Learning to Program with F#
Exercises
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0.1 Continued Fractions

0.1.1 Teacher's guide

Emne Rekursion

Sværhedsgrad Middel

0.1.2 Introduction

In this assignment, you will work with simple continued fractions[1], henceforth just called continued fractions. Continued fractions are lists of integers which represent real numbers. The list is finite for rational numbers and infinite for irrational numbers.

Continued fractions to decimal numbers A continued fraction is written as $x = [q_0; q_1, q_2, \ldots]$ and the corresponding decimal number is found by the following recursive algorithm:

$$x = q_0 + \frac{1}{q_1 + \frac{1}{q_2 + \ldots}}. \tag{1}$$

The series of fractions continues as long as there are elements in the continued fraction.

For example, $[3; 4, 12, 4] = 3.245$, since:

$$\begin{aligned} x &= 3 + \frac{1}{4 + \frac{1}{12 + \frac{1}{4}}} && (2) \\ &= 3 + \frac{1}{4 + \frac{1}{12.25}} && (3) \\ &= 3 + \frac{1}{4.081632653} && (4) \\ &= 3.245. && (5) \end{aligned}$$

Note that all but the first digit must be larger than 0, e.g., $[1; 0]$ is an illigal number, and that every rational number has exactly 2 representations $[q_0; q_1, \ldots, q_n] = [q_0; q_1, \ldots, (q_n - 1), 1]$ where the first is called the canonical representation. E.g., $[2; 3] = [2; 2, 1]$, since

$$2 + \frac{1}{3} = 2 + \frac{1}{2 + \frac{1}{1}}. \tag{6}$$

Decimal numbers to continued fractions For a given number x on decimal form, its continued fraction $[q_0; q_1, q_2, \ldots]$ can be found using the following algorithm:

[1] `https://en.wikipedia.org/wiki/Continued_fraction`

Let $x_0 = x$ and $i \geq 0$, and calculate

$$q_i = \lfloor x_i \rfloor \tag{7}$$
$$r_i = x_i - q_i \tag{8}$$
$$x_{i+1} = 1/r_i \tag{9}$$
$$\tag{10}$$

recursively until $r_i = 0$. The continued fraction is then the sequences of q_i.

For example, if $x = 3.245$ then

i	x_i	$q_i = \lfloor x_i \rfloor$	$r_i = x_i - q_i$	$x_{i+1} = 1/r_i$
0	3.245	3	0.245	4.081632653...
1	4.081632653...	4	0.081632653	12.25
2	12.25	12	0.25	4
3	4	4	0	-

and hence, the continued fraction is in the third column as $3.245 = [3;4,12,4]$.

0.1.3 Exercise(s)

0.1.3.1: Write a recursive function

```
cfrac2float : lst:int list -> float
```

that takes a list of integers as a continued fraction and returns the corresponding real number.

0.1.3.2: Write a function

```
float2cfrac : x:float -> int list
```

that takes a real number and calculates its continued fraction. Recall that floating-point numbers are inaccurate, so you should check that r_i is reasonably close to 0 instead of comparing it for equality to 0.0. For example, `abs ri < 1e-10`.

0.1.3.3: Skriv en rekursiv funktion

```
frac2cfrac : t:int -> n:int -> int list
```

som tager tæller og nævner i brøken t/n og udregner dens repræsentation som kædebrøk udelukkende ved brug af heltalstyper.

0.1.3.4: Skriv en rekursiv funktion

```
cfrac2frac : lst:int list -> i:int -> int * int
```

som tager en kædebrøk og et index og returnerer t_i/n_i approximationen som tuplen `(ti, ni)`.

0.1.3.5: Collect the above functions in a library as the interface file `continuedFraction.fsi` and implementation file `continuedFraction.fs`. Make a white- and blackbox test of these functions as the application `continuedFractionTest.fsx`.