Tutorial No 4 +
Tutorial No. 5

A11 + A12

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(1)

1

Inverse z-transform of $\frac{2z^2+3z}{(z+2)(z-4)}$

Solution :

let $F(z) = \frac{2z^2+3z}{(z+2)(z-4)}$

$\frac{F(z)}{z} = \frac{2z+3}{(z+2)(z-4)} \rightarrow (1)$

Integration by Partial fraction,

$\frac{2z+3}{(z+2)(z-4)} = \frac{A}{z+2} + \frac{B}{z-4}$ — (2)

$2z+3 = A(z-4) + B(z+2)$

Substituting,

$z = -2$

$-4+3 = A(-2-4)$

$-6A = -1 \Rightarrow \boxed{A = 1/6}$

;

$z = 4$

$8+3 = B(4+2)$

$6B = 11 \Rightarrow \boxed{B = \frac{11}{6}}$

Substituting in 2

$$\frac{2z+3}{(z+2)(z-4)} = \frac{1/6}{z+2} + \frac{11/6}{z-4}$$

From 1

$$\frac{F(z)}{z} = \frac{1}{6(z+2)} + \frac{11}{6(z-4)}$$

$$F(z) = \frac{z}{6(z+2)} + \frac{11z}{6(z-4)}$$

$$Z\{f(n)\} = \frac{1}{6}\left\{\frac{z}{z+2}\right\} + \frac{11}{6}\left\{\frac{z}{z-4}\right\}$$

Taking inverse,

$$f(n) = \frac{1}{6} Z^{-1}\left\{\frac{z}{z+2}\right\} + \frac{11}{6} Z^{-1}\left\{\frac{z}{z-4}\right\}$$

$$Z^{-1}\left[\frac{2z^2+3z}{(z+2)(z-4)}\right] = \frac{1}{6}(-2)^n + \frac{11}{6}(4)^n$$

2

$$U(z) = \frac{3z^2 + 5z + 14}{(z-1)^3}$$

$u_2 = ?$ $u_3 = ?$

According to the shifting Property :

$$Z(u_{n+k}) = z^k\left[U(z) - u_0 - \frac{u_1}{z} - \frac{u_2}{z} \cdots \frac{u_{k-1}}{z^{k-1}}\right]$$

For u_2

$\cancel{Z(u_2)}$: $$\boxed{Z(u_{n+2}) = z^2\left[U(z) - u_0 - \frac{u_1}{z}\right]} \quad -(1)$$

for u_3 =>

$$\boxed{Z(u_{n+3}) = z^3\left[U(z) - u_0 - \frac{u_1}{z} - \frac{u_2}{z^2}\right]} \quad -(2)$$

By initial Value Theorem :

$$u_0 = \lim_{z\to\infty} U(z)$$

$$= \lim_{z\to\infty}\left\{\frac{3z^2 + 5z + 14}{(z-1)^3}\right\}$$

$$\Rightarrow \lim_{z\to\infty} \frac{z^2(3 - 5/z + 14/z^2)}{z^3(1 - 1/z)^3},$$ Taking z^2 common from numerator & z^3 common from denominator.

$$\Rightarrow \lim_{z \to \infty} \frac{\cancel{z}(3 + 5/z + 14/z^2)}{z\left(1 - \frac{1}{z}\right)^3} = \frac{3}{\infty}$$

$$\therefore \boxed{u_0 = 0}$$

$$u_1 = \lim_{z \to \infty} z\left[U(z) - U_0\right]$$

$$\Rightarrow \lim_{z \to \infty} z\left[\frac{3z^2 + 5z + 14}{(z-1)^3} - 0\right]$$

$$\Rightarrow \lim_{z \to \infty} z\left[\frac{z^2(3 + 5/z + 14/z^2)}{z^3(1 - 1/z)^3}\right]$$

Taken z^2 common from numerator &
z^3 common from denominator.

$$\Rightarrow \lim_{z \to \infty} \frac{\cancel{z^3}}{\cancel{z^3}}\left\{\frac{(3 + 5/z + 14/z^2)}{(1 - 1/z)^3}\right\}$$

$$\Rightarrow \lim_{z \to \infty} \frac{3 + 5/z + 14/z^2}{(1 - 1/z)^3} = 3$$

$$\boxed{u_1 = 3}$$

For finding U_2

$$U_2 = \lim_{z\to\infty} z^2\left(U(z) - U_0 - \frac{U_1}{z}\right) \quad \text{— i}$$
, by initial value theorem.

Substituting the values of U_0 & U_1 in equation i ~~with limits~~

$$U_2 = \lim_{z\to\infty} z^2\left\{\frac{3z^2+5z+14}{(z-1)^3} - 0 - \frac{3}{z}\right\}$$

$$= \lim_{z\to\infty} z^{\cancel{2}}\left\{\frac{3(3z^2+5z+14) - 3(z-1)^3}{\cancel{z}(z-1)^3}\right\}$$

$$\Rightarrow \lim_{z\to\infty} z\left\{\frac{3z^3+5z^2+14z-3(z^3-3z^2+3z-1)}{(z-1)^3}\right\}$$

$$\Rightarrow \lim_{z\to\infty} z\left\{\frac{\cancel{3z^3}+5z^2+14z-\cancel{3z^3}+9z^2-9z+3}{(z-1)3}\right\}$$

$$\Rightarrow \lim_{z\to\infty} z\left(\frac{14z^2+5z+3}{(z-1)^3}\right)$$

Taking out z^2 common from numerator & z^3 from denominator

$$\Rightarrow \lim_{z\to\infty} \frac{\cancel{z^3}}{\cancel{z^3}}\left\{\frac{14+5/z+3/z^2}{\{1-1/z\}^3}\right\} = 14$$

$$\lim_{z\to\infty} \frac{14 + 5/z + 3/z^2}{\left(1-\frac{1}{z}\right)^3} = 14$$

$$\boxed{U_2 = 14}$$

for $U_3 = \lim_{z\to\infty} z^3\left\{U(z) - U_0 - \frac{U_1}{z} - \frac{U_2}{z^2}\right\}$

Substituting values of U_0, U_1 & U_2 in above eq^n

$$U_3 = \lim_{z\to\infty} z^3\left\{\frac{3z^2+5z+14}{(z-1)^3} - 0 - \frac{3}{z} - \frac{14}{z^2}\right\}$$

$$\Rightarrow \lim_{z\to\infty} z^3\left\{\frac{3z^2+5z+14}{(z-1)^3} - \frac{(3z+14)}{z^2}\right\}$$

$$\Rightarrow \lim_{z\to\infty} z^3\left\{\frac{(3z^2+5z+14)z^2 - (3z+14)(z-1)^3}{z^2(z-1)^3}\right\}$$

$$\Rightarrow \lim_{z\to\infty} z\left\{\frac{3z^4+5z^3+14z^2 - (3z+14)(z^3-3z^2+3z-1)}{(z-1)^3}\right\}$$

$$\Rightarrow \lim_{z\to\infty} z\left\{\frac{\cancel{3z^4} + \cancel{5z^3} + 14z^2 - \cancel{3z^4} + \cancel{9z^3} - 9z^2 + 3z - \cancel{14z^3} + 42z^2 - 42z + 14}{(z-1)^3}\right\}$$

$$\Rightarrow \lim_{z\to\infty} \frac{z(47z^2 - 39z + 14)}{(z-1)^3}$$, take z^2 common from numerator & z^3 common from denominator

$$\lim_{z\to\infty} \frac{z^3}{z^2}\left\{\frac{47 - 39/z + 14/z^2}{(1-1/z)^3}\right\} = \lim_{z\to\infty} \frac{47 - 39/z + 14/z^2}{(1-1/z)^2} = 47$$

$$\boxed{U_3 = 47}$$

$\therefore$ $\boxed{\text{Values of } U_2 \text{ \& } U_3 \text{ are } 14,\ 47}$

3

annual rate = 11 % $\quad r = \frac{11}{100} = 0.11$

$n = 30$ years

$P_0 = 10000$

According to compound interest scenario,

$$\boxed{P_{n+1} = P_n + rP_n}$$

$$\Rightarrow P_n + rP_n$$

$$P_{n+1} = P_n + 0.11P_n$$

$$= P_n(1 + 0.11)$$

$$P_{n+1} - \cancel{1.0}1.11 P_n = 0$$

$$\Rightarrow \boxed{P_{n+1} - 1.11P_n = 0}$$

Auxillary Equation

$$r - 1.11 = 0$$

$$\boxed{r = 1.11}$$

$$P_n = C_1 r^n$$

$$P_1 = C_1 (1.11)^n \quad — (1)$$

we ~~k~~know $P_0 = 10000$.

Putting $P = 10,000$ $n = 1$

$$10000 = C_1 \cdot 1$$

Therefore $\boxed{C_1 = 10000}$

Substituting value of C_1 & n in eq^n (i)

~~P_A~~

$$P_n = 10000\,(1.11)^{30}$$

$$= 10000 \times 22.8922$$

$$= 228922.96$$

$$P_n \approx 228923$$

$\therefore$ P_n = Rs 2,28,923 (approximately)

$$\boxed{\text{After 30 years, } P_n = \text{Rs } 2,28,923 \text{ (approx)}}$$ Ans