Notes on "Finite-Dimensional Vector Spaces" by Paul R. Halmos

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Each `\section` corresponds to the scope of one member's assignment, and each `\subsection` corresponds to one theorem or exercise in the textbook, specified in the format $m.n$ where m is the section number and n is the theorem/exercise number. If n is not given, we use $n = 1$ instead.

1 Toga (2022/09/19)

1.1 Exercise 1.1

(a) Since addition is commutative, $0+\alpha = \alpha+0$ holds. We also have $\alpha+0 = \alpha$ by definition, hence $0 + \alpha = \alpha$.

2 Mohehe

2.1 Exercise 1.1

(b) Since addition is commutative, $\alpha + \beta + (-\alpha) = \beta + \alpha + (-\alpha)$.

(c)

(d)

(e)

(f)

(g)

3 Joh (2022/09/19)

3.1 Exercise 1.2

(a) The set of positive integers is not a field since there is no additive inverse for 1.

(b) The set of integers is not a field since there is no multiplicative inverse for 2.

(c) There exists a bijective map φ from $\mathbb{N}$ (or $\mathbb{Z}$) to $\mathbb{Q}$ [1], where $\mathbb{Q}$ is a field [2]. We can make $\mathbb{N}$ a field by re-defining (i) addition by $a \oplus b = \varphi^{-1}(\varphi(a) + \varphi(b))$ and (ii) multiplication by $a \otimes b = \varphi^{-1}(\varphi(a)\varphi(b))$ for each $a, b \in \mathbb{N}$. Note that the additive and multiplicative identities become $\varphi^{-1}(0)$ and $\varphi^{-1}(1)$, respectively. For each $\alpha \in \mathbb{N}$, the additive inverse becomes $\varphi^{-1}(-\varphi(\alpha))$, and the multiplicative inverse becomes $\varphi^{-1}(1/\varphi(\alpha))$ if $\alpha \neq \varphi^{-1}(0)$.

References

[1] https://proofwiki.org/wiki/Rational_Numbers_are_Countably_Infinite

[2] https://proofwiki.org/wiki/Rational_Numbers_form_Field