# Solution for Problem Set 2

## HANU - Faculty of Information and Technology
## Probability & Statistics

**Problem 1:** [3, Exercise 1.52]
A box contains 2 red and 3 blue marbles. Find the probability that if two marbles are drawn at random (without replacement)

**a.** both are blue,
Solution:

$$P(A) = \frac{3}{5} * \frac{2}{4} = \frac{3}{10} = 0.3$$

**b.** both are red,
Solution:

$$P(B) = \frac{2}{5} * \frac{1}{4} = \frac{1}{10} = 0.1$$

**c.** one is red and one is blue.
Solution:

$$P(C) = \frac{2}{5} * \frac{3}{4} + \frac{3}{5} * \frac{2}{4} = \frac{3}{5} = 0.6$$

**Problem 2:** [3, Exercise 1.53]
Find the probability of drawing 3 aces at random from a deck of 52 ordinary cards if the cards are

**a.** replaced,
Solution:

$$P(A) = \frac{4 * 4 * 4}{52 * 52 * 52} = \frac{1}{2197}$$

**b.** not replaced.
Solution:

$$P(B) = \frac{4 * 3 * 2}{52 * 51 * 50} = \frac{1}{17576}$$

**Problem 3:** [3, Exercise 1.54]
If at least one child in a family with 2 children is a boy, what is the probability that both children are boys?
Solution:
2 children and at least one is a boy = $\{BB, BG, GB\}$

$$\Rightarrow P(both\ are\ boys) = \frac{1}{3}$$

**Problem 4:** [3, Exercise 1.55]
Box $I$ contains 3 red and 5 white balls, while Box $II$ contains 4 red and 2 white balls. A ball is chosen at random from the first box and placed in the second box without observing its color. Then a ball is drawn from the second box. Find the probability that it is white.
Solution:
W1: White ball was chosen from Box 1 $\Rightarrow P(W1) = \frac{5}{8}$
R1: Red ball was chosen from Box 1 $\Rightarrow P(R1) = \frac{3}{8}$
W2: White ball was chosen from Box 2

$$\begin{aligned} P(W2) &= P(W1) * P(W2|W1) + P(R1) * P(W2|R1) \\ &= \frac{5}{8} * \frac{3}{7} + \frac{3}{8} * \frac{2}{7} = \frac{15+6}{56} = \frac{21}{56} = \frac{3}{8} \end{aligned}$$

**Problem 5:** [3, Exercise 1.56]
A box contains 3 blue and 2 red marbles while another box contains 2 blue and 5 red marbles. A marble drawn at random from one of the boxes turns out to be blue. What is the probability that it came from the first box?
Solution:
B1: Marble came from Box 1 $\Rightarrow P(B1) = \frac{1}{2}$
B2: Marble came from Box 2 $\Rightarrow P(B2) = \frac{1}{2}$
Based on Bayes' Theorem:

$$P(B1|Blue) = \frac{P(B1) * P(Blue|B1)}{P(B1) * P(Blue|B1) + P(B2) * P(Blue|B2)} = \frac{\frac{1}{2} * \frac{3}{5}}{\frac{1}{2} * \frac{3}{5} + \frac{1}{2} * \frac{2}{7}} = \frac{21}{31}$$

**Problem 6:** [3, Exercise 1.57]
Each of three identical jewelry boxes has two drawers. In each drawer of the first box there is a gold watch. In each drawer of the second box there is a silver watch. In one drawer of the third box there is a gold watch while in the other there is a silver watch. If we select a box at random, open one of the drawers and find it to contain a silver watch, what is the probability that the other drawer has the gold watch?
Solution:
B1: Watch drawn from Box 1 $\Rightarrow P(B1) = \frac{1}{3}$
B2: Watch drawn from Box 2 $\Rightarrow P(B2) = \frac{1}{3}$
B3: Watch drawn from Box 3 $\Rightarrow P(B3) = \frac{1}{3}$
S: First draw is the silver watch
G: Second draw is the gold watch

$$P(S) = P(S|B1) * P(B1) + P(S|B2) * P(B2) + P(S|B3) * P(B3)$$

$$= 0 * \frac{1}{3} + 1 * \frac{1}{3} + \frac{1}{2} * \frac{1}{3} = \left(0 + 1 + \frac{1}{2}\right) * \frac{1}{3} = \frac{1}{2}$$

After event S occurs, B1, B2, B3 change.

$$P(B1|S) = \frac{P(B1) * P(S|B1)}{P(S)} = \frac{\frac{1}{2} * 0}{\frac{1}{2}} = 0$$

$$P(B2|S) = \frac{P(B2) * P(S|B2)}{P(S)} = \frac{\frac{1}{3} * 1}{\frac{1}{2}} = \frac{2}{3}$$

$$P(B3|S) = \frac{P(B3) * P(S|B3)}{P(S)} = \frac{\frac{1}{3} * \frac{1}{2}}{\frac{1}{2}} = \frac{1}{3}$$

$$\Rightarrow P(G) = P(B1|S) * P(G|B1S) + P(B2|S) * P(G|B2S) + P(B3|S) * P(G|B3S)$$

$$= 0 * 0 + \frac{2}{3} * 0 + \frac{1}{3} * 1 = \frac{1}{3}$$

**Problem 7:** [3, Exercise 1.58]
Urn *I* has 2 white and 3 black balls; Urn *II*, 4 white and 1 black; and Urn *III*, 3 white and 4 black. An urn is selected at random and a ball drawn at random is found to be white. Find the probability that Urn *I* was selected.
Solution:
U1: Ball drawn from Urn I $\Rightarrow P(U1) = \frac{1}{3}$
U2: Ball drawn from Urn II $\Rightarrow P(U2) = \frac{1}{3}$
U3: Ball drawn from Urn III $\Rightarrow P(U3) = \frac{1}{3}$

$$P(U1|White) = \frac{P(U1) * P(White|U1)}{P(U1) * P(White|U1) + P(U2) * P(White|U2) + P(U3) * P(White|U3)}$$

$$= \frac{\frac{1}{3} * \frac{2}{5}}{\frac{1}{3} * \frac{2}{5} * + \frac{1}{3} * \frac{4}{5} * + \frac{1}{3} * \frac{3}{7}} = \frac{\frac{2}{15}}{\frac{19}{35}} = \frac{14}{57}$$

**Problem 8:** [3, Exercise 1.96]
The probability that a man will hit a target is $\frac{2}{3}$. If he shoots at the target until he hits it for the first time, find the probability that it will take him 5 shots to hit the target.
Solution:

$$P(A) = \left(\frac{1}{3}\right)^4 * \left(\frac{2}{3}\right)^1 = \frac{2}{243}$$

**Problem 9:** [2, Exercise 5, Section 2.1]
A box contains $r$ red balls and $b$ blue balls. One ball is selected at random and its color is observed. The ball is then returned to the box and $k$ additional balls of the same color are also put into the box. A second ball is then selected at random, its color is observed, and it is returned to the box together with $k$ additional balls of the same color. Each time another ball is selected, the process is repeated. If four balls are selected, what is the probability that the first three balls will be red and the fourth ball will be blue?
Solution:
Let $R_i$ be the event that the red ball is drawn on the $i$th draw, and let $B_i$ be the event that the a blue ball is drawn on the $i$th draw for i = 1,...,4. Then:

$$Pr(R1) = \frac{r}{r+b}$$

$$Pr(R2|R1) = \frac{r+k}{r+b+k}$$

$$Pr(R3|R1R2) = \frac{r+2k}{r+b+2k}$$

$$Pr(B4|R1R2R3) = \frac{b}{r+b+3k}$$

The desire probability is the product of these four probabilities, namely:

$$\frac{r(r+k)(r+2k)b}{(r+b)(r+b+k)(r+b+2k)(r+b+3k)}$$

**Problem 10:** [2, Exercise 9, Section 2.2]
Consider an experiment in which a fair coin is tossed until a head is obtained for the first time. If this experiment is performed three times, what is the probability that exactly the same number of tosses will be required for each of the three performances?
Solution:
The probability that exactly n tosses will be required on a given performance is:

$$\frac{1}{2^n}$$

Therefore, the probability that exactly n tosses will be required on three performances is:

$$\left(\frac{1}{2^n}\right)^3 = \frac{1}{8^n}$$

The probability that the same number of tosses will be required on three performances is:

$$\sum_{n=1}^{3} \frac{1}{8^n} = \frac{1}{8^1} + \frac{1}{8^2} + \frac{1}{8^3} \approx \frac{1}{7}$$

**Problem 11:** [1, Exercise 2.74, Section 2.6]
A class in advanced physics is composed of 10 juniors, 30 seniors, and 10 graduate students. The final grades show that 3 of the juniors, 10 of the seniors, and 5 of the graduate students received an A for the course. If a student is chosen at random from this class and is found to have earned an A, what is the probability that he or she is a senior?
Solution:

$$P(S|A) = \frac{10}{10+3+5} = \frac{10}{18} = \frac{5}{9}$$

**Problem 12:** [2, Exercise 5, Section 2.2]
Suppose that the probability that the control system used in a spaceship will malfunction on a given flight is 0.001. Suppose further that a duplicate, but completely independent, control system is also installed in the spaceship to take control in case the first system malfunctions. Determine the probability that the spaceship will be under the control of either the original system or the duplicate system on a given flight.
Solution:
The probability that both systems will malfunction is:

$$P(A) = (0.001)^2 = 10^{-6}$$

Therefore, the probability that at least one system will function is:

$$P(B) = 1 - P(A) = 1 - 10^{-6}$$

**Problem 13:** [1, Exercise 2.96, Section 2.6]
Police plan to enforce speed limits by using radar traps at four different locations within the city limits. The radar traps at each of the locations $L1$, $L2$, $L3$, and $L4$ will be operated $40\%, 30\%, 20\%$, and $30\%$ of the time. If a person who is speeding on her way to work has probabilities of 0.2, 0.1, 0.5, and 0.2, respectively, of passing through these locations, what is the probability that she will receive a speeding ticket?
Solution:
Let S1,S2,S3, and S4 represent the events that a person is speeding as she passes through the respective locations and let R represent the event that the radar traps is operating resulting in a speeding ticket. Then the probability that she receives a speeding ticket:

$$P(R) = \sum_{i=1}^{4} P(R|S_i)P(S_i) = (0.4)(0.2) + (0.3)(0.1) + (0.2)(0.5) + (0.3)(0.2) = 0.27$$

**Problem 14:** [2, Exercise 5, Section 2.3]
In a certain city, 30 percent of the people are Conservatives, 50 percent are Liberals, and 20 percent are Independents. Records show that in a particular election, 65 percent of the Conservatives voted, 82 percent of the Liberals voted, and 50 percent of the Independents voted. If a person in the city is selected at random and it is learned that

she did not vote in the last election, what is the probability that she is a Liberal?
Solution:
The desired probability $Pr(Lib.|NoVote)$ can be calculated as follows:

$$\frac{Pr(Lib.)Pr(NoVote|Lib.)}{Pr(Cons.)Pr(NoVote|Cons.) + Pr(Lib.)Pr(NoVote|Lib.) + Pr(Ind.)Pr(NoVote|Ind.)}$$

$$= \frac{(0.5)(0.18)}{(0.3)(0.35) + (0.5)(0.18) + (0.2)(0.5)} = \frac{18}{59}$$

**Problem 15:** [1, Exercise 2.106, Section 2.7]
The probabilities that a service station will pump gas into $0, 1, 2, 3, 4$, or 5 or more cars during a certain 30-minute period are $0.03, 0.18, 0.24, 0.28, 0.10$, and $0.17$, respectively. Find the probability that in this 30-minute period

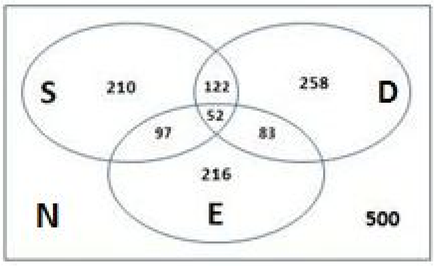
1. more than 2 cars receive gas;
   Solution:
   $$0.28 + 0.10 + 0.17 = 0.55$$
2. at most 4 cars receive gas;
   Solution:
   $$1 - 0.17 = 0.83$$
3. 4 or more cars receive gas.
   Solution:
   $$0.10 + 0.17 = 0.27$$

**Problem 16:** [1, Exercise 2.110, Section 2.7]
The probability that a patient recovers from a delicate heart operation is 0.8. What is the probability that

1. exactly 2 of the next 3 patients who have this operation survive?
   Solution:
   Denote by R the event that a patient survives. Then P(R) = 0.8
   $$P(R1 \cap R2 \cap R'3) + P(R1 \cap R'2 \cap R3) + P(R'1 \cap R2 \cap R3)$$
   $$= P(R1)P(R2)P(R'3) + P(R1)P(R'2)P(R3) + P(R'1)P(R2)P(R3)$$
   $$= (0.8)(0.8)(0.2) + (0.8)(0.2)(0.8) + (0.2)(0.8)(0.8) = (3)(0.8)(0.8)(0.2) = 0.384$$
2. all of the next 3 patients who have this operation survive?
   Solution:
   $$P(R1 \cap R2 \cap R3) = P(R1)P(R2)P(R3) = (0.8)(0.8)(0.8) = 0.512$$

**Problem 17:** [1, Exercise 2.52, Section 2.5]
Suppose that in a senior college class of 500 students it is found that 210 smoke, 258 drink alcoholic beverages, 216 eat between meals, 122 smoke and drink alcoholic beverages, 83 eat between meals and drink alcoholic beverages, 97 smoke and eat between meals,and 52 engage in all three of these bad health practices. If a member of this senior class is selected at random, find the probability that the student

1. smokes but does not drink alcoholic beverages;
   Solution:
$$P(S \cap D') = \frac{S - (S \cap D)}{N} = \frac{210 - 122}{500} = \frac{88}{500} = \frac{22}{125}$$
2. eats between meals and drinks alcoholic beverages but does not smoke;
   Solution:
$$P(E \cap D \cap S') = \frac{(E \cap D) - (S \cap D \cap E)}{N} = \frac{83 - 52}{500} = \frac{31}{500}$$
3. neither smokes nor eats between meals.
   Solution:
$$P(S' \cap E') = \frac{N - (S \cup E)}{N} = \frac{N - (S + E - S \cap E)}{N} = \frac{500 - (210 + 216 - 97)}{500} = \frac{171}{500}$$




**Problem 18:** [2, Exercise 19, Section 2.2]
A box contains 20 red balls, 30 white balls, and 50 blue balls. Suppose that 10 balls are selected at random one at a time, with replacement; that is, each selected ball is replaced in the box before the next selection is made. Determine the probability that at least one color will be missing from the 10 selected balls.
Solution:
A1: No red balls are selected
A2: No white balls are selected
A3: No blue balls are selected
The event A1 will only occur if and only if all 10 selected balls are white or blue.
Since there is probability 0.8 that any given selected ball will be white or blue, we have:
$$Pr(A1) = (0.8)^{10}; Pr(A2) = (0.7)^{10}; Pr(A3) = (0.5)^{10}$$
The event A1A2 will occur if and only if all 10 selected balls are blue. Therefore:
$$Pr(A1A2) = (0.5)^{10}; Pr(A2A3) = (0.2)^{10}; Pr(A1A3) = (0.3)^{10}$$
Finally, the event A1A2A3 cannot possibly occur, so:
$$Pr(A1A2A3) = 0$$
Therefore, the desired probability is:
$$Pr(A1 \cup A2 \cup A3) = [(Pr(A1) + Pr(A2) + Pr(A3)] - [Pr(A1A2) + Pr(A2A3) + Pr(A1A3)]$$
$$= \left[(0.8)^{10} + (0.7)^{10} + (0.5)^{10}\right] - \left[(0.5)^{10} + (0.2)^{10} + (0.3)^{10}\right] \approx 0.1356$$

# References

[1] Walpole, R. E., Myers, R. H., Myers, S. L. and Ye, K., *Probability & Statistics for Engineers & Scientists*, 9th ed., MA, USA: Prentice-Hall, 2012.

[2] DeGroot, M. H. and Schervish, M. J., *Probability and Statistics*, 4th ed., MA, USA: Pearson Education, Inc., 2012.

[3] Murray, R. S., John, J. S. and R, A. Srinivasan, *Probability and Statistics*, 3rd ed., USA: McGraw-Hill, 2009.