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Final Calculus (set 2)

I. Multiple choice Questions:

1. D	6. A	11. A	16. D
2. A	7. A	12. A	17. C
3. B	8. B	13. B	18. B
4. A	9. D	14. A	19. C
5. B	10. D	15. D	20. A

II. Short answer Question:

4) We have $x^2 = x + 2 \Rightarrow x^2 - x - 2 = 0 \Leftrightarrow \begin{cases} x = -1 \\ x = 2 \end{cases}$

$\Rightarrow$ The area of the region enclosed by the parabola $y = 2 - x^2$ and the line $y = -x$ is: $\int_{-1}^{2} |x^2 - x - 2| = \frac{9}{2}$.

1) $\lim_{x \to \infty} x^{1/x} = \lim_{x \to \infty} \left(e^{\frac{1}{x}\ln(x)}\right) = 1$.

3) $\int \frac{2x+4}{x^3 - 2x^2}\,dx = \int \frac{2}{x}\,dx - \int \frac{2}{x^2}\,dx + \int \frac{2}{x-2}\,dx$

$= -2\ln|x| - \left(\frac{-2}{x}\right) + 2\ln|x-2|$

$= -2\ln|x| + \frac{2}{x} + 2\ln|x-2| + C$

5) $\int_0^{+\infty} \frac{dx}{1+x^2} = \lim_{A \to +\infty} \int_0^A \frac{dx}{1+x^2}$

$= \lim_{A \to \infty} \arctan x \Big|_0^A$

$= \lim_{A \to \infty} (\arctan A - \arctan 0)$

$= \frac{\pi}{2} - 0 = \frac{\pi}{2}$.

2) For a function to be differentiable it has to be continuous:

$\Rightarrow f(2^-) = f(2^+) = f(2) \Rightarrow 2a = 4a - 2b + 3$

$\Rightarrow 2a - 2b + 3 = 0 \quad (1)$

$$f'(x) = \begin{cases} a, & x < 2 \\ 2ax - b, & x \geq 2 \end{cases}$$

$\Rightarrow a = 4a - b$

$\Rightarrow b = 3a \quad (2)$

Solving (1) and (2) we get, $a = \frac{3}{4}$, $b = \frac{9}{4}$