Morgan Fissel

S352 HW4

2. a) $-\ell''(p; y_1 \dots y_n)/n \Rightarrow \frac{\bar{y}}{p^2} + \frac{1-\bar{y}}{(1-p)^2} \Rightarrow \frac{\hat{p}}{\hat{p}^2} + \frac{1-\hat{p}}{(1-\hat{p})^2} \Rightarrow$

MLE $\hat{p} = \bar{y}$ $\quad E[Y] = p$

$\Rightarrow \frac{1}{\hat{p}} + \frac{1}{1-\hat{p}} \Rightarrow \frac{1}{\hat{p}(1-\hat{p})}$

limiting value is $\frac{1}{p(1-p)}$ limiting dist. is $N(0, p(1-p))$

b) $-\ell''(p; k_1 \dots k_n)/n \Rightarrow \frac{\bar{k}}{p^2} + \frac{r}{(1-p)^2} \Rightarrow \frac{\bar{k}}{\hat{p}^2} + \frac{r}{(1-\hat{p})^2} \Rightarrow \frac{\hat{p}^3 r}{1-\hat{p}} + \frac{r}{(1-\hat{p})^2} \Rightarrow$

$E[k] = pr/(1-p)$

$\Rightarrow \frac{(1-\hat{p})(\hat{p}^3 r) + r}{(1-\hat{p})^2} \Rightarrow \frac{\hat{p}^3 r - \hat{p}^4 r + r}{(1-\hat{p})^2} \Rightarrow \frac{r(\hat{p}^3 - \hat{p}^4 + 1)}{(1-\hat{p})^2}$

limiting value is $\frac{r(p^3 - p^4 + 1)}{(1-p)^2}$? limiting dist. is $N(0, (1-p)^2)$

c) $-\ell''(\lambda; x_1, \dots, x_n)/n \Rightarrow \frac{1}{\lambda^2} \Rightarrow \frac{1}{(\frac{1}{\bar{x}})^2} \Rightarrow \bar{x}^2 \Rightarrow \frac{1}{\lambda^2}$

limiting value is $\frac{1}{\lambda^2}$, limiting dist. is $N(0, \lambda^2)$

d) $-\ell''(\lambda; y_1, \dots, y_n)/n \Rightarrow \frac{m}{\lambda^2} \Rightarrow \frac{m\bar{y}}{m} \Rightarrow \bar{y} \Rightarrow \frac{m}{\lambda}$

limiting value m/λ, limiting dist. is $N(0, \lambda)$

e) $-l''(\lambda; c_1, \ldots, c_n)/n \Rightarrow \frac{c}{\lambda^2} \Rightarrow \frac{c}{c^2} \to \frac{\lambda}{\lambda^2} = \frac{1}{\lambda}$

limiting value is $\frac{1}{\lambda}$, limiting dist. is $N(0, \lambda)$