MID-TERM TEST : CALCULUS.

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Problem 1:

$$x_1 - 3x_2 - 2x_3 = 0$$
$$2x_1 - x_2 + 3x_3 = 12.$$

$$\begin{bmatrix} 1 & -3 & -2 & 0 \\ 2 & -1 & 3 & 12 \end{bmatrix} \sim \begin{bmatrix} 1 & -3 & -2 & 0 \\ 0 & 5 & 7 & 12 \end{bmatrix}.$$

$$\begin{cases} x_1 - 3x_2 - 2x_3 = 0 \\ 5x_2 - 7x_3 = 12 \end{cases} \rightarrow x_1 = 3x_2 + 2x_3. \ (1)$$

$$5x_2 = 12 + 7x_3.$$
$$x_2 = \frac{12}{5} - \frac{7}{5}x_3. \xrightarrow{(1)} x_1 = 3.\left(\frac{12}{5} - \frac{7}{5}x_3\right) + 2x_3$$
$$= \frac{36}{5} - \frac{11}{5}x_3.$$

We have

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \frac{36}{5} - \frac{11}{5}x_3 \\ \frac{12}{5} - \frac{7}{5}x_3 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} -\frac{11}{5} \\ -\frac{7}{5} \\ 1 \end{bmatrix} + \begin{bmatrix} \frac{36}{5} \\ \frac{12}{5} \\ 0 \end{bmatrix}$$

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Problem 2:

$[A\ 0] = [v_1\ v_2\ v_3\ 0] = \Delta$

$$\Delta = \begin{bmatrix} 2 & -6 & 8 & 0 \\ -4 & 7 & h & 0 \\ 1 & -3 & 4 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -3 & 4 & 0 \\ -4 & 7 & h & 0 \\ 1 & -3 & 4 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -3 & 4 & 0 \\ 0 & -5 & h+16 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

x_1, x_2: basis variable
x_3: Free.

→ Each nonzero value of x_3 determines a nontrivial solution of the matrix.

→ v_1, v_2, v_3 are linearly dependent for all value of h.

$$\begin{bmatrix} 0 & 0 & \frac{102}{5}+\frac{6}{5}h \end{bmatrix} \Leftrightarrow \frac{102}{5}+\frac{6}{5}h=0$$

$$\rightarrow h=-17.$$

Problem 3:

$$A=\begin{bmatrix} 4 & 5 & 9 \\ 6 & 5 & 1 \\ 3 & 4 & 8 \end{bmatrix} \sim \begin{bmatrix} 4 & 5 & 9 \\ 0 & -\frac{5}{2} & -\frac{25}{2} \\ 3 & 4 & 8 \end{bmatrix} \sim \begin{bmatrix} 4 & 5 & 9 \\ 0 & -\frac{5}{2} & -\frac{25}{2} \\ 0 & \frac{1}{4} & \frac{5}{4} \end{bmatrix}$$

$$\sim \begin{bmatrix} 4 & 5 & 9 \\ 0 & -\frac{5}{2} & -\frac{25}{2} \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \text{Basis for Col A:}$$

$$\begin{bmatrix} 4 \\ 6 \\ 3 \end{bmatrix} ; \begin{bmatrix} 5 \\ 5 \\ 4 \end{bmatrix}$$

b)

A has only 2 pivot columns → A is not invertible matrix.

<By the IMT> → dim Nul A = 3

dim Col A = 0.

Problem 4:

a) $A = \begin{bmatrix} 1 & 1 & 3 \\ -2 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

$$\left[\begin{array}{cccccc} 1 & 1 & 3 & 1 & 0 & 0 \\ -2 & 2 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{cccccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 0 & 4 & 7 & 2 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{cccccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 0 & 1 & \frac{7}{4} & \frac{1}{2} & \frac{1}{4} & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{cccccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 0 & 1 & \frac{7}{4} & \frac{1}{2} & \frac{1}{4} & 0 \\ 0 & 0 & \frac{-3}{4} & \frac{-1}{2} & \frac{-1}{4} & 1 \end{array}\right] \sim \left[\begin{array}{cccccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 0 & 1 & \frac{7}{4} & \frac{1}{2} & \frac{1}{4} & 0 \\ 0 & 0 & 1 & \frac{2}{3} & \frac{1}{3} & \frac{-4}{3} \end{array}\right]$$

$$\sim \left[\begin{array}{cccccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 0 & 1 & 0 & \frac{-2}{3} & \frac{-1}{3} & \frac{7}{3} \\ 0 & 0 & 1 & \frac{2}{3} & \frac{1}{3} & \frac{-4}{3} \end{array}\right] \sim \left[\begin{array}{cccccc} 1 & 0 & 0 & \frac{-1}{3} & \frac{-2}{3} & \frac{5}{3} \\ 0 & 1 & 0 & \frac{-2}{3} & \frac{-1}{3} & \frac{7}{3} \\ 0 & 0 & 1 & \frac{2}{3} & \frac{1}{3} & \frac{-4}{3} \end{array}\right]$$

$$\rightarrow A^{-1} = \begin{bmatrix} 1 & 1 & 3 \\ -2 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{-1}{3} & \frac{-2}{3} & \frac{5}{3} \\ \frac{-2}{3} & \frac{-1}{3} & \frac{7}{3} \\ \frac{2}{3} & \frac{1}{3} & \frac{-4}{3} \end{bmatrix}$$

b) Det $(A^{10})^T$

$$A = \begin{bmatrix} 1 & 1 & 3 \\ -2 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$A^2 = A^1 . A^1 = \begin{bmatrix} 1 & 1 & 3 \\ -2 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 3 \\ -2 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 6 & 7 \\ -6 & 3 & -3 \\ -2 & 3 & 2 \end{bmatrix}$$

$$A^4 = A^2 . A^2 = \begin{bmatrix} -1 & 6 & 7 \\ -6 & 3 & -3 \\ -2 & 3 & 2 \end{bmatrix} \begin{bmatrix} -1 & 6 & 7 \\ -6 & 3 & -3 \\ -2 & 3 & 2 \end{bmatrix} = \begin{bmatrix} -49 & 33 & -11 \\ -6 & -36 & -57 \\ -20 & 3 & -19 \end{bmatrix}$$

$$A^8 = A^4 . A^4 = \begin{bmatrix} -49 & 33 & -11 \\ -6 & -36 & -57 \\ -20 & 3 & -19 \end{bmatrix} \begin{bmatrix} -49 & 33 & -11 \\ -6 & -36 & -57 \\ -20 & 3 & -19 \end{bmatrix} = \begin{bmatrix} \; \end{bmatrix}$$

$$= \begin{bmatrix} 2423 & -2838 & 1133 \\ 1650 & 927 & 3201 \\ 1342 & -825 & 410 \end{bmatrix}$$

$$A^{10} = A^8 . A^2 = \begin{bmatrix} 2423 & -2838 & -1133 \\ 1650 & 927 & 3201 \\ 1342 & -825 & 410 \end{bmatrix} \begin{bmatrix} -1 & 6 & 7 \\ -6 & 3 & -3 \\ -2 & 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 16871 & 2625 & 23209 \\ -13614 & 22284 & 15171 \\ 2788 & 6807 & 12689 \end{bmatrix}$$

$$(A^{10})^T = \begin{bmatrix} 16871 & 2625 & 23209 \\ -13614 & 22284 & 15171 \\ 2788 & 6807 & 12689 \end{bmatrix}^T = \begin{bmatrix} 16871 & -13614 & 2788 \\ 2625 & 22284 & 6807 \\ 23209 & 15171 & 12689 \end{bmatrix}$$

$\text{Det}(A^{10})^T = ?$

$$(A^{10})^T = \begin{bmatrix} 16871 & -13614 & 2788 \\ 2625 & 22284 & 6807 \\ 23209 & 15171 & 12689 \end{bmatrix}$$

$$= \begin{bmatrix} 16871 & -13614 & 2788 \\ 0 & \frac{411690114}{16871} & \frac{107522397}{16871} \\ 0 & \frac{571917267}{16871} & \frac{149369427}{16871} \end{bmatrix}$$

$$= \begin{bmatrix} 16871 & -13614 & 2788 \\ 0 & \frac{411690114}{16871} & \frac{107522397}{16871} \\ 0 & 0 & \frac{243}{1694198} \end{bmatrix}$$

$$\begin{vmatrix} 16871 & -13614 & 2788 \\ 2625 & 22284 & 6807 \\ 23209 & 15171 & 12689 \end{vmatrix} = \begin{vmatrix} 16871 & -13614 & 2788 \\ 0 & \frac{411690114}{16871} & \frac{107522397}{16871} \\ 0 & 0 & \frac{243}{1694198} \end{vmatrix}$$

$$= 16871 \cdot \left(\frac{411690114}{16871}\right) \cdot \left(\frac{243}{1694198}\right)$$

$$= 59049.$$