Calculus.

Section 2.1:

2. +) $A + 2B = \begin{bmatrix} 2 & 0 & -1 \\ 4 & -5 & 2 \end{bmatrix} + 2.\begin{bmatrix} 7 & -5 & 1 \\ 1 & -4 & -3 \end{bmatrix}$

$= \begin{bmatrix} 2 & 0 & -1 \\ 4 & -5 & 2 \end{bmatrix} + \begin{bmatrix} 14 & -10 & 2 \\ 2 & -8 & -6 \end{bmatrix}$

$= \begin{bmatrix} 16 & -10 & 1 \\ 6 & -13 & -4 \end{bmatrix}$

+) $3C - E = 3.\begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} - \begin{bmatrix} -5 \\ 3 \end{bmatrix}$

The expression $3C - E$ is not defined because $3C$ has two columns and $-E$ has only 1 columns.

+) $CB = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 7 & -5 & 1 \\ 1 & -1 & -3 \end{bmatrix}$

$= \begin{bmatrix} 1.7 + 2.1 & 1.(-5) + 2.(-4) & 1.1 + 2.(-3) \\ -2.7 + 1.1 & (-2).(-5) + 1.(-4) & (-2).1 + 1.(-3) \end{bmatrix}$

$= \begin{bmatrix} 9 & -13 & -5 \\ -13 & 6 & -5 \end{bmatrix}$

+) $E.B = \begin{bmatrix} -5 \\ 3 \end{bmatrix} . \begin{bmatrix} 7 & -5 & 1 \\ 1 & -4 & -3 \end{bmatrix}$

The product EB is not defined because the number of columns of E doesn't match the number of rows of B.

4. +) $A - 5I_3 = \begin{bmatrix} 9 & -1 & 3 \\ -8 & 7 & 6 \\ -4 & 1 & 8 \end{bmatrix} - 5\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 9 & -1 & 3 \\ -8 & 7 & 6 \\ -4 & 1 & 8 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

$= \begin{bmatrix} 4 & 1 & 3 \\ -8 & -2 & -6 \\ 4 & 1 & 3 \end{bmatrix}$

+) $(5I_3) \cdot A = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix} \cdot \begin{bmatrix} 9 & -1 & 3 \\ -8 & 7 & 6 \\ -4 & 1 & 8 \end{bmatrix}$

$= \begin{bmatrix} 45 & -5 & 15 \\ -45 & 35 & -30 \\ -20 & 5 & 40 \end{bmatrix}$

9. We have :

+) $AB = \begin{bmatrix} 2 & 5 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} 4 & -5 \\ 3 & k \end{bmatrix} = \begin{bmatrix} 23 & -10+5k \\ -9 & 15+k \end{bmatrix}$

+) $BA = \begin{bmatrix} 4 & -5 \\ 3 & k \end{bmatrix} \begin{bmatrix} 2 & 5 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} 23 & 15 \\ 6-3k & 15+k \end{bmatrix}$

$\Rightarrow AB = BA \Leftrightarrow \begin{cases} -10 + 5k = 15 \\ 6 - 3k = -9 \end{cases} \Leftrightarrow k = 5.$

Thus , $k = 5$.

27. $u = \begin{bmatrix} -2 \\ 3 \\ -4 \end{bmatrix}$, $v = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ $\Rightarrow v^T = \begin{bmatrix} a & b & c \end{bmatrix}$

$\Rightarrow u^T = \begin{bmatrix} -2 & 3 & -4 \end{bmatrix}$

+, $u^T v = \begin{bmatrix} -2 & 3 & -4 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} -2a & 3b & -4c \end{bmatrix}$

+, $v^T u = \begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} -2 \\ 3 \\ -4 \end{bmatrix} = \begin{bmatrix} -2a & 3b & -4c \end{bmatrix}$

+, $u v^T = \begin{bmatrix} -2 \\ 3 \\ -4 \end{bmatrix} \begin{bmatrix} a & b & c \end{bmatrix} = \begin{bmatrix} -2a & -2b & -2c \\ 3a & 3b & 3c \\ -4a & 4b & -4c \end{bmatrix}$

+, $v u^T = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \begin{bmatrix} -2 & 3 & -4 \end{bmatrix} = \begin{bmatrix} -2a & 3a & -4a \\ -2b & 3b & -4b \\ -2c & 3c & -4c \end{bmatrix}$

Section 2.2:

5. The system is equivalent to $Ax = b$, where $A = \begin{bmatrix} 8 & 6 \\ 5 & 4 \end{bmatrix}$ and $b = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$, and the solution is

$$x = A^{-1} . b = \begin{bmatrix} 2 & -3 \\ -\frac{5}{2} & 4 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 7 \\ -9 \end{bmatrix}$$

$\Rightarrow$ $x_1 = -7$

$x_2 = 9$.

9.

a, True (By definition of invertible).

b, False (By Theorem 6.b).

c, False (If $A = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$, then $ab - cd = 1 - 0 \neq 0$,

but by Theorem 4 show that this matrix isn't invertible because $ab - cd = 0$).

d, True (This follows from Theorem 5, which also says that the solution of $Ax = b$ is unique, for each b).

e, True (By the box just before Ex6).

10.

a, False (The product matrix is invertible, but the product of inverses should be in the reverse order) (By Theorem 6).

b, True (By Theorem 6a).

c, True (By Theorem 4).

d, True (By Theorem 7).

e, False (The last part of Theorem 7 is misstated).

21. Let $v_1, v_2, \ldots v_n$ denote the columns of A, and let there exist scalars $x_1, x_2 \ldots, x_n$ such that:

$$x_1 v_1 + x_2 v_2 + \ldots + x_n v_n = 0.$$

$$\Rightarrow x_1 = x_2 = \ldots = x_n = 0.$$

23. If A is $n \times n$ matrix and $Ax = 0$ has only the trivial solution then every columns of A must be a pivot column. Because every column of A is a pivot column and A has n rows and n column. Then every row of A must contain a pivot. The only possibility is that $A \sim I$.

31. We have:

$$\begin{bmatrix} 1 & 0 & -2 & 1 & 0 & 0 \\ -3 & 1 & 4 & 0 & 1 & 0 \\ 2 & -3 & 4 & 0 & 0 & 1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 3 & 1 & 0 \\ 0 & -3 & 8 & -2 & 0 & 1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 3 & 1 & 0 \\ 0 & 0 & 2 & 7 & 3 & 1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 8 & 3 & 1 \\ 0 & 1 & 0 & 10 & 4 & 1 \\ 0 & 0 & 2 & 7 & 3 & 1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 8 & 3 & 1 \\ 0 & 1 & 0 & 10 & 4 & 1 \\ 0 & 0 & 1 & 7/2 & 3/2 & 1/2 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \begin{bmatrix} 8 & 3 & 1 \\ 10 & 4 & 1 \\ 7/2 & 3/2 & 1/2 \end{bmatrix}$$

Section 2.3:

3. Row reduction to echolon form is trivial because there is really no need for arthmetic calculations:

$$\begin{bmatrix} 5 & 0 & 0 \\ -3 & -7 & 0 \\ 8 & 5 & -1 \end{bmatrix} \sim \begin{bmatrix} 5 & 0 & 0 \\ 0 & -7 & 0 \\ 0 & 5 & -1 \end{bmatrix} \sim \begin{bmatrix} 5 & 0 & 0 \\ 0 & -7 & 0 \\ 0 & 0 & -1 \end{bmatrix}.$$

The 3×3 matrix has 3 pivot position and hence is invertible, by (c) of IMT.

4. The matrix $\begin{bmatrix} -7 & 0 & 4 \\ 3 & 0 & -1 \\ 2 & 0 & 9 \end{bmatrix}$ obviously has linearly dependent columns (because one column is zero), and so the matrix is not invertible by (e) of IMT.

12. a, True (If statement (k) of IMT is true, then so is statement (j)).

b, True (If statement (e) of IMT is true, then so is statement (h)).

c, True (See the remark immediately following the proof of IMT).

d, False (The first part of statement is not part (i) of IMT. In fact, if A is any $n \times n$ matrix, linear transformation $x \mapsto Ax$ maps $\mathbb{R}^n$ into $\mathbb{R}^n$, yet not every such matrix has n pivot position) e, True (By the IMT, if there is a b in $\mathbb{R}^n$ such that the equation $Ax = b$ is inconsistent, then statement (g) of IMT ~~is~~ is false, and hense statement (f) is also false. That is, the transformation $x \mapsto Ax$ cannot be one-to-one).

11. a, True (By the IMT, if statement (d) of IMT is true, then so is statement (b)).

b, True (________ (h) ________

________ (e)).

c, False (Statement (g) of IMT is true only for invertible matrices).

d, True (By the IMT, if the equation $Ax = 0$ has a nontrivial solution, then statement (d) is false. In this case, all the lettered statements in the IMT are false, including statements (c), which mean ~~A~~ that A must have fewer than n pivot position.

e, True (By the IMT, if A^T is not invertible, then statement (1) is false, and hense statement (a) must be also false.

19. The equation $Dx = b$ has at least one solution for each b in $\mathbb{R}^7$, because Theorem 8 "The equation $Ax = b$ has at least one solution for each b in $\mathbb{R}^n$".