Morgan
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5352 HW5

2 a) Bernoulli(p) $\hat{p}=\bar{y}$ $H_0: p_0 = p$ $H_A: p_0 \neq p$

$$\ell(p_0) = n \times [\bar{y}\log(p) + (1-\bar{y})\log(1-p)]$$

$\hookrightarrow n \times [\hat{p}\log(p_0) + (1-\hat{p})\log(1-p_0)]$ H_0

$$\ell(\hat{p}) = n \times [\bar{y}\log(\hat{p}) + (1-\bar{y})\log(1-\hat{p})]$$

$\hookrightarrow n \times [\hat{p}\log(\hat{p}) + (1-\hat{p})\log(1-\hat{p})]$ H_A

$$\lambda = 2(\ell(\hat{p}) - \ell(p_0))$$
$$= 2n \times \left[\hat{p}\log\left(\frac{\hat{p}}{p_0}\right) - (1-\hat{p})\log\left(\frac{1-\hat{p}}{1-p_0}\right)\right] \checkmark$$

b) Neg-Binom(r,p) $\hat{p} = \frac{\bar{k}}{(r+\bar{k})}$ $H_0: p_0 = p$ $H_A: p_0 \neq p$ $\hat{p}r + \hat{p}\bar{k} = \bar{k}$ $\hat{p}r = \bar{k} - \hat{p}\bar{k}$

$\hat{p}r = \bar{k}(1-\hat{p})$

$\frac{\hat{p}r}{(1-\hat{p})} = \bar{k}$

$$\ell(r,\hat{p}) = \sum_{i=1}^{n} \log\left(\binom{r+k_i-1}{k_i}\right) + n \times [\bar{k}\log(\hat{p}) + r\log(1-\hat{p})]$$

$\hookrightarrow \sum_{i=1}^{n} \log\left(\binom{r+k_i-1}{k_i}\right) + n \times \left[\frac{\hat{p}r}{(1-\hat{p})}\log(\hat{p}) + r\log(1-\hat{p})\right]$ H_A

$$\ell(r,p_0) = \sum_{i=1}^{n} \log\left(\binom{r+k_i-1}{k_i}\right) + n \times \left[\frac{\hat{p}r}{(1-\hat{p})}\log(p_0) + r\log(1-p_0)\right] H_0$$

$$\lambda = 2(\ell(\hat{p}) - \ell(p_0))$$
$$= 2n \times \left[\frac{\hat{p}r}{(1-\hat{p})}\log\left(\frac{\hat{p}}{p_0}\right) + r\log\left(\frac{(1-\hat{p})}{(1-p_0)}\right)\right]$$
$$= 2n \times \left[\frac{r\hat{p}}{1-\hat{p}}\log\left(\frac{\hat{p}}{p_0}\right) + r\log\left(\frac{1-\hat{p}}{1-p_0}\right)\right]$$

c) Exponential(λ) $\hat{\lambda} = 1/\bar{x}$ $\bar{x} = 1/\hat{\lambda}$ $H_0: \lambda = \lambda_0$ $H_A: \lambda \neq \lambda_0$

$$\ell(\hat{\lambda}) = nx[\log(\hat{\lambda}) - \hat{\lambda}\bar{x}]$$
$$= nx[\log(\hat{\lambda}) - \hat{\lambda}(1/\hat{\lambda})]$$
$$= nx[\log(\hat{\lambda}) - 1] \quad H_A$$
$$\ell(\lambda_0) = nx[\log(\lambda_0) - \lambda_0\bar{x}]$$
$$= nx[\log(\lambda_0) - \lambda_0(1/\hat{\lambda})]$$
$$= nx[\log(\lambda_0) - \frac{\lambda_0}{\hat{\lambda}}] \quad H_0$$
$$\lambda = 2[\ell(\hat{\lambda}) - \ell(\lambda_0)]$$
$$\lambda = 2nx\left[\log\left(\frac{\hat{\lambda}}{\lambda_0}\right) + \frac{\lambda_0}{\hat{\lambda}} - 1\right] \checkmark$$

d) Gamma(m, λ) $\hat{\lambda} = m/\bar{g}$ $H_0: \lambda = \lambda_0$ $H_A: \lambda \neq \lambda_0$ $\bar{g} = \frac{m}{\hat{\lambda}}$

$$\ell(m, \hat{\lambda}) = (m-1)\sum_{i=1}^{n} \log(g_i) - n\log((m-1)!) + nx[m\log(\hat{\lambda}) - \hat{\lambda}\bar{g}]$$
$$= (m-1)\sum_{i=1}^{n}\log(g_i) - n\log((m-1)!) + nmx[\log(\hat{\lambda}) - 1]$$

$$\ell(m, \lambda_0) = (m-1)\sum_{i=1}^{n}\log(g_i) - n\log((m-1)!) + nx[m\log(\lambda_0) - \frac{\lambda_0 m}{\hat{\lambda}}]$$
$$= (m-1)\sum_{i=1}^{n}\log(g_i) - n\log((m-1)!) + nmx[\log(\lambda_0) - \frac{\lambda_0}{\hat{\lambda}}]$$

$$\lambda = 2[\ell(\hat{\lambda}) - \ell(\lambda_0)]$$
$$= 2nx\left[m\log\left(\frac{\hat{\lambda}}{\lambda_0}\right) + \frac{m\lambda_0}{\hat{\lambda}} - m\right] = 2nmx\left[\log\left(\frac{\hat{\lambda}}{\lambda_0}\right) + \frac{\lambda_0}{\hat{\lambda}} - 1\right]. \checkmark$$

e) Poisson(λ) $\quad \hat{\lambda} = \bar{c}$ $\quad H_0: \lambda = \lambda_0 \quad H_A: \lambda \neq \lambda_0$

$$\ell(\hat{\lambda}) = -\sum_{i=1}^{n} \log(c_i!) + n \times [\log(\hat{\lambda})\bar{c} - \hat{\lambda}]$$
$$= -\sum_{i=1}^{n} \log(c_i!) + n\hat{\lambda} \times [\log(\hat{\lambda}) - 1]$$

$$\ell(\lambda_0) = -\sum_{i=1}^{n} \log(c_i!) + n \times [\log(\lambda_0)\bar{c} - \lambda_0]$$
$$= -\sum_{i=1}^{n} \log(c_i!) + n \times [\hat{\lambda}\log(\lambda_0) - \lambda_0]$$

$$\lambda = 2[\ell(\hat{\lambda}) - \ell(\lambda_0)]$$
$$\lambda = 2n \times \left[\hat{\lambda}\log\left(\frac{\hat{\lambda}}{\lambda_0}\right) + \lambda_0 - \hat{\lambda}\right] \Rightarrow 2n\hat{\lambda} \times \left[\log\left(\frac{\hat{\lambda}}{\lambda_0}\right) + \frac{\lambda_0}{\hat{\lambda}} - 1\right]$$