

Purdue University

# Fearless

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ECNA 2025

November 9, 2025

Contest (1)

```
template.cpp14 lines
#include <bits/stdc++.h>
using namespace std;

#define rep(i, a, b) for(int i = a; i < (b); ++i)
#define all(x) begin(x), end(x)
#define sz(x) (int)(x).size()
#define pb push_back
typedef long long ll;
typedef pair<int, int> pii;
typedef vector<int> vi;

int main() {
    cin.tie(0)->sync_with_stdio(0);
}
```

```
.bashrc10 lines
run () {
    ok=1
    if [[ ! -f $1 || $1 -ot $1.cpp ]]
    then
        g++ $1.cpp -O2 -o $1 -std=c++17 -Wall -Wextra -Wshadow
            -Wconversion -fsanitize=undefined,address || ok=0
    fi
    [[ $ok -eq 1 ]] && ./$1
}

xmodmap -e 'clear Lock' -e 'keycode 0x42 = Escape'
```

```
.vimrc5 lines
set cin aw ai is ts=4 sw=4 tm=50 rnu noeb bg=dark ru cul
mouse=a
sy on | no ; :
" Select region and then type :Hash to hash your selection.
ca Hash w !cpp -dD -P -fpreprocessed \| tr -d '[:space:]' \
\| md5sum \| cut -c-6
```

```
brute.sh12 lines
#!/bin/zsh

sz=100
for ((i=1;;i++)); do
    echo "$i"
    ./gen "$i" "$sz" > input
    ./sol < input > output1
    ./brute < input > output2
    if (! diff output1 output2); then
        break
    fi
done
```

Mathematics (2)

2.1 Equations

$$\begin{matrix} ax + by = e \\ cx + dy = f \end{matrix} \Rightarrow \begin{matrix} x = \frac{ed - bf}{ad - bc} \\ y = \frac{af - ec}{ad - bc} \end{matrix}$$

$$x_i = \frac{\det A'_i}{\det A}$$

2.2 Trigonometry

$$\begin{aligned} \sin(v + w) &= \sin v \cos w + \cos v \sin w \\ \cos(v + w) &= \cos v \cos w - \sin v \sin w \\ \tan(v + w) &= \frac{\tan v + \tan w}{1 - \tan v \tan w} \\ \sin v + \sin w &= 2 \sin \frac{v + w}{2} \cos \frac{v - w}{2} \\ \cos v + \cos w &= 2 \cos \frac{v + w}{2} \cos \frac{v - w}{2} \end{aligned}$$

$(V + W) \tan(v - w)/2 = (V - W) \tan(v + w)/2$   
where  $V, W$  are lengths of sides opposite angles  $v, w$ .

$$\begin{aligned} a \cos x + b \sin x &= r \cos(x - \phi) \\ a \sin x + b \cos x &= r \sin(x + \phi) \end{aligned}$$

where  $r = \sqrt{a^2 + b^2}, \phi = \text{atan2}(b, a)$ .

2.3 Geometry

2.3.1 Triangles

Circumradius:  $R = abc/4A$   
Inradius:  $r = A/p$   
Length of median (divides triangle into two equal-area triangles):  $m_a = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$   
Length of bisector (divides angles in two):

$$s_a = \sqrt{bc \left[ 1 - \left( \frac{a}{b + c} \right)^2 \right]}$$
  
Law of sines:  $\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c} = \frac{1}{2R}$   
Law of cosines:  $a^2 = b^2 + c^2 - 2bc \cos \alpha$   
Law of tangents:  $\frac{a + b}{a - b} = \frac{\tan \frac{\alpha + \beta}{2}}{\tan \frac{\alpha - \beta}{2}}$

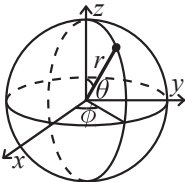
2.3.2 Quadrilaterals

With side lengths  $a, b, c, d$ , diagonals  $e, f$ , diagonals angle  $\theta$ , area  $A$  and magic flux  $F = b^2 + d^2 - a^2 - c^2$ :

$$4A = 2ef \cdot \sin \theta = F \tan \theta = \sqrt{4e^2 f^2 - F^2}$$

For cyclic quadrilaterals the sum of opposite angles is  $180^\circ$ ,  $ef = ac + bd$ , and  $A = \sqrt{(p - a)(p - b)(p - c)(p - d)}$ .

2.3.3 Spherical coordinates



$$\begin{aligned} x &= r \sin \theta \cos \phi & r &= \sqrt{x^2 + y^2 + z^2} \\ y &= r \sin \theta \sin \phi & \theta &= \text{acos}(z/\sqrt{x^2 + y^2 + z^2}) \\ z &= r \cos \theta & \phi &= \text{atan2}(y, x) \end{aligned}$$

2.4 Derivatives/Integrals

$$\begin{aligned} \frac{d}{dx} \arcsin x &= \frac{1}{\sqrt{1 - x^2}} & \frac{d}{dx} \arccos x &= -\frac{1}{\sqrt{1 - x^2}} \\ \frac{d}{dx} \tan x &= 1 + \tan^2 x & \frac{d}{dx} \arctan x &= \frac{1}{1 + x^2} \\ \int \tan ax &= -\frac{\ln |\cos ax|}{a} & \int x \sin ax &= \frac{\sin ax - ax \cos ax}{a^2} \\ \int e^{-x^2} &= \frac{\sqrt{\pi}}{2} \text{erf}(x) & \int x e^{ax} dx &= \frac{e^{ax}}{a^2} (ax - 1) \end{aligned}$$

Integration by parts:

$$\int_a^b f(x)g(x)dx = [F(x)g(x)]_a^b - \int_a^b F(x)g'(x)dx$$

2.5 Sums

$$c^a + c^{a+1} + \dots + c^b = \frac{c^{b+1} - c^a}{c - 1}, c \neq 1$$

$$\begin{aligned} 1 + 2 + 3 + \dots + n &= \frac{n(n + 1)}{2} \\ 1^2 + 2^2 + 3^2 + \dots + n^2 &= \frac{n(2n + 1)(n + 1)}{6} \\ 1^3 + 2^3 + 3^3 + \dots + n^3 &= \frac{n^2(n + 1)^2}{4} \\ 1^4 + 2^4 + 3^4 + \dots + n^4 &= \frac{n(n + 1)(2n + 1)(3n^2 + 3n - 1)}{30} \end{aligned}$$

2.6 Series

$$\begin{aligned} e^x &= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots, (-\infty < x < \infty) \\ \ln(1 + x) &= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots, (-1 < x \leq 1) \\ \sqrt{1 + x} &= 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{2x^3}{32} - \frac{5x^4}{128} + \dots, (-1 \leq x \leq 1) \\ \sin x &= x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots, (-\infty < x < \infty) \\ \cos x &= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots, (-\infty < x < \infty) \end{aligned}$$

2.7 Probability theory

$$\begin{aligned} \sigma^2 &= V(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2 \\ \mathbb{E}(aX + bY) &= a\mathbb{E}(X) + b\mathbb{E}(Y) \\ \text{ind. } X, Y, V(aX + bY) &= a^2V(X) + b^2V(Y). \end{aligned}$$

2.7.1 Discrete distributions

Binomial distribution

$$p(k) = \binom{n}{k} p^k (1 - p)^{n-k}$$
$$\mu = np, \sigma^2 = np(1 - p)$$

Bin(*n*,*p*) is approximately Po(*np*) for small *p*.

Geometric distribution

$$p(k) = p(1 - p)^{k-1}, \; k = 1, 2, \dots$$
$$\mu = \frac{1}{p}, \sigma^2 = \frac{1 - p}{p^2}$$

Poisson distribution

$$p(k) = e^{-\lambda} \frac{\lambda^k}{k!}, \; k = 0, 1, 2, \dots$$
$$\mu = \lambda, \sigma^2 = \lambda$$

2.7.2 Continuous distributions

Uniform distribution

$$f(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & \text{otherwise} \end{cases}$$
$$\mu = \frac{a+b}{2}, \sigma^2 = \frac{(b-a)^2}{12}$$

Exponential distribution

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$
$$\mu = \frac{1}{\lambda}, \sigma^2 = \frac{1}{\lambda^2}$$

Normal distribution

Most real random values with mean  $\mu$  and variance  $\sigma^2$  are well described by  $\mathcal{N}(\mu, \sigma^2)$ ,  $\sigma > 0$ .

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

If  $X_1 \sim \mathcal{N}(\mu_1, \sigma_1^2)$  and  $X_2 \sim \mathcal{N}(\mu_2, \sigma_2^2)$  then

$$aX_1 + bX_2 + c \sim \mathcal{N}(\mu_1 + \mu_2 + c, a^2\sigma_1^2 + b^2\sigma_2^2)$$

2.8 Markov chains

A *Markov chain* is a discrete random process with the property that the next state depends only on the current state. Let  $X_1, X_2, \dots$  be a sequence of random variables generated by the Markov process. Then there is a transition matrix  $\mathbf{P} = (p_{ij})$ , with  $p_{ij} = \Pr(X_n = i | X_{n-1} = j)$ , and  $\mathbf{p}^{(n)} = \mathbf{P}^n \mathbf{p}^{(0)}$  is the probability distribution for  $X_n$  (i.e.,  $p_i^{(n)} = \Pr(X_n = i)$ ), where  $\mathbf{p}^{(0)}$  is the initial distribution.

$\pi$  is a stationary distribution if  $\pi = \pi \mathbf{P}$ . If the Markov chain is *irreducible* (it is possible to get to any state from any state), then  $\pi_i = \frac{1}{\mathbb{E}(T_i)}$  where  $\mathbb{E}(T_i)$  is the expected time between two visits in state  $i$ .  $\pi_j / \pi_i$  is the expected number of visits in state  $j$  between two visits in state  $i$ .

For a connected, undirected and non-bipartite graph, where the transition probability is uniform among all neighbors,  $\pi_i$  is proportional to node  $i$ 's degree.

A Markov chain is *ergodic* if the asymptotic distribution is independent of the initial distribution. A finite Markov chain is ergodic iff it is irreducible and *aperiodic* (i.e., the gcd of cycle lengths is 1).  $\lim_{k \rightarrow \infty} \mathbf{P}^k = \mathbf{1}\pi$ .

A Markov chain is an **A**-chain if the states can be partitioned into two sets **A** and **G**, such that all states in **A** are absorbing ( $p_{ii} = 1$ ), and all states in **G** leads to an absorbing state in **A**. The probability for absorption in state  $i \in \mathbf{A}$ , when the initial state is  $j$ , is  $a_{ij} = p_{ij} + \sum_{k \in \mathbf{G}} a_{ik} p_{kj}$ . The expected time until absorption, when the initial state is  $i$ , is  $t_i = 1 + \sum_{k \in \mathbf{G}} p_{ki} t_k$ .

Data structures (3)

OrderStatisticTree.h

Description: A set (not multiset!) with support for finding the n'th element, and finding the index of an element. To get a map, change null\_type.

Time:  $\mathcal{O}(\log N)$

#include <bits/extc++.h> //893

using namespace \_\_gnu\_pbds;

template<class T>

using Tree = tree<T, null\_type, less<T>, rb\_tree\_tag,

tree\_order\_statistics\_node\_update>; //988

void example() {

Tree<int> t, t2; t.insert(8);

auto it = t.insert(10).first;

assert(it == t.lower\_bound(9)); //6bd

assert(t.order\_of\_key(10) == 1);

assert(t.order\_of\_key(11) == 2);

assert(\*t.find\_by\_order(0) == 8);

t.join(t2); // assuming T < T2 or T > T2, merge t2 into t

//cbb

HashMap.h

Description: Hash map with mostly the same API as unordered\_map, but ~3x faster. Uses 1.5x memory. Initial capacity must be a power of 2 (if provided).

d77092, 7 lines

#include <bits/extc++.h> //1e4

// To use most bits rather than just the lowest ones:

struct chash { // large odd number for C

const uint64\_t C = 11(4e18 \* acos(0)) | 71;

ll operator()(ll x) const { return \_\_builtin\_bswap64(x\*C)

;

}; //198

\_\_gnu\_pbds::gp\_hash\_table<ll, int, chash> h({}, {}, {}, {}, {

1<<16});

Matrix.h

Description: Basic operations on square matrices.

Usage: Matrix<int, 3> A;

A.d = {{{{1,2,3}}, {{4,5,6}}, {{7,8,9}}}};

vector<int> vec = {1,2,3};

vec = (A\*N) \* vec;

c43c7d, 26 lines

template<class T, int N> struct Matrix { //1aa

typedef Matrix M;

array<array<T, N>, N> d{};

M operator\*(const M& m) const {

M a;

rep(i,0,N) rep(j,0,N) //683

rep(k,0,N) a.d[i][j] += d[i][k]\*m.d[k][j];

return a;

}

vector<T> operator\*(const vector<T>& vec) const {

vector<T> ret(N); //9bd

rep(i,0,N) rep(j,0,N) ret[i] += d[i][j] \* vec[j];

return ret;

}

M operator^(ll p) const {

assert(p >= 0); //358

M a, b(\*this);

rep(i,0,N) a.d[i][i] = 1;

while (p) {

if (p&1) a = a\*b;

b = b\*b; //1d8

p >>= 1;

}

return a;

}

}; //214

LineContainer.h

Description: Container where you can add lines of the form kx+m, and query maximum values at points x. Useful for dynamic programming (“convex hull trick”).

Time:  $\mathcal{O}(\log N)$

Sec1c7, 30 lines

struct Line { //7e3

mutable ll k, m, p;

bool operator<(const Line& o) const { return k < o.k; }

bool operator<(ll x) const { return p < x; }

};

//d77

struct LineContainer : multiset<Line, less<>> {

// (for doubles, use inf = 1/.0, div(a,b) = a/b)

static const ll inf = LLONG\_MAX;

ll div(ll a, ll b) { // floored division

return a / b - ((a ^ b) < 0 && a % b); } //66e

bool isect(iterator x, iterator y) {

if (y == end()) return x->p = inf, 0;

if (x->k == y->k) x->p = x->m > y->m ? inf : -inf;

else x->p = div(y->m - x->m, x->k - y->k);

return x->p >= y->p; //bec

}

void add(ll k, ll m) {

```

    auto z = insert({k, m, 0}), y = z++, x = y;
    while (isect(y, z)) z = erase(z);
    if (x != begin() && isect(--x, y)) isect(x, y = erase(y)); //890
    while ((y = x) != begin() && (--x)->p >= y->p)
        isect(x, erase(y));
}
ll query(ll x) {
    assert(!empty()); //b07
    auto l = *lower_bound(x);
    return l.k * x + l.m;
}
};
```

### Treap.h

**Description:** A short self-balancing tree. It acts as a sequential container with log-time splits/joins, and is easy to augment with additional data.

**Time:**  $\mathcal{O}(\log N)$

```

struct Node { //e9f
    Node *l = 0, *r = 0;
    int val, y, c = 1;
    Node(int v) : val(v), y(rand()) {}
    void recalc();
}; //3ef
```

```

int cnt(Node* n) { return n ? n->c : 0; }
void Node::recalc() { c = cnt(l) + cnt(r) + 1; }
```

```

template<class F> void each(Node* n, F f) { //5d5
    if (n) { each(n->l, f); f(n->val); each(n->r, f); }
}
```

```

pair<Node*, Node*> split(Node* n, int k) {
    if (!n) return {}; //ca5
    if (cnt(n->l) >= k) { // "n->val >= k" for lower_bound(k)
        auto pa = split(n->l, k);
        n->l = pa.second;
        n->recalc();
        return {pa.first, n}; //b54
    } else {
        auto pa = split(n->r, k - cnt(n->l) - 1); // and just "k"
        n->r = pa.first;
        n->recalc();
        return {n, pa.second}; //86d
    }
}
```

```

Node* merge(Node* l, Node* r) {
    if (!l) return r; //fbf
    if (!r) return l;
    if (l->y > r->y) {
        l->r = merge(l->r, r);
        l->recalc();
        return l; //780
    } else {
        r->l = merge(l, r->l);
        r->recalc();
        return r;
    } //96d
}
```

```

Node* ins(Node* t, Node* n, int pos) {
    auto pa = split(t, pos);
    return merge(merge(pa.first, n), pa.second); //99b
}
```

```

// Example application: move the range [l, r) to index k
void move(Node& t, int l, int r, int k) {
```

```

    Node *a, *b, *c; //99c
    tie(a,b) = split(t, l); tie(b,c) = split(b, r - 1);
    if (k <= l) t = merge(ins(a, b, k), c);
    else t = merge(a, ins(c, b, k - r));
}
```

### FenwickTree.h

**Description:** Computes partial sums  $a[0] + a[1] + \dots + a[\text{pos} - 1]$ , and updates single elements  $a[i]$ , taking the difference between the old and new value.

**Time:** Both operations are  $\mathcal{O}(\log N)$ .

```

struct FT { //711
    vector<ll> s;
    FT(int n) : s(n) {}
    void update(int pos, ll dif) { // a[pos] += dif
        for (; pos < sz(s); pos |= pos + 1) s[pos] += dif;
    } //cc4
    ll query(int pos) { // sum of values in [0, pos)
        ll res = 0;
        for (; pos > 0; pos &= pos - 1) res += s[pos-1];
        return res;
    } //477
    int lower_bound(ll sum) { // min pos st sum of [0, pos) >= sum
        // Returns n if no sum is >= sum, or -1 if empty sum is
        .
        if (sum <= 0) return -1;
        int pos = 0;
        for (int pw = 1 << 25; pw; pw >= 1) { //fc5
            if (pos + pw <= sz(s) && s[pos + pw-1] < sum)
                pos += pw, sum -= s[pos-1];
        }
        return pos;
    } //e03
};
```

### FenwickTree2d.h

**Description:** Computes sums  $a[i,j]$  for all  $i < I, j < J$ , and increases single elements  $a[i,j]$ . Requires that the elements to be updated are known in advance (call `fakeUpdate()` before `init()`).

**Time:**  $\mathcal{O}(\log^2 N)$ . (Use persistent segment trees for  $\mathcal{O}(\log N)$ .)

```

"FenwickTree.h"
struct FT2 { //4ce
    vector<vi> ys; vector<FT> ft;
    FT2(int limx) : ys(limx) {}
    void fakeUpdate(int x, int y) {
        for (; x < sz(ys); x |= x + 1) ys[x].pb(y);
    } //57f
    void init() {
        for (vi& v : ys) sort(all(v)), ft.emplace_back(sz(v));
    }
    int ind(int x, int y) {
        return (int)(lower_bound(all(ys[x]), y) - ys[x].begin())
        ); //358
    void update(int x, int y, ll dif) {
        for (; x < sz(ys); x |= x + 1)
            ft[x].update(ind(x, y), dif);
    }
    ll query(int x, int y) { //688
        ll sum = 0;
        for (; x; x &= x - 1)
            sum += ft[x-1].query(ind(x-1, y));
        return sum;
    } //e03
};
```

### RMQ.h

**Description:** Range Minimum Queries on an array. Returns  $\min(V[a], V[a + 1], \dots V[b - 1])$  in constant time.

**Usage:** `RMQ rmq(values);`  
`rmq.query(inclusive, exclusive);`  
**Time:**  $\mathcal{O}(|V| \log |V| + Q)$

510c32, 16 lines

```

template<class T> //722
struct RMQ {
    vector<vector<T>> jmp;
    RMQ(const vector<T>& V) : jmp(1, V) {
        for (int pw = 1, k = 1; pw * 2 <= sz(V); pw *= 2, ++k)
            jmp.emplace_back(sz(V) - pw * 2 + 1); //f6c
        rep(j, 0, sz(jmp[k]))
            jmp[k][j] = min(jmp[k - 1][j], jmp[k - 1][j + pw]);
    }
}
T query(int a, int b) { //a3d
    assert(a < b); // or return inf if a == b
    int dep = 31 - __builtin_clz(b - a);
    return min(jmp[dep][a], jmp[dep][b - (1 << dep)]);
}
}; //214
```

### MoQueries.h

**Description:** Answer interval or tree path queries by finding an approximate TSP through the queries, and moving from one query to the next by adding/removing points at the ends. If values are on tree edges, change step to add/remove the edge  $(a, c)$  and remove the initial add call (but keep in).

**Time:**  $\mathcal{O}(N\sqrt{Q})$

a12ef4, 49 lines

```

void add(int ind, int end) { ... } // add a[ind] (end = 0 or 1) //342
void del(int ind, int end) { ... } // remove a[ind]
int calc() { ... } // compute current answer
```

```

vi mo(vector<pii> Q) {
    int L = 0, R = 0, blk = 350; // ~N/sqrt(Q) //cb0
    vi s(sz(Q)), res = s;
    #define K(x) pii(x.first/blk, x.second ^ -(x.first/blk & 1))
    iota(all(s), 0);
    sort(all(s), [&](int s, int t){ return K(Q[s]) < K(Q[t]); });
    for (int qi : s) { //623
        pii q = Q[qi];
        while (L > q.first) add(--L, 0);
        while (R < q.second) add(R++, 1);
        while (L < q.first) del(L++, 0);
        while (R > q.second) del(--R, 1); //d22
        res[qi] = calc();
    }
    return res;
}
//842
vi moTree(vector<array<int, 2>> Q, vector<vi>& ed, int root = 0) {
    int N = sz(ed), pos[2] = {}, blk = 350; // ~N/sqrt(Q)
    vi s(sz(Q)), res = s, I(N), L(N), R(N), in(N), par(N);
    add(0, 0), in[0] = 1;
    auto dfs = [&](int x, int p, int dep, auto& f) -> void {
        //263
        par[x] = p;
        L[x] = N;
        if (dep) I[x] = N++;
        for (int y : ed[x]) if (y != p) f(y, x, !dep, f);
        if (!dep) I[x] = N++; //23e
        R[x] = N;
    };
    dfs(root, -1, 0, dfs);
    #define K(x) pii(I[x[0]] / blk, I[x[1]] ^ -(I[x[0]] / blk & 1))
    iota(all(s), 0); //064
```

```
sort(all(s), [&](int s, int t){ return K(Q[s]) < K(Q[t]);
});
for (int qi : s) rep(end,0,2) {
    int &a = pos[end], b = Q[qi][end], i = 0;
#define step(c) { if (in[c]) { del(a, end); in[a] = 0; } \
                else { add(c, end); in[c] = 1; } a = c; } \
                //440
    while (!(L[b] <= L[a] && R[a] <= R[b]))
        I[i++] = b, b = par[b];
    while (a != b) step(par[a]);
    while (i--) step(I[i]);
    if (end) res[qi] = calc();//695
}
return res;
}
```

### SegmentTree.h

**Description:** Zero-indexed max-tree. Bounds are inclusive to the left and exclusive to the right. Can be changed by modifying T, f and unit.  
**Time:**  $\mathcal{O}(\log N)$

0f4bdb, 19 lines

```
struct Tree { //026
    typedef int T;
    static constexpr T unit = INT_MIN;
    T f(T a, T b) { return max(a, b); } // (any associative fn)
    vector<T> s; int n;
    Tree(int n = 0, T def = unit) : s(2*n, def), n(n) {} //c86
    void update(int pos, T val) {
        for (s[pos += n] = val; pos /= 2;)
            s[pos] = f(s[pos * 2], s[pos * 2 + 1]);
    }
    T query(int b, int e) { // query [b, e) //e90
        T ra = unit, rb = unit;
        for (b += n, e += n; b < e; b /= 2, e /= 2) {
            if (b % 2) ra = f(ra, s[b++]);
            if (e % 2) rb = f(s[--e], rb);
        } //490
        return f(ra, rb);
    }
};
```

### LazySegmentTree.h

**Description:** Range set and add lazy segtree  
**Time:**  $\mathcal{O}(\log N)$ .

6f1390, 50 lines

```
struct segtree { //b1e
#define lc 2*v
#define rc 2*v+1
#define m (l + r) / 2
    int n;
    vector<ll> st; //68e
    vector<bool> toset;
    vector<ll> lazy;
    segtree(int _n) : n(_n), st(4 * n), toset(4 * n), lazy(4 * n) {}
    void push(int v, int l, int r) {
        if (toset[v]) { //dcb
            if (l != r) {
                toset[lc] = toset[rc] = true;
                lazy[lc] = lazy[v];
                lazy[rc] = lazy[v];
            } //c8c
            st[v] = lazy[v];
            toset[v] = false;
        }
    }
    void upd(int l, int r, ll val) { upd(l, 0, n-1, l, r, val); } //a32
    void upd(int v, int l, int r, int x, int y, ll val) {
        push(v, l, r);
```

```
        if (r < x || y < l) return;
        if (x <= l && r <= y) {
            toset[v] = true; //50b
            lazy[v] = val;
            push(v, l, r);
        } else {
            upd(lc, l, m, x, y, val);
            upd(rc, m+1, r, x, y, val); //aef
            st[v] = max(st[lc], st[rc]);
        }
    }
    ll qry(int x, int y) { return qry(l, 0, n-1, x, y); }
    ll qry(int v, int l, int r, int x, int y) { //901
        push(v, l, r);
        if (r < x || y < l) return 0;
        if (x <= l && r <= y) {
            return st[v];
        } else { //f9e
            auto v1 = qry(lc, l, m, x, y);
            auto v2 = qry(rc, m+1, r, x, y);
            return max(v1, v2);
        }
    } //f2e
#undef lc
#undef rc
#undef m
};
```

### UnionFindRollback.h

**Description:** Disjoint-set data structure with undo. If undo is not needed, skip st, time() and rollback().  
**Usage:** int t = uf.time(); ...; uf.rollback(t);  
**Time:**  $\mathcal{O}(\log(N))$

84e98b, 21 lines

```
struct RollbackUF { //f73
    vi e; vector<pii> st;
    RollbackUF(int n) : e(n, -1) {}
    int size(int x) { return -e[find(x)]; }
    int find(int x) { return e[x] < 0 ? x : find(e[x]); }
    int time() { return sz(st); } //cbd
    void rollback(int t) {
        for (int i = time(); i --> t;)
            e[st[i].first] = st[i].second;
        st.resize(t);
    } //e73
    bool join(int a, int b) {
        a = find(a), b = find(b);
        if (a == b) return false;
        if (e[a] > e[b]) swap(a, b);
        st.pb({a, e[a]}); //0d8
        st.pb({b, e[b]});
        e[a] += e[b]; e[b] = a;
        return true;
    }
} //214
```

### OfflineDeletion.h

**Description:** Delete from a data structure given insertions and rollbacks ds needs: void push(U u), void pop(), Q query()  
**Time:**  $\mathcal{O}(T(n)\log(n))$

b156e0, 32 lines

```
template<typename D, typename U, typename Q> //bf5
struct offline_deletion : public D {
    vector<Q> ans;
    vector<vector<U>> updates;
    int q;
    offline_deletion(int queries) : q(queries), ans(q) { //e0a
        int lg = 0; while((1 << lg) < q) lg++;
        updates.resize(1 << (lg + 1));
    }
    void update(int i, int l, int r, int L, int R, U u) {
```

```
        if (r < L || R < l) return; //731
        if (L <= l && r <= R) {
            updates[i].pb(u); return;
        }
        int m = (l + r) / 2;
        update(2 * i + 1, l, m, L, R, u); //087
        update(2 * i + 2, m + 1, r, L, R, u);
    }
    void insert(U u, int l, int r) { update(0, 0, q - 1, l, r, u); }
    void insert(U u, int l) { insert(u, l, q - 1); }
    void solve(int i, int l, int r) { //fa5
        for (auto &u : updates[i]) D::push(u);
        if (l == r) ans[l] = D::query();
        else {
            int m = (l + r) / 2;
            solve(2 * i + 1, l, m); //eb8
            solve(2 * i + 2, m + 1, r);
        }
        for (int j = 0; j < (int) updates[i].size(); j++) D::pop();
    }
    void solve() { solve(0, 0, q - 1); } //dbe
};
```

### NoamQueue.h

**Description:** Online queue-like deletion from a data structure supporting stack-like deletion  
**Time:**  $\mathcal{O}(T(n)\log(n))$

635417, 22 lines

```
struct update { //46a
    bool type;
    update() { type = 0; }
};
template<typename D, typename U>
struct noam : public D { //409
    vector<U> s;
    void push(const U &u) { D::push(u); s.pb(u); }
    void pop() {
        auto i = s.end(); int c = 0;
        do { //31c
            c += (--i)->type ? 1 : -1; D::pop();
        } while (c < 0 && i != begin(s));
        auto j = stable_partition(i, s.end(), [](auto &x) {
            return !x.type; });
        if (i == begin(s)) {
            reverse(i, j); //cdd
            for_each(i, j, [](auto &x) { x.type = 1; });
        }
        s.pop_back();
        while (i != s.end()) D::push(*i), i++;
    } //e03
};
```

## Numerical (4)

### 4.1 Polynomials and recurrences

#### Polynomial.h

c9b7b0, 17 lines

```
struct Poly { //1b7
    vector<double> a;
    double operator()(double x) const {
        double val = 0;
        for (int i = sz(a); i--;) (val += x) += a[i];
        return val; //06d
    }
    void diff() {
        rep(i,1,sz(a)) a[i-1] = i*a[i];
        a.pop_back();
    }
```

```

} //b82
void divroot(double x0) {
    double b = a.back(), c; a.back() = 0;
    for(int i=sz(a)-1; i--;) c = a[i], a[i] = a[i+1]*x0+b,
        b=c;
    a.pop_back();
} //e03
};
```

PolyRoots.h

**Description:** Finds the real roots to a polynomial.  
**Usage:** polyRoots({{2,-3,1}},-1e9,1e9) // solve x^2-3x+2 = 0  
**Time:**  $\mathcal{O}(n^2 \log(1/\epsilon))$

"Polynomial.h"	fbf066, 23 lines
----------------	------------------

```
vector<double> polyRoots(Poly p, double xmin, double xmax)
{ //840
    if (sz(p.a) == 2) { return {-p.a[0]/p.a[1]}; }
    vector<double> ret;
    Poly der = p;
    der.diff();
    auto dr = polyRoots(der, xmin, xmax); //ec1
    dr.pb(xmin-1);
    dr.pb(xmax+1);
    sort(all(dr));
    rep(i,0,sz(dr)-1) {
        double l = dr[i], h = dr[i+1]; //189
        bool sign = p(l) > 0;
        if (sign ^ (p(h) > 0)) {
            rep(it,0,60) { // while (h - l > 1e-8)
                double m = (l + h) / 2, f = p(m);
                if ((f <= 0) ^ sign) l = m; //810
                else h = m;
            }
            ret.pb((l + h) / 2);
        }
    } //808
    return ret;
}
```

PolyInterpolate.h

**Description:** Given  $n$  points  $(x[i], y[i])$ , computes an  $n$ -1-degree polynomial  $p$  that passes through them:  $p(x) = a[0]*x^0 + \dots + a[n-1]*x^{n-1}$ . For numerical precision, pick  $x[k] = c*\cos(k/(n-1)*\pi), k = 0 \dots n-1$ .  
**Time:**  $\mathcal{O}(n^2)$

	08bf48, 13 lines
--	------------------

```
typedef vector<double> vd; //159
vd interpolate(vd x, vd y, int n) {
    vd res(n), temp(n);
    rep(k,0,n-1) rep(i,k+1,n)
        y[i] = (y[i] - y[k]) / (x[i] - x[k]);
    double last = 0; temp[0] = 1; //746
    rep(k,0,n) rep(i,0,n) {
        res[i] += y[k] * temp[i];
        swap(last, temp[i]);
        temp[i] -= last * x[k];
    } //0e1
    return res;
}
```

BerlekampMassey.h

**Description:** Recovers any  $n$ -order linear recurrence relation from the first  $2n$  terms of the recurrence. Useful for guessing linear recurrences after brute-forcing the first terms. Should work on any field, but numerical stability for floats is not guaranteed. Output will have size  $\leq n$ .  
**Usage:** berlekampMassey({0, 1, 1, 3, 5, 11}) // {1, 2}  
**Time:**  $\mathcal{O}(N^2)$

"../number-theory/ModPow.h"	96548b, 20 lines
-----------------------------	------------------

```
vector<ll> berlekampMassey(vector<ll> s) { //b21
    int n = sz(s), L = 0, m = 0;
    vector<ll> C(n), B(n), T;
```

```
    C[0] = B[0] = 1;

    ll b = 1; //4c7
    rep(i,0,n) { ++m;
        ll d = s[i] % mod;
        rep(j,1,L+1) d = (d + C[j] * s[i - j]) % mod;
        if (!d) continue;
        T = C; ll coef = d * modpow(b, mod-2) % mod; //1b2
        rep(j,m,n) C[j] = (C[j] - coef * B[j - m]) % mod;
        if (2 * L > i) continue;
        L = i + 1 - L; B = T; b = d; m = 0;
    }
    //255
    C.resize(L + 1); C.erase(C.begin());
    for (ll& x : C) x = (mod - x) % mod;
    return C;
}
```

LinearRecurrence.h

**Description:** Generates the  $k$ 'th term of an  $n$ -order linear recurrence  $S[i] = \sum_j S[i-j-1]tr[j]$ , given  $S[0 \dots \geq n-1]$  and  $tr[0 \dots n-1]$ . Faster than matrix multiplication. Useful together with Berlekamp-Massey.  
**Usage:** linearRec({0, 1}, {1, 1}, k) //  $k$ 'th Fibonacci number  
**Time:**  $\mathcal{O}(n^2 \log k)$

	f4e444, 26 lines
--	------------------

```
typedef vector<ll> Poly; //bb1
ll linearRec(Poly S, Poly tr, ll k) {
    int n = sz(tr);

    auto combine = [&](Poly a, Poly b) {
        Poly res(n * 2 + 1); //251
        rep(i,0,n+1) rep(j,0,n+1)
            res[i + j] = (res[i + j] + a[i] * b[j]) % mod;
        for (int i = 2 * n; i > n; --i) rep(j,0,n)
            res[i - 1 - j] = (res[i - 1 - j] + res[i] * tr[j]) %
                mod;
        res.resize(n + 1); //12f
        return res;
    };

    Poly pol(n + 1), e(pol);
    pol[0] = e[1] = 1; //df7

    for (++k; k; k /= 2) {
        if (k % 2) pol = combine(pol, e);
        e = combine(e, e);
    } //c0e

    ll res = 0;
    rep(i,0,n) res = (res + pol[i + 1] * S[i]) % mod;
    return res;
} //cbb
```

4.2 Optimization

GoldenSectionSearch.h

**Description:** Finds the argument minimizing the function  $f$  in the interval  $[a,b]$  assuming  $f$  is unimodal on the interval, i.e. has only one local minimum and no local maximum. The maximum error in the result is  $eps$ . Works equally well for maximization with a small change in the code. See TernarySearch.h in the Various chapter for a discrete version.  
**Usage:** double func(double x) { return 4\*x+.3\*x\*x; }  
double xmin = gss(-1000,1000,func);  
**Time:**  $\mathcal{O}(\log((b-a)/\epsilon))$

	d7b114, 15 lines
--	------------------

```
template<class F> //5c6
double gss(double a, double b, F f) {
    double r = (sqrt(5)-1)/2, eps = 1e-7;
    double x1 = b - r*(b-a), x2 = a + r*(b-a);
    double f1 = f(x1), f2 = f(x2);
```

```
    while (b-a > eps) //905
        if (f1 < f2) { //change to > to find maximum
            b = x2; x2 = x1; f2 = f1;
            x1 = b - r*(b-a); f1 = f(x1);
        } else {
            a = x1; x1 = x2; f1 = f2; //00c
            x2 = a + r*(b-a); f2 = f(x2);
        }
    return a;
}
```

Integrate.h

**Description:** Simple integration of a function over an interval using Simpson's rule. The error should be proportional to  $h^4$ , although in practice you will want to verify that the result is stable to desired precision when epsilon changes.

	4756fc, 7 lines
--	-----------------

```
template<class F> //e93
double quad(double a, double b, F f, const int n = 1000) {
    double h = (b - a) / 2 / n, v = f(a) + f(b);
    rep(i,1,n*2)
        v += f(a + i*h) * (i&1 ? 4 : 2);
    return v * h / 3; //2d2
}
```

IntegrateAdaptive.h

**Description:** Fast integration using an adaptive Simpson's rule.  
**Usage:** double sphereVolume = quad(-1, 1, [](double x) { return quad(-1, 1, [&](double y) { return quad(-1, 1, [&](double z) { return x\*x + y\*y + z\*z < 1; }});});});});

	92dd79, 15 lines
--	------------------

```
typedef double d; //e70
#define S(a,b) (f(a) + 4*f((a+b) / 2) + f(b)) * (b-a) / 6

template <class F>
d rec(F& f, d a, d b, d eps, d S) {
    d c = (a + b) / 2; //b17
    d S1 = S(a, c), S2 = S(c, b), T = S1 + S2;
    if (abs(T - S) <= 15 * eps || b - a < 1e-10)
        return T + (T - S) / 15;
    return rec(f, a, c, eps / 2, S1) + rec(f, c, b, eps / 2, S2);
} //836
template<class F>
d quad(d a, d b, F f, d eps = 1e-8) {
    return rec(f, a, b, eps, S(a, b));
}
```

Simplex.h

**Description:** Solves a general linear maximization problem: maximize  $c^T x$  subject to  $Ax \leq b, x \geq 0$ . Returns -inf if there is no solution, inf if there are arbitrarily good solutions, or the maximum value of  $c^T x$  otherwise. The input vector is set to an optimal  $x$  (or in the unbounded case, an arbitrary solution fulfilling the constraints). Numerical stability is not guaranteed. For better performance, define variables such that  $x = 0$  is viable.  
**Usage:** vvd A = {{1,-1}, {-1,1}, {-1,-2}}; vvd b = {1,1,-4}, c = {-1,-1}, x; T val = LPSolver(A, b, c).solve(x);  
**Time:**  $\mathcal{O}(NM * \#pivots)$ , where a pivot may be e.g. an edge relaxation.  $\mathcal{O}(2^n)$  in the general case.

	aa8530, 68 lines
--	------------------

```
typedef double T; // long double, Rational, double + modP
>...//629
typedef vector<T> vd;
typedef vector<vd> vvd;

const T eps = 1e-8, inf = 1/.0;
#define MP make_pair //94e
```

```
#define ltj(X) if(s == -1 || MP(X[j],N[j]) < MP(X[s],N[s]))
    s=j

struct LPSolver {
    int m, n;
    vi N, B;//282
    vvd D;

    LPSolver(const vvd& A, const vd& b, const vd& c) :
        m(sz(b)), n(sz(c)), N(n+1), B(m), D(m+2, vd(n+2)) {
            rep(i,0,m) rep(j,0,n) D[i][j] = A[i][j];//108
            rep(i,0,m) { B[i] = n+i; D[i][n] = -1; D[i][n+1] = b[i];}
            rep(j,0,n) { N[j] = j; D[m][j] = -c[j]; }
            N[n] = -1; D[m+1][n] = 1;
        }
    //9c3
    void pivot(int r, int s) {
        T *a = D[r].data(), inv = 1 / a[s];
        rep(i,0,m+2) if (i != r && abs(D[i][s]) > eps) {
            T *b = D[i].data(), inv2 = b[s] * inv;
            rep(j,0,n+2) b[j] -= a[j] * inv2;//d0d
            b[s] = a[s] * inv2;
        }
        rep(j,0,n+2) if (j != s) D[r][j] *= inv;
        rep(i,0,m+2) if (i != r) D[i][s] *= -inv;
        D[r][s] = inv;//aa5
        swap(B[r], N[s]);
    }

    bool simplex(int phase) {
        int x = m + phase - 1;//c51
        for (;;) {
            int s = -1;
            rep(j,0,n+1) if (N[j] != -phase) ltj(D[x]);
            if (D[x][s] >= -eps) return true;
            int r = -1;//bc0
            rep(i,0,m) {
                if (D[i][s] <= eps) continue;
                if (r == -1 || MP(D[i][n+1] / D[i][s], B[i])
                    < MP(D[r][n+1] / D[r][s], B[r])) r = i;
            }
            //00c
            if (r == -1) return false;
            pivot(r, s);
        }
    }
    //d2f
    T solve(vd &x) {
        int r = 0;
        rep(i,1,m) if (D[i][n+1] < D[r][n+1]) r = i;
        if (D[r][n+1] < -eps) {
            pivot(r, n);//f81
            if (!simplex(2) || D[m+1][n+1] < -eps) return -inf;
            rep(i,0,m) if (B[i] == -1) {
                int s = 0;
                rep(j,1,n+1) ltj(D[i]);
                pivot(i, s);//866
            }
        }
        bool ok = simplex(1); x = vd(n);
        rep(i,0,m) if (B[i] < n) x[B[i]] = D[i][n+1];
        return ok ? D[m][n+1] : inf;//401
    }
};
```

HillClimbing.h  
Description: Poor man’s optimization for unimodal functions.  
8ccaf, 14 lines

```
typedef array<double, 2> P;//68a
```

```
template<class F> pair<double, P> hillClimb(P start, F f) {
    pair<double, P> cur(f(start), start);
    for (double jmp = 1e9; jmp > 1e-20; jmp /= 2) {
        rep(j,0,100) rep(dx,-1,2) rep(dy,-1,2) {//2dc
            P p = cur.second;
            p[0] += dx*jmp;
            p[1] += dy*jmp;
            cur = min(cur, make_pair(f(p), p));
        }//a63
    }
    return cur;
}
```

### 4.3 Matrices

Determinant.h  
Description: Calculates determinant of a matrix. Destroys the matrix.  
Time:  $O(N^3)$   
bd5cec, 15 lines

```
double det(vector<vector<double>>& a) {//309
    int n = sz(a); double res = 1;
    rep(i,0,n) {
        int b = i;
        rep(j,i+1,n) if (fabs(a[j][i]) > fabs(a[b][i])) b = j;
        if (i != b) swap(a[i], a[b]), res *= -1;//454
        res *= a[i][i];
        if (res == 0) return 0;
        rep(j,i+1,n) {
            double v = a[j][i] / a[i][i];
            if (v != 0) rep(k,i+1,n) a[j][k] -= v * a[i][k];//07b
        }
    }
    return res;
}
```

IntDeterminant.h  
Description: Calculates determinant using modular arithmetics. Modules can also be removed to get a pure-integer version.  
Time:  $O(N^3)$   
3313dc, 18 lines

```
const ll mod = 12345;//cab
ll det(vector<vector<ll>>& a) {
    int n = sz(a); ll ans = 1;
    rep(i,0,n) {
        rep(j,i+1,n) {
            while (a[j][i] != 0) { // gcd step//c65
                ll t = a[i][i] / a[j][i];
                if (t) rep(k,i,n)
                    a[i][k] = (a[i][k] - a[j][k] * t) % mod;
                swap(a[i], a[j]);
                ans *= -1;//bc6
            }
        }
        ans = ans * a[i][i] % mod;
        if (!ans) return 0;
    }//b19
    return (ans + mod) % mod;
}
```

SolveLinear.h  
Description: Solves  $A * x = b$ . If there are multiple solutions, an arbitrary one is returned. Returns rank, or -1 if no solutions. Data in  $A$  and  $b$  is lost.  
Time:  $O(n^2m)$   
44c9ab, 38 lines

```
typedef vector<double> vd;//2cf
const double eps = 1e-12;

int solveLinear(vector<vd>& A, vd& b, vd& x) {
    int n = sz(A), m = sz(x), rank = 0, br, bc;
    if (n) assert(sz(A[0]) == m);//940
    vi col(m); iota(all(col), 0);
```

```
rep(i,0,n) {
    double v, bv = 0;
    rep(r,i,n) rep(c,i,m)//ddb
        if ((v = fabs(A[r][c])) > bv)
            br = r, bc = c, bv = v;
    if (bv <= eps) {
        rep(j,i,n) if (fabs(b[j]) > eps) return -1;
        break;//de0
    }
    swap(A[i], A[br]);
    swap(b[i], b[br]);
    swap(col[i], col[bc]);
    rep(j,0,n) swap(A[j][i], A[j][bc]);//328
    bv = 1/A[i][i];
    rep(j,i+1,n) {
        double fac = A[j][i] * bv;
        b[j] -= fac * b[i];
        rep(k,i+1,m) A[j][k] -= fac*A[i][k];//af1
    }
    rank++;
}

x.assign(m, 0);//3c5
for (int i = rank; i--;) {
    b[i] /= A[i][i];
    x[col[i]] = b[i];
    rep(j,0,i) b[j] -= A[j][i] * b[i];
}//807
return rank; // (multiple solutions if rank < m)
}
```

SolveLinear2.h  
Description: To get all uniquely determined values of  $x$  back from SolveLinear, make the following changes:

```
"SolveLinear.h"
08e495, 7 lines

rep(j,0,n) if (j != i) // instead of rep(j,i+1,n)//22b
// ... then at the end:
x.assign(m, undefined);
rep(i,0,rank) {
    rep(j,rank,m) if (fabs(A[i][j]) > eps) goto fail;
    x[col[i]] = b[i] / A[i][i];//4e3
fail;; }
```

SolveLinearBinary.h  
Description: Solves  $Ax = b$  over  $\mathbb{F}_2$ . If there are multiple solutions, one is returned arbitrarily. Returns rank, or -1 if no solutions. Destroys  $A$  and  $b$ .  
Time:  $O(n^2m)$   
fa2d7a, 34 lines

```
typedef bitset<1000> bs;//d90

int solveLinear(vector<bs>& A, vi& b, bs& x, int m) {
    int n = sz(A), rank = 0, br;
    assert(m <= sz(x));
    vi col(m); iota(all(col), 0);//2c9
    rep(i,0,n) {
        for (br=i; br<n; ++br) if (A[br].any()) break;
        if (br == n) {
            rep(j,i,n) if (b[j]) return -1;
            break;//13e
        }
        int bc = (int)A[br]._Find_next(i-1);
        swap(A[i], A[br]);
        swap(b[i], b[br]);
        swap(col[i], col[bc]);//b88
        rep(j,0,n) if (A[j][i] != A[j][bc]) {
            A[j].flip(i); A[j].flip(bc);
        }
        rep(j,i+1,n) if (A[j][i]) {
            b[j] ^= b[i];//76c
        }
    }
}
```

```

    A[j] ^= A[i];
}
rank++;
}
//7a7
x = bs();
for (int i = rank; i--;) {
    if (!b[i]) continue;
    x[col[i]] = 1;
    rep(j,0,i) b[j] ^= A[j][i]; //df7
}
return rank; // (multiple solutions if rank < m)
}
```

### MatrixInverse.h

**Description:** Invert matrix  $A$ . Returns rank; result is stored in  $A$  unless singular (rank <  $n$ ). Can easily be extended to prime moduli; for prime powers, repeatedly set  $A^{-1} = A^{-1}(2I - AA^{-1}) \pmod{p^k}$  where  $A^{-1}$  starts as the inverse of  $A \bmod p$ , and  $k$  is doubled in each step.  
**Time:**  $\mathcal{O}(n^3)$

ebfff6, 35 lines

```
int matInv(vector<vector<double>>& A) { //9a9
    int n = sz(A); vi col(n);
    vector<vector<double>> tmp(n, vector<double>(n));
    rep(i,0,n) tmp[i][i] = 1, col[i] = i;

    rep(i,0,n) { //214
        int r = i, c = i;
        rep(j,i,n) rep(k,i,n)
            if (fabs(A[j][k]) > fabs(A[r][c]))
                r = j, c = k;
        if (fabs(A[r][c]) < 1e-12) return i; //e5b
        A[i].swap(A[r]); tmp[i].swap(tmp[r]);
        rep(j,0,n)
            swap(A[j][i], A[j][c]), swap(tmp[j][i], tmp[j][c]);
        swap(col[i], col[c]);
        double v = A[i][i]; //afc
        rep(j,i+1,n) {
            double f = A[j][i] / v;
            A[j][i] = 0;
            rep(k,i+1,n) A[j][k] -= f*A[i][k];
            rep(k,0,n) tmp[j][k] -= f*tmp[i][k]; //c80
        }
        rep(j,i+1,n) A[i][j] /= v;
        rep(j,0,n) tmp[i][j] /= v;
        A[i][i] = 1;
    } //bfb

    for (int i = n-1; i > 0; --i) rep(j,0,i) {
        double v = A[j][i];
        rep(k,0,n) tmp[j][k] -= v*tmp[i][k];
    } //e74

    rep(i,0,n) rep(j,0,n) A[col[i]][col[j]] = tmp[i][j];
    return n;
}
```

### MatrixInverse-mod.h

**Description:** Invert matrix  $A$  modulo a prime. Returns rank; result is stored in  $A$  unless singular (rank <  $n$ ). For prime powers, repeatedly set  $A^{-1} = A^{-1}(2I - AA^{-1}) \pmod{p^k}$  where  $A^{-1}$  starts as the inverse of  $A \bmod p$ , and  $k$  is doubled in each step.  
**Time:**  $\mathcal{O}(n^3)$

..../number-theory/ModPow.h 0b7b13, 37 lines

```
int matInv(vector<vector<ll>>& A) { //ebd
    int n = sz(A); vi col(n);
    vector<vector<ll>> tmp(n, vector<ll>(n));
    rep(i,0,n) tmp[i][i] = 1, col[i] = i;

    rep(i,0,n) { //79d
```

```

        int r = i, c = i;
        rep(j,i,n) rep(k,i,n) if (A[j][k]) {
            r = j; c = k; goto found;
        }
        return i; //4e3
    found:
    A[i].swap(A[r]); tmp[i].swap(tmp[r]);
    rep(j,0,n)
        swap(A[j][i], A[j][c]), swap(tmp[j][i], tmp[j][c]);
    swap(col[i], col[c]); //416
    ll v = modpow(A[i][i], mod - 2);
    rep(j,i+1,n) {
        ll f = A[j][i] * v % mod;
        A[j][i] = 0;
        rep(k,i+1,n) A[j][k] = (A[j][k] - f*A[i][k]) % mod; //9f7
        rep(k,0,n) tmp[j][k] = (tmp[j][k] - f*tmp[i][k]) % mod;

        rep(j,i+1,n) A[i][j] = A[i][j] * v % mod;
        rep(j,0,n) tmp[i][j] = tmp[i][j] * v % mod;
        A[i][i] = 1; //e3d
    }

    for (int i = n-1; i > 0; --i) rep(j,0,i) {
        ll v = A[j][i];
        rep(k,0,n) tmp[j][k] = (tmp[j][k] - v*tmp[i][k]) % mod; //4b2
    }

    rep(i,0,n) rep(j,0,n)
        A[col[i]][col[j]] = tmp[i][j] % mod + (tmp[i][j] < 0) * mod;
    return n; //400
}
```

### Tridiagonal.h

**Description:**  $x = \text{tridiagonal}(d, p, q, b)$  solves the equation system

$$\begin{pmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_{n-1} \end{pmatrix} = \begin{pmatrix} d_0 & p_0 & 0 & 0 & \cdots & 0 \\ q_0 & d_1 & p_1 & 0 & \cdots & 0 \\ 0 & q_1 & d_2 & p_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & q_{n-3} & d_{n-2} & p_{n-2} \\ 0 & 0 & \cdots & 0 & q_{n-2} & d_{n-1} \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_{n-1} \end{pmatrix}$$

This is useful for solving problems on the type

$$a_i = b_i a_{i-1} + c_i a_{i+1} + d_i, 1 \leq i \leq n,$$

where  $a_0, a_{n+1}, b_i, c_i$  and  $d_i$  are known.  $a$  can then be obtained from

$$\{a_i\} = \text{tridiagonal}(\{1, -1, -1, \dots, -1, 1\}, \{0, c_1, c_2, \dots, c_n\}, \{b_1, b_2, \dots, b_n, 0\}, \{a_0, d_1, d_2, \dots, d_n, a_{n+1}\}).$$

Fails if the solution is not unique.

If  $|d_i| > |p_i| + |q_{i-1}|$  for all  $i$ , or  $|d_i| > |p_{i-1}| + |q_i|$ , or the matrix is positive definite, the algorithm is numerically stable and neither `tr` nor the check for `diag[i] == 0` is needed.

**Time:**  $\mathcal{O}(N)$

8f9fa8, 26 lines

```
typedef double T; //399
vector<T> tridiagonal(vector<T> diag, const vector<T>&
    super,
    const vector<T>& sub, vector<T> b) {
    int n = sz(b); vi tr(n);
    rep(i,0,n-1) {
        if (abs(diag[i]) < 1e-9 * abs(super[i])) { // diag[i]
            = 0//464
            b[i+1] -= b[i] * diag[i+1] / super[i];
            if (i+2 < n) b[i+2] -= b[i] * sub[i+1] / super[i];
            diag[i+1] = sub[i]; tr[++i] = 1;
        } else {
```

```

            diag[i+1] -= super[i]*sub[i]/diag[i]; //d50
            b[i+1] -= b[i]*sub[i]/diag[i];
        }
    }
    for (int i = n; i--;) {
        if (tr[i]) { //054
            swap(b[i], b[i-1]);
            diag[i-1] = diag[i];
            b[i] /= super[i-1];
        } else {
            b[i] /= diag[i]; //20b
            if (i) b[i-1] -= b[i]*super[i-1];
        }
    }
    return b;
} //cbb
```

## 4.4 Fourier transforms

### FastFourierTransform.h

**Description:** `fft(a)` computes  $\hat{f}(k) = \sum_x a[x] \exp(2\pi i \cdot kx/N)$  for all  $k$ .  $N$  must be a power of 2. Useful for convolution: `conv(a, b) = c`, where  $c[x] = \sum a[i]b[x-i]$ . For convolution of complex numbers or more than two vectors: FFT, multiply pointwise, divide by  $n$ , reverse(`start+1, end`), FFT back. Rounding is safe if  $(\sum a_i^2 + \sum b_i^2) \log_2 N < 9 \cdot 10^{14}$  (in practice  $10^{16}$ ; higher for random inputs). Otherwise, use NTT/FFT-Mod.

**Time:**  $\mathcal{O}(N \log N)$  with  $N = |A| + |B|$  ( $\sim 1s$  for  $N = 2^{22}$ )  
00ced6, 35 lines

```
typedef complex<double> C; //1ec
typedef vector<double> vd;
void fft(vector<C>& a) {
    int n = sz(a), L = 31 - __builtin_clz(n);
    static vector<complex<long double>> R(2, 1);
    static vector<C> rt(2, 1); // (^ 10% faster if double) //c50
    for (static int k = 2; k < n; k *= 2) {
        R.resize(n); rt.resize(n);
        auto x = polar(1.0L, acos(-1.0L) / k);
        rep(i,k,2*k) rt[i] = R[i] = i&1 ? R[i/2] * x : R[i/2];
    } //292
    vi rev(n);
    rep(i,0,n) rev[i] = (rev[i / 2] | (i & 1) << L) / 2;
    rep(i,0,n) if (i < rev[i]) swap(a[i], a[rev[i]]);
    for (int k = 1; k < n; k *= 2)
        for (int i = 0; i < n; i += 2 * k) rep(j,0,k) { //577
            C z = rt[j+k] * a[i+j+k]; // (25% faster if hand-rolled)
            a[i + j + k] = a[i + j] - z;
            a[i + j] += z;
        }
    } //15f
    vd conv(const vd& a, const vd& b) {
        if (a.empty() || b.empty()) return {};
        vd res(sz(a) + sz(b) - 1);
        int L = 32 - __builtin_clz(sz(res)), n = 1 << L;
        vector<C> in(n), out(n); //d93
        copy(all(a), begin(in));
        rep(i,0,sz(b)) in[i].imag(b[i]);
        fft(in);
        for (C& x : in) x *= x;
        rep(i,0,n) out[i] = in[-i & (n - 1)] - conj(in[i]); //36e
        fft(out);
        rep(i,0,sz(res)) res[i] = imag(out[i]) / (4 * n);
        return res;
    }
}
```

### FastFourierTransformMod.h

**Description:** Higher precision FFT, can be used for convolutions modulo arbitrary integers as long as  $N \log_2 N \cdot \text{mod} < 8.6 \cdot 10^{14}$  (in practice  $10^{16}$  or higher). Inputs must be in  $[0, \text{mod})$ .



**Time:**  $\mathcal{O}(N \log N)$ , where  $N = |A| + |B|$  (twice as slow as NTT or FFT)

```
"FastFourierTransform.h"                                     b82773, 22 lines
typedef vector<ll> vl; //2c4
template<int M> vl convMod(const vl &a, const vl &b) {
    if (a.empty() || b.empty()) return {};
    vl res(sz(a) + sz(b) - 1);
    int B=32-__builtin_clz(sz(res)), n=1<<B, cut=int(sqrt(M))
        ;
    vector<C> L(n), R(n), outs(n), outl(n); //c4f
    rep(i,0,sz(a)) L[i] = C((int)a[i] / cut, (int)a[i] % cut)
        ;
    rep(i,0,sz(b)) R[i] = C((int)b[i] / cut, (int)b[i] % cut)
        ;
    fft(L), fft(R);
    rep(i,0,n) {
        int j = -i & (n - 1); //3eb
        outl[j] = (L[i] + conj(L[j])) * R[i] / (2.0 * n);
        outs[j] = (L[i] - conj(L[j])) * R[i] / (2.0 * n) / li;
    }
    fft(outl), fft(outs);
    rep(i,0,sz(res)) { //58f
        ll av = ll(real(outl[i])+.5), cv = ll(imag(outs[i])+.5)
            ;
        ll bv = ll(imag(outl[i])+.5) + ll(real(outs[i])+.5);
        res[i] = ((av % M * cut + bv) % M * cut + cv) % M;
    }
    return res; //510
}
```

NumberTheoreticTransform.h

**Description:** ntt(a) computes  $\hat{f}(k) = \sum_x a[x]g^{xk}$  for all  $k$ , where  $g = \text{root}^{(mod-1)/N}$ .  $N$  must be a power of 2. Useful for convolution modulo specific nice primes of the form  $2^ab + 1$ , where the convolution result has size at most  $2^a$ . For arbitrary modulo, see FFTMod.  $\text{conv}(a, b) = c$ , where  $c[x] = \sum a[i]b[x - i]$ . For manual convolution: NTT the inputs, multiply pointwise, divide by n, reverse(start+1, end), NTT back. Inputs must be in  $[0, \text{mod})$ . **Time:**  $\mathcal{O}(N \log N)$

```
"../number-theory/ModPow.h"                                ced03d, 35 lines
const ll mod = (119 << 23) + 1, root = 62; // =
998244353/0ca
// For p < 2^30 there is also e.g. 5 << 25, 7 << 26, 479 <<
21
// and 483 << 21 (same root). The last two are > 10^9.
typedef vector<ll> vl;
void ntt(vl &a) {
    int n = sz(a), L = 31 - __builtin_clz(n); //cc5
    static vl rt(2, 1);
    for (static int k = 2, s = 2; k < n; k *= 2, s++) {
        rt.resize(n);
        ll z[] = {1, modpow(root, mod >> s)};
        rep(i,k,2*k) rt[i] = rt[i / 2] * z[i & 1] % mod; //4a0
    }
    vi rev(n);
    rep(i,0,n) rev[i] = (rev[i / 2] | (i & 1) << L) / 2;
    rep(i,0,n) if (i < rev[i]) swap(a[i], a[rev[i]]);
    for (int k = 1; k < n; k *= 2) //ed7
        for (int i = 0; i < n; i += 2 * k) rep(j,0,k) {
            ll z = rt[j + k] * a[i + j + k] % mod, &ai = a[i + j]
                ;
            a[i + j + k] = ai - z + (z > ai ? mod : 0);
            ai += (ai + z >= mod ? z - mod : z);
        } //dfc
    }
    vl conv(const vl &a, const vl &b) {
        if (a.empty() || b.empty()) return {};
        int s = sz(a) + sz(b) - 1, B = 32 - __builtin_clz(s),
            n = 1 << B; //d58
        int inv = modpow(n, mod - 2);
```

```
vl L(a), R(b), out(n);
L.resize(n), R.resize(n);
ntt(L), ntt(R);
rep(i,0,n) //f18
    out[-i & (n - 1)] = (ll)L[i] * R[i] % mod * inv % mod;
ntt(out);
return {out.begin(), out.begin() + s};
}
```

FastSubsetTransform.h

**Description:** Transform to a basis with fast convolutions of the form  $c[z] = \sum_{z=x \oplus y} a[x] \cdot b[y]$ , where  $\oplus$  is one of AND, OR, XOR. The size of  $a$  must be a power of two. **Time:**  $\mathcal{O}(N \log N)$

```
464cf3, 16 lines
void FST(vi& a, bool inv) //ae8
    for (int n = sz(a), step = 1; step < n; step *= 2) {
        for (int i = 0; i < n; i += 2 * step) rep(j,i,i+step) {
            int &u = a[j], &v = a[j + step]; tie(u, v) =
                inv ? pii(v - u, u) : pii(v, u + v); // AND
                inv ? pii(v, u - v) : pii(u + v, u); // OR//0af
                pii(u + v, u - v); // XOR
        }
    }
    if (inv) for (int& x : a) x /= sz(a); // XOR only
} //dc4
vi conv(vi a, vi b) {
    FST(a, 0); FST(b, 0);
    rep(i,0,sz(a)) a[i] *= b[i];
    FST(a, 1); return a;
} //cbb
```

4.5 Polynomial

PolyBase.h

**Description:** A FFT based Polynomial class.

```
"../number-theory/ModularArithmetic.h", "FastFourierTransform.h",
"FastFourierTransformMod.h", "NumberTheoreticTransform.h"    dd1be7, 35 lines
typedef Mod num; //810
typedef vector<num> poly;
poly &operator+=(poly &a, const poly &b) {
    a.resize(max(sz(a), sz(b)));
    rep(i, 0, sz(b)) a[i] = a[i] + b[i];
    return a; //8a9
}
poly &operator--=(poly &a, const poly &b) {
    a.resize(max(sz(a), sz(b)));
    rep(i, 0, sz(b)) a[i] = a[i] - b[i];
    return a; //e10
}
poly &operator*=(poly &a, const poly &b) {
    if (sz(a) + sz(b) < 100) {
        poly res(sz(a) + sz(b) - 1); //025
        rep(i,0,sz(a)) rep(j,0,sz(b))
            res[i + j] = (res[i + j] + a[i] * b[j]);
        return (a = res);
    }
    // auto res = convMod<mod>(vl(all(a)), vl(all(b)));//0cb
    auto res = conv(vl(all(a)), vl(all(b)));
    return (a = poly(all(res)));
}
poly operator*(poly a, const num b) {
    poly c = a; //41c
    for(auto& i : c) i = i * b;
    return c;
}
#define OP(o, oe) \
    poly operator o(poly a, poly b) { //f19
        poly c = a; \
        return c o##= b; \
```

```
}
OP(*, *) OP(+, +=) OP(-, -=);
```

PolyEvaluate.h

**Description:** Multi-point evaluation. Evaluates a given polynomial  $A$  at  $A(x_0), \dots A(x_n)$ . **Time:**  $\mathcal{O}(n \log^2 n)$

```
"PolyBase.h", "PolyMod.h"                                dc2cdf, 14 lines
vector<num> eval(const poly &a, const vector<num> &x) {//9
    fa
    int n = sz(x);
    if (!n) return {};
    vector<poly> up(2 * n);
    rep(i, 0, n) up[i + n] = poly({num(0) - x[i], 1});
    for (int i = n - 1; i > 0; i--) //923
        up[i] = up[2 * i] * up[2 * i + 1];
    vector<poly> down(2 * n);
    down[1] = a % up[1];
    rep(i, 2, 2 * n) down[i] = down[i / 2] % up[i];
    vector<num> y(n); //835
    rep(i, 0, n) y[i] = down[i + n][0];
    return y;
}
```

PolyIntegDeriv.h

**Description:** Calculate  $\frac{da}{dx}$  and  $\int a dx$ .

```
Time:  $\mathcal{O}(|a|)$ 
"PolyBase.h"                                               803fd5, 14 lines
poly deriv(poly a) {//aba
    if (a.empty()) return {};
    poly b(sz(a) - 1);
    rep(i, 1, sz(a)) b[i - 1] = a[i] * num(i);
    return b;
} //6f9
poly integr(poly a) {
    if (a.empty()) return {0};
    poly b(sz(a) + 1);
    b[1] = num(1);
    rep(i, 2, sz(b)) b[i] = b[mod%i]*Mod(-mod/i+mod); //176
    rep(i, 1, sz(b)) b[i] = a[i-1] * b[i];
    return b;
}
```

PolyInterpolate2.h

**Description:** Given  $n$  points  $(x[i], y[i])$ , computes an  $n$ -1-degree polynomial  $p$  that passes through them:  $p(x) = a[0] \cdot x^0 + \dots + a[n-1] \cdot x^{n-1}$ . **Time:**  $\mathcal{O}(n \log^2 n)$

```
"PolyBase.h", "PolyIntegDeriv.h", "PolyEvaluate.h"        b911f5, 11 lines
poly interp(vector<num> x, vector<num> y) {//74d
    int n=sz(x);
    vector<poly> up(n*2);
    rep(i,0,n) up[i+n] = poly({num(0)-x[i], num(1)});
    for(int i=n-1; i>0;i--) up[i] = up[2*i]*up[2*i+1];
    vector<num> a = eval(deriv(up[1]), x); //6dd
    vector<poly> down(2*n);
    rep(i,0,n) down[i+n] = poly({y[i]*num(1)/a[i]});
    for(int i=n-1;i>0;i--) down[i] = down[i*2] * up[i*2+1] +
        down[i*2+1] * up[i*2];
    return down[1];
} //cbb
```

PolyInverse.h

**Description:** Calculate the first  $|a|$  coefficients of  $a^{-1}$ .

```
Time:  $\mathcal{O}(n \log n)$ .
"PolyBase.h"                                               703c16, 7 lines
poly modK(poly a, int k) { return {a.begin(), a.begin() +
    min(k, sz(a))}; } //140
poly inverse(poly A) {
    poly B = poly({num(1) / A[0]});
```

```
while (sz(B) < sz(A))
    B = modK(B * (poly({num(2)}) - modK(A, 2*sz(B)) * B), 2
        * sz(B));
return modK(B, sz(A));//556
}
```

PolyLogExp.h

**Description:** Calculate the first  $|a|$  coefficients of  $a^{-1}$  of log  $a$  and exp  $a$ .  
**Time:**  $\mathcal{O}(n \log n)$ .

"PolyBase.h", "PolyInverse.h", "PolyIntegDeriv.h"	83ea75, 14 lines
poly log(poly a) <i>{<i>//9c1</i></i>	
return modK(integr(deriv(a) * inverse(a)), sz(a));	
}	
poly exp(poly a) {	
poly b(1, num(1));	
if (a.empty()) <i>//8ff</i>	
return b;	
while (sz(b) < sz(a)) {	
b.resize(sz(b) * 2);	
b *= (poly({num(1)}) + modK(a, sz(b)) - log(b));	
b.resize(sz(b) / 2 + 1); <i>//1f2</i>	
}	
return modK(b, sz(a));	
}	

PolyMod.h

**Description:** Calculate the remainder and quotient of the Euclidean division  $\frac{a}{b}$ .  
**Time:**  $\mathcal{O}(n \log n)$ .

"PolyBase.h", "PolyInverse.h"	264551, 20 lines
poly &operator=(poly &a, poly b) <i>{<i>//b9b</i></i>	
if (sz(a) < sz(b))	
return a = {};	
int s = sz(a) - sz(b) + 1;	
reverse(all(a)), reverse(all(b));	
a.resize(s), b.resize(s); <i>//e15</i>	
a = a * inverse(b);	
a.resize(s), reverse(all(a));	
return a;	
}	
OP(/, /=) <i>//9fe</i>	
poly &operator%=(poly &a, poly &b) {	
if (sz(a) < sz(b))	
return a;	
poly c = (a / b) * b;	
a.resize(sz(b) - 1); <i>//f62</i>	
rep(i, 0, sz(a)) a[i] = a[i] - c[i];	
return a;	
}	
OP(%, %=)	

PolyPow.h

**Description:** Calculate the first  $|a|$  coeffients of  $a^m$ .  
**Time:**  $\mathcal{O}(n \log n)$ .

"PolyBase.h", "PolyLogExp.h"	f0005c, 13 lines
poly pow(poly a, ll m) <i>{<i>//760</i></i>	
int p = 0, n = sz(a);	
while (p < sz(a) && a[p].v == 0)	
++p;	
if (ll(m)*p >= sz(a)) return poly(sz(a));	
num j = a[p]; <i>//a78</i>	
a = {a.begin() + p, a.end()};	
a = a * (num(1) / j);	
a.resize(n);	
auto res = exp(log(a) * num(m)) * (j ^ m);	
res.insert(res.begin(), p*m, 0); <i>//6a6</i>	
return {res.begin(), res.begin()+n};	
}	

PolyRoots.h

**Description:** Finds the real roots to a polynomial.  
**Usage:** polyRoots({{2,-3,1}},-1e9,1e9) *// solve*  $x^2-3x+2 = 0$   
**Time:**  $\mathcal{O}(n^2 \log(1/\epsilon))$

"Polynomial.h"	fbf066, 23 lines
vector<double> polyRoots(Poly p, double xmin, double xmax) <i>{<i>//840</i></i>	
if (sz(p.a) == 2) { return {-p.a[0]/p.a[1]}; }	
vector<double> ret;	
Poly der = p;	
der.diff();	
auto dr = polyRoots(der, xmin, xmax); <i>//ec1</i>	
dr.pb(xmin-1);	
dr.pb(xmax+1);	
sort(all(dr));	
rep(i,0,sz(dr)-1) {	
double l = dr[i], h = dr[i+1]; <i>//189</i>	
bool sign = p(l) > 0;	
if (sign ^ (p(h) > 0)) {	
rep(it,0,60) { <i>// while (h - l &gt; 1e-8)</i>	
double m = (l + h) / 2, f = p(m);	
if ((f <= 0) ^ sign) l = m; <i>//810</i>	
else h = m;	
}	
ret.pb((l + h) / 2);	
}	
} <i>//808</i>	
return ret;	
}	

BostanMori.h

**Description:** Calculate  $[x^k] \frac{p(x)}{q(x)}$ .  
**Time:**  $\mathcal{O}(n \log n \log k)$ .

"PolyBase.h"	5f9ba1, 10 lines
num BostanMori(poly p, poly q, ll k) <i>{<i>//9b3</i></i>	
if (k == 0) return p[0] / q[0];	
poly nq = q;	
for (int i = 1; i < sz(nq); i += 2) nq[i] = num(0) - nq[i]	
l;	
poly u = p * nq, v = q * nq;	
poly nu, nv; <i>//79a</i>	
for (int i = k % 2; i < sz(u); i += 2) nu.pb(u[i]);	
for (int i = 0; i < sz(v); i += 2) nv.pb(v[i]);	
return BostanMori(nu, nv, k / 2);	
};	

Number theory (5)

5.1 Modular arithmetic

ModInverse.h

**Description:** Pre-computation of modular inverses. Assumes LIM ≤ mod and that mod is a prime.

	b4a981, 2 lines
ll* inv = new ll[LIM] - 1; inv[1] = 1; <i>//b4a</i>	
rep(i,2,LIM) inv[i] = mod - (mod / i) * inv[mod % i] % mod;	
ModPow.h	
	7b7908, 8 lines
const int mod = 1000000007; <i>// faster if const//dce</i>	
ll modpow(ll b, ll e) {	
ll ans = 1;	
for (; e; b = b * b % mod, e /= 2)	
if (e & 1) ans = ans * b % mod; <i>//7e5</i>	
return ans;	
}	

ModLog.h

**Description:** Returns the smallest  $x > 0$  s.t.  $a^x = b \pmod m$ , or  $-1$  if no such  $x$  exists. modLog(a,l,m) can be used to calculate the order of  $a$ .

<b>Time:</b> $\mathcal{O}(\sqrt{m})$	c040b8, 11 lines
ll modLog(ll a, ll b, ll m) <i>{<i>//260</i></i>	
ll n = (ll) sqrt(m) + 1, e = 1, f = 1, j = 1;	
unordered_map<ll, ll> A;	
while (j <= n && (e = f = e * a % m) != b % m)	
A[e * b % m] = j++;	
if (e == b % m) return j; <i>//d16</i>	
if (__gcd(m, e) == __gcd(m, b))	
rep(i,2,n+2) if (A.count(e = e * f % m))	
return n * i - A[e];	
return -1;	
} <i>//cbb</i>	

ModSum.h

**Description:** Sums of mod'ed arithmetic progressions.  
modsum(to, c, k, m) =  $\sum_{i=0}^{to-1} (ki + c) \% m$ . divsum is similar but for floored division.  
**Time:**  $\log(m)$ , with a large constant.

	5c5bc5, 16 lines
typedef unsigned long long ull; <i>//df3</i>	
ull sumsq(ull to) { return to / 2 * ((to-1)   1); }	
ull divsum(ull to, ull c, ull k, ull m) {	
ull res = k / m * sumsq(to) + c / m * to;	
k %= m; c %= m; <i>//e1a</i>	
if (!k) return res;	
ull to2 = (to * k + c) / m;	
return res + (to - 1) * to2 - divsum(to2, m-1 - c, m, k);	
}	
<i>//1ae</i>	
ll modsum(ull to, ll c, ll k, ll m) {	
c = ((c % m) + m) % m;	
k = ((k % m) + m) % m;	
return to * c + k * sumsq(to) - m * divsum(to, c, k, m);	
} <i>//cbb</i>	

ModMulLL.h

**Description:** Calculate  $a \cdot b \pmod c$  (or  $a^b \pmod c$ ) for  $0 \leq a, b \leq c \leq 7.2 \cdot 10^8$ .  
**Time:**  $\mathcal{O}(1)$  for modmul,  $\mathcal{O}(\log b)$  for modpow

	bbbd8f, 11 lines
typedef unsigned long long ull; <i>//a9c</i>	
ull modmul(ull a, ull b, ull M) {	
ll ret = a * b - M * ull(1.L / M * a * b);	
return ret + M * (ret < 0) - M * (ret >= (ll)M);	
}	
ull modpow(ull b, ull e, ull mod) <i>{<i>//51d</i></i>	
ull ans = 1;	
for (; e; b = modmul(b, b, mod), e /= 2)	
if (e & 1) ans = modmul(ans, b, mod);	
return ans;	
} <i>//cbb</i>	

ModSqrt.h

**Description:** Tonelli-Shanks algorithm for modular square roots. Finds  $x$  s.t.  $x^2 = a \pmod p$  ( $-x$  gives the other solution).  
**Time:**  $\mathcal{O}(\log^2 p)$  worst case,  $\mathcal{O}(\log p)$  for most  $p$

"ModPow.h"	19a793, 24 lines
ll sqrt(ll a, ll p) <i>{<i>//473</i></i>	
a %= p; if (a < 0) a += p;	
if (a == 0) return 0;	
assert(modpow(a, (p-1)/2, p) == 1); <i>// else no solution</i>	
if (p % 4 == 3) return modpow(a, (p+1)/4, p);	
<i>// a^(n+3)/8 or 2^(n+3)/8 * 2^(n-1)/4 works if p % 8 == 5//a48</i>	
ll s = p - 1, n = 2;	

```
int r = 0, m;
while (s % 2 == 0)
    ++r, s /= 2;
while (modpow(n, (p - 1) / 2, p) != p - 1) ++n;//c4b
ll x = modpow(a, (s + 1) / 2, p);
ll b = modpow(a, s, p), g = modpow(n, s, p);
for (; r = m) {
    ll t = b;
    for (m = 0; m < r && t != 1; ++m)//faf
        t = t * t % p;
    if (m == 0) return x;
    ll gs = modpow(g, 1LL << (r - m - 1), p);
    g = gs * gs % p;
    x = x * gs % p;//a28
    b = b * g % p;
}
}
```

## 5.2 Primality

### FastEratosthenes.h

**Description:** Prime sieve for generating all primes smaller than LIM.  
**Time:** LIM=1e9  $\approx$  1.5s

```
const int LIM = 1e6;//058
bitset<LIM> isPrime;
vi eratosthenes() {
    const int S = (int)round(sqrt(LIM)), R = LIM / 2;
    vi pr = {2}, sieve(S+1); pr.reserve(int(LIM/log(LIM)*1.1)
    );
    vector<pii> cp;//86b
    for (int i = 3; i <= S; i += 2) if (!sieve[i]) {
        cp.pb({i, i * i / 2});
        for (int j = i * i; j <= S; j += 2 * i) sieve[j] = 1;
    }
    for (int L = 1; L <= R; L += S) {//62d
        array<bool, S> block{};
        for (auto &[p, idx] : cp)
            for (int i=idx; i < S+L; idx = (i+=p)) block[i-L] = 1;
        rep(i,0,min(S, R - L))
            if (!block[i]) pr.pb((L + i) * 2 + 1);//0b9
    }
    for (int i : pr) isPrime[i] = 1;
    return pr;
}
```

### MillerRabin.h

**Description:** Deterministic Miller-Rabin primality test. Guaranteed to work for numbers up to  $7 \cdot 10^{18}$ ; for larger numbers, use Python and extend A randomly.

**Time:** 7 times the complexity of  $a^b \bmod c$ .

```
"ModMuLL.h"
60dcd1, 12 lines
bool isPrime(u1ll n) {//60a
    if (n < 2 || n % 6 % 4 != 1) return (n | 1) == 3;
    ull A[] = {2, 325, 9375, 28178, 450775, 9780504,
        1795265022},
        s = __builtin_ctzll(n-1), d = n >> s;
    for (ull a : A) { // ^ count trailing zeroes
        ull p = modpow(a%n, d, n), i = s;//81c
        while (p != 1 && p != n - 1 && a % n && i--)
            p = modmul(p, p, n);
        if (p != n-1 && i != s) return 0;
    }
    return 1;//84a
}
```

### Factor.h

**Description:** Pollard-rho randomized factorization algorithm. Returns prime factors of a number, in arbitrary order (e.g. 2299 -> {11, 19, 11}).

**Time:**  $\mathcal{O}\left(n^{1/4}\right)$ , less for numbers with small factors.

```
"ModMuLL.h", "MillerRabin.h"
21a173, 18 lines
ull pollard(ull n) {//c81
    ull x = 0, y = 0, t = 30, prd = 2, i = 1, q;
    auto f = [&](ull a) { return modmul(a, a, n) + i; };
    while (t++ % 40 || __gcd(prd, n) == 1) {
        if (x == y) x = ++i, y = f(x);
        if ((q = modmul(prd, max(x,y) - min(x,y), n))) prd = q;
        //049
        x = f(x), y = f(f(y));
    }
    return __gcd(prd, n);
}
vector<ull> factor(ull n) {//c19
    if (n == 1) return {};
    if (isPrime(n)) return {n};
    ull x = pollard(n);
    auto l = factor(x), r = factor(n / x);
    l.insert(l.end(), all(r));//363
    return l;
}
}
```

## 5.3 Divisibility

### euclid.h

**Description:** Finds two integers  $x$  and  $y$ , such that  $ax+by = \gcd(a, b)$ . If you just need gcd, use the built in `__gcd` instead. If  $a$  and  $b$  are coprime, then  $x$  is the inverse of  $a \pmod b$ .

```
ll euclid(ll a, ll b, ll &x, ll &y) {//33b
    if (!b) return x = 1, y = 0, a;
    ll d = euclid(b, a % b, y, x);
    return y -= a/b * x, d;
}
```

### CRT.h

**Description:** Chinese Remainder Theorem.  
crt( $a, m, b, n$ ) computes  $x$  such that  $x \equiv a \pmod m$ ,  $x \equiv b \pmod n$ . If  $|a| < m$  and  $|b| < n$ ,  $x$  will obey  $0 \leq x < \text{lcm}(m, n)$ . Assumes  $mn < 2^{62}$ .

**Time:**  $\log(n)$

```
"euclid.h"
04d93a, 7 lines
ll crt(ll a, ll m, ll b, ll n) {//eae
    if (n > m) swap(a, b), swap(m, n);
    ll x, y, g = euclid(m, n, x, y);
    assert((a - b) % g == 0); // else no solution
    x = (b - a) % n * x % n / g * m + a;
    return x < 0 ? x + mn/g : x;//6ac
}
```

### 5.3.1 Bézout’s identity

For  $a \neq, b \neq 0$ , then  $d = \gcd(a, b)$  is the smallest positive integer for which there are integer solutions to

$$ax + by = d$$

If  $(x, y)$  is one solution, then all solutions are given by

$$\left(x + \frac{kb}{\gcd(a,b)}, y - \frac{ka}{\gcd(a,b)}\right), \quad k \in \mathbb{Z}$$

### phiFunction.h

**Description:** Euler’s  $\phi$  function is defined as  $\phi(n) := \#$  of positive integers  $\leq n$  that are coprime with  $n$ .  $\phi(1) = 1$ ,  $p$  prime  $\Rightarrow \phi(p^k) = (p - 1)p^{k-1}$ ,  $m, n$  coprime  $\Rightarrow \phi(mn) = \phi(m)\phi(n)$ . If  $n = p_1^{k_1}p_2^{k_2} \dots p_r^{k_r}$  then  $\phi(n) = (p_1 - 1)p_1^{k_1-1} \dots (p_r - 1)p_r^{k_r-1}$ .  $\phi(n) = n \cdot \prod_{p|n} (1 - 1/p)$ .  $\sum_{d|n} \phi(d) = n$ ,  $\sum_{1 \leq k \leq n, \gcd(k, n) = 1} k = n\phi(n)/2, n > 1$

**Euler’s thm:**  $a, n$  coprime  $\Rightarrow a^{\phi(n)} \equiv 1 \pmod n$ .

**Fermat’s little thm:**  $p$  prime  $\Rightarrow a^{p-1} \equiv 1 \pmod p \ \forall a$ .

```
const int LIM = 5000000;//70b
int phi[LIM];
```

```
void calculatePhi() {
    rep(i,0,LIM) phi[i] = i&1 ? i : i/2;
    for (int i = 3; i < LIM; i += 2) if(phi[i] == i)//103
        for (int j = i; j < LIM; j += i) phi[j] -= phi[j] / i;
}
```

## 5.4 Fractions

### ContinuedFractions.h

**Description:** Given  $N$  and a real number  $x \geq 0$ , finds the closest rational approximation  $p/q$  with  $p, q \leq N$ . It will obey  $|p/q - x| \leq 1/qN$ . For consecutive convergents,  $p_{k+1}q_k - q_{k+1}p_k = (-1)^k$ .  $(p_k/q_k$  alternates between  $> x$  and  $< x$ ). If  $x$  is rational,  $y$  eventually becomes  $\infty$ ; if  $x$  is the root of a degree 2 polynomial the  $a$ ’s eventually become cyclic.  
**Time:**  $\mathcal{O}(\log N)$

```
typedef double d; // for N ~ 1e7; long double for N ~ 1e9
//32b
pair<ll, ll> approximate(d x, ll N) {
    ll LP = 0, LQ = 1, P = 1, Q = 0, inf = LLONG_MAX; d y = x
    ;
    for (;) {
        ll lim = min(P ? (N-LP) / P : inf, Q ? (N-LQ) / Q : inf
        ),
            a = (ll)floor(y), b = min(a, lim),//5ad
            NP = b*P + LP, NQ = b*Q + LQ;
        if (a > b) {
            // If b > a/2, we have a semi-convergent that gives
            us a
            // better approximation; if b = a/2, we *may* have
            one.
            // Return {P, Q} here for a more canonical
            approximation.//fcb
            return (abs(x - (d)NP / (d)NQ) < abs(x - (d)P / (d)Q)
            ) ?
                make_pair(NP, NQ) : make_pair(P, Q);
        }
        if (abs(y = 1/(y - (d)a)) > (d)N*3) {
            return {NP, NQ};//5c7
        }
        LP = P; P = NP;
        LQ = Q; Q = NQ;
    }
}
//cbb
```

### FracBinarySearch.h

**Description:** Given  $f$  and  $N$ , finds the smallest fraction  $p/q \in [0, 1]$  such that  $f(p/q)$  is true, and  $p, q \leq N$ . You may want to throw an exception from  $f$  if it finds an exact solution, in which case  $N$  can be removed.

**Usage:** fracBS({}(Frac f) { return f.p>=3\*f.q; }, 10); // {1,3}

**Time:**  $\mathcal{O}(\log(N))$

```
27ab3e, 25 lines
struct Frac { ll p, q; };//386

template<class F>
Frac fracBS(F f, ll N) {
    bool dir = 1, A = 1, B = 1;
    Frac lo{0, 1}, hi{1, 1}; // Set hi to 1/0 to search (0, N
    ]//262
    if (f(lo)) return lo;
    assert(f(hi));
    while (A || B) {
        ll adv = 0, step = 1; // move hi if dir, else lo
        for (int si = 0; step; (step *= 2) >= si) {//7e2
```

```
    adv += step;
    Frac mid{lo.p * adv + hi.p, lo.q * adv + hi.q};
    if (abs(mid.p) > N || mid.q > N || dir == !f(mid)) {
        adv -= step; si = 2;
    } //bfo
}
hi.p += lo.p * adv;
hi.q += lo.q * adv;
dir = !dir;
swap(lo, hi); //f58
A = B; B = !!adv;
}
return dir ? hi : lo;
}
```

5.5 Pythagorean Triples

The Pythagorean triples are uniquely generated by

$a = k \cdot (m^2 - n^2), \quad b = k \cdot (2mn), \quad c = k \cdot (m^2 + n^2),$

with  $m > n > 0, k > 0, m \perp n$ , and either  $m$  or  $n$  even.

5.6 Primes

$p = 962592769$  is such that  $2^{21} \mid p - 1$ , which may be useful. For hashing use 970592641 (31-bit number), 31443539979727 (45-bit), 3006703054056749 (52-bit). There are 78498 primes less than 1 000 000.

Primitive roots exist modulo any prime power  $p^a$ , except for  $p = 2, a > 2$ , and there are  $\phi(\phi(p^a))$  many. For  $p = 2, a > 2$ , the group  $\mathbb{Z}_{2^a}^\times$  is instead isomorphic to  $\mathbb{Z}_2 \times \mathbb{Z}_{2^{a-2}}$ .

5.7 Mobius Function

$\sum_{d \mid n} \mu(d) = [n = 1]$  (very useful)

$g(n) = \sum_{n \mid d} f(d) \Leftrightarrow f(n) = \sum_{n \mid d} \mu(d/n) g(d)$

$g(n) = \sum_{1 \leq m \leq n} f(\lfloor \frac{n}{m} \rfloor) \Leftrightarrow f(n) = \sum_{1 \leq m \leq n} \mu(m) g(\lfloor \frac{n}{m} \rfloor)$

Combinatorial (6)

6.1 Permutations

6.1.1 Factorial

```
IntPerm.h
Description: Permutation -> integer conversion. (Not order preserv-
ing.) Integer -> permutation can use a lookup table.
Time: O(n)
044568, 6 lines

int permToInt(vi& v) { //cf9
    int use = 0, i = 0, r = 0;
    for(int x:v) r = r * ++i + __builtin_popcount(use & -(1<<
        x)),
        use |= 1 << x; // (note: minus, not
        ~!)
    return r;
} //cbb
```

6.1.2 Derangements

Permutations of a set such that none of the elements appear in their original position.

$D(n) = (n-1)(D(n-1)+D(n-2)) = nD(n-1)+(-1)^n = \left\lfloor \frac{n!}{e} \right\rfloor$

6.1.3 Burnside’s lemma

Given a group  $G$  of symmetries and a set  $X$ , the number of elements of  $X$  up to symmetry equals

$$\frac{1}{|G|} \sum_{g \in G} |X^g|,$$

where  $X^g$  are the elements fixed by  $g$  ( $g.x = x$ ).

If  $f(n)$  counts “configurations” (of some sort) of length  $n$ , we can ignore rotational symmetry using  $G = \mathbb{Z}_n$  to get

$$g(n) = \frac{1}{n} \sum_{k=0}^{n-1} f(\gcd(n, k)) = \frac{1}{n} \sum_{k \mid n} f(k) \phi(n/k).$$

6.2 Partitions and subsets

6.2.1 Partition function

Number of ways of writing  $n$  as a sum of positive integers, disregarding the order of the summands.

$$p(0) = 1, \quad p(n) = \sum_{k \in \mathbb{Z} \setminus \{0\}} (-1)^{k+1} p(n - k(3k - 1)/2)$$

$$p(n) \sim 0.145/n \cdot \exp(2.56\sqrt{n})$$

$n$	5	10	15	20	25	30	35	40	50	100
$p(n)$	7	42	176	627	1958	5604	14883	37338	$\sim 2e5$	$\sim 2e8$

6.2.2 Lucas’ Theorem

Let  $n, m$  be non-negative integers and  $p$  a prime. Write  $n = n_k p^k + \dots + n_1 p + n_0$  and  $m = m_k p^k + \dots + m_1 p + m_0$ . Then  $\binom{n}{m} \equiv \prod_{i=0}^k \binom{n_i}{m_i} \pmod{p}$ .

6.3 General purpose numbers

6.3.1 Bernoulli numbers

EGF of Bernoulli numbers is  $B(t) = \frac{t}{e^t - 1}$  (FFT-able).  $B[0, \dots] = [1, -\frac{1}{2}, \frac{1}{6}, 0, -\frac{1}{30}, 0, \frac{1}{42}, \dots]$

Sums of powers:

$$\sum_{i=1}^n i^m = \frac{1}{m+1} \sum_{k=0}^m \binom{m+1}{k} B_k \cdot (n+1)^{m+1-k}$$

Euler-Maclaurin formula for infinite sums:

$$\begin{aligned} \sum_{i=m}^\infty f(i) &= \int_m^\infty f(x) dx - \sum_{k=1}^\infty \frac{B_k}{k!} f^{(k-1)}(m) \\ &\approx \int_m^\infty f(x) dx + \frac{f(m)}{2} - \frac{f'(m)}{12} + \frac{f'''(m)}{720} + O(f^{(5)}(m)) \end{aligned}$$

6.3.2 Stirling numbers of the first kind

Number of permutations on  $n$  items with  $k$  cycles.

$c(n, k) = c(n - 1, k - 1) + (n - 1)c(n - 1, k), \quad c(0, 0) = 1$

$$\sum_{k=0}^n c(n, k) x^k = x(x + 1) \dots (x + n - 1)$$

$c(8, k) = 8, 0, 5040, 13068, 13132, 6769, 1960, 322, 28, 1$   
 $c(n, 2) = 0, 0, 1, 3, 11, 50, 274, 1764, 13068, 109584, \dots$

6.3.3 Eulerian numbers

Number of permutations  $\pi \in S_n$  in which exactly  $k$  elements are greater than the previous element.  $k$  j:s s.t.  $\pi(j) > \pi(j + 1)$ ,  $k + 1$  j:s s.t.  $\pi(j) \geq j$ ,  $k$  j:s s.t.  $\pi(j) > j$ .

$$E(n, k) = (n - k)E(n - 1, k - 1) + (k + 1)E(n - 1, k)$$

$$E(n, 0) = E(n, n - 1) = 1$$

$$E(n, k) = \sum_{j=0}^k (-1)^j \binom{n+1}{j} (k+1-j)^n$$

6.3.4 Stirling numbers of the second kind

Partitions of  $n$  distinct elements into exactly  $k$  groups.

$$S(n, k) = S(n - 1, k - 1) + kS(n - 1, k)$$

$$S(n, 1) = S(n, n) = 1$$

$$S(n, k) = \frac{1}{k!} \sum_{j=0}^k (-1)^{k-j} \binom{k}{j} j^n$$

6.3.5 Bell numbers

Total number of partitions of  $n$  distinct elements.  $B(n) = 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, \dots$  For  $p$  prime,

$$B(p^m + n) \equiv mB(n) + B(n + 1) \pmod{p}$$

6.3.6 Labeled unrooted trees

# on  $n$  vertices:  $n^{n-2}$   
# on  $k$  existing trees of size  $n_i$ :  $n_1 n_2 \dots n_k n^{k-2}$   
# with degrees  $d_i$ :  $(n - 2)! / ((d_1 - 1)! \dots (d_n - 1)!)$

6.3.7 Catalan numbers

$$C_n = \frac{1}{n+1} \binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n+1} = \frac{(2n)!}{(n+1)!n!}$$

$$C_0 = 1, \quad C_{n+1} = \frac{2(2n+1)}{n+2} C_n, \quad C_{n+1} = \sum C_i C_{n-i}$$

$$C_n = 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, \dots$$

- sub-diagonal monotone paths in an  $n \times n$  grid.
- strings with  $n$  pairs of parenthesis, correctly nested.
- binary trees with with  $n + 1$  leaves (0 or 2 children).
- ordered trees with  $n + 1$  vertices.
- ways a convex polygon with  $n + 2$  sides can be cut into triangles by connecting vertices with straight lines.
- permutations of  $[n]$  with no 3-term increasing subseq.

### 6.3.8 Gambler’s Ruin

$$P_i = \begin{cases} \frac{1-(q/p)^i}{1-(q/p)^N} & \text{if } p \neq q \\ i/N & \text{if } p = q = 0.5 \end{cases}$$

## Graph (7)

### 7.1 Network flow

#### PushRelabel.h

**Description:** Push-relabel using the highest label selection rule and the gap heuristic. Quite fast in practice. To obtain the actual flow, look at positive values only.

**Time:**  $\mathcal{O}\left(V^2\sqrt{E}\right)$

91e61c, 48 lines

```
struct PushRelabel {//d82
    struct Edge {
        int dest, back;
        ll f, c;
    };
    vector<vector<Edge>> g;//bef
    vector<ll> ec;
    vector<Edge*> cur;
    vector<vi> hs; vi H;
    PushRelabel(int n) : g(n), ec(n), cur(n), hs(2*n), H(n) {}
//a7b
    void addEdge(int s, int t, ll cap, ll rcap=0) {
        if (s == t) return;
        g[s].pb({t, sz(g[t]), 0, cap});
        g[t].pb({s, sz(g[s])-1, 0, rcap});
    }//7a1

    void addFlow(Edge& e, ll f) {
        Edge &back = g[e.dest][e.back];
        if (!ec[e.dest] && f) hs[H[e.dest]].pb(e.dest);
        e.f += f; e.c -= f; ec[e.dest] += f;//124
        back.f -= f; back.c += f; ec[back.dest] -= f;
    }
    ll calc(int s, int t) {
        int v = sz(g); H[s] = v; ec[t] = 1;
        vi co(2*v); co[0] = v-1;//a96
        rep(i,0,v) cur[i] = g[i].data();
        for (Edge& e : g[s]) addFlow(e, e.c);

        for (int hi = 0;;) {
            while (hs[hi].empty()) if (!hi--) return -ec[s];//e2e
            int u = hs[hi].back(); hs[hi].pop_back();
            while (ec[u] > 0) // discharge u
                if (cur[u] == g[u].data() + sz(g[u])) {
                    H[u] = 1e9;
                    for (Edge& e : g[u]) if (e.c && H[u] > H[e.dest]
                        +1)//9ff
                        H[u] = H[e.dest]+1, cur[u] = &e;
                    if (++co[H[u]],!--co[hi] && hi < v)
                        rep(i,0,v) if (hi < H[i] && H[i] < v)
                            --co[H[i]], H[i] = v + 1;
                    hi = H[u];//7ed
                } else if (cur[u]->c && H[u] == H[cur[u]->dest]+1)
                    addFlow(*cur[u], min(ec[u], cur[u]->c));
                else ++cur[u];
            }
        }//a5b
    bool leftOfMinCut(int a) { return H[a] >= sz(g); }
};
```

#### MinCut.h

**Description:** After running max-flow, the left side of a min-cut from  $s$  to  $t$  is given by all vertices reachable from  $s$ , only traversing edges with positive residual capacity.

**Time:**  $\mathcal{O}(E)$

#### MinCostMaxFlow.h

**Description:** Min-cost max-flow.

**Time:**  $\mathcal{O}(F * SPFA)$ ,  $F$  is flow,  $SPFA$  is the cost to run SPFA (usually  $\mathcal{O}(E)$ )

a4b013, 50 lines

```
struct MCMF{//2fc
    const ll inf = LLONG_MAX >> 2;
    struct edge{ int v; ll cap,flow,cost; };
    int n;
    vector<edge> edges;
    vector<vector<int>> adj; vector<pair<int,int>> par;
        vector<int> in_q;//764
    vector<ll> dist;
    MCMF( int n ) : n(n), adj(n), dist(n), par(n), in_q
        (n) {}
    void add_edge(int u,int v, ll c,ll f){
        int idx = edges.size();
        edges.push_back({v,f,0,c});//a83
        edges.push_back({u,f,f,-c});
        adj[u].push_back(idx);
        adj[v].push_back(idx^1);
    }
    bool find_path(int s,int t)//36d
        for( auto& e: dist ) e = inf;
        for( auto& e: in_q ) e = 0;
        queue<int> q; q.push(s);
        dist[s] = 0; in_q[s] = 1;
        while( !q.empty() )//a03
            int cur = q.front(); q.pop();
            in_q[cur] = 0;
            for( int idx: adj[cur] ){
                auto [nxt,cap,fl,wt] =
                    edges[idx];
                ll nxtD = dist[cur] + wt;//
                    eda
                if( fl >= cap || nxtD >=
                    dist[nxt] ) continue;
                dist[nxt] = nxtD;
                par[nxt] = {cur,idx};
                if( in_q[nxt] ) continue;
                q.push(nxt); in_q[nxt] = 1;
                //6e8
            }
        }
        return dist[t] < inf;
    }
    pair<ll,ll> calc(int s,int t){//6aa
        ll flow = 0, cost = 0;
        while( find_path(s,t) ){
            ll f = inf;
            for( int i,u,v=t; tie(u,i)=par[v],v
                != s; v = u )
                f = min(f,edges[i].cap -
                    edges[i].flow );//57f
            flow += f;
            for( int i,u,v=t; tie(u,i)=par[v],v
                !=s; v = u )
                edges[i].flow += f, edges[i
                    ^1].flow -= f;
        }
        rep(i,0,edges.size()/2)//4a3
            cost += edges[i<=1].cost * edges[i
                <=1].flow;
        return {flow,cost};
    }
};
```

};

#### GlobalMinCut.h

**Description:** Find a global minimum cut in an undirected graph, as represented by an adjacency matrix.

**Time:**  $\mathcal{O}(V^3)$

8b0e19, 21 lines

```
pair<int, vi> globalMinCut(vector<vi> mat) {//f64
    pair<int, vi> best = {INT_MAX, {}};
    int n = sz(mat);
    vector<vi> co(n);
    rep(i,0,n) co[i] = {i};
    rep(ph,1,n) {//c8f
        vi w = mat[0];
        size_t s = 0, t = 0;
        rep(it,0,n-ph) { // O(V^2) -> O(E log V) with prio.
            queue
                w[t] = INT_MIN;
                s = t, t = max_element(all(w)) - w.begin();//0bb
                rep(i,0,n) w[i] += mat[t][i];
        }
        best = min(best, {w[t] - mat[t][t], co[t]});
        co[s].insert(co[s].end(), all(co[t]));
        rep(i,0,n) mat[s][i] += mat[t][i];//a2c
        rep(i,0,n) mat[i][s] = mat[s][i];
        mat[0][t] = INT_MIN;
    }
    return best;
}//cbb
```

#### GomoryHu.h

**Description:** Given a list of edges representing an undirected flow graph, returns edges of the Gomory-Hu tree. The max flow between any pair of vertices is given by minimum edge weight along the Gomory-Hu tree path.

**Time:**  $\mathcal{O}(V)$  Flow Computations

1ec6c8, 13 lines

```
"PushRelabel.h"
struct Edge { int from, to; ll cap; };//81a
vector<Edge> gomoryHu(int N, vector<Edge> ed) {
    vector<Edge> tree;
    vi par(N);
    rep(i,1,N) {
        PushRelabel D(N); // Dinic also works//489
        for (Edge t : ed) D.addEdge(t.from, t.to, t.cap, t.cap)
            ;
        tree.pb({i, par[i], D.calc(i, par[i])});
        rep(j,i+1,N)
            if (par[j] == par[i] && D.leftOfMinCut(j)) par[j] = i
                ;
    }//ecc
    return tree;
}
```

#### FlowWithDemands.h

**Description:** Add a new source  $s'$  and sink  $t'$ , new edges from  $s'$  to everything, and new edges from everything to  $t'$ . Define:

- $c'((s',v)) = \sum_{u \in V} d((u,v))$  for each edge  $(s',v)$
- $c'((v,t')) = \sum_{w \in V} d((v,w))$  for each edge  $(v,t')$
- $c'((u,v)) = c((u,v)) - d((u,v))$  for each old edge  $(u,v)$
- $c'((t,s)) = \infty$

**Time:**  $\mathcal{O}(VE \log U)$

**Time:**  $\mathcal{O}(\min(E^{1/2}, V^{2/3})E)$  if  $U = 1$ ;  $\mathcal{O}(\sqrt{VE})$  for bipar-

d41d8c, 1 lines

```
//d41
```

Dinic.h

Description: Flow algorithm with complexity  $\mathcal{O}(VE \log U)$  where  $U = \max|cap|$ .  $\mathcal{O}(\min(E^{1/2}, V^{2/3})E)$  if  $U = 1$ ;  $\mathcal{O}(\sqrt{VE})$  for bipar-

tite matching.

b2af4f, 42 lines

```
struct Dinic {//299
    struct Edge {
        int to, rev;
        ll c, oc;
        ll flow(t) { return max(oc - c, 0LL); } // if you need flows
    };//8ec
    vi lvl, ptr, q;
    vector<vector<Edge>> adj;
    Dinic(int n) : lvl(n), ptr(n), q(n), adj(n) {}
    void addEdge(int a, int b, ll c, ll rcap = 0) {
        adj[a].pb({b, sz(adj[b]), c, c});//76f
        adj[b].pb({a, sz(adj[a]) - 1, rcap, rcap});
    }
    ll dfs(int v, int t, ll f) {
        if (v == t || !f) return f;
        for (int& i = ptr[v]; i < sz(adj[v]); i++) //b2a
            Edge& e = adj[v][i];
            if (lvl[e.to] == lvl[v] + 1)
                if (ll p = dfs(e.to, t, min(f, e.c))) {
                    e.c -= p, adj[e.to][e.rev].c += p;
                    return p;//f3e
                }
    }
    return 0;
}
ll calc(int s, int t) //b4c
ll flow = 0; q[0] = s;
rep(L,0,31) do { // 'int L=30' maybe faster for random data
    lvl = ptr = vi(sz(q));
    int qi = 0, qe = lvl[s] = 1;
    while (qi < qe && !lvl[t]) //796
        int v = q[qi++];
        for (Edge e : adj[v])
            if (!lvl[e.to] && e.c >> (30 - L))
                q[qe++] = e.to, lvl[e.to] = lvl[v] + 1;
    }//4ca
    while (ll p = dfs(s, t, LLONG_MAX)) flow += p;
} while (lvl[t]);
return flow;
}
bool leftOfMinCut(int a) { return lvl[a] != 0; }//b90
};
```

EdmondsKarp.h

**Description:** Flow algorithm with guaranteed complexity  $O(VE^2)$ . To get edge flow values, compare capacities before and after, and take the positive values only.

```
482fe0, 36 lines
template<class T> T edmondsKarp(vector<unordered_map<int, T>>& //324
    graph, int source, int sink) {
    assert(source != sink);
    T flow = 0;
    vi par(sz(graph)), q = par;
//cf9
    for (;;) {
        fill(all(par), -1);
        par[source] = 0;
        int ptr = 1;
        q[0] = source;//623

        rep(i,0,ptr) {
            int x = q[i];
            for (auto e : graph[x]) {
                if (par[e.first] == -1 && e.second > 0) //3a4
                    par[e.first] = x;
                    q[ptr++] = e.first;
                    if (e.first == sink) goto out;
            }
        }
    }
}
```

```
//3cd
    }
    return flow;
out:
    T inc = numeric_limits<T>::max();
    for (int y = sink; y != source; y = par[y])//d19
        inc = min(inc, graph[par[y]][y]);

    flow += inc;
    for (int y = sink; y != source; y = par[y]) {
        int p = par[y];//b79
        if ((graph[p][y] -= inc) <= 0) graph[p].erase(y);
        graph[y][p] += inc;
    }
}
//cbb
```

7.1.1 Flow Notes

König’s Theorem: For bipartite graph,  
|minimum node cover| = |maximum matching|

Nodes not in a minimum node cover form a maximum independent set

Node-disjoint Path Cover: Each node belongs to exactly one path

- $\{a_1, \dots, a_n, b_1, \dots, b_n\}$  where  $(a_u \rightarrow b_v)$  exists for each edges  $(u \rightarrow v)$  in DAG  $G$

(General) Path Cover: Each node belongs to at least one path

- Same construction as node-disjoint but with transitive closure edges

Dilworth’s Theorem: In a DAG,  
|minimum general path cover| = |maximum antichain|

7.2 Matching

hopcroftKarp.h

**Description:** Fast bipartite matching algorithm. Graph  $g$  should be a list of neighbors of the left partition, and  $btoa$  should be a vector full of -1’s of the same size as the right partition. Returns the size of the matching.  $btoa[i]$  will be the match for vertex  $i$  on the right side, or  $-1$  if it’s not matched.

**Usage:** vi btoa(m, -1); hopcroftKarp(g, btoa);

**Time:**  $O(\sqrt{VE})$

*d93347, 42 lines*

```
bool dfs(int a, int L, vector<vi>& g, vi& btoa, vi& A, vi& B) //d9e
    if (A[a] != L) return 0;
    A[a] = -1;
    for (int b : g[a]) if (B[b] == L + 1) {
        B[b] = 0;
        if (btoa[b] == -1 || dfs(btoa[b], L + 1, g, btoa, A, B)) //613
            return btoa[b] = a, 1;
    }
    return 0;
}
//ad4
int hopcroftKarp(vector<vi>& g, vi& btoa) {
    int res = 0;
    vi A(g.size()), B(btoa.size()), cur, next;
    for (;;) {
        fill(all(A), 0);//d58
```

```
        fill(all(B), 0);
        cur.clear();
        for (int a : btoa) if (a != -1) A[a] = -1;
        rep(a,0,sz(g)) if (A[a] == 0) cur.pb(a);
        for (int lay = 1; ; lay++) //559
            bool islast = 0;
            next.clear();
            for (int a : cur) for (int b : g[a]) {
                if (btoa[b] == -1) {
                    B[b] = lay;//1ca
                    islast = 1;
                }
                else if (btoa[b] != a && !B[b]) {
                    B[b] = lay;
                    next.pb(btoa[b]);//c66
                }
            }
            if (islast) break;
            if (next.empty()) return res;
            for (int a : next) A[a] = lay;//4f3
            cur.swap(next);
        }
        rep(a,0,sz(g))
            res += dfs(a, 0, g, btoa, A, B);
    }//67c
}
```

DFSMatching.h

**Description:** Simple bipartite matching algorithm. Graph  $g$  should be a list of neighbors of the left partition, and  $btoa$  should be a vector full of -1’s of the same size as the right partition. Returns the size of the matching.  $btoa[i]$  will be the match for vertex  $i$  on the right side, or  $-1$  if it’s not matched.

**Usage:** vi btoa(m, -1); dfsMatching(g, btoa);

**Time:**  $O(VE)$

*522b98, 22 lines*

```
bool find(int j, vector<vi>& g, vi& btoa, vi& vis) //400
    if (btoa[j] == -1) return 1;
    vis[j] = 1; int di = btoa[j];
    for (int e : g[di])
        if (!vis[e] && find(e, g, btoa, vis)) {
            btoa[e] = di;//a0e
            return 1;
        }
    return 0;
}
int dfsMatching(vector<vi>& g, vi& btoa) //52f
    vi vis;
    rep(i,0,sz(g)) {
        vis.assign(sz(btoa), 0);
        for (int j : g[i])
            if (find(j, g, btoa, vis)) //e5b
                btoa[j] = i;
                break;
    }
    return sz(btoa) - (int)count(all(btoa), -1);//ff5
}
```

MinimumVertexCover.h

**Description:** Finds a minimum vertex cover in a bipartite graph. The size is the same as the size of a maximum matching, and the complement is a maximum independent set.

*"DFSMatching.h"*

*62c4ec, 22 lines*

```
vi cover(vector<vi>& g, int n, int m) //60f
    vi match(m, -1);
    int res = dfsMatching(g, match);
    vector<bool> lfound(n, true), seen(m);
    for (int it : match) if (it != -1) lfound[it] = false;
    vi q, cover;//2da
    rep(i,0,n) if (lfound[i]) q.pb(i);
```

```
while (!q.empty()) {
    int i = q.back(); q.pop_back();
    lfound[i] = 1;
    for (int e : g[i]) if (!seen[e] && match[e] != -1) {
        ed
        seen[e] = true;
        q.pb(match[e]);
    }
}
rep(i,0,n) if (!lfound[i]) cover.pb(i); //a72
rep(i,0,m) if (seen[i]) cover.pb(n+i);
assert(sz(cover) == res);
return cover;
}
```

### MaximumIndependentSet.h

**Description:** To obtain a maximum independent set of a graph, find a max clique of the complement. If the graph is bipartite, see MinimumVertexCover.

d41d8c, 1 lines

```
//d41
```

### WeightedMatching.h

**Description:** Given a weighted bipartite graph, matches every node on the left with a node on the right such that no nodes are in two matchings and the sum of the edge weights is minimal. Takes cost[N][M], where cost[i][j] = cost for L[i] to be matched with R[j] and returns (min cost, match), where L[i] is matched with R[match[i]]. Negate costs for max cost. Requires  $N \leq M$ .

**Time:**  $\mathcal{O}(N^2M)$

df0677, 31 lines

```
pair<int, vi> hungarian(const vector<vi> &a) { //64f
    if (a.empty()) return {0, {}};
    int n = sz(a) + 1, m = sz(a[0]) + 1;
    vi u(n), v(m), p(m), ans(n - 1);
    rep(i,1,n) {
        p[0] = i; //0b5
        int j0 = 0; // add "dummy" worker 0
        vi dist(m, INT_MAX), pre(m, -1);
        vector<bool> done(m + 1);
        do { // dijkstra
            done[j0] = true; //bd1
            int i0 = p[j0], j1 = -1, delta = INT_MAX;
            rep(j,1,m) if (!done[j]) {
                auto cur = a[i0 - 1][j - 1] - u[i0] - v[j];
                if (cur < dist[j]) dist[j] = cur, pre[j] = j0;
                if (dist[j] < delta) delta = dist[j], j1 = j; //865
            }
            rep(j,0,m) {
                if (done[j]) u[p[j]] += delta, v[j] -= delta;
                else dist[j] -= delta;
            } //aa1
            j0 = j1;
        } while (p[j0]);
        while (j0) { // update alternating path
            int j1 = pre[j0];
            p[j0] = p[j1], j0 = j1; //88f
        }
    }
    rep(j,1,m) if (p[j]) ans[p[j] - 1] = j - 1;
    return {-v[0], ans}; // min cost
} //cbb
```

### GeneralMatching.h

**Description:** Matching for general graphs. Fails with probability  $N/mod$ .

**Time:**  $\mathcal{O}(N^3)$

1e40dd, 40 lines

```
vector<pii> generalMatching(int N, vector<pii>& ed) { //19e
    vector<vector<ll>> mat(N, vector<ll>(N)), A;
    for (pii pa : ed) {
```

```
        int a = pa.first, b = pa.second, r = rand() % mod;
        mat[a][b] = r, mat[b][a] = (mod - r) % mod;
    } //063

    int r = matInv(A = mat), M = 2*N - r, fi, fj;
    assert(r % 2 == 0);

    if (M != N) do { //f88
        mat.resize(M, vector<ll>(M));
        rep(i,0,N) {
            mat[i].resize(M);
            rep(j,N,M) {
                int rr = rand() % mod; //b47
                mat[i][j] = rr, mat[j][i] = (mod - rr) % mod;
            }
        }
        while (matInv(A = mat) != M);
    } //92b
    vi has(M, 1); vector<pii> ret;
    rep(it,0,M/2) {
        rep(i,0,M) if (has[i])
            rep(j,i+1,M) if (A[i][j] && mat[i][j]) {
                fi = i; fj = j; goto done; //e0a
            }
        assert(0); done:
        if (fj < N) ret.emplace_back(fi, fj);
        has[fi] = has[fj] = 0;
        rep(sw,0,2) {
            ll a = modpow(A[fi][fj], mod-2); //b7f
            rep(i,0,M) if (has[i] && A[i][fj]) {
                ll b = A[i][fj] * a % mod;
                rep(j,0,M) A[i][j] = (A[i][j] - A[fi][j] * b) % mod;
            }
            swap(fi,fj); //3c7
        }
    }
    return ret;
}
```

### MatroidIntersection.h

**Description:** Given two matroids, finds the largest common independent set. Pass the matroid with more expensive add/clear operations to M1.

**Time:**  $R^2N(M2.add + M1.check + M2.check) + R^3M1.add + R^2M1.clear + RN M2.clear$

02fb70, 52 lines

```
struct ColorMat { //b9c
    vi cnt, clr;
    ColorMat(int n, vector<int> clr) : cnt(n), clr(clr) {}
    bool check(int x) { return !cnt[clr[x]]; }
    void add(int x) { cnt[clr[x]]++; }
    void clear() { fill(all(cnt), 0); } //540
};

template <class M1, class M2> struct MatroidIsect {
    int n;
    vector<char> iset;
    M1 m1; M2 m2; //286
    MatroidIsect(M1 m1, M2 m2, int n) : n(n), iset(n + 1), m1(m1), m2(m2) {}

    vi solve() {
        rep(i,0,n) if (m1.check(i) && m2.check(i))
            iset[i] = true, m1.add(i), m2.add(i);
        while (augment()); //b25
        vi ans;
        rep(i,0,n) if (iset[i]) ans.push_back(i);
        return ans;
    }

    bool augment() { //110
        vector<int> frm(n, -1);
        queue<int> q({n}); // starts at dummy node
        auto fwdE = [&](int a) {
```

```
        vi ans;
        m1.clear(); //2d7
        rep(v, 0, n) if (iset[v] && v != a) m1.add(v);
        rep(b, 0, n) if (!iset[b] && frm[b] == -1 && m1.check(b))
            ans.push_back(b), frm[b] = a;
        return ans;
    } //20c
    auto backE = [&](int b) {
        m2.clear();
        rep(cas, 0, 2) rep(v, 0, n)
            if ((v == b || iset[v]) && (frm[v] == -1) == cas) {
                if (!m2.check(v)) //45e
                    return cas ? q.push(v), frm[v] = b, v : -1;
                m2.add(v);
            }
        return n;
    } //c4c
    while (!q.empty()) {
        int a = q.front(), c; q.pop();
        for (int b : fwdE(a))
            while((c = backE(b)) >= 0) if (c == n) {
                while (b != n) iset[b] ^= 1, b = frm[b]; //d9b
                return true;
            }
        return false;
    } //e03
};
```

### 7.2.1 Matroid Notes

$$\text{rank}(A) + \text{rank}(B) \geq \text{rank}(A \cup B) + \text{rank}(A \cap B)$$

Matroid Intersection steps:

- Build exchange graph  $D_{M_1, M_2}(S)$  - Find “free to include” sets  $Y_1$  and  $Y_2$  - Find augmenting path  $P$  without shortcuts (BFS) from an element in  $Y_1$  to an element in  $Y_2$  - Alternate inclusion into  $S$  of all element in  $P$

Exchange Graph construction:

- Left side contains nodes in  $S$ , right side contains nodes not in  $S$
- Edge  $(x \rightarrow y)$  where  $x \in S$  and  $y \notin S$  exists iff removing  $S \setminus \{x\} \cup \{y\}$  is independent in  $M_1$
- Edge  $(x \leftarrow y)$  where  $x \in S$  and  $y \notin S$  exists iff removing  $S \setminus \{x\} \cup \{y\}$  is independent in  $M_2$
- $y \in Y_1$  iff  $S \cup \{y\}$  is independent in  $M_1$
- $y \in Y_2$  iff  $S \cup \{y\}$  is independent in  $M_2$

Weighted Matroid Intersection:

- Assign weights to nodes ( $w(x)$  for  $x \in S$  and  $-w(x)$  o.w.)
- Find shortest augmenting path first by sum of weights, then by # of edges

### 7.3 DFS algorithms

#### SCC.h

**Description:** Finds strongly connected components in a directed graph. If vertices  $u, v$  belong to the same component, we can reach  $u$  from  $v$  and vice versa.

**Usage:** scc(graph, [&](vi& v) { ... }) visits all components in reverse topological order. comp[i] holds the component index of a node (a component only has edges to components with lower index). ncomps will contain the number of components.  
**Time:**  $\mathcal{O}(E + V)$

c16b3c, 24 lines

```
template<class F> vi scc(const vector<vi> &adj, F f) {  
    int n = sz(adj);  
    vi val(n), comp(n, -1), z, cont;  
    int time = 0, ncomps = 0;  
    auto dfs = [&](auto &&self, int u) -> int {  
        int low = val[u] = ++time, x; z.push_back(u);  
        for (auto e : adj[u]) if (comp[e] < 0)  
            low = min(low, val[e] ?: self(self, e));  
        if (low == val[u]) {  
            do {  
                x = z.back(); z.pop_back();  
                comp[x] = ncomps;  
                cont.push_back(x);  
            } while (x != u);  
            f(cont); cont.clear();  
            ncomps++;  
        }  
        return val[u] = low;  
    };  
    rep(i, 0, n) {  
        if (comp[i] < 0) dfs(dfs, i);  
    }  
    return comp;  
}
```

BiconnectedComponents.h

**Description:** Finds all biconnected components in an undirected graph, and runs a callback for the edges in each. In a biconnected component there are at least two distinct paths between any two nodes. Note that a node can be in several components. An edge which is not in a component is a bridge, i.e., not part of any cycle.

**Usage:** int eid = 0; ed.resize(N);  
for each edge (a,b) {  
ed[a].emplace\_back(b, eid);  
ed[b].emplace\_back(a, eid++); }  
bicomps([&](const vi& edgelist) {...});

**Time:**  $\mathcal{O}(E + V)$

389e66, 28 lines

```
template<class F> //c2c  
void bicomps(vector<vector<pii>> &ed, F f) {  
    vi num(sz(ed)), st;  
    int t=0;  
    auto dfs = [&](auto &&self, int at, int par) -> int {  
        int me = num[at] = ++t, top = me;  
        for (auto [y, e] : ed[at]) if (e != par) {  
            if (num[y]) {  
                top = min(top, num[y]);  
                if (num[y] < me)  
                    st.pb(e);  
            } else {  
                int si = sz(st);  
                int up = self(self, y, e);  
                top = min(top, up);  
                if (up == me) {  
                    st.pb(e);  
                    f(vi(st.begin() + si, st.end()));  
                    st.resize(si);  
                }  
                else if (up < me) st.pb(e);  
                else /* e is a bridge */  
            }  
        }  
        return top;  
    }  
}
```

```
};  
rep(i,0,sz(ed)) if (!num[i]) dfs(dfs, i, -1);  
}
```

Articulation.h

**Description:** Finds articulation points (removal separates graph)  
**Time:**  $\mathcal{O}(n + m)$

a7b0ba, 25 lines

```
vector<bool> cutpoints(const vector<vi> &adj) {  
    int timer=0, n=sz(adj);  
    vi tin(n,-1), low(n,-1);  
    vector<bool> vis(n);  
    vector<bool> iscut(n);  
    auto dfs = [&](auto &&self, int v, int p) -> void {  
        vis[v] = true;  
        tin[v] = low[v] = timer++;  
        int ch = 0;  
        for (int to : adj[v]) {  
            if (to == p) continue;  
            if (vis[to])  
                low[v] = min(low[v], tin[to]);  
            else {  
                self(self, to, v);  
                low[v] = min(low[v], low[to]);  
                if (low[to] >= tin[v] && p!=-1) iscut[v]=1;  
                ++ch;  
            }  
        }  
        if(p == -1 && ch > 1) iscut[v]=1;  
    };  
    rep(i,0,n) if (!vis[i]) dfs(dfs, i, -1);  
    return iscut;  
}
```

2sat.h

**Description:** Calculates a valid assignment to boolean variables a, b, c,... to a 2-SAT problem, so that an expression of the type (a||b)&&(!a||c)&&(d||!b)&&... becomes true, or reports that it is unsatisfiable. Negated variables are represented by bit-inversions (~x).

**Usage:** TwoSat ts(number of boolean variables);  
ts.either(0, ~3); // Var 0 is true or var 3 is false  
ts.setValue(2); // Var 2 is true  
ts.atMostOne({0,~1,2}); // <= 1 of vars 0, ~1 and 2 are true  
ts.solve(); // Returns true iff it is solvable  
ts.values[0..N-1] holds the assigned values to the vars  
**Time:**  $\mathcal{O}(N + E)$ , where N is the number of boolean variables, and E is the number of clauses.

35fbf7, 56 lines

```
struct TwoSat {  
    int N;  
    vector<vi> gr;  
    vi values; // 0 = false, 1 = true  
  
    TwoSat(int n = 0) : N(n), gr(2*n) {}  
  
    int addVar() { // (optional)  
        gr.emplace_back();  
        gr.emplace_back();  
        return N++;  
    }  
  
    void either(int f, int j) {  
        f = max(2*f, -1-2*f);  
        j = max(2*j, -1-2*j);  
        gr[f].pb(j^1);  
        gr[j].pb(f^1);  
    }  
  
    void setValue(int x) { either(x, x); }  
  
    void atMostOne(const vi& li) { // (optional)
```

```
    if (sz(li) <= 1) return;  
    int cur = ~li[0];  
    rep(i,2,sz(li)) {  
        int next = addVar();  
        either(cur, ~li[i]);  
        either(cur, next);  
        either(~li[i], next);  
        cur = ~next;  
    }  
    either(cur, ~li[1]);  
}
```

```
vi val, comp, z; int time = 0;  
int dfs(int i) {  
    int low = val[i] = ++time, x; z.pb(i);  
    for(int e : gr[i]) if (!comp[e])  
        low = min(low, val[e] ?: dfs(e));  
    if (low == val[i]) do {  
        x = z.back(); z.pop_back();  
        comp[x] = low;  
        if (values[x>>1] == -1)  
            values[x>>1] = x&1;  
    } while (x != i);  
    return val[i] = low;  
}
```

```
bool solve() {  
    values.assign(N, -1);  
    val.assign(2*N, 0); comp = val;  
    rep(i,0,2*N) if (!comp[i]) dfs(i);  
    rep(i,0,N) if (comp[2*i] == comp[2*i+1]) return 0;  
    return 1;  
}  
};
```

EulerWalk.h

**Description:** Eulerian undirected/directed path/cycle algorithm. Input should be a vector of (dest, global edge index), where for undirected graphs, forward/backward edges have the same index. Returns a list of nodes in the Eulerian path/cycle with src at both start and end, or empty list if no cycle/path exists. To get edge indices back, add .second to s and ret.

**Time:**  $\mathcal{O}(V + E)$

3e0eb1, 15 lines

```
vi eulerWalk(vector<vector<pii>>& gr, int nedges, int src = 0) {  
    int n = sz(gr);  
    vi D(n), its(n), eu(nedges), ret, s = {src};  
    D[src]++; // to allow Euler paths, not just cycles  
    while (!s.empty()) {  
        int x = s.back(), y, e, &it = its[x], end = sz(gr[x]);  
        if (it == end){ ret.pb(x); s.pop_back(); continue; }  
        tie(y, e) = gr[x][it++];  
        if (!eu[e]) {  
            D[x]--, D[y]++;  
            eu[e] = 1; s.pb(y);  
        }  
    }  
    for (int x : D) if (x < 0 || sz(ret) != nedges+1) return {};  
    return {ret.rbegin(), ret.rend()};  
}
```

7.4 Coloring

EdgeColoring.h

**Description:** Given a simple, undirected graph with max degree D, computes a (D+1)-coloring of the edges such that no neighboring edges share a color. (D-coloring is NP-hard, but can be done for bipartite graphs by repeated matchings of max-degree nodes.)

**Time:**  $\mathcal{O}(NM)$

ca07a0, 31 lines



```
vi edgeColoring(int N, vector<pii> eds) {//d26
    vi cc(N + 1), ret(sz(eds)), fan(N), free(N), loc;
    for (pii e : eds) ++cc[e.first], ++cc[e.second];
    int u, v, ncols = *max_element(all(cc)) + 1;
    vector<vi> adj(N, vi(ncols, -1));
    for (pii e : eds) //945
        tie(u, v) = e;
        fan[0] = v;
        loc.assign(ncols, 0);
        int at = u, end = u, d, c = free[u], ind = 0, i = 0;
        while (d = free[v], !loc[d] && (v = adj[u][d]) != -1)//665
            loc[d] = ++ind, cc[ind] = d, fan[ind] = v;
            cc[loc[d]] = c;
            for (int cd = d; at != -1; cd ^= c ^ d, at = adj[at][cd])
                swap(adj[at][cd], adj[end = at][cd ^ c ^ d]);
            while (adj[fan[i]][d] != -1) //f7e
                int left = fan[i], right = fan[++i], x = cc[i];
                adj[u][x] = left;
                adj[left][x] = u;
                adj[right][x] = -1;
                free[right] = x;//e59
        }
        adj[u][d] = fan[i];
        adj[fan[i]][d] = u;
        for (int y : {fan[0], u, end})
            for (int& z = free[y] = 0; adj[y][z] != -1; z++)//b06
        }
        rep(i, 0, sz(eds))
            for (tie(u, v) = eds[i]; adj[u][ret[i]] != v;) ++ret[i];
        return ret;
    }//cbb
```

## 7.5 Heuristics

### MaximalCliques.h

**Description:** Runs a callback for all maximal cliques in a graph (given as a symmetric bitset matrix; self-edges not allowed). Callback is given a bitset representing the maximal clique.

**Time:**  $\mathcal{O}\left(3^{n/3}\right)$ , much faster for sparse graphs

```
typedef bitset<128> B;//abb
template<class F>
void cliques(vector<B>& eds, F f, B P = ~B(), B X={}, B R={}) {
    if (!P.any()) { if (!X.any()) f(R); return; }
    auto q = (P | X)._Find_first();
    auto cand = P & ~eds[q];//7d8
    rep(i, 0, sz(eds)) if (cands[i]) {
        R[i] = 1;
        cliques(eds, f, P & eds[i], X & eds[i], R);
        R[i] = P[i] = 0; X[i] = 1;
    }//67c
}
```

### MaximumClique.h

**Description:** Quickly finds a maximum clique of a graph (given as symmetric bitset matrix; self-edges not allowed). Can be used to find a maximum independent set by finding a clique of the complement graph.

**Time:** Runs in about 1s for n=155 and worst case random graphs (p=.90). Runs faster for sparse graphs.

```
typedef vector<bitset<200>> vb;//b92
struct Maxclique {
    double limit=0.025, pk=0;
    struct Vertex { int i, d=0; };
    typedef vector<Vertex> vv;
    vb e;//5b2
```

```
vv V;
vector<vi> C;
vi qmax, q, S, old;
void init(vv& r) {
    for (auto& v : r) v.d = 0;//dab
    for (auto& v : r) for (auto j : r) v.d += e[v.i][j.i];
    sort(all(r), [](auto a, auto b) { return a.d > b.d; });
    int mxD = r[0].d;
    rep(i, 0, sz(r)) r[i].d = min(i, mxD) + 1;
}//aba
void expand(vv& R, int lev = 1) {
    S[lev] += S[lev - 1] - old[lev];
    old[lev] = S[lev - 1];
    while (sz(R)) {
        if (sz(q) + R.back().d <= sz(qmax)) return;//62e
        q.pb(R.back().i);
        vv T;
        for (auto v:R) if (e[R.back().i][v.i]) T.pb({v.i});
        if (sz(T)) {
            if (S[lev]++ / ++pk < limit) init(T);//feb
            int j = 0, mxk = 1, mnk = max(sz(qmax) - sz(q) + 1, 1);
            C[1].clear(), C[2].clear();
            for (auto v : T) {
                int k = 1;
                auto f = [&](int i) { return e[v.i][i]; };//94f
                while (any_of(all(C[k]), f)) k++;
                if (k > mxk) mxk = k, C[mxk + 1].clear();
                if (k < mnk) T[j++].i = v.i;
                C[k].pb(v.i);
            }//08b
            if (j > 0) T[j - 1].d = 0;
            rep(k, mnk, mxk + 1) for (int i : C[k])
                T[j].i = i, T[j++].d = k;
            expand(T, lev + 1);
        } else if (sz(q) > sz(qmax)) qmax = q;//15f
        q.pop_back(), R.pop_back();
    }
}
vi maxClique() { init(V), expand(V); return qmax; }
Maxclique(vb conn) : e(conn), C(sz(e)+1), S(sz(C)), old(S) {}//83c
    rep(i, 0, sz(e)) V.pb({i});
};
```

## 7.6 Trees

### CompressTree.h

**Description:** Given a rooted tree and a subset S of nodes, compute the minimal subtree that contains all the nodes by adding all (at most  $|S| - 1$ ) pairwise LCA's and compressing edges. Returns a list of (par, orig.index) representing a tree rooted at 0. The root points to itself.

**Time:**  $\mathcal{O}(|S|\log|S|)$

```
"LCA.h"
typedef vector<pair<int, int>> vpi;//386
vpi compressTree(LCA& lca, const vi& subset) {
    static vi rev; rev.resize(sz(lca.time));
    vi li = subset, &T = lca.time;
    auto cmp = [&](int a, int b) { return T[a] < T[b]; };
    sort(all(li), cmp);//3b2
    int m = sz(li)-1;
    rep(i, 0, m) {
        int a = li[i], b = li[i+1];
        li.pb(lca.lca(a, b));
    }//c76
    sort(all(li), cmp);
    li.erase(unique(all(li)), li.end());
    rep(i, 0, sz(li)) rev[li[i]] = i;
    vpi ret = {pii(0, li[0])};
    rep(i, 0, sz(li)-1) //ff8
```

```
    int a = li[i], b = li[i+1];
    ret.emplace_back(rev[lca.lca(a, b)], b);
}
return ret;
}//cbb
```

### HLD.h

**Description:** Decomposes a tree into vertex disjoint heavy paths and light edges such that the path from any leaf to the root contains at most  $\log(n)$  light edges. Takes as input the full adjacency list. op edges being true means that values are stored in the edges, as opposed to the nodes.

**Time:**  $\mathcal{O}(\log N)$

```
template<bool op_edges = false>//1ab
struct hld {
    vector<vi> adj;
    vi par, size, in, head, d;
    hld(int n) : adj(n), par(n), size(n), in(n), head(n), d(n) {}
}//c22
void add_edge(int u, int v) {
    adj[u].pb(v);
    adj[v].pb(u);
}
//9ee
void dfs_size(int v = 0, int p = 0) {
    size[v] = 1;
    for (int &e : adj[v]) {
        if (e != p) {
            d[e] = d[v] + 1;//a74
            par[e] = v;
            dfs_size(e, v);
            size[v] += size[e];
            if (size[e] > size[adj[v][0]] || adj[v][0] == p)
                swap(e, adj[v][0]);
        }//96d
    }
}

void dfs_hld(int v = 0, int p = 0) {
    static int t = 0;//978
    in[v] = t++;
    for (int e : adj[v]) {
        if (e != p) {
            if (e == adj[v][0]) {
                head[e] = head[v];//c3e
            } else {
                head[e] = e;
            }
            dfs_hld(e, v);
        }//47a
    }
}
```

```
template<typename F>
void op_path(int x, int y, F op) //b81
    while (head[x] != head[y]) {
        if (d[head[x]] > d[head[y]]) swap(x, y);
        op(in[head[y]], in[y] + 1);
        y = par[head[y]];
    }//387
    if (d[x] > d[y]) swap(x, y);
    op(in[x] + (op_edges ? 1 : 0), in[y] + 1);
}
```

```
template<typename F>//d25
void op_subtree(int x, F op) {
    op(in[x] + (op_edges ? 1 : 0), in[x] + size[x]);
}
};
```

LinkCutTree.h

**Description:** Represents a forest of unrooted trees. You can add and remove edges (as long as the result is still a forest), and check whether two nodes are in the same tree.

**Time:** All operations take amortized  $\mathcal{O}(\log N)$ . 0fb462, 90 lines

```
struct Node { // Splay tree. Root's pp contains tree's
    parent.//44e
    Node *p = 0, *pp = 0, *c[2];
    bool flip = 0;
    Node() { c[0] = c[1] = 0; fix(); }
    void fix() {
        if (c[0]) c[0]->p = this; //b8f
        if (c[1]) c[1]->p = this;
        // (+ update sum of subtree elements etc. if wanted)
    }
    void pushFlip() {
        if (!flip) return; //dfd
        flip = 0; swap(c[0], c[1]);
        if (c[0]) c[0]->flip ^= 1;
        if (c[1]) c[1]->flip ^= 1;
    }
    int up() { return p ? p->c[1] == this : -1; } //3a9
    void rot(int i, int b) {
        int h = i ^ b;
        Node *x = c[i], *y = b == 2 ? x : x->c[h], *z = b ? y :
            x;
        if ((y->p == p)) p->c[up()] = y;
        c[i] = z->c[i ^ 1]; //eb7
        if (b < 2) {
            x->c[h] = y->c[h ^ 1];
            y->c[h ^ 1] = x;
        }
        z->c[i ^ 1] = this; //430
        fix(); x->fix(); y->fix();
        if (p) p->fix();
        swap(pp, y->pp);
    }
    void splay() { //4c8
        for (pushFlip(); p; ) {
            if (p->p) p->p->pushFlip();
            p->pushFlip(); pushFlip();
            int c1 = up(), c2 = p->up();
            if (c2 == -1) p->rot(c1, 2); //9e8
            else p->p->rot(c2, c1 != c2);
        }
    }
    Node* first() {
        pushFlip(); //828
        return c[0] ? c[0]->first() : (splay(), this);
    }
};

struct LinkCut { //d99
    vector<Node> node;
    LinkCut(int N) : node(N) {}

    void link(int u, int v) { // add an edge (u, v)
        assert(!connected(u, v)); //166
        makeRoot(&node[u]);
        node[u].pp = &node[v];
    }
    void cut(int u, int v) { // remove an edge (u, v)
        Node *x = &node[u], *top = &node[v]; //0b9
        makeRoot(top); x->splay();
        assert(top == (x->pp ? x->c[0]));
        if (x->pp) x->pp = 0;
        else {
            x->c[0] = top->p = 0; //158
            x->fix();
        }
    }
};
```

```
bool connected(int u, int v) { // are u, v in the same
    tree?
    Node* nu = access(&node[u])->first(); //781
    return nu == access(&node[v])->first();
}

void makeRoot(Node* u) {
    access(u);
    u->splay(); //09d
    if (u->c[0]) {
        u->c[0]->p = 0;
        u->c[0]->flip ^= 1;
        u->c[0]->pp = u;
        u->c[0] = 0; //41e
        u->fix();
    }
}

Node* access(Node* u) {
    u->splay(); //4e7
    while (Node* pp = u->pp) {
        pp->splay(); u->pp = 0;
        if (pp->c[1]) {
            pp->c[1]->p = 0; pp->c[1]->pp = pp;
            pp->c[1] = u; pp->fix(); u = pp; //f4d
        }
        return u;
    }
};
```

DirectedMST.h

**Description:** Finds a minimum spanning tree/arborescence of a directed graph, given a root node. If no MST exists, returns -1.

**Time:**  $\mathcal{O}(E \log V)$  "../data-structures/UnionFindRollback.h" 057d96, 60 lines

```
struct Edge { int a, b; ll w(); }; //4d9
struct Node {
    Edge key;
    Node *l=0, *r=0;
    ll delta{};
    void prop() { //936
        key.w += delta;
        if (l) l->delta += delta;
        if (r) r->delta += delta;
        delta = 0;
    } //5dc
    Edge top() { prop(); return key; }
};

Node *merge(Node *a, Node *b) {
    if (!a || !b) return a ? b;
    a->prop(), b->prop(); //72a
    if (a->key.w > b->key.w) swap(a, b);
    swap(a->l, (a->r = merge(b, a->r)));
    return a;
}

void pop(Node*& a) { a->prop(); a = merge(a->l, a->r); } //8e9

pair<ll, vi> dmst(int n, int r, vector<Edge>& g) {
    RollbackUF uf(n);
    vector<Node*> heap(n);
    for (Edge e : g) heap[e.b] = merge(heap[e.b], new Node{e}); //0f3
    ll res = 0;
    vi seen(n, -1), path(n), par(n);
    seen[r] = r;
    vector<Edge> Q(n, in(n, {-1, -1})), comp;
    deque<tuple<int, int, vector<Edge>>> cys; //4c6
    rep(s, 0, n) {
        int u = s, qi = 0, w;
        while (seen[u] < 0) {
            if (!heap[u]) return {-1, {}};
            Edge e = heap[u]->top(); //2b0
```

```
        heap[u]->delta -= e.w, pop(heap[u]);
        Q[qi] = e, path[qi++] = u, seen[u] = s;
        res += e.w, u = uf.find(e.a);
        if (seen[u] == s) {
            Node* cyc = 0; //fff
            int end = qi, time = uf.time();
            do cyc = merge(cyc, heap[w = path[--qi]]);
            while (uf.join(u, w));
            u = uf.find(u), heap[u] = cyc, seen[u] = -1;
            cys.push_front({u, time, {&Q[qi], &Q[end]}}); //984
        }
    }
    rep(i, 0, qi) in[uf.find(Q[i].b)] = Q[i];
} //eba
for (auto& [u, t, cc] : cys) { // restore sol (optional)
    uf.rollback(t);
    Edge inEdge = in[u];
    for (auto& e : cc) in[uf.find(e.b)] = e;
    in[uf.find(inEdge.b)] = inEdge; //ffd
}
rep(i, 0, n) par[i] = in[i].a;
return {res, par};
}
```

LCA.h

**Description:** Data structure for computing lowest common ancestors in a tree (with 0 as root). C should be an adjacency list of the tree, either directed or undirected.

**Time:**  $\mathcal{O}(N \log N + Q)$  "../data-structures/RMQ.h" aa4761, 21 lines

```
struct LCA { //169
    int T = 0;
    vi time, path, ret;
    RMQ<int> rmq;

    LCA(vector<vi>& C) : time(sz(C)), rmq((dfs(C, 0, -1), ret)) {} //1e9
    void dfs(vector<vi>& C, int v, int par) {
        time[v] = T++;
        for (int y : C[v]) if (y != par) {
            path.pb(v), ret.push_back(time[v]);
            dfs(C, y, v); //3f8
        }
    }

    int lca(int a, int b) {
        if (a == b) return a; //3f5
        tie(a, b) = minmax(time[a], time[b]);
        return path[rmq.query(a, b)];
    }
    //dist(a,b){return depth[a] + depth[b] - 2*depth[lca(a,b)];}
}; //214
```

7.7 Math

7.7.1 Number of Spanning Trees

Create an  $N \times N$  matrix  $\text{mat}$ , and for each edge  $a \rightarrow b \in G$ , do  $\text{mat}[a][b]--$ ,  $\text{mat}[b][b]++$  (and  $\text{mat}[b][a]--$ ,  $\text{mat}[a][a]++$  if  $G$  is undirected). Remove the  $i$ th row and column and take the determinant; this yields the number of directed spanning trees rooted at  $i$  (if  $G$  is undirected, remove any row/column).

### 7.7.2 Erdős–Gallai theorem

A simple graph with node degrees  $d_1 \geq \cdots \geq d_n$  exists iff  $d_1 + \cdots + d_n$  is even and for every  $k = 1 \dots n$ ,

$$\sum_{i=1}^k d_i \leq k(k-1) + \sum_{i=k+1}^n \min(d_i, k).$$

## Geometry (8)

### 8.1 Geometric primitives

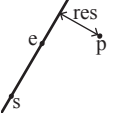
**Point.h**  
**Description:** Class to handle points in the plane. T can be e.g. double or long long. (Avoid int.)

```
634da7, 29 lines
template <class T> int sgn(T x) { return (x > 0) - (x < 0); }
} //fa7
template<class T>
struct Point {
    typedef Point P;
    T x, y;
    explicit Point(T _x=0, T _y=0) : x(_x), y(_y) {} //a5f
    bool operator<(P p) const { return tie(x,y) < tie(p.x,p.y); }
    bool operator==(P p) const { return tie(x,y)==tie(p.x,p.y); }
    P operator+(P p) const { return P(x+p.x, y+p.y); }
    P operator-(P p) const { return P(x-p.x, y-p.y); }
    P operator*(T d) const { return P(x*d, y*d); } //e11
    P operator/(T d) const { return P(x/d, y/d); }
    T dot(P p) const { return x*p.x + y*p.y; }
    T cross(P p) const { return x*p.y - y*p.x; } // + => p on right
    T cross(P a, P b) const { return (a-*this).cross(b-*this); }
    bool half() const { return y < 0 || (y == 0 && x < 0); } //053
    T dist2() const { return x*x + y*y; }
    double dist() const { return sqrt((double)dist2()); }
    // angle to x-axis in interval [-pi, pi]
    double angle() const { return atan2(y, x); }
    P unit() const { return *this/dist(); } // makes dist() =1//8da
    P perp() const { return P(-y, x); } // rotates +90 degrees
    P normal() const { return perp().unit(); }
    // returns point rotated 'a' radians ccw around the origin
    P rotate(double a) const {
        return P(x*cos(a)-y*sin(a),x*sin(a)+y*cos(a)); } //ad4
    friend ostream& operator<<(ostream& os, P p) {
        return os << "(" << p.x << ", " << p.y << ")"; }
};
```

**AngleSort.h**  
**Description:** Sorts points radially across the origin. To sort around a point, sort a-p and b-p.

```
c10d46, 7 lines
Point.h
template<class P> //159
void anglesort(vector<P> &v, P p=P(0, 0)) {
    sort(all(v), [p](P a, P b) {
        a = a - p, b = b - p;
        return a.half() == b.half() ? a.cross(b) > 0 : a.half() < b.half();
    }); //b97
}
```

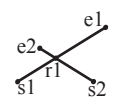
**lineDistance.h**  
**Description:**  
Returns the signed distance between point p and the line containing points a and b. Positive value on left side and negative on right as seen from a towards b. a==b gives nan. P is supposed to be Point<T> or Point3D<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long. Using Point3D will always give a non-negative distance. For Point3D, call .dist on the result of the cross product.



```
f6bf6b, 4 lines
Point.h
template<class P> //f6b
double lineDist(const P& a, const P& b, const P& p) {
    return (double) (b-a).cross(p-a) / (b-a).dist();
}

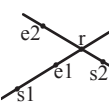
SegmentDistance.h
Description:
Returns the shortest distance between point p and the line segment from point s to e.
Usage: Point<double> a, b(2,2), p(1,1);
bool onSegment = segDist(a,b,p) < 1e-10;
Point.h
5c88f4, 6 lines
typedef Point<double> P; //b95
double segDist(P& s, P& e, P& p) {
    if (s==e) return (p-s).dist();
    auto d = (e-s).dist2(), t = min(d,max(.0, (p-s).dot(e-s)));
    ;
    return ((p-s)*d-(e-s)*t).dist()/d;
} //cbb
```

**SegmentIntersection.h**  
**Description:**  
If a unique intersection point between the line segments going from s1 to e1 and from s2 to e2 exists then it is returned. If no intersection point exists an empty vector is returned. If infinitely many exist a vector with 2 elements is returned, containing the endpoints of the common line segment. The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or long long.



```
Usage: vector<P> inter = segInter(s1,e1,s2,e2);
if (sz(inter)==1)
cout << "segments intersect at " << inter[0] << endl;
Point.h", "OnSegment.h"
9d57f2, 13 lines
template<class P> vector<P> segInter(P a, P b, P c, P d) {
    //dec
    auto oa = c.cross(d, a), ob = c.cross(d, b),
        oc = a.cross(b, c), od = a.cross(b, d);
    // Checks if intersection is single non-endpoint point.
    if (sgn(oa) * sgn(ob) < 0 && sgn(oc) * sgn(od) < 0)
        return {(a * ob - b * oa) / (ob - oa)}; //8a0
    set<P> s;
    if (onSegment(c, d, a)) s.insert(a);
    if (onSegment(c, d, b)) s.insert(b);
    if (onSegment(a, b, c)) s.insert(c);
    if (onSegment(a, b, d)) s.insert(d); //814
```

**lineIntersection.h**  
**Description:**  
If a unique intersection point of the lines going through s1,e1 and s2,e2 exists {1, point} is returned. If no intersection point exists {0, (0,0)} is returned and if infinitely many exists {-1, (0,0)} is returned. The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or ll.



```
Usage: auto res = lineInter(s1,e1,s2,e2);
if (res.first == 1)
cout << "intersection point at " << res.second << endl;
Point.h
a01f81, 8 lines
template<class P> //47e
pair<int, P> lineInter(P s1, P e1, P s2, P e2) {
    auto d = (e1 - s1).cross(e2 - s2);
    if (d == 0) // if parallel
        return {-(s1.cross(e1, s2) == 0), P(0, 0)};
    auto p = s2.cross(e1, e2), q = s2.cross(e2, s1); //16d
    return {1, (s1 * p + e1 * q) / d};
}
```

**sideOf.h**  
**Description:** Returns where p is as seen from s towards e. 1/0/-1 ⇔ left/on line/right. If the optional argument eps is given 0 is returned if p is within distance eps from the line. P is supposed to be Point<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long.  
**Usage:** bool left = sideOf(p1,p2,q)==1;

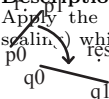
```
Point.h
3af81c, 9 lines
template<class P> //059
int sideOf(P s, P e, P p) { return sgn(s.cross(e, p)); }

template<class P>
int sideOf(const P& s, const P& e, const P& p, double eps)
{
    auto a = (e-s).cross(p-s); //7c7
    double l = (e-s).dist()*eps;
    return (a > l) - (a < -l);
}
```

**OnSegment.h**  
**Description:** Returns true iff p lies on the line segment from s to e. Use (segDist(s,e,p)<=epsilon) instead when using Point<double>.

```
Point.h
c597e8, 3 lines
template<class P> bool onSegment(P s, P e, P p) { //c59
    return p.cross(s, e) == 0 && (s - p).dot(e - p) <= 0;
}
```

**linearTransformation.h**  
**Description:**  
Apply the linear transformation (translation, rotation and scaling) which takes line p0-p1 to line q0-q1 to point r.



```
Point.h
03a306, 6 lines
typedef Point<double> P; //d52
P linearTransformation(const P& p0, const P& p1,
    const P& q0, const P& q1, const P& r) {
    P dp = p1-p0, dq = q1-q0, num(dp.cross(dq), dp.dot(dq));
```

```
    return q0 + P((r-p0).cross(num), (r-p0).dot(num))/dp.
        dist2();
} //cbb
```

### LineProjectionReflection.h

**Description:** Projects point p onto line ab. Set refl=true to get reflection of point p across line ab instead. The wrong point will be returned if P is an integer point and the desired point doesn't have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow.

"Point.h"	b5562d, 5 lines
-----------	-----------------

```
template<class P> //b55
P lineProj(P a, P b, P p, bool refl=false) {
    P v = b - a;
    return p - v.perp()*(1+refl)*v.cross(p-a)/v.dist2();
}
```

Equation of line  $Ax + By + C = 0$  from two points  $(x_1, y_1)$  and  $(x_2, y_2)$ :  $A = y_1 - y_2$ ,  $B = x_2 - x_1$ , and  $C = x_1y_2 - y_1x_2$ .

## 8.2 Circles

### CircleIntersection.h

**Description:** Computes the pair of points at which two circles intersect. Returns false in case of no intersection.

"Point.h"	28e539, 11 lines
-----------	------------------

```
typedef Point<double> P; //deb
bool circleInter(P a,P b,double r1,double r2,pair<P, P>*
    out) {
    if (a == b) { assert(r1 != r2); return false; }
    P vec = b - a;
    double d2 = vec.dist2(), sum = r1+r2, dif = r1-r2,
        p = (d2 + r1*r1 - r2*r2)/(d2*2), h2 = r1*r1 - p*p*d2
            ; //347
    if (sum*sum < d2 || dif*dif > d2) return false;
    P mid = a + vec*p, per = vec.perp() * sqrt(fmax(0, h2) /
        d2);
    *out = {mid - per, mid + per};
    return true;
} //cbb
```

### CircleTangents.h

**Description:** Finds the external tangents of two circles, or internal if r2 is negated. Can return 0, 1, or 2 tangents – 0 if one circle contains the other (or overlaps it, in the internal case, or if the circles are the same); 1 if the circles are tangent to each other (in which case .first = .second and the tangent line is perpendicular to the line between the centers). .first and .second give the tangency points at circle 1 and 2 respectively. To find the tangents of a circle with a point set r2 to 0.

"Point.h"	31cca4, 13 lines
-----------	------------------

```
template<class P> //c18
vector<pair<P, P>> tangents(P c1, double r1, P c2, double
    r2) {
    P d = c2 - c1;
    double dr = r1 - r2, d2 = d.dist2(), h2 = d2 - dr * dr;
    if (d2 == 0 || h2 < 0) return {};
    vector<pair<P, P>> out; //5c8
    for (double sign : {-1, 1}) {
        P v = (d * dr + d.perp() * sqrt(h2) * sign) / d2;
        out.pb({c1 + v * r1, c2 + v * r2});
    }
    if (h2 == 0) out.pop_back(); //918
    return out;
}
```

### CircleLine.h

**Description:** Finds the intersection between a circle and a line. Returns a vector of either 0, 1, or 2 intersection points. P is intended to be Point<double>.

"Point.h"	e0cfba, 9 lines
-----------	-----------------

```
template<class P> //64a
vector<P> circleLine(P c, double r, P a, P b) {
    P ab = b - a, p = a + ab * (c-a).dot(ab) / ab.dist2();
    double s = a.cross(b, c), h2 = r*r - s*s / ab.dist2();
    if (h2 < 0) return {};
    if (h2 == 0) return {p}; //fd3
    P h = ab.unit() * sqrt(h2);
    return {p - h, p + h};
}
```

### CirclePolygonIntersection.h

**Description:** Returns the area of the intersection of a circle with a ccw polygon.

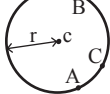
"../..content/geometry/Point.h"	alee63, 19 lines
---------------------------------	------------------

```
typedef Point<double> P; //a6c
#define arg(p, q) atan2(p.cross(q), p.dot(q))
double circlePoly(P c, double r, vector<P> ps) {
    auto tri = [&](P p, P q) {
        auto r2 = r * r / 2;
        P d = q - p; //eda
        auto a = d.dot(p)/d.dist2(), b = (p.dist2()-r*r)/d.
            dist2();
        auto det = a * a - b;
        if (det <= 0) return arg(p, q) * r2;
        auto s = max(0., -a-sqrt(det)), t = min(1., -a+sqrt(det
            ));
        if (t < 0 || 1 <= s) return arg(p, q) * r2; //174
        P u = p + d * s, v = p + d * t;
        return arg(p,u) * r2 + u.cross(v)/2 + arg(v,q) * r2;
    };
    auto sum = 0.0;
    rep(i,0,sz(ps)) //a61
        sum += tri(ps[i] - c, ps[(i + 1) % sz(ps)] - c);
    return sum;
}
```

### circumcircle.h

**Description:**

The circumcirle of a triangle is the circle intersecting all three vertices. ccRadius returns the radius of the circle going through points A, B and C and ccCenter returns the center of the same circle.



"Point.h"	1caa3a, 9 lines
-----------	-----------------

```
typedef Point<double> P; //032
double ccRadius(const P& A, const P& B, const P& C) {
    return (B-A).dist()*(C-B).dist()*(A-C).dist()/
        abs((B-A).cross(C-A))/2;
}
P ccCenter(const P& A, const P& B, const P& C) { //793
    P b = C-A, c = B-A;
    return A + (b*c.dist2()-c*b.dist2()).perp()/b.cross(c)/2;
}
```

### MinimumEnclosingCircle.h

**Description:** Computes the minimum circle that encloses a set of points.

"Point.h"	09dd0a, 17 lines
-----------	------------------

```
pair<P, double> mec(vector<P> ps) { //b50
    shuffle(all(ps), mt19937(time(0)));
}
```

```
P o = ps[0];
double r = 0, EPS = 1 + 1e-8;
rep(i,0,sz(ps)) if ((o - ps[i]).dist() > r * EPS) {
    o = ps[i], r = 0; //d54
    rep(j,0,i) if ((o - ps[j]).dist() > r * EPS) {
        o = (ps[i] + ps[j]) / 2;
        r = (o - ps[i]).dist();
        rep(k,0,j) if ((o - ps[k]).dist() > r * EPS) {
            o = ccCenter(ps[i], ps[j], ps[k]); //4ec
            r = (o - ps[i]).dist();
        }
    }
}
return {o, r}; //2ac
}
```

## 8.3 Polygons

### InsidePolygon.h

**Description:** Returns true if p lies within the polygon. If strict is true, it returns false for points on the boundary. The algorithm uses products in intermediate steps so watch out for overflow.

**Usage:** vector<P> v = {P{4,4}, P{1,2}, P{2,1}}; bool in = inPolygon(v, P{3, 3}, false); **Time:**  $\mathcal{O}(n)$

"Point.h", "OnSegment.h", "SegmentDistance.h"	2bf504, 11 lines
---	------------------

```
template<class P> //1c1
bool inPolygon(vector<P> &p, P a, bool strict = true) {
    int cnt = 0, n = sz(p);
    rep(i,0,n) {
        P q = p[(i + 1) % n];
        if (onSegment(p[i], q, a) return !strict; //fa7
            //or: if (segDist(p[i], q, a) <= eps) return !strict;
        cnt ^= ((a.y<p[i].y) - (a.y<q.y)) * a.cross(p[i], q) >
            0;
    }
    return cnt;
} //cbb
```

### PolygonArea.h

**Description:** Returns twice the signed area of a polygon. Clockwise enumeration gives negative area. Watch out for overflow if using int as T!

"Point.h"	f12300, 6 lines
-----------	-----------------

```
template<class T> //b19
T polygonArea2(vector<Point<T>& v) {
    T a = v.back().cross(v[0]);
    rep(i,0,sz(v)-1) a += v[i].cross(v[i+1]);
    return a;
} //cbb
```

### PolygonCenter.h

**Description:** Returns the center of mass for a polygon. **Time:**  $\mathcal{O}(n)$

"Point.h"	9706dc, 9 lines
-----------	-----------------

```
typedef Point<double> P; //082
P polygonCenter(const vector<P>& v) {
    P res(0, 0); double A = 0;
    for (int i = 0, j = sz(v) - 1; i < sz(v); j = i++) {
        res = res + (v[i] + v[j]) * v[j].cross(v[i]);
        A += v[j].cross(v[i]); //168
    }
    return res / A / 3;
}
```

### PolygonCut.h

**Description:**  
Returns a vector with the vertices of a polygon with everything to the left of the line going from s to e cut away.



```
Usage: vector<P> p = ...;
p = polygonCut(p, P(0,0), P(1,0));

"Point.h", "lineIntersection.h" 056a39, 13 lines

typedef Point<double> P; //366
vector<P> polygonCut(const vector<P>& poly, P s, P e) {
    vector<P> res;
    rep(i,0,sz(poly)) {
        P cur = poly[i], prev = i ? poly[i-1] : poly.back();
        bool side = s.cross(e, cur) < 0; //c08
        if (side != (s.cross(e, prev) < 0))
            res.pb(lineInter(s, e, cur, prev).second);
        if (side)
            res.pb(cur);
    } //0e1
    return res;
}
```

PolygonUnion.h

**Description:** Calculates the area of the union of  $n$  polygons (not necessarily convex). The points within each polygon must be given in CCW order. (Epsilon checks may optionally be added to sideOf/sgn, but shouldn't be needed.)

**Time:**  $O(N^2)$ , where  $N$  is the total number of points

```
"Point.h", "sideOf.h" 3931c6, 33 lines

typedef Point<double> P; //49c
double rat(P a, P b) { return sgn(b.x) ? a.x/b.x : a.y/b.y;
}

double polyUnion(vector<vector<P>>& poly) {
    double ret = 0;
    rep(i,0,sz(poly)) rep(v,0,sz(poly[i])) {
        P A = poly[i][v], B = poly[i][(v + 1) % sz(poly[i])]; //896
        vector<pair<double, int>> segs = {{0, 0}, {1, 0}};
        rep(j,0,sz(poly)) if (i != j) {
            rep(u,0,sz(poly[j])) {
                P C = poly[j][u], D = poly[j][(u + 1) % sz(poly[j])];
                int sc = sideOf(A, B, C), sd = sideOf(A, B, D); //407
                if (sc != sd) {
                    double sa = C.cross(D, A), sb = C.cross(D, B);
                    if (min(sc, sd) < 0)
                        segs.emplace_back(sa / (sa - sb), sgn(sc - sd));
                } else if (!sc && !sd && j < i && sgn((B-A).dot(D-C)) > 0) { //8be
                    segs.emplace_back(rat(C - A, B - A), 1);
                    segs.emplace_back(rat(D - A, B - A), -1);
                }
            }
        } //155
        sort(all(segs));
        for (auto& s : segs) s.first = min(max(s.first, 0.0), 1.0);
        double sum = 0;
        int cnt = segs[0].second;
        rep(j,1,sz(segs)) { //88e
            if (!cnt) sum += segs[j].first - segs[j - 1].first;
            cnt += segs[j].second;
        }
        ret += A.cross(B) * sum;
    } //f48
    return ret / 2;
}
```

ConvexHull.h

**Description:**  
Returns a vector of the points of the convex hull in counter-clockwise order. Points on the edge of the hull between two other points are not considered part of the hull.



```
Time: O(n log n)

"Point.h" 310954, 13 lines

typedef Point<ll> P; //3e3
vector<P> convexHull(vector<P> pts) {
    if (sz(pts) <= 1) return pts;
    sort(all(pts));
    vector<P> h(sz(pts)+1);
    int s = 0, t = 0; //f18
    for (int it = 2; it--; s = --t, reverse(all(pts)))
        for (P p : pts) {
            while (t >= s + 2 && h[t-2].cross(h[t-1], p) <= 0) t--;
            h[t++] = p;
        } //aa0
    return {h.begin(), h.begin() + t - (t == 2 && h[0] == h[1])};
}
```

HullDiameter.h

**Description:** Returns the two points with max distance on a convex hull (ccw, no duplicate/collinear points).

**Time:**  $O(n)$

```
"Point.h" c571b8, 12 lines

typedef Point<ll> P; //5c7
array<P, 2> hullDiameter(vector<P> S) {
    int n = sz(S), j = n < 2 ? 0 : 1;
    pair<ll, array<P, 2>> res({0, {S[0], S[0]}});
    rep(i,0,j)
        for (; j = (j + 1) % n) { //56c
            res = max(res, {{S[i] - S[j]}.dist2(), {S[i], S[j]}});
            if ((S[(j + 1) % n] - S[j]).cross(S[i + 1] - S[i]) >= 0)
                break;
        }
    return res.second; //52a
}
```

PointInsideHull.h

**Description:** Determine whether a point  $t$  lies inside a convex hull (CCW order, with no collinear points). Returns true if point lies within the hull. If strict is true, points on the boundary aren't included.

**Time:**  $O(\log N)$

```
"Point.h", "sideOf.h", "OnSegment.h" 71446b, 14 lines

typedef Point<ll> P; //7a3

bool inHull(const vector<P>& l, P p, bool strict = true) {
    int a = 1, b = sz(l) - 1, r = !strict;
    if (sz(l) < 3) return r && onSegment(l[0], l.back(), p);
    if (sideOf(l[0], l[a], l[b]) > 0) swap(a, b); //4a6
    if (sideOf(l[0], l[a], p) >= r || sideOf(l[0], l[b], p) <= -r)
        return false;
    while (abs(a - b) > 1) {
        int c = (a + b) / 2;
        (sideOf(l[0], l[c], p) > 0 ? b : a) = c; //0da
    }
    return sgn(l[a].cross(l[b], p)) < r;
}
```

LineHullIntersection.h

**Description:** Line-convex polygon intersection. The polygon must be ccw and have no collinear points. lineHull(line, poly) returns a pair describing the intersection of a line with the polygon:  $\bullet (-1, -1)$  if no collision,  $\bullet (i, -1)$  if touching the corner  $i$ ,  $\bullet (i, i)$  if along side  $(i, i + 1)$ ,  $\bullet (i, j)$  if crossing sides  $(i, i + 1)$  and  $(j, j + 1)$ . In the last case, if a corner  $i$  is crossed, this is treated as happening on side  $(i, i + 1)$ . The points are returned in the same order as the line hits the polygon. extrVertex returns the point of a hull with the max projection onto a line.

**Time:**  $O(\log n)$

```
"Point.h" 7cf45b, 39 lines

#define cmp(i,j) sgn(dir.perp().cross(poly[(i)%n]-poly[(j)%n])) //b9d
#define extr(i) cmp(i + 1, i) >= 0 && cmp(i, i - 1 + n) < 0
template <class P> int extrVertex(vector<P>& poly, P dir) {
    int n = sz(poly), lo = 0, hi = n;
    if (extr(0)) return 0;
    while (lo + 1 < hi) { //51a
        int m = (lo + hi) / 2;
        if (extr(m)) return m;
        int ls = cmp(lo + 1, lo), ms = cmp(m + 1, m);
        (ls < ms || (ls == ms && ls == cmp(lo, m)) ? hi : lo) = m;
    } //e8c
    return lo;
}

#define cmpL(i) sgn(a.cross(poly[i], b))
template <class P> //7fd
array<int, 2> lineHull(P a, P b, vector<P>& poly) {
    int endA = extrVertex(poly, (a - b).perp());
    int endB = extrVertex(poly, (b - a).perp());
    if (cmpL(endA) < 0 || cmpL(endB) > 0)
        return {-1, -1}; //04b
    array<int, 2> res;
    rep(i,0,2) {
        int lo = endB, hi = endA, n = sz(poly);
        while ((lo + 1) % n != hi) {
            int m = ((lo + hi + (lo < hi ? 0 : n)) / 2) % n; //ec0
            (cmpL(m) == cmpL(endB) ? lo : hi) = m;
        }
        res[i] = (lo + !cmpL(hi)) % n;
        swap(endA, endB);
    } //6ab
    if (res[0] == res[1]) return {res[0], -1};
    if (!cmpL(res[0]) && !cmpL(res[1]))
        switch ((res[0] - res[1] + sz(poly) + 1) % sz(poly)) {
            case 0: return {res[0], res[0]};
            case 2: return {res[1], res[1]}; //08a
        }
    return res;
}
```

HullTangents.h

**Description:** Finds the two tangent vertices on the convex hull to some point. Point must be outside. Appears to be left then right.

```
"Point.h" 0bddcf, 22 lines

template<typename P, typename F> //134
int extremeVertex(const P& poly, F direction) {
    int n = sz(poly), l = 0, ls;
    auto vertexCmp = [&](int i, int j) {
        return sgn(direction(poly[j]).cross(poly[j] - poly[i]));
    };
    auto isExtreme = [&](int i, int& is) { //d3d
        return (is = vertexCmp((i+1)%n, i)) >= 0 && vertexCmp(i, (i+n-1)%n) < 0;
    };
    for (int r = isExtreme(0, ls) ? 1 : n; l + 1 < r; ) {
        int m = (l + r) / 2, ms;
        if (isExtreme(m, ms)) return m;
    }
}
```

```
    if (ls != ms ? ls < ms : ls == vertexCmp(l, m)) r = m; //
        beb
    else l = m, ls = ms;
    }
    return l;
}
//d22
template<typename P>
pair<int, int> tangentsConvex(const P &point, const vector<
    P>& poly) {
    return {
        extremeVertex(poly, [&](const P& q) { return q - point; }
        ),
        extremeVertex(poly, [&](const P& q) { return point - q; }
        )}; //fa7
}
```

MinkowskiSum.h

**Description:** Minkowski sum of set of convex ccw polygons.  
**Time:**  $\mathcal{O}(P \log N)$ , where  $P$  is number of points and  $N$  is number of polygons.

"Point.h"	ecfe9a, 33 lines
-----------	------------------

```
typedef Point<ll> P; //657
vector<P> minkowskiSum(vector<vector<P>>> hs) {
    auto cmp = [](P a, P b) {
        return make_pair(a.x < 0 || a.x == 0 && a.y < 0, a.y *
            (ll)b.x)
            < make_pair(b.x < 0 || b.x == 0 && b.y < 0, a.x * (ll)
            )b.y);
    }; //289
    typedef tuple<P, int, int> T;
    auto cmp_tup = [&cmp](T a, T b) {
        auto& [pa, ja, ia] = a;
        auto& [pb, jb, ib] = b;
        if (cmp(pa, pb)) return false; //9d0
        if (cmp(pb, pa)) return true;
        return make_pair(ja, ia) < make_pair(jb, ib);
    };
    priority_queue<T, vector<T>, decltype(cmp_tup)> pq(
        cmp_tup);
    P cur = P(); //404
    int s = 0, t = 0;
    rep(i, 0, sz(hs)) {
        auto& v = hs[i];
        rotate(begin(v), min_element(all(v)), end(v));
        if (sz(v) > 1) pq.push({v[1] - v[0], 0, i}), s += sz(v)
            ; //cee
        cur = cur + v[0];
    }
    vector<P> h(s + 1);
    for (h[t++] = cur; sz(pq);) {
        auto [p, j, i] = pq.top(); pq.pop(); //451
        t -= (t >= 2 && !cmp(h[t - 1] - h[t - 2], p));
        h[t++] = (cur = cur + p);
        auto& v = hs[i];
        if (++j < sz(v)) pq.push({v[(j + 1) % sz(v)] - v[j], j,
            i});
    } //297
    return {h.begin(), h.begin() + t - (t >= 2 && h[0] == h[t
        - 1])};
}
```

Centers.h

**Description:** Common triangle centers  
**Time:** fast

"Point.h"	87d7f4, 16 lines
-----------	------------------

```
typedef Point<double> P; //d2b

P incenter(P a, P b, P c) {
```

```
    double p = (a - b).dist() + (b - c).dist() + (c - a).
        dist();
    return (a * (b - c).dist() + b * (c - a).dist() + c * (
        a - b).dist()) / p;
} //39d

P circumcenter(P a, P b, P c) {
    P p = b - a, q = c - a,
        s(p.dist2() / 2, q.dist() / 2);
    return a + P(s.cross(P(p.y, q.y)), P(p.x, q.x).cross(s))
        / p.cross(q); //4cd
}

P orthocenter(P a, P b, P c) {
    return a + b + c - circumcenter(a, b, c) * 2;
} //cbb
```

8.4 Misc. Point Set Problems

ClosestPair.h

**Description:** Finds the closest pair of points.  
**Time:**  $\mathcal{O}(n \log n)$

"Point.h"	ac41a6, 17 lines
-----------	------------------

```
typedef Point<ll> P; //9e7
pair<P, P> closest(vector<P> v) {
    assert(sz(v) > 1);
    set<P> S;
    sort(all(v), [](P a, P b) { return a.y < b.y; });
    pair<ll, pair<P, P>> ret{LLONG_MAX, {P(), P()}}; //e83
    int j = 0;
    for (P p : v) {
        P d{1 + (ll)sqrt(ret.first), 0};
        while (v[j].y <= p.y - d.x) S.erase(v[j++]);
        auto lo = S.lower_bound(p - d), hi = S.upper_bound(p +
            d); //cb2
        for (; lo != hi; ++lo)
            ret = min(ret, {( *lo - p).dist2(), { *lo, p } });
        S.insert(p);
    }
    return ret.second; //982
}
```

ManhattanMST.h

**Description:** Given  $N$  points, returns up to  $4*N$  edges, which are guaranteed to contain a minimum spanning tree for the graph with edge weights  $w(p, q) = -p.x - q.x - + -p.y - q.y -$ . Edges are in the form (distance, src, dst). Use a standard MST algorithm on the result to find the final MST.  
**Time:**  $\mathcal{O}(N \log N)$

"Point.h"	e2611c, 23 lines
-----------	------------------

```
typedef Point<int> P; //bde
vector<array<int, 3>> manhattanMST(vector<P> ps) {
    vi id(sz(ps));
    iota(all(id), 0);
    vector<array<int, 3>> edges;
    rep(k, 0, 4) { //9bd
        sort(all(id), [&](int i, int j) {
            return (ps[i] - ps[j]).x < (ps[j] - ps[i]).y; });
        map<int, int> sweep;
        for (int i : id) {
            for (auto it = sweep.lower_bound(-ps[i].y); //0bb
                it != sweep.end(); sweep.erase(it++)) {
                int j = it->second;
                P d = ps[i] - ps[j];
                if (d.y > d.x) break;
                edges.pb({d.y + d.x, i, j}); //868
            }
            sweep[-ps[i].y] = i;
        }
    }
    for (P& p : ps) if (k & 1) p.x = -p.x; else swap(p.x, p
        .y);
}
```

```
    } //aa4
    return edges;
}

kdTree.h
Description: KD-tree (2d, can be extended to 3d)
"Point.h" bac5b0, 63 lines

typedef long long T; //632
typedef Point<T> P;
const T INF = numeric_limits<T>::max();

bool on_x(const P& a, const P& b) { return a.x < b.x; }
bool on_y(const P& a, const P& b) { return a.y < b.y; } //
    c56

struct Node {
    P pt; // if this is a leaf, the single point in it
    T x0 = INF, x1 = -INF, y0 = INF, y1 = -INF; // bounds
    Node *first = 0, *second = 0; //5b4

    T distance(const P& p) { // min squared distance to a
        point
        T x = (p.x < x0 ? x0 : p.x > x1 ? x1 : p.x);
        T y = (p.y < y0 ? y0 : p.y > y1 ? y1 : p.y);
        return (P(x,y) - p).dist2(); //a82
    }

    Node(vector<P>&& vp) : pt(vp[0]) {
        for (P p : vp) {
            x0 = min(x0, p.x); x1 = max(x1, p.x); //151
            y0 = min(y0, p.y); y1 = max(y1, p.y);
        }
        if (vp.size() > 1) {
            // split on x if width >= height (not ideal...)
            sort(all(vp), x1 - x0 >= y1 - y0 ? on_x : on_y); //1d2
            // divide by taking half the array for each child (
                not
            // best performance with many duplicates in the
                middle)
            int half = sz(vp)/2;
            first = new Node({vp.begin(), vp.begin() + half});
            second = new Node({vp.begin() + half, vp.end()}); //
                acc
        }
    }
};

struct KDTree { //72b
    Node* root;
    KDTree(const vector<P>& vp) : root(new Node({all(vp)})) {}

    pair<T, P> search(Node *node, const P& p) {
        if (!node->first) //119
            // uncomment if we should not find the point itself:
            // if (p == node->pt) return {INF, P()};
            return make_pair((p - node->pt).dist2(), node->pt);
        }
    } //a89

    Node *f = node->first, *s = node->second;
    T bfirst = f->distance(p), bsec = s->distance(p);
    if (bfirst > bsec) swap(bsec, bfirst), swap(f, s);

    // search closest side first, other side if needed//bfa
    auto best = search(f, p);
    if (bsec < best.first)
        best = min(best, search(s, p));
    return best;
} //13a
```

```
// find nearest point to a point, and its squared
distance
// (requires an arbitrary operator< for Point)
pair<T, P> nearest(const P& p) {
    return search(root, p); //213
}
};
```

FastDelaunay.h

**Description:** Fast Delaunay triangulation. Each circumcircle contains none of the input points. There must be no duplicate points. If all points are on a line, no triangles will be returned. Should work for doubles as well, though there may be precision issues in 'circ'. Returns triangles in order {t[0][0], t[0][1], t[0][2], t[1][0], ... }, all counter-clockwise. Time:  $O(n \log n)$

```
"Point.h"                                04ae3a, 88 lines

typedef Point<ll> P; //503
typedef struct Quad* Q;
typedef __int128_t ll1; // (can be ll if coords are < 2e4)
P arb(LLONG_MAX, LLONG_MAX); // not equal to any other point

struct Quad { //8bb
    Q rot, o; P p = arb; bool mark;
    P& F() { return r()->p; }
    Q& r() { return rot->rot; }
    Q prev() { return rot->o->rot; }
    Q next() { return r()->prev(); } //0bd
} *H;

bool circ(P p, P a, P b, P c) { // is p in the circumcircle
    ?
    ll1 p2 = p.dist2(), A = a.dist2()-p2,
        B = b.dist2()-p2, C = c.dist2()-p2; //520
    return p.cross(a,b)*C + p.cross(b,c)*A + p.cross(c,a)*B >
        0;
}

Q makeEdge(P orig, P dest) {
    Q r = H ? H : new Quad{new Quad{new Quad{new Quad{}}}};
    H = r->o; r->r()->r() = r; //60f
    rep(i,0,4) r = r->rot, r->p = arb, r->o = i & 1 ? r : r->
        r();
    r->p = orig; r->F() = dest;
    return r;
}

void splice(Q a, Q b) { //5b1
    swap(a->o->rot->o, b->o->rot->o); swap(a->o, b->o);
}

Q connect(Q a, Q b) {
    Q q = makeEdge(a->F(), b->p);
    splice(q, a->next()); //3cc
    splice(q->r(), b);
    return q;
}

}

pair<Q,Q> rec(const vector<P>& s) { //a03
    if (sz(s) <= 3) {
        Q a = makeEdge(s[0], s[1]), b = makeEdge(s[1], s.back()
            );
        if (sz(s) == 2) return { a, a->r() };
        splice(a->r(), b);
        auto side = s[0].cross(s[1], s[2]); //d54
        Q c = side ? connect(b, a) : 0;
        return {side < 0 ? c->r() : a, side < 0 ? c : b->r() };
    }

#define H(e) e->F(), e->p //f35
#define valid(e) (e->F().cross(H(base)) > 0)
    Q A, B, ra, rb;
    int half = sz(s) / 2;
    tie(ra, A) = rec({all(s) - half});
    tie(B, rb) = rec({sz(s) - half + all(s)}); //c17
```

FastDelaunay PolyhedronVolume Point3D 3dHull

```
while ((B->p.cross(H(A)) < 0 && (A = A->next()) ||
    (A->p.cross(H(B)) > 0 && (B = B->r()->o)));
Q base = connect(B->r(), A);
if (A->p == ra->p) ra = base->r();
if (B->p == rb->p) rb = base; //a99

#define DEL(e, init, dir) Q e = init->dir; if (valid(e)) \
    while (circ(e->dir->F(), H(base), e->F())) { \
        Q t = e->dir; \
        splice(e, e->prev()); //475
        splice(e->r(), e->r()->prev()); \
        e->o = H; H = e; e = t; \
    }
for (;) {
    DEL(LC, base->r(), o); DEL(RC, base, prev()); //031
    if (!valid(LC) && !valid(RC)) break;
    if (!valid(LC) || (valid(RC) && circ(H(RC), H(LC))))
        base = connect(RC, base->r());
    else
        base = connect(base->r(), LC->r()); //907
}
return { ra, rb };

vector<P> triangulate(vector<P> pts) { //e5d
    sort(all(pts)); assert(unique(all(pts)) == pts.end());
    if (sz(pts) < 2) return {};
    Q e = rec(pts).first;
    vector<Q> q = {e};
    int qi = 0; //dd4
    while (e->o->F().cross(e->F(), e->p) < 0) e = e->o;
#define ADD { Q c = e; do { c->mark = 1; pts.pb(c->p); \
    q.pb(c->r()); c = c->next(); } while (c != e); }
    ADD; pts.clear();
    while (qi < sz(q)) if (!(e = q[qi++])->mark) ADD; //24a
    return pts;
}

8.5 3D
PolyhedronVolume.h
Description: Magic formula for the volume of a polyhedron. Faces
should point outwards.                                3058c3, 6 lines

template<class V, class L> //27c
double signedPolyVolume(const V& p, const L& trilst) {
    double v = 0;
    for (auto i : trilst) v += p[i.a].cross(p[i.b]).dot(p[i.
        c]);
    return v / 6;
} //cbb
```

Point3D.h

```
Description: Class to handle points in 3D space. T can be e.g. double
or long long.                                         6eb43e, 32 lines

template<class T> struct Point3D { //811
    typedef Point3D P;
    typedef const P& R;
    T x, y, z;
    explicit Point3D(T _x=0, T _y=0, T _z=0) : x(_x), y(_y),
        z(_z) {}
    bool operator<(R p) const { //5e8
        return tie(x, y, z) < tie(p.x, p.y, p.z); }
    bool operator==(R p) const {
        return tie(x, y, z) == tie(p.x, p.y, p.z); }
    P operator+(R p) const { return P(x+p.x, y+p.y, z+p.z); }
    P operator-(R p) const { return P(x-p.x, y-p.y, z-p.z); }
        //9b1
    P operator*(T d) const { return P(x*d, y*d, z*d); }
    P operator/(T d) const { return P(x/d, y/d, z/d); }
    T dot(R p) const { return x*p.x + y*p.y + z*p.z; }
```

```
P cross(R p) const {
    return P(y*p.z - z*p.y, z*p.x - x*p.z, x*p.y - y*p.x);
    //58a
}
T dist2() const { return x*x + y*y + z*z; }
double dist() const { return sqrt((double)dist2()); }
//Azimuthal angle (longitude) to x-axis in interval [-pi,
    pi]
double phi() const { return atan2(y, x); } //a2c
//Zenith angle (latitude) to the z-axis in interval [0,
    pi]
double theta() const { return atan2(sqrt(x*x+y*y),z); }
P unit() const { return *this/(T)dist(); } //makes dist()
    =1
//returns unit vector normal to *this and p
P normal(P p) const { return cross(p).unit(); } //e88
//returns point rotated 'angle' radians ccw around axis
P rotate(double angle, P axis) const {
    double s = sin(angle), c = cos(angle); P u = axis.unit
        ();
    return u*dot(u)*(1-c) + (*this)*c - cross(u)*s;
    } //e03
};
```

3dHull.h

**Description:** Computes all faces of the 3-dimension hull of a point set. \*No four points must be coplanar\*, or else random results will be returned. All faces will point outwards. Time:  $O(n^2)$

```
"Point3D.h"                                         ce1872, 49 lines

typedef Point3D<double> P3; //e28

struct PR {
    void ins(int x) { (a == -1 ? a : b) = x; }
    void rem(int x) { (a == x ? a : b) = -1; }
    int cnt() { return (a != -1) + (b != -1); } //c34
    int a, b;
};

struct F { P3 q; int a, b, c; };
//36b
vector<F> hull3d(const vector<P3>& A) {
    assert(sz(A) >= 4);
    vector<vector<PR>> E(sz(A), vector<PR>(sz(A), {-1, -1}));
#define E(x,y) E[f.x][f.y]
    vector<F> FS; //de0
    auto mf = [&](int i, int j, int k, int l) {
        P3 q = (A[j] - A[i]).cross((A[k] - A[i]));
        if (q.dot(A[l]) > q.dot(A[i]))
            q = q * -1;
        F f{q, i, j, k}; //2be
        E(a,b).ins(k); E(a,c).ins(j); E(b,c).ins(i);
        FS.pb(f);
    };
    rep(i,0,4) rep(j,i+1,4) rep(k,j+1,4)
        mf(i, j, k, 6 - i - j - k); //e21

    rep(i,4,sz(A)) {
        rep(j,0,sz(FS)) {
            F f = FS[j];
            if(f.q.dot(A[i]) > f.q.dot(A[f.a])) { //b63
                E(a,b).rem(f.c);
                E(a,c).rem(f.b);
                E(b,c).rem(f.a);
                swap(FS[j--], FS.back());
                FS.pop_back(); //0df
            }
            int nw = sz(FS);
            rep(j,0,nw) {
                F f = FS[j]; //945
```



```
#define C(a, b, c) if (E(a,b).cnt() != 2) mf(f.a, f.b, i, f
    .c);
    C(a, b, c); C(a, c, b); C(b, c, a);
}
}
for (F& it : FS) if ((A[it.b] - A[it.a]).cross(//ab3
    A[it.c] - A[it.a]).dot(it.q) <= 0) swap(it.c, it.b);
return FS;
};
```

### sphericalDistance.h

**Description:** Returns the shortest distance on the sphere with radius radius between the points with azimuthal angles (longitude)  $f_1$  ( $\phi_1$ ) and  $f_2$  ( $\phi_2$ ) from x axis and zenith angles (latitude)  $t_1$  ( $\theta_1$ ) and  $t_2$  ( $\theta_2$ ) from z axis (0 = north pole). All angles measured in radians. The algorithm starts by converting the spherical coordinates to cartesian coordinates so if that is what you have you can use only the two last rows.  $dx \times$ radius is then the difference between the two points in the x direction and  $d \times$ radius is the total distance between the points.

```
double sphericalDistance(double f1, double t1,//6da
    double f2, double t2, double radius) {
    double dx = sin(t2)*cos(f2) - sin(t1)*cos(f1);
    double dy = sin(t2)*sin(f2) - sin(t1)*sin(f1);
    double dz = cos(t2) - cos(t1);
    double d = sqrt(dx*dx + dy*dy + dz*dz);//65e
    return radius*2*asin(d/2);
}
```

### SegmentDistance3D.h

**Description:** returns closest two points from each 3D segment

```
"P3oint3D.h"10f4a2, 40 lines
typedef Point3D<double> P3;//63e
//returns closest two points from each 3d segment
pair<P3, P3> segmentDistance3d(P3 s1, P3 e1, P3 s2, P3 e2)
{
    pair<P3, P3> res{s1, s2};
    auto check = [&res](P3 a, P3 b) {
        if ((b-a).dist() < (res.second-res.first).dist())//4d1
            res = {a, b};
    };
//check endpoint–endpoint
    check(s1, s2);
    check(s1, e2);//9e7
    check(e1, s2);
    check(e1, e2);

    P3 d1 = (e1-s1).unit();
    double t1Max = (e1-s1).dist();//3e0
    P3 d2 = (e2-s2).unit();
    double t2Max = (e2-s2).dist();
//check endpoint–segment, dist from p to the line segment
    s + t*d
    auto pointLine = [&](P3 p, P3 s, P3 d, double tMax) {
        P3 v = p-s;//f4b
        double t = v.dot(d);
        if (0 <= t && t <= tMax)
            check(s+d*t, p);
    };
    pointLine(s1, s2, d2, t2Max);//eea
    pointLine(e1, s2, d2, t2Max);
    pointLine(s2, s1, d1, t1Max);
    pointLine(e2, s1, d1, t1Max);

//check segment–segment//491
    P3 n = d1.cross(d2);
    if (n.dist2() != 0) { //only check if not parallel;
        parallel case is handled by pointLine checks
        double t1 = (d2.cross(n)).dot(s2-s1)/n.dot(n);
        double t2 = (d1.cross(n)).dot(s2-s1)/n.dot(n);
```

```
        if(0 <= t1 && t1 <= t1Max && 0 <= t2 && t2 <= t2Max)//
25f
            check(s1+d1*t1, s2+d2*t2);
        }
        return res;
    }
}
```

### PointToFace.h

**Description:** Finds the distance between a point and a 3d hull face

```
"Point3D.h", "3dHull.h"a9d101, 6 lines
typedef Point3D<double> P3;//f1f
double pointToFace(P3 p, F f) {
    auto dir = f.q.unit() * -1;
    auto vec = p - pts[f.a];
    return vec.dot(dir);
}//cbb
```

To project points onto a plane and map plane to  $xy$ -plane, project point onto the plane, use normalized  $(B - A)$  and  $(C - A)$  as unit vectors, and find projected point as linear combination of basis vectors on  $xy$ -plane.  $(B - A)$  maps to  $(1, 0)$ ,  $(C - A)$  maps to  $(\cos \theta, \sin \theta)$ , where  $\theta$  is the angle between  $B - A$  and  $C - A$ .

## Strings (9)

### KMP.h

**Description:**  $pi[x]$  computes the length of the longest prefix of  $s$  that ends at  $x$ , other than  $s[0...x]$  itself (abacaba -> 0010123). Can be used to find all occurrences of a string.

```
Time:  $\mathcal{O}(n)$ d4f9aa, 16 lines
vi pi(const string& s) //f6d
{
    vi p(sz(s));
    rep(i, 1, sz(s)) {
        int g = p[i-1];
        while (g && s[i] != s[g]) g = p[g-1];
        p[i] = g + (s[i] == s[g]);//0ff
    }
    return p;
}
```

```
vi match(const string& s, const string& pat) //9e6
{
    vi p = pi(pat + '\0' + s), res;
    rep(i, sz(p)-sz(s), sz(p))
        if (p[i] == sz(pat)) res.pb(i - 2 * sz(pat));
    return res;
}//cbb
```

### Zfunc.h

**Description:**  $z[i]$  computes the length of the longest common prefix of  $s[i:]$  and  $s$ , except  $z[0] = 0$ . (abacaba -> 0010301)

```
Time:  $\mathcal{O}(n)$ ee09e2, 12 lines
vi Z(const string& S) //fc3
{
    vi z(sz(S));
    int l = -1, r = -1;
    rep(i, 1, sz(S)) {
        z[i] = i >= r ? 0 : min(r - i, z[i - l]);
        while (i + z[i] < sz(S) && S[i + z[i]] == S[z[i]])//8ec
            z[i]++;
        if (i + z[i] > r)
            l = i, r = i + z[i];
    }
    return z;//939
}
```

### Manacher.h

**Description:** For each position in a string, computes  $p[0][i]$  = half length of longest even palindrome around pos  $i$ ,  $p[1][i]$  = longest odd (half rounded down).

```
Time:  $\mathcal{O}(N)$ e7ad79, 13 lines
array<vi, 2> manacher(const string& s) //510
{
    int n = sz(s);
    array<vi, 2> p = {vi(n+1), vi(n)};
    rep(z, 0, 2) for (int i=0, l=0, r=0; i < n; i++) {
        int t = r-i+!z;
        if (i<r) p[z][i] = min(t, p[z][l+t]);//f50
        int L = i-p[z][i], R = i+p[z][i]-!z;
        while (L>=1 && R+1<n && s[L-1] == s[R+1])
            p[z][i]++, L--, R++;
        if (R>r) l=L, r=R;
    }//291
    return p;
}
```

### MinRotation.h

**Description:** Finds the lexicographically smallest rotation of a string.

**Usage:** rotate(v.begin(), v.begin()+minRotation(v), v.end());

```
Time:  $\mathcal{O}(N)$ d07a42, 8 lines
int minRotation(string s) //20f
{
    int a=0, N=sz(s); s += s;
    rep(b, 0, N) rep(k, 0, N) {
        if (a+k == b || s[a+k] < s[b+k]) {b += max(0, k-1);
            break;}
        if (s[a+k] > s[b+k]) {a = b; break;}
    }//3a8
    return a;
}
```

### SuffixArray.h

**Description:** Builds suffix array for a string.  $sa[i]$  is the starting index of the suffix which is  $i$ 'th in the sorted suffix array. The returned vector is of size  $n + 1$ , and  $sa[0] = n$ . The lcp array contains longest common prefixes for neighbouring strings in the suffix array:  $lcp[i] = lcp(sa[i], sa[i-1])$ ,  $lcp[0] = 0$ . The input string must not contain any zero bytes.

**Time:**  $\mathcal{O}(n \log n)$

```
bc716b, 22 lines
struct SuffixArray //7a7
{
    vi sa, lcp;
    SuffixArray(string& s, int lim=256) { // or basic_string<
        int>
        int n = sz(s) + 1, k = 0, a, b;
        vi x(all(s)), y(n), ws(max(n, lim));
        x.push_back(0), sa = lcp = y, iota(all(sa), 0);//7c9
        for (int j = 0, p = 0; p < n; j = max(1, j * 2), lim =
            p) {
            p = j, iota(all(y), n - j);
            rep(i, 0, n) if (sa[i] >= j) y[p++] = sa[i] - j;
            fill(all(ws), 0);
            rep(i, 0, n) ws[x[i]]++;//f08
            rep(i, 1, lim) ws[i] += ws[i - 1];
            for (int i = n; i--;) sa[--ws[x[y[i]]]] = y[i];
            swap(x, y), p = 1, x[sa[0]] = 0;
            rep(i, 1, n) a = sa[i - 1], b = sa[i], x[b] =
                (y[a] == y[b] && y[a + j] == y[b + j]) ? p - 1 : p
                    ++;//726
        }
        for (int i = 0, j; i < n - 1; lcp[x[i++]] = k)
            for (k && k--, j = sa[x[i] - 1];
                s[i + k] == s[j + k]; k++);
    }//e03
};
```



Hashing.h

**Description:** Self-explanatory methods for string hashing. 5966ff, 44 lines

```
// Arithmetic mod 2^64-1. 2x slower than mod 2^64 and more
//d41
// code, but works on evil test data (e.g. Thue-Morse,
// where
// ABBA... and BAAB... of length 2^10 hash the same mod 2^
// 64).
// "typedef ull H;" instead if you think test data is
// random,
// or work mod 10^9+7 if the Birthday paradox is not a
// problem.
typedef uint64_t ull; //98c
struct H {
    ull x; H(ull x=0) : x(x) {}
    H operator+(H o) { return x + o.x + (x + o.x < x); }
    H operator-(H o) { return *this + ~o.x; }
    H operator*(H o) { auto m = (__uint128_t)x * o.x; //884
        return H((ull)m) + (ull)(m >> 64); }
    ull get() const { return x + !~x; }
    bool operator==(H o) const { return get() == o.get(); }
    bool operator<(H o) const { return get() < o.get(); }
}; //7dd
static const H C = (11)1e11+3; // (order ~ 3e9; random also
// ok)
```

```
struct HashInterval {
    vector<H> ha, pw;
    HashInterval(string& str) : ha(sz(str)+1), pw(ha) { //c1e
        pw[0] = 1;
        rep(i, 0, sz(str))
            ha[i+1] = ha[i] * C + str[i],
            pw[i+1] = pw[i] * C;
    } //b8f
    H hashInterval(int a, int b) { // hash [a, b)
        return ha[b] - ha[a] * pw[b - a];
    }
};
//4b7
vector<H> getHashes(string& str, int length) {
    if (sz(str) < length) return {};
    H h = 0, pw = 1;
    rep(i, 0, length)
        h = h * C + str[i], pw = pw * C; //6b3
    vector<H> ret = {h};
    rep(i, length, sz(str)) {
        ret.pb(h = h * C + str[i] - pw * str[i-length]);
    }
    return ret; //413
}
```

```
H hashString(string& s) { H h{}; for(char c:s) h=h*C+c; return
h; }
```

AhoCorasick.h

**Description:** Constructs Aho-Corasick automaton for given list of words

**Time:** Construction is  $\mathcal{O}(N)$ , move() is amortized  $\mathcal{O}(N)$ . 2d48ca, 34 lines

```
struct AhoCorasick { //be4
    struct Node {
        map<char, int> nxt;
        int lnk = 0, cnt = 0;
    };
    vector<Node> t; //b46
    AhoCorasick(vector<string> words) {
        t.pb({});
        for (string s : words) {
            int cur = 0;
            for (char c : s) { //638
                if (!t[cur].nxt[c]) {
```

```
                    t[cur].nxt[c] = sz(t);
                    t.pb({});
                }
                cur = t[cur].nxt[c]; //05c
            }
            t[cur].cnt++;
        }
        queue<int> q; q.push(0);
        while (!q.empty()) { //b39
            int u = q.front(); q.pop();
            assert(0 <= u && u < sz(t));
            for (auto &p : t[u].nxt) {
                if (u) t[p.second].lnk = move(t[u].lnk, p.first);
                q.push(p.second); //e67
            }
        }
        int move(int u, int c) {
            if (t[u].nxt[c]) return t[u].nxt[c]; //1f8
            return u == 0 ? 0 : t[u].nxt[c] = move(t[u].lnk, c);
        }
    };
};
```

SuffixAutomaton.h

**Description:** Constructs a suffix automaton on string s cnt() constructs an array of equivalence class sizes first() constructs an array of first ocurences for each node

**Time:** All functions are  $\mathcal{O}(N)$ . 24974f, 55 lines

```
struct SuffixAutomaton { //3d3
    struct Node {
        int len = 0, lnk = 0;
        int nxt[26];
    };
    string s; //f20
    vector<Node> t; int last = 0;
    SuffixAutomaton(string s = "") {
        t.pb({0, -1, {}});
        for (char c : s) add(c);
    } //a4a
    void add(char c) { s += c; c -= 'a';
        int u = last; int v = last = sz(t);
        t.pb({t[u].len + 1, 0, {}});
        while (u >= 0 && !t[u].nxt[c])
            t[u].nxt[c] = v, u = t[u].lnk; //b21
        if (u == -1) return;
        int q = t[u].nxt[c];
        if (t[u].len + 1 == t[q].len)
            { t[v].lnk = q; return; }
        int cpy = sz(t); t.pb(t[q]); //aa4
        t[cpy].len = t[u].len + 1;
        while (u >= 0 && t[u].nxt[c] == q)
            t[u].nxt[c] = cpy, u = t[u].lnk;
        t[v].lnk = t[q].lnk = cpy;
    } //2f2
    vector<int> cnt() {
        vector<int> res(sz(t), 0);
        int cur = 0;
        for (char c : s)
            res[cur = t[cur].nxt[c - 'a']]++; //82c
        vector<pair<int, int>> srt;
        for (int i = 1; i < sz(t); i++)
            srt.pb({-t[i].len, i});
        sort(all(srt));
        for (auto &p : srt) //cc8
            res[t[p.second].lnk] += res[p.second];
        return res;
    }
    vector<int> first() {
        vector<int> res(sz(t), sz(s)); //c3b
        int cur = 0;
```

```
        for (int i = 0; i < sz(s); i++) {
            cur = t[cur].nxt[s[i] - 'a'];
            res[cur] = min(res[cur], i);
        } //268
        vector<pair<int, int>> srt;
        for (int i = 1; i < sz(t); i++)
            srt.pb({-t[i].len, i});
        sort(all(srt));
        for (auto &p : srt) //23d
            res[t[p.second].lnk] = min(res[t[p.second].lnk], res[
                p.second]);
        return res;
    }
};
```

SuffixTree.h

**Description:** Ukkonen's algorithm for online suffix tree construction. Each node contains indices [l, r) into the string, and a list of child nodes. Suffixes are given by traversals of this tree, joining [l, r) substrings. The root is 0 (has l = -1, r = 0), non-existent children are -1. To get a complete tree, append a dummy symbol – otherwise it may contain an incomplete path (still useful for substring matching, though).

**Time:**  $\mathcal{O}(26N)$  aae0b8, 50 lines

```
struct SuffixTree { //b1f
    enum { N = 200010, ALPHA = 26 }; // N ~ 2*maxlen+10
    int toi(char c) { return c - 'a'; }
    string a; // v = cur node, q = cur position
    int t[N][ALPHA], l[N], r[N], p[N], s[N], v=0, q=0, m=2;
    //b11
    void ukkadd(int i, int c) { suff:
        if (r[v]<=q) {
            if (t[v][c]==-1) { t[v][c]=m; l[m]=i;
                p[m++]=v; v=s[v]; q=r[v]; goto suff; }
            v=t[v][c]; q=l[v]; //99f
        }
        if (q==-1 || c==toi(a[q])) q++; else {
            l[m+1]=i; p[m+1]=m; l[m]=l[v]; r[m]=q;
            p[m]=p[v]; t[m][c]=m+1; t[m][toi(a[q])]=v;
            l[v]=q; p[v]=m; t[p[m]][toi(a[l[m]])]=m; //604
            v=s[p[m]]; q=l[m];
            while (q<r[m]) { v=t[v][toi(a[q])]; q+=r[v]-l[v]; }
            if (q==r[m]) s[m]=v; else s[m]=m+2;
            q=r[v]-(q-r[m]); m+=2; goto suff;
        } //478
    }
}
```

```
SuffixTree(string a) : a(a) {
    fill(r, r+N, sz(a));
    memset(s, 0, sizeof s); //f11
    memset(t, -1, sizeof t);
    fill(t[1], t[1]+ALPHA, 0);
    s[0] = 1; l[0] = l[1] = -1; r[0] = r[1] = p[0] = p[1] =
        0;
    rep(i, 0, sz(a)) ukkadd(i, toi(a[i]));
} //d1a
```

```
// example: find longest common substring (uses ALPHA =
// 28)
pii best;
int lcs(int node, int i1, int i2, int olen) {
    if (l[node] <= i1 && i1 < r[node]) return 1; //636
    if (l[node] <= i2 && i2 < r[node]) return 2;
    int mask = 0, len = node ? olen + (r[node] - l[node]) :
        0;
    rep(c, 0, ALPHA) if (t[node][c] != -1)
        mask |= lcs(t[node][c], i1, i2, len);
    if (mask == 3) //a3a
        best = max(best, {len, r[node] - len});
    return mask;
}
```

```
static pii LCS(string s, string t) {
    SuffixTree st(s + (char)('z' + 1) + t + (char)('z' + 2)
    );//78c
    st.lcs(0, sz(s), sz(s) + 1 + sz(t), 0);
    return st.best;
}
};
```

### 9.0.1 Lyndon Decomposition

Duval.h

**Description:** Returns a list of starting indices of the strings in the Lyndon factorization of  $s$   
**Time:**  $\mathcal{O}(n)$

<pre>vi duval(string s) {//04b     int i = 0;     vi res;     while (i &lt; sz(s)) {         int j = i, k = i + 1;         while (k &lt; sz(s) &amp;&amp; s[k] &gt;= s[j]) {//7e1             if (s[k] &gt; s[j]) j = i;             else j++;             k++;         }         while (i &lt;= j) res.pb(i), i += k - j;//552     }     return res; }</pre>	125a69, 14 lines
---	------------------

$s$  is a Lyndon word  $\iff s$  is strictly smaller than all of its proper suffixes  $\iff s$  is strictly smaller than all of its nontrivial rotations

Lyndon factorization is the unique set of  $s = w_1 + \dots + w_k$  such that  $w$  is non-increasing and each  $w_i$  is a Lyndon word

$w_k$  is the smallest suffix of  $s$  (different from smallest rotation)

### 9.0.2 Run Enumeration

A run is a tuple  $(l, r, p)$  where  $p$  is the (partial) period of  $s[l, r]$  and  $[l, r]$  is maximal

A string of length  $n$  has at most  $n$  runs

Sum of  $(r - l)/p$  is at most  $3n$

Number of two-repeats is at most  $n \log n$

RunEnumeration.h

**Description:** Returns tuples  $(p, l, r)$ , representing the run  $[l, r]$  with period  $p$   
**Time:**  $\mathcal{O}(n \log n)$

<pre>struct LCP {//897     SuffixArray sa;     vector&lt;int&gt; pos;     RMQ&lt;int&gt; rmq;     LCP(string s) : sa(s), rmq(sa.lcp) {         pos = vector&lt;int&gt;(sz(s) + 1);//cd8         rep(i, 0, sz(s) + 1) pos[sa.sa[i]] = i;     }     int lcp(int i, int j) {         if (i == j) return sz(pos) - i;         int l = pos[i], r = pos[j];//0b3         if (l &gt; r) swap(l, r);         return rmq.query(l+1, r+1);     } };</pre>	2f002c, 42 lines
---	------------------

```
    }
};
//658
vector<vector<int>> run_enumerate(string s) {
    string sI = s;
    for (char &c : sI) c = 127 - c;
    set<vector<int>> runs;
    auto part = [&](string s) {//bb0
        string sR = s; reverse(all(sR));
        LCP lcp(s), lcpR(sR);
        vector<int> suf = lcp.sa.sa;
        vector<int> prefix(sz(s));
        set<int> st; st.insert(sz(s));//9b6
        rep(i, 1, sz(s)+1) {
            prefix[suf[i]] = *st.lower_bound(suf[i]) - suf[i]
            ];
            st.insert(suf[i]);
        }
        rep(i, 0, sz(s)) {//852
            int j = i + prefix[i];
            int k = j + (j == sz(s) ? 0 : lcp.lcp(i, j));
            int l = i - (i == 0 ? 0 : lcpR.lcp(sz(s) - j,
            sz(s) - i));
            if (k - l >= 2 * (j - i)) runs.insert({j - i, l
            , k});
        }
    };//bf7
};
part(s); part(sI);
vector<vector<int>> res;
for (auto &v : runs) res.pb(v);
return res;//510
}
```

## Various (10)

### 10.1 Intervals

IntervalContainer.h

**Description:** Add and remove intervals from a set of disjoint intervals. Will merge the added interval with any overlapping intervals in the set when adding. Intervals are [inclusive, exclusive).  
**Time:**  $\mathcal{O}(\log N)$

<pre>set&lt;pii&gt;::iterator addInterval(set&lt;pii&gt;&amp; is, int L, int R) {     //ba1     if (L == R) return is.end();     auto it = is.lower_bound({L, R}), before = it;     while (it != is.end() &amp;&amp; it-&gt;first &lt;= R) {         R = max(R, it-&gt;second);         before = it = is.erase(it);//ea6     }     if (it != is.begin() &amp;&amp; (--it)-&gt;second &gt;= L) {         L = min(L, it-&gt;first);         R = max(R, it-&gt;second);         is.erase(it);//05d     }     return is.insert(before, {L,R}); }  void removeInterval(set&lt;pii&gt;&amp; is, int L, int R) {//858     if (L == R) return;     auto it = addInterval(is, L, R);     auto r2 = it-&gt;second;     if (it-&gt;first == L) is.erase(it);     else (int&amp;)it-&gt;second = L;//61f     if (R != r2) is.emplace(R, r2); } }</pre>	edce47, 23 lines
--	------------------

IntervalCover.h

**Description:** Compute indices of smallest set of intervals covering another interval. Intervals should be [inclusive, exclusive). To support [inclusive, inclusive], change  $(A)$  to add  $||$   $R.empty()$ . Returns empty set on failure (or if  $G$  is empty).  
**Time:**  $\mathcal{O}(N \log N)$

<pre>template&lt;class T&gt;{//0e2 vi cover(pair&lt;T, T&gt; G, vector&lt;pair&lt;T, T&gt;&gt; I) {     vi S(sz(I)), R;     iota(all(S), 0);     sort(all(S), [&amp;](int a, int b) { return I[a] &lt; I[b]; });     T cur = G.first;//ed8     int at = 0;     while (cur &lt; G.second) { // (A)         pair&lt;T, int&gt; mx = make_pair(cur, -1);         while (at &lt; sz(I) &amp;&amp; I[S[at]].first &lt;= cur) {             mx = max(mx, make_pair(I[S[at]].second, S[at]));//607             at++;         }         if (mx.second == -1) return {};         cur = mx.first;         R.pb(mx.second);//661     }     return R; } }</pre>	a9491c, 19 lines
---	------------------

ConstantIntervals.h

**Description:** Split a monotone function on  $[from, to)$  into a minimal set of half-open intervals on which it has the same value. Runs a call-back  $g$  for each such interval.  
**Usage:** `constantIntervals(0, sz(v), [&](int x){return v[x];}, [&](int lo, int hi, T val){...});`  
**Time:**  $\mathcal{O}(k \log \frac{n}{k})$

<pre>template&lt;class F, class G, class T&gt;{//570 void rec(int from, int to, F&amp; f, G&amp; g, int&amp; i, T&amp; p, T q) {     if (p == q) return;     if (from == to) {         g(i, to, p);         i = to; p = q;//05f     } else {         int mid = (from + to) &gt;&gt; 1;         rec(from, mid, f, g, i, p, f(mid));         rec(mid+1, to, f, g, i, p, q);     } }  template&lt;class F, class G&gt; void constantIntervals(int from, int to, F f, G g) {     if (to &lt;= from) return;     int i = from; auto p = f(i), q = f(to-1);//a6c     rec(from, to-1, f, g, i, p, q);     g(i, to, q); } }</pre>	753a4c, 19 lines
---	------------------

### 10.2 Misc. algorithms

TernarySearch.h

**Description:** Find the smallest  $i$  in  $[a, b]$  that maximizes  $f(i)$ , assuming that  $f(a) < \dots < f(i) \geq \dots \geq f(b)$ . To reverse which of the sides allows non-strict inequalities, change the  $<$  marked with  $(A)$  to  $<=$ , and reverse the loop at  $(B)$ . To minimize  $f$ , change it to  $>$ , also at  $(B)$ .  
**Usage:** `int ind = ternSearch(0, n-1, [&](int i){return a[i];});`  
**Time:**  $\mathcal{O}(\log(b - a))$

<pre>template&lt;class F&gt;{//7d4 int ternSearch(int a, int b, F f) {     assert(a &lt;= b);     while (b - a &gt;= 5) {         int mid = (a + b) / 2;         if (f(mid) &lt; f(mid+1)) a = mid; // (A)//ec4     } }</pre>	9155b4, 11 lines
---	------------------

```
        else b = mid+1;
    }
    rep(i,a+1,b+1) if (f(a) < f(i)) a = i; // (B)
    return a;
} //cbb
```

LIS.h  
**Description:** Compute indices for the longest increasing subsequence.  
**Time:**  $\mathcal{O}(N \log N)$

```
2932a0, 17 lines
template<class I> vi lis(const vector<I>& S) { //47f
    if (S.empty()) return {};
    vi prev(sz(S));
    typedef pair<I, int> p;
    vector<p> res;
    rep(i,0,sz(S)) { //a50
        // change 0 -> i for longest non-decreasing subsequence
        auto it = lower_bound(all(res), p{S[i], 0});
        if (it == res.end()) res.emplace_back(), it = res.end()
            -1;
        *it = {S[i], i};
        prev[i] = it == res.begin() ? 0 : (it-1)->second; //476
    }
    int L = sz(res), cur = res.back().second;
    vi ans(L);
    while (L-->) ans[L] = cur, cur = prev[cur];
    return ans; //342
}
```

FastKnapsack.h  
**Description:** Given N non-negative integer weights w and a non-negative target t, computes the maximum S <= t such that S is the sum of some subset of the weights.  
**Time:**  $\mathcal{O}(N \max(w_i))$

```
b20ccc, 16 lines
int knapsack(vi w, int t) { //e2b
    int a = 0, b = 0, x;
    while (b < sz(w) && a + w[b] <= t) a += w[b++];
    if (b == sz(w)) return a;
    int m = *max_element(all(w));
    vi u, v(2*m, -1); //14a
    v[a+m-t] = b;
    rep(i,b,sz(w)) {
        u = v;
        rep(x,0,m) v[x+w[i]] = max(v[x+w[i]], u[x]);
        for (x = 2*m; --x > m;) rep(j, max(0,u[x]), v[x]) //45b
            v[x-w[j]] = max(v[x-w[j]], j);
    }
    for (a = t; v[a+m-t] < 0; a--);
    return a;
} //cbb
```

### 10.3 Dynamic programming

KnuthDP.h  
**Description:** When doing DP on intervals:  $a[i][j] = \min_{i < k < j} (a[i][k] + a[k][j]) + f(i, j)$ , where the (minimal) optimal  $k$  increases with both  $i$  and  $j$ , one can solve intervals in increasing order of length, and search  $k = p[i][j]$  for  $a[i][j]$  only between  $p[i][j-1]$  and  $p[i+1][j]$ . This is known as Knuth DP. Sufficient criteria for this are if  $f(b, c) \leq f(a, d)$  and  $f(a, c) + f(b, d) \leq f(a, d) + f(b, c)$  for all  $a \leq b \leq c \leq d$ . Consider also: LineContainer (ch. Data structures), monotone queues, ternary search.  
**Time:**  $\mathcal{O}(N^2)$

```
d41d8c, 1 lines
//d41
```

DivideAndConquerDP.h  
**Description:** Given  $a[i] = \min_{lo(i) \leq k < hi(i)} (f(i, k))$  where the (minimal) optimal  $k$  increases with  $i$ , computes  $a[i]$  for  $i = L..R-1$ .  
**Time:**  $\mathcal{O}((N + (hi - lo)) \log N)$

```
d38d2b, 18 lines
```

```
struct DP { // Modify at will: //ff9
    int lo(int ind) { return 0; }
    int hi(int ind) { return ind; }
    ll f(int ind, int k) { return dp[ind][k]; }
    void store(int ind, int k, ll v) { res[ind] = pii(k, v); }
} //ec8
void rec(int L, int R, int LO, int HI) {
    if (L >= R) return;
    int mid = (L + R) >> 1;
    pair<ll, int> best(LLONG_MAX, LO);
    rep(k, max(LO, lo(mid)), min(HI, hi(mid))) //680
        best = min(best, make_pair(f(mid, k), k));
    store(mid, best.second, best.first);
    rec(L, mid, LO, best.second+1);
    rec(mid+1, R, best.second, HI);
} //a30
void solve(int L, int R) { rec(L, R, INT_MIN, INT_MAX); }
};
```

### 10.4 Debugging tricks

- signal(SIGSEGV, [](int) { \_Exit(0); }); converts segfaults into Wrong Answers. Similarly one can catch SIGABRT (assertion failures) and SIGFPE (zero divisions). \_GLIBCXX\_DEBUG failures generate SIGABRT (or SIGSEGV on gcc 5.4.0 apparently).
- feenableexcept(29); kills the program on NaNs (1), 0-divs (4), infinities (8) and denormals (16).

### 10.5 Optimization tricks

\_\_builtin\_ia32\_ldmxcsr(40896); disables denormals (which make floats 20x slower near their minimum value).

#### 10.5.1 Bit hacks

- $x \& -x$  is the least bit in  $x$ .
- for (int x = m; x; ) { --x &= m; ... } loops over all subset masks of m (except m itself).
- $c = x \& -x$ ,  $r = x + c$ ;  $((r \wedge x) >> 2) / c$  | r is the next number after x with the same number of bits set.
- rep(b,0,K) rep(i,0,(1 << K)) if (i & 1 << b) D[i] += D[i^(1 << b)]; computes all sums of subsets.

#### 10.5.2 Pragmas

- #pragma GCC optimize ("Ofast") will make GCC auto-vectorize loops and optimizes floating points better.
- #pragma GCC target ("avx2") can double performance of vectorized code, but causes crashes on old machines.
- #pragma GCC optimize ("trapv") kills the program on integer overflows (but is really slow).

FastMod.h  
**Description:** Compute  $a \% b$  about 5 times faster than usual, where  $b$  is constant but not known at compile time. Returns a value congruent to  $a \pmod b$  in the range  $[0, 2b)$ .

```
751a02, 8 lines
typedef unsigned long long ull; //010
struct FastMod {
    ull b, m;
    FastMod(ull b) : b(b), m(-1ULL / b) {}
    ull reduce(ull a) { // a % b + (0 or b)
        return a - (ull)((__uint128_t(m) * a) >> 64) * b; //430
    }
};
```

FastInput.h  
**Description:** Read an integer from stdin. Usage requires your program to pipe in input from file.  
**Usage:** ./a.out < input.txt  
**Time:** About 5x as fast as cin/scanf.

```
7b3c70, 17 lines
inline char gc() { // like getchar() //c51
    static char buf[1 << 16];
    static size_t bc, be;
    if (bc >= be) {
        buf[0] = 0, bc = 0;
        be = fread(buf, 1, sizeof(buf), stdin); //818
    }
    return buf[bc++]; // returns 0 on EOF
}

int readInt() { //f26
    int a, c;
    while ((a = gc()) < 40);
    if (a == '-') return -readInt();
    while ((c = gc()) >= 48) a = a * 10 + c - 48;
    return a - 48; //d34
}
```

BumpAllocator.h  
**Description:** When you need to dynamically allocate many objects and don't care about freeing them. "new X" otherwise has an overhead of something like 0.05us + 16 bytes per allocation.

```
745db2, 8 lines
// Either globally or in a single class: //c17
static char buf[450 << 20];
void* operator new(size_t s) {
    static size_t i = sizeof(buf);
    assert(s < i);
    return (void*)&buf[i -= s]; //ef5
}
void operator delete(void*) {}
```

SmallPtr.h  
**Description:** A 32-bit pointer that points into BumpAllocator memory.

```
"BumpAllocator.h" 2dd6c9, 10 lines
template<class T> struct ptr { //bda
    unsigned ind;
    ptr(T* p = 0) : ind(p ? unsigned((char*)p - buf) : 0) {
        assert(ind < sizeof(buf);
    }
    T& operator*() const { return *(T*)(buf + ind); } //95f
    T* operator->() const { return &*this; }
    T& operator[](int a) const { return (&this)[a]; }
    explicit operator bool() const { return ind; }
};
```

BumpAllocatorSTL.h  
**Description:** BumpAllocator for STL containers.  
**Usage:** vector<vector<int, small<int>>>> ed(N);

```
bb66d4, 14 lines
```

```
char buf[450 << 20] alignas(16);//2c8
size_t buf_ind = sizeof buf;

template<class T> struct small {
    typedef T value_type;
    small() {}//8ec
    template<class U> small(const U&) {}
    T* allocate(size_t n) {
        buf_ind -= n * sizeof(T);
        buf_ind &= 0 - alignof(T);
        return (T*)(buf + buf_ind);//ad1
    }
    void deallocate(T*, size_t) {}
};
```

Unrolling.h

520e76, 5 lines

```
#define F {...; ++i;}//520
int i = from;
while (i&3 && i < to) F // for alignment, if needed
while (i + 4 <= to) { F F F F }
while (i < to) F
```

SIMD.h

Description: Cheat sheet of SSE/AVX intrinsics, for doing arithmetic on several numbers at once. Can provide a constant factor improvement of about 4, orthogonal to loop unrolling. Operations follow the pattern "\_mm(256)?name\_(si(128|256)|epi(8|16|32|64)|pd|ps)". Not all are described here; grep for \_mm\_ in /usr/lib/gcc/\*/4.9/include/ for more. If AVX is unsupported, try 128-bit operations, "emmintrin.h" and #define \_SSE\_ and \_MMX\_ before including it. For aligned memory use \_mm\_malloc(size, 32) or **int** buf[N] alignas(32), but prefer loadu/storeu.

551b82, 43 lines

```
#pragma GCC target ("avx2") // or sse4.1//c6d
#include "immintrin.h"
```

```
typedef _mm256i mi;
#define L(x) _mm256_loadu_si256((mi*)&(x))
//d41
// High-level/specific methods:
// load(u)?_si256, store(u)?_si256, setzero_si256,
_mm_malloc
// blendv_(epi8|ps|pd) (z?y:x), movemask_epi8 (hibits of
bytes)
// i32gather_epi32(addr, x, 4): map addr[] over 32-b parts
of x
// sad_epu8: sum of absolute differences of u8, outputs 4
xi64//d41
// maddubs_epi16: dot product of unsigned i7's, outputs 16
xi15
// madd_epi16: dot product of signed i16's, outputs 8xi32
// extractf128_si256(, i) (256->128), cvtsi128_si32 (128->
lo32)
// permute2f128_si256(x,x,1) swaps 128-bit lanes
// shuffle_epi32(x, 3*64+2*16+1*4+0) == x for each lane//
d41
// shuffle_epi8(x, y) takes a vector instead of an imm

// Methods that work with most data types (append e.g.
_epi32):
// set1, blend (i8?x:y), add, adds (sat.), mullo, sub, and/
or,
// andnot, abs, min, max, sign(1,x), cmp(gt|eq), unpack(lo|
hi)//512
```

```
int sumi32(mi m) { union {int v[8]; mi m;} u; u.m = m;
    int ret = 0; rep(i,0,8) ret += u.v[i]; return ret; }
mi zero() { return _mm256_setzero_si256(); }
mi one() { return _mm256_set1_epi32(-1); }//28e
bool all_zero(mi m) { return _mm256_testz_si256(m, m); }
```

```
bool all_one(mi m) { return _mm256_testc_si256(m, one()); }

ll example_filteredDotProduct(int n, short* a, short* b) {
    int i = 0; ll r = 0;//730
    mi zero = _mm256_setzero_si256(), acc = zero;
    while (i + 16 <= n) {
        mi va = L(a[i]), vb = L(b[i]); i += 16;
        va = _mm256_and_si256(_mm256_cmpgt_epi16(vb, va), va);
        mi vp = _mm256_madd_epi16(va, vb);//b47
        acc = _mm256_add_epi64(_mm256_unpacklo_epi32(vp, zero),
            _mm256_add_epi64(acc, _mm256_unpackhi_epi32(vp, zero)
            ));
    }
    union {ll v[4]; mi m;} u; u.m = acc; rep(i,0,4) r += u.v[
        i];
    for (;i<n;+i) if (a[i] < b[i]) r += a[i]*b[i]; // <-
equiv//c30
    return r;
}
```

Extra Stuff (11)

Pruefer.h

Description: Helps construct random tree Choose random n-2 length array, values [0, n-1]

Time:  $\mathcal{O}(n)$

295d68, 23 lines

```
vector<pii> pruefer_decode(const vi &code) //865
    int n = sz(code) + 2;
    vi degree(n, 1);
    for (int i : code)
        degree[i]++;
//c77
    set<int> leaves;
    rep(i, 0, n)
        if (degree[i] == 1)
            leaves.insert(i);
//f6a
    vector<pii> edges;
    for (int v : code) {
        int leaf = *leaves.begin();
        leaves.erase(leaves.begin());
//1b3
        edges.emplace_back(leaf, v);
        if (--degree[v] == 1)
            leaves.insert(v);
    }
    edges.emplace_back(*leaves.begin(), n-1);//062
    return edges;
}
```

Knuth.h

Description: DP must be in the form  $dp(i, j) = \min[dp(i, k) + dp(k + 1, j) + C(i, j)]$  such that  $opt(i, j - 1) \leq opt(i, j) \leq opt(i + 1, j)$ . True **if** for  $a \leq b \leq c \leq d$ , then  $C(b, c) \leq C(a, d)$  and  $C(a, c) + C(b, d) \leq C(a, d) + C(b, c)$ .

Time:  $\mathcal{O}(n^2)$

776312, 20 lines

```
template<class C>//e79
int solve(int n, C c) {
    vector dp(n, vi(n));
    auto opt = dp;
    rep(i, 0, n) opt[i][i] = i; // initialize dp[i][i] here
    for (int i = n-2; i >= 0; i--) //157
        rep(j, i+1, n) {
            int mn = INT_MAX;
            int cost = C(i, j);
            for (int k = opt[i][j-1]; k <= min(j-1, opt[i+1][j]);
                k++) {
                if (mn >= dp[i][k] + dp[k+1][j] + cost) //201
```

```
                opt[i][j] = k;
                mn = dp[i][k] + dp[k+1][j] + cost;
            }
            dp[i][j] = mn;//b2a
        }
    }
    return dp[0][n-1];
}
```

CentroidDecomposition.h

Description: Centroid decomposition on tree

Time:  $\mathcal{O}(n \log n)$

918f4f, 38 lines

```
struct CD //e81
    vector<vector<int>> adj;
    vector<int> size, vis;

    int dfs_size(int v, int p) {
        size[v] = 1;//43a
        for (int e : adj[v]) {
            if (e != p && !vis[e]) {
                size[v] += dfs_size(e, v);
            }
        }//ef7
        return size[v];
    }

    int dfs_root(int v, int p, int n) {
        for (int e : adj[v]) //14b
            if (e != p && !vis[e] && 2 * size[e] > n) {
                return dfs_root(e, v, n);
            }
        }
        return v;//4ef
    }

    void centroid(int v, int p) {
        dfs_size(v, p);
        int c = dfs_root(v, p, size[v]);//44f
        vis[c] = true;

        // do processing here
        // make sure to ignore visited nodes
//260
        for (int e : adj[c]) {
            if (!vis[e]) {
                centroid(e, c);
            }
        }//29b
    }
};
```

Eertree.h

Time: "Tree" of all palindromic substrings (there are two roots). Also has suffix links.

4b78a3, 56 lines

```
struct eertree //076
    struct node {
        array<int, 26> nxt;
        int sufflink, len, cnt;
        vector<int> edges;
    };//a3e

    string s;
    vector<node> tree;
    int suff, num;
//929
    eertree(const string &s) : s(s), tree(sz(s)+2), suff(2)
        , num(2) {
        tree[1].len = -1, tree[1].sufflink = 1;
```

```

    tree[2].len = 0, tree[2].sufflink = 1;
    tree[1].edges.pb(2);
    rep(i, 0, sz(s)) add(i);//736
}

void add(int pos) {
    int cur = suff, cur_len = 0;
    char c = s[pos];//989

    while (true) {
        cur_len = tree[cur].len;
        if (pos - 1 - cur_len > -1 && s[pos - 1 - cur_len] ==
            s[pos]) break;
        cur = tree[cur].sufflink;//b02
    }

    if (tree[cur].nxt[c]) {
        suff = tree[cur].nxt[c];
        tree[suff].cnt++;//be0
        return;
    }

    suff = ++num;
    tree[num].len = tree[cur].len + 2;//d5d
    tree[num].cnt = 1;
    tree[cur].nxt[c] = num;

    if (tree[num].len == 1) {
        tree[num].sufflink = 2;//eac
        tree[2].edges.pb(num);
        return;
    }

    while (true) {//4bc
        cur = tree[cur].sufflink;
        cur_len = tree[cur].len;
        if (pos - 1 - cur_len > -1 && s[pos - 1 - cur_len] ==
            s[pos]) {
            tree[num].sufflink = tree[cur].nxt[c];
            tree[tree[cur].nxt[c]].edges.pb(num);//ddd
            break;
        }
    }
}
};//214
```

ModInt.h

**Description:** all operations just work, MOD should fit in int

**Time:** constant for operations,  $\mathcal{O}(\log e)$  for pow

---

constexpr int MOD = 1e9+7;*//d87*

```

struct mi {
    int v;
    mi() : mi(0) {}
    mi(int _v) : v(_v) {//715
        if (v >= MOD) v -= MOD;
        if (v < 0) v += MOD;
    }
    mi(ll _v) : mi((int)(_v % MOD)) {}
    mi operator+(const mi &m2) const { return mi(v + m2.v); }
    //89d
    mi operator-(const mi &m2) const { return mi(v - m2.v); }
    mi operator*(const mi &m2) const { return mi((ll) v * m2.v); }
    mi operator/(const mi &m2) const { return mi((ll) v * m2.inv().v); }
    mi &operator+=(const mi &m2) { return *this = *this + m2; }
    mi &operator-=(const mi &m2) { return *this = *this - m2; }
    //e99
}
```

```

mi &operator*=(const mi &m2) { return *this = *this * m2; }
mi &operator/=(const mi &m2) { return *this = *this / m2; }
}

mi pow(ll e) const {
    mi res = 1;
    mi n = *this;//148
    while (e > 0) {
        if (e & 1) res *= n;
        n *= n;
        e >>= 1;
    }//c3c
    return res;
}

mi inv() const {
    return pow(MOD - 2);
}
//e03
};
```

SPFA.h

**Description:** Fast shortest path algo, negative edges ok

**Time:**  $\mathcal{O}(n)$  usually, but exponential worst case

---

85a103, 24 lines

```

constexpr int INF = 1e9;//979
pair<vi, vi> shortest_paths(int src, vector<vector<pii>> &
    adj) {
    int n=sz(adj);
    vi d(n, INF), p(n, -1), m(n, 2);
    d[src]=0;
    deque<int> q;//024
    while (!q.empty()) {
        int u = q.front(); q.pop_front();
        m[u] = 0;
        for (auto [v, w] : adj[u])
            if (d[v] > d[u] + w) {//888
                d[v] = d[u] + w;
                p[v] = u;
                if (m[v] == 2) {
                    m[v] = 1;
                    q.pb(v);//b34
                } else if (m[v] == 0) {
                    m[v] = 1;
                    q.push_front(v);
                }
            }
    }//8f0
    return {d, p};
}
```

## Dylan’s Templates (12)

WaveletTree.h

**Description:** Range K-th Smallest

**Time:**  $\mathcal{O}(n \log^2(n))$  I think

---

ccdc27, 39 lines

```

struct WaveletTree {//b57
    struct Node {
        vector<int> a, b;
        int l = -1, r = -1;
        bool leaf = 0;
    };//188
    int bits = 30;
    vector<int> arr;
    vector<Node> tree = {{{}};
    void construct(int u, int lo, int hi) {
        if (lo == hi) { tree[u].leaf = 1; return; }//5bb
        int l = sz(tree); tree.pb({});
        int r = sz(tree); tree.pb({});
        tree[u].l = l; tree[u].r = r;
        int mid = (lo + hi) / 2;
```

```

        int pre = 0;//16b
        for (int i = 0; i < sz(tree[u].a); i++) {
            if (arr[tree[u].a[i]] <= mid) {
                tree[l].a.pb(tree[u].a[i]);
                pre++;
            } else tree[r].a.pb(tree[u].a[i]);//b04
            tree[u].b.pb(pre);
        }
        if (!tree[l].a.empty()) construct(l, lo, mid);
        if (!tree[r].a.empty()) construct(r, mid + 1, hi);
    }//102
    WaveletTree(vector<int> a) : arr(a) {
        for (int i = 0; i < sz(arr); i++) tree[0].a.pb(i);
        construct(0, 0, (1 << bits) - 1);
    }
    int query(int l, int r, int k, int u = 0) {//07f
        if (u == -1) return 0;
        int lp = lb(tree[u].a, l), rp = lb(tree[u].a, r + 1) - 1;
        if (tree[u].leaf) return tree[u].a[lp + k - 1];
        int n = tree[u].b[rp] - (lp == 0 ? 0 : tree[u].b[lp - 1]);
        if (n >= k) return query(l, r, k, tree[u].l);//3a4
        return query(l, r, k - n, tree[u].r);
    }
};
```

SMAWK.h

**Description:** Outputs row minima of nxm totally monotone matrix f(r, c1, c2) should output 1 if a[r][c1] <= a[r][c2]

**Time:**  $\mathcal{O}(n)$

---

3ff481, 77 lines

```

template<typename F>//95b
vector<int> smawk(F f, vector<int> rows, vector<int> cols)
{
    vector<int> row_mins;
    if (sz(rows) <= 2 && sz(cols) <= 2) {
        for (int r : rows) {
            int mn = -1;//7ea
            for (int c : cols) {
                if (mn == -1 || f(r, c, mn)) mn = c;
            }
            row_mins.pb(mn);
        }//1e0
    } else if (sz(rows) < sz(cols)) {
        // Reduce
        vector<int> st;
        for (int j : cols) {
            while (!st.empty()) {//f89
                if (f(rows[sz(st) - 1], j, st.back())) st.pop_back();
                else if (sz(st) == sz(rows)) break;
                else { st.push_back(j); break; }
            }
            if (st.empty()) st.pb(j);//97e
        }
        row_mins = smawk(f, rows, st);
    } else {
        // Interpolate
        vector<int> half_rows;//420
        for (int i = 1; i < sz(rows); i += 2) half_rows.pb(rows[i]);
        vector<int> half_row_mins = smawk(f, half_rows, cols);
        vector<int> mn_index;
        for (int i = 0, j = 0; j < sz(cols); j++) {
            while (i < sz(half_row_mins) && cols[j] ==
                half_row_mins[i]) {//943
                mn_index.pb(j);
                i++;
            }
        }
    }
```

```
    for (int i = 0; i < sz(rows); i++) {//605
        if (i % 2) row_mins.pb(half_row_mins[i / 2]);
        else {
            int l = i == 0 ? 0 : mn_index[i / 2 - 1];
            int r = i == sz(rows) - 1 ? sz(cols) - 1 : mn_index[i / 2];
            int mn = cols[l];//57d
            for (int j = l + 1; j <= r; j++) {
                if (f(rows[i], cols[j], mn)) mn = cols[j];
            }
            row_mins.pb(mn);
        }//049
    }
}
//fc4
// Min-plus Convolution (b must be convex)
vector<ll> min_plus(vector<ll> a, vector<ll> b) {
    int n = sz(a), m = sz(b);
    auto get = [&](int r, int c) { return a[c] + b[r - c]; };
    auto comp = [&](int r, int c1, int c2) //792
        if (r - max(c1, c2) < 0) return c1 < c2;
        if (r - min(c1, c2) >= m) return c2 < c1;
        return get(r, c1) <= get(r, c2);
    };
    vector<int> rows, cols;//e94
    for (int i = 0; i < n + m - 1; i++) rows.pb(i);
    for (int j = 0; j < n; j++) cols.pb(j);
    vector<int> row_mins = smawk(comp, rows, cols);
    vector<ll> res;
    for (int i = 0; i < n + m - 1; i++) res.pb(get(i, row_mins[i]));//332
    return res;
}

// Min-plus Convolution (b must be concave)
vector<ll> max_plus(vector<ll> a, vector<ll> b) //325
{
    for (ll &i : a) i = -i;
    for (ll &i : b) i = -i;
    vector<ll> res = min_plus(a, b);
    for (ll &i : res) i = -i;
    return res;//510
}
```

### XORConvolution.h

**Description:** Bitwise XOR Convolution

**Time:**  $\mathcal{O}(n\log(n))$

```
void xorfft(vector<ll> &v, bool inv = 0) //475
{
    for (int k = 0; k < bits; k++) {
        for (int i = 0; i < 1 << bits; i++, i += i & 1 << k) {
            ll a = v[i], b = v[i ^ 1 << k];
            v[i] = a + b, v[i ^ 1 << k] = a - b;
        }//aa7
    }
    for (int i = 0; i < 1 << bits; i++) v[i] = (v[i] % mod + mod) % mod;
    if (inv) {
        ll n = 1; for (int i = 0; i < bits; i++) n = n * 2 % mod;
        n = mInv(n);//e2a
        for (int i = 0; i < 1 << bits; i++) v[i] = v[i] * n % mod;
    }
}
vector<ll> mult(vector<ll> a, vector<ll> b) {
    xorfft(a); xorfft(b); vector<ll> c(1 << bits);//703
    for (int i = 0; i < 1 << bits; i++) c[i] = a[i] * b[i] % mod;
    xorfft(c, 1); return c;
}
```

```
}

SubsetConvolution.h
Description: Subset Convolution
Time:  $\mathcal{O}(n\log^2(n))$ 
```

3b4300, 75 lines

```
struct yint //0e1
{
    int B = 22; ll a[22];
    yint() { for (int i = 0; i < B; i++) a[i] = 0; }
    yint(ll k) { *this = yint(); a[0] = k; }
    yint& operator+=(yint o) {
        for (int i = 0; i < B; i++)//b84
            if ((a[i] += o.a[i]) >= mod) a[i] -= mod;
        return *this; }
    yint& operator=(yint o) {
        for (int i = 0; i < B; i++)
            if ((a[i] -= o.a[i]) < 0) a[i] += mod;//59d
        return *this; }
    yint& operator*=(int k) {
        for (int i = 0; i < B; i++)
            a[i] = a[i] * k % mod;
        return *this; }//585
    yint& operator*=(yint o) {
        ll a2[22]; for (int i = 0; i < B; i++) a2[i] = 0;
        for (int i = 0; i < B; i++) for (int j = 0; i + j < B; j++) {
            a2[i + j] += a[i] * o.a[j] % mod;
            if (a2[i + j] >= mod) a2[i + j] -= mod; }//885
        swap(a, a2); return *this; }
    yint& operator-() {
        for (int i = 0; i < B; i++)
            if ((a[i] = -a[i]) < 0) a[i] += mod;
        return *this; }//40b
    yint& shiftY(int k) {
        if (k > 0) {
            for (int i = B-1; i - k >= 0; i--) a[i] = a[i - k];
            for (int i = 0; i < k; i++) a[i] = 0;
        } else { k = -k;//7c5
            for (int i = 0; i + k < B; i++) a[i] = a[i + k];
            for (int i = B - k; i < B; i++) a[i] = 0;
        } return *this; }
    yint& multY() {
        for (int i = B-1; i > 0; i--) a[i] = a[i - 1];//abd
        a[0] = 0; return *this; }
    yint& divY() {
        for (int i = 0; i < B-1; i++) a[i] = a[i + 1];
        a[B-1] = 0; return *this; }
    friend yint operator+(yint a, yint b) { return a += b; }//fef
    friend yint operator-(yint a, yint b) { return a -= b; }
    friend yint operator*(yint a, yint b) { return a *= b; }
};
```

```
void subsetfft(vector<yint> &v) //123
{
    for (int i = 0; i < 1 << bits; i++) v[i].shiftY(
        __builtin_popcount(i));
    for (int j = 0; j < bits; j++) {
        for (int i = 0; i < 1 << bits; i++, i += i & 1 << j) {
            yint a = v[i], by = v[i ^ 1 << j];
            v[i] = a - by; v[i ^ 1 << j] = a + by;//28f
        }
    }
}
void subsetifft(vector<yint> &v) {
    for (int j = 0; j < bits; j++) //1f2
        for (int i = 0; i < 1 << bits; i++, i += i & 1 << j) {
            yint a = v[i], b = v[i ^ 1 << j];
            v[i] = a + b; v[i ^ 1 << j] = (b - a);
        }
    }//c85
}
```

```
    for (int i = 0; i < 1 << bits; i++) v[i].shiftY(
        __builtin_popcount(i));
    ll p = 1; for (int j = 0; j < bits; j++) p = p * 2 % mod;
    p = mInv(p);
    for (int i = 0; i < 1 << bits; i++) v[i] *= p;
}
vector<ll> subsetMult(vector<ll> a, vector<ll> b) //5a1
{
    vector<yint> aY(1 << bits), bY(1 << bits), cY(1 << bits);
    for (int i = 0; i < 1 << bits; i++) aY[i] = yint(a[i]),
        bY[i] = yint(b[i]);
    subsetfft(aY); subsetfft(bY);
    for (int i = 0; i < 1 << bits; i++) cY[i] = aY[i] * bY[i];
    subsetifft(cY);//743
    vector<ll> c(1 << bits);
    for (int i = 0; i < 1 << bits; i++) c[i] = cY[i].a[0];
    return c;
}
```

### LinearSieveOfMultiplicativeFunction.h

**Description:** computes  $f(1) \dots f(n)$  for multiplicative function  $f$

**Time:**  $\mathcal{O}(n)$

0e8937, 24 lines

```
vector<ll> sieve(int n, ll (*fp)(int, int)) //f36
{
    vector<bool> prime(n, 1);
    vector<int> primes;
    vector<int> cnt(n, 1), part(n, 1);
    vector<ll> f(n, 1);
    for (int i = 2; i < n; i++) //29d
        if (prime[i]) {
            primes.pb(i);
            f[i] = fp(i, 1);
        }
    for (int j = 0; j < sz(primes) && i * primes[j] < n; j++) //73d
        prime[i * primes[j]] = 0;
        if (!(i % primes[j])) {
            cnt[i * primes[j]] = cnt[i] + 1;
            part[i * primes[j]] = part[i];
            f[i * primes[j]] = fp(primes[j], cnt[i * primes[j]] * f[part[i * primes[j]]]);//e47
            break;
        }
        part[i * primes[j]] = i;
        f[i * primes[j]] = f[i] * f[primes[j]];
    }//a2c
    return f;
}
```

### LinearDiophantineSolver.h

**Description:** Solves  $Ax + By = C$ , outputs a solution to  $x$  and  $y$ , outputs step size to  $dx$  and  $dy$

**Time:**  $\mathcal{O}(\text{fast})$

78211e, 11 lines

```
bool diophantine(ll A, ll B, ll C, ll &x, ll &y, ll &dx, ll &dy) //922
{
    ll g = exGCD(abs(A), abs(B), x, y);
    if (C % g != 0) return false;
    x *= C / g; y *= C / g;
    if (A < 0) x = -x;
    if (B < 0) y = -y;//f93
    dx = B / g; dy = A / g;
    if (B < 0) dx = -dx;
    if (A < 0) dy = -dy;
    return true;
}//cbb
```

## LCMConvolution.h

**Description:** LCM Convolution

**Time:** about  $\mathcal{O}(n \log(n))$

d8b1b5, 13 lines

// 1-indexed//665

```
vector<ll> lcmMult(vector<ll> a, vector<ll> b) {
    vector<ll> l(sz(a), 0), r(sz(a), 0), c(sz(a), 0);
    for (int i = 1; i <= sz(a); i++)
        for (int j = i; j <= sz(a); j += i)
            l[j-1] += a[i-1], r[j-1] += b[i-1]; //56c
    for (int g = 1; g <= sz(a); g++) {
        c[g-1] += (l[g-1] % mod) * (r[g-1] % mod) % mod;
        c[g-1] = (c[g-1] % mod + mod) % mod;
        for (int i = g*2; i <= sz(a); i += g) c[i-1] -= c[g-1];
    } //930
    return c;
}
```

## AllRoots.h

**Description:** Tree rerooting template promote() follows a parent edge of a subtree (from original node u and following original edge e) merge() merges two subtrees that just had promote() called on by default this computes the sum of distances from each node to all other nodes

**Time:** solve() is  $\mathcal{O}(N)$ .

2e65b7, 65 lines

```
namespace AllRoots { //36c
    struct Edge { int u, v, cost; };
    struct Node {
        int cost;
        int cnt = 0; ll sum = 0, t = 0, sumT = 0;
        Node merge(Node o) { //cb0
            return {0, cnt + o.cnt, sum + o.sum, t + o.t, sumT + o.sumT};
        }
        Node promote(Node u, Edge e) {
            return {0, 1 + cnt, sum + (ll)(1 + cnt) * e.cost, u.cost + t, sumT + (u.cost + t) * e.cost};
        } //925
    };
    Edge rev(Edge e) { swap(e.u, e.v); return e; }
    static vector<Node> solve(vector<Node> nodes, vector<Edge> edges) {
        int N = sz(nodes);
        vector<vector<int>> adj(N); //eac
        vector<Edge> par(N);
        for (Edge e : edges) {
            adj[e.u].pb(e.v);
            adj[e.v].pb(e.u);
        } //3f3
        queue<int> q; q.push(0);
        vector<int> srt, pos(N);
        while (!q.empty()) {
            int u = q.front(); q.pop();
            pos[u] = sz(srt); //c8a
            srt.pb(u);
            for (int v : adj[u]) {
                adj[v].erase(find(all(adj[v]), u));
                q.push(v);
            } //698
        }
        for (Edge e : edges) {
            if (pos[e.u] < pos[e.v]) swap(e.u, e.v);
            par[e.u] = e;
        } //97d
        reverse(all(srt));
        vector<Node> down(N);
        for (int u : srt) for (int v : adj[u])
            down[u] = down[u].merge(down[v].promote(nodes[v], par[v]));
        reverse(all(srt)); //599
        vector<Node> up(N), res(N);
        for (int u : srt) {
```

```
    vector<Node> left(sz(adj[u])), right(sz(adj[u]));
    for (int i = 0; i < sz(adj[u]); i++) {
        int v = adj[u][i]; //e76
        left[i] = right[i] = down[v].promote(nodes[v], par[v]);
    }
    for (int i = 1; i < sz(adj[u]); i++)
        left[i] = left[i-1].merge(left[i]);
    for (int i = sz(adj[u]) - 2; i >= 0; i--) //9bb
        right[i] = right[i].merge(right[i+1]);
    Node p = up[u].promote(nodes[par[u].v], rev(par[u]));
    for (int i = 0; i < sz(adj[u]); i++) {
        int v = adj[u][i];
        if (u > 0) up[v] = up[v].merge(p); //83d
        if (i > 0) up[v] = left[i-1].merge(up[v]);
        if (i+1 < sz(adj[u])) up[v] = up[v].merge(right[i+1]);
    }
    res[u] = down[u];
    if (u > 0) res[u] = res[u].merge(p); //be1
    return res;
}
```

## SegmentTreeBeats.h

**Description:** preset to allow range chmin, range chmax, and range add queries all at once

**Time:** All operations are  $\mathcal{O}(\log N)$ .

aaaba9, 114 lines

```
struct SegTreeBeats { //c3f
    struct Node {
        ll sum = 0, mx = LLONG_MIN, mx2 = LLONG_MIN, mxC = 0,
            mn = LLONG_MAX, mn2 = LLONG_MAX, mnC = 0;
    };
    struct Upd {
        ll chmin = LLONG_MAX, chmax = LLONG_MIN, add = 0; //9dc
    };
    void apply(Node &v, Upd &u, int l, int r) {
        if (v.mx > u.chmin) {
            v.sum -= (v.mx - u.chmin) * v.mxC;
            if (v.mx == v.mn) v.mn = u.chmin; //50a
            if (v.mx == v.mn2) v.mn2 = u.chmin;
            if (v.mx == v.mx2) v.mx2 = u.chmin;
            v.mx = u.chmin;
        }
        if (v.mn < u.chmax) { //e15
            v.sum += (u.chmax - v.mn) * v.mnC;
            if (v.mn == v.mx) v.mx = u.chmax;
            if (v.mn == v.mx2) v.mx2 = u.chmax;
            if (v.mn == v.mn2) v.mn2 = u.chmax;
            v.mn = u.chmax; //535
        }
        v.sum += u.add * (r - l + 1);
        v.mx += u.add;
        v.mx2 += u.add;
        v.mn += u.add; //cc3
        v.mn2 += u.add;
        v.mx2 = min(v.mx2, v.mx);
        v.mn2 = max(v.mn2, v.mn);
    }
    void combine(Upd &a, Upd &b) { //324
        if (b.chmin != LLONG_MAX) {
            a.chmin = min(a.chmin, b.chmin - a.add);
            a.chmax = min(a.chmax, b.chmin - a.add);
        }
        if (b.chmax != LLONG_MIN) { //831
            a.chmax = max(a.chmax, b.chmax - a.add);
            a.chmin = max(a.chmin, b.chmax - a.add);
        }
        a.add += b.add;
```

```
//c89
Node merge(Node &a, Node &b) {
    Node res;
    res.sum = a.sum + b.sum;
    vector<ll> mxSrt = {a.mx, a.mx2, b.mx, b.mx2};
    vector<ll> mnSrt = {a.mn, a.mn2, b.mn, b.mn2}; //d69
    sort(all(mxSrt)); reverse(all(mxSrt)); sort(all(mnSrt));
    ;
    res.mx = mxSrt[0];
    int i = 1;
    while (i < 3 && mxSrt[i-1] == mxSrt[i]) i++;
    res.mx2 = mxSrt[i]; //500
    res.mn = mnSrt[0];
    i = 1;
    while (i < 3 && mnSrt[i-1] == mnSrt[i]) i++;
    res.mn2 = mnSrt[i];
    if (a.mx == b.mx) res.mxC = a.mxC + b.mxC; //51d
    else if (a.mx > b.mx) res.mxC = a.mxC;
    else res.mxC = b.mxC;
    if (a.mn == b.mn) res.mnC = a.mnC + b.mnC;
    else if (a.mn < b.mn) res.mnC = a.mnC;
    else res.mnC = b.mnC; //8bf
    return res;
}
bool breakCondition(int L, int R, Upd k, int i, int l,
    int r) {
    if (r < L || R < l) return true;
    if (k.chmin != LLONG_MAX) return tree[i].mx <= k.chmin;
    //d3d
    if (k.chmax != LLONG_MIN) return tree[i].mn >= k.chmax;
    return false;
}
bool tagCondition(int L, int R, Upd k, int i, int l, int r) {
    if (!(L <= l && r <= R)) return false; //08e
    if (k.chmin != LLONG_MAX) return tree[i].mx2 < k.chmin || tree[i].mx == tree[i].mx2;
    if (k.chmax != LLONG_MIN) return tree[i].mn2 > k.chmax || tree[i].mn == tree[i].mn2;
    return true;
}
vector<Node> tree; vector<Upd> upd; //933
void push(int i, int l, int r) {
    apply(tree[i], upd[i], l, r);
    if (i * 2 < sz(tree)) {
        combine(upd[i * 2], upd[i]);
        combine(upd[i * 2 + 1], upd[i]); //efd
    }
    upd[i] = {};
}
SegTreeBeats(vector<Node> arr) {
    int M = 1; while (M < sz(arr)) M <= 1; //db0
    tree = vector<Node>(M * 2);
    upd = vector<Upd>(M * 2);
    for (int i = 0; i < sz(arr); i++)
        tree[i + M] = arr[i];
    for (int i = M - 1; i > 0; i--) //372
        tree[i] = merge(tree[i * 2], tree[i * 2 + 1]);
}
void updateRec(int L, int R, Upd k, int i, int l, int r) {
    if (l > r) return;
    push(i, l, r); //8d1
    if (breakCondition(L, R, k, i, l, r)) return;
    if (tagCondition(L, R, k, i, l, r)) { combine(upd[i], k); push(i, l, r); return; }
    updateRec(L, R, k, i * 2, l, (l + r) / 2);
    updateRec(L, R, k, i * 2 + 1, (l + r) / 2 + 1, r);
    tree[i] = merge(tree[i * 2], tree[i * 2 + 1]); //b6d
}
```

```
void rangeUpdate(int l, int r, Upd k) { updateRec(l, r, k, l, 0, sz(tree) / 2 - 1); }
Node queryRec(int L, int R, int i, int l, int r) {
    if (l > r) return {};
    push(i, l, r); //c8b
    if (breakCondition(L, R, {}, i, l, r)) return {};
    if (tagCondition(L, R, {}, i, l, r)) return tree[i];
    Node left = queryRec(L, R, i * 2, l, (l + r) / 2);
    Node right = queryRec(L, R, i * 2 + 1, (l + r) / 2 + 1, r);
    return merge(left, right); //7ff
}
Node rangeQuery(int l, int r) { return queryRec(l, r, 1, 0, sz(tree) / 2 - 1); }
};
```

### HalfCompressedBIT2D.h

**Description:** 2D BIT that is semi-offline (requires update positions ahead of time) addUpdate() must be called ahead of time with all update positions then query() and update() may be called arbitrarily  
**Time:** Operations are  $\mathcal{O}(\log^2(n))$

```
struct HalfCompressedBIT2D { //863
    int N;
    bool built = 0;
    vector<pair<int, int>> updates;
    HalfCompressedBIT2D(int N) : N(N) {}
    void addUpdate(int r, int c) { //bd1
        updates.pb({r, c});
    }
    vector<vector<int>> compress;
    vector<vector<ll>> tree;
    static bool cmp(pair<int, int> &a, pair<int, int> &b) { //00d
        return a.second < b.second;
    }
    void build() {
        sort(all(updates), cmp);
        compress = vector<vector<int>>(N, {}); //7a5
        for (auto &p : updates)
            for (int rr = p.first; rr < sz(compress); rr += rr & -rr)
                compress[rr].pb(p.second);
        tree = vector<vector<ll>>(sz(compress));
        for (int i = 0; i < sz(compress); i++) { //6df
            tree[i] = vector<ll>(sz(compress[i]), 0);
        }
        built = 1;
    }
    void update(int r, int c, int k) { //f54
        if (!built) build();
        for (int rr = r; rr < sz(compress); rr += rr & -rr)
            for (int cc = lb(compress[rr], c); cc < sz(compress[rr]); cc += cc & -cc)
                tree[rr][cc] += k;
    } //c42
    ll query(int r, int c) {
        if (!built) build();
        ll res = 0;
        for (int rr = r; rr; rr -= rr & -rr)
            for (int cc = lb(compress[rr], c + 1) - 1; cc; cc -= cc & -cc) //b9e
                res += tree[rr][cc];
        return res;
    }
};
```

### GCDConvolution.h

**Description:** GCD Convolution  
**Time:**  $\mathcal{O}(n \log(n))$

ca54f7, 12 lines

```
// 1-indexed //cab
vector<ll> gcdMult(vector<ll> a, vector<ll> b) {
    vector<ll> c(sz(a), 0);
    for (int g = sz(a); g >= 1; g--) {
        ll l = 0, r = 0;
        for (int i = g; i <= sz(a); i += g) //f68
            l += a[i-1], r += b[i-1], c[g-1] -= c[i-1];
        c[g-1] += (l % mod) * (r % mod) % mod;
        c[g-1] = (c[g-1] % mod + mod) % mod;
    }
    return c; //df7
}
```

### OfflineRectangleUnionArea.h

**Description:** Area of union of rectangles (does compression internally)  
**Time:**  $\mathcal{O}(n \log(n))$

b35a6c, 44 lines

```
struct OfflineRectangleUnionArea { //ead
    struct Update {
        int row, l, r, k;
    };
    vector<Update> updates;
    void addRect(int r1, int r2, int c1, int c2) { //98d
        updates.pb({r1, c1, c2, 1});
        updates.pb({r2 + 1, c1, c2, -1});
    }
    ll area() {
        vector<int> compressR, compressC; //4f9
        for (auto &update : updates) {
            compressR.pb(update.row - 1);
            compressR.pb(update.row);
            compressC.pb(update.l - 1);
            compressC.pb(update.l); //3eb
            compressC.pb(update.r);
        }
        sort(all(compressR));
        sort(all(compressC));
        int R = sz(compressR), C = sz(compressC); //49f
        vector<vector<Update>> compressedUpdates(R);
        for (auto &update : updates) {
            update.row = lb(compressR, update.row);
            update.l = lb(compressC, update.l);
            update.r = lb(compressC, update.r); //9a4
            compressedUpdates[update.row].pb(update);
        }
        vector<SegTree::Node> start(C);
        for (int i = 0; i < C; i++) start[i].mnCnt = compressC[i] - (i == 0 ? -1 : compressC[i - 1]);
        SegTree st(start); //f56
        ll res = 0;
        for (int i = 0; i < R; i++) {
            int h = compressR[i] - (i == 0 ? -1 : compressR[i - 1]);
            for (auto &update : compressedUpdates[i]) {
                st.rangeUpdate(update.l, update.r, {update.k}); //769
            }
            SegTree::Node full = st.rangeQuery(0, C - 1);
            int outside = full.mn == 0 ? full.mnCnt : 0;
            res += (ll)h * (compressC[C - 1] + 1 - outside);
        } //0c6
        return res;
    }
};
```

### OrConvolution.h

**Description:** Bitwise OR Convolution and Bitwise AND Convolution  
**Time:**  $\mathcal{O}(n \log(n))$

4d2a9a, 22 lines

```
void orfft(vector<ll> &v) { //ece
    for (int j = bits - 1; j >= 0; j--)
```

```
        for (int i = 0; i < 1 << bits; i++, i += i & 1 << j)
            v[i ^ 1 << j] += v[i];
        for (int i = 0; i < 1 << bits; i++) v[i] %= mod;
    } //a47
    void orifft(vector<ll> &v) {
        for (int j = 0; j < bits; j++)
            for (int i = 0; i < 1 << bits; i++, i += i & 1 << j)
                v[i ^ 1 << j] -= v[i];
        for (int i = 0; i < 1 << bits; i++) v[i] = (v[i] % mod + mod) % mod; //2c6
    }
    vector<ll> orMult(vector<ll> a, vector<ll> b) {
        orfft(a); orfft(b); vector<ll> c(1 << bits);
        for (int i = 0; i < 1 << bits; i++) c[i] = a[i] * b[i] % mod;
        orifft(c); return c; //71b
    }
    vector<ll> andMult(vector<ll> a, vector<ll> b) {
        reverse(all(a)); reverse(all(b));
        vector<ll> c = orMult(a, b);
        reverse(all(c)); return c; //d43
    }
}
```

### MinDeque.h

**Description:** Min Deque, useful for implementing min queue w/ roll-back push() is for applying an element to a running sum merge() is for merging both sides of the ds (i.e. when querying)  
**Time:** Amortized  $\mathcal{O}(p)$  for non-query ops where p is the runtime of push()  $\mathcal{O}(m)$  for query ops where m is the runtime of merge()

add1ad, 52 lines

```
template <typename Node, typename Item, typename Query,
          typename Result> //1ed
struct MinDeque {
    Node push(Node n, Item k) {

    }

    Result merge(Node a, Node b, Query q) { //9f0

    }

    Node empty;
    stack<pair<Item, Node>> left, right;
    void push_front(Item k) { //e6e
        Node &n = left.empty() ? empty : left.top().second;
        left.push({k, push(n, k)});
    }
    void push_back(Item k) {
        swap(left, right); //417
        push_front(k);
        swap(left, right);
    }
    Item pop_front() {
        if (left.empty()) { //1aa
            int h = sz(right) / 2;
            stack<Item> tmp;
            for (int i = 0; i < h; i++) {
                tmp.push(right.top().first);
                right.pop(); //72e
            }
            while (!right.empty()) {
                push_front(right.top().first);
                right.pop();
            } //ed1
            while (!tmp.empty()) {
                push_back(tmp.top());
                tmp.pop();
            }
        } //886
        Item res = left.top().first;
        left.pop();
        return res;
    }
}
```



```
Item pop_back() {//126
    swap(left, right);
    Item res = pop_front();
    swap(left, right);
    return res;
}//3fb
Result query(Query q) {
    Node &l = left.empty() ? empty : left.top().second;
    Node &r = right.empty() ? empty : right.top().second;
    return merge(l, r, q);
}//e03
};
```

Wilbert’s Templates (13)

```
lazy.h
Description: lazy segtree f102c3, 74 lines

using T = long long; //a95
using TAG = pair<T,T>;
const T defT = 0;
const TAG defTAG = make_pair(1,0);

const int MAX = 2e5; //81b
struct Lazy{
    int N;
    TAG lazy[MAX];
    T seg[2*MAX];
    int len[2*MAX]; //1d8

    T f(T a, T b){ return a+b; } //combine segments
    void apply( TAG f, int i ){
        TAG g = lazy[i];
        if(i < N) lazy[i] = make_pair( f.first*g.
            first, f.first*g.second + f.second );
        //compose two tags//587
        seg[i] = f.first*seg[i] + f.second*len[i];
        //evaluate tag on segment
    }
    void hammer_down( int i ){
        for( int k = 20; k > 0; k-- ){
            int j = (i>>k); //a75
            if(j==0) continue;
            apply( lazy[j], 2*j );
            apply( lazy[j], 2*j+1 );
            lazy[j] = defTAG;
        } //531
    }
    void boiler_up( int i ){
        i /= 2;
        while(i){
            seg[i] = f(seg[2*i], seg[2*i+1]); //e04
            i /= 2;
        }
    }
    void update( int l, int r, TAG tag ){
        l += N, r += N; //acc
        int l0 = l, r0 = r;
        while(l0%2==0) l0 /= 2;
        while(r0%2==0) r0 /= 2;
        hammer_down(l0);
        hammer_down(r0-1); //bbd
        while( l < r ){
            if(l&1) apply(tag, l++);
            if(r&1) apply(tag, --r);
            l /= 2, r /= 2;
        } //2a5
        boiler_up(l0);
        boiler_up(r0-1);
    }
};
```

```

    }
    T query( int l, int r ){
        l += N, r += N; //de1
        hammer_down(l);
        hammer_down(r-1);
        T m1 = defT, m2 = defT;
        while( l < r ){
            if(l&1) m1 = f(m1, seg[l++]); //263
            if(r&1) m2 = f(seg[--r], m2);
            l /= 2, r /= 2;
        }
        return f(m1, m2);
    } //0fa
    void build( int n, vector<T> &a ){
        N = n;
        for( int i = 0; i < n; i++ ){
            seg[i+n] = a[i];
            len[i+n] = 1; //b2f
        }
        for( int i = n-1; i > 0; i-- ){
            lazy[i] = defTAG;
            seg[i] = f(seg[2*i], seg[2*i+1]);
            len[i] = len[2*i] + len[2*i+1]; //c5d
        }
    }
};
```

```
ahocorasick.py
Description: aho corasick 35 lines

trie = [[0]*26] //0e1

def insert(s):
    cur = 0
    for c in s:
        x = ord(c) - ord('A') //ca5
        if not trie[cur][x]:
            trie[cur][x] = len(trie)
            trie.append( [0]*26 )
            cur = trie[cur][x]
    return cur //38b

#insert all the strings here first

queue = [0]
fail = [0]*len(trie) //691
fail[0] = -1
for i in range(len(trie)):
    c = queue[i]
    for x in range(26):
        if not trie[c][x]: continue //9cd

        p = fail[c]
        while p > -1:
            if trie[p][x]:
                fail[trie[c][x]] = trie[p][x] //21c
                break
            p = fail[p]
        queue.append( trie[c][x] )
fail[0] = 0
//94e
go = [[0]*26 for _ in range(len(trie))]
for c in queue:
    for x in range(26):
        go[c][x] = trie[c][x] if trie[c][x] else go[fail[c]][x]
```