HA NOI UNIVERSITY
Faculty of Information Technology

FINAL EXAM (SET 1)

Subject: K20 Calculus
Duration: 75 minutes

I. Multiple Choice Questions (5 points)

1. **(0.25 point)** The matrix multiplication is associative.
   A. True B. False

2. **(0.25 point)** We can add the matrices of _____.
   A. same order B. same number of columns
   C. same number of rows D. different order

3. **(0.25 point)** A system of linear equations is said to be homogeneous if it can be written in the form _____.

   A. $AX = B$ B. $AX = 0$ C. $AB = X$ D. $X = A^{-1}$

4. **(0.25 point)** The row reduction algorithm applies only to augmented matrices for a linear system.
   A. True B. False

5. **(0.25 point)** Whenever a system has no free variable, the solution set contains many solutions.
   A. True B. False

6. **(0.25 point)** A square matrix $A = \left[a_{ij} \right]$ is upper triangular if and only if $a_{ij} = 0$ for _____.
   A. $i > j$ B. $i < j$ C. $i \le j$ D. $i = j$

7. **(0.25 point)** Compute the determinant of the matrix.

$$\begin{bmatrix} 4 & 2 & 9 \\ 5 & 2 & 9 \\ 7 & 7 & 4 \end{bmatrix}$$

   A. 891 B. -197 C. -55 D. 55

8. **(0.25 point)** Find all values of h such that y will be in the subspace of $\mathbb{R}^3$ spanned by v_1, v_2, v_3 if

$$v_1 = \begin{bmatrix} 1 \\ 2 \\ -4 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 3 \\ 4 \\ -8 \end{bmatrix}, \quad v_3 = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}, \quad \text{and } y = \begin{bmatrix} 4 \\ 2 \\ h \end{bmatrix}.$$

A. $h = -4$ or 0
B. all $h \neq -4$
C. $h = -4$
D. $h = -16$

9. (0.25 point) Determine if the vector u is in the column space of matrix A and whether it is in the null space of A.

$$u = \begin{bmatrix} -2 \\ -5 \\ -2 \end{bmatrix}, \quad A = \begin{bmatrix} 1 & -3 & 4 \\ -1 & 0 & -5 \\ 3 & -3 & 6 \end{bmatrix}$$

A. In Col A and in Nul A
B. In Col A, not in Null A
C. Not in Col A, not in Nul A
D. Not in Col A, in Nul A

10. (0.25 point) Determine the values of the parameters for which the system has unique solution

$$\begin{cases} 5x_1 + 9x_2 = -3 \\ 5x_1 + sx_2 = 4 \end{cases}.$$

A. $s \neq \pm 3$ B. $s \neq 3$ C. $s \neq \pm 5$ D. $s \neq \pm 9$

11. (0.25 point) For the given matrix A, find k such that Nul A is a subspace of R^k and find m such that Col A is a subspace of R^m

$$A = \begin{bmatrix} 4 & 0 & 0 & -1 & 1 & -7 \\ 2 & 6 & -5 & -1 & 0 & 3 \\ -3 & -4 & 4 & -5 & 5 & -3 \end{bmatrix}.$$

A. $k = 6,\ m = 3$
B. $k = 6,\ m = 6$
C. $k = 3,\ m = 3$
D. $k = 3,\ m = 6$

12. (0.25 point) If two row interchanges are made in succession, then the new determinant _____.

A. equals to the old determinant
B. equals to -1 times the old determinant

13. (0.25 point) On what intervals are the function $f(x) = x^{-2/3}$ continuous?
A. $(-\infty, +\infty)$
B. $(0, +\infty)$
C. $[0, +\infty)$
D. $(-\infty, 0)$ and $(0, +\infty)$

14. (0.25 point) Suppose that $f(t)$ *and* $g(t)$ are defined for all t and that $\lim_{t \to t_0} f(t) = -7$ and $\lim_{t \to t_0} g(t) = 0$. The limit as $t \to t_0$ of the function $\frac{f(t)}{g(t)-7}$ is _____.

A. -21 B. 49 C. 0 D. 1

15. **(0.25 point)** The vertical asymptote of the curve $f(x) = \frac{x^2+x-6}{x^2+2x-8}$ is _____.

A. $x = 3$ B. $x = 1$ C. $x = -4$ D. $x = -3$

16. **(0.25 point)** Find the limit of $g(x)$ as x approaches the indicated value

$$\lim_{x\to 0^+} \big(4g(x)\big)^{1/3} = 2.$$

A. 2 B. 1 C. 4 D. -2

17. **(0.25 point)** At what points is the function

$$f(x) = \begin{cases} \dfrac{x^2 - x - 6}{x - 3}, & x \neq 3 \\ 5, & x = 3 \end{cases}$$

continuous?

A. $x = 3$ B. $x \neq 3$ C. All x D. $x = 1$

18. **(0.25 point)** The limit $\lim_{x\to 1} \frac{x^{50}-1}{x-1}$ can be expressed as the derivative of what function and at what value of x.

A. $f(x) = \frac{x^{50}-1}{x-1},\ x = 1$ B. $f(x) = x^{50},\ x = 1$

C. $f(x) = x^{50} - 1,\ x = 1$ D. $f(x) = x^{50},\ x = 0$

19. **(0.25 point)** A particle moves along an $s - axis$ with position function $s = s(t)$ and velocity function $v(t) = s'(t)$. Use the given information to find $s(t)$: $v(t) = 32t;\ \ s(0) = 20.$

A. $s(t) = 16t^2 + 20$ C. $s(t) = 64t^2 + 20$

B. $s(t) = 12t^2 + 20$ D. $s(t) = 36t^2 + 20$

20. **(0.25 point)** Evaluate $\int_0^3 f(x)dx$ if $f(x) = \begin{cases} x^2, & x < 2 \\ 3x - 2, & x \geq 2 \end{cases}$.

A. 49/6 C. 29/6 B. 39/6 D. 19/6

II. Short answer Question (5 points)

1. **(1.0 point)** Find the limit $\lim_{x\to -1} \frac{sin(x^2-x-2)}{x+1}$.
2. **(1.0 point)** Suppose that a curve $y = f(x)$ in the xy plane has the property that at each point (x, y) on the curve, the tangent line has slope x^2. Find an equation for the curve given that its passed through the point $(2,1)$.
3. **(1.0 point)** Find $\frac{d^2y}{dx^2}$ by implicit differentiation for $3x^2 - 4y^2 = 7$.
4. **(1.0 point)** Find the region under the curve $y = 3sinx$ and over the interval $[0, 2\pi/3]$.
5. **(1.0 point)** Evaluate $\int_1^\infty \frac{lnx}{x^2} dx$.