HA NOI UNIVERSITY
Faculty of Information Technology

FINAL EXAM (SET 2)

Subject: K20 Calculus
Duration: 75 minutes

I. Multiple Choice Questions (5 points)

1. **(0.25 point)** Which of the following is not a linear equation?
   A. $x_1+4\,x_2\,+1=x_3$ B. $x_1=3$
   C. $x_1+4\,x_2\,-\sqrt{3}x_3=\sqrt{5}$ D. $x_1+4\,x_1x_2\,-\sqrt{3}x_3=\sqrt{5}$

2. **(0.25 point)** Cramer's rule leads easily to a general formula for _____.
   A. the inverse of n x n matrix A B. the adjugate of an matrix A

3. **(0.25 point)** The transpose of a lower triangular matrix is _____.
   A. Lower triangular matrix B. Upper triangular matrix

4. **(0.25 point)** Let A be a square matrix of order 3x3 with $det\ (A)=21$, then $det\ (2A)$
   A. 168 B. 186 C. 21 D. 126

5. **(0.25 point)** The product of upper triangular matrices is _____.
   A. lower triangular matrix B. upper triangular matrix
   C. diagonal matrix

6. **(0.25 point)** Find the dimensions of the null spcae and the column space of the given matrix
$$A=\begin{bmatrix} 1 & 5 & 1 & 3 & 0 \\ -2 & -4 & -5 & -4 & 1 \end{bmatrix}.$$
   A. dim Nul A = 3, dim Col A = 2 B. dim Nul A = 3, dim Col A = 3
   C. dim Nul A = 2, dim Col A = 3 D. dim Nul A = 4, dim Col A = 1

7. **(0.25 point)** For the given matrix A, find k such that Nul A is a subspace of R^k and find m such that Col A is a subspace of R^m

$$A=\begin{bmatrix}1 & -2\\ 0 & 6\\ -4 & 5\\ -1 & -3\\ 3 & -5\end{bmatrix}.$$

A. $k=2,\ m=5$ B. $k=5,\ m=2$

C. $k=5,\ m=5$ D. $k=2,\ m=2$

8. (0.25 point) Solve using Cramer's rule

$$\begin{cases}3x_1+3x_2=9\\ 2x_1+x_2=1\end{cases}.$$

A. $(-5,-2)$ B. $(-2,5)$ C. $(5,-2)$ D. $(2,-5)$

9. (0.25 point) Find an explicit description of the null space of matrix A by listing vectors that span the null space

$$A=\begin{bmatrix}1 & -2 & -2 & -2\\ 0 & 1 & 3 & 4\end{bmatrix}.$$

A. $\begin{bmatrix}2\\ -3\\ 1\\ 0\end{bmatrix}, \begin{bmatrix}2\\ -4\\ 0\\ 1\end{bmatrix}$ B. $\begin{bmatrix}2\\ 1\\ 0\\ 0\end{bmatrix}, \begin{bmatrix}2\\ -3\\ 1\\ 0\end{bmatrix}, \begin{bmatrix}2\\ -4\\ 0\\ 1\end{bmatrix}$

C. $\begin{bmatrix}-4\\ -3\\ 1\\ 0\end{bmatrix}, \begin{bmatrix}-6\\ -4\\ 0\\ 1\end{bmatrix}$ D. $\begin{bmatrix}2\\ 1\\ 0\\ 0\end{bmatrix}, \begin{bmatrix}-4\\ -3\\ 1\\ 0\end{bmatrix}, \begin{bmatrix}-6\\ -4\\ 0\\ 1\end{bmatrix}$

10. (0.25 point) Determine the values of the parameters for which the system has unique solution

$$\begin{cases}5x_1+9x_2=-3\\ 5x_1+sx_2=4\end{cases}.$$

A. $s\neq\pm 3$ B. $s\neq 3$ C. $s\neq\pm 5$ D. $s\neq\pm 9$

11. (0.25 point) If for a linear transformation the equation T(x) =0 has only the trivial solution then T is _____.

A. One-to-one B. Onto

12. (0.25 point) If A is invertible, then $det(A)det(A^{-1})=1.$

A. True B. False

13. (0.25 point) The horizontal asymptote of the curve $f(x) = \frac{\sqrt{x^2+4}}{x}$ is _____.

A. $y = 2$ B. $y = 1$ C. $y = -4$ D. $y = 0$

14. (0.25 point) Find the limit $\lim_{h\to 0} \frac{(x+h)^2-x^2}{h}$.

A. $2x$ B. $-2x$ C. $2h$ D. $h + 1$

15. (0.25 point) Find the second derivative of the function $f(x) = \left(\frac{1+3x}{3x}\right)(3 - x)$.

A. $2x^3$ B. $3x^2$ C. $2x^3 + 1$ D. $2x^{-3}$

16. (0.25 point) Evaluate the given integrals. $\int_{-1}^{1} |2x - 1|\, dx$.

A. 5/6 B. 1 C. 5/4 D. 5/2

17. (0.25 point) Integrate the function $\int \frac{dx}{\sqrt{x}3\sqrt{x}}$.

A. $2(x-2)^2 + C$ B. $-2(x-2)^3 + C$
C. $-2(x-2)^2 + C$ D. $-2(x-2)^4 + C$

18. (0.25 point) Solve the initial value problem

$$\frac{dy}{dx} = cosx,\ y(0) = 1.$$

A. $y = cosx + 1$ B. $y = sinx + 1$
C. $y = sin\, x - 1$ D. $y = cosx\ - 1$

19. (0.25 point) The equation of the tangent line to the graph of $y = lnx$ at $x = e^2$ is _____.

A. $y = \frac{x}{2e^2} + 1$ B. $y = \frac{x}{3e^2} + 1$
C. $y = \frac{x}{e^2} + 1$ D. $y = \frac{x}{4e^2} + 1$

20. (0.25 point) If $\int_{-2}^{5} f(x)dx = 6$, $\int_{-2}^{5} g(x)dx = 2$, what is the values of $\int_{-2}^{5} \left(\frac{f(x)+g(x)}{5}\right) dx$?

A. $\frac{8}{5}$ B. $\frac{5}{8}$ C. $\frac{3}{5}$ D. $-\frac{5}{8}$

II. Short answer Question (5 points)

1. **(1.0 point)** Find the limit $\lim_{x\to\infty} x^{1/x}$.
2. **(1.0 point)** For what values of a and b will

$$f(x) = \begin{cases} ax, & x < 2 \\ ax^2 - bx + 3, & x \geq 2 \end{cases}$$

be differentiable for all values of x?

3. **(1.0 point)** Evaluate $\int \frac{2x+4}{x^3-2x^2} dx$.
4. **(1.0 point)** Find the area of the region enclosed by the parabola $y = 2 - x^2$ and the line $y = -x$.
5. **(1.0 point)** Evaluate $\int_0^{+\infty} \frac{dx}{1+x^2}$.