The University of Austin at Texas

# those who know

Aaryan Prakash, Dylan Smith, Mark Wen

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# Contest (1)

template.cpp 14 lines

```
#include <bits/stdc++.h>
using namespace std;

#define rep(i, a, b) for(int i = a; i < (b); ++i)
#define all(x) begin(x), end(x)
#define sz(x) (int)(x).size()
#define pb push_back
typedef long long ll;
typedef pair<int, int> pii;
typedef vector<int> vi;

int main() {
  cin.tie(0)->sync_with_stdio(0);
}
```

.bashrc 10 lines

```
run () {
  ok=1
  if [[ ! -f $1 || $1 -ot $1.cpp ]]
  then
    g++ $1.cpp -O2 -o $1 -std=c++17 -Wall -Wextra -Wshadow
         -Wconversion -fsanitize=undefined,address || ok=0
  fi
  [[ $ok -eq 1 ]] && ./$1
}

xmodmap -e 'clear Lock' -e 'keycode 0x42 = Escape'
```

.vimrc 5 lines

```
set cin aw ai is ts=4 sw=4 tm=50 rnu noeb bg=dark ru cul
     mouse=a
sy on | no ; :
" Select region and then type :Hash to hash your selection.
ca Hash w !cpp -dD -P -fpreprocessed \| tr -d '[:space:]' \
 \| md5sum \| cut -c-6
```

brute.sh 12 lines

```
#!/bin/zsh

sz=100
for ((i=1;;i++)); do
    echo "$i"
    ./gen "$i" "$sz" > input
    ./sol < input > output1
    ./brute < input > output2
    if (! diff output1 output2); then
        break
    fi
done
```

# Mathematics (2)

## 2.1 Equations

$$\begin{matrix} ax+by=e \\ cx+dy=f \end{matrix} \Rightarrow \begin{matrix} x=\dfrac{ed-bf}{ad-bc} \\ y=\dfrac{af-ec}{ad-bc} \end{matrix}$$

$$x_i = \frac{\det A_i'}{\det A}$$

## 2.2 Recurrences

If $a_n = c_1 a_{n-1} + \cdots + c_k a_{n-k}$, and $r_1, \ldots, r_k$ are distinct roots of $x^k - c_1 x^{k-1} - \cdots - c_k$, there are $d_1, \ldots, d_k$ s.t.

$$a_n = d_1 r_1^n + \cdots + d_k r_k^n.$$

Non-distinct roots $r$ become polynomial factors, e.g. $a_n = (d_1 n + d_2) r^n$.

## 2.3 Trigonometry

$$\sin(v+w) = \sin v \cos w + \cos v \sin w$$
$$\cos(v+w) = \cos v \cos w - \sin v \sin w$$
$$\tan(v+w) = \frac{\tan v + \tan w}{1 - \tan v \tan w}$$
$$\sin v + \sin w = 2 \sin \frac{v+w}{2} \cos \frac{v-w}{2}$$
$$\cos v + \cos w = 2 \cos \frac{v+w}{2} \cos \frac{v-w}{2}$$

$$(V+W)\tan(v-w)/2 = (V-W)\tan(v+w)/2$$

where $V, W$ are lengths of sides opposite angles $v, w$.

$$a\cos x + b \sin x = r\cos(x-\phi)$$
$$a \sin x + b\cos x = r\sin(x+\phi)$$

where $r = \sqrt{a^2+b^2}, \phi = \operatorname{atan2}(b, a)$.

## 2.4 Geometry

### 2.4.1 Triangles

Circumradius: $R = abc/4A$

Inradius: $r = A/p$

Length of median (divides triangle into two equal-area triangles): $m_a = \frac{1}{2}\sqrt{2b^2+2c^2-a^2}$

Length of bisector (divides angles in two):

$$s_a = \sqrt{bc\left[1 - \left(\frac{a}{b+c}\right)^2\right]}$$

Law of sines: $\dfrac{\sin\alpha}{a} = \dfrac{\sin\beta}{b} = \dfrac{\sin\gamma}{c} = \dfrac{1}{2R}$

Law of cosines: $a^2 = b^2 + c^2 - 2bc\cos\alpha$

Law of tangents: $\dfrac{a+b}{a-b} = \dfrac{\tan\frac{\alpha+\beta}{2}}{\tan\frac{\alpha-\beta}{2}}$

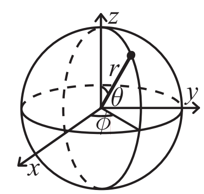
### 2.4.2 Quadrilaterals

With side lengths $a, b, c, d$, diagonals $e, f$, diagonals angle $\theta$, area $A$ and magic flux $F = b^2 + d^2 - a^2 - c^2$:

$$4A = 2ef \cdot \sin\theta = F\tan\theta = \sqrt{4e^2f^2 - F^2}$$

For cyclic quadrilaterals the sum of opposite angles is $180°$, $ef = ac + bd$, and $A = \sqrt{(p-a)(p-b)(p-c)(p-d)}$.

### 2.4.3 Spherical coordinates




$$\begin{aligned} x &= r\sin\theta\cos\phi & r &= \sqrt{x^2+y^2+z^2} \\ y &= r\sin\theta\sin\phi & \theta &= \operatorname{acos}(z/\sqrt{x^2+y^2+z^2}) \\ z &= r\cos\theta & \phi &= \operatorname{atan2}(y, x) \end{aligned}$$

## 2.5 Derivatives/Integrals

$$\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\arccos x = -\frac{1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx}\tan x = 1 + \tan^2 x \qquad \frac{d}{dx}\arctan x = \frac{1}{1+x^2}$$
$$\int \tan ax = -\frac{\ln|\cos ax|}{a} \qquad \int x\sin ax = \frac{\sin ax - ax\cos ax}{a^2}$$
$$\int e^{-x^2} = \frac{\sqrt{\pi}}{2}\operatorname{erf}(x) \qquad \int xe^{ax}dx = \frac{e^{ax}}{a^2}(ax-1)$$

Integration by parts:

$$\int_a^b f(x)g(x)dx = [F(x)g(x)]_a^b - \int_a^b F(x)g'(x)dx$$

## 2.6 Sums

$$c^a + c^{a+1} + \dots + c^b = \frac{c^{b+1} - c^a}{c-1}, c \neq 1$$

$$1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$$
$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(2n+1)(n+1)}{6}$$
$$1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{n^2(n+1)^2}{4}$$
$$1^4 + 2^4 + 3^4 + \dots + n^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$$

## 2.7 Series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots, (-\infty < x < \infty)$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots, (-1 < x \leq 1)$$

$$\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{2x^3}{32} - \frac{5x^4}{128} + \dots, (-1 \leq x \leq 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots, (-\infty < x < \infty)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots, (-\infty < x < \infty)$$

## 2.8 Probability theory

$$\sigma^2 = V(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2$$
$$\mathbb{E}(aX + bY) = a\mathbb{E}(X) + b\mathbb{E}(Y)$$
$$\text{ind. } X, Y, V(aX + bY) = a^2V(X) + b^2V(Y).$$

### 2.8.1 Discrete distributions

**Binomial distribution**

$$p(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$\mu = np, \sigma^2 = np(1-p)$$

$\mathrm{Bin}(n,p)$ is approximately $\mathrm{Po}(np)$ for small $p$.

**Geometric distribution**

$$p(k) = p(1-p)^{k-1}, k = 1, 2, \dots$$

$$\mu = \frac{1}{p}, \sigma^2 = \frac{1-p}{p^2}$$

**Poisson distribution**

$$p(k) = e^{-\lambda}\frac{\lambda^k}{k!}, k = 0, 1, 2, \dots$$

$$\mu = \lambda, \sigma^2 = \lambda$$

### 2.8.2 Continuous distributions

**Uniform distribution**

$$f(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & \text{otherwise} \end{cases}$$

$$\mu = \frac{a+b}{2}, \sigma^2 = \frac{(b-a)^2}{12}$$

**Exponential distribution**

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

$$\mu = \frac{1}{\lambda}, \sigma^2 = \frac{1}{\lambda^2}$$

**Normal distribution**

Most real random values with mean $\mu$ and variance $\sigma^2$ are well described by $\mathcal{N}(\mu, \sigma^2)$, $\sigma > 0$.

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

If $X_1 \sim \mathcal{N}(\mu_1, \sigma_1^2)$ and $X_2 \sim \mathcal{N}(\mu_2, \sigma_2^2)$ then

$$aX_1 + bX_2 + c \sim \mathcal{N}(\mu_1 + \mu_2 + c, a^2\sigma_1^2 + b^2\sigma_2^2)$$

## 2.9 Markov chains

A *Markov chain* is a discrete random process with the property that the next state depends only on the current state. Let $X_1, X_2, \dots$ be a sequence of random variables generated by the Markov process. Then there is a transition matrix $\mathbf{P} = (p_{ij})$, with $p_{ij} = \Pr(X_n = i | X_{n-1} = j)$, and $\mathbf{p}^{(n)} = \mathbf{P}^n \mathbf{p}^{(0)}$ is the probability distribution for $X_n$ (i.e., $p_i^{(n)} = \Pr(X_n = i)$), where $\mathbf{p}^{(0)}$ is the initial distribution.

$\pi$ is a stationary distribution if $\pi = \pi\mathbf{P}$. If the Markov chain is *irreducible* (it is possible to get to any state from any state), then $\pi_i = \frac{1}{\mathbb{E}(T_i)}$ where $\mathbb{E}(T_i)$ is the expected time between two visits in state $i$. $\pi_j / \pi_i$ is the expected number of visits in state $j$ between two visits in state $i$.

For a connected, undirected and non-bipartite graph, where the transition probability is uniform among all neighbors, $\pi_i$ is proportional to node $i$'s degree.

A Markov chain is *ergodic* if the asymptotic distribution is independent of the initial distribution. A finite Markov chain is ergodic iff it is irreducible and *aperiodic* (i.e., the gcd of cycle lengths is 1). $\lim_{k\to\infty} \mathbf{P}^k = \mathbf{1}\pi$.

A Markov chain is an A-chain if the states can be partitioned into two sets $\mathbf{A}$ and $\mathbf{G}$, such that all states in $\mathbf{A}$ are absorbing ($p_{ii} = 1$), and all states in $\mathbf{G}$ leads to an absorbing state in $\mathbf{A}$. The probability for absorption in state $i \in \mathbf{A}$, when the initial state is $j$, is $a_{ij} = p_{ij} + \sum_{k\in\mathbf{G}} a_{ik} p_{kj}$. The expected time until absorption, when the initial state is $i$, is $t_i = 1 + \sum_{k\in\mathbf{G}} p_{ki} t_k$.

# Data structures (3)

### OrderStatisticTree.h

**Description:** A set (not multiset!) with support for finding the n'th element, and finding the index of an element. To get a map, change null_type.

**Time:** $\mathcal{O}(\log N)$

782797, 16 lines

```
#include <bits/extc++.h>//893
using namespace __gnu_pbds;

template<class T>
using Tree = tree<T, null_type, less<T>, rb_tree_tag,
  tree_order_statistics_node_update>;//988

void example() {
  Tree<int> t, t2; t.insert(8);
  auto it = t.insert(10).first;
  assert(it == t.lower_bound(9));//6bd
  assert(t.order_of_key(10) == 1);
  assert(t.order_of_key(11) == 2);
  assert(*t.find_by_order(0) == 8);
  t.join(t2); // assuming T < T2 or T > T2, merge t2 into t
}//cbb
```

### HashMap.h

**Description:** Hash map with mostly the same API as unordered_map, but ~3x faster. Uses 1.5x memory. Initial capacity must be a power of 2 (if provided).

d77092, 7 lines

```
#include <bits/extc++.h>//1e4
// To use most bits rather than just the lowest ones:
struct chash { // large odd number for C
  const uint64_t C = ll(4e18 * acos(0)) | 71;
  ll operator()(ll x) const { return __builtin_bswap64(x*C)
      ; }
};//198
__gnu_pbds::gp_hash_table<ll,int,chash> h({},{},{},{},{
    1<<16});
```

### Matrix.h

**Description:** Basic operations on square matrices.

**Usage:**

```
Matrix<int, 3> A;
A.d = {{{{1,2,3}}, {{4,5,6}}, {{7,8,9}}}};
vector<int> vec = {1,2,3};
vec = (A^N) * vec;
```

c43c7d, 26 lines

```
template<class T, int N> struct Matrix {//1aa
  typedef Matrix M;
  array<array<T, N>, N> d{};
  M operator*(const M& m) const {
    M a;
    rep(i,0,N) rep(j,0,N)//683
      rep(k,0,N) a.d[i][j] += d[i][k]*m.d[k][j];
```

```
    return a;
  }
  vector<T> operator*(const vector<T>& vec) const {
    vector<T> ret(N);//9bd
    rep(i,0,N) rep(j,0,N) ret[i] += d[i][j] * vec[j];
    return ret;
  }
  M operator^(ll p) const {
    assert(p >= 0);//358
    M a, b(*this);
    rep(i,0,N) a.d[i][i] = 1;
    while (p) {
      if (p&1) a = a*b;
      b = b*b;//1d8
      p >>= 1;
    }
    return a;
  }
};//214
```

## LineContainer.h

**Description:** Container where you can add lines of the form kx+m, and query maximum values at points x. Useful for dynamic programming ("convex hull trick").
**Time:** $\mathcal{O}(\log N)$

8ec1c7, 30 lines

```
struct Line {//7e3
  mutable ll k, m, p;
  bool operator<(const Line& o) const { return k < o.k; }
  bool operator<(ll x) const { return p < x; }
};
//d77
struct LineContainer : multiset<Line, less<>> {
  // (for doubles, use inf = 1/.0, div(a,b) = a/b)
  static const ll inf = LLONG_MAX;
  ll div(ll a, ll b) { // floored division
    return a / b - ((a ^ b) < 0 && a % b); }//66e
  bool isect(iterator x, iterator y) {
    if (y == end()) return x->p = inf, 0;
    if (x->k == y->k) x->p = x->m > y->m ? inf : -inf;
    else x->p = div(y->m - x->m, x->k - y->k);
    return x->p >= y->p;//bec
  }
  void add(ll k, ll m) {
    auto z = insert({k, m, 0}), y = z++, x = y;
    while (isect(y, z)) z = erase(z);
    if (x != begin() && isect(--x, y)) isect(x, y = erase(y
        ));//890
    while ((y = x) != begin() && (--x)->p >= y->p)
      isect(x, erase(y));
  }
  ll query(ll x) {
    assert(!empty());//b07
    auto l = *lower_bound(x);
    return l.k * x + l.m;
  }
};
```

## Treap.h

**Description:** A short self-balancing tree. It acts as a sequential container with log-time splits/joins, and is easy to augment with additional data.
**Time:** $\mathcal{O}(\log N)$

30f532, 55 lines

```
struct Node {//e9f
  Node *l = 0, *r = 0;
  int val, y, c = 1;
  Node(int v) : val(v), y(rand()) {}
  void recalc();
};//3ef
int cnt(Node* n) { return n ? n->c : 0; }
void Node::recalc() { c = cnt(l) + cnt(r) + 1; }

template<class F> void each(Node* n, F f) {//5d5
  if (n) { each(n->l, f); f(n->val); each(n->r, f); }
}

pair<Node*, Node*> split(Node* n, int k) {
  if (!n) return {};//ca5
  if (cnt(n->l) >= k) { // "n->val >= k" for lower_bound(k)
    auto pa = split(n->l, k);
    n->l = pa.second;
    n->recalc();
    return {pa.first, n};//b54
  } else {
    auto pa = split(n->r, k - cnt(n->l) - 1); // and just "
        k"
    n->r = pa.first;
    n->recalc();
    return {n, pa.second};//86d
  }
}

Node* merge(Node* l, Node* r) {
  if (!l) return r;//fbf
  if (!r) return l;
  if (l->y > r->y) {
    l->r = merge(l->r, r);
    l->recalc();
    return l;//780
  } else {
    r->l = merge(l, r->l);
    r->recalc();
    return r;
  }//96d
}

Node* ins(Node* t, Node* n, int pos) {
  auto pa = split(t, pos);
  return merge(merge(pa.first, n), pa.second);//99b
}

// Example application: move the range [l, r) to index k
void move(Node*& t, int l, int r, int k) {
  Node *a, *b, *c;//99c
  tie(a,b) = split(t, l); tie(b,c) = split(b, r - l);
  if (k <= l) t = merge(ins(a, b, k), c);
  else t = merge(a, ins(c, b, k - r));
}
```

## FenwickTree.h

**Description:** Computes partial sums a[0] + a[1] + ... + a[pos - 1], and updates single elements a[i], taking the difference between the old and new value.
**Time:** Both operations are $\mathcal{O}(\log N)$.

e62fac, 22 lines

```
struct FT {//711
  vector<ll> s;
  FT(int n) : s(n) {}
  void update(int pos, ll dif) { // a[pos] += dif
    for (; pos < sz(s); pos |= pos + 1) s[pos] += dif;
  }//cc4
  ll query(int pos) { // sum of values in [0, pos)
    ll res = 0;
    for (; pos > 0; pos &= pos - 1) res += s[pos-1];
    return res;
  }//477
  int lower_bound(ll sum) {// min pos st sum of [0, pos] >=
      sum
    // Returns n if no sum is >= sum, or -1 if empty sum is
        .
    if (sum <= 0) return -1;
    int pos = 0;
    for (int pw = 1 << 25; pw; pw >>= 1) {//fc5
      if (pos + pw <= sz(s) && s[pos + pw-1] < sum)
        pos += pw, sum -= s[pos-1];
    }
    return pos;
  }//e03
};
```

## FenwickTree2d.h

**Description:** Computes sums a[i,j] for all i<I, j<J, and increases single elements a[i,j]. Requires that the elements to be updated are known in advance (call fakeUpdate() before init()).
**Time:** $\mathcal{O}(\log^2 N)$. (Use persistent segment trees for $\mathcal{O}(\log N)$.)

"FenwickTree.h" e2f703, 22 lines

```
struct FT2 {//4ce
  vector<vi> ys; vector<FT> ft;
  FT2(int limx) : ys(limx) {}
  void fakeUpdate(int x, int y) {
    for (; x < sz(ys); x |= x + 1) ys[x].pb(y);
  }//57f
  void init() {
    for (vi& v : ys) sort(all(v)), ft.emplace_back(sz(v));
  }
  int ind(int x, int y) {
    return (int)(lower_bound(all(ys[x]), y) - ys[x].begin()
        ); }//358
  void update(int x, int y, ll dif) {
    for (; x < sz(ys); x |= x + 1)
      ft[x].update(ind(x, y), dif);
  }
  ll query(int x, int y) {//688
    ll sum = 0;
    for (; x; x &= x - 1)
      sum += ft[x-1].query(ind(x-1, y));
    return sum;
  }//e03
};
```

## RMQ.h

**Description:** Range Minimum Queries on an array. Returns min(V[a], V[a + 1], ... V[b - 1]) in constant time.
**Usage:** `RMQ rmq(values);`
`rmq.query(inclusive, exclusive);`
**Time:** $\mathcal{O}(|V| \log |V| + Q)$

510c32, 16 lines

```
template<class T>//722
struct RMQ {
  vector<vector<T>> jmp;
  RMQ(const vector<T>& V) : jmp(1, V) {
    for (int pw = 1, k = 1; pw * 2 <= sz(V); pw *= 2, ++k)
        {
      jmp.emplace_back(sz(V) - pw * 2 + 1);//f6c
      rep(j,0,sz(jmp[k]))
        jmp[k][j] = min(jmp[k - 1][j], jmp[k - 1][j + pw]);
    }
  }
  T query(int a, int b) {//a3d
    assert(a < b); // or return inf if a == b
    int dep = 31 - __builtin_clz(b - a);
    return min(jmp[dep][a], jmp[dep][b - (1 << dep)]);
  }
};//214
```

## MoQueries.h

**Description:** Answer interval or tree path queries by finding an approximate TSP through the queries, and moving from one query to the next by adding/removing points at the ends. If values are on tree edges, change step to add/remove the edge $(a,c)$ and remove the initial add call (but keep in).
**Time:** $\mathcal{O}(N\sqrt{Q})$

a12ef4, 49 lines

```
void add(int ind, int end) { ... } // add a[ind] (end = 0
      or 1)//342
void del(int ind, int end) { ... } // remove a[ind]
int calc() { ... } // compute current answer

vi mo(vector<pii> Q) {
  int L = 0, R = 0, blk = 350; // ~N/sqrt(Q)//cb0
  vi s(sz(Q)), res = s;
#define K(x) pii(x.first/blk, x.second ^ -(x.first/blk & 1)
      )
  iota(all(s), 0);
  sort(all(s), [&](int s, int t){ return K(Q[s]) < K(Q[t]);
        });
  for (int qi : s) {//623
    pii q = Q[qi];
    while (L > q.first) add(--L, 0);
    while (R < q.second) add(R++, 1);
    while (L < q.first) del(L++, 0);
    while (R > q.second) del(--R, 1);//d22
    res[qi] = calc();
  }
  return res;
}
//842
vi moTree(vector<array<int, 2>> Q, vector<vi>& ed, int root
      =0){
  int N = sz(ed), pos[2] = {}, blk = 350; // ~N/sqrt(Q)
  vi s(sz(Q)), res = s, I(N), L(N), R(N), in(N), par(N);
  add(0, 0), in[0] = 1;
  auto dfs = [&](int x, int p, int dep, auto& f) -> void {
        //263
    par[x] = p;
    L[x] = N;
    if (dep) I[x] = N++;
    for (int y : ed[x]) if (y != p) f(y, x, !dep, f);
    if (!dep) I[x] = N++;//23e
    R[x] = N;
  };
  dfs(root, -1, 0, dfs);
#define K(x) pii(I[x[0]] / blk, I[x[1]] ^ -(I[x[0]] / blk &
      1))
  iota(all(s), 0);//064
  sort(all(s), [&](int s, int t){ return K(Q[s]) < K(Q[t]);
        });
  for (int qi : s) rep(end,0,2) {
    int &a = pos[end], b = Q[qi][end], i = 0;
#define step(c) { if (in[c]) { del(a, end); in[a] = 0; } \
                  else { add(c, end); in[c] = 1; } a = c; }
                      //440
    while (!(L[b] <= L[a] && R[a] <= R[b]))
      I[i++] = b, b = par[b];
    while (a != b) step(par[a]);
    while (i--) step(I[i]);
    if (end) res[qi] = calc();//695
  }
  return res;
}
```

## NoamQueue.h

**Description:** Online queue-like deletion from a data structure supporting stack-like deletion
**Time:** $\mathcal{O}(T(n)log(n))$

635417, 22 lines

```
struct update {//46a
  bool type;
  update() { type = 0; }
};
template<typename D, typename U>
struct noam : public D {//409
  vector<U> s;
  void push(const U &u) { D::push(u); s.pb(u); }
  void pop() {
    auto i = s.end(); int c = 0;
    do {//31c
      c += (--i)->type ? 1 : -1; D::pop();
    } while (c < 0 && i != begin(s));
    auto j = stable_partition(i, s.end(), [](auto &x) {
        return !x.type; });
    if (i == begin(s)) {
      reverse(i, j);//cdd
      for_each(i, j, [](auto &x) { x.type = 1; });
    }
    s.pop_back();
    while (i != s.end()) D::push(*i), i++;
  }//e03
};
```

## OfflineDeletion.h

**Description:** Delete from a data structure given insertions and rollbacks ds needs: void push(U u), void pop(), Q query()
**Time:** $\mathcal{O}(T(n)log(n))$

b156e0, 32 lines

```
template<typename D, typename U, typename Q>//bf5
struct offline_deletion : public D {
  vector<Q> ans;
  vector<vector<U>> updates;
  int q;
  offline_deletion(int queries) : q(queries), ans(q) {//e0a
    int lg = 0; while((1 << lg) < q) lg++;
    updates.resize(1 << (lg + 1));
  }
  void update(int i, int l, int r, int L, int R, U u) {
    if (r < L || R < l) return;//731
    if (L <= l && r <= R) {
      updates[i].pb(u); return;
    }
    int m = (l + r) / 2;
    update(2 * i + 1, l, m, L, R, u);//087
    update(2 * i + 2, m + 1, r, L, R, u);
  }
  void insert(U u, int l, int r) { update(0, 0, q - 1, l, r
        , u); }
  void insert(U u, int l) { insert(u, l, q - 1); }
  void solve(int i, int l, int r) {//fa5
    for (auto &u : updates[i]) D::push(u);
    if (l == r) ans[l] = D::query();
    else {
      int m = (l + r) / 2;
      solve(2 * i + 1, l, m);//eb8
      solve(2 * i + 2, m + 1, r);
    }
    for (int j = 0; j < (int) updates[i].size(); j++) D::
        pop();
  }
  void solve() { solve(0, 0, q - 1); }//dbe
};
```

## SegmentTree.h

**Description:** Zero-indexed max-tree. Bounds are inclusive to the left and exclusive to the right. Can be changed by modifying T, f and unit.
**Time:** $\mathcal{O}(\log N)$

0f4bdb, 19 lines

```
struct Tree {//026
  typedef int T;
  static constexpr T unit = INT_MIN;
  T f(T a, T b) { return max(a, b); } // (any associative
      fn)
  vector<T> s; int n;
  Tree(int n = 0, T def = unit) : s(2*n, def), n(n) {}//c86
  void update(int pos, T val) {
    for (s[pos += n] = val; pos /= 2;)
      s[pos] = f(s[pos * 2], s[pos * 2 + 1]);
  }
  T query(int b, int e) { // query [b, e)//e90
    T ra = unit, rb = unit;
    for (b += n, e += n; b < e; b /= 2, e /= 2) {
      if (b % 2) ra = f(ra, s[b++]);
      if (e % 2) rb = f(s[--e], rb);
    }//490
    return f(ra, rb);
  }
};
```

## LazySegmentTree.h

**Description:** Segment tree with ability to add or set values of large intervals, and compute max of intervals. Can be changed to other things. Use with a bump allocator for better performance, and SmallPtr or implicit indices to save memory.
**Usage:** `Node* tr = new Node(v, 0, sz(v));`
**Time:** $\mathcal{O}(\log N)$.

"../various/BumpAllocator.h" 34ecf5, 50 lines

```
const int inf = 1e9;//317
struct Node {
  Node *l = 0, *r = 0;
  int lo, hi, mset = inf, madd = 0, val = -inf;
  Node(int lo,int hi):lo(lo),hi(hi){} // Large interval of
      -inf
  Node(vi& v, int lo, int hi) : lo(lo), hi(hi) {//f58
    if (lo + 1 < hi) {
      int mid = lo + (hi - lo)/2;
      l = new Node(v, lo, mid); r = new Node(v, mid, hi);
      val = max(l->val, r->val);
    }//22c
    else val = v[lo];
  }
  int query(int L, int R) {
    if (R <= lo || hi <= L) return -inf;
    if (L <= lo && hi <= R) return val;//2ff
    push();
    return max(l->query(L, R), r->query(L, R));
  }
  void set(int L, int R, int x) {
    if (R <= lo || hi <= L) return;//1bd
    if (L <= lo && hi <= R) mset = val = x, madd = 0;
    else {
      push(), l->set(L, R, x), r->set(L, R, x);
      val = max(l->val, r->val);
    }//f3e
  }
  void add(int L, int R, int x) {
    if (R <= lo || hi <= L) return;
    if (L <= lo && hi <= R) {
      if (mset != inf) mset += x;//415
      else madd += x;
      val += x;
    }
    else {
      push(), l->add(L, R, x), r->add(L, R, x);//cac
      val = max(l->val, r->val);
    }
  }
  void push() {
    if (!l) {//53d
      int mid = lo + (hi - lo)/2;
      l = new Node(lo, mid); r = new Node(mid, hi);
    }
```

```
    if (mset != inf)
      l->set(lo,hi,mset), r->set(lo,hi,mset), mset = inf;//
          333
    else if (madd)
      l->add(lo,hi,madd), r->add(lo,hi,madd), madd = 0;
  }
};
```

### UnionFind.h

**Description:** Disjoint-set data structure.
**Time:** $\mathcal{O}(\alpha(N))$

7aa27c, 14 lines

```
struct UF {//3ab
  vi e;
  UF(int n) : e(n, -1) {}
  bool sameSet(int a, int b) { return find(a) == find(b); }
  int size(int x) { return -e[find(x)]; }
  int find(int x) { return e[x] < 0 ? x : e[x] = find(e[x])
      ; }//e16
  bool join(int a, int b) {
    a = find(a), b = find(b);
    if (a == b) return false;
    if (e[a] > e[b]) swap(a, b);
    e[a] += e[b]; e[b] = a;//9da
    return true;
  }
};
```

### UnionFindRollback.h

**Description:** Disjoint-set data structure with undo. If undo is not needed, skip st, time() and rollback().
**Usage:** int t = uf.time(); ...; uf.rollback(t);
**Time:** $\mathcal{O}(\log(N))$

84e98b, 21 lines

```
struct RollbackUF {//f73
  vi e; vector<pii> st;
  RollbackUF(int n) : e(n, -1) {}
  int size(int x) { return -e[find(x)]; }
  int find(int x) { return e[x] < 0 ? x : find(e[x]); }
  int time() { return sz(st); }//cbd
  void rollback(int t) {
    for (int i = time(); i --> t;)
      e[st[i].first] = st[i].second;
    st.resize(t);
  }//e73
  bool join(int a, int b) {
    a = find(a), b = find(b);
    if (a == b) return false;
    if (e[a] > e[b]) swap(a, b);
    st.pb({a, e[a]});//0d8
    st.pb({b, e[b]});
    e[a] += e[b]; e[b] = a;
    return true;
  }
};//214
```

### SubMatrix.h

**Description:** Calculate submatrix sums quickly, given upper-left and lower-right corners (half-open).
**Usage:** SubMatrix<int> m(matrix);
m.sum(0, 0, 2, 2); // top left 4 elements
**Time:** $\mathcal{O}(N^2 + Q)$

c59ada, 13 lines

```
template<class T>//03e
struct SubMatrix {
  vector<vector<T>> p;
  SubMatrix(vector<vector<T>>& v) {
    int R = sz(v), C = sz(v[0]);
    p.assign(R+1, vector<T>(C+1));//4c9
    rep(r,0,R) rep(c,0,C)
      p[r+1][c+1] = v[r][c] + p[r][c+1] + p[r+1][c] - p[r][
          c];
  }
  T sum(int u, int l, int d, int r) {
    return p[d][r] - p[d][l] - p[u][r] + p[u][l];//286
  }
};
```

# Numerical (4)

## 4.1 Polynomials and recurrences

### Polynomial.h

c9b7b0, 17 lines

```
struct Poly {//1b7
  vector<double> a;
  double operator()(double x) const {
    double val = 0;
    for (int i = sz(a); i--;) (val *= x) += a[i];
    return val;//06d
  }
  void diff() {
    rep(i,1,sz(a)) a[i-1] = i*a[i];
    a.pop_back();
  }//b82
  void divroot(double x0) {
    double b = a.back(), c; a.back() = 0;
    for(int i=sz(a)-1; i--;) c = a[i], a[i] = a[i+1]*x0+b,
        b=c;
    a.pop_back();
  }//e03
};
```

### PolyRoots.h

**Description:** Finds the real roots to a polynomial.
**Usage:** polyRoots({{2,-3,1}},-1e9,1e9) // solve x^2-3x+2 = 0
**Time:** $\mathcal{O}\left(n^2 \log(1/\epsilon)\right)$

"Polynomial.h" fbf066, 23 lines

```
vector<double> polyRoots(Poly p, double xmin, double xmax)
    {//840
  if (sz(p.a) == 2) { return {-p.a[0]/p.a[1]}; }
  vector<double> ret;
  Poly der = p;
  der.diff();
  auto dr = polyRoots(der, xmin, xmax);//ec1
  dr.pb(xmin-1);
  dr.pb(xmax+1);
  sort(all(dr));
  rep(i,0,sz(dr)-1) {
    double l = dr[i], h = dr[i+1];//189
    bool sign = p(l) > 0;
    if (sign ^ (p(h) > 0)) {
      rep(it,0,60) { // while (h - l > 1e-8)
        double m = (l + h) / 2, f = p(m);
        if ((f <= 0) ^ sign) l = m;//810
        else h = m;
      }
      ret.pb((l + h) / 2);
    }
  }//808
  return ret;
}
```

### PolyInterpolate.h

**Description:** Given $n$ points (x[i], y[i]), computes an n-1-degree polynomial $p$ that passes through them: $p(x) = a[0]*x^0 + ... + a[n-1]*x^{n-1}$. For numerical precision, pick $x[k] = c*\cos(k/(n-1)*\pi), k = 0 \ldots n-1$.
**Time:** $\mathcal{O}\left(n^2\right)$

08bf48, 13 lines

```
typedef vector<double> vd;//159
vd interpolate(vd x, vd y, int n) {
  vd res(n), temp(n);
  rep(k,0,n-1) rep(i,k+1,n)
    y[i] = (y[i] - y[k]) / (x[i] - x[k]);
  double last = 0; temp[0] = 1;//746
  rep(k,0,n) rep(i,0,n) {
    res[i] += y[k] * temp[i];
    swap(last, temp[i]);
    temp[i] -= last * x[k];
  }//0e1
  return res;
}
```

### BerlekampMassey.h

**Description:** Recovers any $n$-order linear recurrence relation from the first $2n$ terms of the recurrence. Useful for guessing linear recurrences after brute-forcing the first terms. Should work on any field, but numerical stability for floats is not guaranteed. Output will have size $\leq n$.
**Usage:** berlekampMassey({0, 1, 1, 3, 5, 11}) // {1, 2}
**Time:** $\mathcal{O}(N^2)$

"../number-theory/ModPow.h" 96548b, 20 lines

```
vector<ll> berlekampMassey(vector<ll> s) {//b21
  int n = sz(s), L = 0, m = 0;
  vector<ll> C(n), B(n), T;
  C[0] = B[0] = 1;

  ll b = 1;//4c7
  rep(i,0,n) { ++m;
    ll d = s[i] % mod;
    rep(j,1,L+1) d = (d + C[j] * s[i - j]) % mod;
    if (!d) continue;
    T = C; ll coef = d * modpow(b, mod-2) % mod;//1b2
    rep(j,m,n) C[j] = (C[j] - coef * B[j - m]) % mod;
    if (2 * L > i) continue;
    L = i + 1 - L; B = T; b = d; m = 0;
  }
//255
  C.resize(L + 1); C.erase(C.begin());
  for (ll& x : C) x = (mod - x) % mod;
  return C;
}
```

### LinearRecurrence.h

**Description:** Generates the $k$'th term of an $n$-order linear recurrence $S[i] = \sum_j S[i-j-1]tr[j]$, given $S[0 \ldots \geq n-1]$ and $tr[0 \ldots n-1]$. Faster than matrix multiplication. Useful together with Berlekamp–Massey.
**Usage:** linearRec({0, 1}, {1, 1}, k) // k'th Fibonacci number
**Time:** $\mathcal{O}\left(n^2 \log k\right)$

f4e444, 26 lines

```
typedef vector<ll> Poly;//bb1
ll linearRec(Poly S, Poly tr, ll k) {
  int n = sz(tr);

  auto combine = [&](Poly a, Poly b) {
    Poly res(n * 2 + 1);//251
    rep(i,0,n+1) rep(j,0,n+1)
      res[i + j] = (res[i + j] + a[i] * b[j]) % mod;
    for (int i = 2 * n; i > n; --i) rep(j,0,n)
      res[i - 1 - j] = (res[i - 1 - j] + res[i] * tr[j]) %
          mod;
    res.resize(n + 1);//12f
    return res;
  };

  Poly pol(n + 1), e(pol);
  pol[0] = e[1] = 1;//df7

  for (++k; k; k /= 2) {
    if (k % 2) pol = combine(pol, e);
    e = combine(e, e);
```

```
  }//c0e

  ll res = 0;
  rep(i,0,n) res = (res + pol[i + 1] * S[i]) % mod;
  return res;
}//cbb
```

## 4.2 Optimization

### GoldenSectionSearch.h

**Description:** Finds the argument minimizing the function $f$ in the interval $[a, b]$ assuming $f$ is unimodal on the interval, i.e. has only one local minimum and no local maximum. The maximum error in the result is *eps*. Works equally well for maximization with a small change in the code. See TernarySearch.h in the Various chapter for a discrete version.

**Usage:** double func(double x) { return 4+x+.3*x*x; }
double xmin = gss(-1000,1000,func);

**Time:** $\mathcal{O}(\log((b-a)/\epsilon))$

d7b114, 15 lines

```
template<class F>//5c6
double gss(double a, double b, F f) {
  double r = (sqrt(5)-1)/2, eps = 1e-7;
  double x1 = b - r*(b-a), x2 = a + r*(b-a);
  double f1 = f(x1), f2 = f(x2);
  while (b-a > eps)//905
    if (f1 < f2) { //change to > to find maximum
      b = x2; x2 = x1; f2 = f1;
      x1 = b - r*(b-a); f1 = f(x1);
    } else {
      a = x1; x1 = x2; f1 = f2;//00c
      x2 = a + r*(b-a); f2 = f(x2);
    }
  return a;
}
```

### HillClimbing.h

**Description:** Poor man's optimization for unimodal functions.

8eeeaf, 14 lines

```
typedef array<double, 2> P;//68a

template<class F> pair<double, P> hillClimb(P start, F f) {
  pair<double, P> cur(f(start), start);
  for (double jmp = 1e9; jmp > 1e-20; jmp /= 2) {
    rep(j,0,100) rep(dx,-1,2) rep(dy,-1,2) {//2dc
      P p = cur.second;
      p[0] += dx*jmp;
      p[1] += dy*jmp;
      cur = min(cur, make_pair(f(p), p));
    }//a63
  }
  return cur;
}
```

### Integrate.h

**Description:** Simple integration of a function over an interval using Simpson's rule. The error should be proportional to $h^4$, although in practice you will want to verify that the result is stable to desired precision when epsilon changes.

4756fc, 7 lines

```
template<class F>//e93
double quad(double a, double b, F f, const int n = 1000) {
  double h = (b - a) / 2 / n, v = f(a) + f(b);
  rep(i,1,n*2)
    v += f(a + i*h) * (i&1 ? 4 : 2);
  return v * h / 3;//2d2
}
```

### IntegrateAdaptive.h

**Description:** Fast integration using an adaptive Simpson's rule.

**Usage:** double sphereVolume = quad(-1, 1, [](double x) {
return quad(-1, 1, [&](double y) {
return quad(-1, 1, [&](double z) {
return x*x + y*y + z*z < 1; });});});

92dd79, 15 lines

```
typedef double d;//e70
#define S(a,b) (f(a) + 4*f((a+b) / 2) + f(b)) * (b-a) / 6

template <class F>
d rec(F& f, d a, d b, d eps, d S) {
  d c = (a + b) / 2;//b17
  d S1 = S(a, c), S2 = S(c, b), T = S1 + S2;
  if (abs(T - S) <= 15 * eps || b - a < 1e-10)
    return T + (T - S) / 15;
  return rec(f, a, c, eps / 2, S1) + rec(f, c, b, eps / 2,
      S2);
}//836
template<class F>
d quad(d a, d b, F f, d eps = 1e-8) {
  return rec(f, a, b, eps, S(a, b));
}
```

### Simplex.h

**Description:** Solves a general linear maximization problem: maximize $c^T x$ subject to $Ax \le b$, $x \ge 0$. Returns -inf if there is no solution, inf if there are arbitrarily good solutions, or the maximum value of $c^T x$ otherwise. The input vector is set to an optimal $x$ (or in the unbounded case, an arbitrary solution fulfilling the constraints). Numerical stability is not guaranteed. For better performance, define variables such that $x = 0$ is viable.

**Usage:** vvd A = {{1,-1}, {-1,1}, {-1,-2}};
vd b = {1,1,-4}, c = {-1,-1}, x;
T val = LPSolver(A, b, c).solve(x);

**Time:** $\mathcal{O}(NM * \#pivots)$, where a pivot may be e.g. an edge relaxation. $\mathcal{O}(2^n)$ in the general case.

aa8530, 68 lines

```
typedef double T; // long double, Rational, double + mod<P
      >...//629
typedef vector<T> vd;
typedef vector<vd> vvd;

const T eps = 1e-8, inf = 1/.0;
#define MP make_pair//94e
#define ltj(X) if(s == -1 || MP(X[j],N[j]) < MP(X[s],N[s]))
      s=j

struct LPSolver {
  int m, n;
  vi N, B;//282
  vvd D;

  LPSolver(const vvd& A, const vd& b, const vd& c) :
    m(sz(b)), n(sz(c)), N(n+1), B(m), D(m+2, vd(n+2)) {
      rep(i,0,m) rep(j,0,n) D[i][j] = A[i][j];//108
      rep(i,0,m) { B[i] = n+i; D[i][n] = -1; D[i][n+1] = b[
          i];}
      rep(j,0,n) { N[j] = j; D[m][j] = -c[j]; }
      N[n] = -1; D[m+1][n] = 1;
    }
//9c3
  void pivot(int r, int s) {
    T *a = D[r].data(), inv = 1 / a[s];
    rep(i,0,m+2) if (i != r && abs(D[i][s]) > eps) {
      T *b = D[i].data(), inv2 = b[s] * inv;
      rep(j,0,n+2) b[j] -= a[j] * inv2;//d0d
      b[s] = a[s] * inv2;
    }
    rep(j,0,n+2) if (j != s) D[r][j] *= inv;
    rep(i,0,m+2) if (i != r) D[i][s] *= -inv;
    D[r][s] = inv;//aa5
    swap(B[r], N[s]);
  }

  bool simplex(int phase) {
    int x = m + phase - 1;//c51
    for (;;) {
      int s = -1;
      rep(j,0,n+1) if (N[j] != -phase) ltj(D[x]);
      if (D[x][s] >= -eps) return true;
      int r = -1;//bc0
      rep(i,0,m) {
        if (D[i][s] <= eps) continue;
        if (r == -1 || MP(D[i][n+1] / D[i][s], B[i])
                    < MP(D[r][n+1] / D[r][s], B[r])) r = i
                        ;
      }//00c
      if (r == -1) return false;
      pivot(r, s);
    }
  }
//d2f
  T solve(vd &x) {
    int r = 0;
    rep(i,1,m) if (D[i][n+1] < D[r][n+1]) r = i;
    if (D[r][n+1] < -eps) {
      pivot(r, n);//f81
      if (!simplex(2) || D[m+1][n+1] < -eps) return -inf;
      rep(i,0,m) if (B[i] == -1) {
        int s = 0;
        rep(j,1,n+1) ltj(D[i]);
        pivot(i, s);//866
      }
    }
    bool ok = simplex(1); x = vd(n);
    rep(i,0,m) if (B[i] < n) x[B[i]] = D[i][n+1];
    return ok ? D[m][n+1] : inf;//401
  }
};
```

## 4.3 Matrices

### Determinant.h

**Description:** Calculates determinant of a matrix. Destroys the matrix.

**Time:** $\mathcal{O}(N^3)$

bd5cec, 15 lines

```
double det(vector<vector<double>>& a) {//309
  int n = sz(a); double res = 1;
  rep(i,0,n) {
    int b = i;
    rep(j,i+1,n) if (fabs(a[j][i]) > fabs(a[b][i])) b = j;
    if (i != b) swap(a[i], a[b]), res *= -1;//454
    res *= a[i][i];
    if (res == 0) return 0;
    rep(j,i+1,n) {
      double v = a[j][i] / a[i][i];
      if (v != 0) rep(k,i+1,n) a[j][k] -= v * a[i][k];//07b
    }
  }
  return res;
}
```

### IntDeterminant.h

**Description:** Calculates determinant using modular arithmetics. Modulos can also be removed to get a pure-integer version.

**Time:** $\mathcal{O}(N^3)$

3313dc, 18 lines

```
const ll mod = 12345;//cab
ll det(vector<vector<ll>>& a) {
  int n = sz(a); ll ans = 1;
  rep(i,0,n) {
    rep(j,i+1,n) {
      while (a[j][i] != 0) { // gcd step//c65
        ll t = a[i][i] / a[j][i];
        if (t) rep(k,i,n)
```

```
        a[i][k] = (a[i][k] - a[j][k] * t) % mod;
      swap(a[i], a[j]);
      ans *= -1;//bc6
    }
  }
  ans = ans * a[i][i] % mod;
  if (!ans) return 0;
}//b19
return (ans + mod) % mod;
}
```

## SolveLinear.h

**Description:** Solves $A * x = b$. If there are multiple solutions, an arbitrary one is returned. Returns rank, or -1 if no solutions. Data in $A$ and $b$ is lost.
**Time:** $\mathcal{O}(n^2m)$

44c9ab, 38 lines

```
typedef vector<double> vd;//2cf
const double eps = 1e-12;

int solveLinear(vector<vd>& A, vd& b, vd& x) {
  int n = sz(A), m = sz(x), rank = 0, br, bc;
  if (n) assert(sz(A[0]) == m);//940
  vi col(m); iota(all(col), 0);

  rep(i,0,n) {
    double v, bv = 0;
    rep(r,i,n) rep(c,i,m)//ddb
      if ((v = fabs(A[r][c])) > bv)
        br = r, bc = c, bv = v;
    if (bv <= eps) {
      rep(j,i,n) if (fabs(b[j]) > eps) return -1;
      break;//de0
    }
    swap(A[i], A[br]);
    swap(b[i], b[br]);
    swap(col[i], col[bc]);
    rep(j,0,n) swap(A[j][i], A[j][bc]);//328
    bv = 1/A[i][i];
    rep(j,i+1,n) {
      double fac = A[j][i] * bv;
      b[j] -= fac * b[i];
      rep(k,i+1,m) A[j][k] -= fac*A[i][k];//af1
    }
    rank++;
  }

  x.assign(m, 0);//3c5
  for (int i = rank; i--;) {
    b[i] /= A[i][i];
    x[col[i]] = b[i];
    rep(j,0,i) b[j] -= A[j][i] * b[i];
  }//807
  return rank; // (multiple solutions if rank < m)
}
```

## SolveLinear2.h

**Description:** To get all uniquely determined values of $x$ back from SolveLinear, make the following changes:

"SolveLinear.h" 08e495, 7 lines

```
rep(j,0,n) if (j != i) // instead of rep(j,i+1,n)//22b
// ... then at the end:
x.assign(m, undefined);
rep(i,0,rank) {
  rep(j,rank,m) if (fabs(A[i][j]) > eps) goto fail;
  x[col[i]] = b[i] / A[i][i];//4e3
fail:; }
```

## SolveLinearBinary.h

**Description:** Solves $Ax = b$ over $\mathbb{F}_2$. If there are multiple solutions, one is returned arbitrarily. Returns rank, or -1 if no solutions. Destroys $A$ and $b$.
**Time:** $\mathcal{O}(n^2m)$

fa2d7a, 34 lines

```
typedef bitset<1000> bs;//d90

int solveLinear(vector<bs>& A, vi& b, bs& x, int m) {
  int n = sz(A), rank = 0, br;
  assert(m <= sz(x));
  vi col(m); iota(all(col), 0);//2c9
  rep(i,0,n) {
    for (br=i; br<n; ++br) if (A[br].any()) break;
    if (br == n) {
      rep(j,i,n) if(b[j]) return -1;
      break;//13e
    }
    int bc = (int)A[br]._Find_next(i-1);
    swap(A[i], A[br]);
    swap(b[i], b[br]);
    swap(col[i], col[bc]);//b88
    rep(j,0,n) if (A[j][i] != A[j][bc]) {
      A[j].flip(i); A[j].flip(bc);
    }
    rep(j,i+1,n) if (A[j][i]) {
      b[j] ^= b[i];//76c
      A[j] ^= A[i];
    }
    rank++;
  }
//7a7
  x = bs();
  for (int i = rank; i--;) {
    if (!b[i]) continue;
    x[col[i]] = 1;
    rep(j,0,i) b[j] ^= A[j][i];//df7
  }
  return rank; // (multiple solutions if rank < m)
}
```

## MatrixInverse.h

**Description:** Invert matrix $A$. Returns rank; result is stored in $A$ unless singular (rank < n). Can easily be extended to prime moduli; for prime powers, repeatedly set $A^{-1} = A^{-1}(2I - AA^{-1}) \pmod{p^k}$ where $A^{-1}$ starts as the inverse of A mod p, and k is doubled in each step.
**Time:** $\mathcal{O}(n^3)$

ebfff6, 35 lines

```
int matInv(vector<vector<double>>& A) {//9a9
  int n = sz(A); vi col(n);
  vector<vector<double>> tmp(n, vector<double>(n));
  rep(i,0,n) tmp[i][i] = 1, col[i] = i;

  rep(i,0,n) {//214
    int r = i, c = i;
    rep(j,i,n) rep(k,i,n)
      if (fabs(A[j][k]) > fabs(A[r][c]))
        r = j, c = k;
    if (fabs(A[r][c]) < 1e-12) return i;//e5b
    A[i].swap(A[r]); tmp[i].swap(tmp[r]);
    rep(j,0,n)
      swap(A[j][i], A[j][c]), swap(tmp[j][i], tmp[j][c]);
    swap(col[i], col[c]);
    double v = A[i][i];//afc
    rep(j,i+1,n) {
      double f = A[j][i] / v;
      A[j][i] = 0;
      rep(k,i+1,n) A[j][k] -= f*A[i][k];
      rep(k,0,n) tmp[j][k] -= f*tmp[i][k];//c80
    }
    rep(j,i+1,n) A[i][j] /= v;
    rep(j,0,n) tmp[i][j] /= v;
    A[i][i] = 1;
  }//bfb

  for (int i = n-1; i > 0; --i) rep(j,0,i) {
    double v = A[j][i];
    rep(k,0,n) tmp[j][k] -= v*tmp[i][k];
  }//e74

  rep(i,0,n) rep(j,0,n) A[col[i]][col[j]] = tmp[i][j];
  return n;
}
```

## MatrixInverse-mod.h

**Description:** Invert matrix $A$ modulo a prime. Returns rank; result is stored in $A$ unless singular (rank < n). For prime powers, repeatedly set $A^{-1} = A^{-1}(2I - AA^{-1}) \pmod{p^k}$ where $A^{-1}$ starts as the inverse of A mod p, and k is doubled in each step.
**Time:** $\mathcal{O}(n^3)$

"../number-theory/ModPow.h" 0b7b13, 37 lines

```
int matInv(vector<vector<ll>>& A) {//ebd
  int n = sz(A); vi col(n);
  vector<vector<ll>> tmp(n, vector<ll>(n));
  rep(i,0,n) tmp[i][i] = 1, col[i] = i;

  rep(i,0,n) {//79d
    int r = i, c = i;
    rep(j,i,n) rep(k,i,n) if (A[j][k]) {
      r = j; c = k; goto found;
    }
    return i;//4e3
found:
    A[i].swap(A[r]); tmp[i].swap(tmp[r]);
    rep(j,0,n)
      swap(A[j][i], A[j][c]), swap(tmp[j][i], tmp[j][c]);
    swap(col[i], col[c]);//416
    ll v = modpow(A[i][i], mod - 2);
    rep(j,i+1,n) {
      ll f = A[j][i] * v % mod;
      A[j][i] = 0;
      rep(k,i+1,n) A[j][k] = (A[j][k] - f*A[i][k]) % mod;//
          9f7
      rep(k,0,n) tmp[j][k] = (tmp[j][k] - f*tmp[i][k]) %
          mod;
    }
    rep(j,i+1,n) A[i][j] = A[i][j] * v % mod;
    rep(j,0,n) tmp[i][j] = tmp[i][j] * v % mod;
    A[i][i] = 1;//e3d
  }

  for (int i = n-1; i > 0; --i) rep(j,0,i) {
    ll v = A[j][i];
    rep(k,0,n) tmp[j][k] = (tmp[j][k] - v*tmp[i][k]) % mod;
        //4b2
  }

  rep(i,0,n) rep(j,0,n)
    A[col[i]][col[j]] = tmp[i][j] % mod + (tmp[i][j] < 0)*
        mod;
  return n;//400
}
```

## Tridiagonal.h

**Description:** $x = \text{tridiagonal}(d, p, q, b)$ solves the equation system

$$\begin{pmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_{n-1} \end{pmatrix} = \begin{pmatrix} d_0 & p_0 & 0 & 0 & \cdots & 0 \\ q_0 & d_1 & p_1 & 0 & \cdots & 0 \\ 0 & q_1 & d_2 & p_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & q_{n-3} & d_{n-2} & p_{n-2} \\ 0 & 0 & \cdots & 0 & q_{n-2} & d_{n-1} \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_{n-1} \end{pmatrix}.$$

This is useful for solving problems on the type

$$a_i = b_i a_{i-1} + c_i a_{i+1} + d_i,\ 1 \le i \le n,$$

where $a_0$, $a_{n+1}$, $b_i$, $c_i$ and $d_i$ are known. $a$ can then be obtained from

$$\{a_i\} = \text{tridiagonal}(\{1, -1, -1, ..., -1, 1\}, \{0, c_1, c_2, \dots, c_n\},$$
$$\{b_1, b_2, \dots, b_n, 0\}, \{a_0, d_1, d_2, \dots, d_n, a_{n+1}\}).$$

Fails if the solution is not unique.
If $|d_i| > |p_i| + |q_{i-1}|$ for all $i$, or $|d_i| > |p_{i-1}| + |q_i|$, or the matrix is positive definite, the algorithm is numerically stable and neither `tr` nor the check for `diag[i] == 0` is needed.
**Time:** $\mathcal{O}(N)$

8f9fa8, 26 lines

```
typedef double T;//399
vector<T> tridiagonal(vector<T> diag, const vector<T>&
    super,
    const vector<T>& sub, vector<T> b) {
  int n = sz(b); vi tr(n);
  rep(i,0,n-1) {
    if (abs(diag[i]) < 1e-9 * abs(super[i])) { // diag[i]
        == 0//464
      b[i+1] -= b[i] * diag[i+1] / super[i];
      if (i+2 < n) b[i+2] -= b[i] * sub[i+1] / super[i];
      diag[i+1] = sub[i]; tr[++i] = 1;
    } else {
      diag[i+1] -= super[i]*sub[i]/diag[i];//d50
      b[i+1] -= b[i]*sub[i]/diag[i];
    }
  }
  for (int i = n; i--;) {
    if (tr[i]) {//054
      swap(b[i], b[i-1]);
      diag[i-1] = diag[i];
      b[i] /= super[i-1];
    } else {
      b[i] /= diag[i];//20b
      if (i) b[i-1] -= b[i]*super[i-1];
    }
  }
  return b;
}//cbb
```

## 4.4 Fourier transforms

### FastFourierTransform.h

**Description:** fft(a) computes $\hat{f}(k) = \sum_x a[x] \exp(2\pi i \cdot kx/N)$ for all $k$. N must be a power of 2. Useful for convolution: `conv(a, b) = c`, where $c[x] = \sum a[i]b[x-i]$. For convolution of complex numbers or more than two vectors: FFT, multiply pointwise, divide by n, reverse(start+1, end), FFT back. Rounding is safe if $(\sum a_i^2 + \sum b_i^2)\log_2 N < 9 \cdot 10^{14}$ (in practice $10^{16}$; higher for random inputs). Otherwise, use NTT/FFT-Mod.
**Time:** $\mathcal{O}(N \log N)$ with $N = |A| + |B|$ ($\sim$1s for $N = 2^{22}$)

00ced6, 35 lines

```
typedef complex<double> C;//1ec
typedef vector<double> vd;
void fft(vector<C>& a) {
  int n = sz(a), L = 31 - __builtin_clz(n);
  static vector<complex<long double>> R(2, 1);
  static vector<C> rt(2, 1);  // (^ 10% faster if double)//
      c50
  for (static int k = 2; k < n; k *= 2) {
    R.resize(n); rt.resize(n);
    auto x = polar(1.0L, acos(-1.0L) / k);
    rep(i,k,2*k) rt[i] = R[i] = i&1 ? R[i/2] * x : R[i/2];
  }//292
  vi rev(n);
  rep(i,0,n) rev[i] = (rev[i / 2] | (i & 1) << L) / 2;
  rep(i,0,n) if (i < rev[i]) swap(a[i], a[rev[i]]);
  for (int k = 1; k < n; k *= 2)
    for (int i = 0; i < n; i += 2 * k) rep(j,0,k) {//577
      C z = rt[j+k] * a[i+j+k]; // (25% faster if hand-
          rolled)
      a[i + j + k] = a[i + j] - z;
      a[i + j] += z;
    }
}//15f
vd conv(const vd& a, const vd& b) {
  if (a.empty() || b.empty()) return {};
  vd res(sz(a) + sz(b) - 1);
  int L = 32 - __builtin_clz(sz(res)), n = 1 << L;
  vector<C> in(n), out(n);//d93
  copy(all(a), begin(in));
  rep(i,0,sz(b)) in[i].imag(b[i]);
  fft(in);
  for (C& x : in) x *= x;
  rep(i,0,n) out[i] = in[-i & (n - 1)] - conj(in[i]);//36e
  fft(out);
  rep(i,0,sz(res)) res[i] = imag(out[i]) / (4 * n);
  return res;
}
```

### FastFourierTransformMod.h

**Description:** Higher precision FFT, can be used for convolutions modulo arbitrary integers as long as $N \log_2 N \cdot \text{mod} < 8.6 \cdot 10^{14}$ (in practice $10^{16}$ or higher). Inputs must be in $[0, \text{mod})$.
**Time:** $\mathcal{O}(N \log N)$, where $N = |A| + |B|$ (twice as slow as NTT or FFT)

"FastFourierTransform.h" b82773, 22 lines

```
typedef vector<ll> vl;//2c4
template<int M> vl convMod(const vl &a, const vl &b) {
  if (a.empty() || b.empty()) return {};
  vl res(sz(a) + sz(b) - 1);
  int B=32-__builtin_clz(sz(res)), n=1<<B, cut=int(sqrt(M))
      ;
  vector<C> L(n), R(n), outs(n), outl(n);//c4f
  rep(i,0,sz(a)) L[i] = C((int)a[i] / cut, (int)a[i] % cut)
      ;
  rep(i,0,sz(b)) R[i] = C((int)b[i] / cut, (int)b[i] % cut)
      ;
  fft(L), fft(R);
  rep(i,0,n) {
    int j = -i & (n - 1);//3eb
    outl[j] = (L[i] + conj(L[j])) * R[i] / (2.0 * n);
    outs[j] = (L[i] - conj(L[j])) * R[i] / (2.0 * n) / 1i;
  }
  fft(outl), fft(outs);
  rep(i,0,sz(res)) {//58f
    ll av = ll(real(outl[i])+.5), cv = ll(imag(outs[i])+.5)
        ;
    ll bv = ll(imag(outl[i])+.5) + ll(real(outs[i])+.5);
    res[i] = ((av % M * cut + bv) % M * cut + cv) % M;
  }
  return res;//510
}
```

### NumberTheoreticTransform.h

**Description:** ntt(a) computes $\hat{f}(k) = \sum_x a[x]g^{xk}$ for all $k$, where $g = \text{root}^{(mod-1)/N}$. N must be a power of 2. Useful for convolution modulo specific nice primes of the form $2^a b + 1$, where the convolution result has size at most $2^a$. For arbitrary modulo, see FFTMod. `conv(a, b) = c`, where $c[x] = \sum a[i]b[x-i]$. For manual convolution: NTT the inputs, multiply pointwise, divide by n, reverse(start+1, end), NTT back. Inputs must be in [0, mod).
**Time:** $\mathcal{O}(N \log N)$

"../number-theory/ModPow.h" ced03d, 35 lines

```
const ll mod = (119 << 23) + 1, root = 62; // =
    998244353//0ca
// For p < 2^30 there is also e.g. 5 << 25, 7 << 26, 479 <<
    21
// and 483 << 21 (same root). The last two are > 10^9.
typedef vector<ll> vl;
void ntt(vl &a) {
  int n = sz(a), L = 31 - __builtin_clz(n);//cc5
  static vl rt(2, 1);
  for (static int k = 2, s = 2; k < n; k *= 2, s++) {
    rt.resize(n);
    ll z[] = {1, modpow(root, mod >> s)};
    rep(i,k,2*k) rt[i] = rt[i / 2] * z[i & 1] % mod;//4a0
  }
  vi rev(n);
  rep(i,0,n) rev[i] = (rev[i / 2] | (i & 1) << L) / 2;
  rep(i,0,n) if (i < rev[i]) swap(a[i], a[rev[i]]);
  for (int k = 1; k < n; k *= 2)//ed7
    for (int i = 0; i < n; i += 2 * k) rep(j,0,k) {
      ll z = rt[j + k] * a[i + j + k] % mod, &ai = a[i + j
          ];
      a[i + j + k] = ai - z + (z > ai ? mod : 0);
      ai += (ai + z >= mod ? z - mod : z);
    }//dfc
}
vl conv(const vl &a, const vl &b) {
  if (a.empty() || b.empty()) return {};
  int s = sz(a) + sz(b) - 1, B = 32 - __builtin_clz(s),
    n = 1 << B;//d58
  int inv = modpow(n, mod - 2);
  vl L(a), R(b), out(n);
  L.resize(n), R.resize(n);
  ntt(L), ntt(R);
  rep(i,0,n)//f18
    out[-i & (n - 1)] = (ll)L[i] * R[i] % mod * inv % mod;
  ntt(out);
  return {out.begin(), out.begin() + s};
}
```

### FastSubsetTransform.h

**Description:** Transform to a basis with fast convolutions of the form $c[z] = \sum_{z = x \oplus y} a[x] \cdot b[y]$, where $\oplus$ is one of AND, OR, XOR. The size of $a$ must be a power of two.
**Time:** $\mathcal{O}(N \log N)$

464cf3, 16 lines

```
void FST(vi& a, bool inv) {//ae8
  for (int n = sz(a), step = 1; step < n; step *= 2) {
    for (int i = 0; i < n; i += 2 * step) rep(j,i,i+step) {
      int &u = a[j], &v = a[j + step]; tie(u, v) =
        inv ? pii(v - u, u) : pii(v, u + v); // AND
        inv ? pii(v, u - v) : pii(u + v, u); // OR//0af
        pii(u + v, u - v);                   // XOR
    }
  }
  if (inv) for (int& x : a) x /= sz(a); // XOR only
}//dc4
vi conv(vi a, vi b) {
  FST(a, 0); FST(b, 0);
  rep(i,0,sz(a)) a[i] *= b[i];
  FST(a, 1); return a;
}//cbb
```

## 4.5 Polynomial

### PolyBase.h

**Description:** A FFT based Polynomial class.

"../number-theory/ModularArithmetic.h", "FastFourierTransform.h", "FastFourierTransformMod.h", "NumberTheoreticTransform.h" dd1be7, 35 lines

```
typedef Mod num;//810
typedef vector<num> poly;
poly &operator+=(poly &a, const poly &b) {
  a.resize(max(sz(a), sz(b)));
  rep(i, 0, sz(b)) a[i] = a[i] + b[i];
  return a;//8a9
}
poly &operator-=(poly &a, const poly &b) {
  a.resize(max(sz(a), sz(b)));
  rep(i, 0, sz(b)) a[i] = a[i] - b[i];
  return a;//e10
}

poly &operator*=(poly &a, const poly &b) {
  if (sz(a) + sz(b) < 100){
    poly res(sz(a) + sz(b) - 1);//025
    rep(i,0,sz(a)) rep(j,0,sz(b))
      res[i + j] = (res[i + j] + a[i] * b[j]);
    return (a = res);
  }
  // auto res = convMod<mod>(vl(all(a)), vl(all(b)));//0cb
  auto res = conv(vl(all(a)), vl(all(b)));
  return (a = poly(all(res)));
}
poly operator*(poly a, const num b) {
  poly c = a;//41c
  for(auto& i : c) i = i * b;
  return c;
}
#define OP(o, oe) \
  poly operator o(poly a, poly b) { \//f19
    poly c = a; \
    return c o##= b; \
  }
OP(*, *=) OP(+, +=) OP(-, -=);
```

### PolyEvaluate.h

**Description:** Multi-point evaluation. Evaluates a given polynomial A at $A(x_0), \ldots A(x_n)$.
**Time:** $\mathcal{O}\left(n \log^2 n\right)$

"PolyBase.h", "PolyMod.h" dc2cdf, 14 lines

```
vector<num> eval(const poly &a, const vector<num> &x) {//9fa
  int n = sz(x);
  if (!n) return {};
  vector<poly> up(2 * n);
  rep(i, 0, n) up[i + n] = poly({num(0) - x[i], 1});
  for (int i = n - 1; i > 0; i--)//923
    up[i] = up[2 * i] * up[2 * i + 1];
  vector<poly> down(2 * n);
  down[1] = a % up[1];
  rep(i, 2, 2 * n) down[i] = down[i / 2] % up[i];
  vector<num> y(n);//835
  rep(i, 0, n) y[i] = down[i + n][0];
  return y;
}
```

### PolyIntegDeriv.h

**Description:** Calculate $\frac{da}{dx}$ and $\int a\,dx$.
**Time:** $\mathcal{O}(|a|)$

"PolyBase.h" 803fd5, 14 lines

```
poly deriv(poly a) {//aba
  if (a.empty()) return {};
  poly b(sz(a) - 1);
  rep(i, 1, sz(a)) b[i - 1] = a[i] * num(i);
  return b;
}//6f9
poly integr(poly a) {
  if (a.empty()) return {0};
  poly b(sz(a) + 1);
  b[1] = num(1);
  rep(i, 2, sz(b)) b[i] = b[mod%i]*Mod(-mod/i+mod);//176
  rep(i, 1 ,sz(b)) b[i] = a[i-1] * b[i];
  return b;
}
```

### PolyInterpolate2.h

**Description:** Given $n$ points (x[i], y[i]), computes an n-1-degree polynomial $p$ that passes through them: $p(x) = a[0] \cdot x^0 + \ldots + a[n-1] \cdot x^{n-1}$.
**Time:** $\mathcal{O}\left(n \log^2 n\right)$

"PolyBase.h", "PolyIntegDeriv.h", "PolyEvaluate.h" b911f5, 11 lines

```
poly interp(vector<num> x, vector<num> y) {//74d
  int n=sz(x);
  vector<poly> up(n*2);
  rep(i,0,n) up[i+n] = poly({num(0)-x[i], num(1)});
  for(int i=n-1; i>0;i--) up[i] = up[2*i]*up[2*i+1];
  vector<num> a = eval(deriv(up[1]), x);//6dd
  vector<poly> down(2*n);
  rep(i,0,n) down[i+n] = poly({y[i]*(num(1)/a[i])});
  for(int i=n-1;i>0;i--) down[i] = down[i*2] * up[i*2+1] +
      down[i*2+1] * up[i*2];
  return down[1];
}//cbb
```

### PolyInverse.h

**Description:** Calculate the first $|a|$ coefficients of $a^{-1}$.
**Time:** $\mathcal{O}(n \log n)$.

"PolyBase.h" 703c16, 7 lines

```
poly modK(poly a, int k) { return {a.begin(), a.begin() +
    min(k, sz(a))}; }//140
poly inverse(poly A) {
  poly B = poly({num(1) / A[0]});
  while (sz(B) < sz(A))
    B = modK(B * (poly({num(2)}) - modK(A, 2*sz(B)) * B), 2
        * sz(B));
  return modK(B, sz(A));//556
}
```

### PolyLogExp.h

**Description:** Calculate the first $|a|$ coefficients of $a^{-1}$ of $\log a$ and $\exp a$.
**Time:** $\mathcal{O}(n \log n)$.

"PolyBase.h", "PolyInverse.h", "PolyIntegDeriv.h" 83ea75, 14 lines

```
poly log(poly a) {//9c1
  return modK(integr(deriv(a) * inverse(a)), sz(a));
}
poly exp(poly a) {
  poly b(1, num(1));
  if (a.empty())//8ff
    return b;
  while (sz(b) < sz(a)) {
    b.resize(sz(b) * 2);
    b *= (poly({num(1)}) + modK(a, sz(b)) - log(b));
    b.resize(sz(b) / 2 + 1);//1f2
  }
  return modK(b, sz(a));
}
```

### PolyMod.h

**Description:** Calculate the remainder and quotient of the Euclidean division $\frac{a}{b}$.
**Time:** $\mathcal{O}(n \log n)$.

"PolyBase.h", "PolyInverse.h" 264551, 20 lines

```
poly &operator/=(poly &a, poly b) {//b9b
  if (sz(a) < sz(b))
    return a = {};
  int s = sz(a) - sz(b) + 1;
  reverse(all(a)), reverse(all(b));
  a.resize(s), b.resize(s);//e15
  a = a * inverse(b);
  a.resize(s), reverse(all(a));
  return a;
}
OP(/, /=)//9fe
poly &operator%=(poly &a, poly &b) {
  if (sz(a) < sz(b))
    return a;
  poly c = (a / b) * b;
  a.resize(sz(b) - 1);//f62
  rep(i, 0, sz(a)) a[i] = a[i] - c[i];
  return a;
}
OP(%, %=)
```

### PolyPow.h

**Description:** Calculate the first $|a|$ coeffients of $a^m$.
**Time:** $\mathcal{O}(n \log n)$.

"PolyBase.h", "PolyLogExp.h" f0005c, 13 lines

```
poly pow(poly a, ll m) {//760
  int p = 0, n = sz(a);
  while (p < sz(a) && a[p].v == 0)
    ++p;
  if (ll(m)*p >= sz(a)) return poly(sz(a));
  num j = a[p];//a78
  a = {a.begin() + p, a.end()};
  a = a * (num(1) / j);
  a.resize(n);
  auto res = exp(log(a) * num(m)) * (j ^ m);
  res.insert(res.begin(), p*m, 0);//6a6
  return {res.begin(), res.begin()+n};
}
```

### PolyRoots.h

**Description:** Finds the real roots to a polynomial.
**Usage:** polyRoots({{2,-3,1}},-1e9,1e9) // solve x^2-3x+2 = 0
**Time:** $\mathcal{O}\left(n^2 \log(1/\epsilon)\right)$

"Polynomial.h" fbf066, 23 lines

```
vector<double> polyRoots(Poly p, double xmin, double xmax)
    {//840
  if (sz(p.a) == 2) { return {-p.a[0]/p.a[1]}; }
  vector<double> ret;
  Poly der = p;
  der.diff();
  auto dr = polyRoots(der, xmin, xmax);//ec1
  dr.pb(xmin-1);
  dr.pb(xmax+1);
  sort(all(dr));
  rep(i,0,sz(dr)-1) {
    double l = dr[i], h = dr[i+1];//189
    bool sign = p(l) > 0;
    if (sign ^ (p(h) > 0)) {
      rep(it,0,60) { // while (h - l > 1e-8)
        double m = (l + h) / 2, f = p(m);
        if ((f <= 0) ^ sign) l = m;//810
        else h = m;
      }
      ret.pb((l + h) / 2);
    }
  }//808
  return ret;
}
```

# Number theory (5)

## 5.1 Modular arithmetic

### ModInverse.h

**Description:** Pre-computation of modular inverses. Assumes LIM $\leq$ mod and that mod is a prime.

b4a981, 2 lines

```
ll* inv = new ll[LIM] - 1; inv[1] = 1;//b4a
rep(i,2,LIM) inv[i] = mod - (mod / i) * inv[mod % i] % mod;
```

### ModPow.h

7b7908, 8 lines

```
const int mod = 1000000007; // faster if const//dce

ll modpow(ll b, ll e) {
  ll ans = 1;
  for (; e; b = b * b % mod, e /= 2)
    if (e & 1) ans = ans * b % mod;//7e5
  return ans;
}
```

### ModLog.h

**Description:** Returns the smallest $x > 0$ s.t. $a^x = b \pmod{m}$, or $-1$ if no such $x$ exists. modLog(a,1,m) can be used to calculate the order of $a$.

**Time:** $\mathcal{O}\left(\sqrt{m}\right)$

c040b8, 11 lines

```
ll modLog(ll a, ll b, ll m) {//260
  ll n = (ll) sqrt(m) + 1, e = 1, f = 1, j = 1;
  unordered_map<ll, ll> A;
  while (j <= n && (e = f = e * a % m) != b % m)
    A[e * b % m] = j++;
  if (e == b % m) return j;//d16
  if (__gcd(m, e) == __gcd(m, b))
    rep(i,2,n+2) if (A.count(e = e * f % m))
      return n * i - A[e];
  return -1;
}//cbb
```

### ModSum.h

**Description:** Sums of mod'ed arithmetic progressions.

modsum(to, c, k, m) $= \sum_{i=0}^{\mathrm{to}-1} (ki + c)\%m$. divsum is similar but for floored division.

**Time:** $\log(m)$, with a large constant.

5c5bc5, 16 lines

```
typedef unsigned long long ull;//df3
ull sumsq(ull to) { return to / 2 * ((to-1) | 1); }

ull divsum(ull to, ull c, ull k, ull m) {
  ull res = k / m * sumsq(to) + c / m * to;
  k %= m; c %= m;//e1a
  if (!k) return res;
  ull to2 = (to * k + c) / m;
  return res + (to - 1) * to2 - divsum(to2, m-1 - c, m, k);
}
//1ae
ll modsum(ull to, ll c, ll k, ll m) {
  c = ((c % m) + m) % m;
  k = ((k % m) + m) % m;
  return to * c + k * sumsq(to) - m * divsum(to, c, k, m);
}//cbb
```

### ModMulLL.h

**Description:** Calculate $a \cdot b \bmod c$ (or $a^b \bmod c$) for $0 \leq a, b \leq c \leq 7.2 \cdot 10^{18}$.

**Time:** $\mathcal{O}(1)$ for modmul, $\mathcal{O}(\log b)$ for modpow

bbbd8f, 11 lines

```
typedef unsigned long long ull;//a9c
ull modmul(ull a, ull b, ull M) {
  ll ret = a * b - M * ull(1.L / M * a * b);
  return ret + M * (ret < 0) - M * (ret >= (ll)M);
}
ull modpow(ull b, ull e, ull mod) {//51d
  ull ans = 1;
  for (; e; b = modmul(b, b, mod), e /= 2)
    if (e & 1) ans = modmul(ans, b, mod);
  return ans;
}//cbb
```

### ModSqrt.h

**Description:** Tonelli-Shanks algorithm for modular square roots. Finds $x$ s.t. $x^2 = a \pmod{p}$ ($-x$ gives the other solution).

**Time:** $\mathcal{O}\left(\log^2 p\right)$ worst case, $\mathcal{O}(\log p)$ for most $p$

"ModPow.h" 19a793, 24 lines

```
ll sqrt(ll a, ll p) {//473
  a %= p; if (a < 0) a += p;
  if (a == 0) return 0;
  assert(modpow(a, (p-1)/2, p) == 1); // else no solution
  if (p % 4 == 3) return modpow(a, (p+1)/4, p);
  // a^(n+3)/8 or 2^(n+3)/8 * 2^(n-1)/4 works if p % 8 ==
      5//a48
  ll s = p - 1, n = 2;
  int r = 0, m;
  while (s % 2 == 0)
    ++r, s /= 2;
  while (modpow(n, (p - 1) / 2, p) != p - 1) ++n;//c4b
  ll x = modpow(a, (s + 1) / 2, p);
  ll b = modpow(a, s, p), g = modpow(n, s, p);
  for (;; r = m) {
    ll t = b;
    for (m = 0; m < r && t != 1; ++m)//faf
      t = t * t % p;
    if (m == 0) return x;
    ll gs = modpow(g, 1LL << (r - m - 1), p);
    g = gs * gs % p;
    x = x * gs % p;//a28
    b = b * g % p;
  }
}
```

## 5.2 Primality

### FastEratosthenes.h

**Description:** Prime sieve for generating all primes smaller than LIM.

**Time:** LIM=1e9 $\approx$ 1.5s

9ac0a0, 20 lines

```
const int LIM = 1e6;//058
bitset<LIM> isPrime;
vi eratosthenes() {
  const int S = (int)round(sqrt(LIM)), R = LIM / 2;
  vi pr = {2}, sieve(S+1); pr.reserve(int(LIM/log(LIM)*1.1)
      );
  vector<pii> cp;//86b
  for (int i = 3; i <= S; i += 2) if (!sieve[i]) {
    cp.pb({i, i * i / 2});
    for (int j = i * i; j <= S; j += 2 * i) sieve[j] = 1;
  }
  for (int L = 1; L <= R; L += S) {//62d
    array<bool, S> block{};
    for (auto &[p, idx] : cp)
      for (int i=idx; i < S+L; idx = (i+=p)) block[i-L] =
          1;
    rep(i,0,min(S, R - L))
      if (!block[i]) pr.pb((L + i) * 2 + 1);//0b9
  }
  for (int i : pr) isPrime[i] = 1;
  return pr;
}
```

### MillerRabin.h

**Description:** Deterministic Miller-Rabin primality test. Guaranteed to work for numbers up to $7 \cdot 10^{18}$; for larger numbers, use Python and extend A randomly.

**Time:** 7 times the complexity of $a^b \bmod c$.

"ModMulLL.h" 60dcd1, 12 lines

```
bool isPrime(ull n) {//60a
  if (n < 2 || n % 6 % 4 != 1) return (n | 1) == 3;
  ull A[] = {2, 325, 9375, 28178, 450775, 9780504,
      1795265022},
    s = __builtin_ctzll(n-1), d = n >> s;
  for (ull a : A) {   // ^ count trailing zeroes
    ull p = modpow(a%n, d, n), i = s;//81c
    while (p != 1 && p != n - 1 && a % n && i--)
      p = modmul(p, p, n);
    if (p != n-1 && i != s) return 0;
  }
  return 1;//84a
}
```

### Factor.h

**Description:** Pollard-rho randomized factorization algorithm. Returns prime factors of a number, in arbitrary order (e.g. 2299 -> {11, 19, 11}).

**Time:** $\mathcal{O}\left(n^{1/4}\right)$, less for numbers with small factors.

"ModMulLL.h", "MillerRabin.h" 21a173, 18 lines

```
ull pollard(ull n) {//c81
  ull x = 0, y = 0, t = 30, prd = 2, i = 1, q;
  auto f = [&](ull a) { return modmul(a, a, n) + i; };
  while (t++ % 40 || __gcd(prd, n) == 1) {
    if (x == y) x = ++i, y = f(x);
    if ((q = modmul(prd, max(x,y) - min(x,y), n))) prd = q;
        //049
    x = f(x), y = f(f(y));
  }
  return __gcd(prd, n);
}
vector<ull> factor(ull n) {//c19
  if (n == 1) return {};
  if (isPrime(n)) return {n};
  ull x = pollard(n);
  auto l = factor(x), r = factor(n / x);
  l.insert(l.end(), all(r));//363
  return l;
}
```

## 5.3 Divisibility

### euclid.h

**Description:** Finds two integers $x$ and $y$, such that $ax + by = \gcd(a, b)$. If you just need gcd, use the built in __gcd instead. If $a$ and $b$ are coprime, then $x$ is the inverse of $a \pmod{b}$.

33ba8f, 5 lines

```
ll euclid(ll a, ll b, ll &x, ll &y) {//33b
  if (!b) return x = 1, y = 0, a;
  ll d = euclid(b, a % b, y, x);
  return y -= a/b * x, d;
}
```

### Euclid.java

**Description:** Finds {x, y, d} s.t. ax + by = d = gcd(a, b).

6aba01, 11 lines

```
static BigInteger[] euclid(BigInteger a, BigInteger b) {//6
    fc
  BigInteger x = BigInteger.ONE, yy = x;
  BigInteger y = BigInteger.ZERO, xx = y;
  while (b.signum() != 0) {
    BigInteger q = a.divide(b), t = b;
    b = a.mod(b); a = t;//5fe
    t = xx; xx = x.subtract(q.multiply(xx)); x = t;
    t = yy; yy = y.subtract(q.multiply(yy)); y = t;
  }
  return new BigInteger[]{x, y, a};
```

```
}//cbb
```

CRT.h

**Description:** Chinese Remainder Theorem.
`crt(a, m, b, n)` computes $x$ such that $x \equiv a \pmod m$, $x \equiv b \pmod n$. If $|a| < m$ and $|b| < n$, $x$ will obey $0 \le x < \text{lcm}(m, n)$. Assumes $mn < 2^{62}$.
**Time:** $\log(n)$

"euclid.h" 04d93a, 7 lines

```
ll crt(ll a, ll m, ll b, ll n) {//eae
  if (n > m) swap(a, b), swap(m, n);
  ll x, y, g = euclid(m, n, x, y);
  assert((a - b) % g == 0); // else no solution
  x = (b - a) % n * x % n / g * m + a;
  return x < 0 ? x + m*n/g : x;//6ac
}
```

### 5.3.1 Bézout's identity

For $a \neq$, $b \neq 0$, then $d = gcd(a, b)$ is the smallest positive integer for which there are integer solutions to

$$ax + by = d$$

If $(x, y)$ is one solution, then all solutions are given by

$$\left(x + \frac{kb}{\gcd(a,b)}, y - \frac{ka}{\gcd(a,b)}\right), \quad k \in \mathbb{Z}$$

phiFunction.h

**Description:** *Euler's* $\phi$ function is defined as $\phi(n) := \#$ of positive integers $\le n$ that are coprime with $n$. $\phi(1) = 1$, $p$ prime $\Rightarrow \phi(p^k) = (p-1)p^{k-1}$, $m, n$ coprime $\Rightarrow \phi(mn) = \phi(m)\phi(n)$. If $n = p_1^{k_1} p_2^{k_2} ... p_r^{k_r}$ then $\phi(n) = (p_1 - 1)p_1^{k_1 - 1} ... (p_r - 1)p_r^{k_r - 1}$. $\phi(n) = n \cdot \prod_{p|n}(1 - 1/p)$. $\sum_{d|n} \phi(d) = n$, $\sum_{1 \le k \le n, \gcd(k,n)=1} k = n\phi(n)/2, n > 1$
**Euler's thm**: $a, n$ coprime $\Rightarrow a^{\phi(n)} \equiv 1 \pmod n$.
**Fermat's little thm**: $p$ prime $\Rightarrow a^{p-1} \equiv 1 \pmod p$ $\forall a$.

cf7d6d, 8 lines

```
const int LIM = 5000000;//70b
int phi[LIM];

void calculatePhi() {
  rep(i,0,LIM) phi[i] = i&1 ? i : i/2;
  for (int i = 3; i < LIM; i += 2) if(phi[i] == i)//103
    for (int j = i; j < LIM; j += i) phi[j] -= phi[j] / i;
}
```

## 5.4 Fractions

ContinuedFractions.h

**Description:** Given $N$ and a real number $x \ge 0$, finds the closest rational approximation $p/q$ with $p, q \le N$. It will obey $|p/q - x| \le 1/qN$.
For consecutive convergents, $p_{k+1}q_k - q_{k+1}p_k = (-1)^k$. ($p_k/q_k$ alternates between $> x$ and $< x$.) If $x$ is rational, $y$ eventually becomes $\infty$; if $x$ is the root of a degree 2 polynomial the $a$'s eventually become cyclic.
**Time:** $\mathcal{O}(\log N)$

ea42e2, 21 lines

```
typedef double d; // for N ~ 1e7; long double for N ~ 1e9
    //32b
pair<ll, ll> approximate(d x, ll N) {
  ll LP = 0, LQ = 1, P = 1, Q = 0, inf = LLONG_MAX; d y = x
    ;
  for (;;) {
    ll lim = min(P ? (N-LP) / P : inf, Q ? (N-LQ) / Q : inf
      ),
      a = (ll)floor(y), b = min(a, lim),//5ad
      NP = b*P + LP, NQ = b*Q + LQ;
    if (a > b) {
      // If b > a/2, we have a semi-convergent that gives
        us a
      // better approximation; if b = a/2, we *may* have
        one.
      // Return {P, Q} here for a more canonical
        approximation.//fcb
      return (abs(x - (d)NP / (d)NQ) < abs(x - (d)P / (d)Q)
        ) ?
        make_pair(NP, NQ) : make_pair(P, Q);
    }
    if (abs(y = 1/(y - (d)a)) > (d)N*3) {
      return {NP, NQ};//5c7
    }
    LP = P; P = NP;
    LQ = Q; Q = NQ;
  }
}//cbb
```

FracBinarySearch.h

**Description:** Given $f$ and $N$, finds the smallest fraction $p/q \in [0, 1]$ such that $f(p/q)$ is true, and $p, q \le N$. You may want to throw an exception from $f$ if it finds an exact solution, in which case $N$ can be removed.
**Usage:** `fracBS([](Frac f) { return f.p>=3*f.q; }, 10); // {1,3}`
**Time:** $\mathcal{O}(\log(N))$

27ab3e, 25 lines

```
struct Frac { ll p, q; };//386

template<class F>
Frac fracBS(F f, ll N) {
  bool dir = 1, A = 1, B = 1;
  Frac lo{0, 1}, hi{1, 1}; // Set hi to 1/0 to search (0, N
    ]//262
  if (f(lo)) return lo;
  assert(f(hi));
  while (A || B) {
    ll adv = 0, step = 1; // move hi if dir, else lo
    for (int si = 0; step; (step *= 2) >>= si) {//7e2
      adv += step;
      Frac mid{lo.p * adv + hi.p, lo.q * adv + hi.q};
      if (abs(mid.p) > N || mid.q > N || dir == !f(mid)) {
        adv -= step; si = 2;
      }//bf0
    }
    hi.p += lo.p * adv;
    hi.q += lo.q * adv;
    dir = !dir;
    swap(lo, hi);//f58
    A = B; B = !!adv;
  }
  return dir ? hi : lo;
}
```

## 5.5 Pythagorean Triples

The Pythagorean triples are uniquely generated by

$$a = k \cdot (m^2 - n^2), \quad b = k \cdot (2mn), \quad c = k \cdot (m^2 + n^2),$$

with $m > n > 0$, $k > 0$, $m \perp n$, and either $m$ or $n$ even.

## 5.6 Primes

$p = 962592769$ is such that $2^{21} \mid p - 1$, which may be useful. For hashing use 970592641 (31-bit number), 31443539979727 (45-bit), 3006703054056749 (52-bit). There are 78498 primes less than 1 000 000.

Primitive roots exist modulo any prime power $p^a$, except for $p = 2, a > 2$, and there are $\phi(\phi(p^a))$ many. For $p = 2, a > 2$, the group $\mathbb{Z}_{2^a}^{\times}$ is instead isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_{2^{a-2}}$.

## 5.7 Estimates

$\sum_{d|n} d = O(n \log \log n)$.

The number of divisors of $n$ is at most around 100 for $n < 5e4$, 500 for $n < 1e7$, 2000 for $n < 1e10$, 200 000 for $n < 1e19$.

## 5.8 Mobius Function

$$\mu(n) = \begin{cases} 0 & n \text{ is not square free} \\ 1 & n \text{ has even number of prime factors} \\ -1 & n \text{ has odd number of prime factors} \end{cases}$$

Mobius Inversion:

$$g(n) = \sum_{d|n} f(d) \Leftrightarrow f(n) = \sum_{d|n} \mu(d) g(n/d)$$

Other useful formulas/forms:

$\sum_{d|n} \mu(d) = [n = 1]$ (very useful)

$g(n) = \sum_{n|d} f(d) \Leftrightarrow f(n) = \sum_{n|d} \mu(d/n) g(d)$

$g(n) = \sum_{1 \le m \le n} f(\lfloor \frac{n}{m} \rfloor) \Leftrightarrow f(n) = \sum_{1 \le m \le n} \mu(m) g(\lfloor \frac{n}{m} \rfloor)$

# Combinatorial (6)

## 6.1 Permutations

### 6.1.1 Factorial

| $n$ | 1 2 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|
| $n!$ | 1 2 6 | 24 | 120 | 720 | 5040 | 40320 | 362880 | 3628800 |

| $n$ | 11 | 12 | 13 | 14 | 15 | 16 | 17 |
|---|---|---|---|---|---|---|---|
| $n!$ | 4.0e7 | 4.8e8 | 6.2e9 | 8.7e10 | 1.3e12 | 2.1e13 | 3.6e14 |

| $n$ | 20 | 25 | 30 | 40 | 50 | 100 | 150 | 171 |
|---|---|---|---|---|---|---|---|---|
| $n!$ | 2e18 | 2e25 | 3e32 | 8e47 | 3e64 | 9e157 | 6e262 | >DBL_MAX |

IntPerm.h

**Description:** Permutation -> integer conversion. (Not order preserving.) Integer -> permutation can use a lookup table.
**Time:** $\mathcal{O}(n)$

044568, 6 lines

```
int permToInt(vi& v) {//cf9
  int use = 0, i = 0, r = 0;
  for(int x:v) r = r * ++i + __builtin_popcount(use & -(1<<
      x)),
    use |= 1 << x;                    // (note: minus, not
      ~!)
  return r;
}//cbb
```

### 6.1.2 Cycles

Let $g_S(n)$ be the number of $n$-permutations whose cycle lengths all belong to the set $S$. Then

$$\sum_{n=0}^{\infty} g_S(n)\frac{x^n}{n!} = \exp\left(\sum_{n\in S}\frac{x^n}{n}\right)$$

### 6.1.3 Derangements

Permutations of a set such that none of the elements appear in their original position.

$$D(n) = (n-1)(D(n-1)+D(n-2)) = nD(n-1)+(-1)^n = \left\lfloor \frac{n!}{e} \right\rceil$$

### 6.1.4 Burnside's lemma

Given a group $G$ of symmetries and a set $X$, the number of elements of $X$ *up to symmetry* equals

$$\frac{1}{|G|}\sum_{g\in G}|X^g|,$$

where $X^g$ are the elements fixed by $g$ ($g.x = x$).

If $f(n)$ counts "configurations" (of some sort) of length $n$, we can ignore rotational symmetry using $G = \mathbb{Z}_n$ to get

$$g(n) = \frac{1}{n}\sum_{k=0}^{n-1} f(\gcd(n,k)) = \frac{1}{n}\sum_{k|n} f(k)\phi(n/k).$$

## 6.2 Partitions and subsets

### 6.2.1 Partition function

Number of ways of writing $n$ as a sum of positive integers, disregarding the order of the summands.

$$p(0) = 1,\ p(n) = \sum_{k\in\mathbb{Z}\setminus\{0\}} (-1)^{k+1} p(n - k(3k-1)/2)$$

$$p(n) \sim 0.145/n \cdot \exp(2.56\sqrt{n})$$

| $n$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 20 | 50 | 100 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $p(n)$ | 1 | 1 | 2 | 3 | 5 | 7 | 11 | 15 | 22 | 30 | 627 | ~2e5 | ~2e8 |

### 6.2.2 Lucas' Theorem

Let $n, m$ be non-negative integers and $p$ a prime. Write $n = n_k p^k + ... + n_1 p + n_0$ and $m = m_k p^k + ... + m_1 p + m_0$. Then $\binom{n}{m} \equiv \prod_{i=0}^{k}\binom{n_i}{m_i} \pmod{p}$.

### 6.2.3 Binomials

multinomial.h

**Description:** Computes $\binom{k_1+\cdots+k_n}{k_1,k_2,\ldots,k_n} = \frac{(\sum k_i)!}{k_1!k_2!...k_n!}$. a0a312, 5 lines

```
ll multinomial(vi& v) {//a0a
  ll c = 1, m = v.empty() ? 1 : v[0];
  rep(i,1,sz(v)) rep(j,0,v[i]) c = c * ++m / (j+1);
  return c;
}
```

## 6.3 General purpose numbers

### 6.3.1 Bernoulli numbers

EGF of Bernoulli numbers is $B(t) = \frac{t}{e^t-1}$ (FFT-able). $B[0,\ldots] = [1, -\frac{1}{2}, \frac{1}{6}, 0, -\frac{1}{30}, 0, \frac{1}{42}, \ldots]$

Sums of powers:

$$\sum_{i=1}^{n} n^m = \frac{1}{m+1}\sum_{k=0}^{m}\binom{m+1}{k}B_k \cdot (n+1)^{m+1-k}$$

Euler-Maclaurin formula for infinite sums:

$$\sum_{i=m}^{\infty} f(i) = \int_m^{\infty} f(x)dx - \sum_{k=1}^{\infty}\frac{B_k}{k!}f^{(k-1)}(m)$$

$$\approx \int_m^{\infty} f(x)dx + \frac{f(m)}{2} - \frac{f'(m)}{12} + \frac{f'''(m)}{720} + O(f^{(5)}(m))$$

### 6.3.2 Stirling numbers of the first kind

Number of permutations on $n$ items with $k$ cycles.

$$c(n,k) = c(n-1,k-1) + (n-1)c(n-1,k),\ c(0,0) = 1$$
$$\textstyle\sum_{k=0}^{n} c(n,k)x^k = x(x+1)\ldots(x+n-1)$$

$c(8,k) = 8, 0, 5040, 13068, 13132, 6769, 1960, 322, 28, 1$
$c(n,2) = 0, 0, 1, 3, 11, 50, 274, 1764, 13068, 109584, \ldots$

### 6.3.3 Eulerian numbers

Number of permutations $\pi \in S_n$ in which exactly $k$ elements are greater than the previous element. $k$ $j$:s s.t. $\pi(j) > \pi(j+1)$, $k+1$ $j$:s s.t. $\pi(j) \geq j$, $k$ $j$:s s.t. $\pi(j) > j$.

$$E(n,k) = (n-k)E(n-1,k-1) + (k+1)E(n-1,k)$$

$$E(n,0) = E(n,n-1) = 1$$

$$E(n,k) = \sum_{j=0}^{k}(-1)^j\binom{n+1}{j}(k+1-j)^n$$

### 6.3.4 Stirling numbers of the second kind

Partitions of $n$ distinct elements into exactly $k$ groups.

$$S(n,k) = S(n-1,k-1) + kS(n-1,k)$$

$$S(n,1) = S(n,n) = 1$$

$$S(n,k) = \frac{1}{k!}\sum_{j=0}^{k}(-1)^{k-j}\binom{k}{j}j^n$$

### 6.3.5 Bell numbers

Total number of partitions of $n$ distinct elements. $B(n) = 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, \ldots$. For $p$ prime,

$$B(p^m + n) \equiv mB(n) + B(n+1) \pmod{p}$$

### 6.3.6 Labeled unrooted trees

\# on $n$ vertices: $n^{n-2}$
\# on $k$ existing trees of size $n_i$: $n_1 n_2 \cdots n_k n^{k-2}$
\# with degrees $d_i$: $(n-2)!/((d_1-1)!\cdots(d_n-1)!)$

### 6.3.7 Catalan numbers

$$C_n = \frac{1}{n+1}\binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n+1} = \frac{(2n)!}{(n+1)!n!}$$

$$C_0 = 1,\ C_{n+1} = \frac{2(2n+1)}{n+2}C_n,\ C_{n+1} = \sum C_i C_{n-i}$$

$C_n = 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, \ldots$

- sub-diagonal monotone paths in an $n \times n$ grid.
- strings with $n$ pairs of parenthesis, correctly nested.
- binary trees with with $n+1$ leaves (0 or 2 children).
- ordered trees with $n+1$ vertices.
- ways a convex polygon with $n+2$ sides can be cut into triangles by connecting vertices with straight lines.
- permutations of $[n]$ with no 3-term increasing subseq.

# Graph (7)

## 7.1 Fundamentals

BellmanFord.h

**Description:** Calculates shortest paths from $s$ in a graph that might have negative edge weights. Unreachable nodes get dist = inf; nodes reachable through negative-weight cycles get dist = -inf. Assumes $V^2 \max|w_i| < \sim 2^{63}$.
**Time:** $\mathcal{O}(VE)$ 830a8f, 23 lines

```
const ll inf = LLONG_MAX;//019
struct Ed { int a, b, w, s() { return a < b ? a : -a; }};
struct Node { ll dist = inf; int prev = -1; };

void bellmanFord(vector<Node>& nodes, vector<Ed>& eds, int
    s) {
  nodes[s].dist = 0;//3a0
  sort(all(eds), [](Ed a, Ed b) { return a.s() < b.s(); });

  int lim = sz(nodes) / 2 + 2; // /3+100 with shuffled
      vertices
  rep(i,0,lim) for (Ed ed : eds) {
    Node cur = nodes[ed.a], &dest = nodes[ed.b];//e21
    if (abs(cur.dist) == inf) continue;
    ll d = cur.dist + ed.w;
    if (d < dest.dist) {
      dest.prev = ed.a;
      dest.dist = (i < lim-1 ? d : -inf);//69b
    }
  }
  rep(i,0,lim) for (Ed e : eds) {
    if (nodes[e.a].dist == -inf)
      nodes[e.b].dist = -inf;//943
  }
}
```

TopoSort.h

**Description:** Topological sorting. Given is an oriented graph. Output is an ordering of vertices, such that there are edges only from left to right. If there are cycles, the returned list will have size smaller than $n$ – nodes reachable from cycles will not be returned.

**Time:** $\mathcal{O}(|V|+|E|)$

c191e9, 8 lines

```
vi topoSort(const vector<vi>& gr) {//c31
  vi indeg(sz(gr)), q;
  for (auto& li : gr) for (int x : li) indeg[x]++;
  rep(i,0,sz(gr)) if (indeg[i] == 0) q.pb(i);
  rep(j,0,sz(q)) for (int x : gr[q[j]])
    if (--indeg[x] == 0) q.pb(x);//aab
  return q;
}
```

## 7.2 Network flow

### PushRelabel.h

**Description:** Push-relabel using the highest label selection rule and the gap heuristic. Quite fast in practice. To obtain the actual flow, look at positive values only.

**Time:** $\mathcal{O}\left(V^2\sqrt{E}\right)$

91e61c, 48 lines

```
struct PushRelabel {//d82
  struct Edge {
    int dest, back;
    ll f, c;
  };
  vector<vector<Edge>> g;//bef
  vector<ll> ec;
  vector<Edge*> cur;
  vector<vi> hs; vi H;
  PushRelabel(int n) : g(n), ec(n), cur(n), hs(2*n), H(n) {
      }
//a7b
  void addEdge(int s, int t, ll cap, ll rcap=0) {
    if (s == t) return;
    g[s].pb({t, sz(g[t]), 0, cap});
    g[t].pb({s, sz(g[s])-1, 0, rcap});
  }//7a1

  void addFlow(Edge& e, ll f) {
    Edge &back = g[e.dest][e.back];
    if (!ec[e.dest] && f) hs[H[e.dest]].pb(e.dest);
    e.f += f; e.c -= f; ec[e.dest] += f;//124
    back.f -= f; back.c += f; ec[back.dest] -= f;
  }
  ll calc(int s, int t) {
    int v = sz(g); H[s] = v; ec[t] = 1;
    vi co(2*v); co[0] = v-1;//a96
    rep(i,0,v) cur[i] = g[i].data();
    for (Edge& e : g[s]) addFlow(e, e.c);

    for (int hi = 0;;) {
      while (hs[hi].empty()) if (!hi--) return -ec[s];//e2e
      int u = hs[hi].back(); hs[hi].pop_back();
      while (ec[u] > 0)  // discharge u
        if (cur[u] == g[u].data() + sz(g[u])) {
          H[u] = 1e9;
          for (Edge& e : g[u]) if (e.c && H[u] > H[e.dest
              ]+1)//9ff
            H[u] = H[e.dest]+1, cur[u] = &e;
          if (++co[H[u]], !--co[hi] && hi < v)
            rep(i,0,v) if (hi < H[i] && H[i] < v)
              --co[H[i]], H[i] = v + 1;
          hi = H[u];//7ed
        } else if (cur[u]->c && H[u] == H[cur[u]->dest]+1)
          addFlow(*cur[u], min(ec[u], cur[u]->c));
        else ++cur[u];
    }
  }//a5b
  bool leftOfMinCut(int a) { return H[a] >= sz(g); }
};
```

### MinCostMaxFlow.h

**Description:** Min-cost max-flow. If costs can be negative, call setpi before maxflow, but note that negative cost cycles are not supported. To obtain the actual flow, look at positive values only.

**Time:** $\mathcal{O}(FE\log(V))$ where F is max flow. $\mathcal{O}(VE)$ for setpi.

<bits/extc++.h> bb147c, 77 lines

```
const ll INF = numeric_limits<ll>::max() / 4;//d4e

struct MCMF {
  struct edge {
    int from, to, rev;
    ll cap, cost, flow;//309
  };
  int N;
  vector<vector<edge>> ed;
  vi seen;
  vector<ll> dist, pi;//16a
  vector<edge*> par;

  MCMF(int _N) : N(_N), ed(N), seen(N), dist(N), pi(N), par
      (N) {}

  void addEdge(int from, int to, ll cap, ll cost) {//a9c
    if (from == to) return;
    ed[from].pb(edge{ from,to,sz(ed[to]),cap,cost,0 });
    ed[to].pb(edge{ to,from,sz(ed[from])-1,0,-cost,0 });
  }
//635
  void path(int s) {
    fill(all(seen), 0);
    fill(all(dist), INF);
    dist[s] = 0; ll di;
//17c
    __gnu_pbds::priority_queue<pair<ll, int>> q;
    vector<decltype(q)::point_iterator> its(N);
    q.push({ 0, s });

    while (!q.empty()) {//95a
      s = q.top().second; q.pop();
      seen[s] = 1; di = dist[s] + pi[s];
      for (edge& e : ed[s]) if (!seen[e.to]) {
        ll val = di - pi[e.to] + e.cost;
        if (e.cap - e.flow > 0 && val < dist[e.to]) {//c63
          dist[e.to] = val;
          par[e.to] = &e;
          if (its[e.to] == q.end())
            its[e.to] = q.push({ -dist[e.to], e.to });
          else//dbc
            q.modify(its[e.to], { -dist[e.to], e.to });
        }
      }
    }
    rep(i,0,N) pi[i] = min(pi[i] + dist[i], INF);//02d
  }

  pair<ll, ll> maxflow(int s, int t) {
    ll totflow = 0, totcost = 0;
    while (path(s), seen[t]) {//aa6
      ll fl = INF;
      for (edge* x = par[t]; x; x = par[x->from])
        fl = min(fl, x->cap - x->flow);

      totflow += fl;//21b
      for (edge* x = par[t]; x; x = par[x->from]) {
        x->flow += fl;
        ed[x->to][x->rev].flow -= fl;
      }
    }//cd4
    rep(i,0,N) for(edge& e : ed[i]) totcost += e.cost * e.
         flow;
    return {totflow, totcost/2};
  }

  // If some costs can be negative, call this before
       maxflow://7c7
  void setpi(int s) { // (otherwise, leave this out)
    fill(all(pi), INF); pi[s] = 0;
    int it = N, ch = 1; ll v;
    while (ch-- && it--)
      rep(i,0,N) if (pi[i] != INF)//42d
        for (edge& e : ed[i]) if (e.cap)
          if ((v = pi[i] + e.cost) < pi[e.to])
            pi[e.to] = v, ch = 1;
    assert(it >= 0); // negative cost cycle
  }//e03
};
```

### EdmondsKarp.h

**Description:** Flow algorithm with guaranteed complexity $O(VE^2)$. To get edge flow values, compare capacities before and after, and take the positive values only.

482fe0, 36 lines

```
template<class T> T edmondsKarp(vector<unordered_map<int, T
     >>&//324
    graph, int source, int sink) {
  assert(source != sink);
  T flow = 0;
  vi par(sz(graph)), q = par;
//cf9
  for (;;) {
    fill(all(par), -1);
    par[source] = 0;
    int ptr = 1;
    q[0] = source;//623

    rep(i,0,ptr) {
      int x = q[i];
      for (auto e : graph[x]) {
        if (par[e.first] == -1 && e.second > 0) {//3a4
          par[e.first] = x;
          q[ptr++] = e.first;
          if (e.first == sink) goto out;
        }
      }//3cd
    }
    return flow;
out:
    T inc = numeric_limits<T>::max();
    for (int y = sink; y != source; y = par[y])//d19
      inc = min(inc, graph[par[y]][y]);

    flow += inc;
    for (int y = sink; y != source; y = par[y]) {
      int p = par[y];//b79
      if ((graph[p][y] -= inc) <= 0) graph[p].erase(y);
      graph[y][p] += inc;
    }
  }
}//cbb
```

### Dinic.h

**Description:** Flow algorithm with complexity $O(VE\log U)$ where $U = \max|\text{cap}|$. $O(\min(E^{1/2}, V^{2/3})E)$ if $U = 1$; $O(\sqrt{V}E)$ for bipartite matching.

b2af4f, 42 lines

```
struct Dinic {//299
  struct Edge {
    int to, rev;
    ll c, oc;
    ll flow() { return max(oc - c, 0LL); } // if you need
         flows
  };//8ec
```

```
  vi lvl, ptr, q;
  vector<vector<Edge>> adj;
  Dinic(int n) : lvl(n), ptr(n), q(n), adj(n) {}
  void addEdge(int a, int b, ll c, ll rcap = 0) {
    adj[a].pb({b, sz(adj[b]), c, c});//76f
    adj[b].pb({a, sz(adj[a]) - 1, rcap, rcap});
  }
  ll dfs(int v, int t, ll f) {
    if (v == t || !f) return f;
    for (int& i = ptr[v]; i < sz(adj[v]); i++) {//b2a
      Edge& e = adj[v][i];
      if (lvl[e.to] == lvl[v] + 1)
        if (ll p = dfs(e.to, t, min(f, e.c))) {
          e.c -= p, adj[e.to][e.rev].c += p;
          return p;//f3e
        }
    }
    return 0;
  }
  ll calc(int s, int t) {//b4c
    ll flow = 0; q[0] = s;
    rep(L,0,31) do { // 'int L=30' maybe faster for random
        data
      lvl = ptr = vi(sz(q));
      int qi = 0, qe = lvl[s] = 1;
      while (qi < qe && !lvl[t]) {//796
        int v = q[qi++];
        for (Edge e : adj[v])
          if (!lvl[e.to] && e.c >> (30 - L))
            q[qe++] = e.to, lvl[e.to] = lvl[v] + 1;
      }//4ca
      while (ll p = dfs(s, t, LLONG_MAX)) flow += p;
    } while (lvl[t]);
    return flow;
  }
  bool leftOfMinCut(int a) { return lvl[a] != 0; }//b90
};
```

## MinCut.h

**Description:** After running max-flow, the left side of a min-cut from $s$ to $t$ is given by all vertices reachable from $s$, only traversing edges with positive residual capacity.

d41d8c, 1 lines

```
//d41
```

## GlobalMinCut.h

**Description:** Find a global minimum cut in an undirected graph, as represented by an adjacency matrix.
**Time:** $\mathcal{O}\left(V^3\right)$

8b0e19, 21 lines

```
pair<int, vi> globalMinCut(vector<vi> mat) {//f64
  pair<int, vi> best = {INT_MAX, {}};
  int n = sz(mat);
  vector<vi> co(n);
  rep(i,0,n) co[i] = {i};
  rep(ph,1,n) {//c8f
    vi w = mat[0];
    size_t s = 0, t = 0;
    rep(it,0,n-ph) { // O(V^2) -> O(E log V) with prio.
        queue
      w[t] = INT_MIN;
      s = t, t = max_element(all(w)) - w.begin();//0bb
      rep(i,0,n) w[i] += mat[t][i];
    }
    best = min(best, {w[t] - mat[t][t], co[t]});
    co[s].insert(co[s].end(), all(co[t]));
    rep(i,0,n) mat[s][i] += mat[t][i];//a2c
    rep(i,0,n) mat[i][s] = mat[s][i];
    mat[0][t] = INT_MIN;
  }
  return best;
}//cbb
```

## GomoryHu.h

**Description:** Given a list of edges representing an undirected flow graph, returns edges of the Gomory-Hu tree. The max flow between any pair of vertices is given by minimum edge weight along the Gomory-Hu tree path.
**Time:** $\mathcal{O}(V)$ Flow Computations

"PushRelabel.h" 1ec6c8, 13 lines

```
struct Edge { int from, to; ll cap; };//81a
vector<Edge> gomoryHu(int N, vector<Edge> ed) {
  vector<Edge> tree;
  vi par(N);
  rep(i,1,N) {
    PushRelabel D(N); // Dinic also works//489
    for (Edge t : ed) D.addEdge(t.from, t.to, t.cap, t.cap)
        ;
    tree.pb({i, par[i], D.calc(i, par[i])});
    rep(j,i+1,N)
      if (par[j] == par[i] && D.leftOfMinCut(j)) par[j] = i
          ;
  }//eec
  return tree;
}
```

# 7.3 Matching

## hopcroftKarp.h

**Description:** Fast bipartite matching algorithm. Graph $g$ should be a list of neighbors of the left partition, and *btoa* should be a vector full of -1's of the same size as the right partition. Returns the size of the matching. *btoa*[*i*] will be the match for vertex $i$ on the right side, or $-1$ if it's not matched.
**Usage:** `vi btoa(m, -1); hopcroftKarp(g, btoa);`
**Time:** $\mathcal{O}\left(\sqrt{V}E\right)$

d93347, 42 lines

```
bool dfs(int a, int L, vector<vi>& g, vi& btoa, vi& A, vi&
    B) {//d9e
  if (A[a] != L) return 0;
  A[a] = -1;
  for (int b : g[a]) if (B[b] == L + 1) {
    B[b] = 0;
    if (btoa[b] == -1 || dfs(btoa[b], L + 1, g, btoa, A, B)
        )//613
      return btoa[b] = a, 1;
  }
  return 0;
}
//ad4
int hopcroftKarp(vector<vi>& g, vi& btoa) {
  int res = 0;
  vi A(g.size()), B(btoa.size()), cur, next;
  for (;;) {
    fill(all(A), 0);//d58
    fill(all(B), 0);
    cur.clear();
    for (int a : btoa) if(a != -1) A[a] = -1;
    rep(a,0,sz(g)) if(A[a] == 0) cur.pb(a);
    for (int lay = 1;; lay++) {//559
      bool islast = 0;
      next.clear();
      for (int a : cur) for (int b : g[a]) {
        if (btoa[b] == -1) {
          B[b] = lay;//1ca
          islast = 1;
        }
        else if (btoa[b] != a && !B[b]) {
          B[b] = lay;
          next.pb(btoa[b]);//c66
        }
      }
      if (islast) break;
      if (next.empty()) return res;
      for (int a : next) A[a] = lay;//4f3
      cur.swap(next);
    }
    rep(a,0,sz(g))
      res += dfs(a, 0, g, btoa, A, B);
  }//67c
}
```

## DFSMatching.h

**Description:** Simple bipartite matching algorithm. Graph $g$ should be a list of neighbors of the left partition, and *btoa* should be a vector full of -1's of the same size as the right partition. Returns the size of the matching. *btoa*[*i*] will be the match for vertex $i$ on the right side, or $-1$ if it's not matched.
**Usage:** `vi btoa(m, -1); dfsMatching(g, btoa);`
**Time:** $\mathcal{O}(VE)$

522b98, 22 lines

```
bool find(int j, vector<vi>& g, vi& btoa, vi& vis) {//400
  if (btoa[j] == -1) return 1;
  vis[j] = 1; int di = btoa[j];
  for (int e : g[di])
    if (!vis[e] && find(e, g, btoa, vis)) {
      btoa[e] = di;//a0e
      return 1;
    }
  return 0;
}
int dfsMatching(vector<vi>& g, vi& btoa) {//52f
  vi vis;
  rep(i,0,sz(g)) {
    vis.assign(sz(btoa), 0);
    for (int j : g[i])
      if (find(j, g, btoa, vis)) {//e5b
        btoa[j] = i;
        break;
      }
  }
  return sz(btoa) - (int)count(all(btoa), -1);//ff5
}
```

## MinimumVertexCover.h

**Description:** Finds a minimum vertex cover in a bipartite graph. The size is the same as the size of a maximum matching, and the complement is a maximum independent set.

"DFSMatching.h" 62c4ec, 20 lines

```
vi cover(vector<vi>& g, int n, int m) {//60f
  vi match(m, -1);
  int res = dfsMatching(g, match);
  vector<bool> lfound(n, true), seen(m);
  for (int it : match) if (it != -1) lfound[it] = false;
  vi q, cover;//2da
  rep(i,0,n) if (lfound[i]) q.pb(i);
  while (!q.empty()) {
    int i = q.back(); q.pop_back();
    lfound[i] = 1;
    for (int e : g[i]) if (!seen[e] && match[e] != -1) {//4
        ed
      seen[e] = true;
      q.pb(match[e]);
    }
  }
  rep(i,0,n) if (!lfound[i]) cover.pb(i);//a72
  rep(i,0,m) if (seen[i]) cover.pb(n+i);
  assert(sz(cover) == res);
  return cover;
}
```

## WeightedMatching.h

**Description:** Given a weighted bipartite graph, matches every node on the left with a node on the right such that no nodes are in two matchings and the sum of the edge weights is minimal. Takes cost[N][M], where cost[i][j] = cost for L[i] to be matched with R[j] and returns (min cost, match), where L[i] is matched with R[match[i]]. Negate costs for max cost. Requires $N \leq M$.
**Time:** $\mathcal{O}\left(N^2M\right)$

df0677, 31 lines

```
pair<int, vi> hungarian(const vector<vi> &a) {//64f
  if (a.empty()) return {0, {}};
  int n = sz(a) + 1, m = sz(a[0]) + 1;
  vi u(n), v(m), p(m), ans(n - 1);
  rep(i,1,n) {
    p[0] = i;//0b5
    int j0 = 0; // add "dummy" worker 0
    vi dist(m, INT_MAX), pre(m, -1);
    vector<bool> done(m + 1);
    do { // dijkstra
      done[j0] = true;//bd1
      int i0 = p[j0], j1 = -1, delta = INT_MAX;
      rep(j,1,m) if (!done[j]) {
        auto cur = a[i0 - 1][j - 1] - u[i0] - v[j];
        if (cur < dist[j]) dist[j] = cur, pre[j] = j0;
        if (dist[j] < delta) delta = dist[j], j1 = j;//865
      }
      rep(j,0,m) {
        if (done[j]) u[p[j]] += delta, v[j] -= delta;
        else dist[j] -= delta;
      }//aa1
      j0 = j1;
    } while (p[j0]);
    while (j0) { // update alternating path
      int j1 = pre[j0];
      p[j0] = p[j1], j0 = j1;//88f
    }
  }
  rep(j,1,m) if (p[j]) ans[p[j] - 1] = j - 1;
  return {-v[0], ans}; // min cost
}//cbb
```

## GeneralMatching.h

**Description:** Matching for general graphs. Fails with probability $N/mod$.
**Time:** $\mathcal{O}\left(N^3\right)$

"../numerical/MatrixInverse-mod.h" 1e40dd, 40 lines

```
vector<pii> generalMatching(int N, vector<pii>& ed) {//19e
  vector<vector<ll>> mat(N, vector<ll>(N)), A;
  for (pii pa : ed) {
    int a = pa.first, b = pa.second, r = rand() % mod;
    mat[a][b] = r, mat[b][a] = (mod - r) % mod;
  }//063

  int r = matInv(A = mat), M = 2*N - r, fi, fj;
  assert(r % 2 == 0);

  if (M != N) do {//f88
    mat.resize(M, vector<ll>(M));
    rep(i,0,N) {
      mat[i].resize(M);
      rep(j,N,M) {
        int rr = rand() % mod;//b47
        mat[i][j] = rr, mat[j][i] = (mod - rr) % mod;
      }
    }
  } while (matInv(A = mat) != M);
//92b
  vi has(M, 1); vector<pii> ret;
  rep(it,0,M/2) {
    rep(i,0,M) if (has[i])
      rep(j,i+1,M) if (A[i][j] && mat[i][j]) {
        fi = i; fj = j; goto done;//e0a
    } assert(0); done:
    if (fj < N) ret.emplace_back(fi, fj);
    has[fi] = has[fj] = 0;
    rep(sw,0,2) {
      ll a = modpow(A[fi][fj], mod-2);//b7f
      rep(i,0,M) if (has[i] && A[i][fj]) {
        ll b = A[i][fj] * a % mod;
        rep(j,0,M) A[i][j] = (A[i][j] - A[fi][j] * b) % mod
            ;
      }
      swap(fi,fj);//3c7
    }
  }
  return ret;
}
```

# 7.4 DFS algorithms

## SCC.h

**Description:** Finds strongly connected components in a directed graph. If vertices $u, v$ belong to the same component, we can reach $u$ from $v$ and vice versa.
**Usage:** `scc(graph, [&](vi& v) { ... })` visits all components
in reverse topological order. comp[i] holds the component index of a node (a component only has edges to components with
lower index). ncomps will contain the number of components.
**Time:** $\mathcal{O}\left(E+V\right)$

c16b3c, 24 lines

```
template<class F> vi scc(const vector<vi> &adj, F f) {//496
  int n = sz(adj);
  vi val(n), comp(n, -1), z, cont;
  int time = 0, ncomps = 0;
  auto dfs = [&](auto &&self, int u) -> int {
    int low = val[u] = ++time, x; z.push_back(u);//ad5
    for (auto e : adj[u]) if (comp[e] < 0)
      low = min(low, val[e] ?: self(self, e));
    if (low == val[u]) {
      do {
        x = z.back(); z.pop_back();//4f1
        comp[x] = ncomps;
        cont.push_back(x);
      } while (x != u);
      f(cont); cont.clear();
      ncomps++;//cda
    }
    return val[u] = low;
  };
  rep(i, 0, n) {
    if (comp[i] < 0) dfs(dfs, i);//418
  }
  return comp;
}
```

## BiconnectedComponents.h

**Description:** Finds all biconnected components in an undirected graph, and runs a callback for the edges in each. In a biconnected component there are at least two distinct paths between any two nodes. Note that a node can be in several components. An edge which is not in a component is a bridge, i.e., not part of any cycle.
**Usage:**
```
int eid = 0; ed.resize(N);
for each edge (a,b) {
ed[a].emplace_back(b, eid);
ed[b].emplace_back(a, eid++); }
bicomps([&](const vi& edgelist) {...});
```
**Time:** $\mathcal{O}\left(E+V\right)$

389e66, 28 lines

```
template<class F>//c2c
void bicomps(vector<vector<pii>> &ed, F f) {
  vi num(sz(ed)), st;
  int t=0;
  auto dfs = [&](auto &&self, int at, int par) -> int {
    int me = num[at] = ++t, top = me;//b12
    for (auto [y, e] : ed[at]) if (e != par) {
      if (num[y]) {
        top = min(top, num[y]);
        if (num[y] < me)
          st.pb(e);//6e9
      } else {
        int si = sz(st);
        int up = self(self, y, e);
        top = min(top, up);
        if (up == me) {//2cf
          st.pb(e);
          f(vi(st.begin() + si, st.end()));
          st.resize(si);
        }
        else if (up < me) st.pb(e);//51c
        else { /* e is a bridge */ }
      }
    }
    return top;
  };//835
  rep(i,0,sz(ed)) if (!num[i]) dfs(dfs, i, -1);
}
```

## Articulation.h

**Description:** Finds articulation points (removal separates graph)
**Time:** $\mathcal{O}\left(n+m\right)$

a7b0ba, 25 lines

```
vector<bool> cutpoints(const vector<vi> &adj) {//259
  int timer=0, n=sz(adj);
  vi tin(n,-1), low(n,-1);
  vector<bool> vis(n);
  vector<bool> iscut(n);
  auto dfs = [&](auto &&self, int v, int p) -> void {//7a1
    vis[v] = true;
    tin[v] = low[v] = timer++;
    int ch = 0;
    for (int to : adj[v]) {
      if (to == p) continue;//b9d
      if (vis[to])
        low[v] = min(low[v], tin[to]);
      else {
        self(self, to, v);
        low[v] = min(low[v], low[to]);//914
        if (low[to] >= tin[v] && p!=-1) iscut[v]=1;
        ++ch;
      }
    }
    if(p == -1 && ch > 1) iscut[v]=1;//4e2
  };
  rep(i,0,n) if (!vis[i]) dfs(dfs, i, -1);
  return iscut;
}
```

## 2sat.h

**Description:** Calculates a valid assignment to boolean variables a, b, c,... to a 2-SAT problem, so that an expression of the type $(a||b)\&\&(!a||c)\&\&(d||!b)\&\&...$ becomes true, or reports that it is unsatisfiable. Negated variables are represented by bit-inversions (~x).
**Usage:**
```
TwoSat ts(number of boolean variables);
ts.either(0, ~3); // Var 0 is true or var 3 is false
ts.setValue(2); // Var 2 is true
ts.atMostOne({0,~1,2}); // <= 1 of vars 0, ~1 and 2 are true
ts.solve(); // Returns true iff it is solvable
ts.values[0..N-1] holds the assigned values to the vars
```
**Time:** $\mathcal{O}\left(N+E\right)$, where N is the number of boolean variables, and E is the number of clauses.

35fbf7, 56 lines

```
struct TwoSat {//7c0
  int N;
  vector<vi> gr;
  vi values; // 0 = false , 1 = true

  TwoSat(int n = 0) : N(n), gr(2*n) {}//54e

  int addVar() { // (optional)
    gr.emplace_back();
    gr.emplace_back();
    return N++;//662
  }

  void either(int f, int j) {
    f = max(2*f, -1-2*f);
    j = max(2*j, -1-2*j);//2d3
    gr[f].pb(j^1);
    gr[j].pb(f^1);
  }
  void setValue(int x) { either(x, x); }
//41c
  void atMostOne(const vi& li) { // (optional)
    if (sz(li) <= 1) return;
    int cur = ~li[0];
    rep(i,2,sz(li)) {
      int next = addVar();//f5e
      either(cur, ~li[i]);
      either(cur, next);
      either(~li[i], next);
      cur = ~next;
    }//276
    either(cur, ~li[1]);
  }

  vi val, comp, z; int time = 0;
  int dfs(int i) {//1e9
    int low = val[i] = ++time, x; z.pb(i);
    for(int e : gr[i]) if (!comp[e])
      low = min(low, val[e] ?: dfs(e));
    if (low == val[i]) do {
      x = z.back(); z.pop_back();//0c0
      comp[x] = low;
      if (values[x>>1] == -1)
        values[x>>1] = x&1;
    } while (x != i);
    return val[i] = low;//749
  }

  bool solve() {
    values.assign(N, -1);
    val.assign(2*N, 0); comp = val;//4fa
    rep(i,0,2*N) if (!comp[i]) dfs(i);
    rep(i,0,N) if (comp[2*i] == comp[2*i+1]) return 0;
    return 1;
  }
};//214
```

### EulerWalk.h

**Description:** Eulerian undirected/directed path/cycle algorithm. Input should be a vector of (dest, global edge index), where for undirected graphs, forward/backward edges have the same index. Returns a list of nodes in the Eulerian path/cycle with src at both start and end, or empty list if no cycle/path exists. To get edge indices back, add .second to s and ret.
**Time:** $\mathcal{O}(V+E)$

3e0eb1, 15 lines

```
vi eulerWalk(vector<vector<pii>>& gr, int nedges, int src
    =0) {//fda
  int n = sz(gr);
  vi D(n), its(n), eu(nedges), ret, s = {src};
  D[src]++; // to allow Euler paths, not just cycles
  while (!s.empty()) {
    int x = s.back(), y, e, &it = its[x], end = sz(gr[x]);
        //39a
    if (it == end){ ret.pb(x); s.pop_back(); continue; }
    tie(y, e) = gr[x][it++];
    if (!eu[e]) {
      D[x]--, D[y]++;
      eu[e] = 1; s.pb(y);//f91
    }}
  for (int x : D) if (x < 0 || sz(ret) != nedges+1) return
      {};
  return {ret.rbegin(), ret.rend()};
}
```

## 7.5 Coloring

### EdgeColoring.h

**Description:** Given a simple, undirected graph with max degree $D$, computes a $(D+1)$-coloring of the edges such that no neighboring edges share a color. ($D$-coloring is NP-hard, but can be done for bipartite graphs by repeated matchings of max-degree nodes.)
**Time:** $\mathcal{O}(NM)$

ca07a0, 31 lines

```
vi edgeColoring(int N, vector<pii> eds) {//d26
  vi cc(N + 1), ret(sz(eds)), fan(N), free(N), loc;
  for (pii e : eds) ++cc[e.first], ++cc[e.second];
  int u, v, ncols = *max_element(all(cc)) + 1;
  vector<vi> adj(N, vi(ncols, -1));
  for (pii e : eds) {//945
    tie(u, v) = e;
    fan[0] = v;
    loc.assign(ncols, 0);
    int at = u, end = u, d, c = free[u], ind = 0, i = 0;
    while (d = free[v], !loc[d] && (v = adj[u][d]) != -1)//
        665
      loc[d] = ++ind, cc[ind] = d, fan[ind] = v;
    cc[loc[d]] = c;
    for (int cd = d; at != -1; cd ^= c ^ d, at = adj[at][cd
        ])
      swap(adj[at][cd], adj[end = at][cd ^ c ^ d]);
    while (adj[fan[i]][d] != -1) {//f7e
      int left = fan[i], right = fan[++i], x = cc[i];
      adj[u][x] = left;
      adj[left][x] = u;
      adj[right][x] = -1;
      free[right] = x;//e59
    }
    adj[u][d] = fan[i];
    adj[fan[i]][d] = u;
    for (int y : {fan[0], u, end})
      for (int& z = free[y] = 0; adj[y][z] != -1; z++);//
          b06
  }
  rep(i,0,sz(eds))
    for (tie(u, v) = eds[i]; adj[u][ret[i]] != v;) ++ret[i
        ];
  return ret;
}//cbb
```

## 7.6 Heuristics

### MaximalCliques.h

**Description:** Runs a callback for all maximal cliques in a graph (given as a symmetric bitset matrix; self-edges not allowed). Callback is given a bitset representing the maximal clique.
**Time:** $\mathcal{O}\left(3^{n/3}\right)$, much faster for sparse graphs

b0d5b1, 12 lines

```
typedef bitset<128> B;//abb
template<class F>
void cliques(vector<B>& eds, F f, B P = ~B(), B X={}, B R={
    }) {
  if (!P.any()) { if (!X.any()) f(R); return; }
  auto q = (P | X)._Find_first();
  auto cands = P & ~eds[q];//7d8
  rep(i,0,sz(eds)) if (cands[i]) {
    R[i] = 1;
    cliques(eds, f, P & eds[i], X & eds[i], R);
    R[i] = P[i] = 0; X[i] = 1;
  }//67c
}
```

### MaximumClique.h

**Description:** Quickly finds a maximum clique of a graph (given as symmetric bitset matrix; self-edges not allowed). Can be used to find a maximum independent set by finding a clique of the complement graph.
**Time:** Runs in about 1s for n=155 and worst case random graphs (p=.90). Runs faster for sparse graphs.

108bd4, 49 lines

```
typedef vector<bitset<200>> vb;//b92
struct Maxclique {
  double limit=0.025, pk=0;
  struct Vertex { int i, d=0; };
  typedef vector<Vertex> vv;
  vb e;//5b2
  vv V;
  vector<vi> C;
  vi qmax, q, S, old;
  void init(vv& r) {
    for (auto& v : r) v.d = 0;//dab
    for (auto& v : r) for (auto j : r) v.d += e[v.i][j.i];
    sort(all(r), [](auto a, auto b) { return a.d > b.d; });
    int mxD = r[0].d;
    rep(i,0,sz(r)) r[i].d = min(i, mxD) + 1;
  }//a6a
  void expand(vv& R, int lev = 1) {
    S[lev] += S[lev - 1] - old[lev];
    old[lev] = S[lev - 1];
    while (sz(R)) {
      if (sz(q) + R.back().d <= sz(qmax)) return;//62e
      q.pb(R.back().i);
      vv T;
      for(auto v:R) if (e[R.back().i][v.i]) T.pb({v.i});
      if (sz(T)) {
        if (S[lev]++ / ++pk < limit) init(T);//feb
        int j = 0, mxk = 1, mnk = max(sz(qmax) - sz(q) + 1,
            1);
        C[1].clear(), C[2].clear();
        for (auto v : T) {
          int k = 1;
          auto f = [&](int i) { return e[v.i][i]; };//94f
          while (any_of(all(C[k]), f)) k++;
          if (k > mxk) mxk = k, C[mxk + 1].clear();
          if (k < mnk) T[j++].i = v.i;
          C[k].pb(v.i);
        }//08b
        if (j > 0) T[j - 1].d = 0;
        rep(k,mnk,mxk + 1) for (int i : C[k])
          T[j].i = i, T[j++].d = k;
        expand(T, lev + 1);
      } else if (sz(q) > sz(qmax)) qmax = q;//15f
      q.pop_back(), R.pop_back();
    }
  }
  vi maxClique() { init(V), expand(V); return qmax; }
  Maxclique(vb conn) : e(conn), C(sz(e)+1), S(sz(C)), old(S
      ) {//83c
    rep(i,0,sz(e)) V.pb({i});
  }
};
```

### MaximumIndependentSet.h

**Description:** To obtain a maximum independent set of a graph, find a max clique of the complement. If the graph is bipartite, see MinimumVertexCover.

d41d8c, 1 lines

```
//d41
```

## 7.7 Trees

### CompressTree.h

**Description:** Given a rooted tree and a subset S of nodes, compute the minimal subtree that contains all the nodes by adding all (at most $|S|-1$) pairwise LCA's and compressing edges. Returns a list of (par, orig_index) representing a tree rooted at 0. The root points to itself.
**Time:** $\mathcal{O}(|S| \log |S|)$

"LCA.h" cea406, 21 lines

```
typedef vector<pair<int, int>> vpi;//386
vpi compressTree(LCA& lca, const vi& subset) {
  static vi rev; rev.resize(sz(lca.time));
  vi li = subset, &T = lca.time;
  auto cmp = [&](int a, int b) { return T[a] < T[b]; };
  sort(all(li), cmp);//3b2
  int m = sz(li)-1;
  rep(i,0,m) {
    int a = li[i], b = li[i+1];
    li.pb(lca.lca(a, b));
  }//c76
  sort(all(li), cmp);
  li.erase(unique(all(li)), li.end());
  rep(i,0,sz(li)) rev[li[i]] = i;
  vpi ret = {pii(0, li[0])};
  rep(i,0,sz(li)-1) {//ff8
    int a = li[i], b = li[i+1];
    ret.emplace_back(rev[lca.lca(a, b)], b);
  }
  return ret;
}//cbb
```

### HLD.h

**Description:** Decomposes a tree into vertex disjoint heavy paths and light edges such that the path from any leaf to the root contains at most $\log(n)$ light edges. Takes as input the full adjacency list. op edges being true means that values are stored in the edges, as opposed to the nodes.
**Time:** $\mathcal{O}(\log N)$

f6f4ce, 55 lines

```
template<bool op_edges = false>//1ab
struct hld {
  vector<vi> adj;
  vi par, size, in, head, d;
  hld(int n) : adj(n), par(n), size(n), in(n), head(n), d(n
      ) {}
//c22
  void add_edge(int u, int v) {
    adj[u].pb(v);
    adj[v].pb(u);
  }
//9ee
  void dfs_size(int v = 0, int p = 0) {
    size[v] = 1;
    for (int &e : adj[v]) {
      if (e != p) {
        d[e] = d[v] + 1;//a74
        par[e] = v;
        dfs_size(e, v);
        size[v] += size[e];
        if (size[e] > size[adj[v][0]] || adj[v][0] == p)
            swap(e, adj[v][0]);
      }//96d
    }
  }

  void dfs_hld(int v = 0, int p = 0) {
    static int t = 0;//978
    in[v] = t++;
    for (int e : adj[v]) {
      if (e != p) {
        if (e == adj[v][0]) {
          head[e] = head[v];//c3e
        } else {
          head[e] = e;
        }
        dfs_hld(e, v);
      }//47a
    }
  }

  template<typename F>
  void op_path(int x, int y, F op) {//b81
    while (head[x] != head[y]) {
      if (d[head[x]] > d[head[y]]) swap(x, y);
      op(in[head[y]], in[y] + 1);
      y = par[head[y]];
    }//387
    if (d[x] > d[y]) swap(x, y);
    op(in[x] + (op_edges ? 1 : 0), in[y] + 1);
  }

  template<typename F>//d25
  void op_subtree(int x, F op) {
    op(in[x] + (op_edges ? 1 : 0), in[x] + size[x]);
  }
};
```

### LinkCutTree.h

**Description:** Represents a forest of unrooted trees. You can add and remove edges (as long as the result is still a forest), and check whether two nodes are in the same tree.
**Time:** All operations take amortized $\mathcal{O}(\log N)$.

0fb462, 90 lines

```
struct Node { // Splay tree. Root's pp contains tree's
    parent.//a4e
  Node *p = 0, *pp = 0, *c[2];
  bool flip = 0;
  Node() { c[0] = c[1] = 0; fix(); }
  void fix() {
    if (c[0]) c[0]->p = this;//b8f
    if (c[1]) c[1]->p = this;
    // (+ update sum of subtree elements etc. if wanted)
  }
  void pushFlip() {
    if (!flip) return;//dfd
    flip = 0; swap(c[0], c[1]);
    if (c[0]) c[0]->flip ^= 1;
    if (c[1]) c[1]->flip ^= 1;
  }
  int up() { return p ? p->c[1] == this : -1; }//3a9
  void rot(int i, int b) {
    int h = i ^ b;
    Node *x = c[i], *y = b == 2 ? x : x->c[h], *z = b ? y :
        x;
    if ((y->p = p)) p->c[up()] = y;
    c[i] = z->c[i ^ 1];//eb7
    if (b < 2) {
      x->c[h] = y->c[h ^ 1];
      y->c[h ^ 1] = x;
    }
    z->c[i ^ 1] = this;//430
    fix(); x->fix(); y->fix();
    if (p) p->fix();
    swap(pp, y->pp);
  }
  void splay() {//4c8
    for (pushFlip(); p; ) {
      if (p->p) p->p->pushFlip();
      p->pushFlip(); pushFlip();
      int c1 = up(), c2 = p->up();
      if (c2 == -1) p->rot(c1, 2);//9e8
      else p->p->rot(c2, c1 != c2);
    }
  }
  Node* first() {
    pushFlip();//828
    return c[0] ? c[0]->first() : (splay(), this);
  }
};

struct LinkCut {//d99
  vector<Node> node;
  LinkCut(int N) : node(N) {}

  void link(int u, int v) { // add an edge (u, v)
    assert(!connected(u, v));//166
    makeRoot(&node[u]);
    node[u].pp = &node[v];
  }
  void cut(int u, int v) { // remove an edge (u, v)
    Node *x = &node[u], *top = &node[v];//0b9
    makeRoot(top); x->splay();
    assert(top == (x->pp ?: x->c[0]));
    if (x->pp) x->pp = 0;
    else {
      x->c[0] = top->p = 0;//158
      x->fix();
    }
  }
  bool connected(int u, int v) { // are u, v in the same
      tree?
    Node* nu = access(&node[u])->first();//781
    return nu == access(&node[v])->first();
  }
  void makeRoot(Node* u) {
    access(u);
    u->splay();//09d
    if(u->c[0]) {
      u->c[0]->p = 0;
      u->c[0]->flip ^= 1;
      u->c[0]->pp = u;
      u->c[0] = 0;//41e
      u->fix();
    }
  }
  Node* access(Node* u) {
    u->splay();//4e7
    while (Node* pp = u->pp) {
      pp->splay(); u->pp = 0;
      if (pp->c[1]) {
        pp->c[1]->p = 0; pp->c[1]->pp = pp; }
      pp->c[1] = u; pp->fix(); u = pp;//f4d
    }
    return u;
  }
};
```

### DirectedMST.h

**Description:** Finds a minimum spanning tree/arborescence of a directed graph, given a root node. If no MST exists, returns -1.
**Time:** $\mathcal{O}(E \log V)$

"../data-structures/UnionFindRollback.h" 057d96, 60 lines

```
struct Edge { int a, b; ll w{}; };//4d9
struct Node {
  Edge key;
  Node *l=0, *r=0;
  ll delta{};
  void prop() {//936
    key.w += delta;
```

```
    if (l) l->delta += delta;
    if (r) r->delta += delta;
    delta = 0;
  }//5dc
  Edge top() { prop(); return key; }
};
Node *merge(Node *a, Node *b) {
  if (!a || !b) return a ?: b;
  a->prop(), b->prop();//72a
  if (a->key.w > b->key.w) swap(a, b);
  swap(a->l, (a->r = merge(b, a->r)));
  return a;
}
void pop(Node*& a) { a->prop(); a = merge(a->l, a->r); }//8
    e9

pair<ll, vi> dmst(int n, int r, vector<Edge>& g) {
  RollbackUF uf(n);
  vector<Node*> heap(n);
  for (Edge e : g) heap[e.b] = merge(heap[e.b], new Node{e}
      );//0f3
  ll res = 0;
  vi seen(n, -1), path(n), par(n);
  seen[r] = r;
  vector<Edge> Q(n), in(n, {-1,-1}), comp;
  deque<tuple<int, int, vector<Edge>>> cycs;//4c6
  rep(s,0,n) {
    int u = s, qi = 0, w;
    while (seen[u] < 0) {
      if (!heap[u]) return {-1,{}};
      Edge e = heap[u]->top();//2b0
      heap[u]->delta -= e.w, pop(heap[u]);
      Q[qi] = e, path[qi++] = u, seen[u] = s;
      res += e.w, u = uf.find(e.a);
      if (seen[u] == s) {
        Node* cyc = 0;//fff
        int end = qi, time = uf.time();
        do cyc = merge(cyc, heap[w = path[--qi]]);
        while (uf.join(u, w));
        u = uf.find(u), heap[u] = cyc, seen[u] = -1;
        cycs.push_front({u, time, {&Q[qi], &Q[end]}});//984
      }
    }
    rep(i,0,qi) in[uf.find(Q[i].b)] = Q[i];
  }
//eba
  for (auto& [u,t,cc] : cycs) { // restore sol (optional)
    uf.rollback(t);
    Edge inEdge = in[u];
    for (auto& e : cc) in[uf.find(e.b)] = e;
    in[uf.find(inEdge.b)] = inEdge;//ffd
  }
  rep(i,0,n) par[i] = in[i].a;
  return {res, par};
}
```

### BinaryLifting.h

**Description:** Calculate power of two jumps in a tree, to support fast upward jumps and LCAs. Assumes the root node points to itself.
**Time:** construction $\mathcal{O}(N \log N)$, queries $\mathcal{O}(\log N)$

bfce85, 25 lines

```
vector<vi> treeJump(vi& P){//bcb
  int on = 1, d = 1;
  while(on < sz(P)) on *= 2, d++;
  vector<vi> jmp(d, P);
  rep(i,1,d) rep(j,0,sz(P))
    jmp[i][j] = jmp[i-1][jmp[i-1][j]];//47a
  return jmp;
}

int jmp(vector<vi>& tbl, int nod, int steps){
  rep(i,0,sz(tbl))//66f
    if(steps&(1<<i)) nod = tbl[i][nod];
  return nod;
}

int lca(vector<vi>& tbl, vi& depth, int a, int b) {//57b
  if (depth[a] < depth[b]) swap(a, b);
  a = jmp(tbl, a, depth[a] - depth[b]);
  if (a == b) return a;
  for (int i = sz(tbl); i--;) {
    int c = tbl[i][a], d = tbl[i][b];//30e
    if (c != d) a = c, b = d;
  }
  return tbl[0][a];
}
```

### LCA.h

**Description:** Data structure for computing lowest common ancestors in a tree (with 0 as root). C should be an adjacency list of the tree, either directed or undirected.
**Time:** $\mathcal{O}(N \log N + Q)$

"../data-structures/RMQ.h" aa4761, 21 lines

```
struct LCA {//169
  int T = 0;
  vi time, path, ret;
  RMQ<int> rmq;

  LCA(vector<vi>& C) : time(sz(C)), rmq((dfs(C,0,-1), ret))
      {}//1e9
  void dfs(vector<vi>& C, int v, int par) {
    time[v] = T++;
    for (int y : C[v]) if (y != par) {
      path.pb(v), ret.push_back(time[v]);
      dfs(C, y, v);//3f8
    }
  }

  int lca(int a, int b) {
    if (a == b) return a;//3f5
    tie(a, b) = minmax(time[a], time[b]);
    return path[rmq.query(a, b)];
  }
  //dist(a,b){return depth[a] + depth[b] - 2*depth[lca(a,b)
      ];}
};//214
```

## 7.8 Math

### 7.8.1 Number of Spanning Trees

Create an $N \times N$ matrix mat, and for each edge $a \to b \in G$, do mat[a][b]--, mat[b][b]++ (and mat[b][a]--, mat[a][a]++ if $G$ is undirected). Remove the $i$th row and column and take the determinant; this yields the number of directed spanning trees rooted at $i$ (if $G$ is undirected, remove any row/column).

### 7.8.2 Erdős–Gallai theorem

A simple graph with node degrees $d_1 \geq \cdots \geq d_n$ exists iff $d_1 + \cdots + d_n$ is even and for every $k = 1 \ldots n$,

$$\sum_{i=1}^{k} d_i \leq k(k-1) + \sum_{i=k+1}^{n} \min(d_i, k).$$

# Geometry (8)

## 8.1 Geometric primitives

### Point.h

**Description:** Class to handle points in the plane. T can be e.g. double or long long. (Avoid int.)

634da7, 29 lines

```
template <class T> int sgn(T x) { return (x > 0) - (x < 0);
    }//fa7
template<class T>
struct Point {
  typedef Point P;
  T x, y;
  explicit Point(T _x=0, T _y=0) : x(_x), y(_y) {}//a5f
  bool operator<(P p) const { return tie(x,y) < tie(p.x,p.y
      ); }
  bool operator==(P p) const { return tie(x,y)==tie(p.x,p.y
      ); }
  P operator+(P p) const { return P(x+p.x, y+p.y); }
  P operator-(P p) const { return P(x-p.x, y-p.y); }
  P operator*(T d) const { return P(x*d, y*d); }//e11
  P operator/(T d) const { return P(x/d, y/d); }
  T dot(P p) const { return x*p.x + y*p.y; }
  T cross(P p) const { return x*p.y - y*p.x; } // + => p on
      right
  T cross(P a, P b) const { return (a-*this).cross(b-*this)
      ; }
  bool half() const { return y < 0 || (y == 0 && x < 0); }
      //053
  T dist2() const { return x*x + y*y; }
  double dist() const { return sqrt((double)dist2()); }
  // angle to x-axis in interval [-pi, pi]
  double angle() const { return atan2(y, x); }
  P unit() const { return *this/dist(); } // makes dist()
      =1//8da
  P perp() const { return P(-y, x); } // rotates +90
      degrees
  P normal() const { return perp().unit(); }
  // returns point rotated 'a' radians ccw around the
      origin
  P rotate(double a) const {
    return P(x*cos(a)-y*sin(a),x*sin(a)+y*cos(a)); }//ad4
  friend ostream& operator<<(ostream& os, P p) {
    return os << "(" << p.x << "," << p.y << ")"; }
};
```

### AngleSort.h

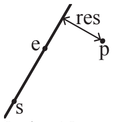
**Description:** Sorts points radially across the origin. To sort around a point, sort a-p and b-p.

"Point.h" c10d46, 7 lines

```
template<class P>//159
void anglesort(vector<P> &v, P p=P(0, 0)) {
  sort(all(v), [p](P a, P b) {
    a = a - p, b = b - p;
    return a.half() == b.half() ? a.cross(b) > 0 : a.half()
        < b.half();
  });//b97
}
```

### lineDistance.h

**Description:**
Returns the signed distance between point p and the line containing points a and b. Positive value on left side and negative on right as seen from a towards b. a==b gives nan. P is supposed to be Point<T> or Point3D<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long. Using Point3D will always give a non-negative distance. For Point3D, call .dist on the result of the cross product.

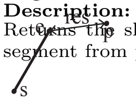



"Point.h" f6bf6b, 4 lines

```
template<class P>//f6b
double lineDist(const P& a, const P& b, const P& p) {
  return (double)(b-a).cross(p-a)/(b-a).dist();
}
```

## SegmentDistance.h

**Description:**
Returns the shortest distance between point p and the line segment from point s to e.



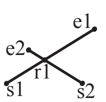

**Usage:** `Point<double> a, b(2,2), p(1,1);`
`bool onSegment = segDist(a,b,p) < 1e-10;`

"Point.h" 5c88f4, 6 lines

```
typedef Point<double> P;//b95
double segDist(P& s, P& e, P& p) {
  if (s==e) return (p-s).dist();
  auto d = (e-s).dist2(), t = min(d,max(.0,(p-s).dot(e-s)))
      ;
  return ((p-s)*d-(e-s)*t).dist()/d;
}//cbb
```

## SegmentIntersection.h

**Description:**
If a unique intersection point between the line segments going from s1 to e1 and from s2 to e2 exists then it is returned. If no intersection point exists an empty vector is returned. If infinitely many exist a vector with 2 elements is returned, containing the endpoints of the common line segment. The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or long long.



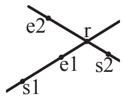

**Usage:** `vector<P> inter = segInter(s1,e1,s2,e2);`
`if (sz(inter)==1)`
`cout << "segments intersect at " << inter[0] << endl;`

"Point.h", "OnSegment.h" 9d57f2, 13 lines

```
template<class P> vector<P> segInter(P a, P b, P c, P d) {
    //dec
  auto oa = c.cross(d, a), ob = c.cross(d, b),
       oc = a.cross(b, c), od = a.cross(b, d);
  // Checks if intersection is single non-endpoint point.
  if (sgn(oa) * sgn(ob) < 0 && sgn(oc) * sgn(od) < 0)
    return {(a * ob - b * oa) / (ob - oa)};//8a0
  set<P> s;
  if (onSegment(c, d, a)) s.insert(a);
  if (onSegment(c, d, b)) s.insert(b);
  if (onSegment(a, b, c)) s.insert(c);
  if (onSegment(a, b, d)) s.insert(d);//814
  return {all(s)};
}
```

## lineIntersection.h

**Description:**
If a unique intersection point of the lines going through s1,e1 and s2,e2 exists {1, point} is returned. If no intersection point exists {0, (0,0)} is returned and if infinitely many exists {-1, (0,0)} is returned. The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or ll.




**Usage:** `auto res = lineInter(s1,e1,s2,e2);`
`if (res.first == 1)`
`cout << "intersection point at " << res.second << endl;`

"Point.h" a01f81, 8 lines

```
template<class P>//47e
pair<int, P> lineInter(P s1, P e1, P s2, P e2) {
  auto d = (e1 - s1).cross(e2 - s2);
  if (d == 0) // if parallel
    return {-(s1.cross(e1, s2) == 0), P(0, 0)};
  auto p = s2.cross(e1, e2), q = s2.cross(e2, s1);//16d
  return {1, (s1 * p + e1 * q) / d};
}
```

## sideOf.h

**Description:** Returns where $p$ is as seen from $s$ towards $e$. 1/0/-1 $\Leftrightarrow$ left/on line/right. If the optional argument *eps* is given 0 is returned if $p$ is within distance *eps* from the line. P is supposed to be Point<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long.
**Usage:** `bool left = sideOf(p1,p2,q)==1;`

"Point.h" 3af81c, 9 lines

```
template<class P>//059
int sideOf(P s, P e, P p) { return sgn(s.cross(e, p)); }

template<class P>
int sideOf(const P& s, const P& e, const P& p, double eps)
    {
  auto a = (e-s).cross(p-s);//7c7
  double l = (e-s).dist()*eps;
  return (a > l) - (a < -l);
}
```

## OnSegment.h

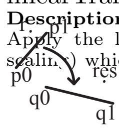
**Description:** Returns true iff p lies on the line segment from s to e. Use `(segDist(s,e,p)<=epsilon)` instead when using Point<double>.

"Point.h" c597e8, 3 lines

```
template<class P> bool onSegment(P s, P e, P p) {//c59
  return p.cross(s, e) == 0 && (s - p).dot(e - p) <= 0;
}
```

## linearTransformation.h

**Description:**
Apply the linear transformation (translation, rotation and scaling) which takes line p0-p1 to line q0-q1 to point r.




"Point.h" 03a306, 6 lines

```
typedef Point<double> P;//d52
P linearTransformation(const P& p0, const P& p1,
    const P& q0, const P& q1, const P& r) {
  P dp = p1-p0, dq = q1-q0, num(dp.cross(dq), dp.dot(dq));
  return q0 + P((r-p0).cross(num), (r-p0).dot(num))/dp.
      dist2();
}//cbb
```

## LineProjectionReflection.h

**Description:** Projects point p onto line ab. Set refl=true to get reflection of point p across line ab instead. The wrong point will be returned if P is an integer point and the desired point doesn't have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow.

"Point.h" b5562d, 5 lines

```
template<class P>//b55
P lineProj(P a, P b, P p, bool refl=false) {
  P v = b - a;
  return p - v.perp()*(1+refl)*v.cross(p-a)/v.dist2();
}
```

## Angle.h

**Description:** A class for ordering angles (as represented by int points and a number of rotations around the origin). Useful for rotational sweeping. Sometimes also represents points or vectors.
**Usage:** `vector<Angle> v = {w[0], w[0].t360() ...}; // sorted`
`int j = 0; rep(i,0,n) { while (v[j] < v[i].t180()) ++j; }`
`// sweeps j such that (j-i) represents the number of positively oriented triangles with vertices at 0 and i`

28b5e9, 35 lines

```
struct Angle {//717
  int x, y;
  int t;
  Angle(int _x, int _y, int _t=0) : x(_x), y(_y), t(_t) {}
  Angle operator-(Angle b) const { return {x-b.x, y-b.y, t}
      ; }
  int half() const {//a5b
    assert(x || y);
    return y < 0 || (y == 0 && x < 0);
  }
  Angle t90() const { return {-y, x, t + (half() && x >= 0)
      }; }
  Angle t180() const { return {-x, -y, t + half()}; }//de0
  Angle t360() const { return {x, y, t + 1}; }
};
bool operator<(Angle a, Angle b) {
  // add a.dist2() and b.dist2() to also compare distances
  return make_tuple(a.t, a.half(), a.y * (ll)b.x) <//41b
      make_tuple(b.t, b.half(), a.x * (ll)b.y);
}

// Given two points, this calculates the smallest angle
    between
// them, i.e., the angle that covers the defined line
    segment.//f86
pair<Angle, Angle> segmentAngles(Angle a, Angle b) {
  if (b < a) swap(a, b);
  return (b < a.t180() ?
      make_pair(a, b) : make_pair(b, a.t360()));
}//b11
Angle operator+(Angle a, Angle b) { // point a + vector b
  Angle r(a.x + b.x, a.y + b.y, a.t);
  if (a.t180() < r) r.t--;
  return r.t180() < a ? r.t360() : r;
}//073
Angle angleDiff(Angle a, Angle b) { // angle b - angle a
  int tu = b.t - a.t; a.t = b.t;
  return {a.x*b.x + a.y*b.y, a.x*b.y - a.y*b.x, tu - (b < a
      )};
}
```

## 8.2 Circles

### CircleIntersection.h

**Description:** Computes the pair of points at which two circles intersect. Returns false in case of no intersection.

"Point.h" 84d6d3, 11 lines

```
typedef Point<double> P;//deb
bool circleInter(P a,P b,double r1,double r2,pair<P, P>*
    out) {
  if (a == b) { assert(r1 != r2); return false; }
  P vec = b - a;
  double d2 = vec.dist2(), sum = r1+r2, dif = r1-r2,
         p = (d2 + r1*r1 - r2*r2)/(d2*2), h2 = r1*r1 - p*p*d2
             ;//367
  if (sum*sum < d2 || dif*dif > d2) return false;
  P mid = a + vec*p, per = vec.perp() * sqrt(fmax(0, h2) /
       d2);
  *out = {mid + per, mid - per};
  return true;
}//cbb
```

### CircleTangents.h

**Description:** Finds the external tangents of two circles, or internal if r2 is negated. Can return 0, 1, or 2 tangents – 0 if one circle contains the other (or overlaps it, in the internal case, or if the circles are the same); 1 if the circles are tangent to each other (in which case .first = .second and the tangent line is perpendicular to the line between the centers). .first and .second give the tangency points at circle 1 and 2 respectively. To find the tangents of a circle with a point set r2 to 0.

"Point.h" 31cca4, 13 lines

```
template<class P>//c18
vector<pair<P, P>> tangents(P c1, double r1, P c2, double
    r2) {
  P d = c2 - c1;
  double dr = r1 - r2, d2 = d.dist2(), h2 = d2 - dr * dr;
  if (d2 == 0 || h2 < 0)  return {};
  vector<pair<P, P>> out;//5c8
  for (double sign : {-1, 1}) {
    P v = (d * dr + d.perp() * sqrt(h2) * sign) / d2;
    out.pb({c1 + v * r1, c2 + v * r2});
  }
  if (h2 == 0) out.pop_back();//918
  return out;
}
```

### CircleLine.h

**Description:** Finds the intersection between a circle and a line. Returns a vector of either 0, 1, or 2 intersection points. P is intended to be Point<double>.

"Point.h" e0cfba, 9 lines

```
template<class P>//64a
vector<P> circleLine(P c, double r, P a, P b) {
  P ab = b - a, p = a + ab * (c-a).dot(ab) / ab.dist2();
  double s = a.cross(b, c), h2 = r*r - s*s / ab.dist2();
  if (h2 < 0) return {};
  if (h2 == 0) return {p};//fd3
  P h = ab.unit() * sqrt(h2);
  return {p - h, p + h};
}
```

### CirclePolygonIntersection.h

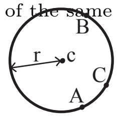
**Description:** Returns the area of the intersection of a circle with a ccw polygon.
**Time:** $\mathcal{O}(n)$

"../../content/geometry/Point.h" a1ee63, 19 lines

```
typedef Point<double> P;//a6c
#define arg(p, q) atan2(p.cross(q), p.dot(q))
double circlePoly(P c, double r, vector<P> ps) {
  auto tri = [&](P p, P q) {
    auto r2 = r * r / 2;
    P d = q - p;//eda
    auto a = d.dot(p)/d.dist2(), b = (p.dist2()-r*r)/d.
        dist2();
    auto det = a * a - b;
    if (det <= 0) return arg(p, q) * r2;
    auto s = max(0., -a-sqrt(det)), t = min(1., -a+sqrt(det
        ));
    if (t < 0 || 1 <= s) return arg(p, q) * r2;//174
    P u = p + d * s, v = p + d * t;
    return arg(p,u) * r2 + u.cross(v)/2 + arg(v,q) * r2;
  };
  auto sum = 0.0;
  rep(i,0,sz(ps))//a61
    sum += tri(ps[i] - c, ps[(i + 1) % sz(ps)] - c);
  return sum;
}
```

### circumcircle.h

**Description:**
The circumcirle of a triangle is the circle intersecting all three vertices. ccRadius returns the radius of the circle going through points A, B and C and ccCenter returns the center of the same circle.




"Point.h" 1caa3a, 9 lines

```
typedef Point<double> P;//032
double ccRadius(const P& A, const P& B, const P& C) {
  return (B-A).dist()*(C-B).dist()*(A-C).dist()/
      abs((B-A).cross(C-A))/2;
}
P ccCenter(const P& A, const P& B, const P& C) {//793
  P b = C-A, c = B-A;
  return A + (b*c.dist2()-c*b.dist2()).perp()/b.cross(c)/2;
}
```

### MinimumEnclosingCircle.h

**Description:** Computes the minimum circle that encloses a set of points.
**Time:** expected $\mathcal{O}(n)$

"circumcircle.h" 09dd0a, 17 lines

```
pair<P, double> mec(vector<P> ps) {//b50
  shuffle(all(ps), mt19937(time(0)));
  P o = ps[0];
  double r = 0, EPS = 1 + 1e-8;
  rep(i,0,sz(ps)) if ((o - ps[i]).dist() > r * EPS) {
    o = ps[i], r = 0;//d54
    rep(j,0,i) if ((o - ps[j]).dist() > r * EPS) {
      o = (ps[i] + ps[j]) / 2;
      r = (o - ps[i]).dist();
      rep(k,0,j) if ((o - ps[k]).dist() > r * EPS) {
        o = ccCenter(ps[i], ps[j], ps[k]);//4ec
        r = (o - ps[i]).dist();
      }
    }
  }
  return {o, r};//2ac
}
```

## 8.3 Polygons

### InsidePolygon.h

**Description:** Returns true if p lies within the polygon. If strict is true, it returns false for points on the boundary. The algorithm uses products in intermediate steps so watch out for overflow.
**Usage:** `vector<P> v = {P{4,4}, P{1,2}, P{2,1}};`
`bool in = inPolygon(v, P{3, 3}, false);`
**Time:** $\mathcal{O}(n)$

"Point.h", "OnSegment.h", "SegmentDistance.h" 2bf504, 11 lines

```
template<class P>//1c1
bool inPolygon(vector<P> &p, P a, bool strict = true) {
  int cnt = 0, n = sz(p);
  rep(i,0,n) {
    P q = p[(i + 1) % n];
    if (onSegment(p[i], q, a)) return !strict;//fa7
    //or: if (segDist(p[i], q, a) <= eps) return !strict;
    cnt ^= ((a.y<p[i].y) - (a.y<q.y)) * a.cross(p[i], q) >
        0;
  }
  return cnt;
}//cbb
```

### PolygonArea.h

**Description:** Returns twice the signed area of a polygon. Clockwise enumeration gives negative area. Watch out for overflow if using int as T!

"Point.h" f12300, 6 lines

```
template<class T>//b19
T polygonArea2(vector<Point<T>>& v) {
  T a = v.back().cross(v[0]);
  rep(i,0,sz(v)-1) a += v[i].cross(v[i+1]);
  return a;
}//cbb
```

### PolygonCenter.h

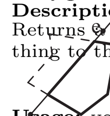
**Description:** Returns the center of mass for a polygon.
**Time:** $\mathcal{O}(n)$

"Point.h" 9706dc, 9 lines

```
typedef Point<double> P;//082
P polygonCenter(const vector<P>& v) {
  P res(0, 0); double A = 0;
  for (int i = 0, j = sz(v) - 1; i < sz(v); j = i++) {
    res = res + (v[i] + v[j]) * v[j].cross(v[i]);
    A += v[j].cross(v[i]);//168
  }
  return res / A / 3;
}
```

### PolygonCut.h

**Description:**
Returns a vector with the vertices of a polygon with everything to the left of the line going from s to e cut away.



**Usage:** `vector<P> p = ...;`
`p = polygonCut(p, P(0,0), P(1,0));`

"Point.h", "lineIntersection.h" 056a39, 13 lines

```
typedef Point<double> P;//366
vector<P> polygonCut(const vector<P>& poly, P s, P e) {
  vector<P> res;
  rep(i,0,sz(poly)) {
    P cur = poly[i], prev = i ? poly[i-1] : poly.back();
    bool side = s.cross(e, cur) < 0;//c08
    if (side != (s.cross(e, prev) < 0))
      res.pb(lineInter(s, e, cur, prev).second);
    if (side)
      res.pb(cur);
  }//0e1
  return res;
}
```

### PolygonUnion.h

**Description:** Calculates the area of the union of $n$ polygons (not necessarily convex). The points within each polygon must be given in CCW order. (Epsilon checks may optionally be added to sideOf/sgn, but shouldn't be needed.)

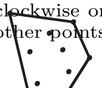
**Time:** $\mathcal{O}(N^2)$, where $N$ is the total number of points

"Point.h", "sideOf.h" 3931c6, 33 lines

```
typedef Point<double> P;//49c
double rat(P a, P b) { return sgn(b.x) ? a.x/b.x : a.y/b.y;
    }
double polyUnion(vector<vector<P>>& poly) {
  double ret = 0;
  rep(i,0,sz(poly)) rep(v,0,sz(poly[i])) {
    P A = poly[i][v], B = poly[i][(v + 1) % sz(poly[i])];//
        896
    vector<pair<double, int>> segs = {{0, 0}, {1, 0}};
    rep(j,0,sz(poly)) if (i != j) {
      rep(u,0,sz(poly[j])) {
        P C = poly[j][u], D = poly[j][(u + 1) % sz(poly[j])
            ];
        int sc = sideOf(A, B, C), sd = sideOf(A, B, D);//
            407
        if (sc != sd) {
          double sa = C.cross(D, A), sb = C.cross(D, B);
          if (min(sc, sd) < 0)
            segs.emplace_back(sa / (sa - sb), sgn(sc - sd))
                ;
        } else if (!sc && !sd && j<i && sgn((B-A).dot(D-C))
            >0){//8be
          segs.emplace_back(rat(C - A, B - A), 1);
          segs.emplace_back(rat(D - A, B - A), -1);
        }
      }
    }//155
    sort(all(segs));
    for (auto& s : segs) s.first = min(max(s.first, 0.0),
        1.0);
    double sum = 0;
    int cnt = segs[0].second;
    rep(j,1,sz(segs)) {//88e
      if (!cnt) sum += segs[j].first - segs[j - 1].first;
      cnt += segs[j].second;
    }
    ret += A.cross(B) * sum;
  }//f48
  return ret / 2;
}
```

## ConvexHull.h

**Description:**
Returns a vector of the points of the convex hull in counter-clockwise order. Points on the edge of the hull between two other points are not considered part of the hull.



**Time:** $\mathcal{O}(n \log n)$

"Point.h" 310954, 13 lines

```
typedef Point<ll> P;//3e3
vector<P> convexHull(vector<P> pts) {
  if (sz(pts) <= 1) return pts;
  sort(all(pts));
  vector<P> h(sz(pts)+1);
  int s = 0, t = 0;//f18
  for (int it = 2; it--; s = --t, reverse(all(pts)))
    for (P p : pts) {
      while (t >= s + 2 && h[t-2].cross(h[t-1], p) <= 0) t
          --;
      h[t++] = p;
    }//aa0
  return {h.begin(), h.begin() + t - (t == 2 && h[0] == h
      [1])};
}
```

## HullDiameter.h

**Description:** Returns the two points with max distance on a convex hull (ccw, no duplicate/collinear points).

**Time:** $\mathcal{O}(n)$

"Point.h" c571b8, 12 lines

```
typedef Point<ll> P;//5c7
array<P, 2> hullDiameter(vector<P> S) {
  int n = sz(S), j = n < 2 ? 0 : 1;
  pair<ll, array<P, 2>> res({0, {S[0], S[0]}});
  rep(i,0,j)
    for (;; j = (j + 1) % n) {//56c
      res = max(res, {(S[i] - S[j]).dist2(), {S[i], S[j]}})
          ;
      if ((S[(j + 1) % n] - S[j]).cross(S[i + 1] - S[i]) >=
          0)
        break;
    }
  return res.second;//52a
}
```

## PointInsideHull.h

**Description:** Determine whether a point t lies inside a convex hull (CCW order, with no collinear points). Returns true if point lies within the hull. If strict is true, points on the boundary aren't included.

**Time:** $\mathcal{O}(\log N)$

"Point.h", "sideOf.h", "OnSegment.h" 71446b, 14 lines

```
typedef Point<ll> P;//7a3

bool inHull(const vector<P>& l, P p, bool strict = true) {
  int a = 1, b = sz(l) - 1, r = !strict;
  if (sz(l) < 3) return r && onSegment(l[0], l.back(), p);
  if (sideOf(l[0], l[a], l[b]) > 0) swap(a, b);//4a6
  if (sideOf(l[0], l[a], p) >= r || sideOf(l[0], l[b], p)<=
      -r)
    return false;
  while (abs(a - b) > 1) {
    int c = (a + b) / 2;
    (sideOf(l[0], l[c], p) > 0 ? b : a) = c;//0da
  }
  return sgn(l[a].cross(l[b], p)) < r;
}
```

## LineHullIntersection.h

**Description:** Line-convex polygon intersection. The polygon must be ccw and have no collinear points. lineHull(line, poly) returns a pair describing the intersection of a line with the polygon: • $(-1,-1)$ if no collision, • $(i,-1)$ if touching the corner $i$, • $(i,i)$ if along side $(i,i+1)$, • $(i,j)$ if crossing sides $(i,i+1)$ and $(j,j+1)$. In the last case, if a corner $i$ is crossed, this is treated as happening on side $(i,i+1)$. The points are returned in the same order as the line hits the polygon. `extrVertex` returns the point of a hull with the max projection onto a line.

**Time:** $\mathcal{O}(\log n)$

"Point.h" 7cf45b, 39 lines

```
#define cmp(i,j) sgn(dir.perp().cross(poly[(i)%n]-poly[(j)%
    n]))//b9d
#define extr(i) cmp(i + 1, i) >= 0 && cmp(i, i - 1 + n) < 0
template <class P> int extrVertex(vector<P>& poly, P dir) {
  int n = sz(poly), lo = 0, hi = n;
  if (extr(0)) return 0;
  while (lo + 1 < hi) {//51a
    int m = (lo + hi) / 2;
    if (extr(m)) return m;
    int ls = cmp(lo + 1, lo), ms = cmp(m + 1, m);
    (ls < ms || (ls == ms && ls == cmp(lo, m)) ? hi : lo) =
        m;
  }//e8c
  return lo;
}

#define cmpL(i) sgn(a.cross(poly[i], b))
template <class P>//7fd
array<int, 2> lineHull(P a, P b, vector<P>& poly) {
  int endA = extrVertex(poly, (a - b).perp());
  int endB = extrVertex(poly, (b - a).perp());
  if (cmpL(endA) < 0 || cmpL(endB) > 0)
    return {-1, -1};//04b
  array<int, 2> res;
  rep(i,0,2) {
    int lo = endB, hi = endA, n = sz(poly);
    while ((lo + 1) % n != hi) {
      int m = ((lo + hi + (lo < hi ? 0 : n)) / 2) % n;//ec0
      (cmpL(m) == cmpL(endB) ? lo : hi) = m;
    }
    res[i] = (lo + !cmpL(hi)) % n;
    swap(endA, endB);
  }//6ab
  if (res[0] == res[1]) return {res[0], -1};
  if (!cmpL(res[0]) && !cmpL(res[1]))
    switch ((res[0] - res[1] + sz(poly) + 1) % sz(poly)) {
      case 0: return {res[0], res[0]};
      case 2: return {res[1], res[1]};//08a
    }
  return res;
}
```

## HullTangents.h

**Description:** Finds the two tangent vertices on the convex hull to some point. Point must be outside. Appears to be left then right.

"Point.h" 0bdfcf, 22 lines

```
template<typename P, typename F>//134
int extremeVertex(const P& poly, F direction) {
  int n = sz(poly), l = 0, ls;
  auto vertexCmp = [&](int i, int j) {
  return sgn(direction(poly[j]).cross(poly[j] - poly[i]));
      };
  auto isExtreme = [&](int i, int& is) {//d3d
  return (is = vertexCmp((i+1)%n, i)) >= 0 && vertexCmp(i,
      (i+n-1)%n) < 0; };
  for (int r = isExtreme(0, ls) ? 1 : n; l + 1 < r;) {
  int m = (l + r) / 2, ms;
  if (isExtreme(m, ms)) return m;
  if (ls != ms ? ls < ms : ls == vertexCmp(l, m)) r = m;//
      beb
  else l = m, ls = ms;
  }
  return l;
}
//d22
template<typename P>
pair<int, int> tangentsConvex(const P &point, const vector<
    P>& poly) {
  return {
  extremeVertex(poly, [&](const P& q) { return q - point; }
      ),
  extremeVertex(poly, [&](const P& q) { return point - q; }
      )};//fa7
}
```

## MinkowskiSum.h

**Description:** Returns the set of all sums of points of two convex polygons.

**Time:** $\mathcal{O}(n+m)$

"Point.h" e065ea, 29 lines

```
typedef Point<ll> P;//9c1
void reorder_polygon(vector<P> &p) {
  int pos = 0;
  for (int i = 1; i < sz(p); i++) {
    if (p[i].y < p[pos].y || (p[i].y == p[pos].y && p[i].x
        < p[pos].x))
      pos = i;//bf2
```

```
  }
  rotate(p.begin(), p.begin() + pos, p.end());
}

vector<P> minkowski(vector<P> p, vector<P> q) {//ad1
  reorder_polygon(p);
  reorder_polygon(q);

  p.pb(p[0]);
  p.pb(p[1]);//26e
  q.pb(q[0]);
  q.pb(q[1]);

  vector<P> result;
  int i = 0, j = 0;//257
  while (i < sz(p) - 2 || j < sz(q) - 2) {
    result.pb(p[i] + q[j]);
    auto cross = (p[i + 1] - p[i]).cross(q[j + 1] - q[j]);
    if (cross >= 0 && i < sz(p) - 2) ++i;
    if (cross <= 0 && j < sz(q) - 2) ++j;//19e
  }
  return result;
}
```

## 8.4 Misc. Point Set Problems

### ClosestPair.h

**Description:** Finds the closest pair of points.
**Time:** $\mathcal{O}(n \log n)$

"Point.h" ac41a6, 17 lines

```
typedef Point<ll> P;//9e7
pair<P, P> closest(vector<P> v) {
  assert(sz(v) > 1);
  set<P> S;
  sort(all(v), [](P a, P b) { return a.y < b.y; });
  pair<ll, pair<P, P>> ret{LLONG_MAX, {P(), P()}};//e83
  int j = 0;
  for (P p : v) {
    P d{1 + (ll)sqrt(ret.first), 0};
    while (v[j].y <= p.y - d.x) S.erase(v[j++]);
    auto lo = S.lower_bound(p - d), hi = S.upper_bound(p +
        d);//cb2
    for (; lo != hi; ++lo)
      ret = min(ret, {(*lo - p).dist2(), {*lo, p}});
    S.insert(p);
  }
  return ret.second;//982
}
```

### ManhattanMST.h

**Description:** Given N points, returns up to 4*N edges, which are guaranteed to contain a minimum spanning tree for the graph with edge weights w(p, q) = —p.x - q.x— + —p.y - q.y—. Edges are in the form (distance, src, dst). Use a standard MST algorithm on the result to find the final MST.
**Time:** $\mathcal{O}(N \log N)$

"Point.h" e2611c, 23 lines

```
typedef Point<int> P;//bde
vector<array<int, 3>> manhattanMST(vector<P> ps) {
  vi id(sz(ps));
  iota(all(id), 0);
  vector<array<int, 3>> edges;
  rep(k,0,4) {//9bd
    sort(all(id), [&](int i, int j) {
       return (ps[i]-ps[j]).x < (ps[j]-ps[i]).y;});
    map<int, int> sweep;
    for (int i : id) {
      for (auto it = sweep.lower_bound(-ps[i].y);//0bb
            it != sweep.end(); sweep.erase(it++)) {
        int j = it->second;
        P d = ps[i] - ps[j];
        if (d.y > d.x) break;
        edges.pb({d.y + d.x, i, j});//868
      }
      sweep[-ps[i].y] = i;
    }
    for (P& p : ps) if (k & 1) p.x = -p.x; else swap(p.x, p
        .y);
  }//aa4
  return edges;
}
```

### kdTree.h

**Description:** KD-tree (2d, can be extended to 3d)

"Point.h" bac5b0, 63 lines

```
typedef long long T;//632
typedef Point<T> P;
const T INF = numeric_limits<T>::max();

bool on_x(const P& a, const P& b) { return a.x < b.x; }
bool on_y(const P& a, const P& b) { return a.y < b.y; }//
    c56

struct Node {
  P pt; // if this is a leaf, the single point in it
  T x0 = INF, x1 = -INF, y0 = INF, y1 = -INF; // bounds
  Node *first = 0, *second = 0;//5b4

  T distance(const P& p) { // min squared distance to a
      point
    T x = (p.x < x0 ? x0 : p.x > x1 ? x1 : p.x);
    T y = (p.y < y0 ? y0 : p.y > y1 ? y1 : p.y);
    return (P(x,y) - p).dist2();//a82
  }

  Node(vector<P>&& vp) : pt(vp[0]) {
    for (P p : vp) {
      x0 = min(x0, p.x); x1 = max(x1, p.x);//151
      y0 = min(y0, p.y); y1 = max(y1, p.y);
    }
    if (vp.size() > 1) {
      // split on x if width >= height (not ideal...)
      sort(all(vp), x1 - x0 >= y1 - y0 ? on_x : on_y);//1d2
      // divide by taking half the array for each child (
          not
      // best performance with many duplicates in the
          middle)
      int half = sz(vp)/2;
      first = new Node({vp.begin(), vp.begin() + half});
      second = new Node({vp.begin() + half, vp.end()});//
          ace
    }
  }
};

struct KDTree {//72b
  Node* root;
  KDTree(const vector<P>& vp) : root(new Node({all(vp)})) {
      }

  pair<T, P> search(Node *node, const P& p) {
    if (!node->first) {//119
      // uncomment if we should not find the point itself:
      // if (p == node->pt) return {INF, P()};
      return make_pair((p - node->pt).dist2(), node->pt);
    }
//a89
    Node *f = node->first, *s = node->second;
    T bfirst = f->distance(p), bsec = s->distance(p);
    if (bfirst > bsec) swap(bsec, bfirst), swap(f, s);

    // search closest side first, other side if needed//bfa
    auto best = search(f, p);
    if (bsec < best.first)
      best = min(best, search(s, p));
    return best;
  }//13a

  // find nearest point to a point, and its squared
      distance
  // (requires an arbitrary operator< for Point)
  pair<T, P> nearest(const P& p) {
    return search(root, p);//213
  }
};
```

### FastDelaunay.h

**Description:** Fast Delaunay triangulation. Each circumcircle contains none of the input points. There must be no duplicate points. If all points are on a line, no triangles will be returned. Should work for doubles as well, though there may be precision issues in 'circ'. Returns triangles in order {t[0][0], t[0][1], t[0][2], t[1][0], ...}, all counter-clockwise.
**Time:** $\mathcal{O}(n \log n)$

"Point.h" 04ae3a, 88 lines

```
typedef Point<ll> P;//503
typedef struct Quad* Q;
typedef __int128_t lll; // (can be ll if coords are < 2e4)
P arb(LLONG_MAX,LLONG_MAX); // not equal to any other point

struct Quad {//8bb
  Q rot, o; P p = arb; bool mark;
  P& F() { return r()->p; }
  Q& r() { return rot->rot; }
  Q prev() { return rot->o->rot; }
  Q next() { return r()->prev(); }//0bd
} *H;

bool circ(P p, P a, P b, P c) { // is p in the circumcircle
    ?
  lll p2 = p.dist2(), A = a.dist2()-p2,
    B = b.dist2()-p2, C = c.dist2()-p2;//520
  return p.cross(a,b)*C + p.cross(b,c)*A + p.cross(c,a)*B >
      0;
}
Q makeEdge(P orig, P dest) {
  Q r = H ? H : new Quad{new Quad{new Quad{new Quad{0}}}};
  H = r->o; r->r()->r() = r;//60f
  rep(i,0,4) r = r->rot, r->p = arb, r->o = i & 1 ? r : r->
      r();
  r->p = orig; r->F() = dest;
  return r;
}
void splice(Q a, Q b) {//5b1
  swap(a->o->rot->o, b->o->rot->o); swap(a->o, b->o);
}
Q connect(Q a, Q b) {
  Q q = makeEdge(a->F(), b->p);
  splice(q, a->next());//3cc
  splice(q->r(), b);
  return q;
}

pair<Q,Q> rec(const vector<P>& s) {//a03
  if (sz(s) <= 3) {
    Q a = makeEdge(s[0], s[1]), b = makeEdge(s[1], s.back()
        );
    if (sz(s) == 2) return { a, a->r() };
    splice(a->r(), b);
    auto side = s[0].cross(s[1], s[2]);//d54
    Q c = side ? connect(b, a) : 0;
    return {side < 0 ? c->r() : a, side < 0 ? c : b->r() };
  }
```

```
#define H(e) e->F(), e->p//f35
#define valid(e) (e->F().cross(H(base)) > 0)
  Q A, B, ra, rb;
  int half = sz(s) / 2;
  tie(ra, A) = rec({all(s) - half});
  tie(B, rb) = rec({sz(s) - half + all(s)});//c17
  while ((B->p.cross(H(A)) < 0 && (A = A->next())) ||
       (A->p.cross(H(B)) > 0 && (B = B->r()->o)));
  Q base = connect(B->r(), A);
  if (A->p == ra->p) ra = base->r();
  if (B->p == rb->p) rb = base;//a99

#define DEL(e, init, dir) Q e = init->dir; if (valid(e)) \
    while (circ(e->dir->F(), H(base), e->F())) { \
      Q t = e->dir; \
      splice(e, e->prev()); \//475
      splice(e->r(), e->r()->prev()); \
      e->o = H; H = e; e = t; \
    }
  for (;;) {
    DEL(LC, base->r(), o);  DEL(RC, base, prev());//031
    if (!valid(LC) && !valid(RC)) break;
    if (!valid(LC) || (valid(RC) && circ(H(RC), H(LC))))
      base = connect(RC, base->r());
    else
      base = connect(base->r(), LC->r());//907
  }
  return { ra, rb };
}

vector<P> triangulate(vector<P> pts) {//e5d
  sort(all(pts));  assert(unique(all(pts)) == pts.end());
  if (sz(pts) < 2) return {};
  Q e = rec(pts).first;
  vector<Q> q = {e};
  int qi = 0;//dd4
  while (e->o->F().cross(e->F(), e->p) < 0) e = e->o;
#define ADD { Q c = e; do { c->mark = 1; pts.pb(c->p); \
  q.pb(c->r()); c = c->next(); } while (c != e); }
  ADD; pts.clear();
  while (qi < sz(q)) if (!(e = q[qi++])->mark) ADD;//24a
  return pts;
}
```

## 8.5 3D

### PolyhedronVolume.h

**Description:** Magic formula for the volume of a polyhedron. Faces should point outwards.

3058c3, 6 lines

```
template<class V, class L>//27c
double signedPolyVolume(const V& p, const L& trilist) {
  double v = 0;
  for (auto i : trilist) v += p[i.a].cross(p[i.b]).dot(p[i.
      c]);
  return v / 6;
}//cbb
```

### Point3D.h

**Description:** Class to handle points in 3D space. T can be e.g. double or long long.

6eb43e, 32 lines

```
template<class T> struct Point3D {//811
  typedef Point3D P;
  typedef const P& R;
  T x, y, z;
  explicit Point3D(T _x=0, T _y=0, T _z=0) : x(_x), y(_y),
      z(_z) {}
  bool operator<(R p) const {//5e8
    return tie(x, y, z) < tie(p.x, p.y, p.z); }
  bool operator==(R p) const {
    return tie(x, y, z) == tie(p.x, p.y, p.z); }
  P operator+(R p) const { return P(x+p.x, y+p.y, z+p.z); }
  P operator-(R p) const { return P(x-p.x, y-p.y, z-p.z); }
      //9b1
  P operator*(T d) const { return P(x*d, y*d, z*d); }
  P operator/(T d) const { return P(x/d, y/d, z/d); }
  T dot(R p) const { return x*p.x + y*p.y + z*p.z; }
  P cross(R p) const {
    return P(y*p.z - z*p.y, z*p.x - x*p.z, x*p.y - y*p.x);
        //58a
  }
  T dist2() const { return x*x + y*y + z*z; }
  double dist() const { return sqrt((double)dist2()); }
  //Azimuthal angle (longitude) to x-axis in interval [-pi,
      pi]
  double phi() const { return atan2(y, x); }//a2c
  //Zenith angle (latitude) to the z-axis in interval [0,
      pi]
  double theta() const { return atan2(sqrt(x*x+y*y),z); }
  P unit() const { return *this/(T)dist(); } //makes dist()
      =1
  //returns unit vector normal to *this and p
  P normal(P p) const { return cross(p).unit(); }//e88
  //returns point rotated 'angle' radians ccw around axis
  P rotate(double angle, P axis) const {
    double s = sin(angle), c = cos(angle); P u = axis.unit
        ();
    return u*dot(u)*(1-c) + (*this)*c - cross(u)*s;
  }//e03
};
```

### 3dHull.h

**Description:** Computes all faces of the 3-dimension hull of a point set. *No four points must be coplanar*, or else random results will be returned. All faces will point outwards.

**Time:** $\mathcal{O}\left(n^2\right)$

"Point3D.h" ce1872, 49 lines

```
typedef Point3D<double> P3;//e28

struct PR {
  void ins(int x) { (a == -1 ? a : b) = x; }
  void rem(int x) { (a == x ? a : b) = -1; }
  int cnt() { return (a != -1) + (b != -1); }//c34
  int a, b;
};

struct F { P3 q; int a, b, c; };
//36b
vector<F> hull3d(const vector<P3>& A) {
  assert(sz(A) >= 4);
  vector<vector<PR>> E(sz(A), vector<PR>(sz(A), {-1, -1}));
#define E(x,y) E[f.x][f.y]
  vector<F> FS;//de0
  auto mf = [&](int i, int j, int k, int l) {
    P3 q = (A[j] - A[i]).cross((A[k] - A[i]));
    if (q.dot(A[l]) > q.dot(A[i]))
      q = q * -1;
    F f{q, i, j, k};//2be
    E(a,b).ins(k); E(a,c).ins(j); E(b,c).ins(i);
    FS.pb(f);
  };
  rep(i,0,4) rep(j,i+1,4) rep(k,j+1,4)
    mf(i, j, k, 6 - i - j - k);//e21

  rep(i,4,sz(A)) {
    rep(j,0,sz(FS)) {
      F f = FS[j];
      if(f.q.dot(A[i]) > f.q.dot(A[f.a])) {//b63
        E(a,b).rem(f.c);
        E(a,c).rem(f.b);
        E(b,c).rem(f.a);
        swap(FS[j--], FS.back());
        FS.pop_back();//0df
      }
    }
    int nw = sz(FS);
    rep(j,0,nw) {
      F f = FS[j];//945
#define C(a, b, c) if (E(a,b).cnt() != 2) mf(f.a, f.b, i, f
    .c);
      C(a, b, c); C(a, c, b); C(b, c, a);
    }
  }
  for (F& it : FS) if ((A[it.b] - A[it.a]).cross(//ab3
    A[it.c] - A[it.a]).dot(it.q) <= 0) swap(it.c, it.b);
  return FS;
};
```

### sphericalDistance.h

**Description:** Returns the shortest distance on the sphere with radius radius between the points with azimuthal angles (longitude) f1 ($\phi_1$) and f2 ($\phi_2$) from x axis and zenith angles (latitude) t1 ($\theta_1$) and t2 ($\theta_2$) from z axis (0 = north pole). All angles measured in radians. The algorithm starts by converting the spherical coordinates to cartesian coordinates so if that is what you have you can use only the two last rows. dx*radius is then the difference between the two points in the x direction and d*radius is the total distance between the points.

611f07, 8 lines

```
double sphericalDistance(double f1, double t1,//6da
    double f2, double t2, double radius) {
  double dx = sin(t2)*cos(f2) - sin(t1)*cos(f1);
  double dy = sin(t2)*sin(f2) - sin(t1)*sin(f1);
  double dz = cos(t2) - cos(t1);
  double d = sqrt(dx*dx + dy*dy + dz*dz);//65e
  return radius*2*asin(d/2);
}
```

# Strings (9)

### KMP.h

**Description:** pi[x] computes the length of the longest prefix of s that ends at x, other than s[0...x] itself (abacaba -> 0010123). Can be used to find all occurrences of a string.

**Time:** $\mathcal{O}(n)$

d4f9aa, 16 lines

```
vi pi(const string& s) {//f6d
  vi p(sz(s));
  rep(i,1,sz(s)) {
    int g = p[i-1];
    while (g && s[i] != s[g]) g = p[g-1];
    p[i] = g + (s[i] == s[g]);//0ff
  }
  return p;
}

vi match(const string& s, const string& pat) {//9e6
  vi p = pi(pat + '\0' + s), res;
  rep(i,sz(p)-sz(s),sz(p))
    if (p[i] == sz(pat)) res.pb(i - 2 * sz(pat));
  return res;
}//cbb
```

### Zfunc.h

**Description:** z[i] computes the length of the longest common prefix of s[i:] and s, except z[0] = 0. (abacaba -> 0010301)

**Time:** $\mathcal{O}(n)$

ee09e2, 12 lines

```
vi Z(const string& S) {//fc3
  vi z(sz(S));
  int l = -1, r = -1;
  rep(i,1,sz(S)) {
    z[i] = i >= r ? 0 : min(r - i, z[i - 1]);
```

```
    while (i + z[i] < sz(S) && S[i + z[i]] == S[z[i]])//8ec
      z[i]++;
    if (i + z[i] > r)
      l = i, r = i + z[i];
  }
  return z;//939
}
```

## Manacher.h

**Description:** For each position in a string, computes p[0][i] = half length of longest even palindrome around pos i, p[1][i] = longest odd (half rounded down).

**Time:** $\mathcal{O}(N)$

e7ad79, 13 lines

```
array<vi, 2> manacher(const string& s) {//510
  int n = sz(s);
  array<vi,2> p = {vi(n+1), vi(n)};
  rep(z,0,2) for (int i=0,l=0,r=0; i < n; i++) {
    int t = r-i+!z;
    if (i<r) p[z][i] = min(t, p[z][l+t]);//f50
    int L = i-p[z][i], R = i+p[z][i]-!z;
    while (L>=1 && R+1<n && s[L-1] == s[R+1])
      p[z][i]++, L--, R++;
    if (R>r) l=L, r=R;
  }//291
  return p;
}
```

## MinRotation.h

**Description:** Finds the lexicographically smallest rotation of a string.

**Usage:** `rotate(v.begin(), v.begin()+minRotation(v), v.end());`

**Time:** $\mathcal{O}(N)$

d07a42, 8 lines

```
int minRotation(string s) {//20f
  int a=0, N=sz(s); s += s;
  rep(b,0,N) rep(k,0,N) {
    if (a+k == b || s[a+k] < s[b+k]) {b += max(0, k-1);
        break;}
    if (s[a+k] > s[b+k]) { a = b; break; }
  }//3a8
  return a;
}
```

## SuffixArray.h

**Description:** Builds suffix array for a string. `sa[i]` is the starting index of the suffix which is $i$'th in the sorted suffix array. The returned vector is of size $n+1$, and `sa[0] = n`. The `lcp` array contains longest common prefixes for neighbouring strings in the suffix array: `lcp[i] = lcp(sa[i], sa[i-1])`, `lcp[0] = 0`. The input string must not contain any zero bytes.

**Time:** $\mathcal{O}(n \log n)$

bc716b, 22 lines

```
struct SuffixArray {//7a7
  vi sa, lcp;
  SuffixArray(string& s, int lim=256) { // or basic_string<
      int>
    int n = sz(s) + 1, k = 0, a, b;
    vi x(all(s)), y(n), ws(max(n, lim));
    x.push_back(0), sa = lcp = y, iota(all(sa), 0);//7c9
    for (int j = 0, p = 0; p < n; j = max(1, j * 2), lim =
        p) {
      p = j, iota(all(y), n - j);
      rep(i,0,n) if (sa[i] >= j) y[p++] = sa[i] - j;
      fill(all(ws), 0);
      rep(i,0,n) ws[x[i]]++;//f08
      rep(i,1,lim) ws[i] += ws[i - 1];
      for (int i = n; i--;) sa[--ws[x[y[i]]]] = y[i];
      swap(x, y), p = 1, x[sa[0]] = 0;
      rep(i,1,n) a = sa[i - 1], b = sa[i], x[b] =
        (y[a] == y[b] && y[a + j] == y[b + j]) ? p - 1 : p
            ++;//726
    }
    for (int i = 0, j; i < n - 1; lcp[x[i++]] = k)
      for (k && k--, j = sa[x[i] - 1];
          s[i + k] == s[j + k]; k++);
  }//e03
};
```

## SuffixTree.h

**Description:** Ukkonen's algorithm for online suffix tree construction. Each node contains indices [l, r) into the string, and a list of child nodes. Suffixes are given by traversals of this tree, joining [l, r) substrings. The root is 0 (has l = -1, r = 0), non-existent children are -1. To get a complete tree, append a dummy symbol – otherwise it may contain an incomplete path (still useful for substring matching, though).

**Time:** $\mathcal{O}(26N)$

aae0b8, 50 lines

```
struct SuffixTree {//b1f
  enum { N = 200010, ALPHA = 26 }; // N ~ 2*maxlen+10
  int toi(char c) { return c - 'a'; }
  string a; // v = cur node, q = cur position
  int t[N][ALPHA],l[N],r[N],p[N],s[N],v=0,q=0,m=2;
//b11
  void ukkadd(int i, int c) { suff:
    if (r[v]<=q) {
      if (t[v][c]==-1) { t[v][c]=m;  l[m]=i;
        p[m++]=v; v=s[v]; q=r[v];  goto suff; }
      v=t[v][c]; q=l[v];//99f
    }
    if (q==-1 || c==toi(a[q])) q++; else {
      l[m+1]=i;  p[m+1]=m;  l[m]=l[v];  r[m]=q;
      p[m]=p[v];  t[m][c]=m+1;  t[m][toi(a[q])]=v;
      l[v]=q;  p[v]=m;  t[p[m]][toi(a[l[m]])]=m;//604
      v=s[p[m]];  q=l[m];
      while (q<r[m]) { v=t[v][toi(a[q])];  q+=r[v]-l[v]; }
      if (q==r[m])  s[m]=v;  else s[m]=m+2;
      q=r[v]-(q-r[m]);  m+=2;  goto suff;
    }//478
  }

  SuffixTree(string a) : a(a) {
    fill(r,r+N,sz(a));
    memset(s, 0, sizeof s);//f11
    memset(t, -1, sizeof t);
    fill(t[1],t[1]+ALPHA,0);
    s[0] = 1; l[0] = l[1] = -1; r[0] = r[1] = p[0] = p[1] =
        0;
    rep(i,0,sz(a)) ukkadd(i, toi(a[i]));
  }//d1a

  // example: find longest common substring (uses ALPHA =
      28)
  pii best;
  int lcs(int node, int i1, int i2, int olen) {
    if (l[node] <= i1 && i1 < r[node]) return 1;//636
    if (l[node] <= i2 && i2 < r[node]) return 2;
    int mask = 0, len = node ? olen + (r[node] - l[node]) :
        0;
    rep(c,0,ALPHA) if (t[node][c] != -1)
      mask |= lcs(t[node][c], i1, i2, len);
    if (mask == 3)//a3a
      best = max(best, {len, r[node] - len});
    return mask;
  }
  static pii LCS(string s, string t) {
    SuffixTree st(s + (char)('z' + 1) + t + (char)('z' + 2)
        );//78c
    st.lcs(0, sz(s), sz(s) + 1 + sz(t), 0);
    return st.best;
  }
};
```

## Hashing.h

**Description:** Self-explanatory methods for string hashing.

5966ff, 44 lines

```
// Arithmetic mod 2^64-1. 2x slower than mod 2^64 and more
     //d41
// code, but works on evil test data (e.g. Thue-Morse,
     where
// ABBA... and BAAB... of length 2^10 hash the same mod 2^
     64).
// "typedef ull H;" instead if you think test data is
     random,
// or work mod 10^9+7 if the Birthday paradox is not a
     problem.
typedef uint64_t ull;//98c
struct H {
  ull x; H(ull x=0) : x(x) {}
  H operator+(H o) { return x + o.x + (x + o.x < x); }
  H operator-(H o) { return *this + ~o.x; }
  H operator*(H o) { auto m = (__uint128_t)x * o.x;//884
    return H((ull)m) + (ull)(m >> 64); }
  ull get() const { return x + !~x; }
  bool operator==(H o) const { return get() == o.get(); }
  bool operator<(H o) const { return get() < o.get(); }
};//7dd
static const H C = (ll)1e11+3; // (order ~ 3e9; random also
      ok)

struct HashInterval {
  vector<H> ha, pw;
  HashInterval(string& str) : ha(sz(str)+1), pw(ha) {//c1e
    pw[0] = 1;
    rep(i,0,sz(str))
      ha[i+1] = ha[i] * C + str[i],
      pw[i+1] = pw[i] * C;
  }//b8f
  H hashInterval(int a, int b) { // hash [a, b)
    return ha[b] - ha[a] * pw[b - a];
  }
};
//4b7
vector<H> getHashes(string& str, int length) {
  if (sz(str) < length) return {};
  H h = 0, pw = 1;
  rep(i,0,length)
    h = h * C + str[i], pw = pw * C;//6b3
  vector<H> ret = {h};
  rep(i,length,sz(str)) {
    ret.pb(h = h * C + str[i] - pw * str[i-length]);
  }
  return ret;//413
}

H hashString(string& s){H h{}; for(char c:s) h=h*C+c;return
      h;}
```

## AhoCorasick.h

**Description:** Aho-Corasick automaton, used for multiple pattern matching. Initialize with AhoCorasick ac(patterns); the automaton start node will be at index 0. find(word) returns for each position the index of the longest word that ends there, or -1 if none. findAll(−, word) finds all words (up to $N\sqrt{N}$ many if no duplicate patterns) that start at each position (shortest first). Duplicate patterns are allowed; empty patterns are not. To find the longest words that start at each position, reverse all input. For large alphabets, split each symbol into chunks, with sentinel bits for symbol boundaries.

**Time:** construction takes $\mathcal{O}(26N)$, where $N$ = sum of length of patterns. find(x) is $\mathcal{O}(N)$, where N = length of x. findAll is $\mathcal{O}(NM)$.

85943d, 66 lines

```
struct AhoCorasick {//724
```

```
enum {alpha = 26, first = 'A'}; // change this!
struct Node {
  // (nmatches is optional)
  int back, next[alpha], start = -1, end = -1, nmatches =
      0;
  Node(int v) { memset(next, v, sizeof(next)); }//cc2
};
vector<Node> N;
vi backp;
void insert(string& s, int j) {
  assert(!s.empty());//757
  int n = 0;
  for (char c : s) {
    int& m = N[n].next[c - first];
    if (m == -1) { n = m = sz(N); N.emplace_back(-1); }
    else n = m;//7ab
  }
  if (N[n].end == -1) N[n].start = j;
  backp.pb(N[n].end);
  N[n].end = j;
  N[n].nmatches++;//77c
}
AhoCorasick(vector<string>& pat) : N(1, -1) {
  rep(i,0,sz(pat)) insert(pat[i], i);
  N[0].back = sz(N);
  N.emplace_back(0);//12a

  queue<int> q;
  for (q.push(0); !q.empty(); q.pop()) {
    int n = q.front(), prev = N[n].back;
    rep(i,0,alpha) {//57b
      int &ed = N[n].next[i], y = N[prev].next[i];
      if (ed == -1) ed = y;
      else {
        N[ed].back = y;
        (N[ed].end == -1 ? N[ed].end : backp[N[ed].start
            ])//338
          = N[y].end;
        N[ed].nmatches += N[y].nmatches;
        q.push(ed);
      }
    }//c05
  }
}
vi find(string word) {
  int n = 0;
  vi res; // ll count = 0;//1d3
  for (char c : word) {
    n = N[n].next[c - first];
    res.pb(N[n].end);
    // count += N[n].nmatches;
  }//bb1
  return res;
}
vector<vi> findAll(vector<string>& pat, string word) {
  vi r = find(word);
  vector<vi> res(sz(word));//82a
  rep(i,0,sz(word)) {
    int ind = r[i];
    while (ind != -1) {
      res[i - sz(pat[ind]) + 1].pb(ind);
      ind = backp[ind];//8f0
    }
  }
  return res;
}
};//214
```

# Various (10)

## 10.1 Intervals

### IntervalContainer.h

**Description:** Add and remove intervals from a set of disjoint intervals. Will merge the added interval with any overlapping intervals in the set when adding. Intervals are [inclusive, exclusive).
**Time:** $\mathcal{O}(\log N)$

edce47, 23 lines

```
set<pii>::iterator addInterval(set<pii>& is, int L, int R)
    {//ba1
  if (L == R) return is.end();
  auto it = is.lower_bound({L, R}), before = it;
  while (it != is.end() && it->first <= R) {
    R = max(R, it->second);
    before = it = is.erase(it);//ea6
  }
  if (it != is.begin() && (--it)->second >= L) {
    L = min(L, it->first);
    R = max(R, it->second);
    is.erase(it);//05d
  }
  return is.insert(before, {L,R});
}

void removeInterval(set<pii>& is, int L, int R) {//858
  if (L == R) return;
  auto it = addInterval(is, L, R);
  auto r2 = it->second;
  if (it->first == L) is.erase(it);
  else (int&)it->second = L;//61f
  if (R != r2) is.emplace(R, r2);
}
```

### IntervalCover.h

**Description:** Compute indices of smallest set of intervals covering another interval. Intervals should be [inclusive, exclusive). To support [inclusive, inclusive], change (A) to add `|| R.empty()`. Returns empty set on failure (or if G is empty).
**Time:** $\mathcal{O}(N \log N)$

a9491c, 19 lines

```
template<class T>//0e2
vi cover(pair<T, T> G, vector<pair<T, T>> I) {
  vi S(sz(I)), R;
  iota(all(S), 0);
  sort(all(S), [&](int a, int b) { return I[a] < I[b]; });
  T cur = G.first;//ed8
  int at = 0;
  while (cur < G.second) { // (A)
    pair<T, int> mx = make_pair(cur, -1);
    while (at < sz(I) && I[S[at]].first <= cur) {
      mx = max(mx, make_pair(I[S[at]].second, S[at]));//607
      at++;
    }
    if (mx.second == -1) return {};
    cur = mx.first;
    R.pb(mx.second);//661
  }
  return R;
}
```

### ConstantIntervals.h

**Description:** Split a monotone function on [from, to) into a minimal set of half-open intervals on which it has the same value. Runs a callback g for each such interval.
**Usage:** `constantIntervals(0, sz(v), [&](int x){return v[x];}, [&](int lo, int hi, T val){...});`
**Time:** $\mathcal{O}\left(k \log \frac{n}{k}\right)$

753a4c, 19 lines

```
template<class F, class G, class T>//570
void rec(int from, int to, F& f, G& g, int& i, T& p, T q) {
  if (p == q) return;
  if (from == to) {
    g(i, to, p);
    i = to; p = q;//05f
  } else {
    int mid = (from + to) >> 1;
    rec(from, mid, f, g, i, p, f(mid));
    rec(mid+1, to, f, g, i, p, q);
  }//729
}
template<class F, class G>
void constantIntervals(int from, int to, F f, G g) {
  if (to <= from) return;
  int i = from; auto p = f(i), q = f(to-1);//a6c
  rec(from, to-1, f, g, i, p, q);
  g(i, to, q);
}
```

## 10.2 Misc. algorithms

### TernarySearch.h

**Description:** Find the smallest i in $[a, b]$ that maximizes $f(i)$, assuming that $f(a) < \dots < f(i) \geq \dots \geq f(b)$. To reverse which of the sides allows non-strict inequalities, change the $<$ marked with (A) to $<=$, and reverse the loop at (B). To minimize $f$, change it to $>$, also at (B).
**Usage:** `int ind = ternSearch(0,n-1,[&](int i){return a[i];});`
**Time:** $\mathcal{O}(\log(b-a))$

9155b4, 11 lines

```
template<class F>//7d4
int ternSearch(int a, int b, F f) {
  assert(a <= b);
  while (b - a >= 5) {
    int mid = (a + b) / 2;
    if (f(mid) < f(mid+1)) a = mid; // (A)//ec4
    else b = mid+1;
  }
  rep(i,a+1,b+1) if (f(a) < f(i)) a = i; // (B)
  return a;
}//cbb
```

### LIS.h

**Description:** Compute indices for the longest increasing subsequence.
**Time:** $\mathcal{O}(N \log N)$

2932a0, 17 lines

```
template<class I> vi lis(const vector<I>& S) {//47f
  if (S.empty()) return {};
  vi prev(sz(S));
  typedef pair<I, int> p;
  vector<p> res;
  rep(i,0,sz(S)) {//a50
    // change 0 -> i for longest non-decreasing subsequence
    auto it = lower_bound(all(res), p{S[i], 0});
    if (it == res.end()) res.emplace_back(), it = res.end()
        -1;
    *it = {S[i], i};
    prev[i] = it == res.begin() ? 0 : (it-1)->second;//476
  }
  int L = sz(res), cur = res.back().second;
  vi ans(L);
  while (L--) ans[L] = cur, cur = prev[cur];
  return ans;//342
}
```

### FastKnapsack.h

**Description:** Given N non-negative integer weights w and a non-negative target t, computes the maximum S <= t such that S is the sum of some subset of the weights.
**Time:** $\mathcal{O}(N \max(w_i))$

b20ccc, 16 lines

```
int knapsack(vi w, int t) {//e2b
```

```
  int a = 0, b = 0, x;
  while (b < sz(w) && a + w[b] <= t) a += w[b++];
  if (b == sz(w)) return a;
  int m = *max_element(all(w));
  vi u, v(2*m, -1);//14a
  v[a+m-t] = b;
  rep(i,b,sz(w)) {
    u = v;
    rep(x,0,m) v[x+w[i]] = max(v[x+w[i]], u[x]);
    for (x = 2*m; --x > m;) rep(j, max(0,u[x]), v[x])//45b
      v[x-w[j]] = max(v[x-w[j]], j);
  }
  for (a = t; v[a+m-t] < 0; a--) ;
  return a;
}//cbb
```

## 10.3 Dynamic programming

### KnuthDP.h

**Description:** When doing DP on intervals: $a[i][j] = \min_{i<k<j}(a[i][k]+a[k][j]) + f(i,j)$, where the (minimal) optimal $k$ increases with both $i$ and $j$, one can solve intervals in increasing order of length, and search $k = p[i][j]$ for $a[i][j]$ only between $p[i][j-1]$ and $p[i+1][j]$. This is known as Knuth DP. Sufficient criteria for this are if $f(b,c) \le f(a,d)$ and $f(a,c) + f(b,d) \le f(a,d) + f(b,c)$ for all $a \le b \le c \le d$. Consider also: LineContainer (ch. Data structures), monotone queues, ternary search.
**Time:** $\mathcal{O}(N^2)$

d41d8c, 1 lines

```
//d41
```

### DivideAndConquerDP.h

**Description:** Given $a[i] = \min_{lo(i)\le k<hi(i)}(f(i,k))$ where the (minimal) optimal $k$ increases with $i$, computes $a[i]$ for $i = L..R-1$.
**Time:** $\mathcal{O}((N + (hi - lo)) \log N)$

d38d2b, 18 lines

```
struct DP { // Modify at will://ff9
  int lo(int ind) { return 0; }
  int hi(int ind) { return ind; }
  ll f(int ind, int k) { return dp[ind][k]; }
  void store(int ind, int k, ll v) { res[ind] = pii(k, v);
      }
//ec8
  void rec(int L, int R, int LO, int HI) {
    if (L >= R) return;
    int mid = (L + R) >> 1;
    pair<ll, int> best(LLONG_MAX, LO);
    rep(k, max(LO,lo(mid)), min(HI,hi(mid)))//680
      best = min(best, make_pair(f(mid, k), k));
    store(mid, best.second, best.first);
    rec(L, mid, LO, best.second+1);
    rec(mid+1, R, best.second, HI);
  }//a30
  void solve(int L, int R) { rec(L, R, INT_MIN, INT_MAX); }
};
```

## 10.4 Debugging tricks

- `signal(SIGSEGV, [](int) { _Exit(0); });` converts segfaults into Wrong Answers. Similarly one can catch SIGABRT (assertion failures) and SIGFPE (zero divisions). `_GLIBCXX_DEBUG` failures generate SIGABRT (or SIGSEGV on gcc 5.4.0 apparently).
- `feenableexcept(29);` kills the program on NaNs (`1`), 0-divs (`4`), infinities (`8`) and denormals (`16`).

## 10.5 Optimization tricks

`__builtin_ia32_ldmxcsr(40896);` disables denormals (which make floats 20x slower near their minimum value).

### 10.5.1 Bit hacks

- `x & -x` is the least bit in `x`.
- `for (int x = m; x; ) { --x &= m; ... }` loops over all subset masks of `m` (except `m` itself).
- `c = x&-x, r = x+c; (((r^x) >> 2)/c) | r` is the next number after `x` with the same number of bits set.
- `rep(b,0,K) rep(i,0,(1 << K))`
  `if (i & 1 << b) D[i] += D[i^(1 << b)];`
  computes all sums of subsets.

### 10.5.2 Pragmas

- `#pragma GCC optimize ("Ofast")` will make GCC auto-vectorize loops and optimizes floating points better.
- `#pragma GCC target ("avx2")` can double performance of vectorized code, but causes crashes on old machines.
- `#pragma GCC optimize ("trapv")` kills the program on integer overflows (but is really slow).

### FastMod.h

**Description:** Compute $a\%b$ about 5 times faster than usual, where $b$ is constant but not known at compile time. Returns a value congruent to $a \pmod b$ in the range $[0, 2b)$.

751a02, 8 lines

```
typedef unsigned long long ull;//010
struct FastMod {
  ull b, m;
  FastMod(ull b) : b(b), m(-1ULL / b) {}
  ull reduce(ull a) { // a % b + (0 or b)
    return a - (ull)((__uint128_t(m) * a) >> 64) * b;//430
  }
};
```

### FastInput.h

**Description:** Read an integer from stdin. Usage requires your program to pipe in input from file.
**Usage:** `./a.out < input.txt`
**Time:** About 5x as fast as cin/scanf.

7b3c70, 17 lines

```
inline char gc() { // like getchar()//c51
  static char buf[1 << 16];
  static size_t bc, be;
  if (bc >= be) {
    buf[0] = 0, bc = 0;
    be = fread(buf, 1, sizeof(buf), stdin);//818
  }
  return buf[bc++]; // returns 0 on EOF
}

int readInt() {//f26
  int a, c;
  while ((a = gc()) < 40);
  if (a == '-') return -readInt();
  while ((c = gc()) >= 48) a = a * 10 + c - 480;
  return a - 48;//d34
}
```

### BumpAllocator.h

**Description:** When you need to dynamically allocate many objects and don't care about freeing them. "new X" otherwise has an overhead of something like 0.05us + 16 bytes per allocation.

745db2, 8 lines

```
// Either globally or in a single class://c17
static char buf[450 << 20];
void* operator new(size_t s) {
  static size_t i = sizeof buf;
  assert(s < i);
  return (void*)&buf[i -= s];//ef5
}
void operator delete(void*) {}
```

### SmallPtr.h

**Description:** A 32-bit pointer that points into BumpAllocator memory.

"BumpAllocator.h" 2dd6c9, 10 lines

```
template<class T> struct ptr {//bda
  unsigned ind;
  ptr(T* p = 0) : ind(p ? unsigned((char*)p - buf) : 0) {
    assert(ind < sizeof buf);
  }
  T& operator*() const { return *(T*)(buf + ind); }//95f
  T* operator->() const { return &**this; }
  T& operator[](int a) const { return (&**this)[a]; }
  explicit operator bool() const { return ind; }
};
```

### BumpAllocatorSTL.h

**Description:** BumpAllocator for STL containers.
**Usage:** `vector<vector<int, small<int>>> ed(N);`

bb66d4, 14 lines

```
char buf[450 << 20] alignas(16);//2c8
size_t buf_ind = sizeof buf;

template<class T> struct small {
  typedef T value_type;
  small() {}//8ec
  template<class U> small(const U&) {}
  T* allocate(size_t n) {
    buf_ind -= n * sizeof(T);
    buf_ind &= 0 - alignof(T);
    return (T*)(buf + buf_ind);//ad1
  }
  void deallocate(T*, size_t) {}
};
```

### Unrolling.h

520e76, 5 lines

```
#define F {...; ++i;}//520
int i = from;
while (i&3 && i < to) F // for alignment, if needed
while (i + 4 <= to) { F F F F }
while (i < to) F
```

### SIMD.h

**Description:** Cheat sheet of SSE/AVX intrinsics, for doing arithmetic on several numbers at once. Can provide a constant factor improvement of about 4, orthogonal to loop unrolling. Operations follow the pattern `"_mm(256)?_name_(si(128|256)|epi(8|16|32|64)|pd|ps)"`. Not all are described here; grep for `_mm_` in `/usr/lib/gcc/*/4.9/include/` for more. If AVX is unsupported, try 128-bit operations, "emmintrin.h" and #define `__SSE__` and `__MMX__` before including it. For aligned memory use `_mm_malloc(size, 32)` or `int buf[N] alignas(32)`, but prefer loadu/storeu.

551b82, 43 lines

```
#pragma GCC target ("avx2") // or sse4.1//c6d
#include "immintrin.h"
```

```
typedef __m256i mi;
#define L(x) _mm256_loadu_si256((mi*)&(x))
//d41
// High-level/specific methods:
// load(u)?_si256, store(u)?_si256, setzero_si256,
    _mm_malloc
// blendv_(epi8|ps|pd) (z?y:x), movemask_epi8 (hibits of
    bytes)
// i32gather_epi32(addr, x, 4): map addr[] over 32-b parts
    of x
// sad_epu8: sum of absolute differences of u8, outputs 4
    xi64//d41
// maddubs_epi16: dot product of unsigned i7's, outputs 16
    xi15
// madd_epi16: dot product of signed i16's, outputs 8xi32
// extractf128_si256(, i) (256->128), cvtsi128_si32 (128->
    lo32)
// permute2f128_si256(x,x,1) swaps 128-bit lanes
// shuffle_epi32(x, 3*64+2*16+1*4+0) == x for each lane//
    d41
// shuffle_epi8(x, y) takes a vector instead of an imm

// Methods that work with most data types (append e.g.
    _epi32):
// set1, blend (i8?x:y), add, adds (sat.), mullo, sub, and/
    or,
// andnot, abs, min, max, sign(1,x), cmp(gt|eq), unpack(lo|
    hi)//512

int sumi32(mi m) { union {int v[8]; mi m;} u; u.m = m;
  int ret = 0; rep(i,0,8) ret += u.v[i]; return ret; }
mi zero() { return _mm256_setzero_si256(); }
mi one() { return _mm256_set1_epi32(-1); }//28e
bool all_zero(mi m) { return _mm256_testz_si256(m, m); }
bool all_one(mi m) { return _mm256_testc_si256(m, one()); }

ll example_filteredDotProduct(int n, short* a, short* b) {
  int i = 0; ll r = 0;//730
  mi zero = _mm256_setzero_si256(), acc = zero;
  while (i + 16 <= n) {
    mi va = L(a[i]), vb = L(b[i]); i += 16;
    va = _mm256_and_si256(_mm256_cmpgt_epi16(vb, va), va);
    mi vp = _mm256_madd_epi16(va, vb);//b47
    acc = _mm256_add_epi64(_mm256_unpacklo_epi32(vp, zero),
      _mm256_add_epi64(acc, _mm256_unpackhi_epi32(vp, zero)
          ));
  }
  union {ll v[4]; mi m;} u; u.m = acc; rep(i,0,4) r += u.v[
      i];
  for (;i<n;++i) if (a[i] < b[i]) r += a[i]*b[i]; // <-
      equiv//c30
  return r;
}
```

# Extra Stuff (11)

## CentroidDecomposition.h

**Description:** Centroid decomposition on tree
**Time:** $\mathcal{O}(nlogn)$

918f4f, 38 lines

```
struct CD {//e81
  vector<vector<int>> adj;
  vector<int> size, vis;

  int dfs_size(int v, int p) {
    size[v] = 1;//43a
    for (int e : adj[v]) {
      if (e != p && !vis[e]) {
        size[v] += dfs_size(e, v);
      }
    }//ef7
    return size[v];
  }

  int dfs_root(int v, int p, int n) {
    for (int e : adj[v]) {//14b
      if (e != p && !vis[e] && 2 * size[e] > n) {
        return dfs_root(e, v, n);
      }
    }
    return v;//4ef
  }

  void centroid(int v, int p) {
    dfs_size(v, p);
    int c = dfs_root(v, p, size[v]);//44f
    vis[c] = true;

    // do processing here
    // make sure to ignore visited nodes
//260
    for (int e : adj[c]) {
      if (!vis[e]) {
        centroid(e, c);
      }
    }//29b
  }
};
```

## Eertree.h

**Time:** "Tree" of all palindromic substrings (there are two roots). Also has suffix links.

4b78a3, 56 lines

```
struct eertree {//076
  struct node {
    array<int, 26> nxt;
    int sufflink, len, cnt;
    vector<int> edges;
  };//a3e

  string s;
  vector<node> tree;
  int suff, num;
//929
  eertree(const string &_s) : s(_s), tree(sz(s)+2), suff(2)
      , num(2)  {
    tree[1].len = -1, tree[1].sufflink = 1;
    tree[2].len = 0, tree[2].sufflink = 1;
    tree[1].edges.pb(2);
    rep(i, 0, sz(s)) add(i);//736
  }

  void add(int pos) {
    int cur = suff, cur_len = 0;
    char c = s[pos];//989

    while (true) {
      cur_len = tree[cur].len;
      if (pos - 1 - cur_len > -1 && s[pos - 1 - cur_len] ==
          s[pos]) break;
      cur = tree[cur].sufflink;//b02
    }

    if (tree[cur].nxt[c]) {
      suff = tree[cur].nxt[c];
      tree[suff].cnt++;//be0
      return;
    }

    suff = ++num;
    tree[num].len = tree[cur].len + 2;//d5d
    tree[num].cnt = 1;
    tree[cur].nxt[c] = num;

    if (tree[num].len == 1) {
      tree[num].sufflink = 2;//eac
      tree[2].edges.pb(num);
      return;
    }

    while (true) {//4bc
      cur = tree[cur].sufflink;
      cur_len = tree[cur].len;
      if (pos - 1 - cur_len > -1 && s[pos - 1 - cur_len] ==
          s[pos]) {
        tree[num].sufflink = tree[cur].nxt[c];
        tree[tree[cur].nxt[c]].edges.pb(num);//ddd
        break;
      }
    }
  }
};//214
```

## Knuth.h

**Description:** DP must be in the form dp(i, j) = min[dp(i, k) + dp(k + 1, j) + C(i, j)] such that opt(i, j - 1) <= opt(i, j) <= opt(i + 1, j). True if for a <= b <= c <= d, then C(b, c) <= C(a, d) and C(a, c) + C(b, d) <= C(a, d) + C(b, c).
**Time:** $\mathcal{O}(n^2)$

776312, 20 lines

```
template<class C>//e79
int solve(int n, C c) {
  vector dp(n, vi(n));
  auto opt = dp;
  rep(i, 0, n) opt[i][i] = i; // initialize dp[i][i] here
  for (int i = n-2; i >= 0; i--) {//157
    rep(j, i+1, n) {
      int mn = INT_MAX;
      int cost = C(i, j);
      for (int k = opt[i][j-1]; k <= min(j-1, opt[i+1][j]);
          k++) {
        if (mn >= dp[i][k] + dp[k+1][j] + cost) {//201
          opt[i][j] = k;
          mn = dp[i][k] + dp[k+1][j] + cost;
        }
      }
      dp[i][j] = mn;//b2a
    }
  }
  return dp[0][n-1];
}
```

## ModInt.h

**Description:** all operations just work, MOD should fit in int
**Time:** constant for operations, $\mathcal{O}(loge)$ for pow

209e30, 32 lines

```
constexpr int MOD = 1e9+7;//d87

struct mi {
  int v;
  mi() : mi(0) {}
  mi(int _v) : v(_v) {//715
    if (v >= MOD) v -= MOD;
    if (v < 0) v += MOD;
  }
  mi(ll _v) : mi((int)(_v % MOD)) {}
  mi operator+(const mi &m2) const { return mi(v + m2.v); }
      //89d
  mi operator-(const mi &m2) const { return mi(v - m2.v); }
  mi operator*(const mi &m2) const { return mi((ll) v * m2.
      v); }
```

```
  mi operator/(const mi &m2) const { return mi((ll) v * m2.
      inv().v); }
  mi &operator+=(const mi &m2) { return *this = *this + m2;
       }
  mi &operator-=(const mi &m2) { return *this = *this - m2;
       }//e99
  mi &operator*=(const mi &m2) { return *this = *this * m2;
       }
  mi &operator/=(const mi &m2) { return *this = *this / m2;
       }
  mi pow(ll e) const {
    mi res = 1;
    mi n = *this;//148
    while (e > 0) {
      if (e & 1) res *= n;
      n *= n;
      e >>= 1;
    }//c3c
    return res;
  }
  mi inv() const {
    return pow(MOD - 2);
  }//e03
};
```

### Pruefer.h

**Description:** Helps construct random tree Choose random n-2 length array, values [0, n-1]
**Time:** $\mathcal{O}(n)$

295d68, 23 lines

```
vector<pii> pruefer_decode(const vi &code) {//865
  int n = sz(code) + 2;
  vi degree(n, 1);
  for (int i : code)
    degree[i]++;
//c77
  set<int> leaves;
  rep(i, 0, n)
    if (degree[i] == 1)
      leaves.insert(i);
//f6a
  vector<pii> edges;
  for (int v : code) {
    int leaf = *leaves.begin();
    leaves.erase(leaves.begin());
//1b3
    edges.emplace_back(leaf, v);
    if (--degree[v] == 1)
      leaves.insert(v);
  }
  edges.emplace_back(*leaves.begin(), n-1);//062
  return edges;
}
```

### SuffixAutomaton.h

**Description:** Builds suffix automaton for a string. Each node corresponds to a class of substrings which end at the same indices.
**Time:** $\mathcal{O}(n)$

bebd7b, 55 lines

```
struct SuffixAutomaton {//3d3
  struct Node {
    int len = 0, lnk = 0;
    int nxt[26];
  };
  string s;//c66
  vector<Node> t; int last = 0;
  SuffixAutomaton(string _s = "") {
    t.pb({0, -1, {}});
    for (char c : _s) add(c);
  }//c4c
  void add(int c) { s += (char) c; c -= 'a';
    int u = last; int v = last = sz(t);
    t.pb({t[u].len + 1, 0, {}});
    while (u >= 0 && !t[u].nxt[c])
      t[u].nxt[c] = v, u = t[u].lnk;//b21
    if (u == -1) return;
    int q = t[u].nxt[c];
    if (t[u].len + 1 == t[q].len)
      { t[v].lnk = q; return; }
    int cpy = sz(t); t.pb(t[q]);//aa4
    t[cpy].len = t[u].len + 1;
    while (u >= 0 && t[u].nxt[c] == q)
      t[u].nxt[c] = cpy, u = t[u].lnk;
    t[v].lnk = t[q].lnk = cpy;
  }//c74
  vi cnt() {
    vi res(sz(t), 0);
    int cur = 0;
    for (char c : s)
      res[cur = t[cur].nxt[c - 'a']]++;//e36
    vector<pii> srt;
    rep(i, 1, sz(t))
      srt.pb({-t[i].len, i});
    sort(all(srt));
    for (auto &p : srt)//df9
      res[t[p.second].lnk] += res[p.second];
    return res;
  }
  vi first() {
    vi res(sz(t), sz(s));//b72
    int cur = 0;
    for (int i = 0; i < sz(s); i++) {
      cur = t[cur].nxt[s[i] - 'a'];
      res[cur] = min(res[cur], i);
    }//18b
    vector<pii> srt;
    rep(i, 1, sz(t))
      srt.pb({-t[i].len, i});
    sort(all(srt));
    for (auto &p : srt)//23d
      res[t[p.second].lnk] = min(res[t[p.second].lnk], res[
          p.second]);
    return res;
  }
};
```

# Dylan's Template (12)

### AhoCorasick.h

**Description:** Constructs Aho-Corasick automaton for given list of words
**Time:** Construction is $\mathcal{O}(N)$, move() is amortized $\mathcal{O}(N)$

2d48ca, 34 lines

```
struct AhoCorasick {//be4
  struct Node {
    map<char, int> nxt;
    int lnk = 0, cnt = 0;
  };
  vector<Node> t;//b46
  AhoCorasick(vector<string> words) {
    t.pb({});
    for (string s : words) {
      int cur = 0;
      for (char c : s) {//638
        if (!t[cur].nxt[c]) {
          t[cur].nxt[c] = sz(t);
          t.pb({});
        }
        cur = t[cur].nxt[c];//05c
      }
      t[cur].cnt++;
    }
    queue<int> q; q.push(0);
    while (!q.empty()) {//b39
      int u = q.front(); q.pop();
      assert(0 <= u && u < sz(t));
      for (auto &p : t[u].nxt) {
        if (u) t[p.second].lnk = move(t[u].lnk, p.first);
        q.push(p.second);//e67
      }
    }
  }
  int move(int u, int c) {
    if (t[u].nxt[c]) return t[u].nxt[c];//1f8
    return u == 0 ? 0 : t[u].nxt[c] = move(t[u].lnk, c);
  }
};
```

### AllRoots.h

**Description:** Tree rerooting template promote() follows a parent edge of a subtree (from original node u and following original edge e) merge() merges two subtrees that just had promote() called on by default this computes the sum of distances from each node to all other nodes
**Time:** solve() is $\mathcal{O}(N)$.

2e65b7, 65 lines

```
namespace AllRoots {//36c
  struct Edge { int u, v, cost; };
  struct Node {
    int cost;
    int cnt = 0; ll sum = 0, t = 0, sumT = 0;
    Node merge(Node o) {//cb0
      return {0, cnt + o.cnt, sum + o.sum, t + o.t, sumT +
          o.sumT};
    }
    Node promote(Node u, Edge e) {
      return {0, 1 + cnt, sum + (ll)(1 + cnt) * e.cost, u.
          cost + t, sumT + (u.cost + t) * e.cost};
    }//925
  };
  Edge rev(Edge e) { swap(e.u, e.v); return e; }
  static vector<Node> solve(vector<Node> nodes, vector<Edge
      > edges) {
    int N = sz(nodes);
    vector<vector<int>> adj(N);//eac
    vector<Edge> par(N);
    for (Edge e : edges) {
      adj[e.u].pb(e.v);
      adj[e.v].pb(e.u);
    }//3f3
    queue<int> q; q.push(0);
    vector<int> srt, pos(N);
    while (!q.empty()) {
      int u = q.front(); q.pop();
      pos[u] = sz(srt);//c8a
      srt.pb(u);
      for (int v : adj[u]) {
        adj[v].erase(find(all(adj[v]), u));
        q.push(v);
      }//698
    }
    for (Edge e : edges) {
      if (pos[e.u] < pos[e.v]) swap(e.u, e.v);
      par[e.u] = e;
    }//97d
    reverse(all(srt));
    vector<Node> down(N);
    for (int u : srt) for (int v : adj[u])
      down[u] = down[u].merge(down[v].promote(nodes[v], par
          [v]));
    reverse(all(srt));//599
    vector<Node> up(N), res(N);
    for (int u : srt) {
```

```
      vector<Node> left(sz(adj[u])), right(sz(adj[u]));
      for (int i = 0; i < sz(adj[u]); i++) {
        int v = adj[u][i];//e76
        left[i] = right[i] = down[v].promote(nodes[v], par[
            v]);
      }
      for (int i = 1; i < sz(adj[u]); i++)
        left[i] = left[i - 1].merge(left[i]);
      for (int i = sz(adj[u]) - 2; i >= 0; i--)//9bb
        right[i] = right[i].merge(right[i + 1]);
      Node p = up[u].promote(nodes[par[u].v], rev(par[u]));
      for (int i = 0; i < sz(adj[u]); i++) {
        int v = adj[u][i];
        if (u > 0) up[v] = up[v].merge(p);//83d
        if (i > 0) up[v] = left[i - 1].merge(up[v]);
        if (i + 1 < sz(adj[u])) up[v] = up[v].merge(right[i
            + 1]);
      }
      res[u] = down[u];
      if (u > 0) res[u] = res[u].merge(p);//be1
    }
    return res;
  }
}
```

## HalfCompressedBIT2D.h

**Description:** 2D BIT that is semi-offline (requires update positions ahead of time) addUpdate() must be called ahead of time with all update positions then query() and update() may be called arbitrarily
**Time:** Operations are $\mathcal{O}\left(log^2(n)\right)$

e34ff9, 40 lines

```
struct HalfCompressedBIT2D {//863
  int N;
  bool built = 0;
  vector<pair<int, int>> updates;
  HalfCompressedBIT2D(int N) : N(N) {}
  void addUpdate(int r, int c) {//bd1
    updates.pb({r, c});
  }
  vector<vector<int>> compress;
  vector<vector<ll>> tree;
  static bool cmp(pair<int, int> &a, pair<int, int> &b) {//
      00d
    return a.second < b.second;
  }
  void build() {
    sort(all(updates), cmp);
    compress = vector<vector<int>>(N, {0});//7a5
    for (auto &p : updates)
      for (int rr = p.first; rr < sz(compress); rr += rr &
          -rr)
        compress[rr].pb(p.second);
    tree = vector<vector<ll>>(sz(compress));
    for (int i = 0; i < sz(compress); i++) {//6df
      tree[i] = vector<ll>(sz(compress[i]), 0);
    }
    built = 1;
  }
  void update(int r, int c, int k) {//f54
    if (!built) build();
    for (int rr = r; rr < sz(compress); rr += rr & -rr)
      for (int cc = lb(compress[rr], c); cc < sz(compress[
          rr]); cc += cc & -cc)
        tree[rr][cc] += k;
  }//c42
  ll query(int r, int c) {
    if (!built) build();
    ll res = 0;
    for (int rr = r; rr; rr -= rr & -rr)
      for (int cc = lb(compress[rr], c + 1) - 1; cc; cc -=
          cc & -cc)//b9e
        res += tree[rr][cc];
    return res;
  }
};
```

## LazySegmentTree.h

**Description:** Segment tree that performs lazy operations The Node struct, Upd struct, apply(), combine(), and merge() may be customized apply() should apply update u to node v, and l and r represent the range corresponding to node v combine() should merge update b into update a merge() should merge nodes a and b and return the result by default this supports range addition, range set, range multiply, and range sum queries
**Time:** Construction is $\mathcal{O}(N)$, queries and updates are all $\mathcal{O}(\log N)$

77cc61, 66 lines

```
struct SegTree {//91a
  struct Node {
    ll sum = 0;
  };
  struct Upd {
    ll a = 1, b = 0;//782
  };
  void apply(Node &v, Upd &u, int l, int r) {
    v.sum = v.sum * u.a + u.b * (r - l + 1);
  }
  void combine(Upd &a, Upd &b) {//486
    a.b = a.b * b.a + b.b;
    a.a = a.a * b.a;
  }
  Node merge(Node &a, Node &b) {
    return {a.sum + b.sum};//4d7
  }
  vector<Node> tree; vector<Upd> upd;
  void push(int i, int l, int r) {
    apply(tree[i], upd[i], l, r);
    if (i * 2 < sz(tree)) {//bc7
      combine(upd[i * 2], upd[i]);
      combine(upd[i * 2 + 1], upd[i]);
    }
    upd[i] = {};
  }//924
  SegTree(vector<Node> arr) {
    int M = 1; while (M < sz(arr)) M <<= 1;
    tree = vector<Node>(M * 2);
    upd = vector<Upd>(M * 2);
    for (int i = 0; i < sz(arr); i++)//2fc
      tree[i + M] = arr[i];
    for (int i = M - 1; i > 0; i--)
      tree[i] = merge(tree[i * 2], tree[i * 2 + 1]);
  }
  void updateRec(int L, int R, Upd k, int i, int l, int r)
      {//8a6
    if (l > r) return;
    push(i, l, r);
    if (r < L || R < l) return;
    if (L <= l && r <= R) { combine(upd[i], k); push(i, l,
        r); return; }
    updateRec(L, R, k, i * 2, l, (l + r) / 2);//100
    updateRec(L, R, k, i * 2 + 1, (l + r) / 2 + 1, r);
    tree[i] = merge(tree[i * 2], tree[i * 2 + 1]);
  }
  void rangeUpdate(int l, int r, Upd k) { updateRec(l, r, k
      , 1, 0, sz(tree) / 2 - 1); }
  Node queryRec(int L, int R, int i, int l, int r) {//6f7
    if (l > r) return {};
    push(i, l, r);
    if (r < L || R < l) return {};
    if (L <= l && r <= R) return tree[i];
    Node left = queryRec(L, R, i * 2, l, (l + r) / 2);//37e
    Node right = queryRec(L, R, i * 2 + 1, (l + r) / 2 + 1,
        r);
    return merge(left, right);
  }
  Node rangeQuery(int l, int r) { return queryRec(l, r, 1,
      0, sz(tree) / 2 - 1); }
  /*int kthRec(int K, int i, int l, int r) {//d41
    if (l > r) return {};
    push(i, l, r);
    if (tree[i].sum < K) return -1;
    if (l == r) return l;
    int m = kthRec(K, i * 2, l, (l + r) / 2);//860
    if (m == -1) return kthRec(K - tree[i * 2].sum, i * 2 +
        1, (l + r) / 2 + 1, r);
    return m;
  }
  int kth(int K) { return kthRec(K, 1, 0, sz(tree) / 2 - 1)
      ; }*/
};//214
```

## LinearDiophantineSolver.h

**Description:** Solves Ax + By = C, outputs a solution to x and y, outputs step size to dx and dy
**Time:** $\mathcal{O}(fast)$

78211e, 11 lines

```
bool diophantine(ll A, ll B, ll C, ll &x, ll &y, ll &dx, ll
    &dy) {//922
  ll g = exGCD(abs(A), abs(B), x, y);
  if (C % g != 0) return false;
  x *= C / g; y *= C / g;
  if (A < 0) x = -x;
  if (B < 0) y = -y;//f93
  dx = B / g; dy = A / g;
  if (B < 0) dx = -dx;
  if (A < 0) dy = -dy;
  return true;
}//cbb
```

## LinearSieveOfMultiplicativeFunction.h

**Description:** computes f(1) ... f(n) for multiplicative function f
**Time:** $\mathcal{O}(n)$

0e8937, 24 lines

```
vector<ll> sieve(int n, ll (*fp)(int, int)) {//f36
  vector<bool> prime(n, 1);
  vector<int> primes;
  vector<int> cnt(n, 1), part(n, 1);
  vector<ll> f(n, 1);
  for (int i = 2; i < n; i++) {//29d
    if (prime[i]) {
      primes.pb(i);
      f[i] = fp(i, 1);
    }
    for (int j = 0; j < sz(primes) && i * primes[j] < n; j
        ++) {//73d
      prime[i * primes[j]] = 0;
      if (!(i % primes[j])) {
        cnt[i * primes[j]] = cnt[i] + 1;
        part[i * primes[j]] = part[i];
        f[i * primes[j]] = fp(primes[j], cnt[i * primes[j
            ]]) * f[part[i * primes[j]]];//e47
        break;
      }
      part[i * primes[j]] = i;
      f[i * primes[j]] = f[i] * f[primes[j]];
    }//a2c
  }
  return f;
}
```

## MinDeque.h

**Description:** Min Deque, useful for implementing min queue w/ rollback push() is for applying an element to a running sum merge() is for merging both sides of the ds (i.e. when querying)

**Time:** Amortized $\mathcal{O}(p)$ for non-query ops where p is the runtime of push() $\mathcal{O}(m)$ for query ops where m is the runtime of merge()

dd81ad, 52 lines

```
template <typename Node, typename Item, typename Query,
    typename Result>//1ed
struct MinDeque {
  Node push(Node n, Item k) {

  }
  Result merge(Node a, Node b, Query q) {//9f0

  }
  Node empty;
  stack<pair<Item, Node>> left, right;
  void push_front(Item k) {//e6e
    Node &n = left.empty() ? empty : left.top().second;
    left.push({k, push(n, k)});
  }
  void push_back(Item k) {
    swap(left, right);//417
    push_front(k);
    swap(left, right);
  }
  Item pop_front() {
    if (left.empty()) {//1aa
      int h = sz(right) / 2;
      stack<Item> tmp;
      for (int i = 0; i < h; i++) {
        tmp.push(right.top().first);
        right.pop();//72e
      }
      while (!right.empty()) {
        push_front(right.top().first);
        right.pop();
      }//ed1
      while (!tmp.empty()) {
        push_back(tmp.top());
        tmp.pop();
      }
    }//886
    Item res = left.top().first;
    left.pop();
    return res;
  }
  Item pop_back() {//126
    swap(left, right);
    Item res = pop_front();
    swap(left, right);
    return res;
  }//3fb
  Result query(Query q) {
    Node &l = left.empty() ? empty : left.top().second;
    Node &r = right.empty() ? empty : right.top().second;
    return merge(l, r, q);
  }//e03
};
```

## OfflineRectangleUnionArea.h

**Description:** Area of union of rectangles (does compression internally)

**Time:** $\mathcal{O}(n\log(n))$

b35a6c, 44 lines

```
struct OfflineRectangleUnionArea {//ead
  struct Update {
    int row, l, r, k;
  };
  vector<Update> updates;
  void addRect(int r1, int r2, int c1, int c2) {//98d
    updates.pb({r1, c1, c2, 1});
    updates.pb({r2 + 1, c1, c2, -1});
  }
  ll area() {
    vector<int> compressR, compressC;//4f9
    for (auto &update : updates) {
      compressR.pb(update.row - 1);
      compressR.pb(update.row);
      compressC.pb(update.l - 1);
      compressC.pb(update.l);//3eb
      compressC.pb(update.r);
    }
    sort(all(compressR));
    sort(all(compressC));
    int R = sz(compressR), C = sz(compressC);//49f
    vector<vector<Update>> compressedUpdates(R);
    for (auto &update : updates) {
      update.row = lb(compressR, update.row);
      update.l = lb(compressC, update.l);
      update.r = lb(compressC, update.r);//9a4
      compressedUpdates[update.row].pb(update);
    }
    vector<SegTree::Node> start(C);
    for (int i = 0; i < C; i++) start[i].mnCnt = compressC[
        i] - (i == 0 ? -1 : compressC[i - 1]);
    SegTree st(start);//f56
    ll res = 0;
    for (int i = 0; i < R; i++) {
      int h = compressR[i] - (i == 0 ? -1 : compressR[i -
          1]);
      for (auto &update : compressedUpdates[i]) {
        st.rangeUpdate(update.l, update.r, {update.k});//
            769
      }
      SegTree::Node full = st.rangeQuery(0, C - 1);
      int outside = full.mn == 0 ? full.mnCnt : 0;
      res += (ll)h * (compressC[C - 1] + 1 - outside);
    }//0c6
    return res;
  }
};
```

## SegmentTreeBeats.h

**Description:** preset to allow range chmin, range chmax, and range add queries all at once

**Time:** All operations are $\mathcal{O}(\log N)$.

aaaba9, 114 lines

```
struct SegTreeBeats {//c3f
  struct Node {
    ll sum = 0, mx = LLONG_MIN, mx2 = LLONG_MIN, mxC = 0,
        mn = LLONG_MAX, mn2 = LLONG_MAX, mnC = 0;
  };
  struct Upd {
    ll chmin = LLONG_MAX, chmax = LLONG_MIN, add = 0;//9dc
  };
  void apply(Node &v, Upd &u, int l, int r) {
    if (v.mx > u.chmin) {
      v.sum -= (v.mx - u.chmin) * v.mxC;
      if (v.mx == v.mn) v.mn = u.chmin;//50a
      if (v.mx == v.mn2) v.mn2 = u.chmin;
      if (v.mx == v.mx2) v.mx2 = u.chmin;
      v.mx = u.chmin;
    }
    if (v.mn < u.chmax) {//e15
      v.sum += (u.chmax - v.mn) * v.mnC;
      if (v.mn == v.mx) v.mx = u.chmax;
      if (v.mn == v.mx2) v.mx2 = u.chmax;
      if (v.mn == v.mn2) v.mn2 = u.chmax;
      v.mn = u.chmax;//535
    }
    v.sum += u.add * (r - l + 1);
    v.mx += u.add;
    v.mx2 += u.add;
    v.mn += u.add;//cc3
    v.mn2 += u.add;
    v.mx2 = min(v.mx2, v.mx);
    v.mn2 = max(v.mn2, v.mn);
  }
  void combine(Upd &a, Upd &b) {//324
    if (b.chmin != LLONG_MAX) {
      a.chmin = min(a.chmin, b.chmin - a.add);
      a.chmax = min(a.chmax, b.chmin - a.add);
    }
    if (b.chmax != LLONG_MIN) {//831
      a.chmax = max(a.chmax, b.chmax - a.add);
      a.chmin = max(a.chmin, b.chmax - a.add);
    }
    a.add += b.add;
  }//c89
  Node merge(Node &a, Node &b) {
    Node res;
    res.sum = a.sum + b.sum;
    vector<ll> mxSrt = {a.mx, a.mx2, b.mx, b.mx2};
    vector<ll> mnSrt = {a.mn, a.mn2, b.mn, b.mn2};//d69
    sort(all(mxSrt)); reverse(all(mxSrt)); sort(all(mnSrt))
        ;
    res.mx = mxSrt[0];
    int i = 1;
    while (i < 3 && mxSrt[i - 1] == mxSrt[i]) i++;
    res.mx2 = mxSrt[i];//500
    res.mn = mnSrt[0];
    i = 1;
    while (i < 3 && mnSrt[i - 1] == mnSrt[i]) i++;
    res.mn2 = mnSrt[i];
    if (a.mx == b.mx) res.mxC = a.mxC + b.mxC;//51d
    else if (a.mx > b.mx) res.mxC = a.mxC;
    else res.mxC = b.mxC;
    if (a.mn == b.mn) res.mnC = a.mnC + b.mnC;
    else if (a.mn < b.mn) res.mnC = a.mnC;
    else res.mnC = b.mnC;//8bf
    return res;
  }
  bool breakCondition(int L, int R, Upd k, int i, int l,
      int r) {
    if (r < L || R < l) return true;
    if (k.chmin != LLONG_MAX) return tree[i].mx <= k.chmin;
        //d3d
    if (k.chmax != LLONG_MIN) return tree[i].mn >= k.chmax;
    return false;
  }
  bool tagCondition(int L, int R, Upd k, int i, int l, int
      r) {
    if (!(L <= l && r <= R)) return false;//08e
    if (k.chmin != LLONG_MAX) return tree[i].mx2 < k.chmin
        || tree[i].mx == tree[i].mx2;
    if (k.chmax != LLONG_MIN) return tree[i].mn2 > k.chmax
        || tree[i].mn == tree[i].mn2;
    return true;
  }
  vector<Node> tree; vector<Upd> upd;//933
  void push(int i, int l, int r) {
    apply(tree[i], upd[i], l, r);
    if (i * 2 < sz(tree)) {
      combine(upd[i * 2], upd[i]);
      combine(upd[i * 2 + 1], upd[i]);//efd
    }
    upd[i] = {};
  }
  SegTreeBeats(vector<Node> arr) {
    int M = 1; while (M < sz(arr)) M <<= 1;//db0
    tree = vector<Node>(M * 2);
    upd = vector<Upd>(M * 2);
    for (int i = 0; i < sz(arr); i++)
```

```
    tree[i + M] = arr[i];
  for (int i = M - 1; i > 0; i--)//372
    tree[i] = merge(tree[i * 2], tree[i * 2 + 1]);
}
void updateRec(int L, int R, Upd k, int i, int l, int r)
    {
  if (l > r) return;
  push(i, l, r);//8d1
  if (breakCondition(L, R, k, i, l, r)) return;
  if (tagCondition(L, R, k, i, l, r)) { combine(upd[i], k
      ); push(i, l, r); return; }
  updateRec(L, R, k, i * 2, l, (l + r) / 2);
  updateRec(L, R, k, i * 2 + 1, (l + r) / 2 + 1, r);
  tree[i] = merge(tree[i * 2], tree[i * 2 + 1]);//b6d
}
void rangeUpdate(int l, int r, Upd k) { updateRec(l, r, k
    , 1, 0, sz(tree) / 2 - 1); }
Node queryRec(int L, int R, int i, int l, int r) {
  if (l > r) return {};
  push(i, l, r);//c8b
  if (breakCondition(L, R, {}, i, l, r)) return {};
  if (tagCondition(L, R, {}, i, l, r)) return tree[i];
  Node left = queryRec(L, R, i * 2, l, (l + r) / 2);
  Node right = queryRec(L, R, i * 2 + 1, (l + r) / 2 + 1,
       r);
  return merge(left, right);//7ff
}
Node rangeQuery(int l, int r) { return queryRec(l, r, 1,
    0, sz(tree) / 2 - 1); }
};
```

## SMAWK.h

**Description:** Outputs row minima of nxm totally monotone matrix f(r, c1, c2) should output 1 if a[r][c1] <= a[r][c2]

**Time:** $\mathcal{O}(n)$

3ff481, 77 lines

```
template<typename F>//95b
vector<int> smawk(F f, vector<int> rows, vector<int> cols)
    {
  vector<int> row_mins;
  if (sz(rows) <= 2 && sz(cols) <= 2) {
    for (int r : rows) {
      int mn = -1;//7ea
      for (int c : cols) {
        if (mn == -1 || f(r, c, mn)) mn = c;
      }
      row_mins.pb(mn);
    }//1e0
  } else if (sz(rows) < sz(cols)) {
    // Reduce
    vector<int> st;
    for (int j : cols) {
      while (!st.empty()) {//f89
        if (f(rows[sz(st) - 1], j, st.back())) st.pop_back
            ();
        else if (sz(st) == sz(rows)) break;
        else { st.push_back(j); break; }
      }
      if (st.empty()) st.pb(j);//97e
    }
    row_mins = smawk(f, rows, st);
  } else {
    // Interpolate
    vector<int> half_rows;//420
    for (int i = 1; i < sz(rows); i += 2) half_rows.pb(rows
        [i]);
    vector<int> half_row_mins = smawk(f, half_rows, cols);
    vector<int> mn_index;
    for (int i = 0, j = 0; j < sz(cols); j++) {
      while (i < sz(half_row_mins) && cols[j] ==
          half_row_mins[i]) {//943
        mn_index.pb(j);
        i++;
      }
    }
    for (int i = 0; i < sz(rows); i++) {//605
      if (i % 2) row_mins.pb(half_row_mins[i / 2]);
      else {
        int l = i == 0 ? 0 : mn_index[i / 2 - 1];
        int r = i == sz(rows) - 1 ? sz(cols) - 1 : mn_index
            [i / 2];
        int mn = cols[l];//57d
        for (int j = l + 1; j <= r; j++) {
          if (f(rows[i], cols[j], mn)) mn = cols[j];
        }
        row_mins.pb(mn);
      }//049
    }
  }
  return row_mins;
}
//fc4
// Min-plus Convolution (b must be convex)
vector<ll> min_plus(vector<ll> a, vector<ll> b) {
  int n = sz(a), m = sz(b);
  auto get = [&](int r, int c) { return a[c] + b[r - c]; };
  auto comp = [&](int r, int c1, int c2) {//792
    if (r - max(c1, c2) < 0) return c1 < c2;
    if (r - min(c1, c2) >= m) return c2 < c1;
    return get(r, c1) <= get(r, c2);
  };
  vector<int> rows, cols;//e94
  for (int i = 0; i < n + m - 1; i++) rows.pb(i);
  for (int j = 0; j < n; j++) cols.pb(j);
  vector<int> row_mins = smawk(comp, rows, cols);
  vector<ll> res;
  for (int i = 0; i < n + m - 1; i++) res.pb(get(i,
      row_mins[i]));//332
  return res;
}

// Min-plus Convolution (b must be concave)
vector<ll> max_plus(vector<ll> a, vector<ll> b) {//325
  for (ll &i : a) i = -i;
  for (ll &i : b) i = -i;
  vector<ll> res = min_plus(a, b);
  for (ll &i : res) i = -i;
  return res;//510
}
```

## SuffixAutomaton.h

**Description:** Constructs a suffix automaton on string s cnt() constructs an array of equivalence class sizes first() constructs an array of first occurences for each node

**Time:** All functions are $\mathcal{O}(N)$.

24974f, 55 lines

```
struct SuffixAutomaton {//3d3
  struct Node {
    int len = 0, lnk = 0;
    int nxt[26];
  };
  string s;//f20
  vector<Node> t; int last = 0;
  SuffixAutomaton(string s = "") {
    t.pb({0, -1, {}});
    for (char c : s) add(c);
  }//a4a
  void add(char c) { s += c; c -= 'a';
    int u = last; int v = last = sz(t);
    t.pb({t[u].len + 1, 0, {}});
    while (u >= 0 && !t[u].nxt[c])
      t[u].nxt[c] = v, u = t[u].lnk;//b21
    if (u == -1) return;
    int q = t[u].nxt[c];
    if (t[u].len + 1 == t[q].len)
      { t[v].lnk = q; return; }
    int cpy = sz(t); t.pb(t[q]);//aa4
    t[cpy].len = t[u].len + 1;
    while (u >= 0 && t[u].nxt[c] == q)
      t[u].nxt[c] = cpy, u = t[u].lnk;
    t[v].lnk = t[q].lnk = cpy;
  }//2f2
  vector<int> cnt() {
    vector<int> res(sz(t), 0);
    int cur = 0;
    for (char c : s)
      res[cur = t[cur].nxt[c - 'a']]++;//82c
    vector<pair<int, int>> srt;
    for (int i = 1; i < sz(t); i++)
      srt.pb({-t[i].len, i});
    sort(all(srt));
    for (auto &p : srt)//cc8
      res[t[p.second].lnk] += res[p.second];
    return res;
  }
  vector<int> first() {
    vector<int> res(sz(t), sz(s));//c3b
    int cur = 0;
    for (int i = 0; i < sz(s); i++) {
      cur = t[cur].nxt[s[i] - 'a'];
      res[cur] = min(res[cur], i);
    }//268
    vector<pair<int, int>> srt;
    for (int i = 1; i < sz(t); i++)
      srt.pb({-t[i].len, i});
    sort(all(srt));
    for (auto &p : srt)//23d
      res[t[p.second].lnk] = min(res[t[p.second].lnk], res[
          p.second]);
    return res;
  }
};
```

## WaveletTree.h

**Description:** Range K-th Smallest

**Time:** $\mathcal{O}\left(n\log^2(n)\right)$ I think

ccdc27, 39 lines

```
struct WaveletTree {//b57
  struct Node {
    vector<int> a, b;
    int l = -1, r = -1;
    bool leaf = 0;
  };//188
  int bits = 30;
  vector<int> arr;
  vector<Node> tree = {{}};
  void construct(int u, int lo, int hi) {
    if (lo == hi) { tree[u].leaf = 1; return; }//5bb
    int l = sz(tree); tree.pb({});
    int r = sz(tree); tree.pb({});
    tree[u].l = l; tree[u].r = r;
    int mid = (lo + hi) / 2;
    int pre = 0;//16b
    for (int i = 0; i < sz(tree[u].a); i++) {
      if (arr[tree[u].a[i]] <= mid) {
        tree[l].a.pb(tree[u].a[i]);
        pre++;
      } else tree[r].a.pb(tree[u].a[i]);//b04
      tree[u].b.pb(pre);
    }
    if (!tree[l].a.empty()) construct(l, lo, mid);
    if (!tree[r].a.empty()) construct(r, mid + 1, hi);
  }//102
```

```
  WaveletTree(vector<int> a) : arr(a) {
    for (int i = 0; i < sz(arr); i++) tree[0].a.pb(i);
    construct(0, 0, (1 << bits) - 1);
  }
  int query(int l, int r, int k, int u = 0) {//07f
    if (u == -1) return 0;
    int lp = lb(tree[u].a, l), rp = lb(tree[u].a, r + 1) -
        1;
    if (tree[u].leaf) return tree[u].a[lp + k - 1];
    int n = tree[u].b[rp] - (lp == 0 ? 0 : tree[u].b[lp -
        1]);
    if (n >= k) return query(l, r, k, tree[u].l);//3a4
    return query(l, r, k - n, tree[u].r);
  }
};
```

## 12.1 Convolutions

### GCDConvolution.h

**Description:** GCD Convolution
**Time:** $\mathcal{O}(n\log(n))$

ca54f7, 12 lines

```
// 1-indexed//cab
vector<ll> gcdMult(vector<ll> a, vector<ll> b) {
  vector<ll> c(sz(a), 0);
  for (int g = sz(a); g >= 1; g--) {
    ll l = 0, r = 0;
    for (int i = g; i <= sz(a); i += g)//f68
      l += a[i-1], r += b[i-1], c[g-1] -= c[i-1];
    c[g-1] += (l % mod) * (r % mod) % mod;
    c[g-1] = (c[g-1] % mod + mod) % mod;
  }
  return c;//df7
}
```

### LCMConvolution.h

**Description:** LCM Convolution
**Time:** about $\mathcal{O}(n\log(n))$

d8b1b5, 13 lines

```
// 1-indexed//665
vector<ll> lcmMult(vector<ll> a, vector<ll> b) {
  vector<ll> l(sz(a), 0), r(sz(a), 0), c(sz(a), 0);
  for (int i = 1; i <= sz(a); i++)
    for (int j = i; j <= sz(a); j += i)
      l[j-1] += a[i-1], r[j-1] += b[i-1];//56c
  for (int g = 1; g <= sz(a); g++) {
    c[g-1] += (l[g-1] % mod) * (r[g-1] % mod) % mod;
    c[g-1] = (c[g-1] % mod + mod) % mod;
    for (int i = g*2; i <= sz(a); i += g) c[i-1] -= c[g-1];
  }//930
  return c;
}
```

### OrConvolution.h

**Description:** Bitwise OR Convolution and Bitwise AND Convolution
**Time:** $\mathcal{O}(n\log(n))$

4d2a9a, 22 lines

```
void orfft(vector<ll> &v) {//ece
  for (int j = bits - 1; j >= 0; j--)
    for (int i = 0; i < 1 << bits; i++, i += i & 1 << j)
      v[i ^ 1 << j] += v[i];
  for (int i = 0; i < 1 << bits; i++) v[i] %= mod;
}//a47
void orifft(vector<ll> &v) {
  for (int j = 0; j < bits; j++)
    for (int i = 0; i < 1 << bits; i++, i += i & 1 << j)
      v[i ^ 1 << j] -= v[i];
  for (int i = 0; i < 1 << bits; i++) v[i] = (v[i] % mod +
      mod) % mod;//2c6
}
vector<ll> orMult(vector<ll> a, vector<ll> b) {
  orfft(a); orfft(b); vector<ll> c(1 << bits);
  for (int i = 0; i < 1 << bits; i++) c[i] = a[i] * b[i] %
      mod;
  orifft(c); return c;//71b
}
vector<ll> andMult(vector<ll> a, vector<ll> b) {
  reverse(all(a)); reverse(all(b));
  vector<ll> c = orMult(a, b);
  reverse(all(c)); return c;//d43
}
```

### SubsetConvolution.h

**Description:** Subset Convolution
**Time:** $\mathcal{O}(n\log^2(n))$

3b4300, 75 lines

```
struct yint {//0e1
  int B = 22; ll a[22];
  yint() { for (int i = 0; i < B; i++) a[i] = 0; }
  yint(ll k) { *this = yint(); a[0] = k; }
  yint& operator+=(yint o) {
    for (int i = 0; i < B; i++)//b84
      if ((a[i] += o.a[i]) >= mod) a[i] -= mod;
    return *this; }
  yint& operator-=(yint o) {
    for (int i = 0; i < B; i++)
      if ((a[i] -= o.a[i]) < 0) a[i] += mod;//59d
    return *this; }
  yint& operator*=(int k) {
    for (int i = 0; i < B; i++)
      a[i] = a[i] * k % mod;
    return *this; }//585
  yint& operator*=(yint o) {
    ll a2[22]; for (int i = 0; i < B; i++) a2[i] = 0;
    for (int i = 0; i < B; i++) for (int j = 0; i + j < B;
        j++) {
      a2[i + j] += a[i] * o.a[j] % mod;
      if (a2[i + j] >= mod) a2[i + j] -= mod; }//885
    swap(a, a2); return *this; }
  yint& operator-() {
    for (int i = 0; i < B; i++)
      if ((a[i] = -a[i]) < 0) a[i] += mod;
    return *this; }//40b
  yint& shiftY(int k) {
    if (k > 0) {
      for (int i = B-1; i - k >= 0; i--) a[i] = a[i - k];
      for (int i = 0; i < k; i++) a[i] = 0;
    } else { k = -k;//7c5
      for (int i = 0; i + k < B; i++) a[i] = a[i + k];
      for (int i = B - k; i < B; i++) a[i] = 0;
    } return *this; }
  yint& multY() {
    for (int i = B-1; i > 0; i--) a[i] = a[i - 1];//abd
    a[0] = 0; return *this; }
  yint& divY() {
    for (int i = 0; i < B-1; i++) a[i] = a[i + 1];
    a[B-1] = 0; return *this; }
  friend yint operator+(yint a, yint b) { return a += b; }
      //fef
  friend yint operator-(yint a, yint b) { return a -= b; }
  friend yint operator*(yint a, yint b) { return a *= b; }
};

void subsetfft(vector<yint> &v) {//123
  for (int i = 0; i < 1 << bits; i++) v[i].shiftY(
      __builtin_popcount(i));
  for (int j = 0; j < bits; j++) {
    for (int i = 0; i < 1 << bits; i++, i += i & 1 << j) {
      yint a = v[i], by = v[i ^ 1 << j];
      v[i] = a - by; v[i ^ 1 << j] = a + by;//28f
    }
  }
}
void subsetifft(vector<yint> &v) {
  for (int j = 0; j < bits; j++) {//1f2
    for (int i = 0; i < 1 << bits; i++, i += i & 1 << j) {
      yint a = v[i], b = v[i ^ 1 << j];
      v[i] = a + b; v[i ^ 1 << j] = (b - a);
    }
  }//c85
  for (int i = 0; i < 1 << bits; i++) v[i].shiftY(-
      __builtin_popcount(i));
  ll p = 1; for (int j = 0; j < bits; j++) p = p * 2 % mod;
     p = mInv(p);
  for (int i = 0; i < 1 << bits; i++) v[i] *= p;
}
vector<ll> subsetMult(vector<ll> a, vector<ll> b) {//5a1
  vector<yint> aY(1 << bits), bY(1 << bits), cY(1 << bits);
  for (int i = 0; i < 1 << bits; i++) aY[i] = yint(a[i]),
      bY[i] = yint(b[i]);
  subsetfft(aY); subsetfft(bY);
  for (int i = 0; i < 1 << bits; i++) cY[i] = aY[i] * bY[i
      ];
  subsetifft(cY);//743
  vector<ll> c(1 << bits);
  for (int i = 0; i < 1 << bits; i++) c[i] = cY[i].a[0];
  return c;
}
```

### XORConvolution.h

**Description:** Bitwise XOR Convolution
**Time:** $\mathcal{O}(n\log(n))$

c3f3d6, 19 lines

```
void xorfft(vector<ll> &v, bool inv = 0) {//475
  for (int k = 0; k < bits; k++) {
    for (int i = 0; i < 1 << bits; i++, i += i & 1 << k) {
      ll a = v[i], b = v[i ^ 1 << k];
      v[i] = a + b, v[i ^ 1 << k] = a - b;
    }//aa7
  }
  for (int i = 0; i < 1 << bits; i++) v[i] = (v[i] % mod +
      mod) % mod;
  if (inv) {
    ll n = 1; for (int i = 0; i < bits; i++) n = n * 2 %
        mod;
    n = mInv(n);//e2a
    for (int i = 0; i < 1 << bits; i++) v[i] = v[i] * n %
        mod;
  }
}
vector<ll> mult(vector<ll> a, vector<ll> b) {
  xorfft(a); xorfft(b); vector<ll> c(1 << bits);//703
  for (int i = 0; i < 1 << bits; i++) c[i] = a[i] * b[i] %
      mod;
  xorfft(c, 1); return c;
}
```

# Techniques (A)

## techniques.txt

159 lines

```
Recursion
Divide and conquer
  Finding interesting points in N log N
Algorithm analysis
  Master theorem
  Amortized time complexity
Greedy algorithm
  Scheduling
  Max contiguous subvector sum
  Invariants
  Huffman encoding
Graph theory
  Dynamic graphs (extra book-keeping)
  Breadth first search
  Depth first search
  * Normal trees / DFS trees
  Dijkstra's algorithm
  MST: Prim's algorithm
  Bellman-Ford
  Konig's theorem and vertex cover
  Min-cost max flow
  Lovasz toggle
  Matrix tree theorem
  Maximal matching, general graphs
  Hopcroft-Karp
  Hall's marriage theorem
  Graphical sequences
  Floyd-Warshall
  Euler cycles
  Flow networks
  * Augmenting paths
  * Edmonds-Karp
  Bipartite matching
  Min. path cover
  Topological sorting
  Strongly connected components
  2-SAT
  Cut vertices, cut-edges and biconnected components
  Edge coloring
  * Trees
  Vertex coloring
  * Bipartite graphs (=> trees)
  * 3^n (special case of set cover)
  Diameter and centroid
  K'th shortest path
  Shortest cycle
Dynamic programming
  Knapsack
  Coin change
  Longest common subsequence
  Longest increasing subsequence
  Number of paths in a dag
  Shortest path in a dag
  Dynprog over intervals
  Dynprog over subsets
  Dynprog over probabilities
  Dynprog over trees
  3^n set cover
  Divide and conquer
  Knuth optimization
  Convex hull optimizations
  RMQ (sparse table a.k.a 2^k-jumps)
  Bitonic cycle
  Log partitioning (loop over most restricted)
Combinatorics
  Computation of binomial coefficients
  Pigeon-hole principle
  Inclusion/exclusion
  Catalan number
  Pick's theorem
Number theory
  Integer parts
  Divisibility
  Euclidean algorithm
  Modular arithmetic
  * Modular multiplication
  * Modular inverses
  * Modular exponentiation by squaring
  Chinese remainder theorem
  Fermat's little theorem
  Euler's theorem
  Phi function
  Frobenius number
  Quadratic reciprocity
  Pollard-Rho
  Miller-Rabin
  Hensel lifting
  Vieta root jumping
Game theory
  Combinatorial games
  Game trees
  Mini-max
  Nim
  Games on graphs
  Games on graphs with loops
  Grundy numbers
  Bipartite games without repetition
  General games without repetition
  Alpha-beta pruning
Probability theory
Optimization
  Binary search
  Ternary search
  Unimodality and convex functions
  Binary search on derivative
Numerical methods
  Numeric integration
  Newton's method
  Root-finding with binary/ternary search
  Golden section search
Matrices
  Gaussian elimination
  Exponentiation by squaring
Sorting
  Radix sort
Geometry
  Coordinates and vectors
  * Cross product
  * Scalar product
  Convex hull
  Polygon cut
  Closest pair
  Coordinate-compression
  Quadtrees
  KD-trees
  All segment-segment intersection
Sweeping
  Discretization (convert to events and sweep)
  Angle sweeping
  Line sweeping
  Discrete second derivatives
Strings
  Longest common substring
  Palindrome subsequences
  Knuth-Morris-Pratt
  Tries
  Rolling polynomial hashes
  Suffix array
  Suffix tree
  Aho-Corasick
  Manacher's algorithm
  Letter position lists
Combinatorial search
  Meet in the middle
  Brute-force with pruning
  Best-first (A*)
  Bidirectional search
  Iterative deepening DFS / A*
Data structures
  LCA (2^k-jumps in trees in general)
  Pull/push-technique on trees
  Heavy-light decomposition
  Centroid decomposition
  Lazy propagation
  Self-balancing trees
  Convex hull trick (wcipeg.com/wiki/Convex_hull_trick)
  Monotone queues / monotone stacks / sliding queues
  Sliding queue using 2 stacks
  Persistent segment tree
```