MAT-2001

Tutorial 2

20BCE10077

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A11 + A12

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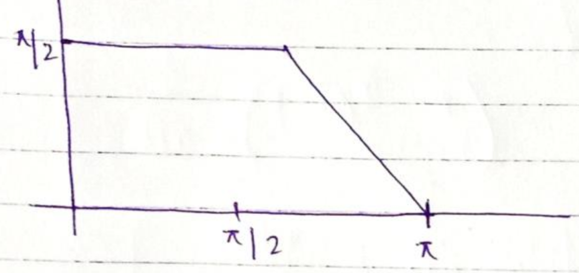
Q1



function would be

$$f(x) = \begin{cases} \pi/2 & 0 < x < \pi/2 \\ \pi - x & \pi/2 < x < \pi \end{cases}$$

the fourier series is written as

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{2n\pi t}{T} + \sum_{n=1}^{\infty} b_n \sin \frac{2n\pi t}{T}$$

here $a = 0 \quad b = \pi$

$T = b - a = \pi$

$$\boxed{f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos 2nx + \sum_{n=1}^{\infty} b_n \sin 2nx}$$

Calculating a_0

$$a_0 = \frac{2}{T}\int_0^b f(t)\,dt$$

$$\Rightarrow \frac{2}{\pi}\int_0^{\pi} f(x)\,dx$$

$$\Rightarrow 2/\pi\left[\int_0^{\pi/2} \pi/2\,dx + \int_{\pi/2}^{\pi} \pi - x\,dx\right]$$

$$\Rightarrow \frac{2}{\pi}\left[\left[\frac{\pi}{2}x\right]_0^{\pi/2} + \left[\pi x - \frac{x^2}{2}\right]_{\pi/2}^{\pi}\right]$$

$$\Rightarrow \frac{2}{\pi}\left[\frac{\pi^2}{4} + \left\{\left[\pi^2 - \frac{\pi^2}{2} - \frac{\pi^2}{2} + \frac{\pi^2}{8}\right]\right\}\right]$$

$$\Rightarrow \frac{2}{\pi}\left[\frac{\pi^2}{4} + \left[\frac{8\pi^2 - 4\pi^2 - 4\pi^2 + \pi^2}{8}\right]\right]$$

$$\Rightarrow \frac{2}{\pi}\left[\frac{\pi^2}{4} + \frac{\pi^2}{8}\right] \Rightarrow \frac{2}{\pi} \times \frac{3\pi^2}{8}$$

$$\Rightarrow \frac{3\pi}{4}$$

$$a_n = \frac{2}{T}\int_0^b f(t)\cos\frac{2n\pi t}{T}\,dt$$

$$a_n = \frac{2}{\pi}\left[\int_0^{\pi/2} \pi/2 \cos 2nx \, dx + \int_{\pi/2}^{\pi} (\pi - x)\cos 2nx \, dx\right.$$

$$\Rightarrow \frac{2}{\pi}\left\{\left[\frac{\pi}{2}\frac{\sin 2nx}{2n}\right]_0^{\pi/2} + \left[(\pi-x)\frac{\sin 2nx}{2n} - (-1)\left(\frac{-\cos 2nx}{4n^2}\right)\right]_{\pi/2}^{\pi}\right.$$

$$\Rightarrow \frac{2}{\pi}\left[\frac{\pi}{2}(0) + \right.$$

$$\Rightarrow \frac{2}{\pi}\left(\frac{-1}{4n^2} + \frac{(-1)^n}{4n^2}\right)$$

$$= \frac{2}{4n^2\pi}\left((-1)^n - 1\right)$$

$$b_n = \frac{2}{\pi}\left[\int_0^{\pi/2} \frac{\pi}{2}\sin 2nx \, dx + \int_{\pi/2}^{\pi} (\pi - x)\sin 2nx \, dx\right.$$

$$\Rightarrow \frac{2}{\pi}\left[-\frac{\pi}{2}\left(\frac{\cos 2nx}{2n}\right)_0^{\pi/2} + \left[(\pi - x)\left(\frac{-\cos 2nx}{2n}\right) + n\left(\frac{\sin 2nx}{4n^2}\right)\right]_{\pi/2}^{\pi}\right.$$

$$b_n = \frac{2}{\pi}\left[-\pi((\cos n\pi - 1) - \frac{\pi}{2}\left(\frac{-\cos n\pi}{2n}\right)\right]$$

$$b_n = \frac{1}{2n}$$

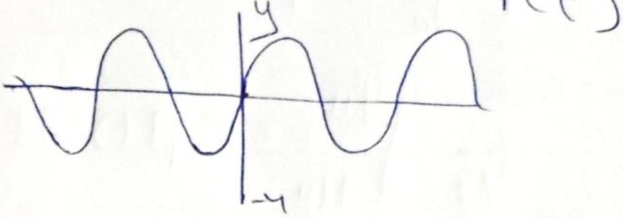
$$\therefore f(x) = \frac{3\pi}{8} + \sum_{n=1}^{\infty} \frac{1}{2n^2\pi}\left((-1)^n - 1\right)\frac{\cos 2n\pi x}{\pi^2} +$$

$$\sum_{n=1}^{\infty} \frac{1}{2n}\sin\frac{2n\pi x}{\pi}.$$

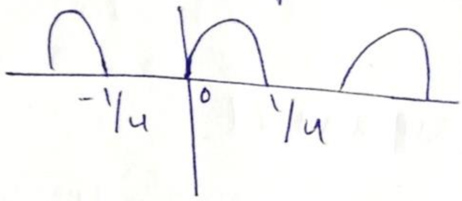
2

$f(t) = I_0 \sin 4\pi t$ (1)

for a sin function



After passing through a half-wave rectifier the graph becomes



now the new $f_1(t) = I_0 \sin 4\pi t$

~~Ksinwt~~
Equation - $K \sin \omega t$ ~~whe~~ & $T = 2\pi/\omega$
on comparing this with eq (1)
we get $\boxed{\omega = 4\pi}$ & $\boxed{T = 1/2}$

and $a = -1/4$ (leftmost)

$b = 1/4$ (right most)

Now the function becomes

$$f_1(t) = \begin{cases} 0 & -1/4 < x < 0 \\ I_0 \sin 4\pi t, & 0 < x < 1/4 \end{cases}$$

To find a_0

$$a_0 = \frac{2}{T}\int_a^b f(t)\,dt$$

$$= \frac{2}{1/2}\int_{-1/4}^{1/4} f(t)\,dt$$

$$= 4\left\{\int_{-1/4}^{0} 0\,dt + \int_0^{1/4} I_0 \sin 4\pi t\,dt\right\}$$

$$= 4I_0\int_0^{1/4} \sin 4\pi t\,dt$$

$$\Rightarrow 4I_0\left[-\frac{\cos 4\pi t}{4\pi}\right]_0^{1/4}$$

$$\Rightarrow 4I_0\left[-\frac{\cos\pi}{4\pi} + \frac{\cos 0}{4\pi}\right] \Rightarrow 4I_0\left[\frac{2}{4\pi}\right]$$

$$\Rightarrow \boxed{a_n = \frac{2I_0}{\pi}}$$

$$a_n = \frac{2}{T}\int_a^b f(t)\cos\frac{2n\pi t}{T}\,dt$$

$$= \frac{2}{1/2}\int_{-1/4}^{1/4} f(t)\cos\frac{2n\pi t}{T}\,dt$$

$$= 4\int_{-1/4}^{0} 0\cos\frac{2n\pi t}{T}\,dt + 4\int_0^{1/4} I_0\sin 4\pi t\cos\frac{2n\pi t}{T}\,dt$$

$$\Rightarrow 4I_0 \int_0^{1/4} \sin 4\pi t \cos \frac{2n\pi t}{1/2} \, dt$$

$$\Rightarrow 4I_0 \int_0^{1/4} \sin 4\pi t \cos 4n\pi t \, dt$$

using formula $\sin a \cos b = \frac{1}{2}\left(\sin(a+b) + \sin(a-b)\right)$

$$\Rightarrow 2I_0 \int_0^{1/4} \sin(1+n)4\pi t + \sin(1-n)4\pi t \, dt$$

$$\Rightarrow 2I_0 \left[\frac{-\cos (1+n) 4\pi t}{(1+n) 4\pi} - \frac{\cos (1-n) 4\pi t}{(1-n) 4\pi} \right]_0^{1/4}$$

$$\Rightarrow \boxed{\frac{-I_0}{2\pi} \left(\frac{(-1)^{n+1} - 1}{n+1} + \frac{(-1)^{1-n} - 1}{1-n} \right)}$$

Calculating b_n

$$= \frac{2}{T} \int_a^b f(t) \sin \frac{2n\pi t}{T} \, dt$$

$$\Rightarrow 4I_0 \int_0^{1/4} \sin 4\pi t \sin \frac{2n\pi t}{1/2} \, dt$$

$$\Rightarrow 4I_0 \int_0^{1/4} \sin 4\pi t \sin 4n\pi t \, dt$$

$$\Rightarrow 4\pi \quad \frac{4I_0}{\pi} \int_0^{1/4} \cos(1-n)4\pi t - \cos(1+n)4\pi t \, dt$$

$$\Rightarrow 2I_0 \left[\frac{\sin(1-n)4\pi t}{(1-n)4\pi} - \frac{\sin(1+n)4\pi t}{(1+n)4\pi} \right]_0^{1/4}$$

now since we know

$\sin n\pi = 0$, where $n = 1, 3, 7,$ odd numbers

& $\sin n\pi = 0$, where $n =$ even numbers

therefore $\left. \begin{array}{l} \sin(1-n)\pi = 0 \ \& \\ \sin(1+n)\pi = 0 \end{array} \right\}$ as

$$\therefore \quad b_n = 2I_0 \times 0$$

$$\boxed{b_n = 0}$$

Thus fourier series is given by

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{2\pi n t}{T}\right) + b_n \sin\left(\frac{2\pi n t}{T}\right)$$

$$\boxed{f_1(t) = \frac{I_0}{\pi} + \sum_{n=1}^{\infty} \frac{-I_0}{2\pi}\left(\frac{(-1)^{n+1} - 1}{1+n} + \frac{(-1)^{1-n} - 1}{(1-n)}\right)}$$

answer

where $T = 1/2$ $a = -1/4$ $b = 1/4$