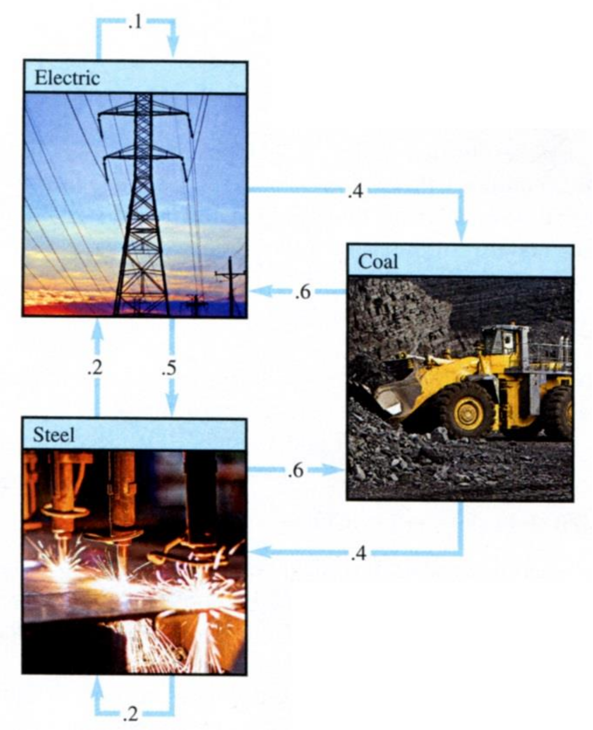
Applications of Linear Systems

Applications

- Economics
- Chemistry
- Network flow

A Homogeneous System in Economics

- Leontief "input-output" model



Suppose an economy consists of the Coal, Electric (power), and Steel sectors, and the output of each sector is distributed among the various sectors as shown in Table 1, where the entries in a column represent the fractional parts of a sector's total output.

TABLE 1 A Simple Economy

Distribution of Output from:			
Coal	**Electric**	**Steel**	**Purchased by:**
.0	.4	.6	Coal
.6	.1	.2	Electric
.4	.5	.2	Steel

- Denote the prices (i.e., dollar values) of the total annual outputs of the Coal, Electric, and Steel sectors by p_C, p_E, and p_S, respectively.
- p_C=.4pE+.6pS
- p_E=.6pC +.1pE+.2pS
- p_S=.4pC +.5pE+.2pS

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$$
\begin{aligned}
p_C - .4p_E - .6p_S &= 0 \\
-.6p_C + .9p_E - .2p_S &= 0 \\
-.4p_C - .5p_E + .8p_S &= 0
\end{aligned}
$$

$$
\mathbf{p} = \begin{bmatrix} p_C \\ p_E \\ p_S \end{bmatrix} = \begin{bmatrix} .94p_S \\ .85p_S \\ p_S \end{bmatrix} = p_S \begin{bmatrix} .94 \\ .85 \\ 1 \end{bmatrix}
$$

Balancing Chemical Equations

- $(x1)C_3H_8+(x2)O_2 \rightarrow (x3)CO_2+(x4)H_2O$

$$C_3H_8: \begin{bmatrix} 3 \\ 8 \\ 0 \end{bmatrix},\ O_2: \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix},\ CO_2: \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix},\ H_2O: \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} \begin{array}{l} \leftarrow \text{Carbon} \\ \leftarrow \text{Hydrogen} \\ \leftarrow \text{Oxygen} \end{array}$$

$$x_1 \begin{bmatrix} 3 \\ 8 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} = x_3 \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$$

$$x_1 \begin{bmatrix} 3 \\ 8 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} + x_3 \begin{bmatrix} -1 \\ 0 \\ -2 \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ -2 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_1 = \tfrac{1}{4}x_4,\ x_2 = \tfrac{5}{4}x_4,\ x_3 = \tfrac{3}{4}x_4,\ \text{ with } x_4 \text{ free}$$

$$C_3H_8 + 5O_2 \rightarrow 3CO_2 + 4H_2O$$

Network Flow

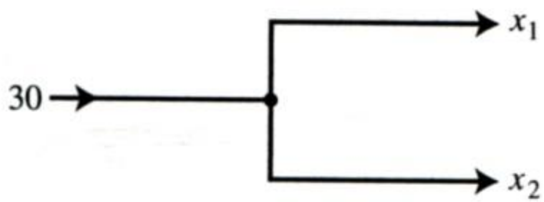

FIGURE 1

A junction, or node.

Intersection	Flow in		Flow out
A	$300 + 500$	$=$	$x_1 + x_2$
B	$x_2 + x_4$	$=$	$300 + x_3$
C	$100 + 400$	$=$	$x_4 + x_5$
D	$x_1 + x_5$	$=$	600

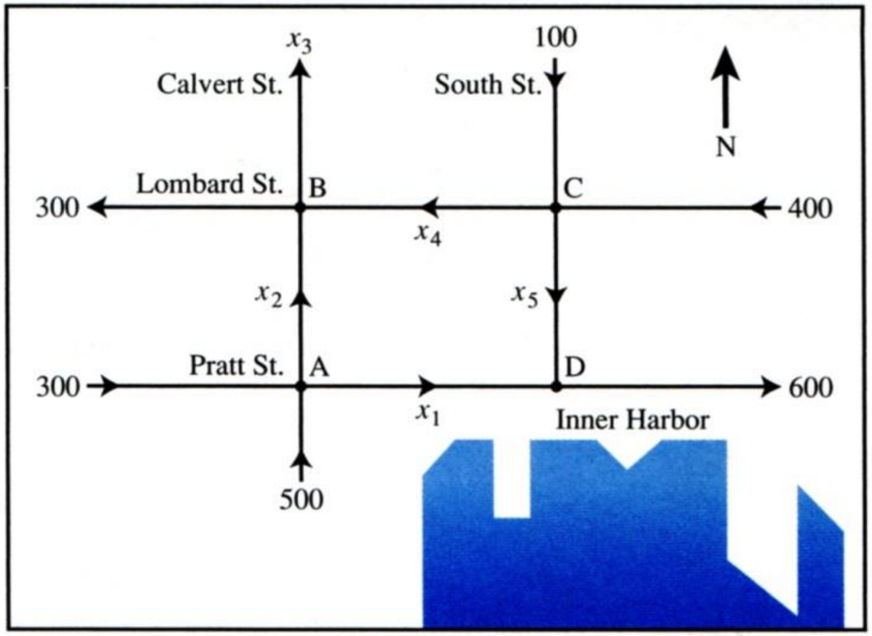

FIGURE 2 Baltimore streets.

$$\begin{cases} x_1 = 600 - x_5 \\ x_2 = 200 + x_5 \\ x_3 = 400 \\ x_4 = 500 - x_5 \\ x_5 \text{ is free} \end{cases}$$