# CONTINUOUS PROBABILITY DISTRIBUTIONS

## Normal Probability Distribution

### Bell curves and z-scores

**7.1** Identify the three defining characteristics of the normal probability distribution.

The normal probability distribution is a bell-shaped continuous distribution that fulfills the following conditions:

- The distribution is symmetrical around the mean.
- The mean, median, and mode are the same value.
- The total area under the curve is equal to one.

*Normal*


$$z = \frac{x - \mu}{\sigma}$$

$$P(X < x) \Rightarrow \text{Use Z-Chart}$$

## Continuous Uniform Distribution

### Going from a bell-shaped to a box-shaped distribution

**7.38** Identify the defining characteristics of the continuous uniform distribution.

The continuous probability distribution is defined by two values, $a$ and $b$, that represent the minimum and maximum values respectively. All intervals of the same length between $a$ and $b$ are equally probable.

To calculate the probability that a random variable will lie between $x_1$ and $x_2$ in the distribution, apply the following equation.

$$P\left(x_1 \le x \le x_2\right) = \frac{x_2 - x_1}{b - a}$$

*Uniform*


$$\mu = \frac{a + b}{2} \qquad \sigma = \sqrt{\frac{(b - a)^2}{12}}$$

$$P(X < x) = \frac{x - a}{b - a}$$

# Using the Normal Distribution to Approximate the Binomial Distribution

## Another binomial probability shortcut

**7.30** Describe the conditions under which the normal distribution can be used to approximate the binomial distribution.

If $n$ represents the number of trials in which only outcomes $p$ and $q$ may occur, the normal distribution can be used to approximate the binomial distribution as long as $np \geq 5$ and $nq \geq 5$.

Continuity correction is used when a continuous distribution (such as the normal distribution) is used to approximate a discrete distribution (such as the binomial distribution). To correct for continuity, add 0.5 to a boundary of $x$ or subtract 0.5 from a boundary of $x$ as directed below:

- Subtract 0.5 from the $x$-value representing the left boundary under the normal curve.
- Add 0.5 to the $x$-value representing the right boundary under the normal curve.

Note that continuity correction is unnecessary when $n > 100$.

Calculate the mean and standard deviation of the binomial distribution.

$$\mu = np$$
$$\sigma = \sqrt{npq}$$

The problem asks you to calculate $P(7 \leq x \leq 8)$. Apply the continuity correction to adjust the boundaries: $P(6.5 \leq x \leq 8.5)$.

Calculate the $z$-scores for endpoints $x = 6.5$ and 8.5.

## Exponential Distribution

*Like the Poisson distribution, but continuous*

**7.45** Identify the defining characteristics of the exponential distribution.

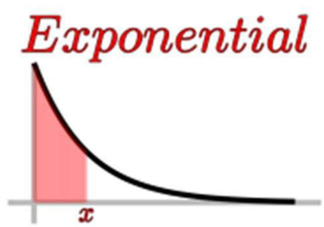
The exponential probability distribution is a continuous distribution commonly used to measure the time between events of interest, such as the time between customer arrivals at a retail store or the time between failures in a process.

In Chapter 6, the variable $\lambda$ was used to represent the mean of the Poisson distribution, a discrete distribution that counted the number of times an event occurred during a specific time period. The mean and the standard deviation of the exponential distribution are both $\frac{1}{\lambda}$.

The exponential distribution is the continuous counterpart of the discrete Poisson distribution. For example, if a random variable follows the Poisson distribution with an average occurrence of two times per minute ($\lambda = 2$), then the same random variable also follows the exponential distribution with a mean and standard deviation of $\frac{1}{\lambda} = \frac{1}{2} = 0.5$.

If $x$ is a random variable that follows the exponential distribution, then the probability that $x \geq t$ is $e^{-\lambda t}$.

$$P(x \geq t) = e^{-\lambda t}$$




$$\mu = \frac{1}{\gamma} \qquad \sigma = \frac{1}{\gamma}$$

$$P(X < x) = 1 - e^{-\gamma x}$$