

S352

HWOA

1. a) Gamma(α, β) density has kernel:

$$f(x|u) = \frac{f(u|x)f(x)}{f(u)} \propto f(u|x)f(x)$$

$$\propto x^2 u^{2-1} e^{-ux} \cdot e^{-x}$$

$$\propto x^2 e^{-x(u+1)}$$

$$\propto x^{3-1} e^{-x(u+1)}$$

$$\rightarrow x|u \sim \text{Gamma}(3, u+1)$$

b) $f(u) = \int f(u|x)f(x) dx$

this is a function we can use again

$$I(W, X) = f(u) = \int_0^\infty x^2 u e^{-ux} e^{-x} dx = u \int_0^\infty x^2 e^{-(u+1)x} dx$$

$$u \left[x^2 \int_0^\infty e^{-(u+1)x} dx - \int_0^\infty 2x \left(\int_0^\infty e^{-(u+1)x} dx \right) dx \right]$$

$$u \left[x^2 \left(\frac{e^{-(u+1)x}}{-(u+1)} \right) - \int_0^\infty 2x \left(\frac{e^{-(u+1)x}}{-(u+1)} \right) dx \right] = u \left[x^2 e^{-(u+1)x} \right]_0^\infty - \frac{2}{u+1} \int_0^\infty x^2 e^{-(u+1)x} dx$$

Another int. by parts, I can't. $f(u) = \frac{2}{u^2-1}$

c) Gamma(α, β) density has kernel: $x^{\alpha-1} e^{-x/\beta} \leftarrow (1+u)^3$

$$f(u|W, X) = \frac{f(u|w, x)f(w, x)}{f(u)} \propto f(u|w, x)f(w, x)$$

$$\text{Gamma}(3, X+W)$$

$$\propto x^{3-1} e^{-x(x+w)}$$

$$\propto x^2 e^{-x(x+w)}$$

$$u|W, X \sim \text{Gamma}(3, X+W)$$

d) $I(W, X) = f(w|x) = \int f(w|u, x)f(u, x) dx$

2x integration by parts using $I(W, X)$,

$$I(W, X) = f(w|x) = \frac{2x^2}{(w+x)^3}$$