Vietnam National University Ho Chi Minh City, University of Science
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Topic 4:

Data Encryption Standard và Advanced Encryption Standard

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Contents

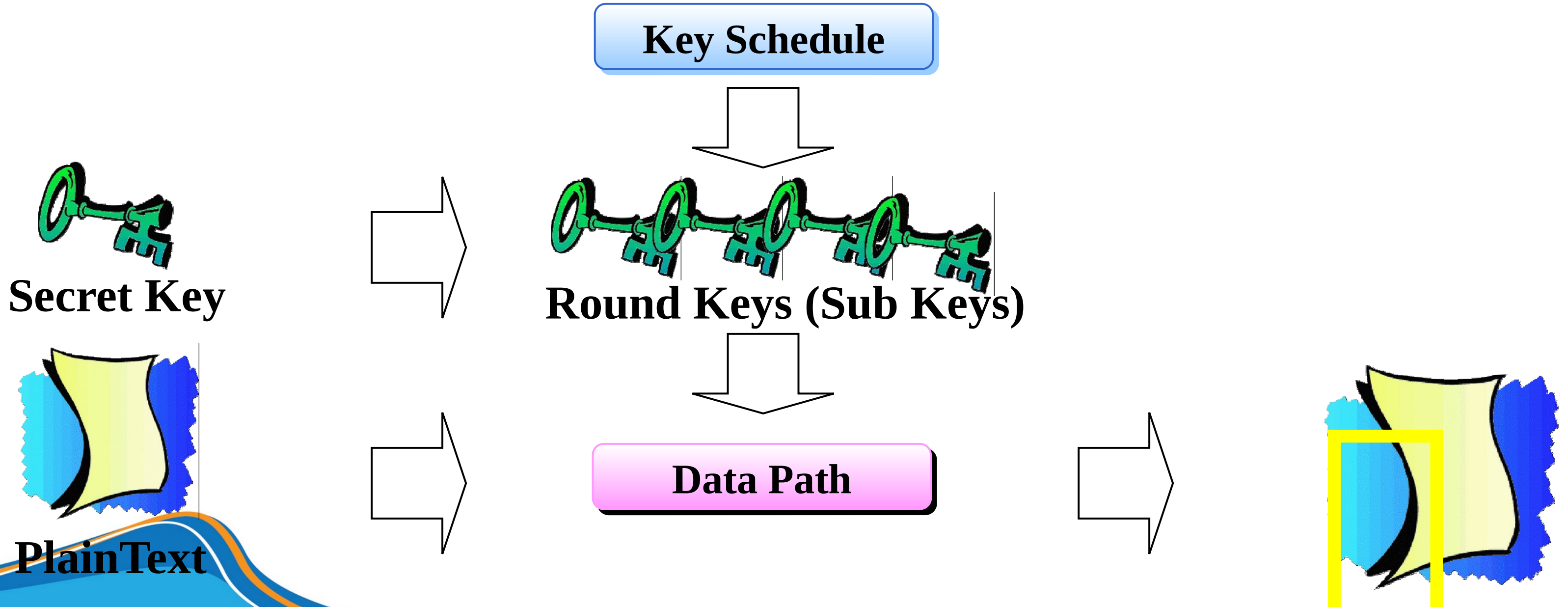
- **D**ata **E**ncryption **S**tandard (DES)
- **A**dvanced **E**ncryption **S**tandard (AES)
  - Guidance of AES128

Product Cipher

- Encryption using only substitution or transposition is not safe (due to the nature of the language)
- Consecutive use of simple encryption operations creates a more secure way of encrypting information
  - **Substitution** combined with **Substitution** is more secure than one **Substitution**
  - **Transposition** combined with **Transposition** is more secure than one **Transposition**
  - **Substitution** combined with **Transposition** is much safer than using only one type of operation (replace or swap)
- This is the prelude to modern encryption methods

Block-cipher

- Data Path: Usually, the encryption process consists of several consecutive encryption cycles (rounds); Each cycle consists of many encryption operations
- Key Schedule: From a secret-key, produce (with rule) key values to be used in each encryption cycle (round key)



Cipher architecture

- Common architectures:
  - Fiestel: for example, Blowfish, Camellia, CAST-128, DES, FEAL, KASUMI, LOKI97, Lucifer, MARS, MAGENTA, MISTY1, RC5, TEA, Triple DES, Twofish, and XTEA
  - SPN: for example, Rijndael – AES, Anubis…
- Data Encryption Standard: block-cipher
  - Ideas: product cipher with key-size 56 bits and block-size: 64 bits
  - IBM developed from **Lucifer** and published in **1975**
  - Chosen to be Federal Information Processing Standard-FIPS) in **1976**
  - Encryption and decryption algorithm published
  - Mathematical and cryptographic basis of DES design: secret information

Feistel architecture - encryption

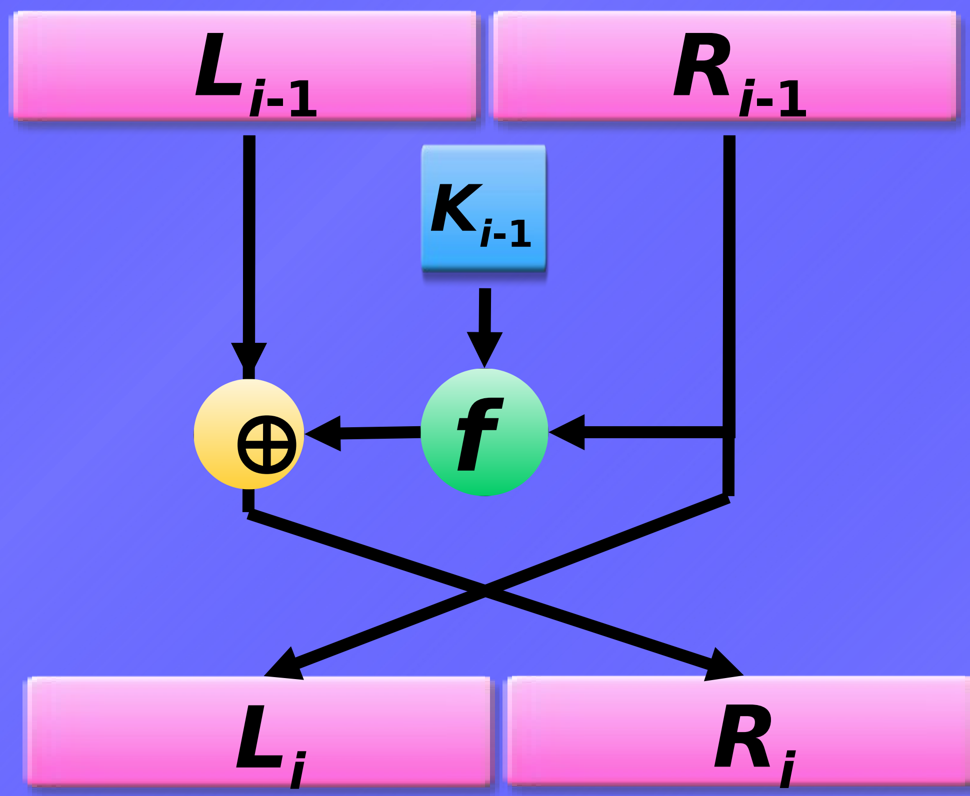
Round 1

...

Round *i*

...

Round *Nr*



$$L_i = R_{i-1}$$

$$R_i = L_{i-1} \oplus f\,(R_{i-1}, K_{i-1})$$

Feistel architecture - decryption

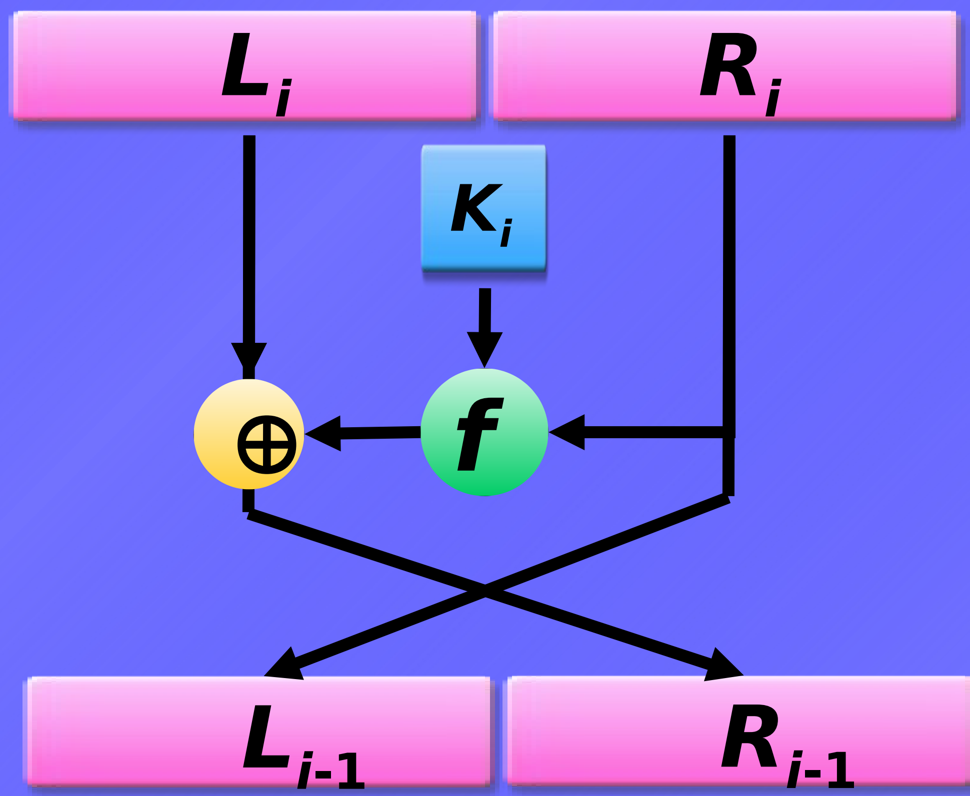
Round *Nr*

...

Round *i*

...

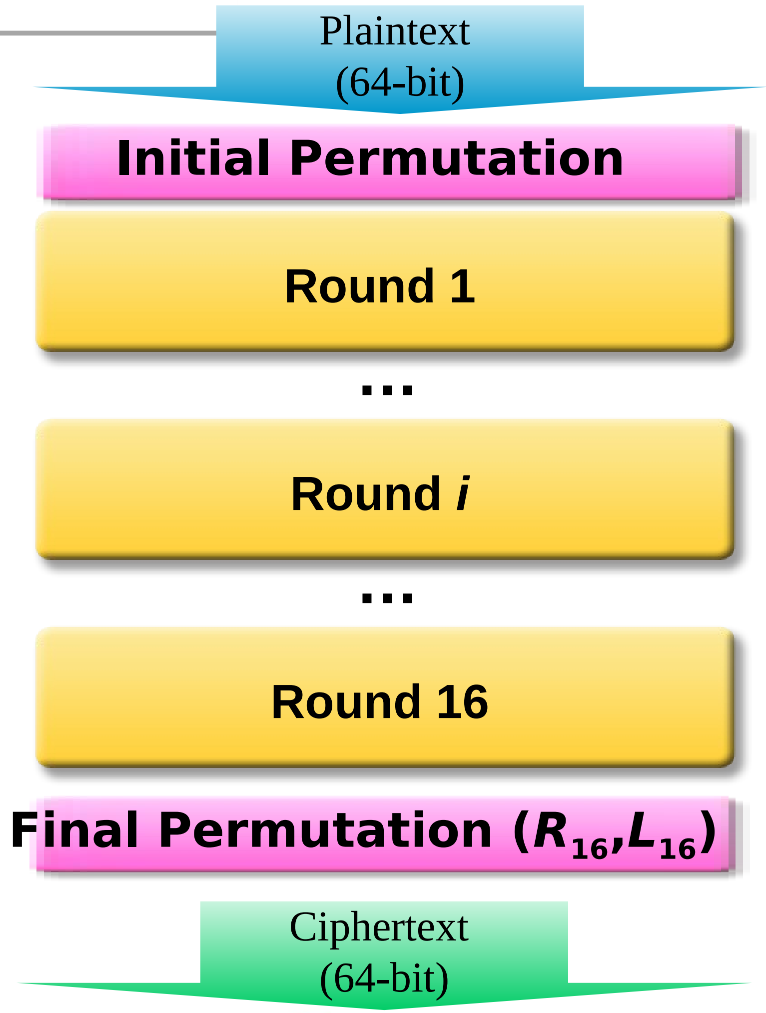
Round *1*



$$R_{i\text{-}1} = L_i$$

$$L_{i\text{-}1} = R_i \oplus f\,(L_i, K_i)$$

The Encryption Process of the DES



IP: Initial Permutation

FP: Final Permutation

FP = IP^{-1}

<u>Notes</u>:

FP and IP have no cryptographic significance, only for loading data into and out of data blocks (in the mid-1970s hardware mechanism!!!)

Initial Permutation

IP							
58	50	42	34	26	18	10	2
60	52	44	36	28	20	12	4
62	54	46	38	30	22	14	6
64	56	48	40	32	24	16	8
57	49	41	33	25	17	9	1
59	51	43	35	27	19	11	3
61	53	45	37	29	21	13	5
63	55	47	39	31	23	15	7

Example: 58^{th} bit of x (input) becomes the 1^{st} bit of IP(x) (output)
50^{th} bit of x (input) becomes the 2^{nd} bit of IP(x) (output)

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Final Permutation

FP = IP^{-1}							
40	8	48	16	56	24	64	32
39	7	47	15	55	23	63	31
38	6	46	14	54	22	62	30
37	5	45	13	53	21	61	29
36	4	44	12	52	20	60	28
35	3	43	11	51	19	59	27
34	2	42	10	50	18	58	26
33	1	41	9	49	17	57	25

Example: The 1st bit of x (input) becomes 58th bit of IP$^{-1}(x)$ (output)
The 2nd bit of x (input) becomes 50th bit of IP$^{-1}(x)$ (output)

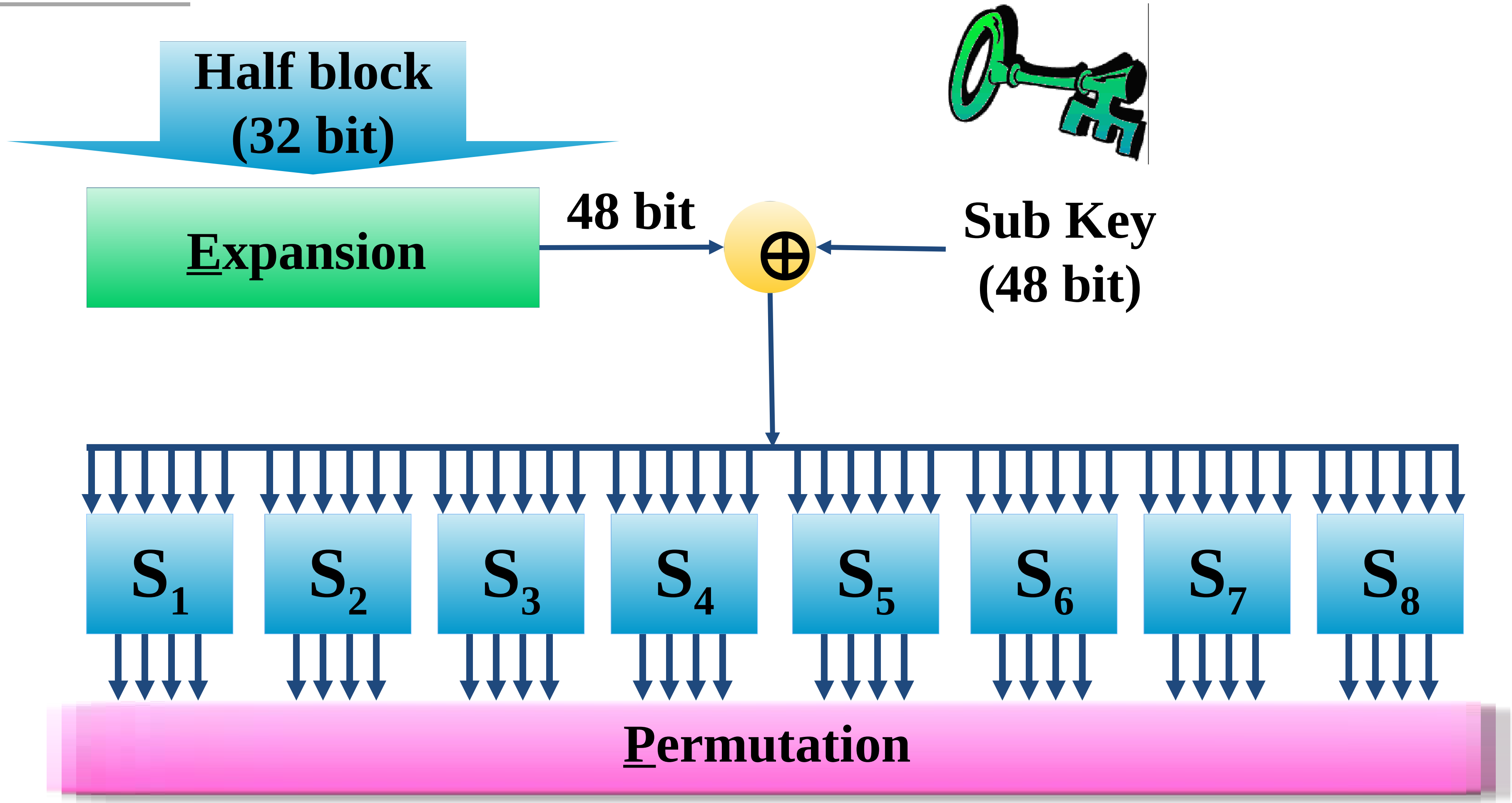
Expansion

Table E: extend from 32 bits to 48 bits

Table of choosing bit					
32	1	2	3	4	5
4	5	6	7	8	9
8	9	10	11	12	13
12	13	14	15	16	17
16	17	18	19	20	21
20	21	22	23	24	25
24	25	26	27	28	29
28	29	30	31	32	1

Example: 32nd bit of x (input) becomes the 1st bit of E(x) (output)

Function *f* of DES



S-box

S_1															
14	4	13	1	2	15	11	8	3	10	6	12	5	9	0	7
0	15	7	4	14	2	13	1	10	6	12	11	9	5	3	8
4	1	14	8	13	6	2	11	15	12	9	7	3	10	5	0
15	12	8	2	4	9	1	7	5	11	3	14	10	0	6	13

S_2															
15	1	8	14	6	11	3	4	9	7	2	13	12	0	5	10
3	13	4	7	15	2	8	14	12	0	1	10	6	9	11	5
0	14	7	11	10	4	13	1	5	8	12	6	9	3	2	15
13	8	10	1	3	15	4	2	11	6	7	12	0	5	14	9

Example: $B_j = b_1b_2b_3b_4b_5b_6$ then $S_j(B_i) = S_j[b_1b_6][b_2b_3b_4b_5]$

S-box

S_3															
10	0	9	14	6	3	15	5	1	13	12	7	11	4	2	8
13	7	0	9	3	4	6	10	2	8	5	14	12	11	15	1
13	6	4	9	8	15	3	0	11	1	2	12	5	10	14	7
1	10	13	0	6	9	8	7	4	15	14	3	11	5	2	12

S_4															
7	13	14	3	0	6	9	10	1	2	8	5	11	12	4	15
13	8	11	5	6	15	0	3	4	7	2	12	1	10	14	9
10	6	9	0	12	11	7	13	15	1	3	14	5	2	8	4
3	15	0	6	10	1	13	8	9	4	5	11	12	7	2	14

S-box

S_5															
2	12	4	1	7	10	11	6	8	5	3	15	13	0	14	9
14	11	2	12	4	7	13	1	5	0	15	10	3	9	8	6
4	2	1	11	10	13	7	8	15	9	12	5	6	3	0	14
11	8	12	7	1	14	2	13	6	15	0	9	10	4	5	3

S_6															
12	1	10	15	9	2	6	8	0	13	3	4	14	7	5	11
10	15	4	2	7	12	9	5	6	1	13	14	0	11	3	8
9	14	15	5	2	8	12	3	7	0	4	10	1	13	11	6
4	3	2	12	9	5	15	10	11	14	1	7	6	0	8	13

S-box

S_7															
4	11	2	14	15	0	8	13	3	12	9	7	5	10	6	1
13	0	11	7	4	9	1	10	14	3	5	12	2	15	8	6
1	4	11	13	12	3	7	14	10	15	6	8	0	5	9	2
6	11	13	8	1	4	10	7	9	5	0	15	14	2	3	12

S_8															
13	2	8	4	6	15	11	1	10	9	3	14	5	0	12	7
1	15	13	8	10	3	7	4	12	5	6	11	0	14	9	2
7	11	4	1	9	12	14	2	0	6	10	13	15	3	5	8
2	1	14	7	4	10	8	13	15	12	9	0	3	5	6	11

Permutation table

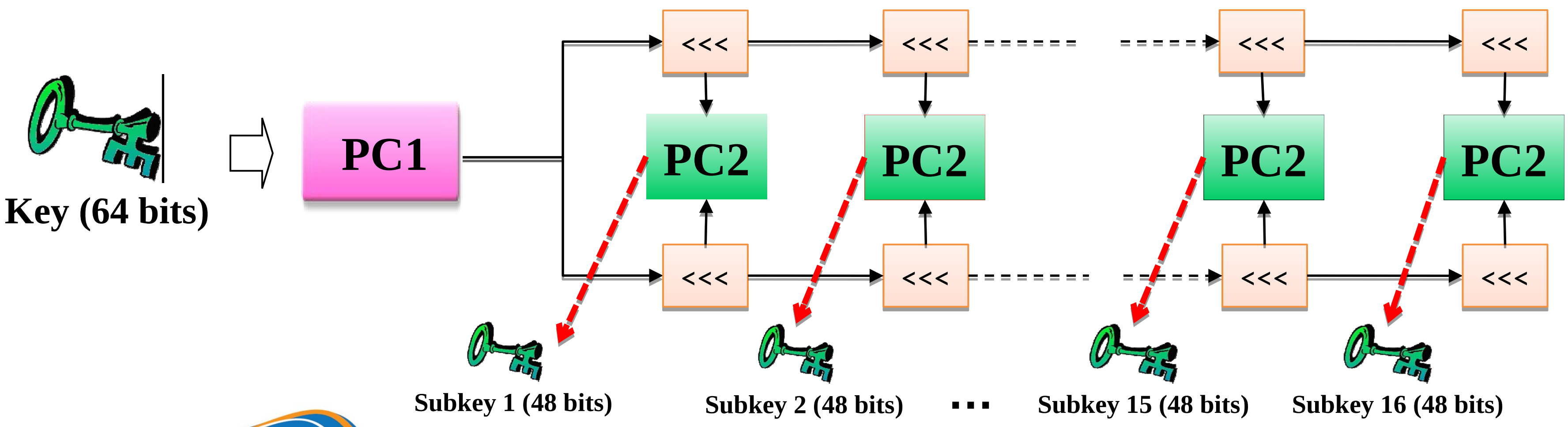
P			
16	7	20	21
29	12	28	17
1	15	23	26
5	18	31	10
2	8	24	14
32	27	3	9
19	13	30	6
22	11	4	25

Notes:

- Input of 32 bits (includes 8 S-box, each S-box has 4 bits)
- Output 32 bits are permuted
- Example: the 1^{st} bit of **P**(x) (output) is 16^{th} bit of x (input)

Key Schedule

- Bit rotation operation
  - <<<: Rotate to the left
  - >>>: Rotate to the right
- With subkey in {1, 2, 9, 16}: rotate to the left one position
- With remaining subkey: rotate to the left two positions



Permutations in Key Schedule

PC-1

57	49	41	33	25	17	9
1	58	50	42	34	26	18
10	2	59	51	43	35	27
19	11	3	60	52	44	36
63	55	47	39	31	23	15
7	62	54	46	38	30	22
14	6	61	53	45	37	29
21	13	5	28	20	12	4

Choose 56 bits

(leave 8, 16, 24, 32, 40, 48, 56, 64)

PC-2

14	17	11	24	1	5
3	28	15	6	21	10
23	19	12	4	26	8
16	7	27	20	13	2
41	52	31	37	47	55
30	40	51	45	33	48
44	49	39	56	34	53
46	42	50	36	29	32

Choose 48 bits

(leave 9, 18, 22, 25, 35, 38, 43, 54)

Example: 50^{th} bit of x (input) becomes the 1^{st} bit of PC-1(x) (output)

Explain: $\mathbf{57} = 8 \times \mathbf{7} + 1 \rightarrow \mathbf{50} = \mathbf{57} - \mathbf{7}$

Comments

- 4 weak keys:
  - All bits 0
  - All bits 1
  - ½ are bit 0 (consecutive), ½ are bit 1 (consecutive)
- 12 semi-weak keys
  - Property: $\text{Encrypt}_k(P) = P$
  - Form of such key: 7 bit 0 (consecutive), 7 bit 1 (consecutive)
- Complement key
  - $\text{Encrypt}_k(P) = C \rightarrow \text{Encrypt}_{k^*}(P^*) = C^*$
  - x^* is formed by reversing bits of x

Pseudocode DES

- Encryption(K, M) // $|K| = 56$ and $|M| = 64$
  - $(K_1, \ldots, K_{16}) \leftarrow \mathit{CreateSubKey}(K)$ // $|K_i| = 48$
  - $M \leftarrow \mathbf{IP}(M)$
  - $M \rightarrow L_0 \,\|\, R_0$
  - Loop $r = 1, \ldots, 16$
    - $L_r \leftarrow R_{r-1}$; $R_r \leftarrow \boldsymbol{f}(K_r, R_{r-1}) \oplus L_{r-1}$
  - $C \leftarrow \mathbf{FP}(L_{16} \,\|\, R_{16})$
  - Return C

- CreateSubKey(K) // $|K| = 56$ bits
  - $K \leftarrow \textbf{PC-1}(K)$
  - $K \rightarrow C_0 \,\|\, D_0$
  - Loop $r = 1, \ldots, 16$
    - If $r \in \{1, 2, 9, 16\}$ then $j \leftarrow 1$, else $j \leftarrow 2$
    - $C_r \leftarrow C_{r-1} \lll j$; $D_r \leftarrow D_{r-1} \lll j$
    - $K_r \leftarrow \textbf{PC-2}(C_r \,\|\, D_r)$
  - Return $(K_1, \ldots, K_{16})$

Rijndael method

- Rijndael method proposed by Vincent Rijmen and Joan Daeman
- The National Institute of Standards and Technology (NIST) has adopted it as the Advanced Encryption Standard since October 2, 2000.
- The block cipher method has a flexible block size and key cipher with values of 128, 192 or 256 bits.
- This method is suitable for applications on a variety of systems from smart cards to personal computers.

Some Math Concepts

- Unit of information processed in Rijndael algorithm is the byte
- Each byte is treated as an element of the field Galois $GF(2^8)$ equipped with addition (denote $\oplus$) and multiplication (denote $\bullet$)
- Each byte is represented in different ways:
  - Binary: $\{b_7 b_6 b_5 b_4 b_3 b_2 b_1 b_0\}$ and hexadecimal: $\{h_1 h_0\}$
  - Polynomial with binary coefficients: $\sum_{i=0}^{7} b_i x^i$
- **Addition in $GF(2^8) = \{0, 1, \ldots, 255\}$**
  - $\{a_7 a_6 a_5 a_4 a_3 a_2 a_1 a_0\} \oplus \{b_7 b_6 b_5 b_4 b_3 b_2 b_1 b_0\} = \{c_7 c_6 c_5 c_4 c_3 c_2 c_1 c_0\}$
  - for $c_i = a_i \oplus b_i$, $0 \le i \le 7$
- Multiplication in $GF(2^8)$:
  - $a(x) \bullet b(x) \equiv a(x) \times b(x) \mod (x^8 + x^4 + x^3 + x + 1)$

Some Math Concepts

- Example of multiplication of {53} = 01010011 & {CA} = 11001010
  - $(x^6 + x^4 + x + 1) \bullet (x^7 + x^6 + x^3 + x)$
  - $= x^{13} + x^{12} + x^{11} + x^{10} + x^9 + x^8 + x^6 + x^5 + x^4 + x^3 + x^2 + x \bmod x^8 + x^4 + x^3 + x + 1$
  - $= 1$
  - = {01}
- Example of addition of {53} = 01010011 & {CA} = 11001010
  - $(x^6 + x^4 + x + 1) \oplus (x^7 + x^6 + x^3 + x)$
  - $= x^7 + x^4 + x^3 + 1$
  - = {99}

Polynomials with coefficients in $GF(2^8)$

- Addition of two polynomials:
  - $a(x) = \sum_{i=0}^{3} a_i x^i = a_0 + a_1 x + a_2 x^2 + a_3 x^3$
  - $b(x) = \sum_{i=0}^{3} b_i x^i = b_0 + b_1 x + b_2 x^2 + b_3 x^3 \; (a_i, b_i \in GF(2^8))$
  - $a(x) + b(x) = \sum_{i=0}^{3} (a_i \oplus b_i) x^i$

$$= (a_0 \oplus b_0) + (a_1 \oplus b_1) x + (a_2 \oplus b_2) x^2 + (a_3 \oplus b_3) x^3$$

- Multiplication of 2 polynomials: $c(x) = a(x) \otimes b(x) = a(x) \bullet b(x) \bmod x^4 + 1$

$$\begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} a_0 & a_3 & a_2 & a_1 \\ a_1 & a_0 & a_3 & a_2 \\ a_2 & a_1 & a_0 & a_3 \\ a_3 & a_2 & a_1 & a_0 \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

Polynomials with coefficients in $GF(2^8)$

- Two polynomials: $a(x) = \{3\}x^3 + \{1\}x^2 + \{1\}x + \{2\}$ and $b(x) = \{1\}x^2 + \{8\}x + \{1\}$
- Example of addition:
  - $a(x) + b(x) = \{3\}x^3 + \{9\}x + \{3\}$
- Example of multiplication:
  - $a(x) \otimes b(x) = a(x) \bullet b(x) \bmod x^4 + 1$
  - Consider multiplication step:
    - $a(x) \bullet b(x) = (\{3\}x^3 + \{1\}x^2 + \{1\}x + \{2\}) \bullet (\{1\}x^2 + \{8\}x + \{1\})$
    - $= (\{3\}\bullet\{1\})x^5 + (\{3\}\bullet\{8\})x^4 + (\{3\}\bullet\{1\})x^3 + (\{1\}\bullet\{1\})x^4 + (\{1\}\bullet\{8\})x^3 + (\{1\}\bullet\{1\})x^2 + (\{1\}\bullet\{1\})x^3 + (\{1\}\bullet\{8\})x^2 + (\{1\}\bullet\{1\})x + (\{2\}\bullet\{1\})x^2 + (\{2\}\bullet\{8\})x + (\{2\}\bullet\{1\})$
    - $= (\{3\}\bullet\{1\})x^5 + (\{3\}\bullet\{8\} \oplus \{1\}\bullet\{1\})x^4 + (\{3\}\bullet\{1\} \oplus \{1\}\bullet\{8\} \oplus \{1\}\bullet\{1\})x^3 + (\{1\}\bullet\{1\} \oplus \{1\}\bullet\{8\} \oplus \{2\}\bullet\{1\})x^2 + (\{1\}\bullet\{1\} \oplus \{2\}\bullet\{8\})x + (\{2\}\bullet\{1\})$

Polynomials with coefficients in $GF(2^8)$

- Example of multiplication: $a(x) \otimes b(x) = a(x) \bullet b(x) \bmod x^4 + 1$
  - Consider modulo step:
    - $c(x) = (\{3\}\bullet\{1\})x^5 + (\{3\}\bullet\{8\} \oplus \{1\}\bullet\{1\})x^4 + (\{3\}\bullet\{1\} \oplus \{1\}\bullet\{8\} \oplus \{1\}\bullet\{1\})x^3 + (\{1\}\bullet\{1\} \oplus \{1\}\bullet\{8\} \oplus \{2\}\bullet\{1\})x^2 + (\{1\}\bullet\{1\} \oplus \{2\}\bullet\{8\})x + (\{2\}\bullet\{1\})$
    - $c(x) \bmod x^4 + 1 = (\{3\}\bullet\{1\})x^{5 \bmod 4} + (\{3\}\bullet\{8\} \oplus \{1\}\bullet\{1\})x^{4 \bmod 4} + (\{3\}\bullet\{1\} \oplus \{1\}\bullet\{8\} \oplus \{1\}\bullet\{1\})x^{3 \bmod 4} + (\{1\}\bullet\{1\} \oplus \{1\}\bullet\{8\} \oplus \{2\}\bullet\{1\})x^{2 \bmod 4} + (\{1\}\bullet\{1\} \oplus \{2\}\bullet\{8\})x + (\{2\}\bullet\{1\})$
    - $c(x) \bmod x^4 + 1 = (\{3\}\bullet\{1\})x + (\{3\}\bullet\{8\} \oplus \{1\}\bullet\{1\}) + (\{3\}\bullet\{1\} \oplus \{1\}\bullet\{8\} \oplus \{1\}\bullet\{1\})x^3 + (\{1\}\bullet\{1\} \oplus \{1\}\bullet\{8\} \oplus \{2\}\bullet\{1\})x^2 + (\{1\}\bullet\{1\} \oplus \{2\}\bullet\{8\})x + (\{2\}\bullet\{1\})$
    - $c(x) \bmod x^4 + 1 = (\{3\}\bullet\{1\} \oplus \{1\}\bullet\{8\} \oplus \{1\}\bullet\{1\})x^3 + (\{1\}\bullet\{1\} \oplus \{1\}\bullet\{8\} \oplus \{2\}\bullet\{1\})x^2 + (\{3\}\bullet\{1\} \oplus \{1\}\bullet\{1\} \oplus \{2\}\bullet\{8\})x + (\{3\}\bullet\{8\} \oplus \{1\}\bullet\{1\} \oplus \{2\}\bullet\{1\})$

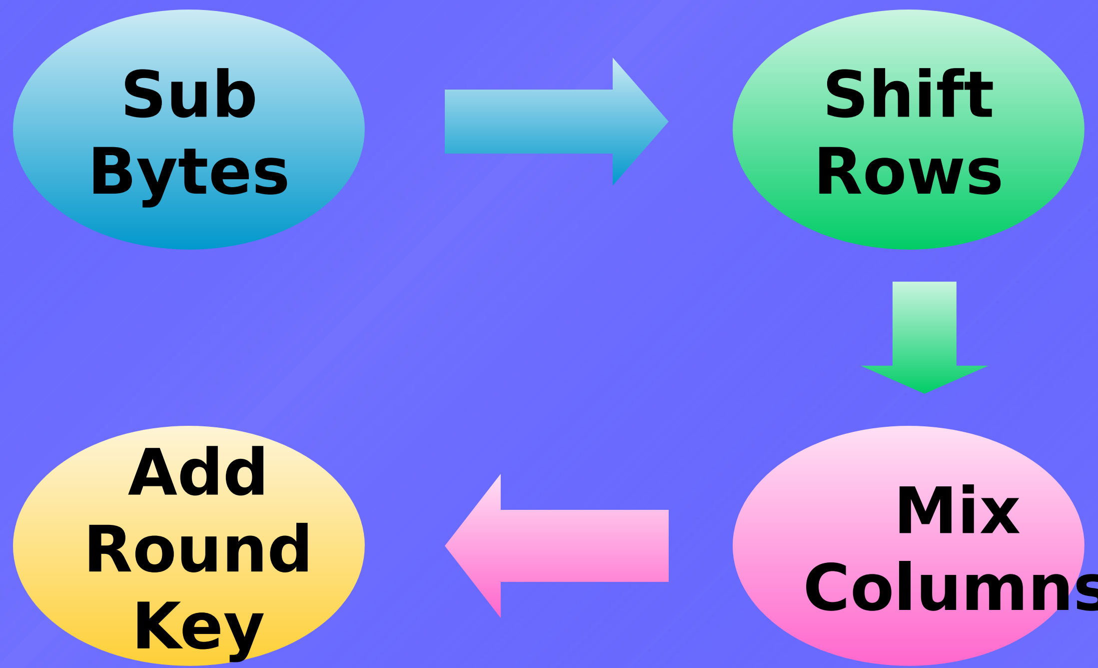
Data block representation and key code

- Intermediate result between the transformation steps is called state.
- A state is represented as a matrix of 4-rows and *Nb*-columns with *Nb* = block-size/32
- Cipher-key is represented as a matrix of 4-rows and *Nk*-columns with *Nk* = key-size/32
- Number of cycles ***Nr*** = *max*{*Nb*, *Nk*} + 6

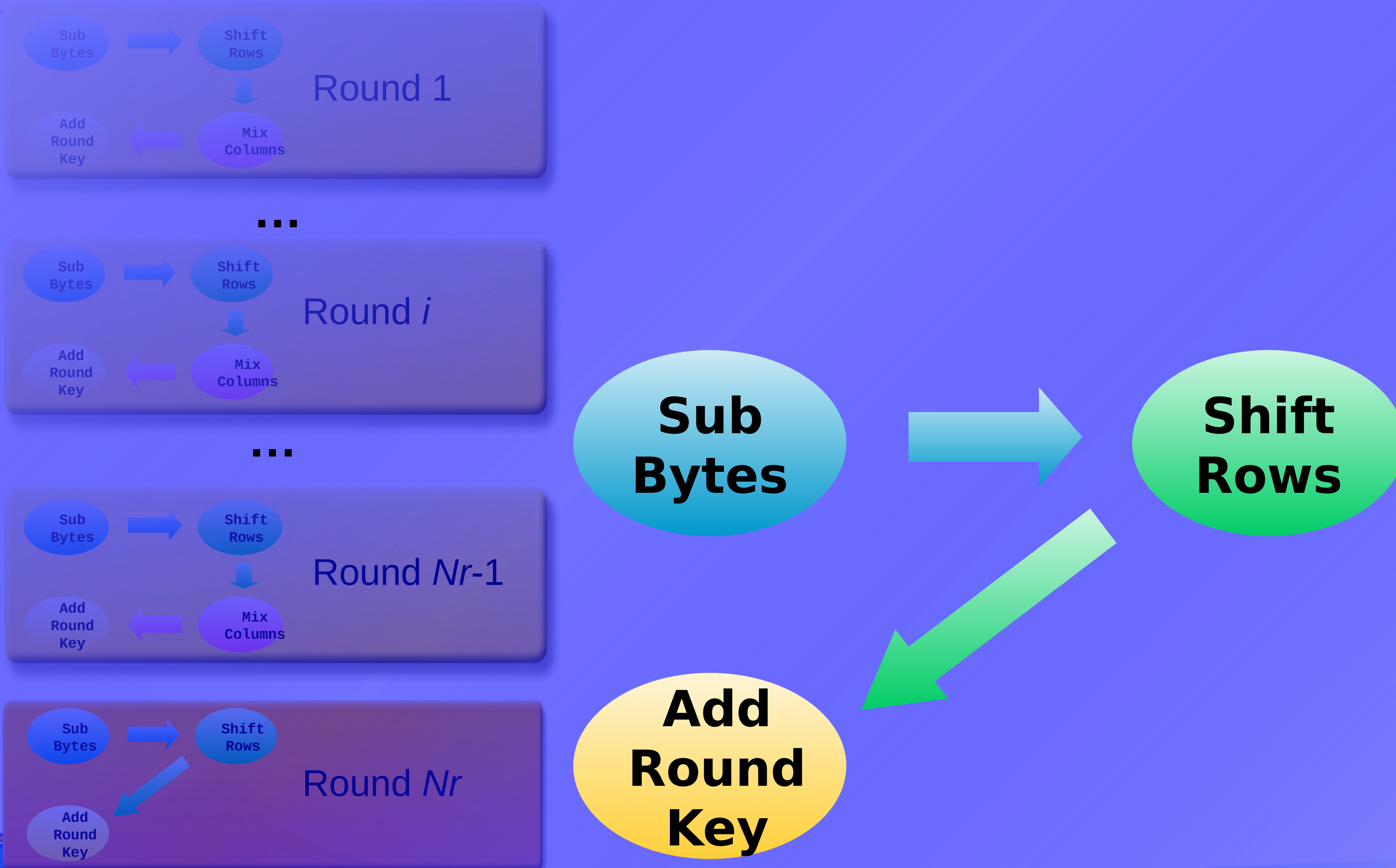
Nb=8

S_{00}	S_{01}	S_{02}	S_{03}	S_{04}	S_{05}	S_{06}	S_{07}
S_{10}	S_{11}	S_{12}	S_{13}	S_{14}	S_{15}	S_{16}	S_{17}
S_{20}	S_{21}	S_{22}	S_{23}	S_{24}	S_{25}	S_{26}	S_{27}
S_{30}	S_{31}	S_{32}	S_{33}	S_{34}	S_{35}	S_{36}	S_{37}

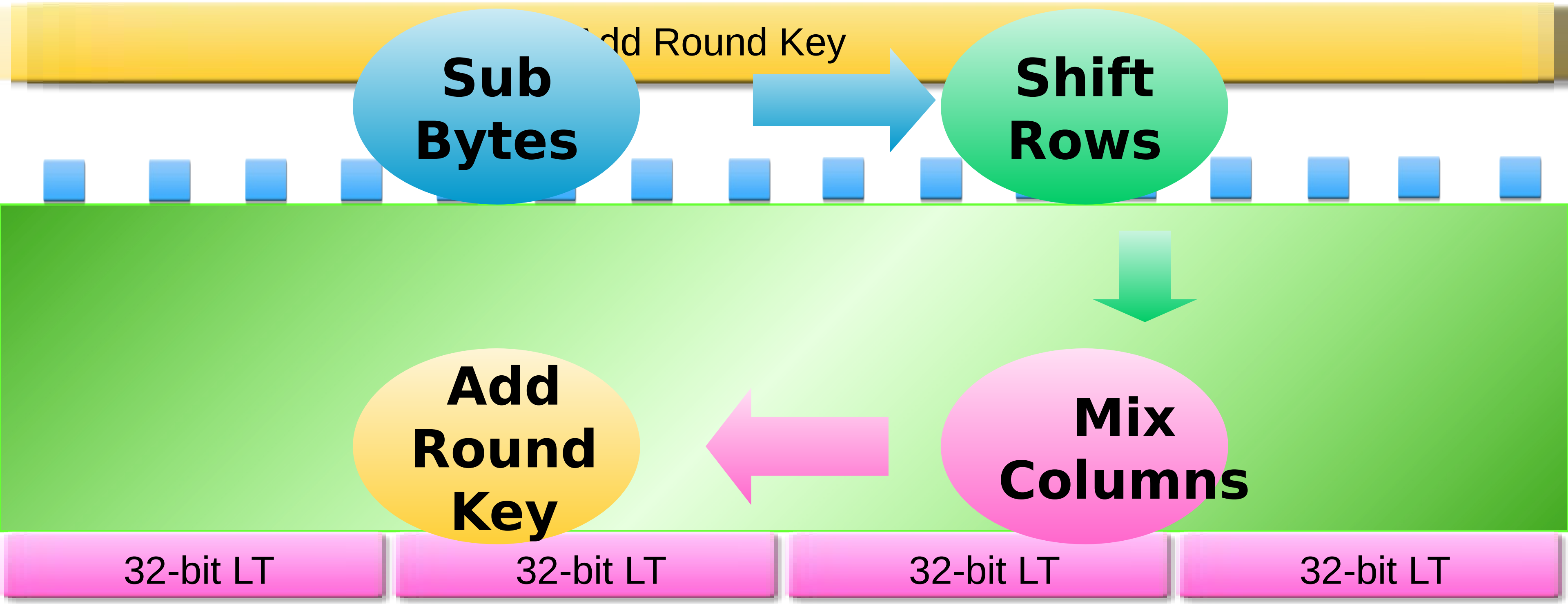
Normal cycles



Last cycle


Sub Bytes
Shift Rows
Mix Columns
Add Round Key
Round 1
...
Sub Bytes
Shift Rows
Mix Columns
Add Round Key
Round *i*
...
Sub Bytes
Shift Rows
Mix Columns
Add Round Key
Round *Nr*-1
Sub Bytes
Shift Rows
Add Round Key
Round *Nr*
Sub Bytes
Shift Rows
Add Round Key

Substitution-Permutation Network architecture

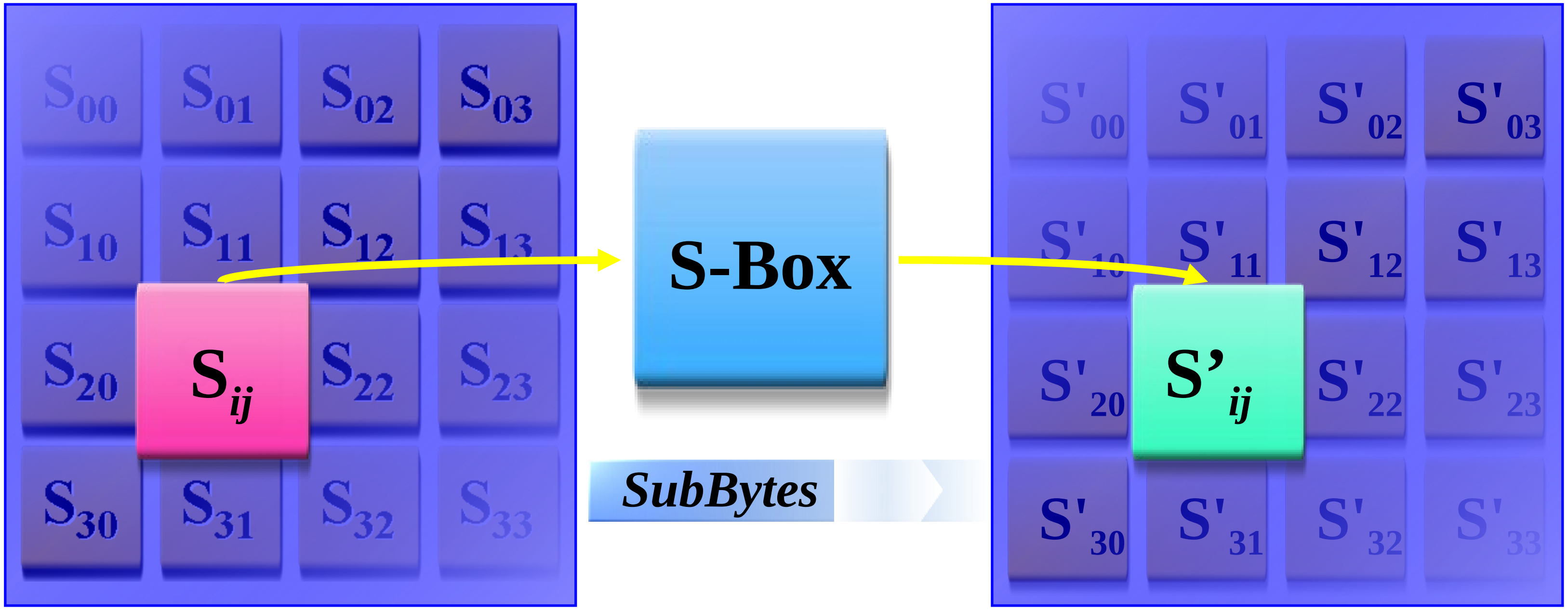


Rijndael algorithm encryption

```
Cipher( byte in[4 * Nb], byte out[4 * Nb], word w[Nb*(Nr + 1)])
begin
    byte      state[4,Nb]
    state =   in
    AddRoundKey(state, w)
    for round = 1 to Nr – 1
        SubBytes(state)
        ShiftRows(state)
        MixColumns(state)
        AddRoundKey(state, w + round * Nb)
    end for
    SubBytes(state)
    ShiftRows(state)
    AddRoundKey(state, w + Nr * Nb)
    out = state
end
```

SubBytes

- Nonlinear byte substitution via substitution table (S-box)
- Acts independently on each byte in the current state



SubBytes

- Process of replacing byte x in **SubBytes**:
  - Determine the inverse element x^{-1} (Binary form is $\{x_7x_6x_5x_4x_3x_2x_1x_0\}$), with $\{00\}^{-1} = \{00\}$
  - Affine: $y_i = x_i \oplus x_{(i+4) \bmod 8} \oplus x_{(i+5) \bmod 8} \oplus x_{(i+6) \bmod 8} \oplus x_{(i+7) \bmod 8} \oplus c_i$
  - with $\{c_7c_6c_5c_4c_3c_2c_1c_0\} = \{63\}$

$$\begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

InvSubBytes

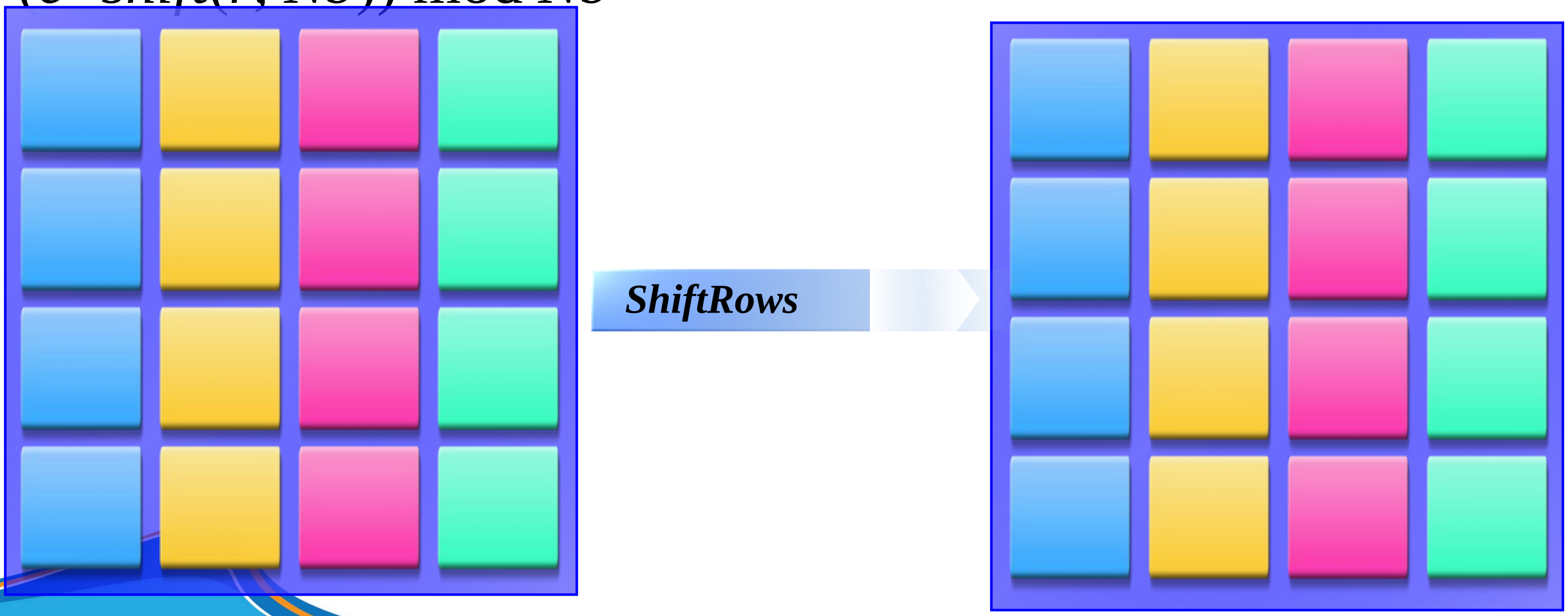
- Process of replacing byte y in **InvSubBytes**:
- Affine:
  - $x_i = y_{(i+2) \bmod 8} \oplus y_{(i+5) \bmod 8} \oplus y_{(i+7) \bmod 8} \oplus d_i$ với $\{d_7d_6d_5d_4d_3d_2d_1d_0\} = \{05\}$
- Determine the inverse element $x^{-1} \in GF(2^8)$ of x ($\{00\}^{-1} = \{00\}$)

$$\begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

ShiftRows

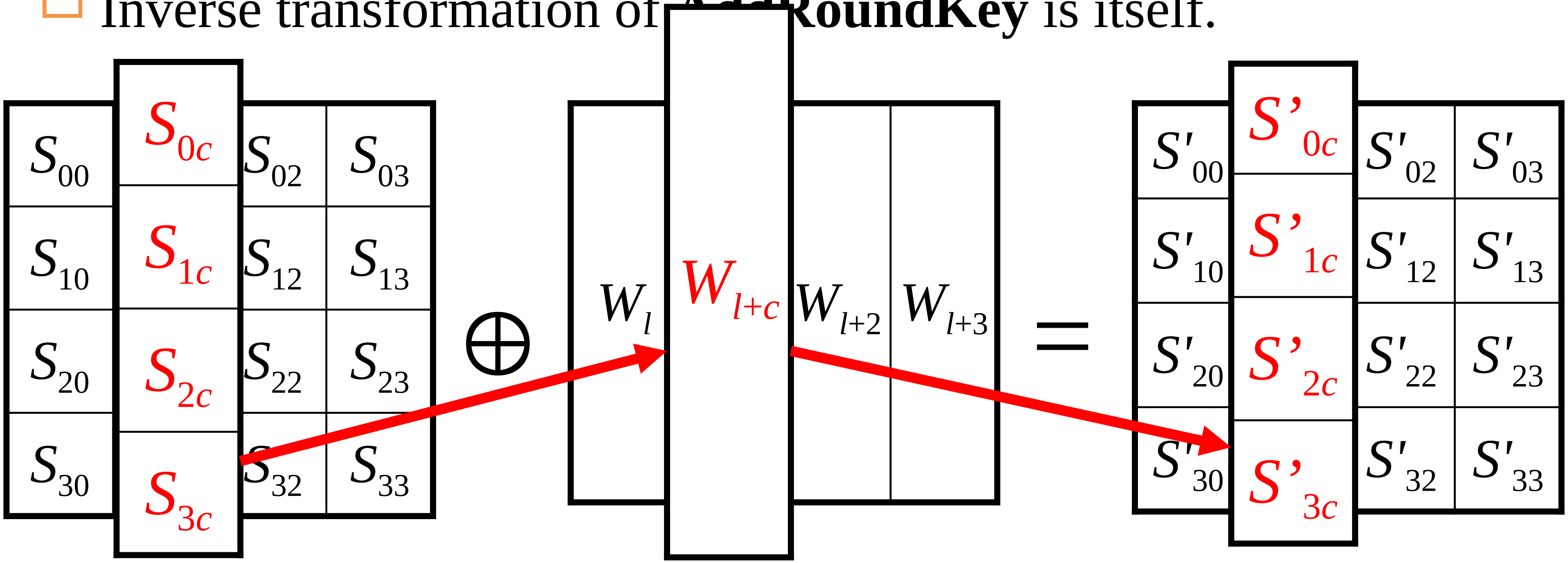
***shift*(*r*, *Nb*)**		***r***		
		1	**2**	**3**
Nb	**4**	**1**	**2**	**3**
	6	**1**	**2**	**3**
	8	**1**	**3**	**4**

- Each line of the current state is rotated by a number of places
- Byte $s_{r,c}$ at row r column c changes to column (c + *shift*(r, Nb)) mod Nb
- **InvShiftRows**: Byte $s_{r,c}$ at row r column c changes to column (c- *shift*(r, Nb)) mod Nb



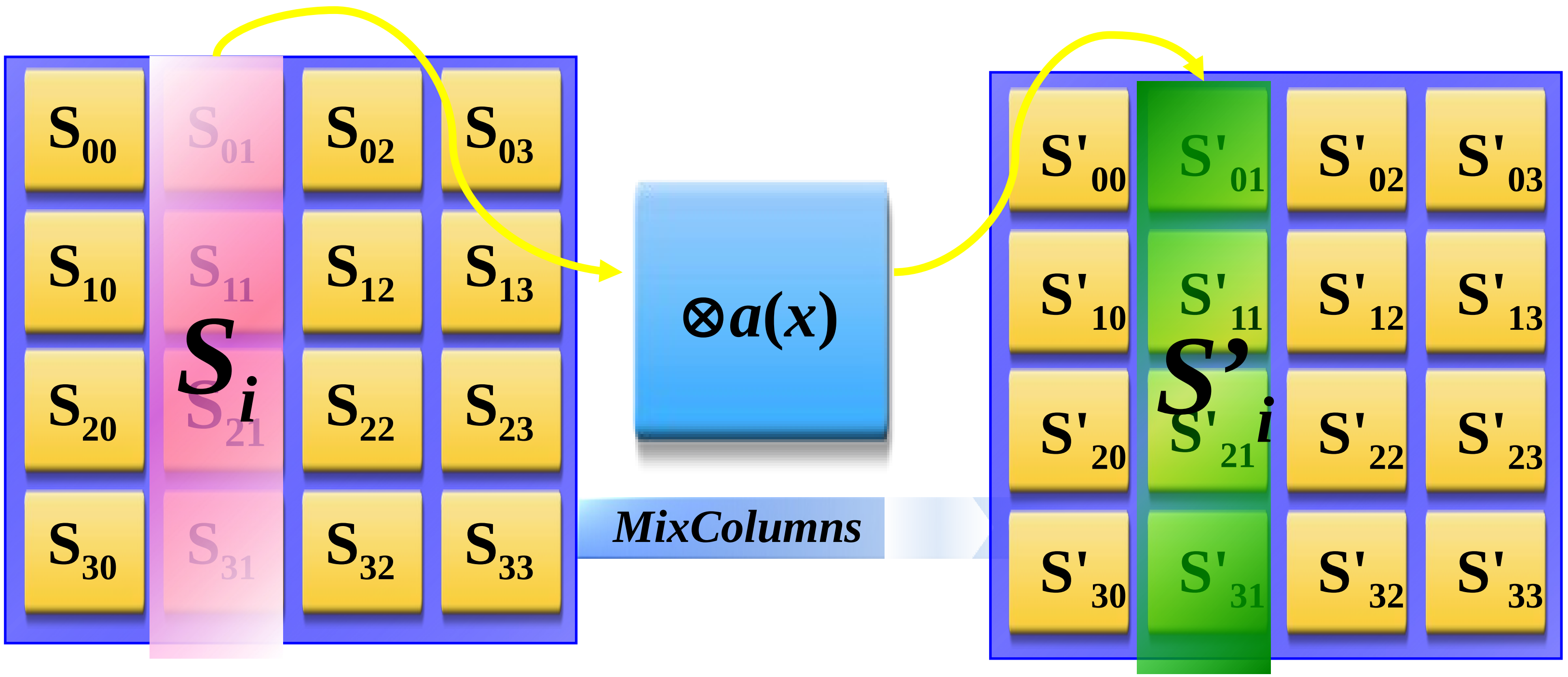
AddRoundKey

- Each byte of the status will XOR with corresponding byte in key-cipher of the current cycle: $s'_{r,c} = k_{r,c} \oplus s_{r,c}$, $0 \le r < 4$, $0 \le c < Nb$
- Inverse transformation of **AddRoundKey** is itself.



$l = round*Nb$

MixColumns


S00
S01
S02
S03
S10
S11
S12
S13
S20
S21
S22
S23
S30
S31
S32
S33
Si
⊗a(x)
MixColumns
S'00
S'01
S'02
S'03
S'10
S'11
S'12
S'13
S'20
S'21
S'22
S'23
S'30
S'31
S'32
S'33
S'i

MixColumns

- Each column of the current state is represented as a polynomial s(x) with coefficients in $GF(2^8)$.
- Perform multiplication: $s'(x) = a(x) \otimes s(x)$
- With: $a(x) = \{03\}x^3+\{01\}x^2+\{01\}x+\{02\}$

$$\begin{bmatrix} s'_{0,c} \\ s'_{1,c} \\ s'_{2,c} \\ s'_{3,c} \end{bmatrix} = \begin{bmatrix} 02 & 03 & 01 & 01 \\ 01 & 02 & 03 & 01 \\ 01 & 01 & 02 & 03 \\ 03 & 01 & 01 & 02 \end{bmatrix} \begin{bmatrix} s_{0,c} \\ s_{1,c} \\ s_{2,c} \\ s_{3,c} \end{bmatrix}$$

InvMixColumns

- Each column of the current state is represented as a polynomial s(x) with coefficients in GF(2^8).
- Perform multiplication: $s'(x) = a^{-1}(x) \otimes s(x)$
- With: $a^{-1}(x) = \{0b\}x^3+\{0d\}x^2+\{09\}x+\{0e\}$

$$\begin{bmatrix} s'_{0,c} \\ s'_{1,c} \\ s'_{2,c} \\ s'_{3,c} \end{bmatrix} = \begin{bmatrix} 0e & 0b & 0d & 09 \\ 09 & 0e & 0b & 0d \\ 0d & 09 & 0e & 0b \\ 0b & 0d & 09 & 0e \end{bmatrix} \begin{bmatrix} s_{0,c} \\ s_{1,c} \\ s_{2,c} \\ s_{3,c} \end{bmatrix}$$

Generate a key-cipher for each cycle

- The extended key table is a 1-dimensional array of words (4 bytes in length).
- Function that generates the extended key cipher table depends on the value *Nk*, that is, on the length of the primary key

4 bytes

w_0	w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8	w_9	w_{10}	w_{11}	w_{12}	w_{13}	w_{14}	w_{15}	w_{16}	w_{17}	…
Key for cycle 0						Key for cycle 1						Key for cycle 2						…

Extended key cipher table and how to determine key of the period (*Nb* = 6 (192-*bit* data), *Nk* = 4 (128-*bit* key))

Generate a key-cipher for each cycle

```
KeyExpansion(byte key[4 * Nk], word w[Nb * (Nr + 1)], Nk)
begin
    i=0
    while (i < Nk)
        w[i] = word[key[4*i], key[4*i+1],  key[4*i+2],key[4*i+3]]
        i = i + 1
    end while
    i = Nk
    while (i < Nb * (Nr + 1))
        word temp = w[i - 1]
        if  (i mod Nk = 0) then
            temp = SubWord(RotWord(temp)) xor Rcon[i / Nk]
        else
            if (Nk = 8) and (i mod Nk = 4) then temp = SubWord(temp)
        end if
        w[i] = w[i - Nk] xor temp
        i = i + 1
    end while
end
```

Generate a key-cipher for each cycle

- The set of constants is generated according to the following rule
  - $Rcon[i] = (RC[i], \{00\}, \{00\}, \{00\})$ với $RC[i] \in GF(2^8)$
  - $RC[i] = \{02\} \bullet RC[i - 1] = x^{(i-1)}$
  - Example
    - $Rcon[1] = (x^0 = \{01\}, \{00\}, \{00\}, \{00\})$
    - $Rcon[2] = (x^1 = \{02\}, \{00\}, \{00\}, \{00\})$
    - $Rcon[3] = (x^2 = \{04\}, \{00\}, \{00\}, \{00\})$
    - …
    - $Rcon[8] = (x^7 = \{80\}, \{00\}, \{00\}, \{00\})$
    - $Rcon[9] = (x^8 = \{1B\}, \{00\}, \{00\}, \{00\})$
      - Explain: $x^8 = \{100000000\} \bmod x^8 + x^4 + x^3 + x + 1 = \{100011011\} = \{1B\}$
    - $Rcon[10] = (x^9 = \{36\}, \{00\}, \{00\}, \{00\})$
      - Explain: $x^9 = \{1000000000\} \bmod x^8 + x^4 + x^3 + x + 1 = \{100011011\} = \{36\}$
    - …

S-box

Sbox		y															
		0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
	0	63	7c	77	7b	f2	6b	6f	c5	30	01	67	2b	fe	d7	ab	76
	1	ca	82	c9	7d	fa	59	47	f0	ad	d4	a2	af	9c	a4	72	c0
	2	b7	fd	93	26	36	3f	f7	cc	34	a5	e5	f1	71	d8	31	15
	3	04	c7	23	c3	18	96	05	9a	07	12	80	e2	eb	27	b2	75
	4	09	83	2c	1a	1b	6e	5a	a0	52	3b	d6	b3	29	e3	2f	84
	5	53	d1	00	ed	20	fc	b1	5b	6a	cb	be	39	4a	4c	58	cf
	6	d0	ef	aa	fb	43	4d	33	85	45	f9	02	7f	50	3c	9f	a8
x	7	51	a3	40	8f	92	9d	38	f5	bc	b6	da	21	10	ff	f3	d2
	8	cd	0c	13	ec	5f	97	44	17	c4	a7	7e	3d	64	5d	19	73
	9	60	81	4f	dc	22	2a	90	88	46	ee	b8	14	de	5e	0b	db
	a	e0	32	3a	0a	49	06	24	5c	c2	d3	ac	62	91	95	e4	79
	b	e7	c8	37	6d	8d	d5	4e	a9	6c	56	f4	ea	65	7a	ae	08
	c	ba	78	25	2e	1c	a6	b4	c6	e8	dd	74	1f	4b	bd	8b	8a
	d	70	3e	B5	66	48	03	f6	0e	61	35	57	b9	86	c1	1d	9e
	e	e1	f8	98	11	69	d9	8e	94	9b	1e	87	e9	ce	55	28	df
	f	8c	a1	89	0d	Bf	e6	42	68	41	99	2d	0f	b0	54	bb	16

fit@hcmus

S-Box^{-1}

Sbox^{-1}		*y*															
		0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
x	0	52	09	6a	d5	30	36	a5	38	bf	40	a3	9e	81	f3	d7	fb
	1	7c	e3	39	82	9b	2f	ff	87	34	8e	43	44	c4	de	e9	cb
	2	54	7b	94	32	a6	c2	23	3d	ee	4c	95	0b	42	fa	c3	4e
	3	08	2e	a1	66	28	d9	24	b2	76	5b	a2	49	6d	8b	d1	25
	4	72	f8	f6	64	86	68	98	16	d4	a4	5c	cc	5d	65	b6	92
	5	6c	70	48	50	fd	ed	b9	da	5e	15	46	57	a7	8d	9d	84
	6	90	d8	ab	00	8c	bc	d3	0a	f7	e4	58	05	b8	b3	45	06
	7	d0	2c	1e	8f	ca	3f	0f	02	c1	af	bd	03	01	13	8a	6b
	8	3a	91	11	41	4f	67	dc	ea	97	f2	cf	ce	f0	b4	e6	73
	9	96	ac	74	22	e7	ad	35	85	e2	f9	37	e8	1c	75	df	6e
	a	47	f1	1a	71	1d	29	c5	89	6f	b7	62	0e	aa	18	be	1b
	b	fc	56	3e	4b	c6	d2	79	20	9a	db	c0	fe	78	cd	5a	f4
	c	1f	dd	a8	33	88	07	c7	31	b1	12	10	59	27	80	ec	5f
	d	60	51	7f	a9	19	b5	4a	0d	2d	e5	7a	9f	93	c9	9c	ef
	e	a0	e0	3b	4d	ae	2a	f5	b0	c8	eb	bb	3c	83	53	99	61
	f	17	2b	04	7e	ba	77	d6	26	e1	69	14	63	55	21	0c	7d

Guidance of AES128

- AES128(M, K) // $|K| = |M| = 128$ bits
  - $(K_0, K_1, \ldots, K_{10}) \leftarrow$ expand(K) // $K_0 \leftarrow K$
  - $s \leftarrow M \oplus K_0$
  - Loop $r = 1 \rightarrow 10$
    - $s \leftarrow$ s-box(s)
    - $s \leftarrow$ shift-rows(s)
    - If $r \leq 9$ then $s \leftarrow$ mix-cols(s)
    - $s \leftarrow s \oplus K_r$
  - Return s

- expand(K) // $|K| = 128$ bits
  - $K_0 \leftarrow K$
  - Loop $i = 1 \rightarrow 10$
    - $K_i[0] \leftarrow K_{i-1}[0] \oplus$ s-box($K_{i-1}[3]$ <<< 8) $\oplus C_i$
    - $K_i[1] \leftarrow K_{i-1}[1] \oplus K_i[0]$
    - $K_i[2] \leftarrow K_{i-1}[2] \oplus K_i[1]$
    - $K_i[3] \leftarrow K_{i-1}[3] \oplus K_i[2]$
  - Return $(K_0, K_1, \ldots, K_{10})$

Guidance of AES128 (expand)

- expand(K) // $|K|$ = 128 bits
  - $K_0 \leftarrow K$
  - Loop $i = 1 \rightarrow 10$
    - $K_i[0] \leftarrow K_{i-1}[0] \oplus$ s-box($K_{i-1}[3]$ <<< 8) $\oplus C_i$
    - $K_i[1] \leftarrow K_{i-1}[1] \oplus K_i[0]$
    - $K_i[2] \leftarrow K_{i-1}[2] \oplus K_i[1]$
    - $K_i[3] \leftarrow K_{i-1}[3] \oplus K_i[2]$
  - Return $(K_0, K_1, \ldots, K_{10})$
- Notes:
  - The input is usually a 16-byte key sequence, ví dụ "Thats my Kung Fu"
  - The algorithm deals with integers, each subkey consists of 4 integers
    - Each integer 4 bytes
    - Use type int

Guidance of AES128 (expand)

- expand(K) // $|K|$ = 128 bits
  - $K_0 \leftarrow K$
  - Loop $i = 1 \rightarrow 10$
    - $K_i[0] \leftarrow K_{i-1}[0] \oplus$ s-box($K_{i-1}[3]$ <<< 8) $\oplus C_i$
    - $K_i[1] \leftarrow K_{i-1}[1] \oplus K_i[0]$
    - $K_i[2] \leftarrow K_{i-1}[2] \oplus K_i[1]$
    - $K_i[3] \leftarrow K_{i-1}[3] \oplus K_i[2]$
  - Return (K_0, K_1, …, K_{10})
- Table of constants C:

i	**Values**
C_0	0x00000000
C_1	0x01000000
C_2	0x02000000
C_3	0x04000000
C_4	0x08000000

C_5	0x10000000
C_6	0x20000000
C_7	0x40000000
C_8	0x80000000
C_9	0x1b000000
C_{10}	0x36000000

Guidance of AES128 (expand)

- Some notes
  - Convert from 4 bytes to 1 int: char *s*[] = "abcd" → int *a* = 0x61626364
  - Convert from 1 int to 4 bytes: int *a* = 0x61626364 → char *s*[] = "abcd"
  - Same for transferring 16 bytes → 4 ints and 4 ints → 16 bytes
- Example to test expand:

Keys	Values
K_0	54 68 61 74 73 20 6d 79 20 4b 75 6e 67 20 46 75
K_1	e2 32 fc f1 91 12 91 88 b1 59 e4 e6 d6 79 a2 93
K_2	56 08 20 07 c7 1a b1 8f 76 43 55 69 a0 3a f7 fa
K_3	d2 60 0d e7 15 7a bc 68 63 39 e9 01 c3 03 1e fb
K_4	a1 12 02 c9 b4 68 be a1 d7 51 57 a0 14 52 49 5b
K_5	b1 29 3b 33 05 41 85 92 d2 10 d2 32 c6 42 9b 69
K_6	bd 3d c2 87 b8 7c 47 15 6a 6c 95 27 ac 2e 0e 4e
K_7	cc 96 ed 16 74 ea aa 03 1e 86 3f 24 b2 a8 31 6a
K_8	8e 51 ef 21 fa bb 45 22 e4 3d 7a 06 56 95 4b 6c
K_9	bf e2 bf 90 45 59 fa b2 a1 64 80 b4 f7 f1 cb d8
K_{10}	28 fd de f8 6d a4 24 4a cc c0 a4 fe 3b 31 6f 26

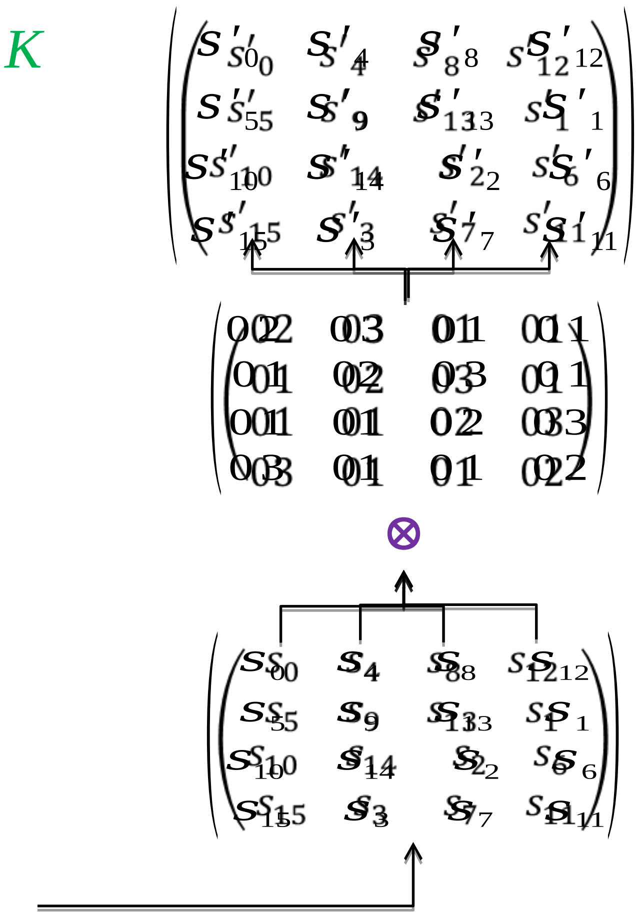
Thats my Kung Fu

Guidance of AES128 (AES128)

- AES128(M, K) // $|K| = |M| = 128$ bits
  - $(K_0, K_1, \ldots, K_{10}) \leftarrow$ expand(K) // $K_0 \leftarrow K$
  - $s \leftarrow M \oplus K_0$
  - Loop $r = 1 \rightarrow 10$
    - $s \leftarrow$ s-box(s)
    - $s \leftarrow$ shift-rows(s)
    - if $r \leq 9$ then $s \leftarrow$ mix-cols(s)
    - $s \leftarrow s \oplus K_r$
  - Return s
- Notes:
  - Value s: $s_0s_1s_2s_3s_4s_5s_6s_7s_8s_9s_{10}s_{11}s_{12}s_{13}s_{14}s_{15}$ stands for a 16-byte string ($s = M \oplus K_0$)
  - Function s-box: $[0, 255] \rightarrow [0, 255]$ follows S-box table

Guidance of AES128 (AES128)

- AES128(M, K) // $|K| = |M| = 128$ bits
  - (K_0, K_1, …, K_{10}) ← expand(K) // $K_0 \leftarrow K$
  - $s \leftarrow M \oplus K_0$
  - Loop $r = 1 \rightarrow 10$
    - $s \leftarrow$ s-box(s)
    - $s \leftarrow$ shift-rows(s)
    - if $r \leq 9$ then $s \leftarrow$ mix-cols(s)
    - $s \leftarrow s \oplus K_r$
  - Return s
- Notes:
  - shift-rows
    - Input: $s_0s_1s_2s_3s_4s_5s_6s_7s_8s_9s_{10}s_{11}s_{12}s_{13}s_{14}s_{15}$
    - Output: $s_0s_5s_{10}s_{15}s_4s_9s_{14}s_3s_8s_{13}s_2s_7s_{12}s_1s_6s_{11}$



Guidance of AES128 (AES128)

- Test-vector for AES128
  - Key: 54|68|61|74|73|20|6d|79|20|4b|75|6e|67|20|46|75 (“Thats my Kung Fu”)
  - Message: 54|77|6f|20|4f|6e|65|20|4e|69|6e|65|20|54|77|6f (“Two One Nine Two”)
  - Cipher-text:
    - C_0: 00 1f 0e 54 3c 4e 08 59 6e 22 1b 0b 47 74 31 1a
    - C_1: 58 47 08 8b 15 b6 1c ba 59 d4 e2 e8 cd 39 df ce
    - C_2: 43 c6 a9 62 0e 57 c0 c8 09 08 eb fe 3d f8 7f 37
    - C_3: 78 76 30 54 70 76 7d 23 99 3c 37 5b 4b 39 34 f1
    - C_4: b1 ca 51 ed 08 fc 54 e1 04 b1 c9 d3 e7 b2 6c 20
    - C_5: 9b 51 20 68 23 5f 22 f0 5d 1c bd 32 2f 38 91 56
    - C_6: 14 93 25 77 8f a4 2b e8 c0 60 24 40 5e 0f 92 75
    - C_7: 53 39 8e 5d 43 06 93 f8 4f 0a 3b 95 85 52 57 bd
    - C_8: 66 25 3c 74 70 ce 5a a8 af d3 0f 0a a3 73 13 54
    - C_9: 09 66 8b 78 a2 d1 9a 65 f0 fc e6 c4 7b 3b 30 89
    - C_{10}: 29 c3 50 5f 57 14 20 f6 40 22 99 b3 1a 02 d7 3a

Intermediate cipher-text of AES128

Final cipher-text of AES128