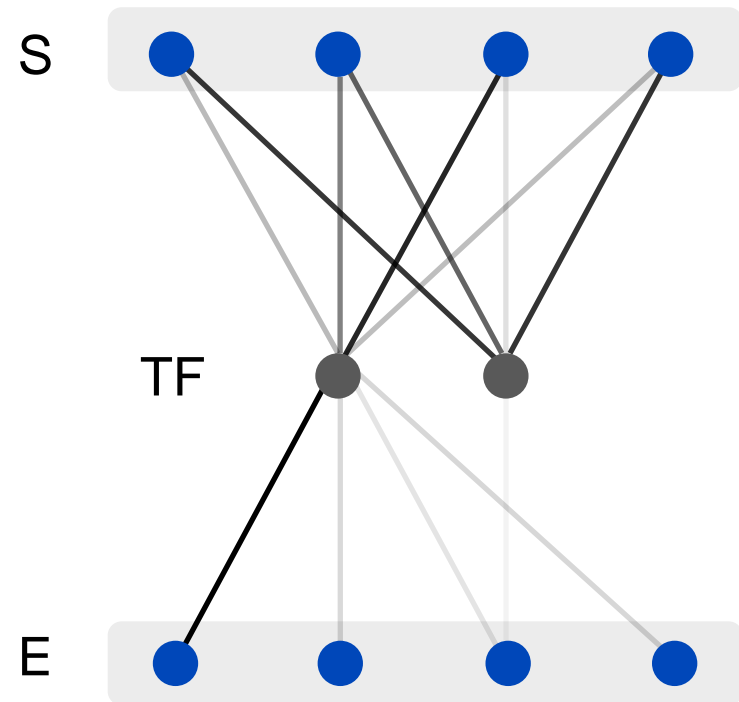
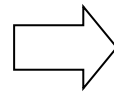


.. to get the calculated **weight matrix**

Example

Weight matrix

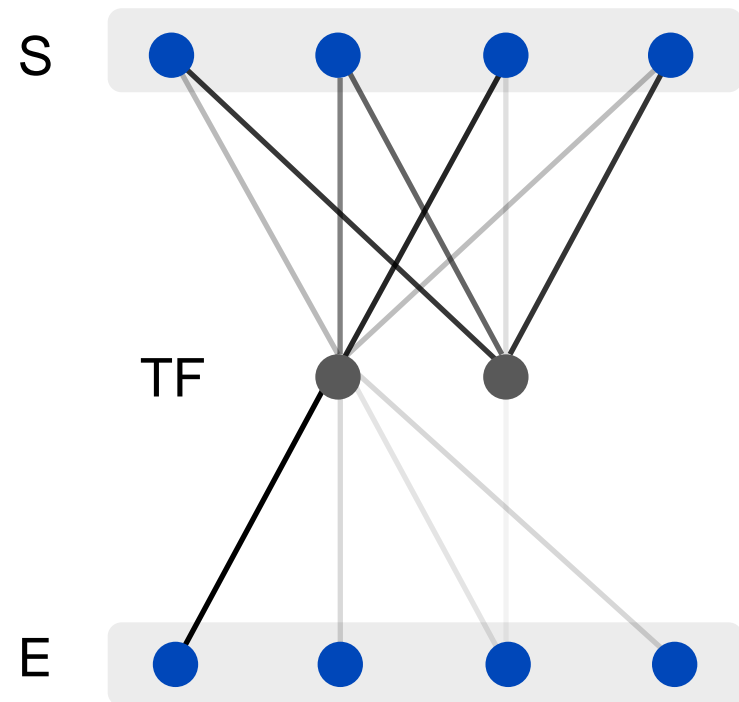
	TF_1	TF_2
S_1	0,3	0,8
S_2	0,5	0,6
S_3	0,9	0,1
S_4	0,3	0,8
E_1	1,0	0,0
E_2	0,1	0,0
E_3	0,1	0,0
E_4	0,2	0,0



The weights are visualized by the **intensity** of the edges

Learning samples

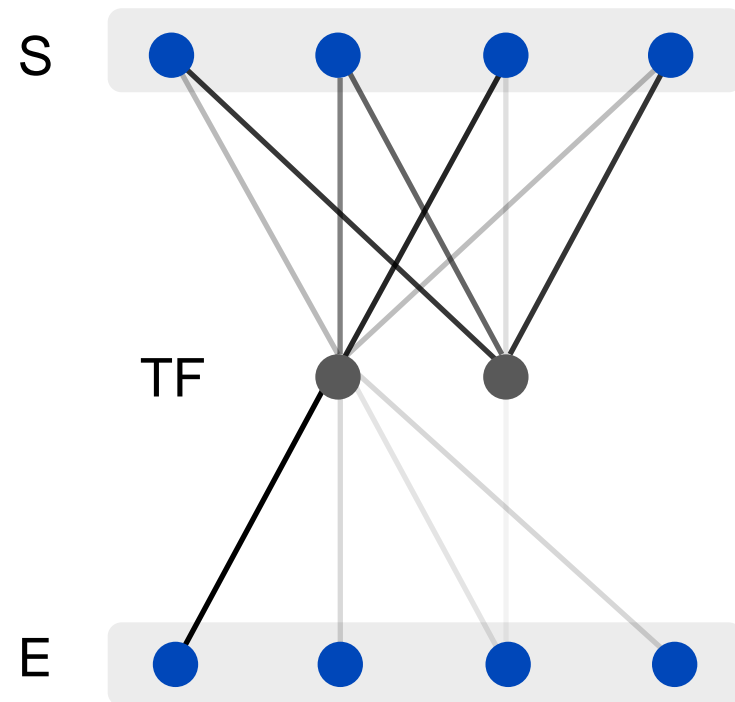
S	E
1,0,0,1	1,0,0,0
1,0,0,1	1,1,0,0
1,0,0,1	1,0,1,0
1,0,0,1	1,0,0,1
1,0,1,1	0,0,0,0
1,0,1,1	0,1,0,0
1,0,1,1	0,0,1,0
1,0,1,1	0,0,0,1



Now we can compare the results with the samples

Learning samples

S	E
1,0, 0 ,1	1 ,0,0,0
1,0, 0 ,1	1 ,1,0,0
1,0, 0 ,1	1 ,0,1,0
1,0, 0 ,1	1 ,0,0,1
1,0, 1 ,1	0 ,0,0,0
1,0, 1 ,1	0 ,1,0,0
1,0, 1 ,1	0 ,0,1,0
1,0, 1 ,1	0 ,0,0,1

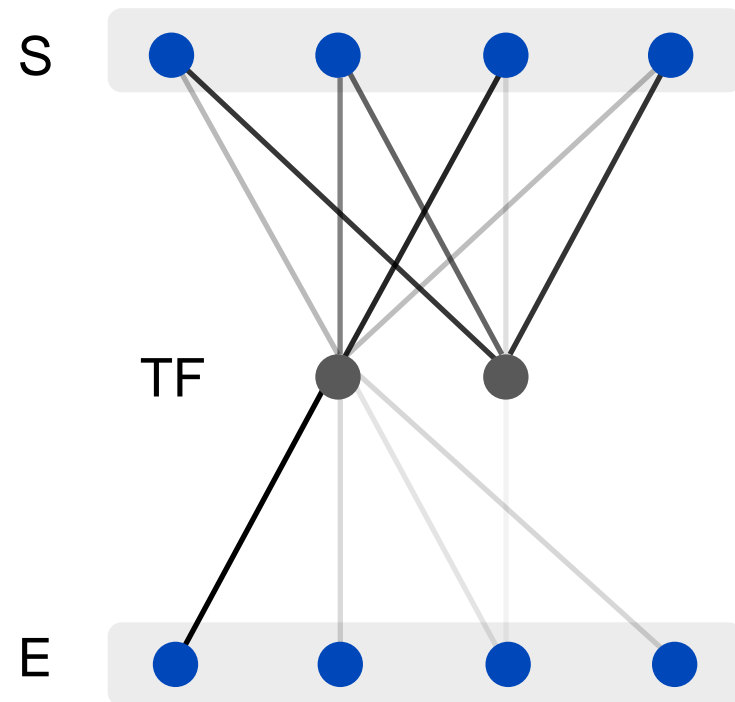


There's a strong dependency between S_3 and E_1

Example

Learning samples

S	E
1,0,0,1	1,0,0,0
1,0,0,1	1,1,0,0
1,0,0,1	1,0,1,0
1,0,0,1	1,0,0,1
1,0,1,1	0,0,0,0
1,0,1,1	0,1,0,0
1,0,1,1	0,0,1,0
1,0,1,1	0,0,0,1



S_1 , S_2 and S_4 do almost not affect the metabolism

Further Objectives

Since 2006 RBMs have successfully be used to train (pre-train) Multi-Layer ANNs (Hinton, Osindero, 2006)

This new branch in machine learning („deep learning“) already has a wide area of applications, including:

- Face recognition / Voice recognition
- Unsupervised detection of features
- Imagetransformation

It has to be asumed that deep learning strategies also provide further capabilities in regulatory analysis