Problem Set 7.

1, a, $\lim\limits_{x\to -1} \dfrac{x^3-x^2}{x-1} = \dfrac{(-1)^3-1}{-1-1} = 1$

b, $\lim\limits_{x\to 1} \dfrac{x^3-x^2}{x-1} = \lim\limits_{x\to 1} \dfrac{x^2(x-1)}{x-1} = \lim\limits_{x\to 1} x^2 = 1$

c, $\lim\limits_{x\to -3} \dfrac{3x+9}{x^2+4x+3} = \lim\limits_{x\to 3} \dfrac{3(x+3)}{(x+1)(x+3)} = \lim\limits_{x\to 3} \dfrac{3}{x+1} = -\dfrac{3}{2}$

d, $\lim\limits_{x\to 2^-} \dfrac{x+2}{x-2}$

We have: $\begin{cases} \lim\limits_{x\to 2^-} (x+2) = 4 > 0 \\ \lim\limits_{x\to 2^-} (x-2) = 0 \\ x-2 < 0, \ \forall x < 2. \end{cases}$

$\Rightarrow \lim\limits_{x\to 2^-} \dfrac{x+2}{x-2} = -\infty$

e, $\lim\limits_{x\to +\infty} \dfrac{(2x-1)^5}{(3x^2+2x-7)(x^3-9x)} = +\infty$

f, $\lim\limits_{x\to 0} \dfrac{\sqrt{x^2+4}-2}{x^2} = \lim\limits_{x\to 0} \left(\dfrac{\sqrt{x^2+4}-2}{x^2} \cdot \dfrac{\sqrt{x^2+4}+2}{\sqrt{x^2+4}+2} \right)$

$= \lim\limits_{x\to 0} \dfrac{1}{\sqrt{x^2+4}+2} = \dfrac{1}{4}$.

2 a, $y = \dfrac{2x-7}{x^2-4x}$

$\lim\limits_{x \to -\infty} \dfrac{2x-7}{x^2-4x} = 0$.

$\lim\limits_{x \to +\infty} \dfrac{2x-7}{x^2-4x} = 0$

$\Rightarrow$ The line $y = 0$ is a horizontal asymptote.

b, $y = \dfrac{x^3 - x^2 + 10}{3x^2 - 4x}$

There isn't ~~ex~~ horizontal asymptote.

c, $y = \dfrac{2x^2 - 6}{x^2 - 5x}$

The line $y = 2$ is a horizontal asymptote.

3. a, $f(x) = \sqrt{5-x}$

+) $\lim\limits_{x \to 0} \sqrt{5-x} = \sqrt{5}$

+) $\lim\limits_{x \to 5^+} \sqrt{5-x}$ doesn't exist because domain $x \leq 5$.

+) $\lim\limits_{x \to 5^-} \sqrt{5-x} = \sqrt{10}$

+) $\lim\limits_{x \to -5} \sqrt{5-x} = \sqrt{10}$

+) $\lim\limits_{x \to +5} \sqrt{5-x}$ doesn't exist because $\lim\limits_{x \to 5^+} \sqrt{5-x}$ doesn't exist

+) $\lim\limits_{x \to -\infty} \sqrt{5-x} = -\infty$.

+) $\lim\limits_{x \to +\infty} \sqrt{5-x}$ doesn't exist.

b, $g(x) = \begin{cases} \dfrac{x-5}{|x-5|}, & x \neq 5 \\ 0, & x = 5 \end{cases}$

+) $\lim_{x \to 0} g(x) = \lim_{x \to 0} \left(\dfrac{x-5}{|x-5|} \right) = -1$.

+) $\lim_{x \to 5^+} g(x) = \lim_{x \to 5^+} \left(\dfrac{x-5}{|x-5|} \right) = 1$.

+) $\lim_{x \to 5^-} g(x) = \lim_{x \to 5^-} \left(\dfrac{x-5}{|x-5|} \right) = -1$.

+) $\lim_{x \to -5} g(x) = \lim_{x \to -5} \left(\dfrac{x-5}{5-x} \right) = -1$

+) $\lim_{x \to 5^+} g(x) \neq \lim_{x \to 5^-} g(x)$

$\Rightarrow \lim_{x \to 5} g(x)$ doesn't exist.

+) $\lim_{x \to +\infty} g(x) = \lim_{x \to +\infty} \left(\dfrac{x-5}{5-x} \right) = -1$.

4 a, $\lim_{x \to 0} \dfrac{\sin 3x}{\tan 3x} = \lim_{x \to 0} \dfrac{\sin 3x}{\frac{\sin 3x}{\cos 3x}} = \lim_{x \to 0} \cos 3x = 1$.

b, $\lim_{x \to 0} \dfrac{x \sin x}{1 - \cos x}$

$= \lim_{x \to 0} \left(\dfrac{x \sin x}{1 - \cos x} \cdot \dfrac{1 + \cos x}{1 + \cos x} \right) = \lim_{x \to 0} \left(\dfrac{x \sin x (1 + \cos x)}{1 - \cos^2 x} \right)$.

$= \lim_{x \to 0} \dfrac{x}{\sin x} \cdot (1 + \cos x) = 2$.

c, $\lim_{x \to 0} \dfrac{3x - \sin(kx)}{x}$, $k \neq 0$.

$= 3 - \lim_{x \to 0} \left(\dfrac{\sin(kx)}{x} \right) = 3 - \lim_{x \to 0} \dfrac{k \cdot \sin(kx)}{kx} = 3 - k$.

7 a, $f(x) = \dfrac{x}{x^2 - 1}$

$f(x)$ is not defined at $x = \pm 1$, and f is continuous elsewhere.

b, $f(x) = |x^3 - 2x^2|$

There is no value valid, and f is continuous elsewhere.

c, $f(x) = \dfrac{x+3}{|x^2 + 3x|}$

$f(x)$ is not defined at $\begin{cases} x = 0 \\ x = -3 \end{cases}$, f is continuous elsewhere.

8 a, $f(x) = \dfrac{x}{|x| - 3}$

$f(x)$ is continuous everywhere except for when $x = \pm 3$.

b, $f(x) = \cos^{-1}\left(\dfrac{1}{x}\right)$

$f(x)$ is defined and continuous for $x \leq -1$, $x \geq 1$

c, $f(x) = e^{\ln x}$

$f(x)$ is defined and continuous for $x > 0$.