EMT 2103:Planar Mechanics

Students are reminded not to treat these lecture notes as a comprehensive and solely sufficient for their studies. These notes are aimed at providing a quick reference and a brief guidance for the students NOT a substitute for attending regular classes and reading recommended textbooks.

Expected Outcomes

At the end of this course, you should be able to;

- carry out position analysis of points in a given planar mechanism.
- solve problems concerning velocity analysis in simple and compound mechanisms.
- carry out acceleration analysis in a given planar mechanism.
- carry out computer simulation in mechanisms analysis.

Course Outline

Mechanism: Definition, terminology, degrees of freedom. Displacement analysis: Displacement; position analysis of linkages, relative motion. Velocity analysis, instant centres, Kennedys Theorem, graphical methods of analysis. Mechanical advantage: Correlation of mechanical advantage and transmission angle, optimal mechanical advantage. Computer simulation. Acceleration analysis: Acceleration difference, relative acceleration, coriolis acceleration, graphical solutions and computer simulation.

Reference Textbooks

1. Hannah, J. and Stephens, R.C., *Mechanics of Machines -Elementary Theory and Examples*, Arnold International Publishers.
2. Hannah, J. and Stephens, R.C, *Mechanics of Machines -Advanced Theory and Examples*, Arnold International Publishers.
3. Ryder, G.H. and Bennett, M.D., *Mechanics of Machines*, McMillan Education Ltd Publishers, 2nd Ed., 1990.
4. Hamilton, H.M and Charles F.R., *Mechanics and Dynamics of Machinery*,
5. Shigley J. and Uicker, *Theory of Machines and Mechanisms*,
6. Norton, R.L., *Design of Machinery - An Introduction to the Synthesis and Analysis of Mechanisms and Machines*, McGraw-Hill Publishers, 3rd Ed., 2004.
7. Grosjean, J., *Kinematics and Dynamics of Mechanisms*, McGraw-Hill Book Co. (UK) Ltd, 1991.
8. Mabie, H. H., *Mechanisms and Dynamics of Machines.*
9. Prentice, J. M., *Dynamics of Mechanical Systems.*

Contents

Chapter 1

Fundamentals of Machines and Mechanisms

1.1 Introduction and Terminologies

1.1.1 Rigid body

A rigid body consists of a set of particles that have a fixed distance. The relationship among these particles does not change with respect to time. A rigid body can also be defined as one which does not change its physical form on application of forces. Although a truly rigid body does not exist, many engineering components are assumed to be rigid because their deformations and distortions are negligible in comparison with their relative movements.

1.1.2 Mechanism and machine

Mechanism is a combination of several rigid bodies formed and connected so that they produce a definite relative motions with respect to one another. For example the assembly of piston, cylinder, connecting rod and crank forms a mechanism called slider-crank mechanism, which converts reciprocating motion of piston into a rotary motion of crank or vice-versa.

On the other hand, a *machine* is the combination of mechanisms which apart from producing relative motion, also does useful work. Examples of machines include, engine, lathe, drill, shaper among others. Hence, a *machine* is a combination of rigid bodies, formed and connected so that they move with definite relative motions and transmit force and couples from the source of power to the resistance to be overcome. A machine has two functions: producing definite relative motion and transmitting force. The difference between machine and mechanism is that machines transform energy to do work, while mechanisms do not necessarily perform this function. The term machinery generally means machines and mechanisms

1.1.3 Kinematics, dynamics and kinetics

Kinematics: Is concerned with motion of the part without considering the forces responsible. Therefore, kinematics deals with the fundamental concepts of space and time and the quantities velocity and acceleration derived there from.
Dynamics: Concerns the forces that acts on the parts (both balanced and unbalanced) and takes into account the masses and acceleration of the parts of as well as the external forces
Kinetics: Deals with action of forces on bodies. This is where the the effects of gravity come into play.

1.1.4 Kinematic link

A link is defined as a rigid body having two or more pairing elements (nodes) which connect it to other bodies for the purpose of transmitting force or motion. Therefore a mechanism consists of links which are joined together by use of mechanical joints (pairs). In mechanisms two types of links are most widely used, that is rigid and flexible links . A rigid link, depending upon its ends with which connection between the other links is formed, can be classified into the following three categories,

(a) Binary link: A link which has two ends for connection (two nodes).

(b) Ternary link: A link which forms three connections (three nodes).

(c) Quaternary link: A link which has four ends for connection (four nodes).

1.1.5 Kinematic pairs

A pair is a joint between the surfaces of two rigid bodies that keeps them in contact and relatively movable. A kinematic pair can be classified according to the following criteria:

(a) Nature of contact between the links: Under this classification, a kinematic pair is either lower or higher pair.

  (i) When two links have a surface or area contact between them while in motion, such a pair is called a lower pair. The relative motion in a lower pair is either purely turning or sliding. Examples include, revolute, prismatic, cylindrical or spherical joints.

  (ii) When two links have point or line contact between them while in motion, the pair so formed is called a higher pair. The relative motion in a higher pair is a combination of turning and sliding. Examples include contacts between , gears, cams and follower, ball/roller bearings.

(b) Nature of relative motion between the links: A kinematic pair is classified into the following,

(i) Sliding pair: when two links are connected such that one is constrained to have a sliding motion relative to the other. Example, prismatic joint.

(ii) Turning pair: when two links are connected in such a way that only a constrained motion of rotation is possible between them. Example, revolute joints, which are further categorized into,

- *Rotational joints (R)*- The rotation motion of one link is about an axis perpendicular to the adjoining link.
- *Twisting joint (T)*- The rotation motion takes place about an axis that is parallel to both links.
- *Revolving joint (V)*- The rotation motion takes place about an axis that is parallel to one of the links. Usually the links are aligned perpendicular to one another , and the rotation involves revolution of one link about the other.

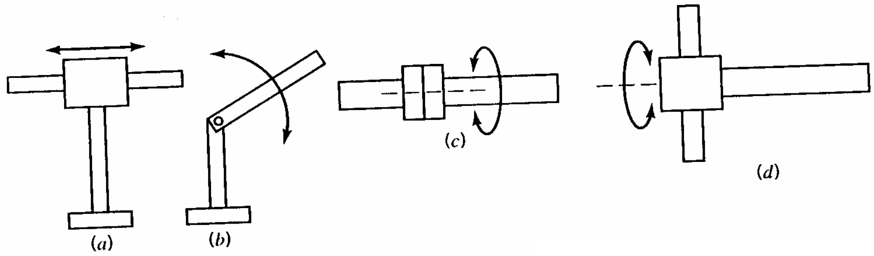


Figure 1.1: (a)Prismatic (b)Rotational (c)Twisting (d)Revolving joints

(iii) Rolling pair: when two links have a rolling motion relative to each other. Example, contact between ball or roller bearing, or pulley in a belt drive.

(iv) Spherical pair: when one link is in the form of a sphere which turns inside a fixed link. Example, a ball and socket joint (spherical joint: in spatial mechanisms).

(c) Screw pair: A pair of screw and threads which has both turning and sliding motion between them. Also called helical pair.

1.1.6 Planar and Spatial mechanisms

Mechanisms can be divided into planar mechanisms and spatial mechanisms, according to the relative motion of the rigid bodies.

In a planar mechanisms, all of the relative motions of the rigid bodies are in one plane or in parallel planes. This implies that all the links of a planar mechanism are joined using prismatic and/or revolute joints. In spatial mechanisms, there is a relative motion that is not in the same plane or in parallel planes.

In other words, planar mechanisms are essentially two dimensional while spatial mechanisms are three dimensional.

1.1.7 Kinematic Analysis and Synthesis

In *kinematic analysis*, a particular given mechanism is investigated based on the mechanism geometry plus other known characteristics (such as input angular velocity, angular acceleration, etc.).

Kinematic synthesis, on the other hand, is the process of designing a mechanism to accomplish a desired task

1.1.8 Kinematic chain

A kinematic chain is an assemblage of links and joints, interconnected in a way to provide a controlled output motion in response to a supplied input motion. Such a chain becomes a *mechanism* when one of the links in the chain is fixed. The fixed link is called a frame or, sometimes, a *base link* (which itself may be in motion).
Kinematic chains or mechanisms may be either open or closed. A closed mechanism will have no open attachment points or nodes and may have one or more degrees of freedom (DOF). An open mechanism of more than one link will always have more than one degree of freedom, thus requiring as many actuators (motors) as it has DOF. A common example of an open mechanism is an industrial robot.

1.1.9 Four bar chain

A four bar (link) chain consists of four kinematic links of different lengths which are connected by turning type of lower pairs. The fixed link is called the frame, and the link which is not connected to frame is called coupler link. Among other two links hinged to the frame, one which receives input is termed as the crank, and the output link is called follower link

The four bar chain is the most common kinematic chain and has a wide engineering applications. Its skeleton outline is represented as shown in figure 1.2,

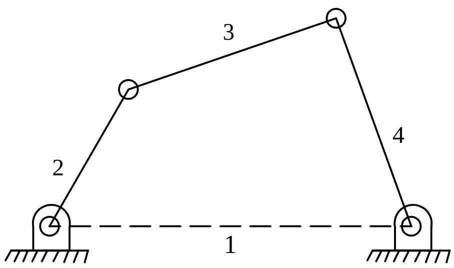


Figure 1.2: A four-bar kinematic chain

A variety of useful mechanisms can be formed from a four-link mechanism through slight variations, such as changing the character of the pairs, proportions of links, etc. The majority of four-link mechanisms fall into one of the following two classes, the typical four-bar linkage mechanism, the slider-crank mechanism.

Inversions of a typical four bar linkage

The process of choosing different links in the chain as frame is known as kinematic inversion. Inversions of the four bar linkage are shown in figure 1.3.

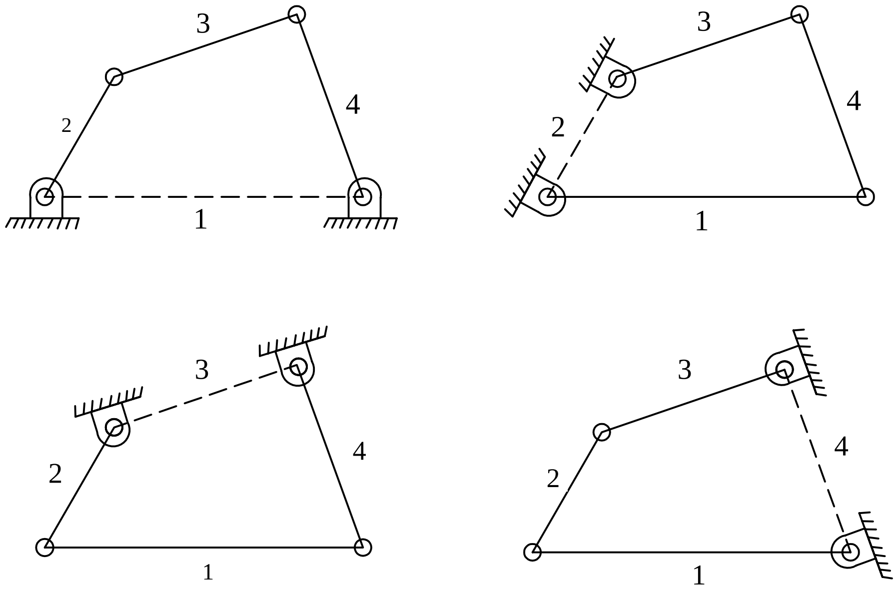


Figure 1.3: Inversions of a four-bar mechanism

Kinematically speaking all the four inversions above are identical. However by changing the proportions of lengths of the links several mechanisms with practical applications are obtained as illustrated below.

Classifications of a typical four bar linkage

In a four-bar linkage, we refer to the line segment between the joints on a given link as a bar. Let,

- s = length of shortest bar.
- l = length of longest bar.
- p, q = lengths of the other bars.

Grashof's theorem states that a four-bar mechanism has at least one revolving link if,

$$s + l \leq p + q$$

otherwise the input and output links will only oscillate. Therefore, four-bar linkage can be classified as,

(a) *Double-crank mechanism:* Both the input and output links make complete revolutions. There are two different forms of this mechanism,

   (i) *Parallel crank mechanism:* The length of the fixed link is equal to that of the coupler, and also links 2 and 4 are of equal length, as shown in figure 1.4.

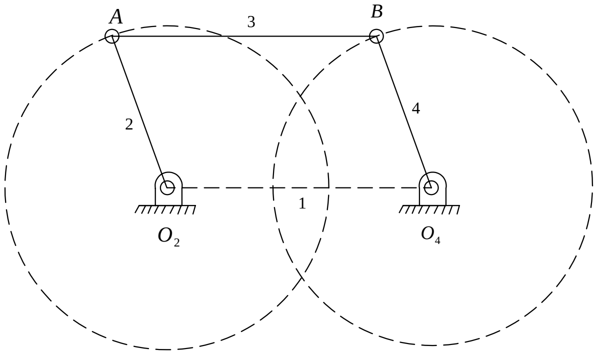


Figure 1.4: Parallel crank mechanism

   (ii) *Drag link mechanism:* This is shown in figure 1.5. It is such that $s + l \leq p + q$ and the fixed link is the shortest.

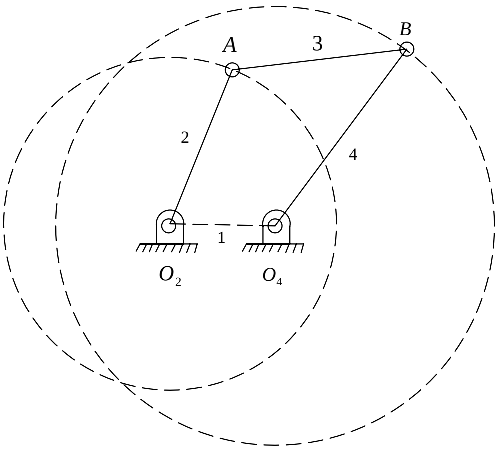


Figure 1.5: Drag-link mechanism

(b) *Double-rocker mechanism:* In this case both the links 2 and 4 can only oscillate. It is such that $s + l > p + q$, that is the Grashof's equation is not satisfied.

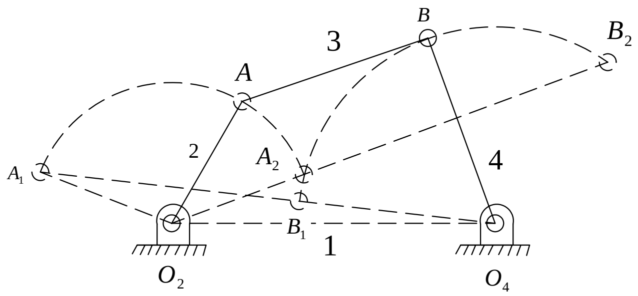


Figure 1.6: Double-rocker mechanism

(c) *Crank-rocker mechanism:* This mechanism is shown in figure 1.7. In this mechanism for every complete revolution of link 2 (crank link), the link 4 (lever link) makes a complete oscillation. It is such that $s + l \leq p + q$ and one of the side links is the shortest.

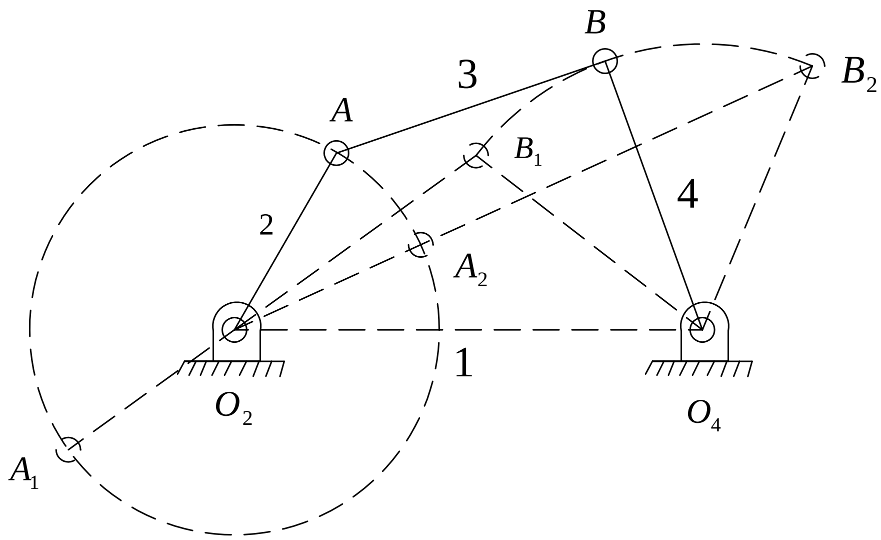


Figure 1.7: Crank-rocker mechanism

Assignment 1

With well-labeled diagrams, describe the inversions of a slider-crank linkage citing examples where each is applied in engineering mechanisms.

1.2 Degrees of Freedom of a rigid body

The concept of degree of freedom (DOF) is fundamental to both the synthesis and analysis of mechanisms. Degree of freedom (also called the mobility M) of a system can be defined as the number of inputs which need to be provided in order to create a predictable output.

1.2.1 Degree of freedom of a rigid body in plane

Consider link AB on a xy coordinate system as shown in figure 1.8 below;

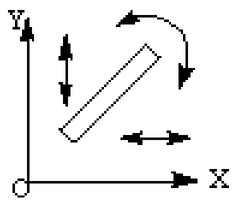


Figure 1.8: Degree of freedom of a rigid body in plane

To determine the dof of this body we must consider how many distinct ways the link can be moved. The link can be translated along the x axis, translated along the y axis, and rotated about the plane.

Therefore *a rigid body in plane has three degrees of freedom.*

1.2.2 Degree of freedom of a rigid body in space

Consider figure 1.9.

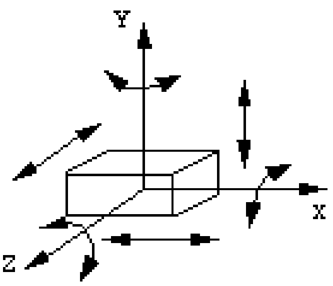


Figure 1.9: Degree of freedom of a rigid body in space

An unrestrained rigid body in space has six degrees of freedom: three translating motions along the x, y and z axes and three rotary motions around the x, y and z axes respectively.

Therefore *a rigid body in space has six degrees of freedom.*

1.2.3 Mobility Criteria

One of the basic steps in kinematic and dynamic analysis of mechanical system is to determine the number of degrees of freedom of the system.

Calculating the degrees of freedom of a rigid body system is straight forward. Any unconstrained rigid body has six degrees of freedom in space and three degrees of freedom in a plane. Adding kinematic constraints between rigid bodies will correspondingly decrease the degrees of freedom of the rigid body system. Mobility criteria is a simple method developed for determining the number of degrees of freedom of a mechanical system.

Mobility criteria for planar mechanisms

As previously seen in figure 1.8, any unconstrained rigid body has three degrees of freedom in a plane.

A planar system that consists of n_b unconstrained bodies therefore has $3n_b$ degrees of freedom. For example, consider two links in plane xy as shown in figure 1.10 below;

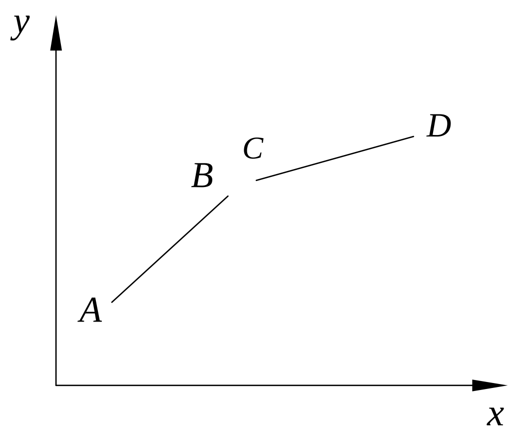


Figure 1.10: Degree of freedom of a rigid body in plane

Each body has 3 dofs wrt the plane. Therefore the total number of dofs of the system is $3 \times 2 = 6$.

If these bodies are connected by joints, then the number of the system's dof decreases. The reduction in the system's dof depends on the number of independent constraints introduced by the joint. Therefore, in planar motion the number of the system's dof can be determined according to the mobility criteria given in equation 1.1.

$$n_d = 3n_b - n_c \tag{1.1}$$

Where;

n_d number of the system's dof.

n_b number of the bodies in the system.

n_c total number of linearly independent constraint equations that describe the joints in the system.

Gruebler represented equation 1.1 in a simpler way as;

$$n_d = 3(n_b - 1) - 2n_l - n_h \tag{1.2}$$

Where;

n_d number of the system's dof.

n_b number of the bodies in the system.

n_l number of lower pairs

n_h number of higher pairs

Equation 1.3 is called Gruebler's equation, and it cant be applied to a mechanism with no fixed link, such as manipulators.

Examples 1.1.2

If the links in figure 1.10 are connected using a revolute joint and end A is connected to a stationary base by a revolute joint, determine the number of dofs of the resulting planar system.

Solution: The resulting system is as shown in figure 1.11

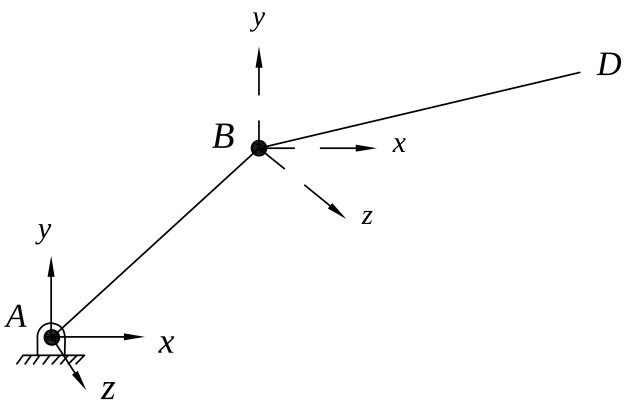


Figure 1.11: Example 1

There are two links,therefore $n_b = 2$.

There are two revolute joint each reducing the number of dofs by two, therefore $n_c = 2 + 2 = 4$.

Using equation 1.1 we get;

$$\begin{aligned} n_d &= 3 \times 2 - 4 \\ &= 2 \end{aligned}$$

The system therefore is a two dof system.

Examples 1.1.3

Determine the number of dofs of a slider-crank mechanism shown in figure 1.12 below;

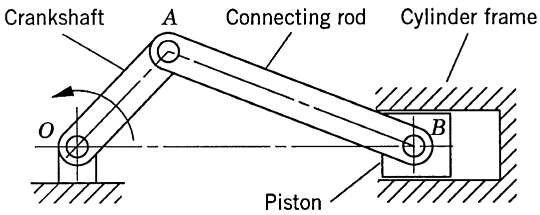


Figure 1.12: Example 2: Slider-crank mechanism

Solution: The mechanism consists of four bodies, that is crank (denoted as OA), connecting rod (denoted as AB), slider block (piston) and the ground (fixed link denoted as OB). Therefore $n_b = 4$

The system has three revolute joints at O, A and B. Each revolute joint reduces the number of dofs by 2, hence these joints reduce the dofs by $2 \times 3 = 6$.

The system has one prismatic joint between the slider block and the cylinder frame. Prismatic joints reduce the dofs by 2, hence this joint reduces the systems dof by $1 \times 2 = 2$.

The fixed joint constraints are 3 since in planar motion two conditions are required in order to eliminate the freedom of the body to translate, and one condition is required to eliminate the freedom of the body to rotate.

Therefore the the constraints reduce the number of system's dofs by $n_c = 6+2+3 = 11$. Using equation 1.1 we get;

$$\begin{aligned} n_d &= 3 \times 4 - 11 \\ &= 1 \end{aligned}$$

The system therefore is a single dof system.

Alternatively, we can use Gruebler's equation 1.3. The number of lower pairs is $n_l = 4$, and there are no higher pairs, hence $n_h = 0$. Therefore;

$$n_d = 3(4-1) - 2 \times 4 - 0 \ = \ 1$$

Mobility criteria for spatial mechanisms

In spatial kinematics, the configuration of a rigid body in space is identified using six coordinates, that is, three angular coordinates θ, ϕ, and ρ each about the xy, xz and yz planes respectively, as well as three linear coordinates x, y, z.

If the system consists of n_b bodies, the mobility criterion in the spatial analysis can be represented as shown in equation 1.3.

$$n_d = 6n_b - n_c \tag{1.3}$$

Examples 1.1.4

Determine the number of dofs of a RSSR spatial mechanism shown in figure 1.13 below;

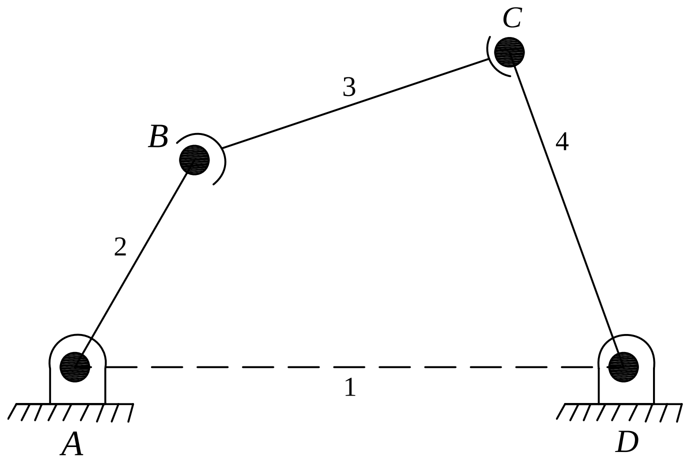


Figure 1.13: Example 3: RSSR mechanism

Solution: The mechanism consists of four bodies, that is link OA, link AB, link BC and the fixed link denoted as OC. Therefore $n_b = 4$

The system has two revolute joints at O and C. Each revolute joint reduces the number of dofs by 5, hence these joints reduce the dofs by $2 \times 5 = 10$.

The system has two spherical joints at A and B. Spherical joints reduce the dofs by 3, hence these joints reduce the system's dof by $2 \times 3 = 6$.

The fixed joint constraints are 6 since in spatial motion three conditions are required in order to eliminate the freedom of the body to translate along any axis, and three conditions are required to eliminate the freedom of the body to rotate about any plane.

Therefore the the constraints reduce the number of system's dofs by $n_c = 10+6+6 = 22$. Using equation 1.3 we get;

$$\begin{aligned} n_d &= 6 \times 4 - 22 \\ &= 2 \end{aligned}$$

The system therefore is a two dof system.

Assignment 2

Determine the number of degrees of freedom of the mechanisms shown shown below;

(a) Damping truck:

(b) Six-bar linkage:

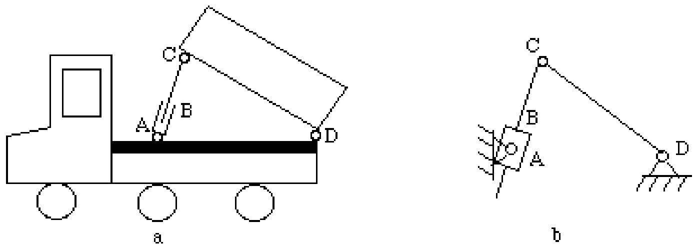


Figure 1.14: Damping truck

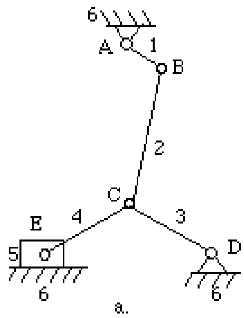


Figure 1.15: Six-bar linkage

(c) Door transom:

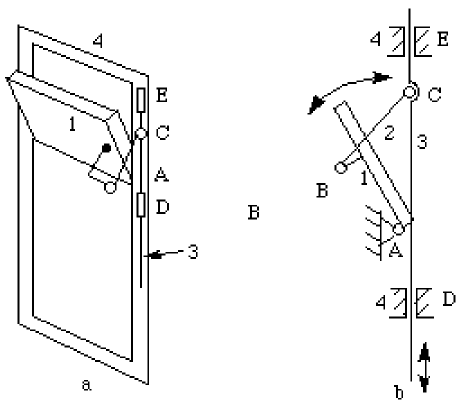


Figure 1.16: Door transom

(d) Fifteen-bar linkage

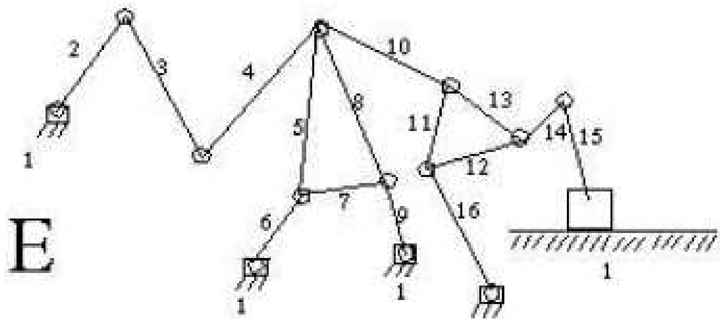


Figure 1.17: Fifteen-bar linkage

Chapter 2

Position/Displacement Analysis

2.1 Definitions

Position

The position of a point in the plane can be defined by the use of a position vector as shown in figure 2.1.

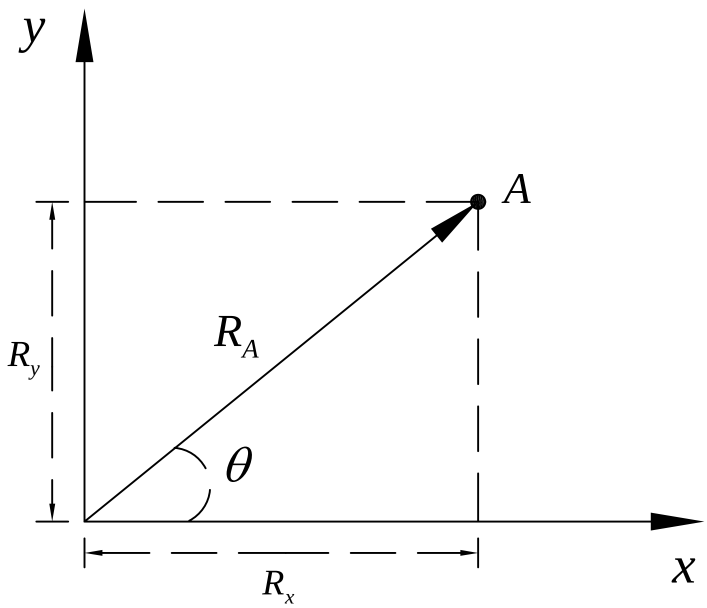


Figure 2.1: The position vector of a point

A 2-D vector has two attributes which can be expressed in either polar or cartesian coordinates. The polar form provides the magnitude and the angle of the vector which is measured from the root of the vector, that is, $|R_A|@\angle\theta$

The cartesian form provides the x and y linear coordinates, that is (R_x, R_y).

Displacement

Displacement of a point is the change in its position and can be defined as the straight line distances between the initial and final positions of a point which has moved in a reference frame.

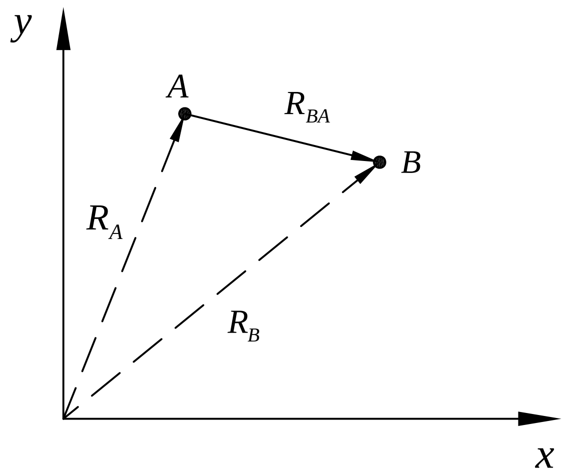


Figure 2.2: The displacement of a point

The position vector R_{BA} defines the displacement of the point B wrt point A and can be expressed in position difference equation as;

$$R_{BA} = R_B - R_A \tag{2.1}$$

Which is read as; *the position of B wrt point A is equal to the absolute position of B minus the absolute position of A, where absolute means wrt the origin of the reference frame.*

If points A and B represent not the same particle but two independent particles moving in the same reference frame, the vector equation 2.1 is still valid, but we now refer to R_{BA} as a relative position, that is, the position of point B relative to point A.

2.2 Position analysis

The position analysis of a kinematic chain requires the determination of the joint positions or the position of the center of gravity of the links. Then there are 3 approaches in position analysis.

i Graphical

ii Absolute cartesian method

iii Vector loop method (Loop closure equation method)

2.2.1 Graphical Analysis

For any 1-dof mechanism, only one parameter is needed to completely define the position of all the links.

Consider a four-bar linkage shown in figure 2.3 below; Only the angle of input link θ_2 is

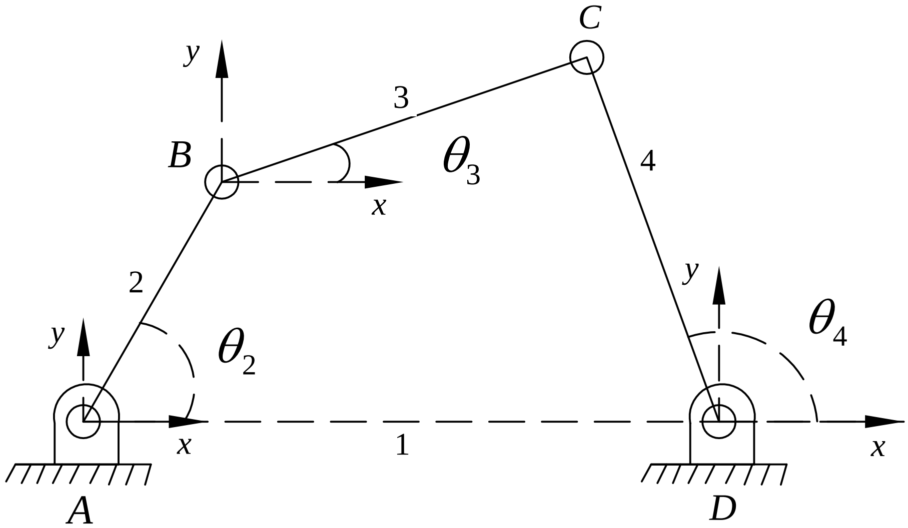


Figure 2.3: Measurement of angles in a four-bar linkage

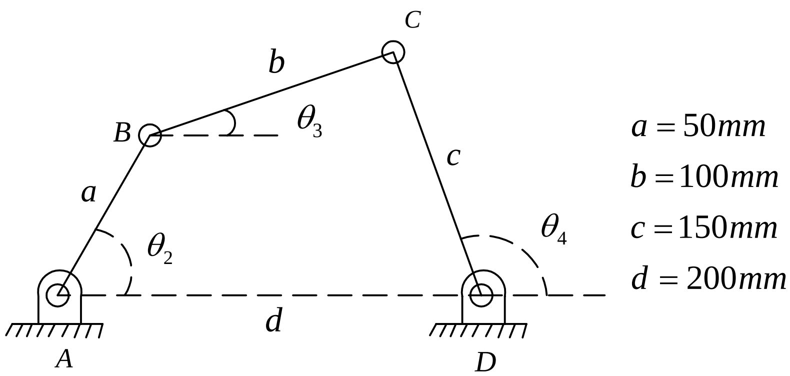
needed to completely define the position of all links.

If we draw the linkage carefully to scale in a particular position of θ_2, then its only necessary to measure the angles of link 3 (θ_3) and of link 4 (θ_4) with a protractor.

The graphical analysis of a mechanism is trivial: that is, the obtained solution is only valid for the particular position of chosen θ_2, and for another additional position analysis we must completely redraw the linkage.

Examples 2.1

For a four-bar linkage shown in figure 2.2.1 below, determine graphically θ_3 and θ_4 if $\theta_2 = 60^0$.



Solution:

Step 1 Choose an arbitrary point A.

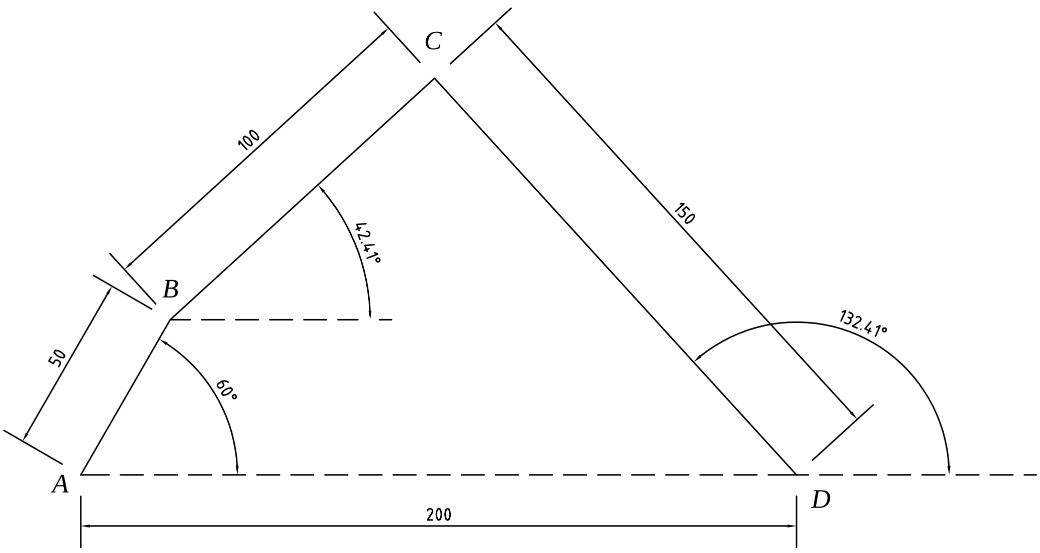
Step 2 Locate point D by drawing a line of length d.

Step 3 Locate point B by drawing a line of length a at an angle of θ_2 from AD.

Step 4 Locate point C which is the point of intersection of arcs of radius equal to b and c drawn from points B and D respectively.

Step 5 Measure angles θ_3 and θ_4.

These solutions are shown in figure 2.2.1 below.



2.2.2 Absolute cartesian method

Example 1

In the planar mechanisms shown, the driver link is the rigid link AB AB=0.15M, BC=0.4M, CD=0.37, CE=0.23M, EF=CE=L_a=0.3M, L_b=0.45M, L_c=CD. At the instant shown ϕ=45^o. Determine the position of the joints.

Position of Joint A

$$x_A = 0, \quad y_A = 0$$

Position of Joint B

$$\begin{aligned} x_A &= AB\cos\phi = 0.106 \\ y_B &= AB\sin\phi = 0.106 \end{aligned}$$

Position of Joint C

Since the lengths of the links BC and CD are constants.

$$\begin{aligned} (x_C - x_B)^2 + (y_C - y_B)^2 &= \overline{BC}^2 \\ (x_C - x_D)^2 + (y_C - y_D)^2 &= \overline{CD}^2 \end{aligned}$$

Solving the system of quadratic equation we have

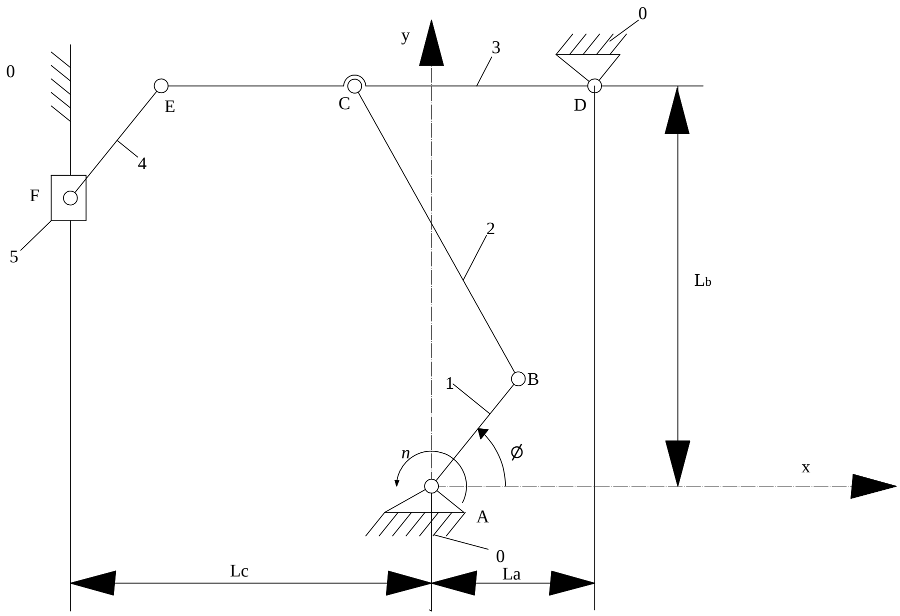


Figure 2.4: Absolute Cartesian method

$$x_{C_1} = -0.069M \quad \text{and} \quad y_{C_1} = 0.465M$$
$$x_{C_2} = 0.504M \quad \text{and} \quad y_{C_2} = 0.141M$$

From the mechanism constraints

$$\begin{aligned} x_C &< x_D \\ x_C &= -0.069M \\ y_C &= 0.465M \end{aligned}$$

<u>Position of Joint E</u>

$$(x_E - x_C)^2 + (y_E - y_C)^2 = \overline{CE}^2 \tag{2.2}$$

Joints E, C and D are located on the same straight line

$$y_D = mx_D + n \tag{2.3}$$
$$y_C = mx_C + n \tag{2.4}$$
$$y_E = mx_E + n \tag{2.5}$$

m is the gradient and n is the y-intercept
Eq 4 and Eq 2.5 gives the value of m and n as

$$\begin{aligned} m &= -0.041 \\ n &= 0.462 \end{aligned}$$

Now solving Eq 2.2 and 2.5

$$\begin{aligned} x_{E_1} &= -0.299M \quad y_{E_1} = 0.474M \\ \text{or} x_{E_2} &= 0.160M \quad y_{E_2} = 0.455M \end{aligned}$$

From mechanism constraint

$$x_E < x_C$$

From which

$$x_E = -0.299M \quad \text{and} \quad y_E = 0.474M$$

<u>Position of Joint F</u>
F is restricted to move in vertical position

$$\begin{aligned} x_F = -L_C &= -0.370M \\ (x_F - x_E)^2 + (y_F - y_E)^2 &= \overline{FC}^2 \\ y_{F_1} &= 0.256M \\ y_{F_2} &= 0.693M \end{aligned}$$

From which

$$\begin{aligned} x_F &= -0.370M \\ y_F &= 0.256M \end{aligned}$$

2.2.3 Loop-closure equation method

Consider a four-bar linkage whose links are drawn as position vectors which form a vector loop as shown in figure 2.5

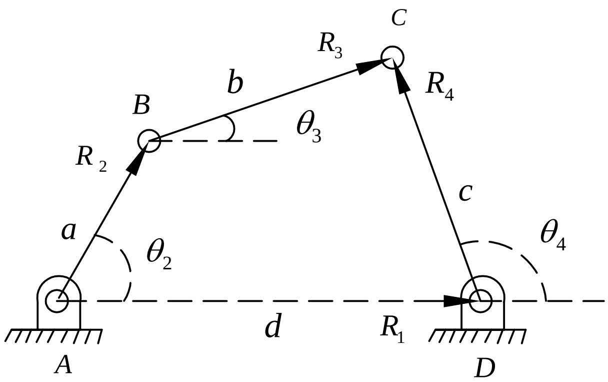


Figure 2.5: Position vector loop for a four-bar linkage

The loop closes on itself, and hence the sum of the vectors in the loop must be equal to zero. The length of the vectors are the link lengths and therefore are known.

The particular linkage position shown is defined by one input angle θ_2, and we want to solve analytical for the unknown angles θ_3 and θ_4.

Note that the directions of the position vectors are chosen so as to define their angles where we want them to be measured. By definition, the angle of a vector ia always measured at its root not its head.

Therefore;

$$R_2 + R_3 - R_4 - R_1 = 0 \tag{2.6}$$

Substituting the complex number notation for each position vector in equation 2.6, we get;

$$ae^{i\theta_2} + be^{i\theta_3} - ce^{i\theta_2} - de^{i\theta_1} = 0 \tag{2.7}$$

The link lengths a, b, c and d are known and constant. The value of the angle of link 1 is fixed to zero,i.e, $\theta_1 = 0$, and θ_2 is the independent variable which is the input. Therefore we are left with θ_3 and θ_4 to be found.

Substituting the Euler-identity for the $e^{i\theta}$ term, i.e., $e^{\pm\theta} = \cos\theta \pm i\sin\theta$ in the vector equation 2.7 and noting that $\theta_1 = 0$, we get;

$$\begin{aligned} 0 &= a(\cos\theta_2 + i\sin\theta_2) + b(\cos\theta_3 + i\sin\theta_3) - c(\cos\theta_4 + i\sin\theta_4) \\ &\quad -d(\cos\theta_1 + i\sin\theta_1) \qquad (2.8) \\ &= a(\cos\theta_2 + i\sin\theta_2) + b(\cos\theta_3 + i\sin\theta_3) - c(\cos\theta_4 + i\sin\theta_4) - d \qquad (2.9) \end{aligned}$$

Equation 2.9 can be split into its imaginary and real parts, and each equated to zero.

Real part;

$$0 = a\cos\theta_2 + b\cos\theta_3 - c\cos\theta_4 - d \qquad (2.10)$$

Imaginary part;

$$\begin{aligned} 0 &= i(a\sin\theta_2 + b\sin\theta_3 - c\sin\theta_4) \\ &= a\sin\theta_2 + b\sin\theta_3 - c\sin\theta_4 \qquad (2.11) \end{aligned}$$

The scalar equations 2.10 and 2.11 can be solved simultaneously for θ_3 and θ_4. Lets first eliminate θ_3 to solve for θ_4 by rewriting equations 2.10 and 2.11 as follows;

$$\begin{aligned} b\cos\theta_3 &= -a\cos\theta_2 + c\cos\theta_4 + d \qquad (2.12) \\ b\sin\theta_3 &= -a\sin\theta_2 + c\sin\theta_4 \qquad (2.13) \end{aligned}$$

Square equations 2.12 and 2.13 and add them.

$$b^2(\cos^2\theta_3 + \sin^2\theta_3) = (-a\cos\theta_2 + c\cos\theta_4 + d)^2 + (-a\sin\theta_2 + c\sin\theta_4)^2 \qquad (2.14)$$

Expand the right hand side of equation 2.14.

$$\begin{aligned} b^2 &= a^2\sin^2\theta_2 + c^2\sin^2\theta_4 - 2ac\sin\theta_2\sin\theta_4 + a^2\cos^2\theta_2 + c^2\cos^2\theta_4 + d^2 - 2ac\cos\theta_2\cos\theta_4 \\ &\quad -2ad\cos\theta_2 + 2cd\cos\theta_4 \\ &= a^2 + c^2 + d^2 - 2ac(sin\theta_2\sin\theta_4 + \cos\theta_2\cos\theta_4) - 2ad\cos\theta_2 + 2cd\cos\theta_4 \qquad (2.15) \end{aligned}$$

Equation 2.15 can be rearranged as;

$$\frac{a^2 - b^2 + c^2 + d^2}{2ac} - \frac{d}{c}\cos\theta_2 + \frac{d}{a}\cos\theta_4 = sin\theta_2\sin\theta_4 + \cos\theta_2\cos\theta_4 \qquad (2.16)$$

To further simplify the equation let;

$K_1 = \frac{d}{a}$

$K_2 = \frac{d}{c}$

$K_2 = \frac{a^2-b^2+c^2+d^2}{2ac}$

Which when substituted in 2.16 we get;

$$K_1\cos\theta_4 - K_2\cos\theta_2 + K_3 = sin\theta_2\sin\theta_4 + \cos\theta_2\cos\theta_4 \qquad (2.17)$$

To reduce equation 2.17 to a more tractable form for solution, it is necessary to substitute the half angle identities, which will convert the $\sin\theta_4$ and $\cos\theta_4$ to $\tan\theta_4$.

$$\begin{aligned}\cos\theta_4 &= \frac{1-\tan^2\frac{\theta_4}{2}}{1+\tan^2\frac{\theta_4}{2}} \\ \sin\theta_4 &= \frac{2\tan\frac{\theta_4}{2}}{1+\tan^2\frac{\theta_4}{2}}\end{aligned}$$

Substituting the identities in equation 2.17 and simplifying, we get an equation of quadratic form.

$$A\tan^2\frac{\theta_4}{2} + B\tan\frac{\theta_4}{2} + C = 0 \tag{2.18}$$

Where;

$A = (1-K_2)cos\theta_2 - K_1 + K_3$

$B = -2sin\theta_2$

$C = K_1 - (1+K_2)cos\theta_2 + K_3$

Solving equation 2.18 we get;

$$\theta_{4_{1,2}} = 2\tan^{-1}\frac{-B\pm\sqrt{B^2-4AC}}{2A} \tag{2.19}$$

Equation 2.18 has two solutions of which as with any quadratic equation may be of three types: real and equal, real and unequal or complex conjugate.

If the solution is complex conjugate then this implies that the link lengths chosen are not capable of connection for the value of the input angle θ_2.

For real and unequal solutions then it means that there are two values of θ_4 corresponding to any one value of θ_2. These are referred to as the crossed and open configuration of the linkage. For a four-bar linkage, the $-\theta_4$ solution gives open configuration while the $+\theta_4$ gives the crossed configuration.

Assignment 2

Starting from a vector-loop closure equation of a four bar linkage shown in figure 2.5, show that;

$$K_1\cos\theta_3 + K_4\cos\theta_2 + K_5 = \cos\theta_2\cos\theta_3 + \sin\theta_2\sin\theta_3 \tag{2.20}$$

Where;

$K_1 = \frac{d}{a}$

$K_4 = \frac{d}{b}$

$K_5 = \frac{c^2-d^2-a^2-b^2}{2ab}$

Also show that equation 2.20 above reduces to a quadratic equation of form;

$$D \tan^2 \frac{\theta_3}{2} + E \tan \frac{\theta_3}{2} + F = 0 \tag{2.21}$$

Where;

$$D = (1 + K_4) \cos \theta_2 - K_1 + K_5$$

$$E = -2 \sin \theta_2$$

$$F = K_1 + (K_4 - 1) \cos \theta_2 + K_5$$

Loop-closure equation of an offset four bar slider crank linkage

Consider an offset four bar slider-crank linkage shown in figure 2.6 below;

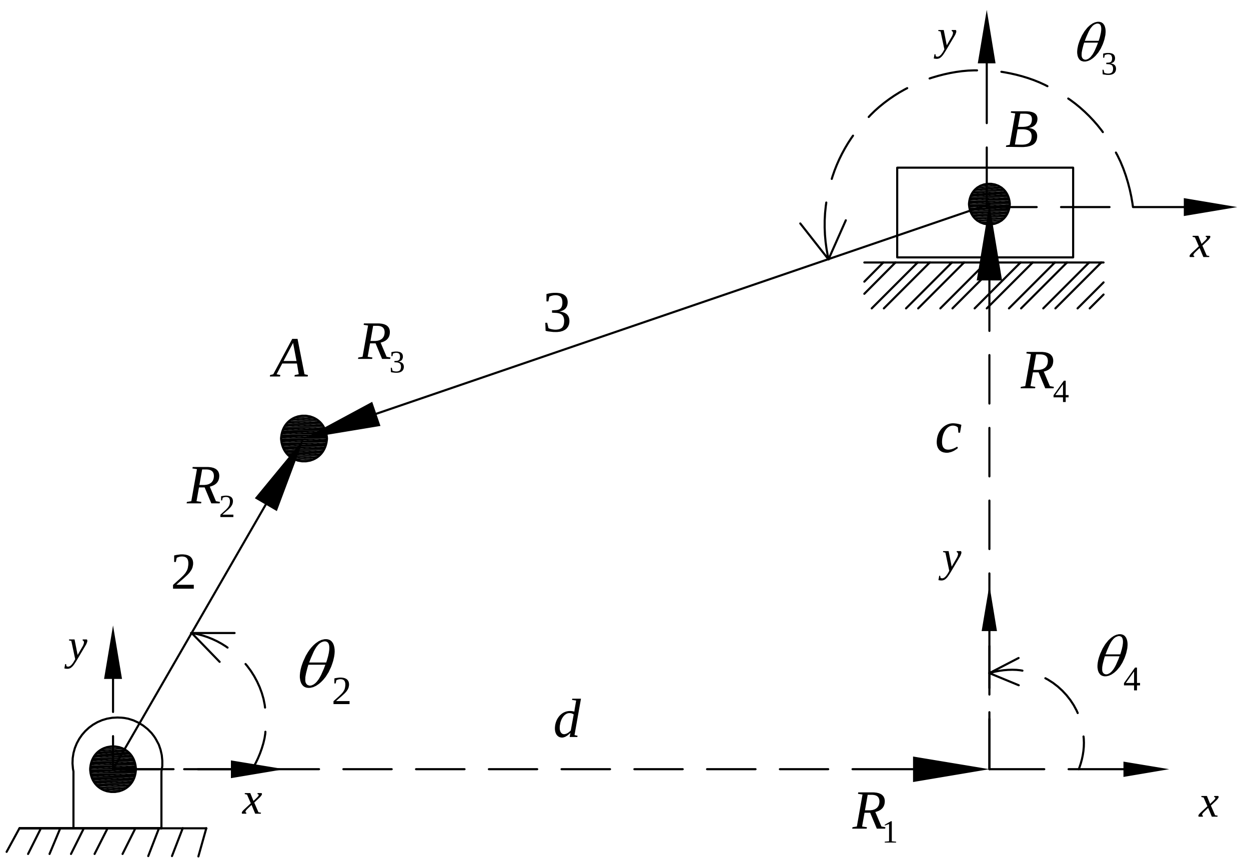


Figure 2.6: Offset slider-crank equation

The term offset means that the slider axis extended does not pass through the crank pivot.

For a non offset slider crank mechanism, the linkage is represented by only three position vectors: R_2, R_3 and $R_1 = R_s$. R_1 only varies in magnitude only.

For an offset slider-crank mechanism it is easier to use four position vectors R_1, R_2, R_3 and R_4. R_1 which is a variable length, constant direction vector represents the slider position with magnitude d, and R_4 which is perpendicular to R_1 defines the constant magnitude offset of the slider.

The position vector arrangement gives a loop equation as;

$$R_2 - R_3 - R_4 - R_1 = 0 \tag{2.22}$$

Substitute the complex number equivalence for the position vectors to equation 2.22;

$$ae^{i\theta_2} - be^{i\theta_3} - ce^{i\theta_4} - de^{i\theta_1} = 0 \tag{2.23}$$

Substitute the Euler equivalents to equation 2.23;

$$a(\cos\theta_2 + i\sin\theta_2) - b(\cos\theta_3 + i\sin\theta_3) - c(\cos\theta_4 + i\sin\theta_4) - d(\cos\theta_1 + i\sin\theta_1) = 0 \tag{2.24}$$

Noting that $\theta_1 = 0^0$ and $\theta_4 = 90^0$, equation 2.24 becomes;

$$a(\cos\theta_2 + i\sin\theta_2) - b(\cos\theta_3 + i\sin\theta_3) - ci - d = 0 \tag{2.25}$$

Separating real and imaginary components of equation 2.25;

$$0 = a\cos\theta_2 - b\cos\theta_3 - d \tag{2.26}$$
$$0 = a\sin\theta_2 - b\sin\theta_3 - c \tag{2.27}$$

From equation 2.27,

$$\theta_3 = \sin^{-1}\left[\frac{a\sin\theta_2 - c}{b}\right] \tag{2.28}$$

Once θ_3 is calculated, we use equation 2.26 to determine d.

$$d = a\cos\theta_2 - b\cos\theta_3 \tag{2.29}$$

NOTE: There are also two valid solutions corresponding to the two branches of the linkage.

Chapter 3

Velocity Analysis

Once a position analysis is done, the next step is to determine the velocities of all links and points in a mechanism, which can be found using graphical or analytical methods.

3.1 Definition of velocity

Velocity is defined as the rate of change of position wrt time. Velocity can be angular (ω) or linear (V), which are defined as;

$$\omega = \frac{d\theta}{dt} \tag{3.1}$$

and

$$V = \frac{dR}{dt} \tag{3.2}$$

Consider link PA pivoted at point A and in pure rotation in the xy plane.

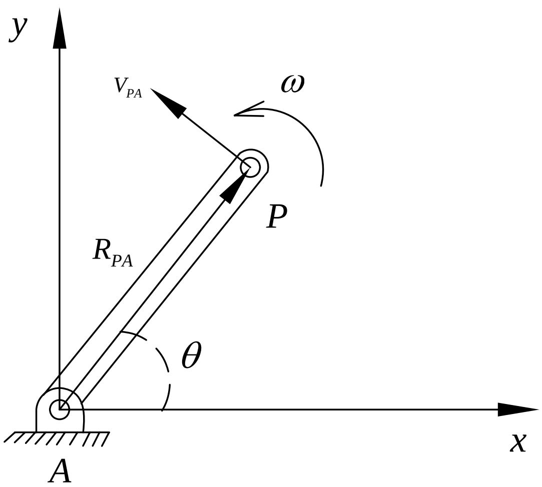


Figure 3.1: A link in pure rotation

We are interested with the velocity of point P when the link if subjected to an angular velocity ω. R_{PA} can be represented as a complex number in polar form as;

$$R_{PA} = pe^{i\theta} \tag{3.3}$$

which when differentiated wrt to time we get;

$$\dot{R}_{PA} = \frac{dR_{PA}}{dt} = pie^{i\theta}\frac{d\theta}{dt} = i\omega pe^{i\theta} = i\omega R_{PA} \tag{3.4}$$

Equation 3.4 shows that the velocity vector is rotated at 90^0 from the angle θ of the position vector. This is just a mathematical verification of what is already known, namely: velocity is always in a direction perpendicular to the radius of rotation and is tangential to the path of rotation.

Velocity V_{PA} in figure 3.1 above can be referred to as an absolute velocity denoted simply as V_p, since it is referenced to point A which is stationary and is the origin of the coordinate system.

3.1.1 Relative velocity

Consider a figure 3.2 in which link AP is in pure rotation and is joined to a block which is in pure translation.

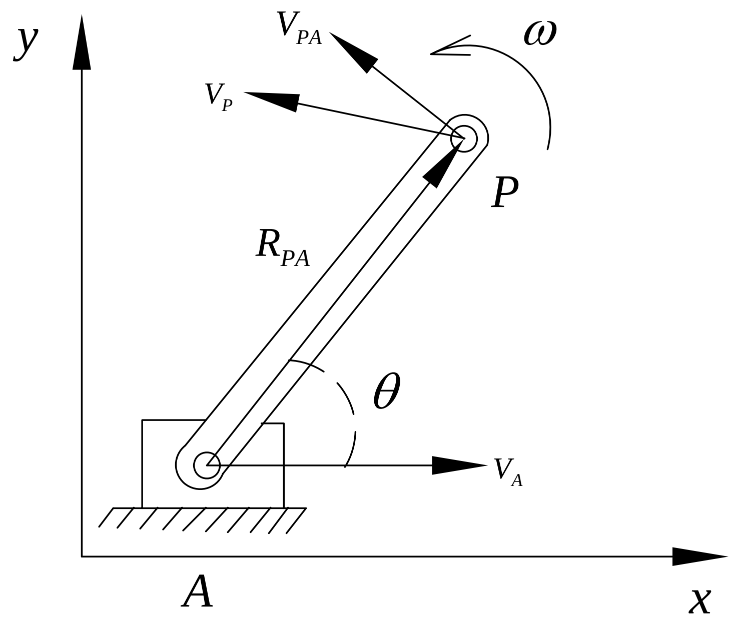


Figure 3.2: Velocity difference

V_A is the linear velocity of the slider block and is known. V_{PA} is the velocity of point P relative to point A, and since A is moving, then V_{PA} cannot be considered as an absolute velocity.

V_{PA} is represented using a velocity difference equation as;

$$V_{PA} = V_P - V_A \tag{3.5}$$

where V_P is the absolute velocity of point P given from equation above as;

$$V_P = V_A + V_P = V_A + \overline{AP}\omega \tag{3.6}$$

3.2 Graphical methods in velocity analysis of linkages

To solve a velocity problem graphically, we need to know the lengths of all the links, the angular positions of all the links and the instantaneous input velocity of any one driving link or point.

We must first do a complete position analysis on the linkage using any method discussed in section 2, but in the preceding analysis we assume that position analysis has been done and that the input angular velocity of the driving link is known.

The graphical methods include;

1. Use of velocity diagrams.
2. Use of instant centers

3.2.1 Velocity diagrams

To illustrate how to use velocity diagram method in velocity analysis problems of linkages, lets consider a four bar linkage shown in figure below.

Steps;

1. Start at the end of the linkage where there is most velocity information. For this example at point B since the magnitude and direction of V_B are known, that is $V_A = \overline{AB}\omega_2$ at a direction perpendicular to link AB.
2. Draw the velocity vector of magnitude equal to V_B at a convenient point, say point a as shown in figure 3.3, and its direction perpendicular to link AB.
3. Move next to a point about which we have some velocity information. For our example, point C since the direction of velocity of point C (V_C) is predictable, because link DC is in pure rotation about point D.
4. Draw a vector V_C which is perpendicular to link DC and rooted at point d which is chosen at a point coincident to point a since point D is fixed. Point a, d is a point of no velocity.
5. Use equation 3.5 to write the velocity difference equation for point C relative to point B in link BC, that is, $V_{CB} = V_C - V_B$, which gives a closed vector equation of form, $V_C = V_B + V_{CB}$.

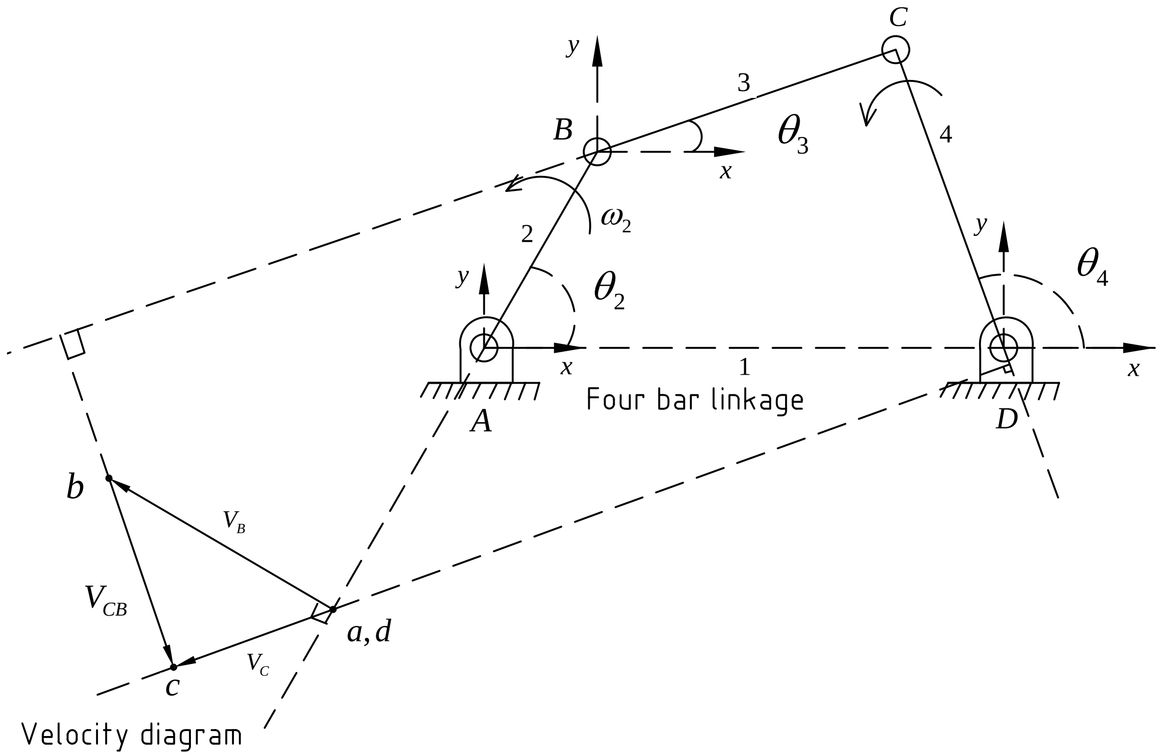


Figure 3.3: Drawing a velocity diagram of a four bar linkage

6. Assuming that link BC is rigid, then there can be no component of V_{CB} directed along link CB since point C cant move towards or away from point B without stretching or shrinking the link. Therefore the direction of V_{CB} must be perpendicular to link BC and rooted at point b as shown in figure 3.3.
7. Where the lines representing vectors V_C and V_{CB} intersect is the point c.
8. From the velocity diagram obtained determine the velocities of the points and links desired,

For link BC

$$\begin{aligned} V_{BC} &= \overline{bc} \\ \omega_3 &= \frac{V_{BC}}{\overline{BC}} \end{aligned}$$

For link DC

$$\begin{aligned} V_C &= \overline{dc} \\ \omega_4 &= \frac{V_C}{\overline{DC}} \end{aligned}$$

Examples 3.1

For a four bar linkage shown in figure 3.4 find using the velocity diagram method the angular velocity of link CB and the absolute velocity of point D in link AB, given that:

$\overline{AD} = \overline{DB} = 0.45m$, $\overline{OA} = 0.5m$, $\overline{PB} = 0.65m$, $\overline{OC} = 1.4m$ and crank OA rotates clockwise with angular velocity of $2.6rad/s$

Solution: Absolute velocity of point A;

$$V_A = \omega_{OA}\overline{OA} = 2.6 \times 0.5 = 1.3m/s$$

V_A is drawn perpendicular to link OA, rooted at a convenient point, say $'o'$ and at a direction dictated to the direction of ω_{OA}. Only the direction of absolute velocity of point B is predictable, that is, draw vector V_B from point, say $'c'$ coincident to point $'o'$ and in a direction perpendicular to link CB. Vector V_{BA} is drawn perpendicular to link AB and rooted at point $'a'$. ADB will produce a geometrically similar velocity image $'adb'$. From the velocity diagram shown in figure 3.4;

Angular velocity of link BC is given by

$$\omega_{BC} = \frac{V_{BC}}{\overline{BC}} = \frac{bc}{\overline{BC}} = \frac{1.02}{0.65} = 1.57rad/s\, clockwise$$

Absolute velocity of point D is given by;

$$V_D = \overline{od} = 1.04m/s \tag{3.7}$$

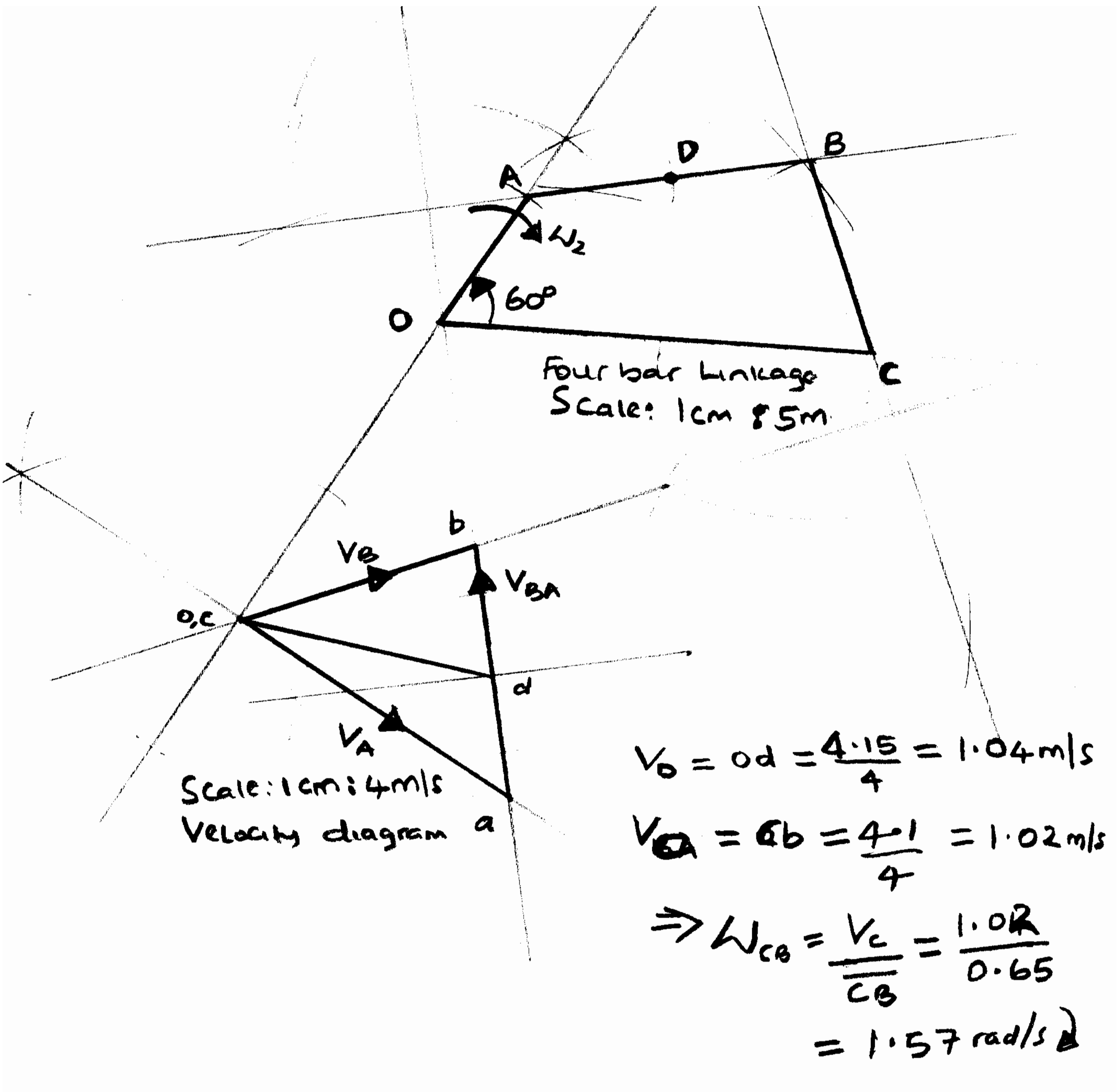


Figure 3.4: Solution to example 1

Examples 3.2

A slider crank mechanism has a crank radius of $160mm$ and a connecting link $500mm$ ling. The crank rotates at a steady speed of $4500RPM$. Using the velocity diagram method find the velocity of a point on the connecting link $360mm$ from the slider, when the crank has turned through an angle of 40^0 as shown in figure 3.5

Solution:

(a) $V_B = 0.16 \times 4500 \times \frac{2\pi}{60} = 75.4m/s$ perpendicular to OB. Draw **ob**

(b) Because of the constraints V_A, that is V_{AO} must be horizontal.

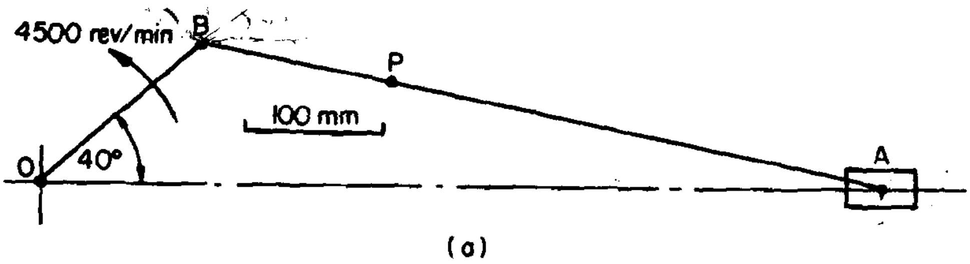


Figure 3.5: Example 2 problem

(c) Draw V_{AB} perpendicular to AB rooted at **b** and V_A parallel to OA rooted at **o** so as to locate point **a** at the point of intersection in the velocity diagram.

(d) APB is geometrically similar to **apb** hence point **p** can be located in the velocity diagram.

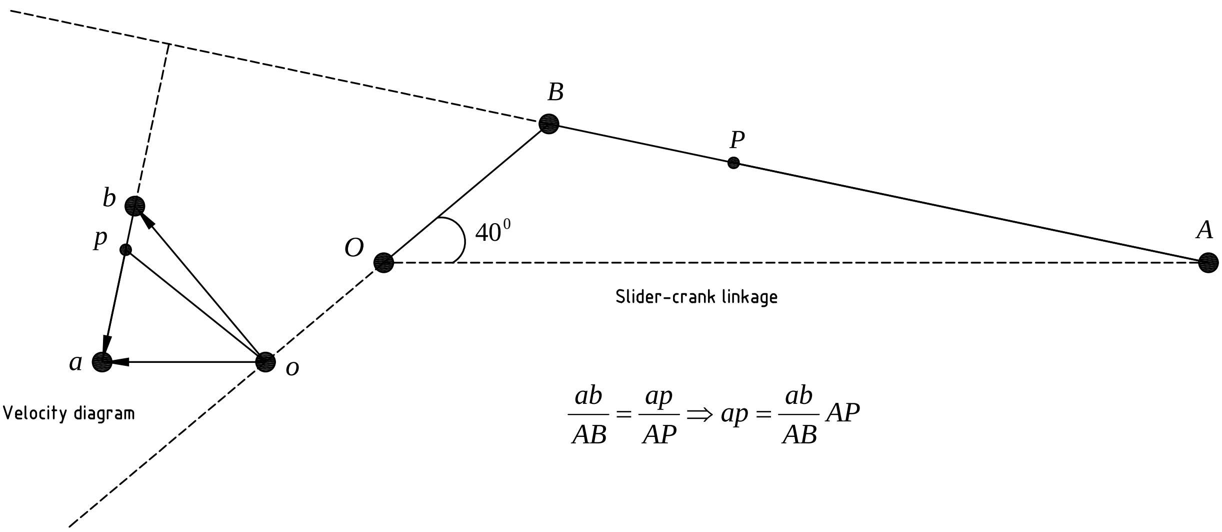


Figure 3.6: Example 2 solution

From the completed velocity diagram in figure 3.6,

The angular velocity of connecting link AB is given as;

$$\omega_{AB} = \frac{V_{AB}}{\overline{AB}} = \frac{\overline{ab}}{\overline{AB}} = \frac{59}{0.5} = 118 rad/s clockwise \tag{3.8}$$

The absolute velocity of point P is given as;

$$V_P = \overline{op} = 66.48 m/s \tag{3.9}$$

Examples 3.3

In the mechanism shown in figure 3.7, D is the slider at the end of the link rotating about center P and C is a fixed point on AB coincident with D at the instant shown. $OA = 0.6m$, $AB = 1.9m$, $BQ = 1.0m$ and $PD = 1.4$. If the crank OA rotates anticlockwise at $120RPM$, determine the angular velocity of member PD, and the speed of sliding at D.

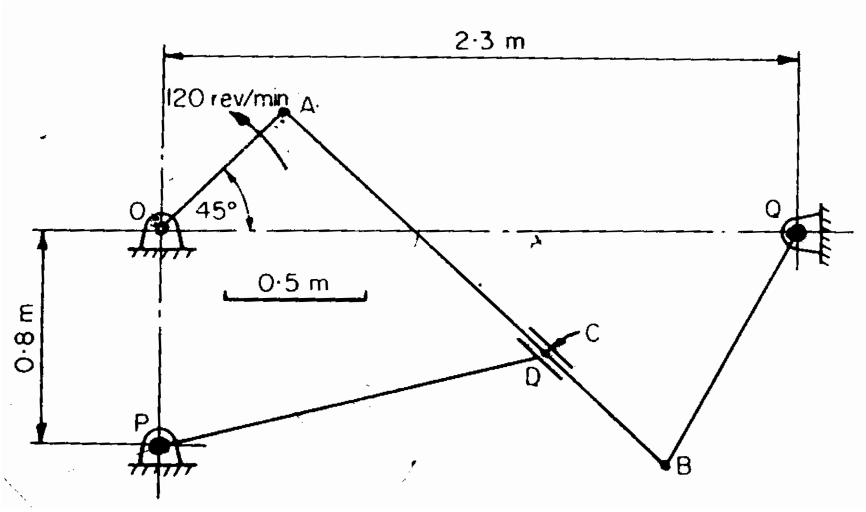


Figure 3.7: Example 3 problem

Solution:

(a) $V_A = 0.6 \times 120 \times \frac{2\pi}{60} = 7.54m/s$ perpendicular to OA. Draw **oa**

(b) Since O, P and Q are fixed points, then **o**, **p** and **q** will be coincident on the velocity diagram.

(c) Draw V_{BQ} perpendicular to BQ rooted at **q** and V_{BA} perpendicular to BA rooted at **a** so as to locate point **b** at the point of intersection in the velocity diagram.

(d) ACB is geometrically similar to **acb** hence point **c** can be located in the velocity diagram.

(e) Draw V_{DP} perpendicular to DP and V_{DC} parallel to AB to locate point **d** in the velocity diagram.

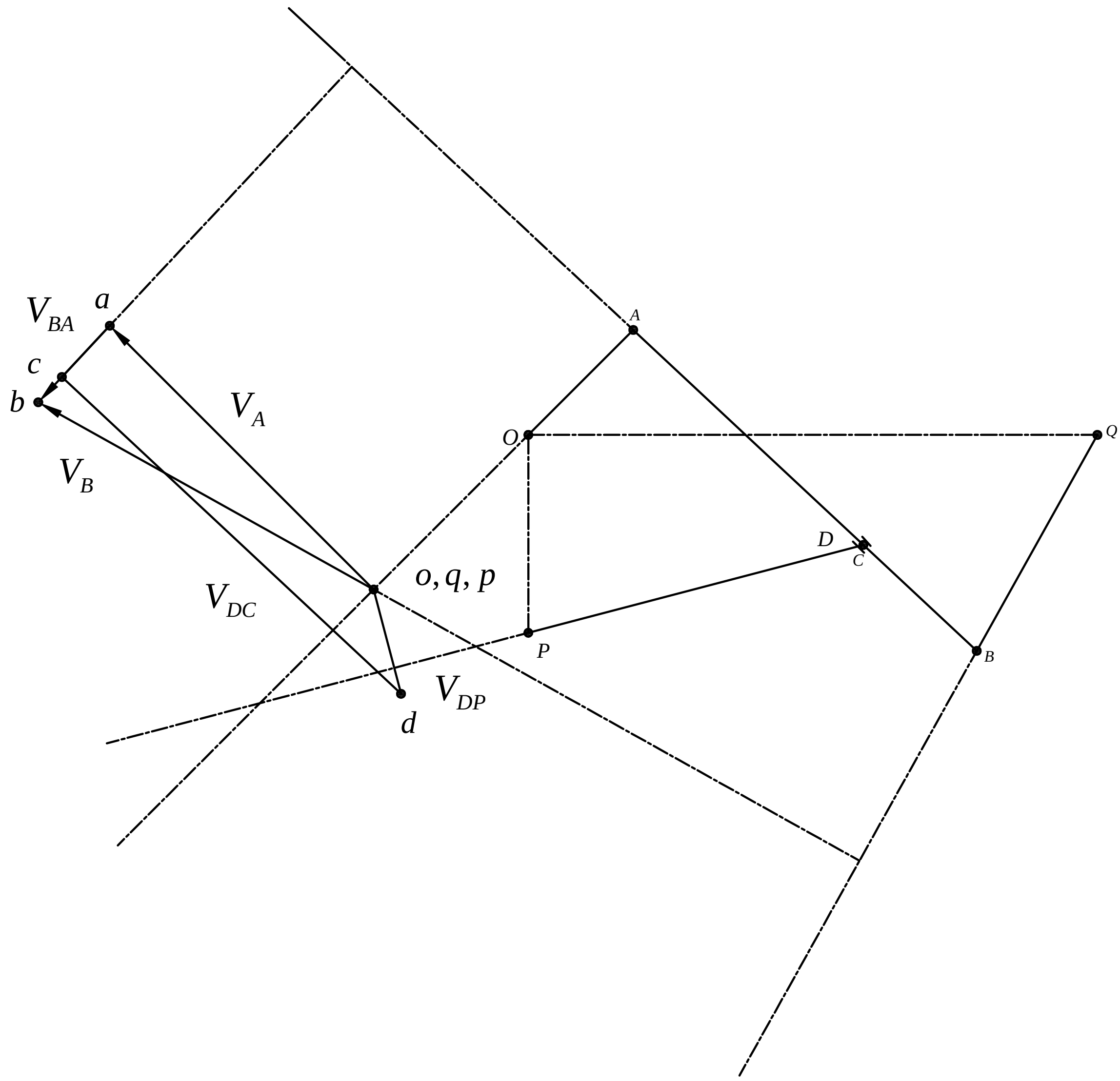


Figure 3.8: Example 3 solution

From the completed velocity diagram,

The angular velocity of link PD is given as;

$$\omega_{PD} \;=\; \frac{V_{DP}}{\overline{PD}} = \frac{\overline{pd}}{\overline{PD}} = \frac{2.2}{1.4} = 1.57 rad/s clockwise \tag{3.10}$$

The sliding velocity at D is given as;

$$V_{DC} \;=\; \overline{cd} = 9.37 m/s \tag{3.11}$$

3.2.2 Use of Instant centers of velocity method

An instant center is defined as, *A point common to two bodies and which has the same linear velocity in both magnitude and direction.*

Any two links in a mechanism have motion relative to one another and thus have a common instant center. Therefore the number of instant centers for a mechanism is equal to all the possible combinations of two links from the total number of links. Let n be the number of links in a mechanism, then the number of instant centers is given by;

$$N_{I_C} = \frac{n(n-1)}{2} \tag{3.12}$$

All instant centers which can be found merely by inspection are called primary instant centers. The remaining instant centers in the mechanism are found by applying **Arnold-Kennedy's theorem** which states that, *Any three bodies whether connected or not, in plane motion relative to one another, have exactly three instant centers which lie in the same straight line.*

Primary instant centers can be summarized as,

- Instant center for a revolute joint - two links connected by a revolute joint have an instant center at the joint.
- Instant center for a prismatic joint - Two links connected with a prismatic joint have an instant center at an infinite distance along a line perpendicular to the point of contact.
- Instant center for a rolling body - If two bodies have a rolling contact, then their instant center lies at the point of contact.

Examples 3.4

Locate all the instant centers for the four bar linkage shown in figure 3.9.

Solution:

1. By inspection, determine all the primary centers, which are I_{12}, I_{14}, I_{23} and I_{34}. According to equation 3.12 there are $N_{I_C} = \frac{4(4-1)}{2} = 6$ instant centers in a four bar linkage. Therefore the other two instant centers are found using Arnold-Kennedy's theorem
2. Draw a circle of an arbitrary radius and place circles to represent all the links approximately equally spaced around the perimeter of the circle as shown in figure 3.10. Since I_{12}, I_{14}, I_{23} and I_{34} are known draw on the circle straight lines connecting the corresponding numbers of links in the circle. For example for I_{12} draw a line connecting link symbols 1 and 2.
3. On the circle draw a dotted line corresponding to a link combination not considered, first say links 1 and 3. This forms two triangles.

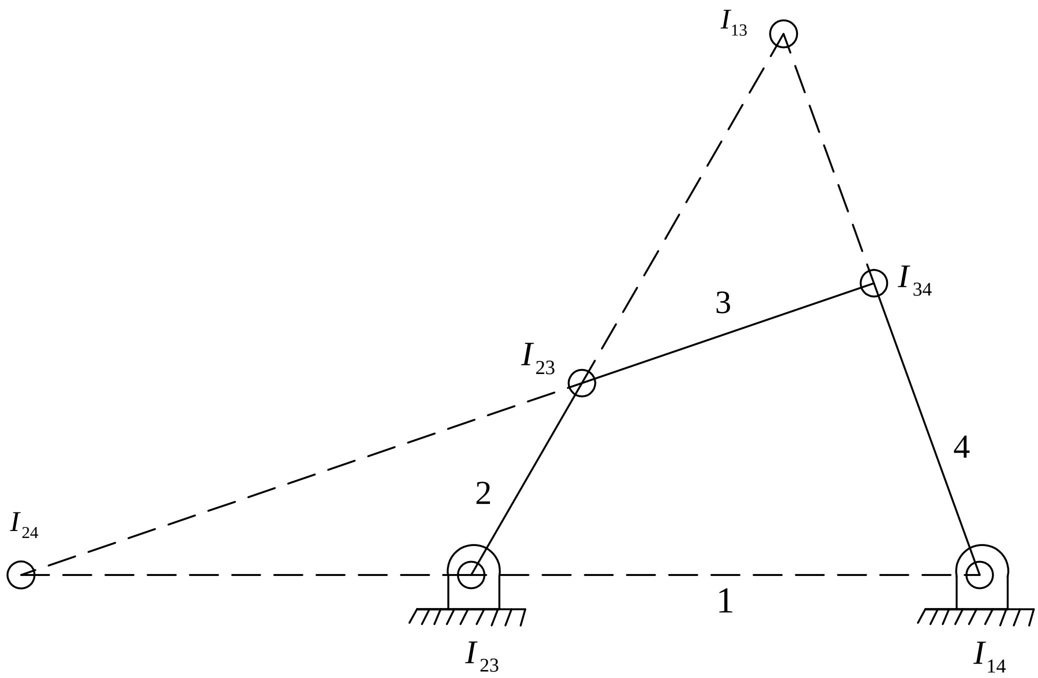


Figure 3.9: Locating instant centers in a four bar linkage

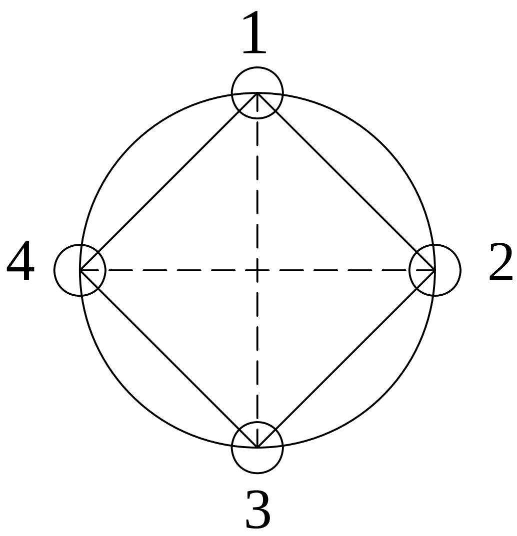


Figure 3.10:

- One triangle with edges 12, 23 and 13. Therefore according to Arnold-Kennedy's theorem, instant centers I_{12}, I_{23} and I_{13} must lie on the same straight line. Extend the line joining instant centers I_{12} and I_{23}, therefore instant center I_{13} must lie somewhere along the line.
- The other triangle has edges 14, 34 and 13. Therefore according to Arnold-Kennedy's theorem, instant centers I_{14}, I_{34} and I_{13} must lie on the same straight line. Extend the line joining instant centers I_{14} and I_{34}, therefore instant center I_{13} must lie somewhere along the line.

Instant center I_{13} is at the point of intersection of the two lines.

4. On the circle draw a dotted line corresponding to the other link combination not considered, that is 2 and 4. This forms two triangles.
   - One triangle with edges 12, 14 and 24. Therefore according to Arnold-Kennedy's theorem, instant centers I_{12}, I_{14} and I_{24} must lie on the same straight line. Extend the line joining instant centers I_{12} and I_{14}, therefore instant center I_{24} must lie somewhere along the line.
   - The other triangle has edges 23, 34 and 24. Therefore according to Arnold-Kennedy's theorem, instant centers I_{23}, I_{34} and I_{24} must lie in the same straight line. Extend the line joining instant centers I_{23} and I_{34}, therefore instant center I_{13} must lie somewhere along the line.

   Instant center I_{24} is at the point of intersection of the two lines.

Therefore, all the six instant centers of the four bar linkage have been located.

Once the instant centers have been found, they can therefore be used to do a very rapid graphical velocity analysis of the linkage. Depending on the particular position of the linkage being analyzed, some of the instant centers may be far away from the link. For example if links 2 and 4 are nearly parallel, their extended lines will intersect at a point far away, and therefore instant center I_{13} will not available for velocity analysis.

Consider figure 3.11 when the crank link 2 is rotating anticlockwise.

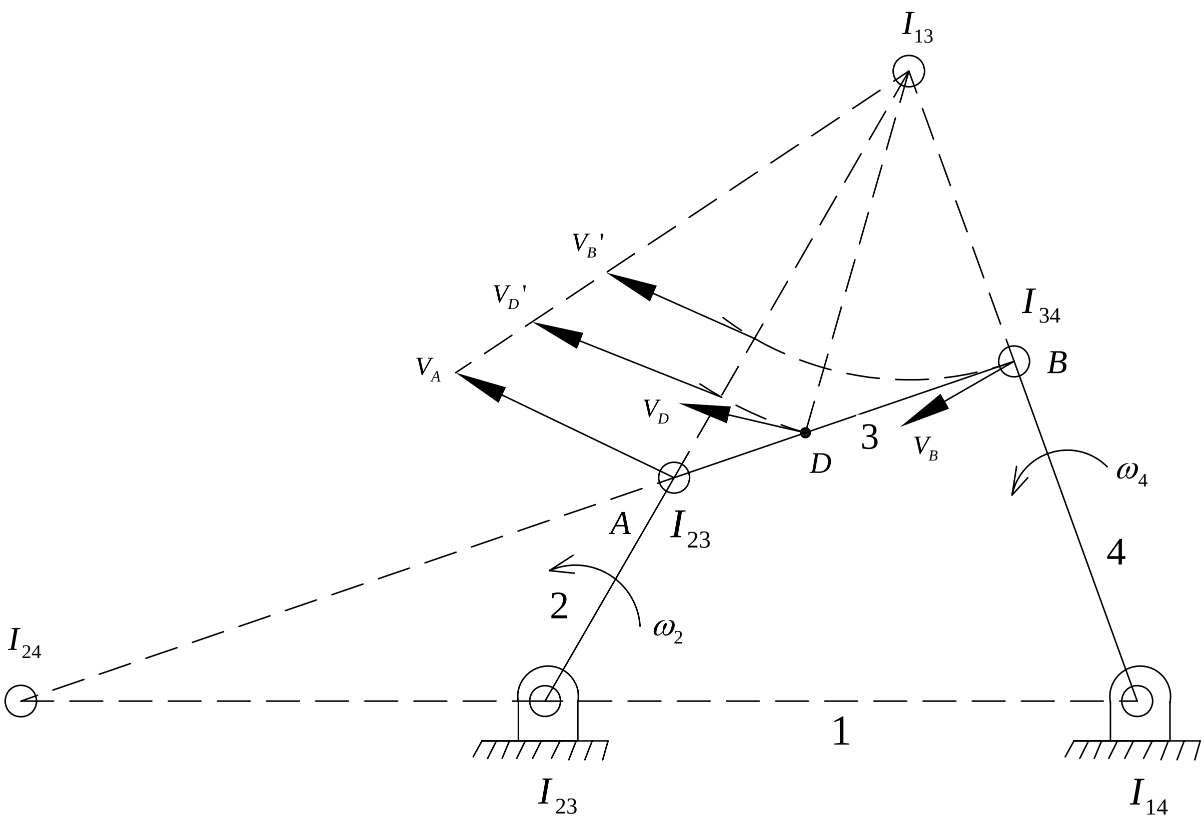


Figure 3.11: Locating instant centers in a four bar linkage

We need to determine the angular velocities of links 3 and 4, and the absolute velocity of point D in the coupler. From the definition of an instant center, both link sharing the instant center will have an identical velocity at that point.

Instant center I_{13} involves link 3 and 1, but link 1 is stationary meaning that all points in this link have a zero velocity. Therefore point I_{13} has zero velocity at this instant, and hence it is considered to be a fixed pivot about which link 3 is in pure rotation about point I_{13}.

The magnitude and direction of V_A is known, therefore,

$$\begin{aligned} V_A &= \omega_2 \bar{OA} = \omega_3 A\bar{I}_{13} \\ \omega_3 &= \frac{V_A}{A\bar{I}_{13}} = \frac{\omega_2 \bar{OA}}{A\bar{I}_{13}} \end{aligned}$$

Once ω_3 is known then we can determine ω_4 and V_D

$$\begin{aligned} V_B &= \omega_4 \bar{BC} = \omega_3 B\bar{I}_{13} \\ \omega_4 &= \frac{V_B}{\bar{BC}} = \frac{\omega_3 B\bar{I}_{13}}{\bar{BC}} \end{aligned}$$

Velocity of point D in the coupler is,

$$V_D = \omega_3 D\bar{I}_{13}$$

Alternatively, the absolute velocities V_B and V_D can be obtained as follows,

(i) First join the tip of V_A to center I_{13}

(ii) Draw arcs centered at I_{13} from points B and C.

(iii) At the points where the arcs intersect line AI_{13}, draw perpendicular lines to intersect the line from I_{13} to the tip of V_A. Then $V_B = V'_B$ and $V_D = V'_D$.

Note: The instant center method is a quick graphical method used to analyze velocities in a mechanism, but it will only work if the instant centers are in a reachable positions.

Examples 3.4

Repeat **Examples 3.1** using the instantaneous center method. i.e. $\overline{AD} = \overline{DB} = 0.45m$, $\overline{OA} = 0.5m$, $\overline{PB} = 0.65m$, $\overline{OC} = 1.4m$ and crank OA rotates clockwise with angular velocity of $2.6rad/s$, find the angular velocity of link CB and the absolute velocity of point D in link AB.

Solution: Draw the linkage to scale and locate all the instant centers.

Linear velocity of point A is,

$$V_A = \omega_{OA} \bar{OA} = 2.6 \times 0.5 = 1.3m/s$$

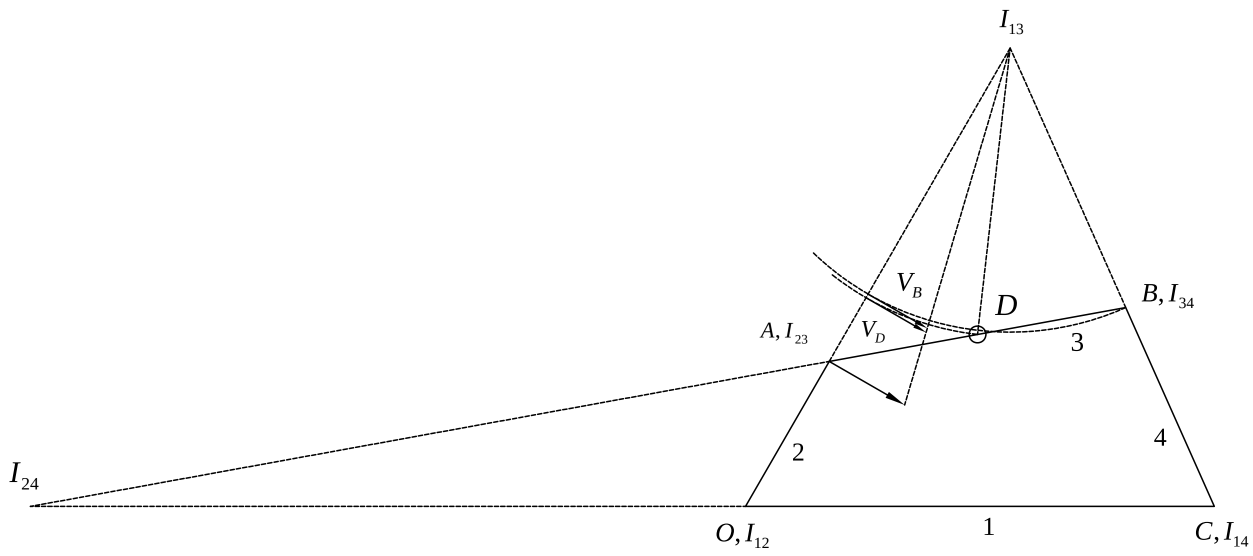


Figure 3.12:

Also from figure above V_A can also be obtained as,

$$
\begin{aligned}
V_A &= \omega_{AB} A\bar{I}_{13} \\
\omega_{AB} &= \frac{V_A}{A\bar{I}_{13}} = \frac{1.3}{1.0806} = 1.203 rad/s
\end{aligned}
$$

The linear velocity of point D is,

$$V_D = \omega_{AB} D\bar{I}_{13} = 1.203 \times 0.861 = 1.04 m/s$$

Angular velocity of link CB is obtained as,

$$
\begin{aligned}
V_B &= \omega_{AB} B\bar{I}_{13} = \omega_{CB} \bar{CB} \\
\omega_{CB} &= \frac{\omega_{AB} B\bar{I}_{13}}{\bar{CB}} \\
&= \frac{1.203 \times 0.8485}{0.65} = 1.57 rad/s
\end{aligned}
$$

Assignment 2

1. Locate all the instant centers of a slider-crank mechanism, and hence solve the **Example 3.2** using the instantaneous center method.

2. In the mechanism shown in figure 3.13, the crank OA is rotating anticlockwise at $100 rev/min$. If $OA = 20mm$, all other links $80mm$, DC is vertical, O is $100mm$ to the left and $80mm$ below D, use the instant center method to determine the absolute velocity of the slider C, and the angular velocities of all the links in the mechanism.

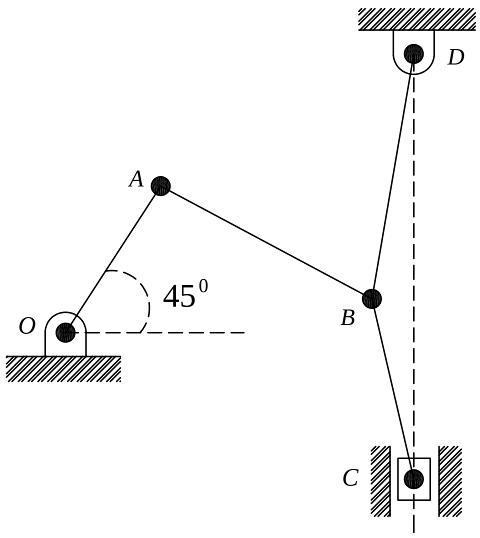


Figure 3.13:

3. Figure 3.14 shows a mechanism (Stephenson type1) with the following dimensions: $A_oB_o = B_oB = AB = DE = 205mm$, $CD = 100mm$, $AC = CB = 110mm$ and $A_oA = 50mm$. It produces an oscillatory output with a dwell when the input rotates at a constant speed. Calculate the absolute velocities of B, C, D and the angular velocity of the output link ED when the crank A_oA makes an angle of 225^0 with A_oB_o using the instantaneous center method. Point E is $50mm$ to the right of point B_o and $150mm$ above A_oB_o.

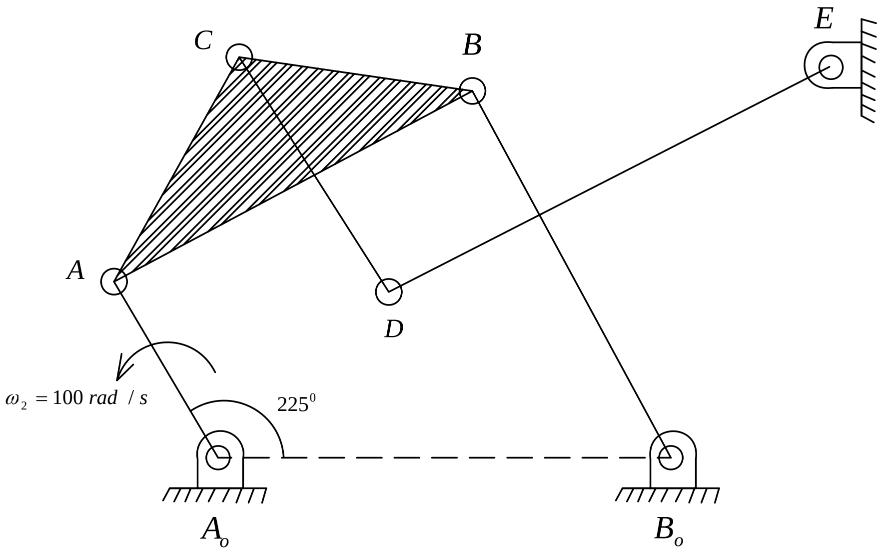


Figure 3.14:

3.2.3 Angular Velocity Ratio and Mechanical Advantage

Angular velocity Ratio

This is the ratio of the angular velocity of the output link to the angular velocity of the input link. Consider figure 3.15.

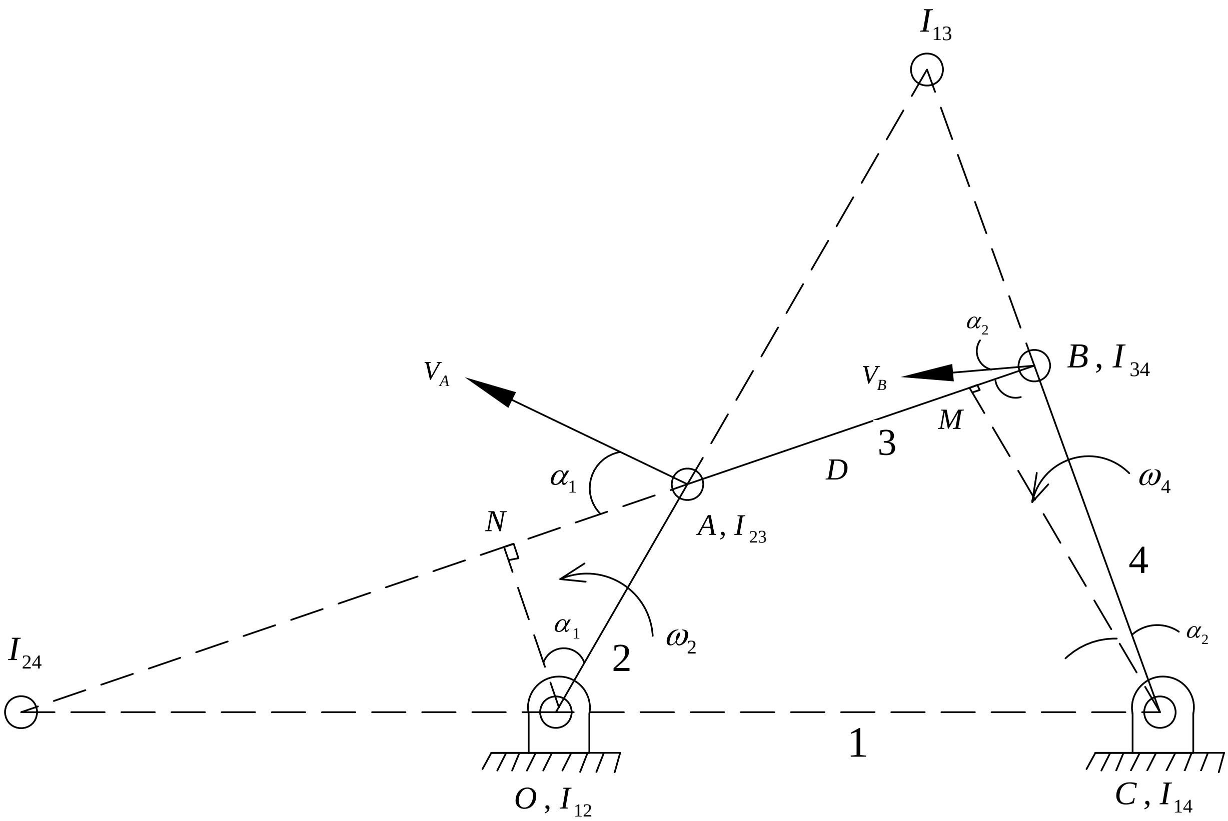


Figure 3.15:

The components of V_A and V_B along AB are,

$$V'_A = \omega_2 OA\cos\alpha_1 = \omega_2 ON \tag{3.13}$$
$$V'_B = \omega_4 CB\cos\alpha_2 = \omega_4 CM \tag{3.14}$$

Since link AB is rigid, then $V'_A = V'_B$, hence

$$\omega_2 ON = \omega_4 CM \tag{3.15}$$
$$\frac{\omega_4}{\omega_2} = \frac{ON}{CM} \tag{3.16}$$

Since triangles $I_{24}ON$ and $I_{24}CM$ are similar, we have $\frac{ON}{I_{24}O} = \frac{CM}{I_{24}C}$ hence the required velocity ratio is,

$$\frac{\omega_4}{\omega_2} = \frac{I_{24}O}{I_{24}C} \tag{3.17}$$

Mechanical Advantage

The power in a translational mechanical system is defined as the product of the force and the velocity. And for a rotational mechanical system, the power is the product of torque and the angular velocity.

Generally, power in=power out+power losses.

Mechanical advantage can be defined as the ratio of the output power (P_{out}) to the input power (P_{in}), that is,

$$\eta = \frac{P_{out}}{P_{in}}$$

Mechanical systems can be very efficient if they are well made with low friction joints.

For a linkage, mechanical advantage (MA) can be defined as the ratio of the output torque (T_{out}) to the input torque (T_{in}), and which for a four-bar linkage is,

$$MA = \frac{T_4}{T_2}$$

Consider figure 3.16,

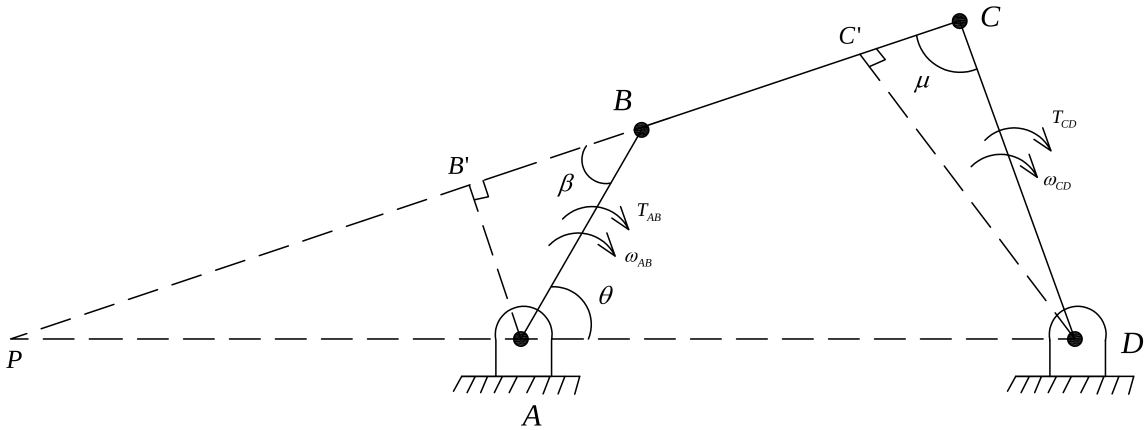


Figure 3.16:

Input power to link AB,

$$P_2 = T_2\omega_2$$

Output power to link CD,

$$P_4 = T_4\omega_4$$

Assuming no friction at joints, then the input power is equal to output power,

$$\begin{aligned} P_2 &= P_4 \\ T_2\omega_2 &= T_4\omega_4 \\ \frac{T_4}{T_2} &= MA = \frac{\omega_2}{\omega_4} = \frac{1}{VR} \end{aligned} \tag{3.18}$$

Relationship between the MA and the transmission angle

The transmission angle for a four bar linkage as shown in figure 3.17 can be obtained as follows,

$$\begin{aligned} AB_o^2 &= a^2 + d^2 - 2ad\cos\theta_2 = b^2 + c^2 - 2bc\cos\mu \\ \cos\mu &= \frac{b^2 + c^2 - a^2 - d^2 + 2ad\cos\theta_2}{2bc} \end{aligned} \tag{3.19}$$

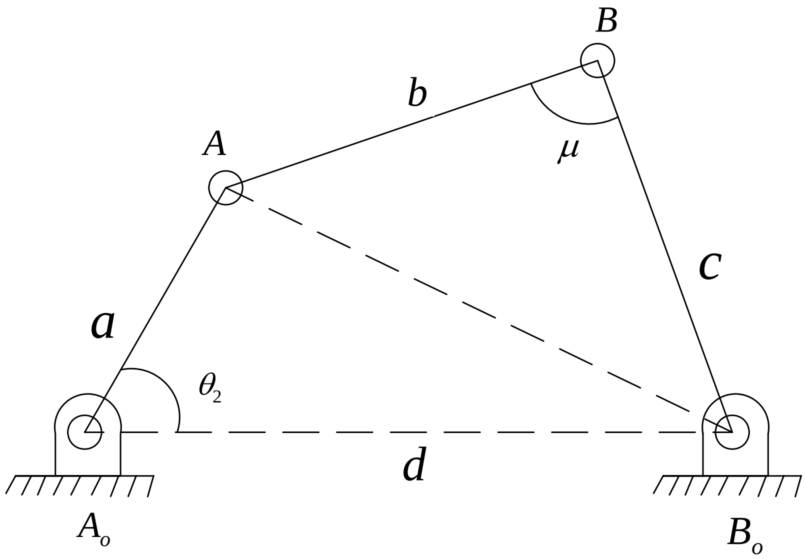


Figure 3.17:

For smooth operation of any mechanism without jerky movements, it is important to ensure that the transmission angle μ lies between 40^0 and 140^0. The ideal value is 90^0 but this cant remain constant during a cycle.

To show how the mechanical advantage and the transmission angle for a four-bar linkage are related consider figure 3.16. The mechanical advantage is given by,

$$MA = \frac{T_4}{T_2} = \frac{\omega_2}{\omega_4}$$

But it has been shown that from the angular velocity ratio,

$$\frac{\omega_2}{\omega_4} = \frac{\overline{DC'}}{\overline{AB'}} \tag{3.20}$$

But from triangles ABB' and DCC', we have;

$$\begin{aligned} AB' &= \overline{AB}\sin\beta \\ DC' &= \overline{DC}\sin\mu \end{aligned}$$

which when substituted in equation 3.20, we have,

$$\frac{\omega_2}{\omega_4} = \frac{\overline{DC}\sin\mu}{\overline{AB}\sin\beta}$$

Hence the mechanical advantage in equation equation 3.18 becomes,

$$MA = \frac{\omega_2}{\omega_4} = \frac{\overline{DC}\sin\mu}{\overline{AB}\sin\beta}$$

The equation above shows that the mechanical advantage for any four-bar linkage depend on the transmission angle μ and angle β.

If μ tends to zero, (which during kinematic synthesis, this is avoided), then the output torque goes to zero regardless of the amount of input torque applied.

But when angle β goes to zero, which it can and occurs twice per cycle in a Grashof linkage, the mechanical advantage tends to infinite. Example of a mechanism which employs this aspect is a rock-crasher mechanism, (draw an illustration). A quite moderate torque applied to input link generates a huge torque on the out put link to crash the rock when $\beta \to 0$.

3.3 Analytical methods for velocity analysis

Once an expression for position of a point or a link is obtained, it can be differentiated with respect to time to get the velocity of the point or link.

For instance we earlier obtained an equation relating θ_2 and θ_4 for a four-bar linkage using vector loop closure equation as,

$$\begin{aligned} K_1\cos\theta_4 - K_2\cos\theta_2 + K_3 &= \cos(\theta_2 - \theta_4) \\ K_1\cos\theta_4 - K_2\cos\theta_2 + K_3 &= \cos\theta_2\cos\theta_4 + \sin\theta_2\sin\theta_4 \end{aligned}$$

Where,

$K_1 = \frac{d}{a}$

$K_2 = \frac{d}{c}$

$K_3 = \frac{a^2-b^2+c^2+d^2}{2ac}$

To obtain the velocity of the output link ω_4, we differentiate this equation with respect to time.

$$\begin{aligned} (-\sin\theta_4)K_1\frac{d\theta_4}{dt} - (-\sin\theta_2)K_2\frac{d\theta_2}{dt} &= -\sin\theta_2\cos\theta_4\frac{d\theta_2}{dt} - \sin\theta_4\cos\theta_2\frac{d\theta_4}{dt} \\ &\quad + \sin\theta_4\cos\theta_2\frac{d\theta_2}{dt} + \sin\theta_2\cos\theta_4\frac{d\theta_4}{dt} \end{aligned}$$

But $\frac{d\theta_2}{dt} = \omega_2$ and $\frac{d\theta_4}{dt} = \omega_4$, then

$$
\begin{aligned}
-\omega_4 K_1 \sin\theta_4 + \omega_2 K_2 \sin\theta_2 &= -\omega_2 \sin\theta_2 \cos\theta_4 - \omega_4 \sin\theta_4 \cos\theta_2 + \omega_2 \sin\theta_4 \cos\theta_2 \\
&\quad +\omega_4 \sin\theta_2 \cos\theta_4 \\
-\omega_4 K_1 \sin\theta_4 + \omega_2 K_2 \sin\theta_2 &= \omega_2 \Big[\sin\theta_4 \cos\theta_2 - \sin\theta_2 \cos\theta_4\Big] + \omega_4 \Big[\sin\theta_2 \cos\theta_4 - \sin\theta_4 \cos\theta_2\Big] \\
-\omega_4 K_1 \sin\theta_4 + \omega_2 K_2 \sin\theta_2 &= \omega_4 \sin(\theta_2 - \theta_4) - \omega_2 \sin(\theta_2 - \theta_4) \\
\omega_4 \Big[\sin(\theta_2 - \theta_4) + K_1 \sin\theta_4\Big] &= \omega_2 \Big[\sin(\theta_2 - \theta_4) + K_2 \sin\theta_2\Big] \\
\omega_4 &= \omega_2 \left[\frac{\sin(\theta_2 - \theta_4) + K_2 \sin\theta_2}{\sin(\theta_2 - \theta_4) + K_1 \sin\theta_4}\right] \qquad (3.21)
\end{aligned}
$$

Similarly, the angular velocity of the coupler can be determined by differentiating equation,

$$K_1 \cos\theta_3 + K_4 \cos\theta_2 + K_5 = \cos\theta_2 \cos\theta_3 + \sin\theta_2 \sin\theta_3$$

where;

$$
\begin{aligned}
K_1 &= \tfrac{d}{a} \\
K_4 &= \tfrac{d}{b} \\
K_5 &= \tfrac{c^2 - d^2 - a^2 - b^2}{2ab}
\end{aligned}
$$

with respect to time and making $\frac{d\theta_3}{dt} = \omega_3$ the subject of formula.

Chapter 4

Acceleration analysis

Once a velocity analysis is done, the next step is to determine the accelerations of all links and points of interest in the mechanism or machine. The accelerations in mechanisms can be found using different methods, such as,

(i) Graphical methods

(ii) Analytical methods

4.1 Definition of acceleration

Acceleration is defined as the rate of change of velocity wrt time. Acceleration can be angular (α) or linear (A), defined as,

$$\alpha = \frac{d\omega}{dt} \tag{4.1}$$

and

$$A = \frac{dV}{dt} \tag{4.2}$$

Consider link PA in pure rotation in the xy plane. We need to determine the acceleration of point P when the link is subjected to an angular velocity ω and an angular acceleration α.

R_{PA} can be represented as a complex number in polar form as;

$$R_{PA} = pe^{i\theta} \tag{4.3}$$

which when differentiated wrt to time we get;

$$\dot{R}_{PA} = \frac{dR_{PA}}{dt} = pie^{i\theta}\frac{d\theta}{dt} = i\omega pe^{i\theta} = i\omega R_{PA} \tag{4.4}$$

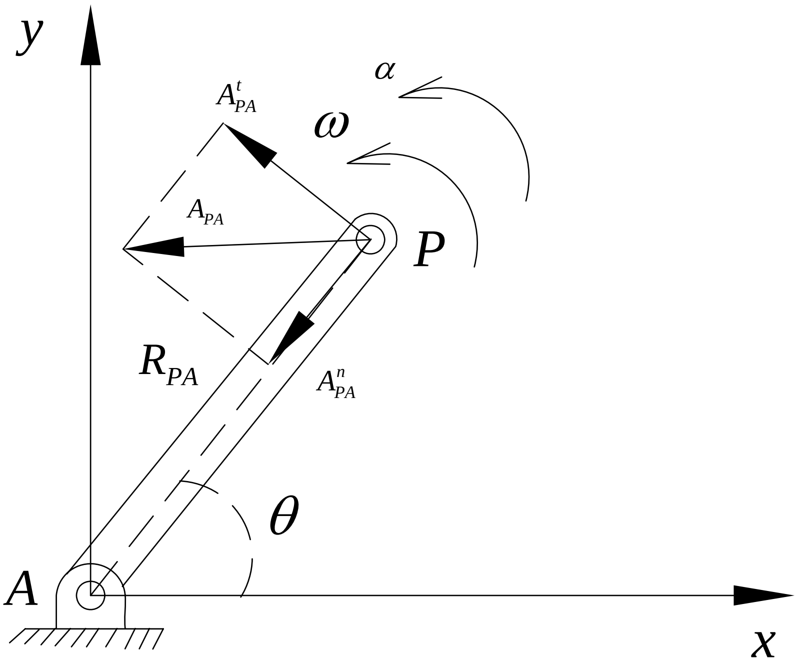


Figure 4.1: Acceleration of a link in pure rotation

Equation 4.4 can be differentiated wrt time to obtain an expression for acceleration of point P.

$$\begin{aligned} A_{AP} &= \frac{d(p\omega i e^{i\theta})}{dt} \\ &= i e^{i\theta} p \frac{d\omega}{dt} + p\omega i^2 e^{i\theta} \frac{d\theta}{dt} \\ &= i p e^{i\theta}\alpha + i^2 p e^{i\theta}\omega^2 \\ &= i R_{PA}\alpha + i^2 R_{PA}\omega^2 \end{aligned} \tag{4.5}$$

As seen in equation 4.5, there are two terms in the expression for acceleration,

- The first term which is multiplied by the constant complex operator i. Therefore this component is at 90^0 to the original position vector, and is called the tangential component of acceleration (A^t_{AP}). The tangential component will be perpendicular to the radius of rotation in a direction dictated by the angular acceleration (α) of the link, and is thus tangential to the path of motion as shown in figure 4.1.

$$A^t_{PA} = R_{PA}\alpha \tag{4.6}$$

- The second term which is multiplied by i^2 or -1. This term is called the normal or centripetal component of acceleration (A^n_{PA}) and is directed towards the center of rotation as shown in figure 4.1.

$$A^n_{PA} = R_{PA}\omega^2 \tag{4.7}$$

Therefore the total acceleration A_{PA} of point P is the sum of the tangential and centripetal components as shown in figure 4.1. A_{PA} for this case can also be referred to as the absolute acceleration A_P of point P since it is referenced to point A which is the origin of the coordinate system.

4.1.1 Relative acceleration

Consider a figure 4.2 in which link AP is in pure rotation and is joined to a block which is in pure translation.

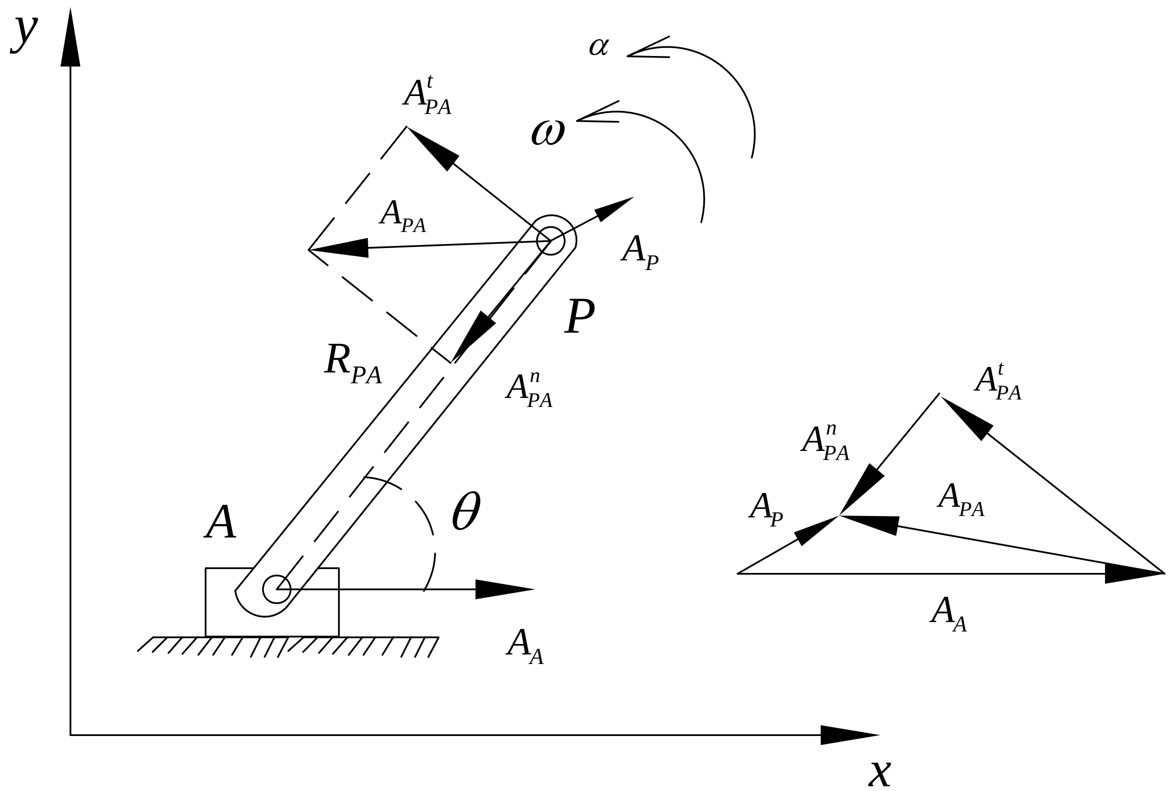


Figure 4.2: Acceleration difference

The absolute acceleration A_P of point P if found using the acceleration difference equation as,

$$A_P = A_A + A_{PA} \tag{4.8}$$

$$(A^t_P + A^n_P) = (A^t_A + A^n_A) + (A^t_{PA} + A^n_{PA}) \tag{4.9}$$

4.2 Graphical acceleration analysis - Acceleration diagrams

To draw an acceleration diagram, we need three equations,

$$\begin{aligned} A_P &= A_A + A_{PA} \\ |A^t| &= r\alpha \\ |A^n| &= r\omega^2 \end{aligned}$$

Four-bar linkage as a typical case

The linkage is shown in a particular position θ of the input in figure 4.3. Let ω_2 and α_2 be the angular velocity and acceleration of link 2, ω_3 and ω_4 the angular velocities of the coupler link 3 and output link 4 respectively.

ω_3 and ω_4 can be obtained from a velocity diagram, or from instant center method.

The point A has two known components of acceleration:

- Tangential component $A_A^t = \alpha_2 a$ perpendicular to link O_1A and in the direction dictated by α_2
- Normal component $A_A^n = \omega_2^2 a$ in the radial direction towards O_1

Similarly the point B considered as point on the link AB has two components of acceleration:

- Tangential component $A_{BA}^t = \alpha_3 b$ perpendicular to link AB.
- Normal component $A_{BA}^n = \omega_3^2 b$ in the direction towards A.

Similarly the point B considered as point on the link O_2B has two components of acceleration:

- Tangential component $A_B^t = \alpha_4 c$ perpendicular to link O_2B.
- Normal component $A_B^n = \omega_4^2 c$ in the radial direction towards O_2.

The vector equation (acceleration difference) for point B becomes,

$$A_B = A_A + A_{BA} \tag{4.10}$$
$$(A_B^t + A_B^n) = (A_A^t + A_A^n) + (A_{BA}^t + A_{BA}^n) \tag{4.11}$$

Equation 4.11 can be solved graphically by drawing an acceleration diagram using the following procedure.

1. Starting from an arbitrary point **o** called a pole, draw the vector **oa'** equal to $A_A^n = \omega_2^2 a$ in a direction parallel to link O_2A to correspond to the fact that the radial direction is from A towards O_1.
2. From point **a'** draw a vector **a'a** equal to $A_A^t = \alpha_2 a$ perpendicular to vector **oa'** to correspond to the fact that the tangential acceleration is perpendicular to link O_1A. This locates point **a**, and then $\overline{oa}$ is the total acceleration of point A in the linkage.
3. From **a** draw vector **ab'** equal to $A_{BA}^n = \omega_3^2 b$ in a direction parallel to link AB towards A. This locates the point **b'**

4. From point **b'** draw a line perpendicular to **ab'**. The point b lies somewhere on this line.

5. From the pole **o** draw the vector **ob"** equal to $A_B^n = \omega_4^2 c$ in a direction parallel to link BO_2 and towards O_2 to locate the point **b"**.

6. From point **b"** draw a line perpendicular to **ob"**, then also b lies somewhere on this line.

7. Finally where the lines **b'b** and **b"b** intersect is the position for point **b**

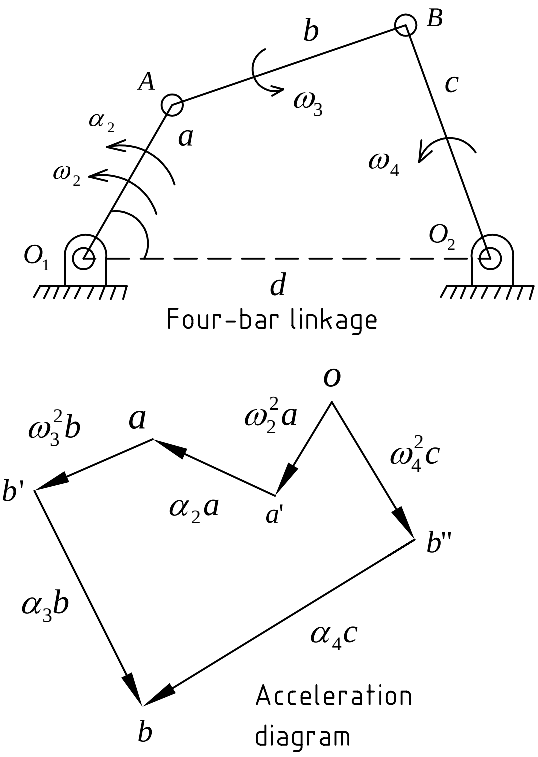


Figure 4.3: Acceleration diagram of a four-bar linkage

Examples 3.5

Figure below shows one of the four-bar linkages used in shoe-stitching machine. For the position shown calculate the angular acceleration of the output link BB_o when the input link AA_o rotates at a constant speed of $500rev/min$. $A_oA = 40mm$, $AB = 100mm$, $A_oB_o = 100mm$, $B_oB = 50mm$ and $\theta_2 = -25^0$.

Solution: Before we draw the acceleration diagram we need the angular velocity ω_3 of the coupler link AB, and the angular velocity ω_4 of the output link BB_o. To obtain these values lets use the velocity diagram method. Therefore we need to draw a velocity diagram for the linkage at the given position.

Absolute velocity of point A is,

$$V_A = \omega_2 \overline{A_oA} = \frac{2\pi \times 500}{60} \times 0.04 = 2.09m/s$$

From the velocity diagram drawn, angular acceleration of the coupler is,

$$\omega_3 = \frac{V_{BA}}{\overline{AB}} = \frac{\overline{ab}}{AB} = \frac{3.54}{0.1} = 35.4rad/s$$

Angular velocity of the output link is

$$\omega_4 = \frac{V_B}{\overline{B_oB}} = \frac{\overline{ob}}{B_oB} = \frac{3.29}{0.05} = 65.8rad/s$$

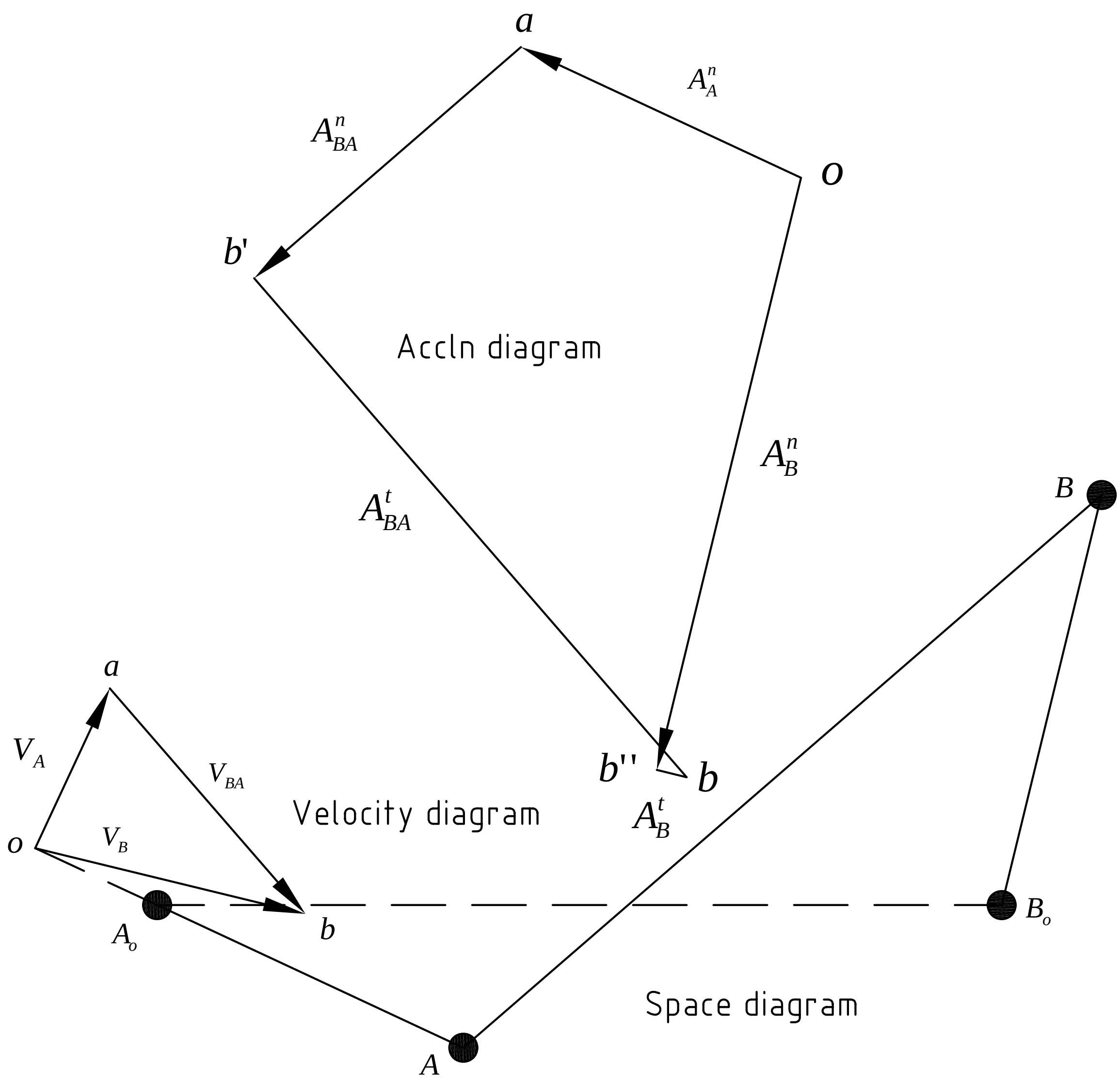


Figure 4.4:

The known acceleration components are,

(i) $A_A^n = \omega_2^2 \overline{A_oA} = 52.36^2 \times 0.04 = 109.7 m/s^2$ directed towards A_o. From a pole point draw **oa'**

(ii) $A_A^t = \alpha_2 \overline{A_oA} = 0$ since the crank is rotating at a constant velocity. Hence point **a** coincide with **a'**.

(ii) $A_{BA}^n = \omega_3^2 \overline{AB} = 35.4^2 \times 0.1 = 125.3 m/s^2$ parallel to AB and directed towards A. From point **a** draw **ab'**.

(iv) $A_B^n = \omega_4^2 \overline{B_oB} = 65.8^2 \times 0.05 = 216.5 m/s^2$ parallel to B_oB and directed towards B_o. From the pole point draw **ob'**

The acceleration diagram is completed by locating point **b**. This is the point of intersections of the lines drawn perpendicular from points **b'** and **b"**. From the acceleration diagram, the angular acceleration of the output link is,

$$\alpha_4 = \frac{A_B^t}{\overline{B_oB}} = \frac{\overline{bb''}}{B_oB} = \frac{12.8}{0.05} = 256 rad/s^2$$

Also the angular acceleration of the coupler is,

$$\alpha_3 = \frac{A_{BA}^t}{\overline{AB}} = \frac{\overline{bb'}}{AB} = \frac{234.6}{0.1} = 2346 rad/s^2$$

Examples 3.6

An engine mechanism shown in figure below has a crank $CB = 100mm$ long, and the connecting rod $BA = 300mm$ long. The center of gravity G of the connecting rod is $100mm$ from B. In the position shown ($< BCA = 120^0$), the crank has a speed of $25\pi rad/s$ and an angular acceleration of $400\pi rad/s^2$ both in the clockwise direction. Find the velocity and acceleration of point G and angular velocity and acceleration of AB.

*Solution:*Absolute velocity of point B is,

$$V_B = \omega_{CB}\overline{BC} = 25\pi \times 0.1 = 7.85 m/s$$

From the velocity diagram the velocity of point G is,

$$V_G = \overline{cg} = 6.92 m/s$$

and the angular velocity of link AB is,

$$\omega_{AB} = \frac{V_{AB}}{\overline{AB}} = \frac{\overline{ab}}{\overline{AB}} = \frac{4.1}{0.3} = 13.67 rad/s$$

The known acceleration components are,

(i) $A_B^n = \omega_{BC}^2\overline{BC} = (25\pi)^2 \times 0.1 = 616.9 m/s^2$ directed towards C. From a pole point draw **cb'**.

(ii) $A_B^t = \alpha_{BC}\overline{BC} = 400\pi \times 0.1 = 125.7 m/s^2$ perpendicular to BC. From point b', draw vector $b'b$ perpendicular to cb'.

(ii) $A_{AB}^n = \omega_{AB}^2\overline{AB} = 13.67^2 \times 0.3 = 46.1 m/s^2$ parallel to AB and directed towards B. From point **b** draw **ba'**.

The acceleration diagram is completed by locating point **a**. This is the point of intersection of the line drawn perpendicular from point **a'** and a line parallel to line AC, which represent the A_{AB}^t and the acceleration of the slider respectively.

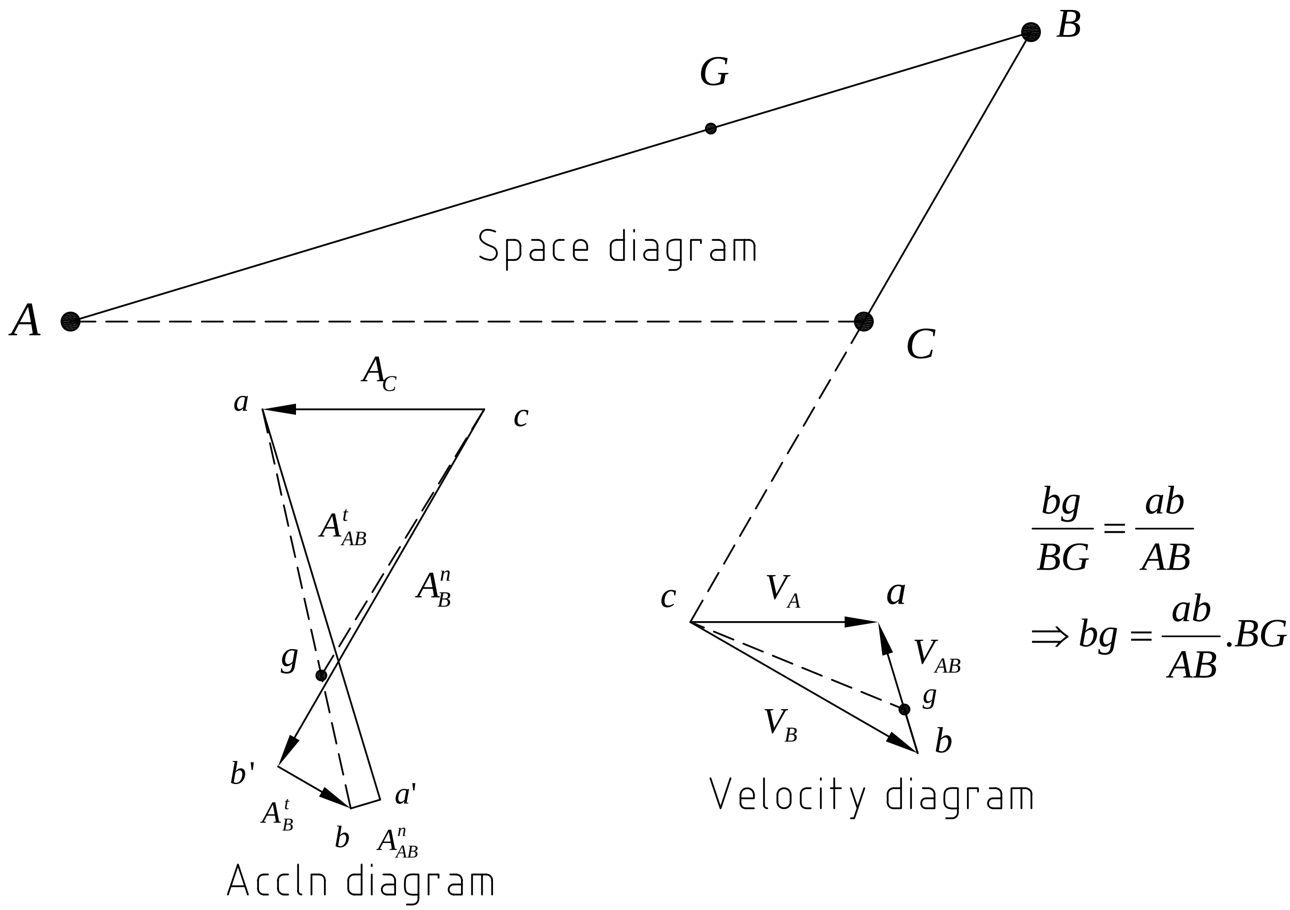


Figure 4.5:

From the acceleration diagram the acceleration of point G is,

$$A_G = \overline{cg} = 466.7 m/s^2$$

and the angular acceleration of link AB is,

$$\alpha_{AB} = \frac{A^t_{AB}}{\overline{AB}} = \frac{\overline{ab}}{\overline{AB}} = \frac{610}{0.3} = 2033.3 rad/s^2$$

Examples 3.7

In the mechanism shown below, D is constrained to move on a horizontal path. Find, for the given configuration, the velocity and acceleration of point D and the angular velocity and acceleration of BD when OC is rotating in a counter-clockwise direction at a speed of $180 rev/min$, increasing at the rate of $50 rad/s^2$.

*Solution:*Absolute velocity of point C is,

$$V_C = \omega_{OC}\overline{OC} = \frac{2\pi \times 180}{60} \times 018 = 3.39 m/s$$

From the velocity diagram the velocity of block D is,

$$V_D = \overline{ad} = 2.05 m/s$$

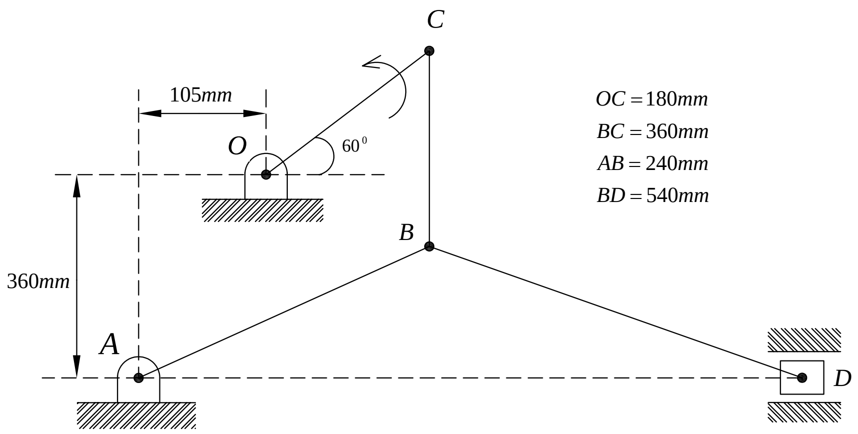


Figure 4.6:

The angular velocity of link BD is,

$$\omega_{BD} = \frac{V_{DB}}{\overline{BD}} = \frac{\overline{bd}}{\overline{BD}} = \frac{2.4}{0.54} = 4.45 rad/s$$

The angular velocity of link AB is,

$$\omega_{AB} = \frac{V_B}{\overline{AB}} = \frac{\overline{ab}}{\overline{AB}} = \frac{2.75}{0.24} = 11.5 rad/s$$

The angular velocity of link BC is,

$$\omega_{BC} = \frac{V_{BC}}{\overline{BC}} = \frac{\overline{bc}}{\overline{BC}} = \frac{0.9}{0.36} = 2.5 rad/s$$

The known acceleration components are,

(i) $A_C^n = \omega_{OC}^2 \overline{OC} = (6\pi)^2 \times 0.18 = 63.96 m/s^2$ directed towards O. From a pole point draw **oc'**

(ii) $A_C^t = \alpha_{OC} \overline{OC} = 50 \times 0.18 = 9 m/s^2$ perpendicular to link OC. From point **c'** draw vector $c'c$.

(ii) $A_{BC}^n = \omega_{BC}^2 \overline{BC} = 2.5^2 \times 0.36 = 2.25 m/s^2$ parallel to BC and directed towards C. From point **c** draw **cb'**.

(iv) $A_B^n = \omega_{AB}^2 \overline{AB} = 11.5^2 \times 0.24 = 31.7 m/s^2$ parallel to AB and directed towards A. From the pole point draw **ob"** NB. Point **b** is the intersection of lines drawn perpendicular to vectors **ob"** and **cb'** rooted at points **b"** and **b'** respectively.

(v) $A_{DB}^n = \omega_{BD}^2 \overline{BD} = 4.45^2 \times 0.54 = 10.7 m/s^2$ parallel to BD and directed towards B. From point **b** draw vector **bd'** NB. Point **d** is the intersection of line drawn perpendicular to vectors **bd'** rooted at d' and the line drawn parallel to AD rooted at pole point **a**.

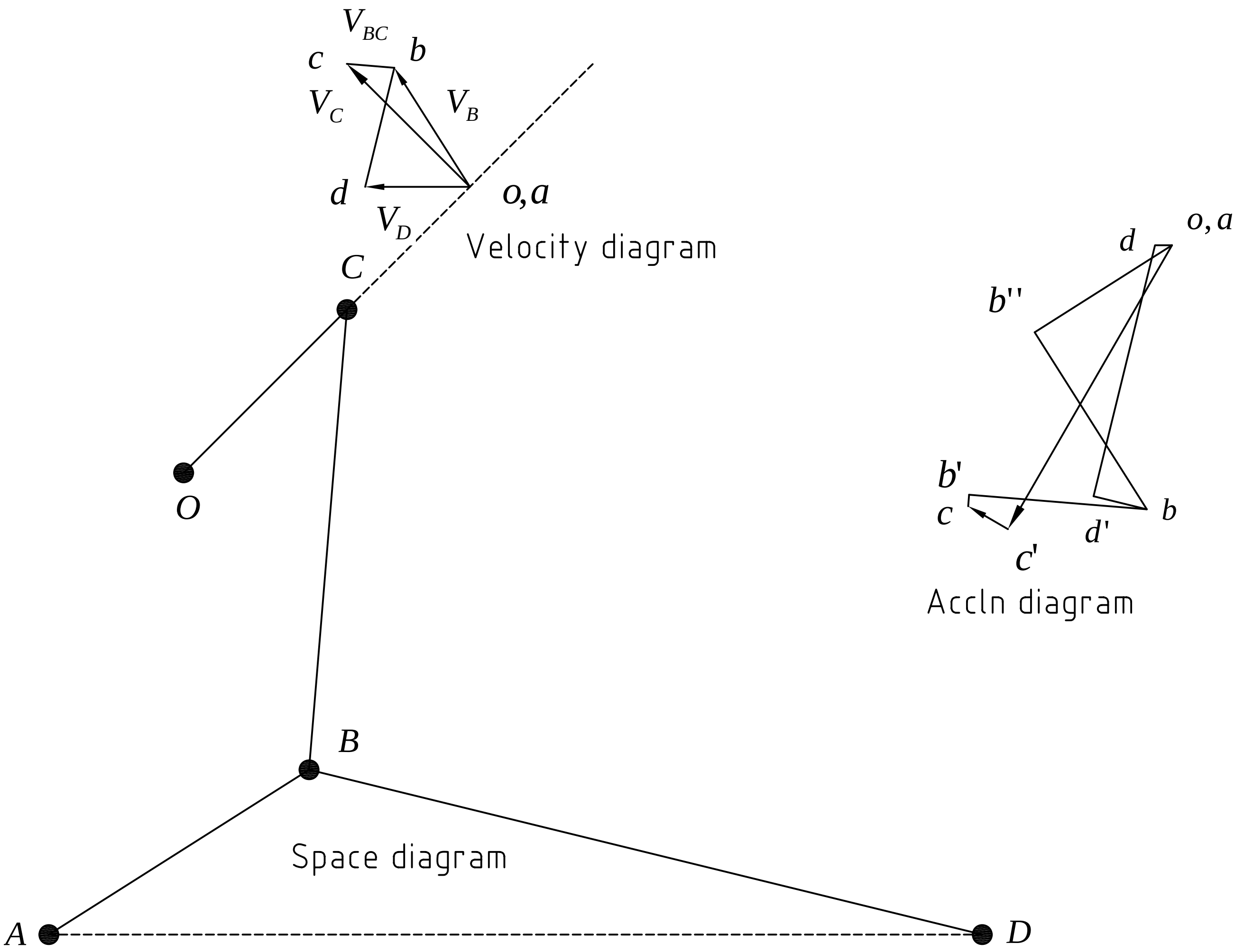


Figure 4.7:

From the acceleration diagram the acceleration of the bloc D is,

$$A_D = \overline{ad} = 3.3m/s^2$$

and the angular acceleration of link BD is,

$$\alpha_{BD} = \frac{A^t_{DB}}{\overline{BD}} = \frac{\overline{d'd}}{\overline{BD}} = \frac{50.4}{0.54} = 93.3rad/s^2$$

4.2.1 Acceleration of a block sliding on a rotating link: Coriolis component of acceleration

Figure 4.8 shows block P sliding relative to the link AB which is in pure rotation. The instantaneous location of the block is defined by a position vector R_P which is both rotating and changing length as the system moves.

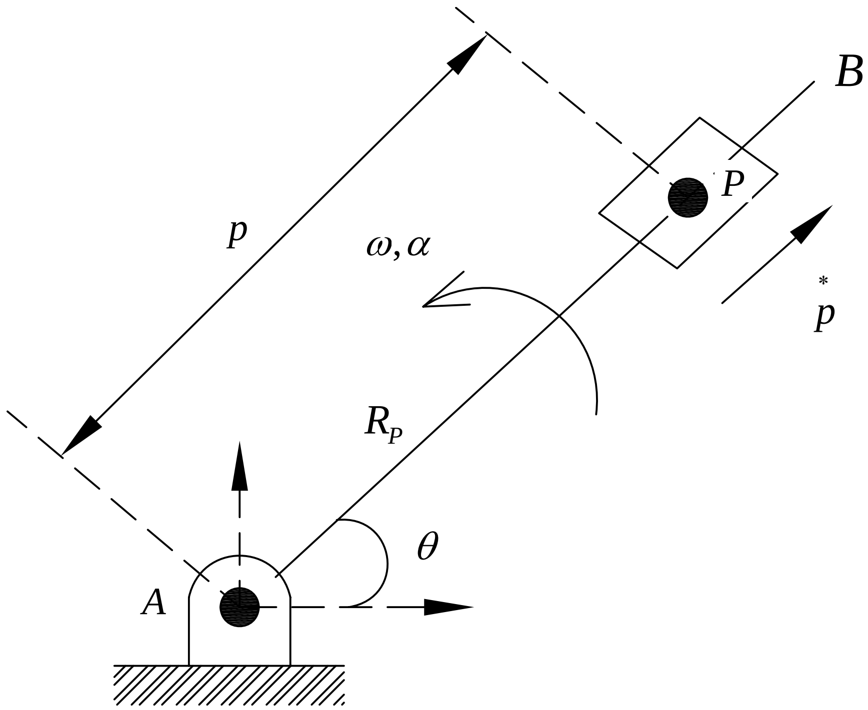


Figure 4.8:

We want to determine the total acceleration at the center of the block P under this combined motion of rotation and sliding. First we represent the the position vector R_P in complex polar form,

$$R_P = pe^{i\theta}$$

There are two functions of time in equation above, that is p and θ. Differentiating equation above we get,

$$V_P = p\omega ie^{i\theta} + \dot{p}e^{i\theta} \tag{4.12}$$

First term $(p\omega)$ is the transmission component and is perpendicular to the radius of rotation, while the second term is the slip component and is directed along the axis of slip.

Differentiating equation 4.12 with respect to time we get the total acceleration of point P as,

$$\begin{aligned} A_P &= ip\alpha e^{i\theta} + i^2 p\omega^2 e^{i\theta} + i\dot{p}\omega e^{i\theta} + i\dot{p}\omega e^{i\theta} + \ddot{p}e^{i\theta} \\ &= ip\alpha e^{i\theta} + i^2 p\omega^2 e^{i\theta} + 2i\dot{p}\omega e^{i\theta} + \ddot{p}e^{i\theta} \end{aligned} \tag{4.13}$$

These terms represent the following components of acceleration,

$$A_P = A_P^{tangential} + A_P^{normal} + A_P^{coriolis} + A_P^{slip} \tag{4.14}$$

Therefore the coriolis component of acceleration is $2\dot{p}\omega$ and is perpendicular to the radius of rotation, and will always be present when there is a velocity of slip associated with any member which has an angular velocity. When doing graphical acceleration analysis one must be alert to recognize the presence of coriolis component, calculate it, and include it in the acceleration diagram.

Examples 3.8

In the figure below, rod PR is constrained to move horizontally, and is driven by a crank OA and a sliding block at P. For the given configuration determine graphically the acceleration of rod PR when OA has an angular velocity of $5rad/s$ in a counter-clockwise direction an angular acceleration of $-35rad/s^2$.

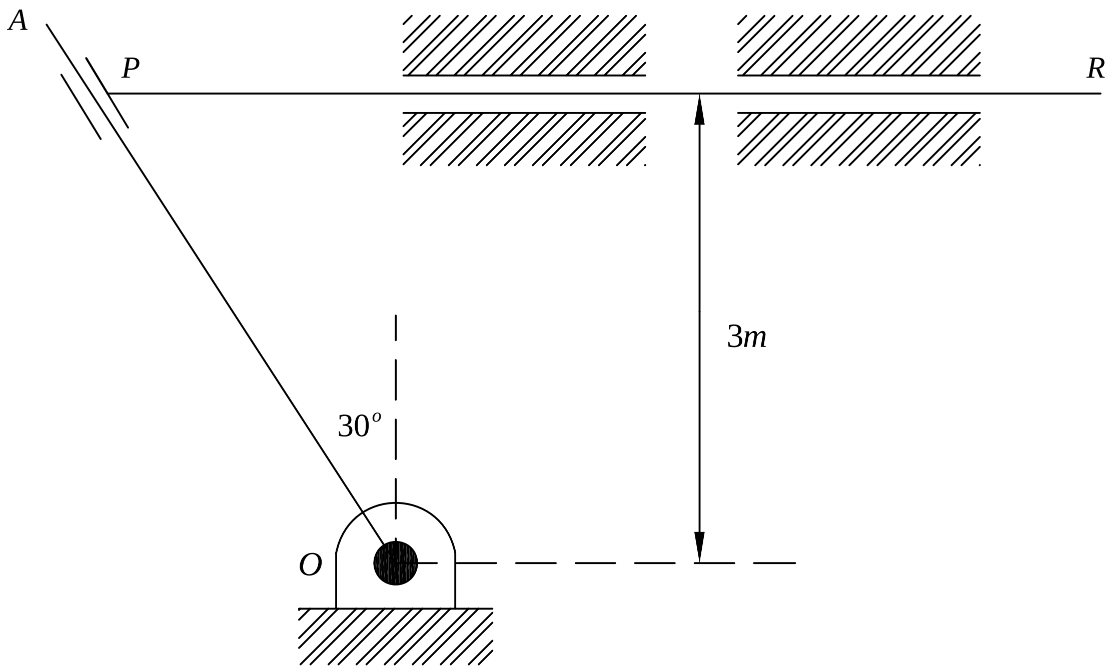


Figure 4.9:

Solution: Let P' be a point on the link OA which is coincident with the block P. First draw the velocity diagram to determine, the sliding velocity of the block relative to link OA.

Velocity of P' relative to O is

$$V_{P'} = \omega_{OA} \times \overline{OP'} = \frac{3}{\cos 30^0} \times 5 = 17.32m/s$$

Draw **op'** $= 17.32m/s$ and perpendicular to link OA.

The sliding velocity of point P relative to P' is parallel to link OA. From point p' draw a line parallel to OA. The velocity diagram is completed by drawing vector V_P parallel to link PR to locate point p at the point of intersection of the two last lines.

From the velocity diagram, the velocity of P relative to P' (the sliding velocity) is,

$$V_{PP'} = \overline{p'p} = 10m/s$$

$A^n_{P'} = \overline{OP'} \times \omega^2_{OA} = 5^2 \times \frac{3}{\cos 30^0} = 86.6m/s^2$. From pole point o draw $op'' = 86.6m/s^2$ parallel to OA and towards pivot O.

$A^t_{P'} = \alpha_{OA} \times \overline{OP'} = 35 \times \frac{3}{\cos 30^0} = 121.2m/s^2$. From point p'' draw $p''p' = 121.2m/s^2$ to locate point p'.

There is a coriolis acceleration which is the acceleration of P relative to P' given by,

$$A^c_{PP'} = 2V_{PP'}\omega_{OP'} = 2 \times 10 \times 5 = 100m/s^2$$

From point p' draw $p'p_1 = 100m/s^2$ perpendicular to link OA.

Sliding acceleration of point P relative to P' ($A_{PP'}$) is parallel to link OA. From point p_1 draw a line parallel to link OA to intersect a line drawn from o parallel to link PR to represent A_{PO}. This locates point p.

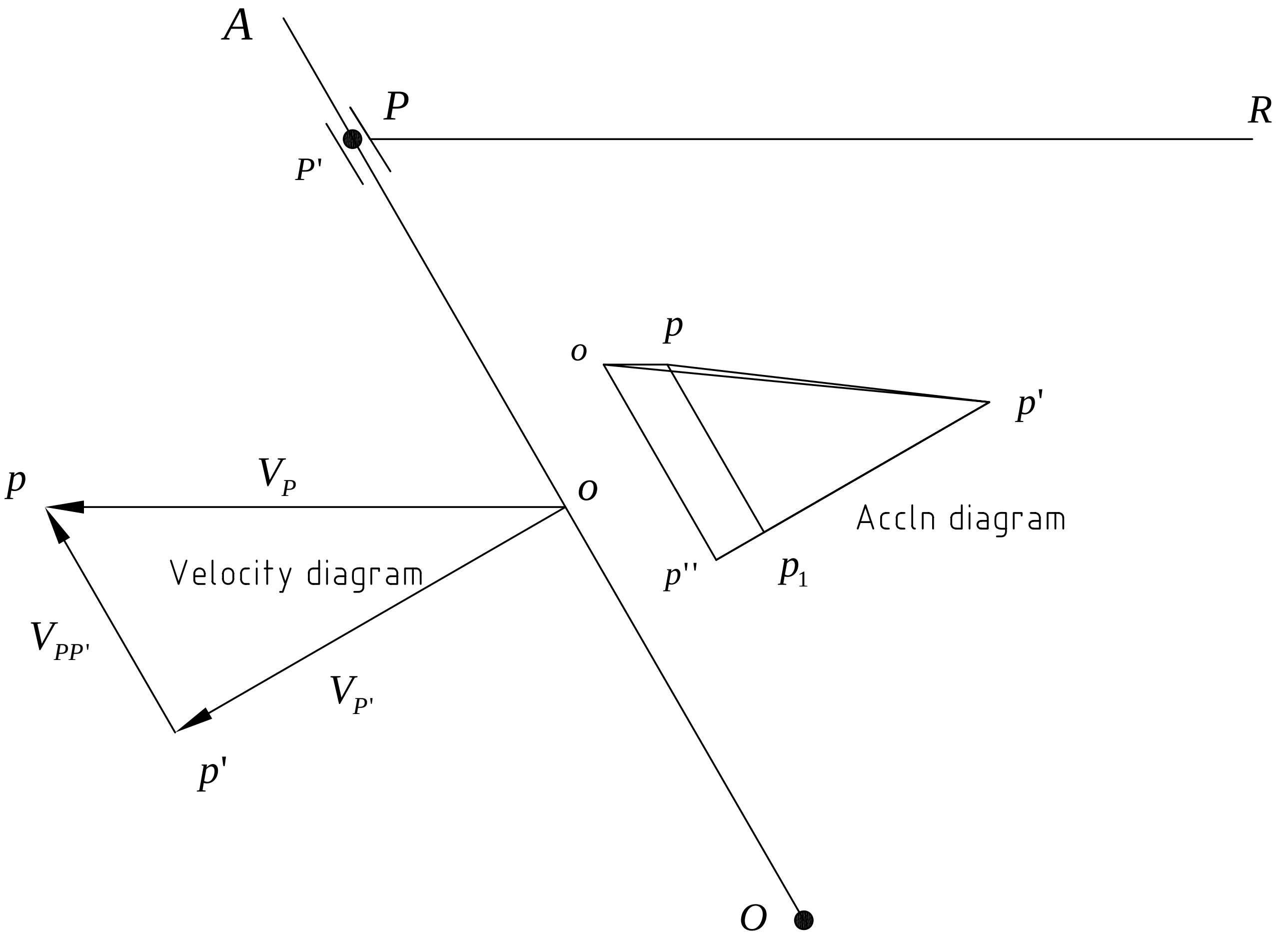


Figure 4.10:

From the acceleration diagram, acceleration of link PR is,

$$A_P = \overline{op} = 24.5m/s^2$$

Examples 3.9

The end A of a bar AB is constrained to move along the vertical path AD and the bar passes through a swivel bearing pivoted at C. Draw velocity and acceleration diagrams for the given configuration when A has a velocity of $3m/s$ towards D and an acceleration of $25m/s^2$ in the opposite direction, and hence determine,

(i) the acceleration and velocity of sliding of the bar through the swivel.

(ii) angular velocity and acceleration of link AB.

Solution: Let point C' be a point on the link AB which is coincident with swivel C.

Draw $da = V_{AD} = 3m/s$ parallel to AD and towards D. $V_{C'A}$ is perpendicular to link AB and rooted at a. Sliding velocity $V_{C'C}$ is parallel to AB and rooted at C.

From the velocity diagram, the sliding velocity of the bar through the swivel is,

$$V_{C'C} = \overline{cc'} = 1.5m/s$$

Angular velocity of the link AB is,

$$\omega_{AB} = \frac{V_{C'A}}{\overline{AC}} = \frac{\overline{ac'}}{AC} = \frac{2.6}{0.461} = 5.63rad/s$$

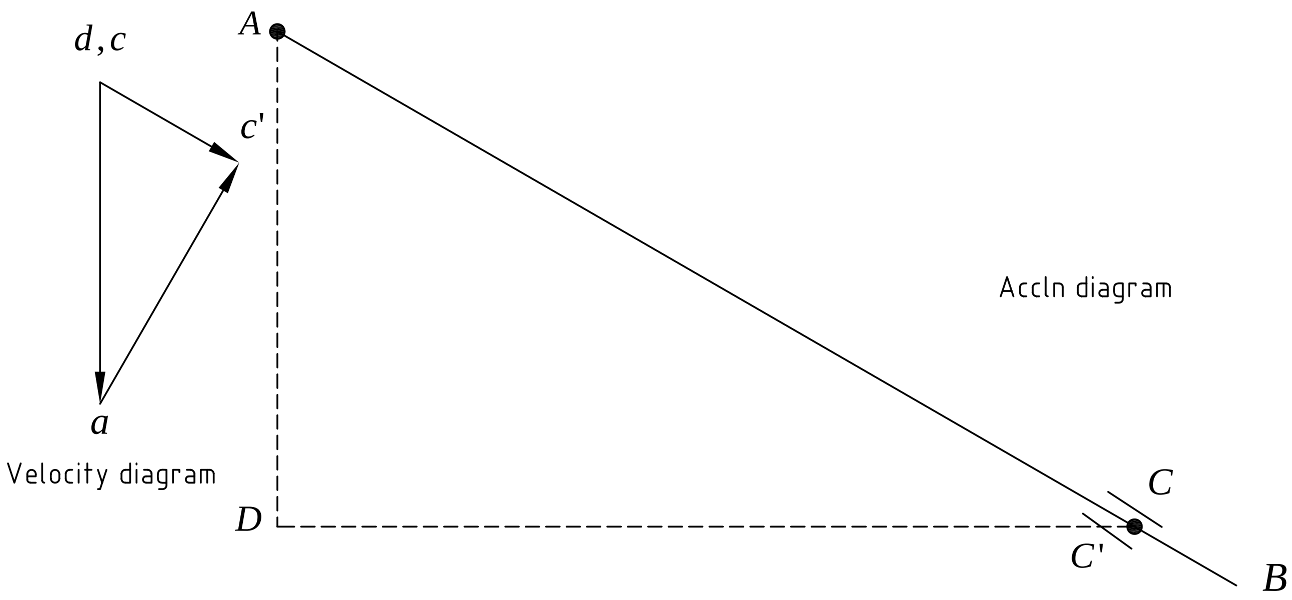


Figure 4.11: