Midterm Calculus

Name : Lương Thị Uý Thiều
ID: 2001040194
Class : A02.

Problem 1:

Row reduce the augmented matrix:

$$\begin{bmatrix} 1 & -3 & -2 & 0 \\ 2 & -1 & 3 & 12 \end{bmatrix} \sim \begin{bmatrix} 1 & -3 & -2 & 0 \\ 0 & 5 & 7 & 12 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -3 & -2 & 0 \\ 0 & 1 & \frac{7}{5} & \frac{12}{5} \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & \frac{11}{5} & \frac{36}{5} \\ 0 & 1 & \frac{7}{5} & \frac{12}{5} \end{bmatrix}$$

$$\Rightarrow x_1 + \frac{11}{5} x_3 = \frac{36}{5} \qquad \Rightarrow x_1 = -\frac{11}{5} x_3 + \frac{36}{5}$$

$$x_2 + \frac{7}{5} x_3 = \frac{12}{5} \qquad \Rightarrow x_2 = \frac{-7}{5} x_3 + \frac{12}{5}$$

The general solution has form

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -\frac{11}{5} x_3 + \frac{36}{5} \\ -\frac{7}{5} x_3 + \frac{12}{5} \\ x_3 \end{bmatrix}$$

$$= x_3 \begin{bmatrix} -\frac{11}{5} \\ -\frac{7}{5} \\ 1 \end{bmatrix} + \begin{bmatrix} \frac{36}{5} \\ \frac{12}{5} \\ 0 \end{bmatrix}$$

Geometrically the solution set is the line through $\begin{bmatrix} \frac{36}{5} \\ \frac{12}{5} \\ 0 \end{bmatrix}$ in the direction of $\begin{bmatrix} -\frac{11}{5} \\ \frac{-7}{5} \\ 1 \end{bmatrix}$

Problem 5:

$$A = \begin{bmatrix} 1 & 1 & 3 \\ -2 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

a, The nine cofactor are:

$$C_{11} = + \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} = 1 \qquad C_{22} = + \begin{vmatrix} 1 & 3 \\ 0 & 1 \end{vmatrix} = 1$$

$$C_{12} = - \begin{vmatrix} -2 & 1 \\ 0 & 1 \end{vmatrix} = 2 \qquad C_{23} = - \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} = -1$$

$$C_{13} = + \begin{vmatrix} -2 & 2 \\ 0 & 1 \end{vmatrix} = -2 \qquad C_{31} = + \begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix} = -5$$

$$C_{21} = - \begin{vmatrix} 1 & 3 \\ 1 & 1 \end{vmatrix} = 2 \qquad C_{32} = - \begin{vmatrix} 1 & 3 \\ -2 & 1 \end{vmatrix} = -7$$

$$C_{33} = + \begin{vmatrix} 1 & 1 \\ -2 & 2 \end{vmatrix} = 4$$

The adjugate matrix is the transpose of the matrix of cofactors.
Thus:

$$\text{adj } A = \begin{bmatrix} 1 & 2 & -5 \\ 2 & 1 & -7 \\ -2 & -1 & 4 \end{bmatrix}$$

With $\det A = -3$, by Theorem 8 in Section 3.3, we have:

$$A^{-1} = \frac{1}{\det A} \cdot \text{adj } A = \frac{1}{-3} \cdot \begin{bmatrix} 1 & 2 & -5 \\ 2 & 1 & -7 \\ -2 & -1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -1/3 & -2/3 & 5/3 \\ -2/3 & -1/3 & 7/3 \\ 2/3 & 1/3 & -4/3 \end{bmatrix}$$

Therefore, the inverse of A is $\begin{bmatrix} -1/3 & -2/3 & 5/3 \\ -2/3 & -1/3 & 7/3 \\ 2/3 & 1/3 & -4/3 \end{bmatrix}$