ÀLGEBRA (EI)
Curs 2012-2013
Determinants

1. Si $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = 8$, calculeu

a) $\begin{vmatrix} a_{31} & a_{32} & a_{33} \\ a_{21} & a_{22} & a_{23} \\ a_{11} & a_{12} & a_{13} \end{vmatrix}$ b) $\begin{vmatrix} a_{31} & a_{32} & a_{33} \\ a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{vmatrix}$ c) $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ 2a_{21} & 2a_{22} & 2a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

d) $\begin{vmatrix} -3a_{11} & -3a_{12} & -3a_{13} \\ 2a_{21} & 2a_{22} & 2a_{23} \\ 5a_{31} & 5a_{32} & 5a_{33} \end{vmatrix}$ e) $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ -a_{11} & -a_{12} & -a_{13} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ f) $\begin{vmatrix} 2a_{11}-3a_{21} & 2a_{12}-3a_{22} & 2a_{13}-3a_{23} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$.

2. Calculeu els determinants següents:

$$\begin{vmatrix} 1 & 4 & 2 \\ 0 & 1 & -1 \\ 1 & 3 & 2 \end{vmatrix}, \quad \begin{vmatrix} 1 & 0 & 3 & 2 \\ -2 & 1 & 1 & 1 \\ 3 & 2 & 4 & -1 \\ 0 & 3 & 2 & 2 \end{vmatrix}, \quad \begin{vmatrix} 1-i & 1 & 0 & 2 \\ 2+i & 3 & 3 & 2 \\ 3+i & 2 & 3 & 2 \\ 1+i & 2 & 4 & 1 \end{vmatrix},$$

$$\begin{vmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 5 & 6 & 7 \\ 4 & 5 & 6 & 7 & 8 \\ 5 & 6 & 7 & 8 & 9 \end{vmatrix}, \quad \begin{vmatrix} -4 & 1 & 1 & 1 & 1 \\ 1 & -4 & 1 & 1 & 1 \\ 1 & 1 & -4 & 1 & 1 \\ 1 & 1 & 1 & -4 & 1 \\ 1 & 1 & 1 & 1 & -4 \end{vmatrix}.$$

3. Calculeu el següents determinants:

a) $\begin{vmatrix} a & b & 0 & 0 \\ c & d & 0 & 0 \\ 0 & 0 & a & -b \\ 0 & 0 & c & d \end{vmatrix}$ b) $\begin{vmatrix} a & 0 & 0 & 0 & 0 \\ 0 & 0 & b & 0 & 0 \\ 0 & 0 & 0 & 0 & c \\ 0 & 0 & 0 & d & 0 \\ 0 & e & 0 & 0 & 0 \end{vmatrix}$ c) $\begin{vmatrix} 0 & -1 & -1 & -1 & -1 \\ 1 & 0 & -1 & -1 & -1 \\ 1 & 1 & a & -1 & -1 \\ 1 & 1 & 1 & 0 & -1 \\ 1 & 1 & 1 & 1 & 0 \end{vmatrix}$.

4. Resoleu l'equació:

$$\begin{vmatrix} x & 1 & 2 \\ x & -1 & 1 \\ 0 & 3 & x \end{vmatrix} = -1.$$

5. Trobeu para quins valors de $\alpha \in \mathbb{R}$ la matriu M és invertible:

$$M = \begin{pmatrix} \alpha+1 & -3 \\ 5 & 1-\alpha \end{pmatrix}, \quad M = \begin{pmatrix} -\alpha & \alpha-1 & \alpha+1 \\ 1 & 2 & 3 \\ 2-\alpha & \alpha+3 & \alpha+7 \end{pmatrix}, \quad M = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}.$$