Assignment 3: Applied Linear Algebra

1. Consider the vector space $C[-\pi, \pi]$. For $f, g \in C[-\pi, \pi]$ an inner product is defined by

$$\langle f, g \rangle = \int_{-\pi}^{\pi} f(x)g(x)dx.$$

   Check whether the set $B = \{\sin(x), \sin(2x), \sin(4x)\}$ is orthogonal or not (with respect to this inner product).

2. Using Gram-Schmidt method convert the basis $B = \{(1, 2, -2), (4, 3, 2), (1, 2, 1)\}$ to an orthogonal basis of $\mathbb{R}^3$. Find the coordinate of $(1, 1, 1)$ with respect to the orthogonal basis.

3. Consider the subspace $W = \{(x, y, z)|x + y - 2z = 0\}$ of $\mathbb{R}^3$. Find a basis of $W^\perp$ (orthogonal complement of W).

4. Consider the subspace $W = \{(x, y, z)|x - y - z = 0\}$ of $\mathbb{R}^3$. Express $x = (2, 3, 5)$ as $x = p + q$ where $p \in W$ and $\langle p, q \rangle = 0$.

5. Consider two orthonormal bases of $B = \{(3/5, 4/5, 0), (-4/5, 3/5, 0), (0, 0, 1)\}$ and $C = \{(3/7, -6/7, -2/7), (2/7, 3/7, -6/7), (6/7, 2/7, 3/7)\}$ of $\mathbb{R}^3$. Show that the transition matrix from B coordinate to C coordinate is an orthogonal matrix.

6. Find the QR-factorization of $A = \begin{bmatrix} 4 & -3 \\ 11 & -2 \\ 5 & -10 \end{bmatrix}$. Using this find the least square solution of $Ax = b$, where $b = (0, 15, -20)^T$.