FINAL EXAM 02 (60 minutes)
Discrete Mathematics - Fall 2021

There are 11 questions in the exam.
Fill in blanks with one phrase or one sentence only.

Q1. (2p) Consider values of (p→ r) ∧ (q→ r) and (p ∨ q) → r

- ✓ (p→ r) ∧ (q→ r) false when …(1)…..... true and …(2)….. false
- ✓ (p ∨ q) → r false when …(3)…… true and …(4)…… false
- ✓ Conclusion: (p→ r) ∧ (q→ r) and (p ∨ q) → r are …(5)….

Q2. (10p) Write a short answer (at most 5 lines) to prove that $\sqrt{3}$ is irrational.

Q3. (10p) $S_n = 5+10+15+\ldots+5n = (5/2)^* A \quad (n \in N+)$

- ✓ What is A? ..... (1).....
- ✓ What is inductive hypothesis? …(2)….
- ✓ In the inductive step, $S_{k+1} = S_k$ + ….(3)....

Q4.

- ✓ (2p) If A ⊂ B, then what is |A ∩ B|? ……(1)……..
- ✓ (1p) If f(a) = f(b) implies that a =b then function f is …(2)……..
  (Choose one of these three: one to one/onto/both one to one and onto).
- ✓ (2p) We have functions f: R-> R and g: R-> R such that (f ∘ g)(x) = $\lfloor (x^2 + 7.1) \rfloor$.
  Find f(x) and g(x). ……(3)……

Q5. (10p) Given: 10x ≡ 47 (mod 129)
We have:

- ✓ gcd(10, 129) =129*s+10*r=…..(1)…..
- ✓ s = ….(2)….
- ✓ r = ….(3)….
- ✓ x_0 = …..(4)….
- ➔ X = {……(5)……}

Q6. (5p) The Lagrange polynomial that passes through the 3 data points is given by

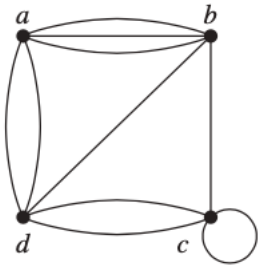
x	15	18	22
y	24	37	25

$f(x) = L_0(x)(24) + L_1(x)(37) + L_2(x)(25)$
$L_1(x)$ is……..(1)……..

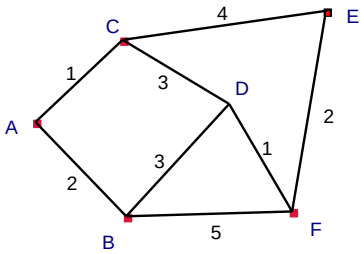
$L_1(x)$ at x=16 is……(2)……..

Q7. (10p) We received data through a transmission line with general error rate is 1 over 5 packages. The received packages are {6, 4, 4, 6, 4} and the encryption employs polynomials over GF (7). Write the system of 5 linear equations corresponding with Q(x) = R(x) * E(x) (over GF7)
(at most 5 lines)

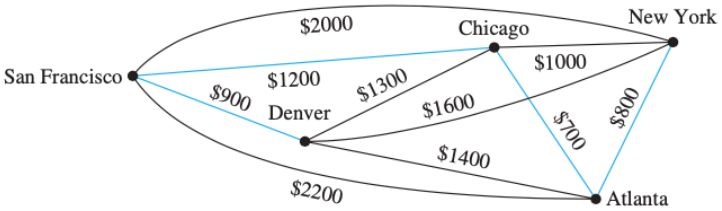
Q8. (5p) Represent the following graph with an adjacency matrix.



Q9. (5p) Use Dijkstra's algorithm to find the shortest path and its length from vertex A to vertex F in the following weighted graph. (Draw a table)



Q10. (5p) Use Prim's algorithm to find a minimum spanning tree in the following weighted graph.



Q11. (5p) Given a data source of 7 characters A, B, C, D, E, F, G with probability of appearance as follows:

A	B	C	D	E	F
0.18	0.29	0.30	0.15	0.07	0.01

- ✓ Draw the Huffman tree (Draw the final result tree)
- ✓ Find the code words for the above characters