Solution for Problem Set 9

HANU - Faculty of Information and Technology
MAT204: Probability & Statistics

May 14, 2015

Problem 1: [1, Exercise 9.54]
A manufacturer of MP3 players conducts a set of comprehensive tests on the electrical functions of its product. All MP3 players must pass all tests prior to being sold. Of a random sample of 500 MP3 players, 15 failed one or more tests. Find a 90% confidence interval for the proportion of MP3 players from the population that pass all tests.
Solution:

$$n = 500$$
$$\widehat{p} = \frac{500 - 15}{500} = \frac{485}{500} = 0.97$$
$$\widehat{q} = 1 - \widehat{p} = 0.03$$
$$1 - \alpha = 90\% \Rightarrow \alpha = 0.1 \Rightarrow z_{\alpha/2} = z_{0.05} = 1.645$$

Therefore:

$$\widehat{p} - z_{\alpha/2}\sqrt{\frac{\widehat{pq}}{n}} < p < \widehat{p} + z_{\alpha/2}\sqrt{\frac{\widehat{pq}}{n}} \Rightarrow \widehat{p} \pm z_{\alpha/2}\sqrt{\frac{\widehat{pq}}{n}}$$
$$\Rightarrow 0.97 \pm (1.645)\sqrt{\frac{(0.97)(0.03)}{500}} = 0.97 \pm 0.013$$
$$\Rightarrow 0.957 < p < 0.983$$

Problem 2: [1, Exercise 9.56]
A geneticist is interested in the proportion of African males who have a certain minor blood disorder. In a random sample of 100 African males, 24 are found to be afflicted.
(a) Compute a 99% confidence interval for the proportion of African males who have this blood disorder.
(b) What can we assert with 99% confidence about the possible size of our error if we estimate the proportion of African males with this blood disorder to be 0.24?
Solution:

$$n = 100$$
$$\widehat{p} = \frac{24}{100} = 0.24$$
$$\widehat{q} = 1 - \widehat{p} = 0.76$$
$$1 - \alpha = 99\% \Rightarrow \alpha = 0.01 \Rightarrow z_{\alpha/2} = z_{0.005} = 2.575$$

a)

$$\widehat{p} \pm z_{\alpha/2}\sqrt{\frac{\widehat{pq}}{n}}$$

$$\Rightarrow 0.24 \pm (2.575)\sqrt{\frac{(0.24)(0.76)}{100}} = 0.24 \pm 0.110$$

$$\Rightarrow 0.130 < p < 0.350$$

b) The possible size of error can be calculated as:

$$e \leq z_{\alpha/2}\sqrt{\frac{\widehat{pq}}{n}}$$

$$\Rightarrow e \leq (2.575)\sqrt{\frac{(0.24)(0.76)}{100}} = 0.110$$

Problem 3: [1, Exercise 9.65]
A certain geneticist is interested in the proportion of males and females in the population who have a minor blood disorder. In a random sample of 1000 males, 250 are found to be afflicted, whereas 275 of 1000 females tested appear to have the disorder. Compute a 95% confidence interval for the difference between the proportions of males and females who have the blood disorder.
Solution:

$$n_1 = n_2 = n_F = n_M = 1000$$

$$\widehat{p}_1 = \widehat{p}_F = \frac{275}{1000} = 0.275$$

$$\widehat{p}_2 = \widehat{p}_M = \frac{250}{1000} = 0.250$$

$$\widehat{q}_1 = 1 - \widehat{p}_1 = 0.725$$

$$\widehat{q}_2 = 1 - \widehat{p}_2 = 0.750$$

$$1 - \alpha = 95\% \Rightarrow \alpha = 0.5 \Rightarrow z_{\alpha/2} = z_{0.025} = 1.96$$

Therefore:

$$(\widehat{p}_1 - \widehat{p}_2) - z_{\alpha/2}\sqrt{\frac{\widehat{p}_1\widehat{q}_1}{n_1} + \frac{\widehat{p}_2\widehat{q}_2}{n_2}} < p_1 - p_2 < (\widehat{p}_1 - \widehat{p}_2) + z_{\alpha/2}\sqrt{\frac{\widehat{p}_1\widehat{q}_1}{n_1} + \frac{\widehat{p}_2\widehat{q}_2}{n_2}}$$

$$\Rightarrow (\widehat{p}_1 - \widehat{p}_2) \pm z_{\alpha/2}\sqrt{\frac{\widehat{p}_1\widehat{q}_1}{n_1} + \frac{\widehat{p}_2\widehat{q}_2}{n_2}}$$

$$\Rightarrow (0.275 - 0.250) \pm (1.96)\sqrt{\frac{(0.250)(0.750)}{1000} + \frac{(0.275)(0.725)}{1000}} = 0.025 \pm 0.039$$

$$\Rightarrow -0.0136 < p_1 - p_2 < 0.0636$$

Problem 4: [1, Exercise 9.66]
Ten engineering schools in the United States were surveyed. The sample contained 250 electrical engineers, 80 being women; 175 chemical engineers, 40 being women. Compute a 90% confidence interval for the difference between the proportions of women in these two fields of engineering. Is there a significant difference between the two proportions?
Solution:

$$n_1 = 250$$
$$n_2 = 175$$
$$\widehat{p}_1 = \frac{80}{250} = 0.32$$
$$\widehat{p}_2 = \frac{40}{175} = 0.2286$$
$$\widehat{q}_1 = 1 - \widehat{p}_1 = 0.68$$
$$\widehat{q}_2 = 1 - \widehat{p}_2 = 0.7714$$
$$1 - \alpha = 90\% \Rightarrow \alpha = 0.1 \Rightarrow z_{\alpha/2} = z_{0.05} = 1.645$$

Therefore:

$$(\widehat{p}_1 - \widehat{p}_2) \pm z_{\alpha/2}\sqrt{\frac{\widehat{p}_1\widehat{q}_1}{n_1} + \frac{\widehat{p}_2\widehat{q}_2}{n_2}}$$
$$\Rightarrow (0.32 - 0.2286) \pm (1.645)\sqrt{\frac{(0.32)(0.68)}{250} + \frac{(0.2286)(0.7714)}{175}} = 0.0914 \pm 0.0713$$
$$\Rightarrow 0.0201 < p_1 - p_2 < 0.1627$$

There is a significantly higher proportion of women in electrical engineering than there is in chemical engineering.

Problem 5: [1, Exercise 9.69]
A survey of 1000 students found that 274 chose professional baseball team A as their favorite team. In a similar survey involving 760 students, 240 of them chose team A as their favorite. Compute a 95% confidence interval for the difference between the proportions of students favoring team A in the two surveys. Is there a significant difference?
Solution:

$$n_1 = 1000$$
$$n_2 = 760$$
$$\widehat{p}_1 = \frac{274}{1000} = 0.2740$$
$$\widehat{p}_2 = \frac{240}{760} = 0.3158$$
$$\widehat{q}_1 = 1 - \widehat{p}_1 = 0.7260$$
$$\widehat{q}_2 = 1 - \widehat{p}_2 = 0.6842$$
$$1 - \alpha = 95\% \Rightarrow \alpha = 0.5 \Rightarrow z_{\alpha/2} = z_{0.025} = 1.96$$

Therefore:

$$(\widehat{p}_1 - \widehat{p}_2) \pm z_{\alpha/2}\sqrt{\frac{\widehat{p}_1\widehat{q}_1}{n_1} + \frac{\widehat{p}_2\widehat{q}_2}{n_2}}$$

$$\Rightarrow (0.2740 - 0.3158) \pm (1.96)\sqrt{\frac{(0.2740)(0.7260)}{1000} + \frac{(0.3158)(0.6842)}{760}} = -0.0418 \pm 0.0431$$

$$\Rightarrow -0.0849 < p_1 - p_2 < 0.0013$$

At the confidence level of 95%, the significance can not be shown.

Problem 6: [1, Exercise 9.70]
According to USA Today (March 17, 1997), women made up 33.7% of the editorial staff at local TV stations in the United States in 1990 and 36.2% in 1994. Assume 20 new employees were hired as editorial staff.
(a) Estimate the number that would have been women in 1990 and 1994, respectively.
(b) Compute a 95% confidence interval to see if there is evidence that the proportion of women hired as editorial staff was higher in 1994 than in 1990.
Solution:

$$n_1 = n_2 = 20$$

$$\widehat{p}_1 = 33.7\% = 0.337$$

$$\widehat{p}_2 = 36.2\% = 0.362$$

$$\widehat{q}_1 = 1 - \widehat{p}_1 = 0.663$$

$$\widehat{q}_2 = 1 - \widehat{p}_2 = 0.638$$

a)
The number that would have been woman in 1990 is:

$$n_1 * \widehat{p}_1 = (20)(0.337) \approx 7$$

The number that would have been woman in 1994 is:

$$n_2 * \widehat{p}_2 = (20)(0.362) \approx 7$$

b)

$$1 - \alpha = 95\% \Rightarrow \alpha = 0.5 \Rightarrow z_{\alpha/2} = z_{0.025} = 1.96$$

$$(\widehat{p}_1 - \widehat{p}_2) \pm z_{\alpha/2}\sqrt{\frac{\widehat{p}_1\widehat{q}_1}{n_1} + \frac{\widehat{p}_2\widehat{q}_2}{n_2}}$$

$$\Rightarrow (0.337 - 0.362) \pm (1.96)\sqrt{\frac{(0.337)(0.663)}{20} + \frac{(0.362)(0.638)}{20}} = -0.025 \pm 0.295$$

$$\Rightarrow -0.320 < p_1 - p_2 < 0.270$$

There is no evidence that at the confidence level of 95%, there is a change in proportions.

Problem 7: [1, Exercise 9.72]
A random sample of 20 students yielded a mean of $\overline{x} = 72$ and a variance of $s^2 = 16$ for scores on a college placement test in mathematics. Assuming the scores to be normally distributed, construct a 98% confidence interval for σ^2 ?
Solution:

$$n = 20$$

$$s^2 = 16$$

$$1 - \alpha = 98\% \Rightarrow \alpha = 0.02$$

$$\upsilon = n - 1 = 20 - 1 = 19 \; (degrees\; of\; freedom)$$

$$\chi^2_{\alpha/2} = \chi^2_{0.01} = 36.191$$

$$\chi^2_{1-\alpha/2} = \chi^2_{0.99} = 7.633$$

Therefore:

$$\frac{(n-1)s^2}{\chi^2_{\alpha/2}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{1-\alpha/2}}$$

$$\Rightarrow \frac{(19)(16)}{36.191} < \sigma^2 < \frac{(19)(16)}{7.633}$$

$$\Rightarrow 8.400 < \sigma^2 < 39.827$$

Problem 8: [1, Exercise 9.106]
A random sample of 30 firms dealing in wireless products was selected to determine the proportion of such firms that have implemented new software to improve productivity. It turned out that 8 of the 30 had implemented such software. Find a 95% confidence interval on p, the true proportion of such firms that have implemented new software.
Solution:

$$n = 30$$

$$x = 8$$

$$\widehat{p} = \frac{8}{30} = \frac{4}{15}$$

$$\widehat{q} = 1 - \widehat{p} = \frac{11}{15}$$

$$1 - \alpha = 95\% \Rightarrow \alpha = 0.05 \Rightarrow z_{\alpha/2} = z_{0.025} = 1.96$$

Therefore:

$$\widehat{p} \pm z_{\alpha/2}\sqrt{\frac{\widehat{p}\widehat{q}}{n}}$$

$$\Rightarrow \frac{4}{15} \pm (1.96)\sqrt{\frac{(4/15)(11/15)}{30}} = \frac{4}{15} \pm 0.158$$

$$\Rightarrow 0.108 < p < 0.425$$

References

[1] WALPOLE, R. E., MYERS, R. H., MYERS, S. L. AND YE, K., *Probability & Statistics for Engineers & Scientists*, 9th ed., MA, USA: Prentice-Hall, 2012.

[2] DEGROOT, M. H. AND SCHERVISH, M. J., *Probability and Statistics*, 4th ed., MA, USA: Pearson Education, Inc., 2012.

[3] MURRAY, R. S., JOHN, J. S. AND R, A. SRINIVASAN, *Probability and Statistics*, 3rd ed., USA: McGraw-Hill, 2009.