**FULL NAME:**

**Problem 1**

Given \(\frac{n-\frac{4n}{n+5}}{\frac{1}{n+5}+\frac{1}{n-5}}=0\) and \(n>0\), find the number of possible values of \(n\).  
(A) 1   
(B) 2  
(C) 3  
(D) 0

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| Given \(\frac{n-\frac{4n}{n+5}}{\frac{1}{n+5}+\frac{1}{n-5}}=0\) and \(n>0\),  \\ find the number of possible values of \(n\)\\  \[  \frac{\frac{n^{2}+5n-{4n}}{\cancel{n+5}}}{\frac{(n-5)+(n+5)}{(\cancel{n+5})(n-5)}} = 0  \]\\  and $n \neq -5, 5, 0$  \[\frac{\frac{n^{2}+n}{1}}{\frac{2n}{n-5}}=0  \] \\ \\  \[\frac{\cancel{n}{(n+1)}{(n-5)}}{2\cancel{n}}=0  \] \\  \[\frac{{(n+1)}{(n-5)}}{2}=0\]\\  \[(n+1)(n-5)=0\]\\  \* n+1= 0 or n-5 = 0 or both to make the whole equation equals zero and the values for n will be n = -1 and n = 5 but -1 cannot be the solution due to the conditions given on the give statement that n must be greater than 0 (n$>$0) and 5 also cannot be the solution because it makes the whole equation  undefined. There for the solution for this equation will be empty set.  \[The Solution Set(SS) = \emptyset\] \\  therefore, the number of possible value of n will be 0(zero) |

**Problem 2**

Let \(\triangle ABC\) be a right triangle such that \(B\) is a right angle. A circle with diameter \(BC\) meets side \(AC\) at \(D\). If \(AD=1\) and \(BD=4\), then what is \(CD\)?

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| We are given a circle with diameter \(BC\) that meets side \(AC\) at \(D\). According to Thales's theorem, if \(B\), \(C\) and \(D\) are distinct points on a circle where the line \(BC\) is a diameter, then \(\angle BDC\) is a right angle.  Since \(D\) lies on \(AC\), we know that \(\angle BDA\) and \(\angle BDC\) are supplementary angles.  Thus, we can find the value of \(\angle BDA\):  \[\begin{align\*}  \angle BDA &= 180^{\circ} - \angle BDC \\  &= 180^{\circ} - 90^{\circ} \\  &= 90^{\circ}  \end{align\*}\]  We are also given that \(\angle ABC= 90^{\circ}\).  So we have three right-angled triangles: \(\triangle ABC\), \(\triangle BDA\) and \(\triangle BDC\).  The Pythagorean theorem states that for any right-angled triangle:  \(a^2+b^2=c^2\), where \(a\), \(b\) and \(c\) are three sides of the triangle and \(c\) is the hypotenuse.  Let's apply the Pythagorean theorem to each triangle.  For \(\triangle ABC\), we have sides \(AB\), \(BC\) and \(AC\), with \(AC\) as the hypothenuse:  \[AB^2 + BC^2 = AC^2\]  Let's name this as Equation 1.  For \(\triangle BDA\), we have sides \(BD\), \(AD\) and \(AB\), with \(AB\) as the hypothenuse:  \[BD^2 + AD^2 = AB^2\]  Let's name this as Equation 2.  For \(\triangle BDC\), we have sides \(BD\), \(DC\) and \(BC\) , with \(BC\) as the hypotenuse:  \begin{equation}  BD^2 + DC^2 = BC^2  \label{eq:example2}  \end{equation}\\  take equation 2 :  \[BD^2 + AD^2 = AB^2\]  \[4^2 + 1^2 = AB^2\]  \[16+1=AB^2\]  \[17=AB^2\]  Substitute this value in the equation 1  \[AB^2 + BC^2 = AC^2\]  \begin{equation}  17+BC^2=AC^2  \label{eq:example2}  \end{equation}\\  but \(BC^2\) is  \[BC^2= BD^2+CD^2 \text{ from equation 3}\],then we can substitute in the equation 4 and we get the following \\  \[17+(BD^2+CD^2)= AC^2\] put the values already known,  \[17+(4^2+x^2)=AC^2\] but, Ac = x+1 \[Ac^2=(x+1)^2\] put this value in the previous equation then we get,  \[17+(16+x^2)=(x+1)^2\]  \[33+x^2=x^2+2x+1\]  \[33+\cancel{x^2}=\cancel{x^2}+2x+1\]  \[33-1=2x\]  \[32=2x\]  \[x=\frac{32}{2}\]  \[x=16\]  \[  \therefore \text{The value of the side CD is 16. }  \] |

**Problem 3**

It is given that \(\int\_{\frac{\pi}{2}}^{\pi} \sin ^2 x \ dx = k\pi\) for some constant \(k\). Use integration by parts to solve for \(k\).

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| To solve for \(k\), we need to first evaluate the definite integral \(\int\_{\frac{\pi}{2}}^{\pi} \sin ^2 x \ dx\).  We can do so using integration by parts, which is given by:  \[\int\_a^b u \ dv = \left[uv\right]\_a^b - \int\_a^b v \ du\]  where \(u\) and \(v\) are functions of \(x\), and \(du\) and \(dv\) are their differentials.  In our case, \(a=\frac{\pi}{2}\) and \(b=\pi\).  Let \(u = \sin x\) and \(dv = \sin x \ dx\).  Then, we have \(du = \cos x \ dx\) and \(v = -\cos x\).  Substituting these values into the integration by parts formula, we get:  \[\begin{align\*}  \int\_{\frac{\pi}{2}}^{\pi} \sin ^2 x \ dx &= \int\_{\frac{\pi}{2}}^{\pi} \sin x \cdot \sin x \ dx\\  &= \left[-\sin x \cos x\right]\_{\frac{\pi}{2}}^{\pi} - \int\_{\frac{\pi}{2}}^{\pi} (-\cos x) \cos x \ dx\\  &= \left[-\sin x \cos x\right]\_{\frac{\pi}{2}}^{\pi} + \int\_{\frac{\pi}{2}}^{\pi} \cos^2 x \ dx\\  &= -\sin \pi \cos \pi - \left(-\sin \frac{\pi}{2} \cos \frac{\pi}{2}\right)+ \int\_{\frac{\pi}{2}}^{\pi} \cos^2 x \ dx\\  &= 0 \times (-1) - (-1) \times 0+ \int\_{\frac{\pi}{2}}^{\pi} \cos^2 x \ dx\\  &= 0+ \int\_{\frac{\pi}{2}}^{\pi} \cos^2 x \ dx\\  &=\int\_{\frac{\pi}{2}}^{\pi} \cos^2 x \ dx  \end{align\*}\]  Next, we use the trigonometric identity \(\cos^2 x = \frac{1 + \cos 2x}{2}\):  \begin{align\*}  &=\int\_{\frac{\pi}{2}}^{\pi} \frac{1+\cos(2x)}{2} \ dx \text{ , split the equation into two as follows and do the integration separately.}\\  &=\int\_{\frac{\pi}{2}}^{\pi} \frac{1}{2} \ dx+\int\_{\frac{\pi}{2}}^{\pi}\frac{\cos(2x)}{2} \ dx\\  &=\left[\frac{1}{2}\right]\_{\frac{\pi}{2}}^{\pi}+\frac{1}{2}\int\_{\frac{\pi}{2}}^{\pi}\cos(2x) \ dx\\  &\quad \text{Let } u = 2x \text{ when } x=\frac{\pi}{2} \text{ , } u =\pi \text{ , when } x=\pi\text{ , } u = 2\pi \text{ then for, } du = 2dx \text{ and } dx = \frac{du}{2}\\  \text{Value substitution }\\  &=\left[\frac{1}{2}\right]\_{\frac{\pi}{2}}^{\pi}+\frac{1}{2}\int\_{\pi}^{2\pi}\cos(u) \ du\\  &=\left[\frac{1}{2}\right]\_{\frac{\pi}{2}}^{\pi}+\frac{1}{2}\left[\sin(u)\right]\_{\pi}^{2\pi} \ du\\  &=\frac{1}{2} (\pi-\frac{\pi}{2}) +\frac{1}{2}\sin(2\pi-\pi)\\  &=\frac{\pi}{2} -\frac{\pi}{4} +\cancel{\frac{1}{2}\sin(2\pi-\pi)} \quad \text{where } \sin{2\pi-\pi}=\sin{\pi} = 0\\ \\  \int\_{\frac{\pi}{2}}^{\pi} \sin ^2 x \ dx&=\frac{\pi}{4}  \end{align\*} |

**Problem 4**

Let \(\mathcal{S} = \mathcal{P}(\mathbb{Z}\_3)\backslash\emptyset\). For \(A,B \in \mathcal{S}\), \(A \sim B\) if and only if there exists a bijection \(f: A \rightarrow B\). How many equivalence classes on \(\mathcal{S}\) are there, determined by \(\sim\)?

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| \text{Given} \(\mathcal{S} = \mathcal{P}(\mathbb{Z}\_3)\backslash\emptyset\)\\  \text{where,}  if we take any number that can divide by three with or without a remainder have the following formula, $n=3k+r$, due to this set $\mathbf{Z\_3}$ \text{ can only have 0 ,1 and 2 as a remainder in the : } \\  \[  \mathbb{Z}\_3=\{0,1,2\}  \]\\  from this we can get the set $\mathcal{P}(\mathbb{Z}\_3)=\{\{\emptyset\},\{0\},\{1\},\{2\},\{0,1\},\{0,2\},\{1,2\},\{0,1,2\}\}$  when we remove the empty set from the $\mathcal{P}(\mathbb{Z}\_3)\backslash\emptyset$, we will get three set of classes:-\\ \\  Set 1: \{\{0\},\{1\},\{2\}\} is create 1 equivalent class of size of 1\\ \\  Set 2: \{\{0,1\},\{0,2\},\{1,2\}\} is create 1 equivalent class of size 2\\ \\  Set 3: \{\{0,1,2\}\} is create 1 equivalent class of size 3\\ \\  from these extraction of subsets we can conclude that , we have 3 equivalent class on S. |

**Problem 5**

The random variable \(X\) has the distribution \(N(50, 8^2)\). The random variable \(Y\) is related to \(X\) by the formula \(Y=aX+b\), where \(a\) and \(b\) are constants with \(a>0\). Also, \(P(Y<74)=P(Y>146)=0.0668\). Without finding the values of \(a\) and \(b\), find the variance of \(Y\).

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| \[X \sim N(50,8^2)\]\\  mean$(\sigma)$ = 50 , Variance ($\sigma^2$)=  $8^2=64$ and The standard Deviation ($\sigma$)=8 \\  Y = aX +b, a $>$ 0  P(Y$<$ 74)= P(Y$>$ 146) = 0.0668,  from the Z- Score table we can get 0.06681 which gives the value of -1.5 that is approximately equal to the value of 0.0668.  \begin{figure}[h]  \centering  \includegraphics[width=0.5\linewidth]{Z.jpg}  \caption{Z-Score Table}  \label{fig:enter-label}  \end{figure}\\  P(Z$>$1.5) $\approx$ 0.0668, which means the Z-score probability will be approximately $\pm$1.5\\  \[  Z = \frac{Y-\mu\_y}{\sigma\_y}  \]  for the first distribution P(Y$<$74) = 0.0668 will represent the Z- score of -1.5.  \[  P(\frac{Y-\mu\_y}{\sigma\_y}<-1.5)= 0.0668  \]  \[  \frac{74-\mu\_y}{\sigma\_y}= -1.5 \quad \text{........eq. 1}  \]  for the second distribution P(y$>$146)=0.0668 will be represent the Z-score 1.5,\\  \[  \frac{146-\mu\_y}{\sigma\_y}= 1.5 \quad \text{........eq. 2}  \]  we are going to equate the two equations and we get :  \[  \frac{-(74-\mu\_y)}{\cancel{\sigma\_y}}= \frac{146-\mu\_y}{\cancel{\sigma\_y}}  \]  \[  -74+\mu\_y = 146-\mu\_y  \]  \[  \mu\_y = 110 ,  \]\\ \text{ substitute this value in the original  equation eq.1 or eq.2 we can do on both equation and it gives us the same answer.}  I choose the second equation and the result shows as follows:  \[  \frac{146-\mu\_y}{\sigma\_y}= 1.5 \quad \text{........eq. 2}  \]  \[  \frac{146-110}{\sigma\_y}=1.5  \]  \[  \sigma\_y=\frac{36}{1.5} = 24  \]  The Next calculation we will do is to find the coefficient a  \[ \sigma\_y = a \* \sigma\_x , \] but we already know the value of the $\sigma\_x$=8 and $\sigma\_y$=24,  \[ 24=a\*8\]  \[a=24\]  Finally, we have every thing to calculate for the variance of Y  \[  Var(Y)=a^2\*Var(X)  \] we already know the value of the Variance of X from the given which is \\Var(X) = $8^2$  \[  Var(Y)=a^2\*8^2  \]  \[  Var(Y)=3^2\*8^2  \]  \[  Var(Y)=9\*64  \]  \[  Var(Y)=576  \] |