HW3

Question 2.

a) $f(y;p) = p^{y}(1-p)^{1-y}$

$\log(f(y;p)) = \log(p^{y}(1-p)^{1-y})$

$l(p) = \sum_{i=1}^{n} \log(f(y_i;p))$

$= \sum_{i=1}^{n} \log(p^{y_i}) + \log(1-p)^{1-y_i}$

$= \sum_{i=1}^{n} y_i \log(p) + (1-y_i)\log(1-p)$

$= n \times [y_i \log(p) + (1-y_i)\log(1-p)]$ ■

b) $f(k_i;p) = \binom{r+k-1}{k}(1-p)^{r} p^{k_i}$

$\log(f(k_i;p)) = \log\left(\binom{r+k-1}{k}(1-p)^{r}p^{k_i}\right)$

$l(p) = \sum_{i=1}^{n} \log(f(k_i;p))$

$= \sum_{i=1}^{n} \underbrace{\log\binom{r+k_i-1}{k_i}}_{=q} + \log((1-p)^{r}p^{k_i})$

$= q + r\log(1-p) + k_i\log(p)$

$= \sum_{i=1}^{n} \log\binom{r+k_i-1}{k_i} + n \times [k_i\log(p) + r\log(1-p)]$ ■

c) $f(x, \lambda) = \lambda \exp(-\lambda x)$

$\log(f(\bar{x}, \lambda)) = \log(\lambda \exp(-\lambda \bar{x}))$

$= \log \lambda + \log(\exp(-\lambda \bar{x}))$

$= \log \lambda - \lambda \bar{x}$

$\ell(\lambda) = \sum_{i=1}^{n} \log \lambda - \lambda \bar{x}$

$= n \times [\log(\lambda) - \lambda \bar{x}]$ ■

d) $f(g; \lambda) = \frac{\lambda^m}{(m-1)!} g^{m-1} \exp(-\lambda g)$

$\log(f(g; \lambda)) = \log\left(\frac{\lambda^m}{(m-1)!}\right) + \log(g^{m-1}) + \log(\exp(-\lambda g))$

$= m \log(\lambda) - \log((m-1)!) + (m-1)\log(g) - \lambda g$

$= (m-1)\log(g) - \log((m-1)!) + m\log(\lambda) - \lambda g$

$\ell(\lambda; g_i) = (m-1)\sum_{i=1}^{n} \log(g_i) - n\log((m-1)!) + n[m\log(\lambda) - \lambda g]$ ■

e) $f(c; \lambda) = \frac{\lambda^c}{c!} \exp(-\lambda)$

$\log(f(c; \lambda)) = \log\left(\frac{\lambda^c}{c!}\right) + \log(\exp(-\lambda))$

$= c\log\lambda - \log(c!) - \lambda$

$= -\log(c!) + \log(\lambda)c - \lambda$

$\ell(\lambda; c_i) = -\sum_{i=1}^{n} \log(c_i!) + n \times [\log(\lambda)\bar{c} - \lambda]$ ■