

# HOME WORK 1

## Probability & Statistics

### Problem 1:

Determine the probability  $p$ , or an estimate of it, for each of the following events: a. A king, ace, jack of clubs, or queen of diamonds appears in drawing a single

card from a well-shuffled ordinary deck of cards. b. The sum 9 appears in a single toss of a pair of fair dice.

c. A non defective bolt will be found next if out of 600 bolts already examined, 12 were defective.

d. A 7 or 11 comes up in a single toss of a pair of fair dice.

e. At least 1 head appears in 3 tosses of a fair coin.

### Problem 2:

A marble is drawn at random from a box containing 10 red, 30 white, 20 blue, and 15 orange marbles. Find the probability that it is

a. Orange or red;

b. Not red or blue;

c. Not blue;

d. White;

e. Red, white, or blue.

### Problem 3:

Two marbles are drawn in succession from the box of Problem 2, replacement being made after each drawing. Find the probability that

a. Both are white; b. The first is red and the second is white; c. Neither is orange; d. They are either red or white or both (red and white);

e. The second is not blue; f. The first is orange; g. At least one is blue; h. At most one is red;

i. The first is white but the second is not; j. Only one is red; k. Only one is blue.

### Problem 4:

Work Problem 3 with no replacement after each drawing.

**Problem 5:**

Suppose that a number  $x$  is to be selected from the real line  $S$ , and let  $A$ ;  $B$ ; and  $C$  be the events represented by the following subsets of  $S$ , where the notation  $\{x : \dots\}$  denotes the set containing every point  $x$  for which the property presented following the colon is satisfied:

$$A = \{x : 1 \leq x \leq 5\}$$

$$B = \{x : 3 \leq x \leq 7\}$$

$$C = \{x : x \leq 0\}$$

Describe each of the following events as a set of real numbers:

- a.  $A'$
- b.  $A \cup B$
- c.  $B \cap C'$
- d.  $A' \cap B' \cap C'$
- e.  $(A \cup B) \cap C$

**Problem 6:**

Three six-sided dice are rolled. The six sides of each die are numbered from 1 to 6. Let  $A$  be the event that the first die shows an even number, let  $B$  be the event that the second die shows an even number, and let  $C$  be the event that the third die shows an even number. Also, for each  $i = 1, \dots, 6$ , let  $A_i$  be the event that the first die shows the number  $i$ , let  $B_i$  be the event that the second die shows the number  $i$ , and let  $C_i$  be the event that the third die shows the number  $i$ . Express each of the following events in terms of the named events described above:

- a. The event that all three dice show even numbers.
- b. The event that no die shows an even number.
- c. The event that at least one die shows an odd number.
- d. The event that at most two dice show odd numbers.
- e. The event that the sum of the three dice is no greater than 5.

**Problem 7:**

Consider two events  $A$  and  $B$  such that  $P(A) = 1/3$  and  $P(B) = 1/2$ . Determine the value of  $P(B \cap A')$  for each of the following conditions:

- a.  $A$  and  $B$  are disjoint;
- b.  $A \subset B$ ;
- c.  $P(A \cap B) = 1/8$

**Problem 8:**

If a man has six different sport-shirts and four different pairs of slacks, how many different combinations can he wear?

**Problem 9:**

A coin is tossed 3 times. Use a tree diagram to determine the various possibilities that can arise.

**Problem 10:**

Three cards are drawn at random (without replacement) from an ordinary deck of 52 cards. Find the number of ways in which one can draw

- a. A diamond and a club and a heart in succession; b. Two hearts and then a club or a spade

**Problem 11:**

In how many ways can 7 books be arranged on a shelf if a. any arrangement is possible; b. 3 particular books must always stand together; c. two particular books must occupy the end.

**Problem 12:**

How many numbers consisting of five different digits each can be made from the digits 1, 2, 3, ..., 9 if

- a. the numbers must be odd;  
b. the first two digits of each number are even.

**Problem 13:**

In how many ways can 3 men and 3 women be seated at a round table if

- a. no restriction is imposed;  
b. 2 particular women must not sit together;  
c. each woman is to be between 2 men.

**Problem 14:**

In how many ways can 2 men, 4 women, 3 boys and 3 girls be selected from 6 men, 8 women, 4 boys and 5 girls if

- a. no restrictions are imposed;  
b. a particular man and woman must be selected.

**Problem 15:**

In how many ways can a group of 10 people be divided into

- a. two groups consisting of 7 and 3 people;
- b. three groups consisting of 5, 3 and 2 people.

**Problem 16:**

An urn contains 6 red and 8 blue marbles. Five marbles are drawn at random from it without replacement. Find the probability that 3 are red and 2 are blue.

**Problem 17:**

- a. Find the probability of getting the sum 7 on at least 1 of 3 tosses of a pair of fair dice;
- b. How many tosses are needed in order that the probability in (a) be greater than 0.95.

**Problem 18:**

A pair of dice is tossed repeatedly. Find the probability that an 11 occurs for the first time on the 6<sup>th</sup> toss.

**Problem 19:**

Four integers are chosen at random between 0 and 9, inclusive. Find the probability that

- a. they are all different;
- b. not more than 2 are the same.

**Problem 20:**

What is the least number of tosses needed in Problem 18 so that the probability of getting an 11 will be greater than

- a. 0.5;
- b. 0.95.