DISCRETE PROBABILITY DISTRIBUTIONS

Binomial Probability Distribution

Using coefficients that are combinations

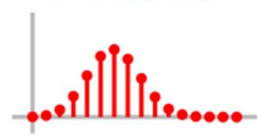
6.1 Define the characteristics of a binomial experiment and provide an example.

A binomial experiment has the following characteristics:

- The experiment consists of a fixed number of trials, n.
- Each trial has only two possible outcomes (for example, success or failure).
- The probabilities of both outcomes are constant throughout the experiment.
- Each trial in the experiment is independent.

Flipping a coin five times and recording the number of heads is one example of a binomial experiment.

Binomial



$$\mu = n \cdot p \qquad \sigma = \sqrt{n \cdot p \cdot (1-p)}$$

$$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$$

Given n = the number of trials, r = the number of successes, p = the probability of a success, and q = the probability of a failure, the binomial probability distribution states that the probability of r successes in n trials is $\binom{n}{r} p^n q^{n-r} = \frac{n!}{(n-r)!r!} p^n q^{n-r}$. Note that the notation $\binom{n}{r}$ is used to represent ${}_nC_r$, the combination of n things, choosing r at a time.

Poisson Probability Distribution

Determining probabilities over specific intervals

6.17 Define the characteristics of a Poisson process and provide an example.

A Poisson process has the following characteristics:

- The experiment counts the number of times an event occurs over a specific period of measurement (such as time, area, or distance).
- The mean of the Poisson distribution is the same for each interval of measurement.
- The number of occurrences in each interval is independent.

An example of a Poisson process would be the number of cars that pass through a tollbooth during one hour.

If x = the number of occurrences per interval, λ = the average number of occurrences per interval, and e is Euler's number, then the probability of x occurrences per interval is $P(x) = \frac{\lambda^x e^{-\lambda}}{x!}$.

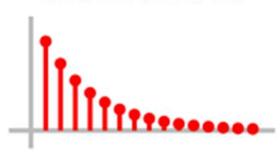
The Poisson Distribution as an Approximation to the Binomial Distribution

A binomial shortcut

6.32 Describe the conditions under which the Poisson distribution can be used as an approximation to the binomial distribution.

The Poisson distribution can be used as an approximation to the binomial distribution when the number of trials n is greater than or equal to 20 and the probability of success p is less than or equal to 0.05. If these conditions are met, the probability is $P(x) = \frac{(np)^x e^{-(np)}}{x!}$.

Geometric



$$\mu = \frac{1}{p} \qquad \sigma = \frac{\sqrt{1-p}}{p}$$

$$P(X = x) = (1-p)^{x-1}p$$

Hypergeometric Probability Distribution

Determining probabilities when events are not independent

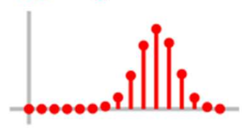
6.39 Define the characteristics of a hypergeometric probability distribution.

Unlike the binomial and Poisson distributions, the hypergeometric distribution does not require that events be independent of one another. Thus, the distribution is useful when samples are taken from small populations without replacement.

Consider an event that has only two possible outcomes, success or failure. Let N equal the population size and X equal the number of successes in the population; let n equal the sample size and x equal the number of successes in the sample. The formula below calculates the probability of x successes in a hypergeometric distribution.

$$P(x) = \frac{\binom{N-X}{n-x}\binom{X}{x}}{\binom{N}{n}}$$

Hypergeometric



$$\mu = n\frac{K}{N} \qquad \sigma = \sqrt{n\frac{K(N-K)(N-n)}{N^2(N-1)}}$$

$$P(X = x) = \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}$$