Assignment 1

Fitsum Sereke Tedla

July 2024

1 Question number 1

Given $\frac{n-\frac{4n}{n+5}}{\frac{1}{n+5}+\frac{1}{n-5}} = 0$ and $n > 0$,
find the number of possible values of n

1.1 Solution 1

$$\frac{\frac{n^2+5n-4n}{\cancel{n+5}}}{\frac{(n-5)+(n+5)}{\cancel{(n+5)}(n-5)}} = 0$$

and $n \neq -5, 5, 0$

$$\frac{\frac{n^2+n}{1}}{\frac{2n}{n-5}} = 0$$

$$\frac{\cancel{n}(n+1)(n-5)}{2\cancel{n}} = 0$$

$$\frac{(n+1)(n-5)}{2} = 0$$

$$(n+1)(n-5) = 0$$

* n+1= 0 or n-5 = 0 or both to make the whole equation equals zero and the values for n will be n = -1 and n = 5 but -1 cannot be the solution due to the conditions given on the give statement that n must be greater than 0 (n>0) and 5 also cannot be the solution because it makes the whole equation undefined. There for the solution for this equation will be empty set.

$$TheSolutionSet(SS) = \emptyset$$

therefore, the number of possible value of n will be 0(zero)

2 Question number 2

Let $\triangle ABC$ be a right triangle such that B is a right angle. A circle with diameter BC meets side AC at D. If $AD = 1$ and $BD = 4$, then what is CD?

We are given a circle with diameter BC that meets side AC at D. According to Thales's theorem, if B, C and D are distinct points on a circle where the line BC is a diameter, then $\angle BDC$ is a right angle.

Since D lies on AC, we know that $\angle BDA$ and $\angle BDC$ are supplementary angles.

Thus, we can find the value of $\angle BDA$:

$$\begin{aligned}\angle BDA &= 180^\circ - \angle BDC \\ &= 180^\circ - 90^\circ \\ &= 90^\circ\end{aligned}$$

We are also given that $\angle ABC = 90^\circ$.

So we have three right-angled triangles: $\triangle ABC$, $\triangle BDA$ and $\triangle BDC$.

The Pythagorean theorem states that for any right-angled triangle:
$a^2 + b^2 = c^2$, where a, b and c are three sides of the triangle and c is the hypotenuse.

2.1 Solution 2

Let's apply the Pythagorean theorem to each triangle.

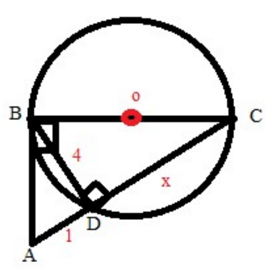

Figure 1:

For $\triangle ABC$, we have sides AB, BC and AC, with AC as the hypotenuse:

$$AB^2 + BC^2 = AC^2 \tag{1}$$

For $\triangle BDA$, we have sides BD, AD and AB, with AB as the hypotenuse:

$$BD^2 + AD^2 = AB^2 \tag{2}$$

For $\triangle BDC$, we have sides BD, DC and BC , with BC as the hypotenuse:

$$BD^2 + DC^2 = BC^2 \tag{3}$$

take equation 2 :

$$BD^2 + AD^2 = AB^2$$
$$4^2 + 1^2 = AB^2$$
$$16 + 1 = AB^2$$
$$17 = AB^2$$

Substitute this value in the equation 1

$$AB^2 + BC^2 = AC^2$$

$$17 + BC^2 = AC^2 \tag{4}$$

but BC^2 is

$$BC^2 = BD^2 + CD^2 \text{ from equation 3}$$

,then we can substitute in the equation 4 and we get the following

$$17 + (BD^2 + CD^2) = AC^2$$

put the values already known,

$$17 + (4^2 + x^2) = AC^2$$

but, Ac = x+1

$$Ac^2 = (x+1)^2$$

put this value in the previous equation then we get,

$$17 + (16 + x^2) = (x+1)^2$$
$$33 + x^2 = x^2 + 2x + 1$$
$$33 + \cancel{x^2} = \cancel{x^2} + 2x + 1$$
$$33 - 1 = 2x$$
$$32 = 2x$$
$$x = \frac{32}{2}$$
$$x = 16$$

$\therefore$ The value of the side CD is 16.

3 Question number 3

It is given that $\int_{\frac{\pi}{2}}^{\pi} \sin^2 x \; dx = k\pi$ for some constant k. Use integration by parts to solve for k. To solve for k, we need to first evaluate the definite integral $\int_{\frac{\pi}{2}}^{\pi} \sin^2 x \; dx$.

We can do so using integration by parts, which is given by:

$$\int_a^b u \; dv = [uv]_a^b - \int_a^b v \; du$$

where u and v are functions of x, and du and dv are their differentials.
In our case, $a = \frac{\pi}{2}$ and $b = \pi$. Let $u = \sin x$ and $dv = \sin x \; dx$.
Then, we have $du = \cos x \; dx$ and $v = -\cos x$.
Substituting these values into the integration by parts formula, we get:

3.1 Solution 3

$$\begin{aligned}
\int_{\frac{\pi}{2}}^{\pi} \sin^2 x \; dx &= \int_{\frac{\pi}{2}}^{\pi} \sin x \cdot \sin x \; dx \\
&= [-\sin x \cos x]_{\frac{\pi}{2}}^{\pi} - \int_{\frac{\pi}{2}}^{\pi} (-\cos x) \cos x \; dx \\
&= [-\sin x \cos x]_{\frac{\pi}{2}}^{\pi} + \int_{\frac{\pi}{2}}^{\pi} \cos^2 x \; dx \\
&= -\sin \pi \cos \pi - \left(-\sin \frac{\pi}{2} \cos \frac{\pi}{2}\right) + \int_{\frac{\pi}{2}}^{\pi} \cos^2 x \; dx \\
&= 0 \times (-1) - (-1) \times 0 + \int_{\frac{\pi}{2}}^{\pi} \cos^2 x \; dx \\
&= 0 + \int_{\frac{\pi}{2}}^{\pi} \cos^2 x \; dx \\
&= \int_{\frac{\pi}{2}}^{\pi} \cos^2 x \; dx
\end{aligned}$$

Next, we use the trigonometric identity $\cos^2 x = \frac{1+\cos 2x}{2}$:

$$= \int_{\frac{\pi}{2}}^{\pi} \frac{1+\cos(2x)}{2}\ dx \text{ , split the equation into two as follows and do the integration separa}$$

$$= \int_{\frac{\pi}{2}}^{\pi} \frac{1}{2}\ dx + \int_{\frac{\pi}{2}}^{\pi} \frac{\cos(2x)}{2}\ dx$$

$$= \left[\frac{1}{2}\right]_{\frac{\pi}{2}}^{\pi} + \frac{1}{2}\int_{\frac{\pi}{2}}^{\pi} \cos(2x)\ dx$$

Let $u = 2x$ when $x = \frac{\pi}{2}$, $u = \pi$, when $x = \pi$, $u = 2\pi$ then for, $du = 2dx$ and $dx = \frac{du}{2}$

Value substitution

$$= \left[\frac{1}{2}\right]_{\frac{\pi}{2}}^{\pi} + \frac{1}{2}\int_{\pi}^{2\pi} \cos(u)\ du$$

$$= \left[\frac{1}{2}\right]_{\frac{\pi}{2}}^{\pi} + \frac{1}{2}\left[\sin(u)\right]_{\pi}^{2\pi}\ du$$

$$= \frac{1}{2}(\pi - \frac{\pi}{2}) + \frac{1}{2}\sin(2\pi - \pi)$$

$$= \frac{\pi}{2} - \frac{\pi}{4} + \cancel{\frac{1}{2}\sin(2\pi - \pi)} \quad \text{where } \sin 2\pi - \pi = \sin\pi = 0$$

$$\int_{\frac{\pi}{2}}^{\pi} \sin^2 x\ dx = \frac{\pi}{4}$$

4 question number 4

Let $\mathcal{S} = \mathcal{P}(\mathbb{Z}_3)\backslash\emptyset$. For $A, B \in \mathcal{S}$, $A \sim B$ if and only if there exists a bijection $f : A \to B$. How many equivalence classes on $\mathcal{S}$ are there, determined by $\sim$?

4.1 Solution 4

Given $\mathcal{S} = \mathcal{P}(\mathbb{Z}_3)\backslash\emptyset$
where, if we take any number that can divide by three with or without a remainder have the following formula, $n = 3k+r$, due to this set $\mathbf{Z_3}$ can only have 0 ,1 and 2 as a remainder in the :

$$\mathbb{Z}_3 = \{0, 1, 2\}$$

from this we can get the set $\mathcal{P}(\mathbb{Z}_3) = \{\{\emptyset\}, \{0\}, \{1\}, \{2\}, \{0,1\}, \{0,2\}, \{1,2\}, \{0,1,2\}\}$
when we remove the empty set from the $\mathcal{P}(\mathbb{Z}_3)\backslash\emptyset$, we will get three set of classes:-

Set 1: {{0},{1},{2}} is create 1 equivalent class of size of 1

Set 2: {{0,1},{0,2},{1,2}} is create 1 equivalent class of size 2

Set 3: {{0,1,2}} is create 1 equivalent class of size 3

from these extraction of subsets we can conclude that , we have 3 equivalent class on S.

5 question numbers 5

The random variable X has the distribution $N(50, 8^2)$. The random variable Y is related to X by the formula $Y = aX + b$, where a and b are constants with $a > 0$. Also, $P(Y < 74) = P(Y > 146) = 0.0668$. Without finding the values of a and b, find the variance of Y.

5.1 Solution 5

$$X \sim N(50, 8^2)$$

mean(σ) = 50 , Variance (σ^2)= 8^2 = 64 and The standard Deviation (σ)=8
Y = aX +b, a > 0 P(Y< 74)= P(Y> 146) = 0.0668, from the Z- Score table we can get 0.06681 which gives the value of -1.5 that is approximately equal to the value of 0.0668.

z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-0	.50000	.49601	.49202	.48803	.48405	.48006	.47608	.47210	.46812	.46414
-0.1	.46017	.45620	.45224	.44828	.44433	.44034	.43640	.43251	.42858	.42465
-0.2	.42074	.41683	.41294	.40905	.40517	.40129	.39743	.39358	.38974	.38591
-0.3	.38209	.37828	.37448	.37070	.36693	.36317	.35942	.35569	.35197	.34827
-0.4	.34458	.34090	.33724	.33360	.32997	.32636	.32276	.31918	.31561	.31207
-0.5	.30854	.30503	.30153	.29806	.29460	.29116	.28774	.28434	.28096	.27760
-0.6	.27425	.27093	.26763	.26435	.26109	.25785	.25463	.25143	.24825	.24510
-0.7	.24196	.23885	.23576	.23270	.22965	.22663	.22363	.22065	.21770	.21476
-0.8	.21186	.20897	.20611	.20327	.20045	.19766	.19489	.19215	.18943	.18673
-0.9	.18406	.18141	.17879	.17619	.17361	.17106	.16853	.16602	.16354	.16109
-1	.15866	.15625	.15386	.15151	.14917	.14686	.14457	.14231	.14007	.13786
-1.1	.13567	.13350	.13136	.12924	.12714	.12507	.12302	.12100	.11900	.11702
-1.2	.11507	.11314	.11123	.10935	.10749	.10565	.10383	.10204	.10027	.09853
-1.3	.09680	.09510	.09342	.09176	.09012	.08851	.08692	.08534	.08379	.08226
-1.4	.08076	.07927	.07780	.07636	.07493	.07353	.07215	.07078	.06944	.06811
-1.5	.06681	.06552	.06426	.06301	.06178	.06057	.05938	.05821	.05705	.05592
-1.6	.05480	.05370	.05262	.05155	.05050	.04947	.04846	.04746	.04648	.04551
-1.7	.04457	.04363	.04272	.04182	.04093	.04006	.03920	.03836	.03754	.03673
-1.8	.03593	.03515	.03438	.03362	.03288	.03216	.03144	.03074	.03005	.02938
-1.9	.02872	.02807	.02743	.02680	.02619	.02559	.02500	.02442	.02385	.02330
-2	.02275	.02222	.02169	.02118	.02068	.02018	.01970	.01923	.01876	.01831

Figure 2: Z-Score Table

P(Z>1.5) $\approx$ 0.0668, which means the Z-score probability will be approximately $\pm$1.5

$$Z = \frac{Y - \mu_y}{\sigma_y}$$

for the first distribution P(Y<74) = 0.0668 will represent the Z- score of -1.5.

$$P(\frac{Y - \mu_y}{\sigma_y} < -1.5) = 0.0668$$

$$\frac{74 - \mu_y}{\sigma_y} = -1.5 \quad \text{........eq. 1}$$

for the second distribution P(y>146)=0.0668 will be represent the Z-score 1.5,

$$\frac{146 - \mu_y}{\sigma_y} = 1.5 \quad ....\text{eq. } 2$$

we are going to equate the two equations and we get :

$$\frac{-(74 - \mu_y)}{\cancel{\sigma_y}} = \frac{146 - \mu_y}{\cancel{\sigma_y}}$$

$$-74 + \mu_y = 146 - \mu_y$$

$$\mu_y = 110,$$

substitute this value in the original equation eq.1 or eq.2 we can do on both equation and it gives us the sam
I choose the second equation and the result shows as follows:

$$\frac{146 - \mu_y}{\sigma_y} = 1.5 \quad ....\text{eq. } 2$$

$$\frac{146 - 110}{\sigma_y} = 1.5$$

$$\sigma_y = \frac{36}{1.5} = 24$$

The Next calculation we will do is to find the coefficient a

$$\sigma_y = a * \sigma_x,$$

but we already know the value of the σ_x=8 and σ_y=24,

$$24 = a * 8$$

$$a = 24$$

Finally, we have every thing to calculate for the variance of Y

$$Var(Y) = a^2 * Var(X)$$

we already know the value of the Variance of X from the given which is
Var(X) = 8^2

$$Var(Y) = a^2 * 8^2$$

$$Var(Y) = 3^2 * 8^2$$

$$Var(Y) = 9 * 64$$

$$Var(Y) = 576$$