Beyond the single biggest fish: Robust inference of species maximum length (L_{max}) from sample maxima

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Abstract

1. Maximum body length (L_{max}) is a key trait for describing an animal species' biological characteristics, ecology and vulnerability to exploitation. This is particularly true for fish species, and L_{max} is widely used in fisheries and is a key parameter in population assessments. Yet L_{max} estimation is strongly contingent on sampling intensity, and uncertainty is rarely quantified or propagated in downstream applications.

2. We apply and develop two complementary estimators of L_{max} and its uncertainty using observations of the largest individuals recorded across multiple samples of approximately similar size. First, using Extreme Value Theory (EVT), a method widely used in insurance and finance, we use the Generalised Extreme Value distribution to model the probability of an extreme event, i.e., the observation of a certain individual body length. Second, we propose a new method, an Exact Finite Sample (EFS) approach, which estimates the most likely parameters of the underlying body-size distribution that gives rise to the observed sample maxima. We use Bayesian inference for both methods to estimate the expected maximum individual body length for a given sampling effort (e.g., the expected maximum from 20 comparable samples) with credible intervals.

3. Sensitivity analyses show that both EVT and EFS recover unbiased L_{max} when samples arise from approximately truncated normal population length-frequency distribution. For heavier tailed length-frequency distributions, both methods tend to underestimate L_{max} (5–15%), with EVT yielding wider uncertainty but less sensitivity to distribution misspecification. For animal, and especially fish, ecology and management applications, we recommend reporting a "20-sample maximum L_{max}", defined as the 95th percentile of the probability density function of the maximum lengths, as a practical benchmark that is comparable across studies and explicitly conditions on sampling effort. As a case-study, we use 14 fishing competition records for Australasian snapper (*Chrysophrys auratus*) and estimate its 20-sample L_{max} as 139 cm (127-151 cm, 80% credible interval) using EVT, and 126 cm (121-133 cm, 80% credible interval) using EFS.

Keywords: Bayesian; Extreme value theory; fish; fisheries; Gumbel

Introduction

Body size represents a fundamental animal species biological trait that influences physiological and ecological processes, including metabolic rate, growth patterns, predation risk, and mortality (Blackburn and Gaston, 1994; Jennings et al., 2001; Peters and Wassenberg, 1983; White et al., 2007). Body size metrics correlate with various life-history parameters and serve as critical inputs in population or community dynamics and management models (Andersen, 2020; Hordyk et al., 2015; Pauly and Morgan, 1987). In fisheries, two particularly important metrics are the asymptotic length (L_∞), estimated from length-at-age growth models such as the von Bertalanffy function (von Bertalanffy, 1960), and the maximum observed length (L_{max}) (Andersen, 2019). While estimation of L_∞ generally requires ageing information, obtaining L_{max} requires only observational data of individual body lengths. As a result, L_{max} is available for most fish species catalogued in repositories such as FishBase (Froese and Pauly, 2025), arguably making it the most important biological parameter defining a fish or other indeterminately growing animal species or population.

Despite the widespread application of L_{max} in ecological analyses and modelling, considerable ambiguity surrounds its estimation methodology and biological interpretation. Various definitions have been proposed, including an observation of the largest individual reported for a given locality (Froese and Binohlan, 2000; which is also the definition used in FishBase), the median of the largest n length observations for a species, a specific percentile of length distribution, or calculated from asymptotic size assuming the L_{max}/L_∞ ratio of 0.95 (Chen et al., 2022). Yet, even after accounting for the well-recognised fact that L_{max} estimates are highly sensitive to population depletion through harvesting, the key issue with these approaches is that the probability of observing exceptionally large individuals depends on sampling intensity. Consequently, a true L_{max} can never be definitively established (unless every individual is measured), and any L_{max} estimate, and its associated uncertainty, is a function of sampling effort. Current approaches for estimating L_{max} neither explicitly account for sample size nor provide uncertainty estimates.

A statistical basis to estimate the probability and uncertainty of rare events is formalised in extreme value theory (EVT) (e.g., Gumbel, 1958). Just like the central limit theorem, which states that means from random samples for any population converges to a normal distribution, EVT proposes that, for any distribution in the exponential family (e.g., but not limited to, normal, gamma, lognormal), the distribution of sample maxima (i.e., the maximum of each sample) also converges to a predictable form. This maxima distribution has been described by the Gumbel distribution (Fisher and Tippett, 1928; Gumbel, 1935), but later to

Generalised Extreme Value (GEV) distribution (Jenkinson, 1955), which incorporates a shape parameter (ξ) to capture diverse tail behaviours of the underlying distribution (exponential, heavy or light). Over the last decades, extreme value theory became widely used in finance (e.g., Rocco, 2014), hydrology (e.g., Katz, 1999), insurance or weather forecasting and is gaining popularity in some ecological disciplines, such as environmental stressors (Gaines and Denny, 1993), paleoecology (Katz et al., 2005), or extinction risk (Burgman et al., 2012). Yet, while the seminal paper by Gaines and Denny (1993) did introduce an example of EVT application to estimating oldest expected age in humans, the theory has rarely been applied to estimate largest body sizes in indeterminate growers, i.e. animal that continue to growth throughout their lifetimes.

In fisheries, the most significant attempt to enhance the statistical rigour of fish L_{max} estimates was undertaken by Formacion (1991), who used the Gumbel distribution to estimate the most probable L_{max} value from a restricted set of samples of large fish (31 purse seine samples and 131 trawl samples). This methodology was subsequently incorporated into the FiSAT stock assessment tool (Gayanilo Jr et al., 1996) and has occasionally been used to estimate maximum fish body sizes from extremely limited observations (e.g., Ulman et al., 2022 using just four angler records). Yet, the current implementation of EVT for fish maximum size estimations has several limitations and has not been adopted more widely. First, the Gumbel distribution represents only one specific case within the three variations (Gumbel, Fréchet and Weibull) of the generalised extreme value distribution (GEV), hence it would be desirable to have a more general approach applied to L_{max} estimations. Second, the EVT approach assumes large samples or knowledge of the sample sizes, none of which is explicitly used in current L_{max} estimations in fisheries. Finally, the current implementation in FiSAT lacks transparency and is restricted to a specific software package, limiting its broader application within the research community.

In this study, we build upon the work of Formacion (1991) to develop a user-friendly L_{max} estimation using both the Gumbel and the GEV distribution and also propose an additional method to estimate L_{max} using a method that jointly estimates sample size and parameters of the underlying distribution generating the extreme values. We term this approach the 'Exact Finite Sample' (EFS) method and compare EFS and EVT performance using simulated data. Unlike the EVT, which is informed only by single extreme observation from each sample, the EFS method, in a slightly modified form, can also be used to estimate of L_{max} from samples where more than the single largest individuals are known. As a case study, we use fishing competition records to estimate L_{max} for Australasian snapper (*Chrysophrys auratus*), a popular recreational angling target species.

Materials and Methods

Given five 'extreme' measured lengths in a fish species – say, 19.8, 20.0, 20.0, 20.1, and 22.1 cm (e.g. see Figure 1A) - our aim is to estimate its maximum length, L_{max}. Taking the strict definition of L_{max}, as the 'largest observed individual in a population or a species' (e.g., Froese & Binohlan, 2000) would result in us defining L_{max} as 22.1 cm. Alternatively, we may take the median of the top three values, and define L_{max} to be 20.1 cm. With no other information these may be the only viable approaches, although it is obvious that more extensive sampling is likely to yield a larger fish at some point. In this study we propose two methods that can use existing knowledge to inform us about the best estimate of L_{max} and, importantly, on its uncertainty.

Firstly, we can apply Extreme Value Theory (EVT), modelling the five largest fish above using the Generalised Extreme Value (GEV) and more specific Gumbel distribution. After estimating the parameters of the GEV or Gumbel distribution we can make predictions about L_{max} and its uncertainty. Alternatively, we can use knowledge about the shape of the underlying fish length frequency distribution from which the five extreme length observations are sampled from. We can then infer the likelihood of observing those five largest fish, given the sample size. Here we term this approach the 'Exact Finite Sample' (EFS) method. The EFS method can readily be extended to situations where we have more observations than just the single largest fish from each of the five samples, such as if 2, 3, 4 or more of the largest individuals in the sample have been recorded. We term this approach with multiple maxima (EFSMM).

The key difference between the EVT and EFS methods is that EVT applies to a broad range of underlying distributions in the large sample limit while EFS applies directly to finite samples from a specific distribution. We describe each method in detail below.

Let x represent an individual's body size, drawn from a probability density function (PDF) $f(x;\theta)$, where θ represents the parameters of the underlying population body length distribution. The cumulative distribution function (CDF), $F(x;\theta)$, gives the probability that an individual is smaller than or equal to x. If we sample n individuals from a population and $x_1, x_2, \dots, x_n$ represent their respective body sizes, the probability that all individuals in our sample, assuming they are independently sampled, are smaller than some threshold, t, is:

$$P(x_1 \le t, x_2 \le t, \dots, x_n \le t) = \prod_{k=1}^{n} F\,(t;\theta) = F(t;\theta)^n \tag{1}$$

Since the probability that all individuals in the sample are smaller than the threshold *t*, is the same as the probability that the largest individual in that sample is less than the threshold *t*, we can say that the PDF of the largest observed individual in the sample is $F(x;\theta)^n$. Here the sample can be defined as a sampling or observation event, where multiple individuals are captured or observed (for example in scientific surveys or fishing competitions). For each of these k sampling events, the maximum is recorded to obtain k number of sample maxima. The EFS or EVT methods can then be used to estimate the probability density function of these sample maxima, which we term $g(x|N)$, equal to $F(x;\theta)^n$. Below the methods are introduced using the fisheries context, but the methods are equally applicable to other indeterminately growing animals or plants.

Exact Finite Sample (EFS) approach

In the EFS approach we assume that the underlying body length distribution $f(x;\theta)$ in a population or a species can be approximated by a known statistical distribution. Heather et al. (2025) showed that empirical length frequency distributions from 800 fish species could be approximated by a truncated-normal distribution, with the truncation at zero to avoid negative body size values. Using the truncated-normal distribution, we will make statistical inference to estimate $F(x;\theta)^n$ and the parameters θ (i.e. mean and standard deviation) of this distribution and sampling intensity required to produce the observed data. As the sample size from which the length maxima from k samples are derived is unknown (Figure 1A), we will assume that sample sizes also follow some statistical distribution. Specifically, since the sample size values are count data, we assume that distribution of sample sizes, n, follows a Poisson distribution, defined by a single parameter, λ.

Here we are using Bayesian methods to estimate parameters λ and θ simultaneously, which means that we estimate sampling intensity and underlying size distribution parameters all at once. The Bayesian framework facilitates the use of prior knowledge about body size and sample size distributions, making the method applicable across different species and management contexts. We will also explore method performance for cases where underlying body size distributions have heavier right-hand tails, such as gamma or lognormal distributions.

Exact Finite Sample approach with multiple maxima (EFSMM)

We can take the EFS approach further to utilise additional available information about fish body sizes in cases where more than one maximum fish observation is available per sample. In the specific example shown in Figure 1, we assume that three largest fish are known in sample 1, two in sample 2, three fish in sample 3 and so on (teal and black dots in Figure 1). More generally, if for a given sample we know the m largest fish, the probability that m of the total n fish are larger than some threshold, t, is:

$$P(x_1 \geq t, x_2 \geq t, \ldots, x_m \geq t) = \binom{n}{m} F(t;\theta)^{n-m}[1 - F(t;\theta)]^m \quad (2)$$

See supplementary material S1 for the likelihood function of this EFS method.

Extreme Value Theory (EVT) approach

When samples sizes from which maxima observations were derived approach infinity, the distribution for these maxima converges to the generalised extreme value (GEV) distribution, describing tail behaviour over a wide range of possible underlying body size distributions.

The cumulative distribution function (CDF) of the GEV, defined by three parameters, is given by:

$$F(x;\mu,\sigma,\xi) = \begin{cases} \exp\left(-\left(1 + \xi\left(\frac{x-\mu}{\sigma}\right)\right)^{-\frac{1}{\xi}}\right), & \xi \neq 0 \\ \exp\left(-\exp\left(-\frac{x-\mu}{\sigma}\right)\right), & \xi = 0 \end{cases} \quad (3)$$

where μ is the location parameter, σ is the scale parameter, and ξ is the shape parameter, which defines the tail behaviour. Under the assumption of truncated-normal underlying distribution, the shape parameter ξ becomes zero and the GEV is equivalent to Gumbel, which is approximated with just two parameters. Both GEV and Gumbel distributions were explored in this study.

Estimating L_{max} from the fitted GEV or Gumbel distribution

Once the GEV or Gumbel distribution is fitted to the sample maxima values, we can estimate the expected largest (extreme) observation in a specified number of samples. In extreme temporal event forecasting this number of samples is usually referred to as 'return time' (Gaines and Denny, 1993) and indicates, for example, the expected number of years between extreme weather events. In our example 'return time' equates to the number of samples (or approximately similar size) needed to observe a certain maximum size of a fish. Or alternatively, using the number of samples taken from the same population, the size of an expected largest individual x_p can be calculated as the p^{th} percentile of the distribution (GEV or Gumbel – if the shape parameter is zero):

$$x_p = \begin{cases} \mu + \frac{\sigma}{\xi}\left[\left(-\log(p)\right)^{-\xi} - 1\right], & \xi \neq 0 \\ \mu - \sigma\log\left(-\log(p)\right), & \xi = 0 \end{cases} \quad (4)$$

where $p = 1 - (1/k)$, and where k is the number of sampling events. Here, the quantile function was implemented using the *qgev* or *qgumbel* functions in the *evd* package in R (Stephenson, 2002).

In the specific example below – if we know the lengths of largest observed fish from five samples (black dots in Figure 1A), as could be the case in a fishing competition with five fishers reporting their largest catch, we can fit the GEV or Gumbel distribution to these five maxima and use the quantile function (Equation 4) to estimate the maximum length for a specified number of samples (e.g. number of fishers). For 20 fishers (samples) this would correspond to the 95th percentile of the fitted distribution, and for 100 fishers it would be the 99th percentile. The choice of the number of samples is arbitrary, but large difference between the number of samples used to fit the GEV or Gumbel distribution (e.g. largest fish observed in 5 samples, as above) and to make predictions (e.g., what would be the largest fish expected in 100 samples) increases the prediction error (Figure 1C).

Furter details on the likelihood function of the GEV distribution is provided in supplementary material S2.

Estimating L_{max} from $g(x|N)$ in the EFS method

$g(x|N)$ is the probability density function of the maximum of a given CDF $F(x;\theta)$ given n samples and is defined as $F(x;\theta)^N$. Under the assumption of the underlying length-frequency distribution is truncated-normal, $F(x;\theta)$ is equal is the CDF of the truncated normal distribution, where θ represents the mean and standard deviation. We can estimate the maximum of 20 samples (each of length λ), by calculating $g(x|N)$ when N = 20λ. Likewise, $g(x|k\lambda)$ provides an estimate of the maximum expected in *k* samples. Note that when *N* is equal to lambda (i.e., *k* = 1), then $g(x|\lambda)$ is analogous to the fitted GEV or Gumbel PDFs.

Method illustration using a simulated data example

To illustrate the fitting procedure, we chose an example scenario and assume that fish are sampled from a truncated-normal distribution with a mean of 10 cm and standard deviation at one-third of the mean length (Heather et al., 2025). From this distribution we take five samples (k), each of sample size n_k, where n_k values were sampled from a Poisson distribution with a mean and variance of 1000 (λ=1000). These parameters resulted in sample sizes of 1045, 1062, 988, 929 and 995 and five observations of largest fish: 22.1, 20.0, 20.0, 19.8, and 20.1 cm. These observations were used to estimate L_{max} using the EVT and EFS method. Additionally, for the EFSMM method we recorded 3, 2, 3, 3, and 4 of the largest fish in each of the five samples respectively and used their values to estimate

L_{max}. The number of largest fish to be recorded for the EFSMM method was sampled randomly from the integers 1 to 4 with equal probability and replacement.

Sensitivity analysis using simulated data

To test the accuracy and precision of each of the L_{max} estimation methods, we simulated multiple datasets for populations with different average body lengths, different shapes of the underlying body length distributions and different sampling intensities. We then applied EVT GEV, EVT Gumbel, EFS and EFSMM to each simulated scenarios to estimate the probability density distribution of 'maximum body length' and compared the estimated L_{max} with the 'true' maximum length. The 'true' maximum was calculated as the mode of $g(x)$ of the known underlying distribution, used to simulate the data. $g(x)$ is the PDF of the maximum of a distribution given *n* total samples, and is calculated as $g(x) = n(F(x)^{n-1})f(x)$, where $f(x)$ is the probability density function and $F(x)$ is the cumulative distribution function for the given distribution. For example, for a normal distribution with a mean of 10 and standard deviation of 3.4 the modal maximum from a sample of 1000 is 22.6 cm.

Simulations were run varying: 1. The number of samples taken (k = 5, 10, 50, 100, and 200 samples, or fishers); 2. The mean of the Poisson distribution (λ = 100, 1000, and 10000) to derive n_k or the size of each sample *k*; 3. The shape of the underlying body size distribution - truncated-normal, gamma and lognormal; 4. The mean length of fish in the underlying population (μ = 10, 50, and 100 cm). Following the findings of Heather et al. (2025) the standard deviation of the underlying body size distribution, σ, was fixed at one-third of the mean, μ. The parameters for each of the three underlying distributions were calculated based on the three mean values and a standard deviation of one third of the mean. This resulted in a total of 135 simulations, each giving one true maximum body size which was compared with the estimated L_{max} using EVT and EFS. Estimation bias was calculated as (*estimated*– *true*)/*true*. See supplementary material S3 for the calculation of the underlying distribution parameters based on the scenario-defined mean and standard deviation.

Method illustration using empirical data from Australasian snapper

Finally, to demonstrate the application of the three methods to estimate L_{max} in a real-world example, we used online reports for fishing competitions records (see supplementary material S4 for more details on the sources) for a popular recreational fisheries species Australasian snapper (*Chrysophrys auratus*, also known as *Pagrus auratus*). We obtained 14 fishing records (91.3, 102, 111, 99, 112, 123,115, 118, 107, 117, 120, 99.2, 95, and 82.2 cm) reported over 40 years from 1975 to 2015 (most values were in the last two years).

Records were reported as either length, weight or both, and in cases where lengths were not given, we used the default length-weight relationship of $W = 0.01L^3$ to convert weights to lengths. This default relationship gave a better match to the data (where both length and weight were known) compared to species-specific length-weight conversion parameters on Fishbase. The obtained 14 values of length maxima were then used to estimate L_{max} of *C. auratus* and the associated uncertainty using EVT and EFS methods. The EFSMM was not used, since only one record per sample was available.

For all methods we estimated parameter values using a Bayesian approach, coded in Stan (v2.36.0; Stan Development Team, 2022), as implemented in the *cmdstanr* package (v0.8.1; Gabry et al., 2024) in R (v4.4.2; R Core Team, 2024). The R code for this analysis is available at online ([anonymous until publication – local zipped copy available]).

Results

The two variants of models (EVT versus EFS) differ in how they approach the same task of estimating L_{max}, and the way their results should be interpreted. EVT describes how the sample maxima are distributed and does not explicitly account for the size of each of the *k* samples. In contrast, EFS and EFSMM estimate the probability density function of underlying body length distribution, $f(x)$, and use that to estimate the $g(x)$ given N observations, where $N = k \cdot \lambda$. The mean (or mode) of the fitted GEV or Gumbel distributions represents the typical (average) largest individual expected in a single sample, whereas the mean (or mode) of $g(x|N)$ represents the estimated largest individual from N observations. For EVT the L_{max} is then estimated as a specific percentile (given the number of samples taken, or for 20 samples) and for EFS, we use the mode of $g(x|N)$ to estimate the L_{max} for any k number of samples.

The method is first illustrated with a simulated data set, where five samples were taken from a population with length frequencies approximated by a truncated-normal distribution with the mean of 10 cm and standard deviation of 3.4 cm. The sample size, n for each sample, k, was sampled from a Poisson distribution with a mean and variance of 1000 ($\lambda = 1000$). The calculated 'true' largest fish from a sample of 5000 individuals (5 samples of around 1000 each) was 22.1 cm, and for a sample of 20000 individuals (20 samples of around 1000) it was 23.3 cm.

The EVT (GEV) method estimated the three parameters of the GEV distribution - location (μ), scale (σ) and shape (ξ), with median of the posterior distributions at 20.1, 0.7 and 0.02, respectively. The largest observed fish expected in five samples was estimated using the quantile function, where for 5 samples we used the 80th percentile. This resulted in the estimated 5-sample L_{max} of 21.1 cm, with 80% credible intervals (CI) between 20.5-22.6 cm. The expected largest fish in 20 samples (i.e., 95th percentile) was 22.3 cm (80% CI of 21.1-24.8 cm). The EVT (Gumbel) method estimated the two parameters of the Gumbel distribution - location (μ), scale (σ), with median of the posterior distributions of estimated at 20.1, 0.6, respectively. The estimated L_{max} for 5 samples was 20.9 cm (80% CI of 20.4-21.9 cm) and the 20-sample L_{max} was 21.8 cm (80% CI of 21.0-23.5 cm).

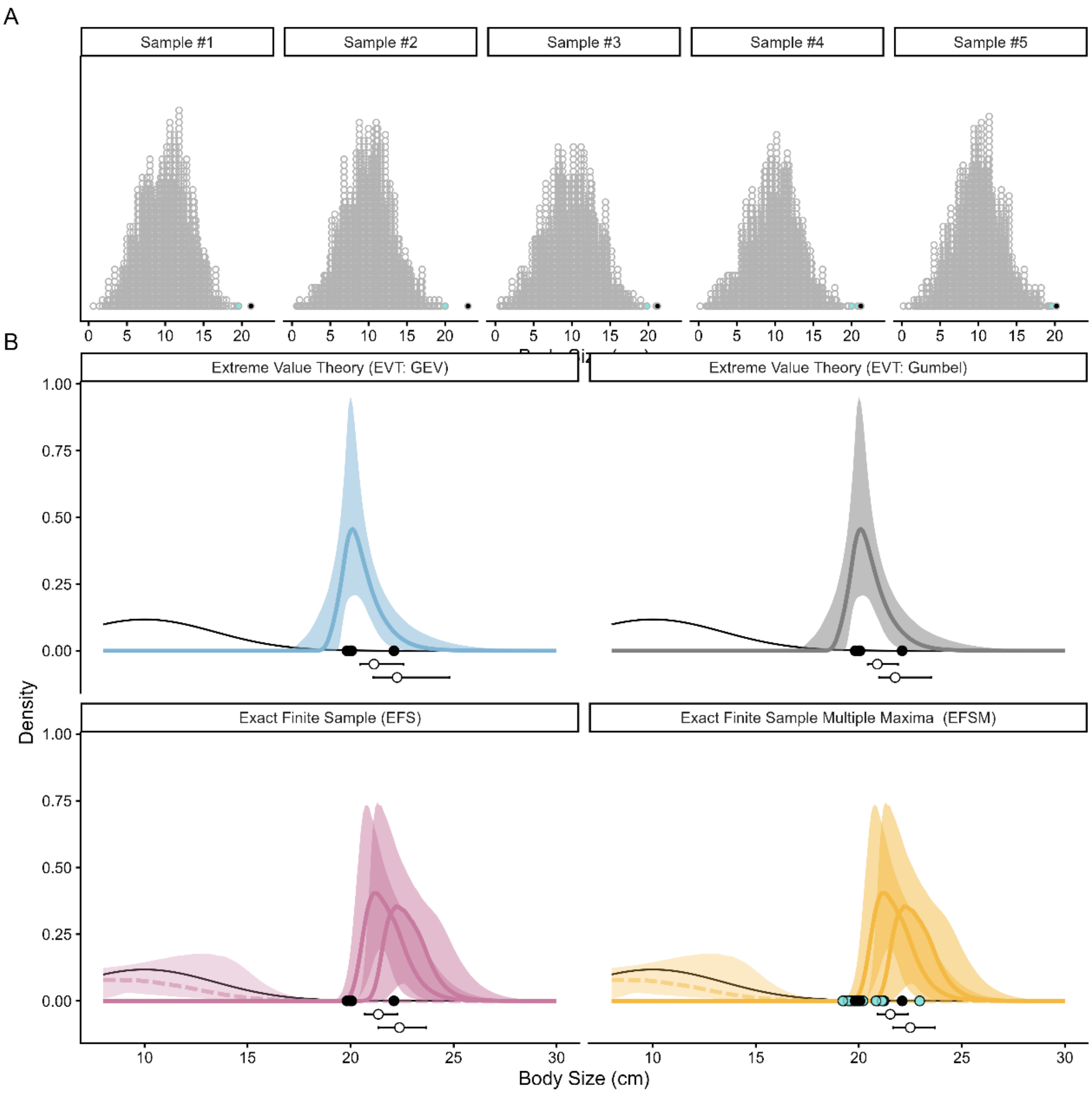

***Figure 1**: Conceptual diagram illustrating the four methods for estimating the maximum individual size in a population, L_{max}. Panel **A** shows five samples (#1-5) of individual length measurements, with each circle representing a single individual, randomly sampled from an underlying distribution (shown with a black thin line in **B**). Black filled dots represent the maxima in each of the five samples, and teal dots represent the largest m of the sample. Only single maxima values are used by EVT, EVT (Gumbel), and EFS, while*

EFSMM can utilise multiple maxima per sample. Panel ***B*** *shows model fitting results. For EVT and EVT Gumbel the blue and grey distribution shows the estimated distributions of maxima, but the shape of the underlying distribution (black line) is not estimated. EFS, EFSMM methods estimate both the underlying distribution (dashed lines and associated uncertainty) and the probability of largest value given the sample size, illustrated here for 5 (left) and 20 samples. The white points under the x-axis show the estimated 5- and 20-sample* L_{max}.

Sensitivity to sample sizes, total size and underlying body size distributions

In the sensitivity analysis we explored method performance assuming different mean length values of the underlying length frequency distribution (i.e. small, medium or very large fish), different number of samples taken (*k*) and shapes of the underlying body size distributions (truncated-normal, lognormal or gamma). In all cases the EFS method assumed that the underlying distribution is truncated-normal, while EVT does not make assumptions about the shape of the underlying distribution. All three methods (EVT, EFS and EFSMM) were not sensitive to the true mean length of the sampled population (10, 50 or 100 cm; see columns in Figure 2), as biases and uncertainty for each method were similar across the three mean length assumptions. As expected, increasing the number of samples (k), generally reduced the uncertainty of the L_{max} estimate. The EFS and EFSMM methods performed very similarly, despite EFSMM utilising more data (i.e., the m maxima per sample). Figure 2 shows the 45 scenarios where the λ is 1000. See supplementary material S5 for other 90 scenarios when λ is 100 and 10000.

The violation of assumptions about the underlying distribution (gamma, lognormal or truncated-normal) had larger impacts on estimation bias. When underlying distribution was truncated-normal, i.e. same as assumed in EFS and EFMM, all methods performed well, but EVT was less precise and had larger uncertainty ranges. When underlying length distribution was approximated by gamma distribution and had a heavier right-hand tail, all three methods tended to underestimate L_{max}values for cases with *k* = 5 or 10. As the number of samples increased to 50, 100 or 200, all methods generally converged closer to the true L_{max}value, although EFS/EFSMM method still tended towards small (ca 5%) underestimation (Fig. 2). Finally, when fish were sampled from an underlying lognormal distribution (i.e. heavy right-hand tail), all three methods underestimated the true L_{max}, with bias being greatest for the EFS methods. For k = 5 and λ = 1000, the median percent error across all scenarios was -1.9%, -2.4%, -7.6% and -9.0% for EVT, EVT (Gumbel), EFS and EFSMM respectively. Adding more samples (from k = 5 to 200) did not consistently reduce the bias in the EFS methods nor in the EVT methods.

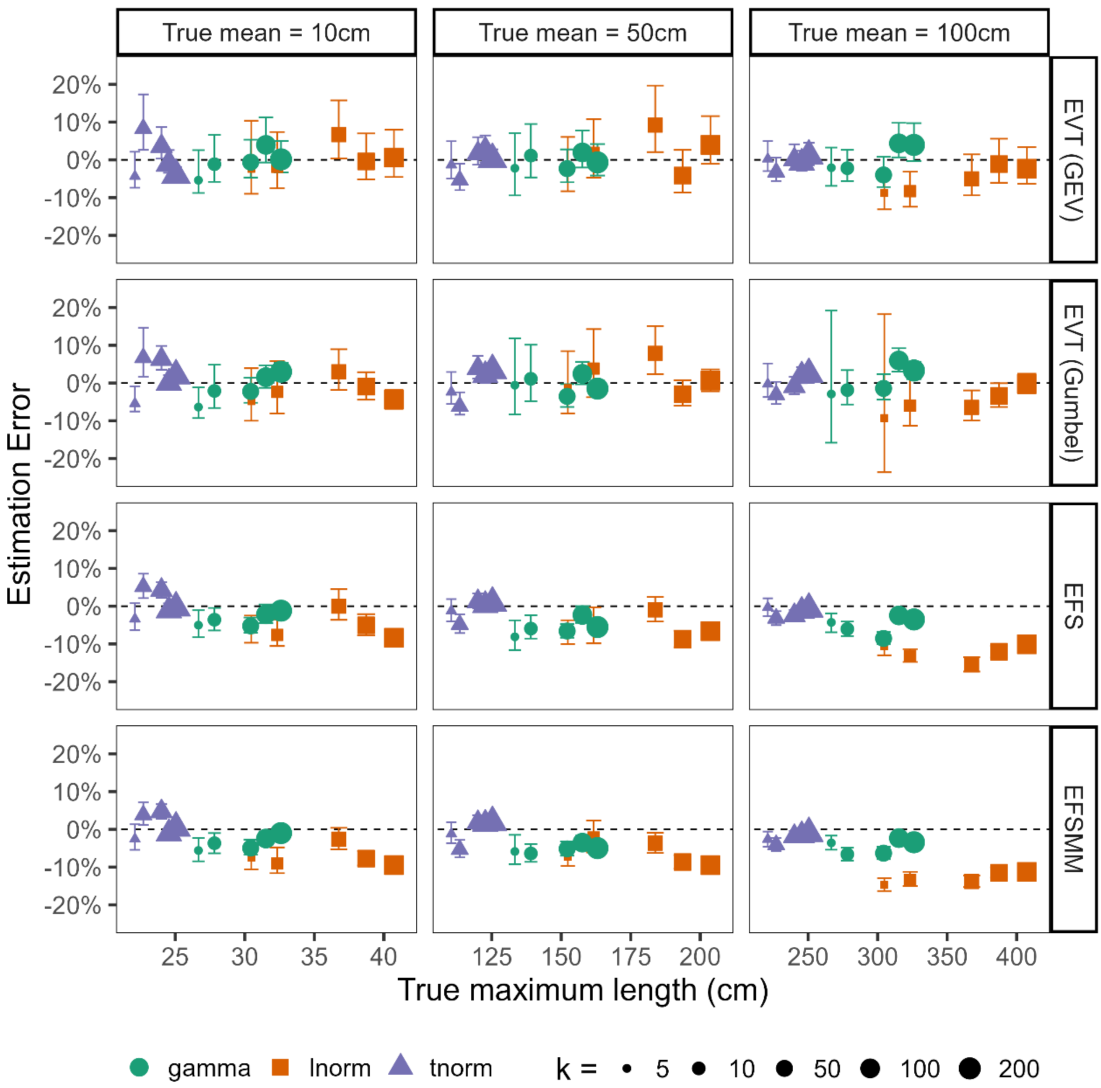

Figure 2: Biases and precision of the four L_{max} estimation methods under three underlying shapes of body length distributions in the population (truncated-normal, gamma and lognormal), five levels of the number of samples (k), and three true mean length of individuals in the populations. Error bars represent 80% credible intervals. Estimation error is calculated as (estimated-truth)/truth.

Estimating L_{max} for Australasian snapper using recreational fishing records

Using the 14 length records of Australian snapper reported by fishers and fishing competitions we estimated the 14-sample and 20-sample L_{max} using EVT (Gumbel) and EFS methods. For EVT approach the estimated L_{max} was at 134 cm (123-146 cm, 80% credible interval) and 139 cm (127-151 cm) for 14 and 20 samples respectively, whereas for EFS method it was 124 cm (118-131 cm) and 126 cm (121-133 cm). Further, the EFS estimated that these reported 14 record lengths came from an underlying distribution with a mean of 47 cm (23-68 cm, 80% CI), standard deviation of 25 cm (21-29, 80% CI), and a sample size of 59 individuals per each of 14 samples (15-350, 80% CI). On FishBase the reported L_{max} for this species is 130 cm.

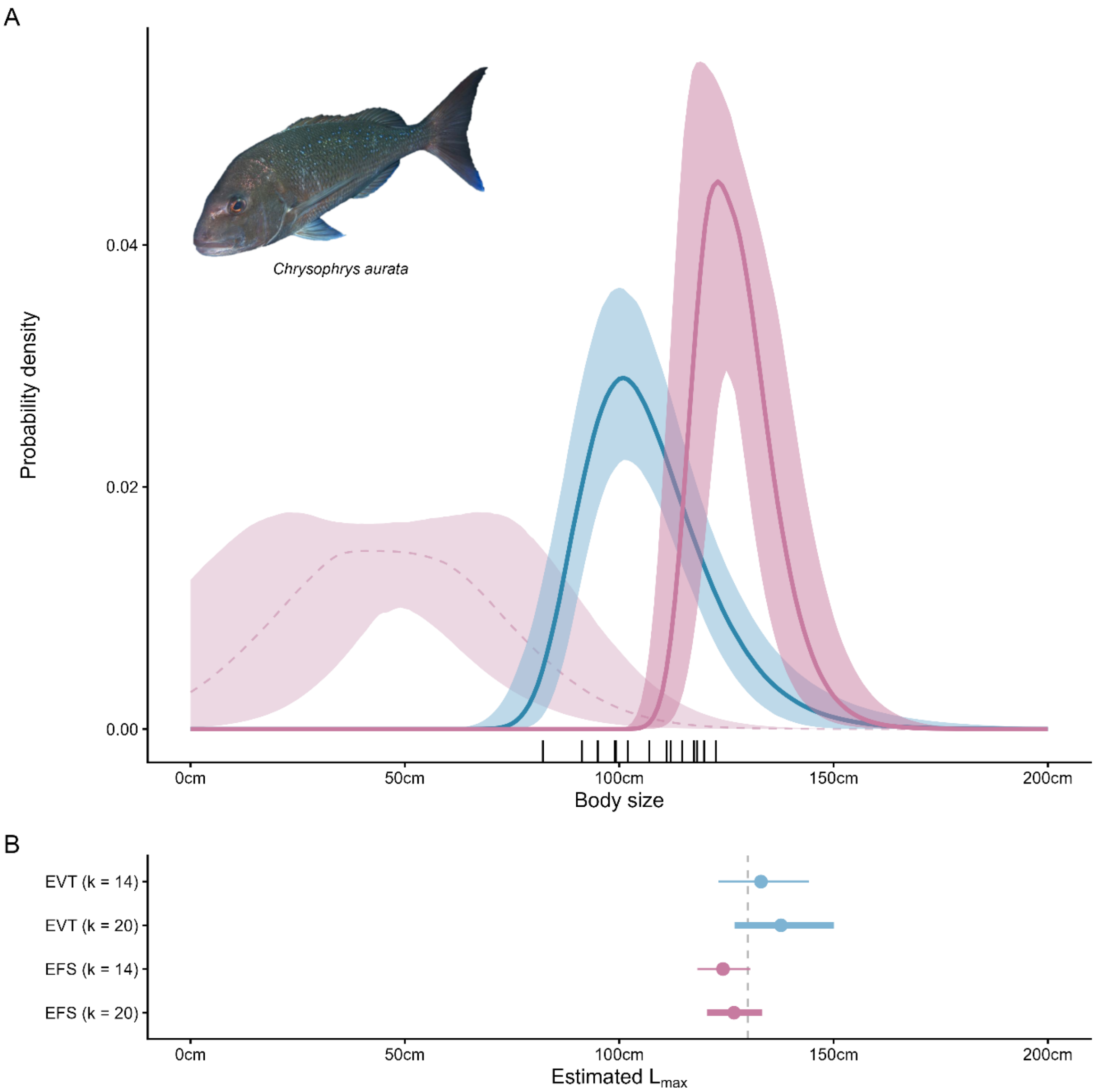

*Figure 3: Maximum length (L_{max}) estimates for Australasian snapper using 14 reported large fish observations (black vertical lines in panel A). Blue line in panel **A** show predictions for the expected largest fish in one sample, with 14- and 20-sample L_{max} values estimated from the corresponding upper percentile (**B**). Red line shows the underlying distribution estimated from available 14 sample (light shaded) and the estimated distribution of 20-sample maxima (dark shaded). The reported L_{max} for this species, according to Fishbase is 130 cm (dotted vertical line in **B**). Error bars and shading indicate 80% credible intervals.*

Discussion

In this study we employed two approaches for statistically estimating maximum length in fishes using data on one or several largest observed individuals across a set of samples. Compared with the common practice of setting L_{max} to the single largest observed fish, our methods have two key advantages: they provide uncertainty ranges and explicitly account for sampling intensity. Specifically, our estimates of L_{max} are tied to the number of samples from which observations are derived (e.g. a 20-sample L_{max}). We show, using Australasian snapper as an example, that we can use readily available data (such as angler records), to improve our estimates of L_{max}. Instead of the FishBase record of 130cm for *C. auratus*, we can report the 20-sample L_{max} from EVT (Gumbel distribution) of 134cm with an 80% credible interval of 123 to 146 cm.

Maximum length, L_{max}, is arguably the most important life-history parameter in fishes: it is often the easiest to obtain (given some length data) and it correlates with other key parameters, including asymptotic length, maturation size and even recruitment rates (Denney et al., 2002; Froese and Binohlan, 2000; Thorson et al., 2017). Comparisons between observed and expected L_{max} values are used as proxies for population status at local to global scales (Uy et al., 2025; Worm et al., 2009). L_{max} also functions as a dominant trait in size-based single- and multi-species models (Andersen, 2019; Scott et al., 2014). Yet, despite its importance, L_{max} estimation procedures rarely acknowledge uncertainty. In most studies (Juan-Jordá et al., 2013; Tewfik et al., 2022; Worm et al., 2009), L_{max} is taken as the single largest known fish from samples, literature or FishBase. Some work explores variability in ratios involving L_{max} (Apostolidis and Stergiou, 2014; Chen et al., 2022), but it is often unclear how much of this variability is due to L_{max} estimation uncertainty versus environmental or biological factors that truly affect fish size. When new data become available, updated “records” are reported (Edelist, 2014), yet without uncertainty estimates it is impossible to test whether new size records are consistent with expected variation or indicate a genuine shift in the underlying length distribution due to population status or growth.

The approach presented here is an important step towards greater statistical rigour in reporting fish and other animal life-history parameters. Making L_{max} uncertainty explicit enables more robust analyses of life-history relationships and their spatiotemporal dynamics. When estimating human impacts on population size structure through time (e.g., Worm et al., 2009), L_{max} values that include uncertainty and specify sampling intensity could be used to provide an estimate for the probability of observing a certain (large) animal size given the sampling intensity. Such probability estimates of observing an unusual event are now common for predicting extreme heat or flood events and usually involve extreme value theory methods. With growing availability of angling records and underwater survey data, it is now also timely to apply similar methods in fisheries and ecology. One rare example of applying EVT to estimate L_{max} in fisheries is from Ulman et al. (2022) who used four angler silver-cheeked toadfish records (~78.5, ~80, 83 and ~120 cm) to estimate L_{max}, using methods implemented in FiSAT software. The estimated L_{max} value was at 110 cm, with 95% confidence ranges at 87–132 cm. If the same data are analysed using approaches presented here, the estimated 20-sample L_{max} is at 117 cm (Bayesian 80% credible intervals at 99-139 cm) with EVT (GEV), 123 cm (80% credible intervals at 100-155 cm) with EVT (assuming Gumbel), and 98 cm (80% credible intervals at 92–103 cm) with exact finite sample method (EFS). Given the small sample size (k=4) and one unusually large observation, such wide uncertainty intervals around the L_{max} estimates reflect the true and limited state of knowledge, and as already highlighted in our results, uncertainty ranges for EVT methods were much wider than for EFS. Notably, the original record of 120 cm was on the extreme upper tail of the posterior from the EFS approach, indicating that it could be an erroneous observation. This was also concluded by Ulman et al. (2022) and closer analysis of photos and original data indeed supports this conclusion, because the reported 8kg weight of the individual with the estimated length of 120cm, aligns more closely with a length of 80-90 cm. While simple, this exercise shows the potential applications of the proposed L_{max} estimation method to quantify uncertainty in maximum length estimates for a given population given the sampling, to assess the legitimacy of record observations and to statistically compare trends through space and time.

The next steps in the application of the proposed methods to fish and fisheries studies could include applying them to time series of angler-record databases, testing for trends in animal maximum sizes across environmental gradients (e.g. temperature; Lavin et al., 2022), and updating FishBase L_{max} entries to include uncertainty estimates. Moreover, although the method development was driven by fish and fisheries related questions, the framework is general and can estimate maximum sizes for any organism, provided the underlying size distribution can be approximated with simple forms. For instance, even protist size distributions follow predictable, near-lognormal patterns (Giometto et al., 2013) and the same is likely to apply across most animal and plant taxa. And in cases where the underlying distribution is unknown, EVT still remains applicable.

Naturally, the approach proposed here has its limitations. The three most important limitations are: (i) assumptions about sample size, (ii) assumptions about the underlying body size distribution, and (iii) a tendency to underestimate L_{max} in the exact finite sample (EFS) method, when the true distribution has a heavier right tail. Directional bias or underestimation was typically larger for EFS (when model assumptions were not met), whereas EVT showed greater random variability and wider uncertainly intervals. This is unsurprising, as EFS gains precision by assuming additional knowledge about the population size distribution. If that prior knowledge is correct, EFS is more accurate and precise; if incorrect, it is more biased. Yet, even in extreme cases, the bias was generally below 15% and if known, the bias could be incorporated into downstream analyses. It is not clear how many species or population are likely to have heavy right-hand tails in the underlying body size distribution, but analysis of length-frequency data from ~800 species showed that ~90% of them were better approximated by truncated-normal rather than lognormal distributions, supporting truncated-normal as a reasonable first approximation when species-specific information is lacking. However, if good evidence for an alternative underlying distribution exists, the estimation method does not need to assume that samples come from a truncated normal distribution. The method can be extended to other assumed underlying distribution (e.g. lognormal or gamma), but new simulations should be conducted to assess method performance under those alternatives.

Our simulations showed that when the assumption about the underlying size distribution is correct, the EFS method can estimate L_{max} with high precision and accuracy. Note that L_{max} estimate depends on estimates of three parameters – mean, variance of the underlying distribution and sample size (λ). However, accurate estimates of each parameter separately are not required to obtain an accurate L_{max} value, since the probability of sampling a large individual depends on the interaction among mean size, variance and sampling. A large fish might arise from a distribution with a large mean and small sample size, or a smaller mean and larger sample size; these parameters should therefore be considered jointly when estimating L_{max}. For example, in Figure 1B the estimated underlying distribution may be outside of the credible intervals of the true underlying distribution, yet the L_{max} estimate was still accurate. In cases where prior knowledge exists - especially about mean length of the population or sample size – incorporating informative priors in EFS should improve method accuracy and precision. Here we used weak priors for sample size ($\lambda \sim \mathcal{N}(10000{,}4000)$), mean length ($\mu \sim \mathcal{N}(50{,}20)$, and standard deviation σ ~ $\mathcal{N}(20{,}5)$), but these values are likely to be more narrowly defined in a real-world scenario.

In conclusion, we show that extreme value theory and exact finite sample methods described here provide two valid approaches to estimate maximum body sizes in animal populations using observations of largest individuals in different sampling regimes. We recommend that these methods are adopted in ecology and management to improve estimates and application of fish and other animal species' life-history parameters.

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