Design & Analysis of Algorithm

20/09/21

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Assignment 1

Slot → B11

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1) Optimal Binary Search tree

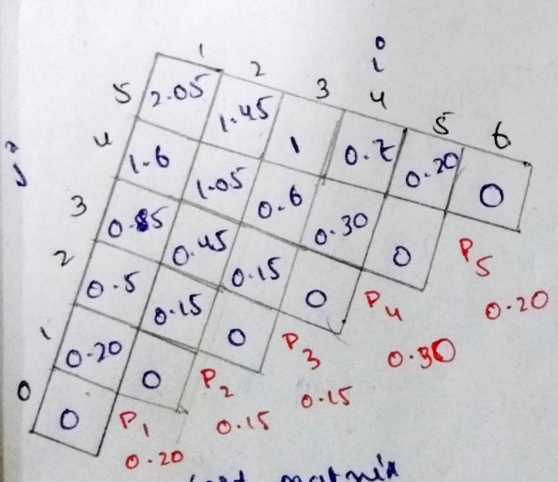
Key Values	P	Q	R	S	T
frequencies	0.20	0.15	0.15	0.30	0.20

Formula :

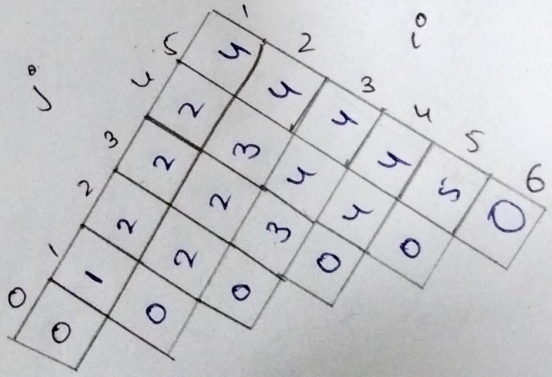
$$\rightarrow c(i,j) = \min_{i \le k \le j} \{ (c(i,k-1) + c(k+1,j) + \sum_{s=i}^{j} p_s$$

for $1 \le i \le j \le n$

$$\rightarrow c(i,i) = p_i \quad \text{for } 1 \le i \le n.$$



Cost matrix



Root matrix

for $1 \le i \le n$

we know $c(i,i) = p_i$, therefore;

$c(1,1) = 0.20$ $\quad$ $c(2,2) = 0.15$ $\quad$ $c(3,3) = 0.15$ $\quad$ $c(4,4) = 0.30$ $\quad$ $c(5,5) = 0.20$

Now starting with, $i = 1, j = 2$

•) $c[1,2] \rightarrow k = 1, 2$

using formula:

$k=1$, $c[1,2] = c[1,0] + c[2,2] + P_1 + P_2 = 0.0 + 0.15 + 0.20 + 0.15$
$= 0.5$

$k=2$, $c[1,2] = c[1,1] + c[3,2] + P_1 + P_2 = 0.20 + 0.0 + 0.20 + 0.15$
$= 0.55$

choosing min. out of 2,

so, $\boxed{c[1,2] = 0.5, \quad R[1,2] = 1}$

•) $c[2,3] \rightarrow k = 2, 3$

using formula:

$k=2$, $c[2,3] = c[2,1] + c[3,3] + P_2 + P_3 = 0.0 + 0.15 + 0.15 + 0.15 = 0.45$

$k=3$, $c[2,3] = c[2,2] + c[4,3] + P_2 + P_3 = 0.15 + 0.0 + 0.15 + 0.15 = 0.45$

choosing min out of 2

so $\boxed{c[2,3] = 0.45, \quad R[2,3] = 2}$

• $c[3,4] \rightarrow k = 3, 4$, using formula;

$k=3$, $c[3,4] = c[3,2] + c[4,4] + P_3 + P_4 = 0.0 + 0.30 + 0.15 + 0.30 = 0.75$

$k=4$, $c[3,4] = c[3,3] + c[5,4] + P_3 + P_4 = 0.15 + 0.0 + 0.15 + 0.30 = 0.6$

so, $\boxed{c[3,4] = 0.6, \quad R[3,4] = 4}$

•) $c[4,5] \rightarrow k = 4, 5$

By formula,

$$c[i,j] = \min_{i \le k \le j} \{ c[i,k-1] + c[k+1,j] + \sum_{s=i}^{j} P_s \}$$

for $1 \le i \le j \le n$

$k=4$, $c[4,5] = c[4,3] + c[5,5] + P_4 + P_5$

$= 0 + 0.20 + 0.30 + 0.20 = 0.7$

$k=5$, $c[4,5] = c[4,4] + c[6,5] + P_4 + P_5$

$= 0.30 + 0 + 0.20 + 0.30 = 0.8$

so, $\boxed{c[4,5] = 0.7 \quad R[4,5] = 4}$

~~$c[4,5] = 0.7 \quad R[4,5] = 4$~~

• $c[1,3] \rightarrow k = 1, 2, 3$

$k=1$, $c[1,3] = c[1,0] + c[2,3] + P_1 + P_2 + P_3 = 0 + 0.45 + 0.5 = 0.95$

$k=2$, $c[1,3] = c[1,1] + c[3,3] + P_1 + P_2 + P_3 = 0.20 + 0.15 + 0.5 = 0.85$

$k=3$, $c[1,3] = c[1,2] + c[4,3] + P_1 + P_2 + P_3 = 1$

$= 0.5 + 0 + 0.5$

Choosing min value out of 3:

$\boxed{c[1,3] = 0.85 \quad R[1,3] = 2}$

•) $c[2,4] \rightarrow k = 2, 3, 4$

$k=2$, $c[2,4] = c[2,1] + c[3,4] + P_2 + P_3 + P_4$

$= 0 + 0.6 + 0.6 = 1.2$

$k=3$, $c[2,4] = c[2,2] + c[4,4] + P_2 + P_3 + P_4$

$= 0.15 + 0.30 + 0.6 = 1.05$

$k=4$, $c[2,4] = c[2,3] + c[5,4] + P_2 + P_3 + P_4$

$= 0.45 + 0.6 = 1.05$

Choosing min;

$C[2,4] = 1.05$

$R[2,4] = 3$

- $C[3,5] \rightarrow k = 3, 4, 5$

using formulas:

$k=3$, $c[3,5] = c[3,3] + c[4,5] + P_3 + P_4 + P_5$

$k=4$, $c[3,5] = c[3,3] + c[5,5] + P_3 + P_4 + P_5$

$k=5$, $c[3,5] = c[3,4] + c[6,5] + P_3 + P_4 + P_5$

$0 + 0.7 + 0.15 + 0.30 + 0.20 = 1.35$

$0.15 + 0.20 + 0.15 + 0.30 + 0.20 = 1$

$0.6 + 0 + 0.15 + 0.30 + 0.20 = 1.25$

$C[3,5] = 1$, $R[3,5] = 4$

- $c[1,4] = k = 1, 2, 3, 4$

1) $k=1$, $c[1,4] = c[1,0] + c[2,4] + P_1 + P_2 + P_3 + P_4$
2) $k=2$, $c[1,4] = c[1,1] + c[3,4] + P_1 + P_2 + P_3 + P_4$
3) $k=3$, $c[1,4] = c[1,2] + c[4,4] + P_1 + P_2 + P_3 + P_4$
4) $k=4$, $c[1,4] = c[1,3] + c[5,4] + P_1 + P_2 + P_3 + P_4$

1) $= 0 + 1.05 + 0.8 = 1.85$
2) $= 0.20 + 0.6 + 0.8 = 1.6$
3) $= 0.5 + 0.30 + 0.8 = 1.6$
4) $= 0.85 + 0 + 0.8 = 1.65$

min out of 4

$c[1,4] = 1.6$, $R[1,4] = 2$

- $c[2,5] \Rightarrow k = 2, 3, 4, 5$, using formulas

$k=2 \Rightarrow c[2,5] = c[2,1] + c[3,5] + P_2 + P_3 + P_4 + P_5$

$k=2$, $c[2,5] = c[2,1] + c[3,5] + 0.8 = 0 + 1 + 0.8 = 1.8$

$k=3$, $c[2,5] = c[2,2] + c[4,5] + 0.8 = 0.15 + 0.7 + 0.8$

$P_2 + P_3 + P_4 + P_5 = 1.65$

$k=4$, $c[2,5] = c[2,3] + c[5,5] + 0.8 = 0.45 + 0.20 + 0.8 = 1.45$

$k=5$, $c[2,5] = c[2,4] + c[6,5] + 0.8 = 1.05 + 0.8 = 1.85$

choosing min of 4:

$$c[2,5] = 1.45, \quad R[2,5] = 4$$

- $c[1,5] \Rightarrow k = 1, 2, 3, 4, 5$

$P_1 + P_2 + P_3 + P_4 + P_5 = 1.0$

$k=1$, $c[1,5] = c[1,0] + c[2,5] + 1 = 0 + 1.45 + 1 = 2.45$

$k=2$, $c[1,5] = c[1,1] + c[3,5] + 1 = 0.20 + 1 + 1 = 2.2$

$k=3$, $c[1,5] = c[1,2] + c[4,5] + 1 = 0.5 + 0.7 + 1 = 2.2$

$k=4$ $c[1,5] = c[1,3] + c[5,5] + 1 = 0.85 + 0.20 + 1 = 2.05$

$k=5$ $c[1,5] = c[1,4] + c[6,5] + 1 = 1.6 + 1 = 2.6$

$$c[1,5] = 2.05 \quad , \quad R[1,5] = 4$$

Constructing optimal Binary Search tree:

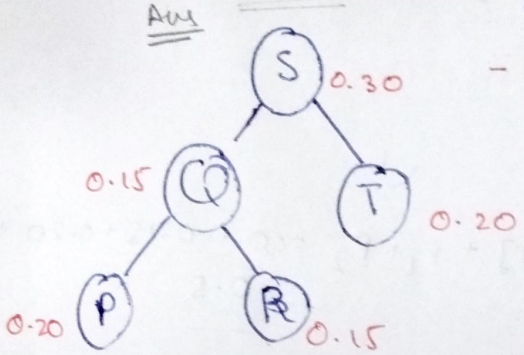
R[1,5] = 4 → Root → S, Left PQR, Right T

R[1,3] = 2 → Root → Q, Left P, Right R

~~PQRST~~
1 2 3 4 5
P Q R S T

Ans

PQR
1 2 3



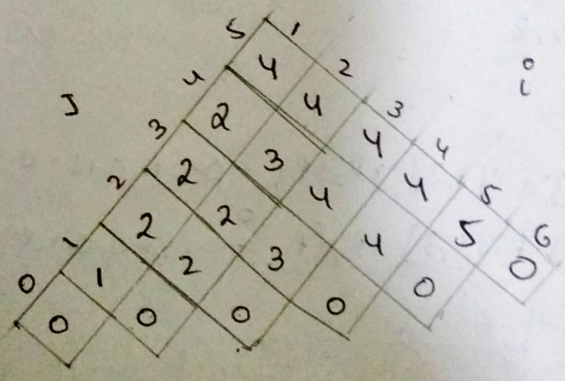
- cost - 0.30

optimal binary search tree

$$S(n) = 1 \times 0.30 + 2 \times 0.15 + 3 \times 0.20 + 3 \times 0.15 + 2 \times 0.20$$

$$= 2.05$$

→ optimal value



Root matrix

Q2

Construct Huffman Tree & predict Huffman code for

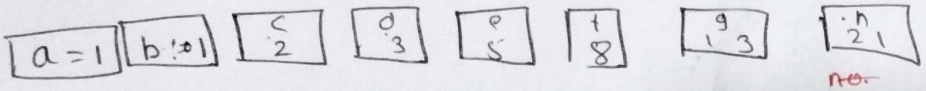
a:1, b:1, c:2, d:3, e:5, f:8, g:13, h:21

Sum of all values → 54

Total no. of characters → 8

Probability → 54/8 = 6.75

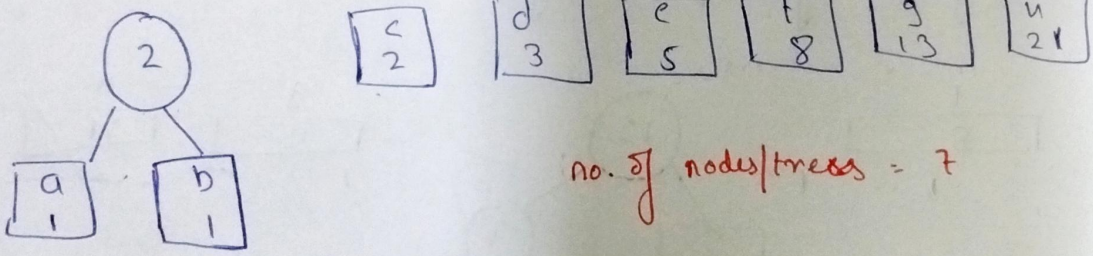
1)



→ The keys & frequencies are already arranged in ascending order.

Choosing 2 min frequencies & joining them, here a:1 & b:1

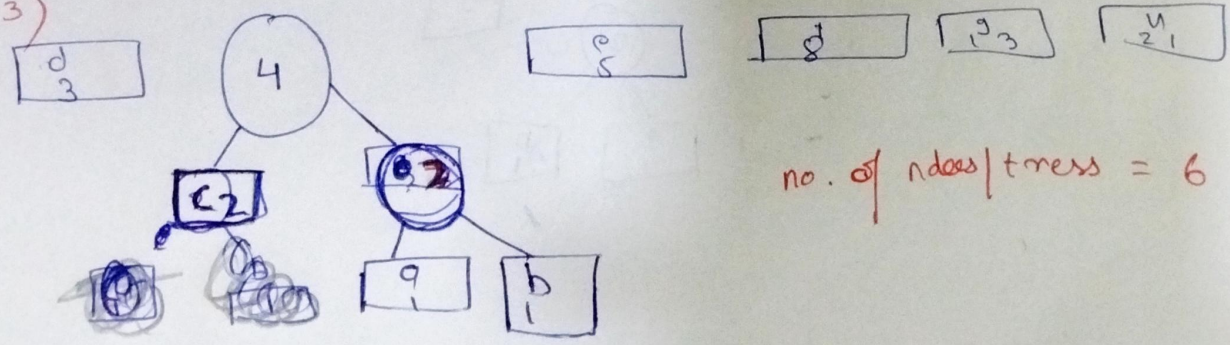
2)



no. of nodes/trees = 7

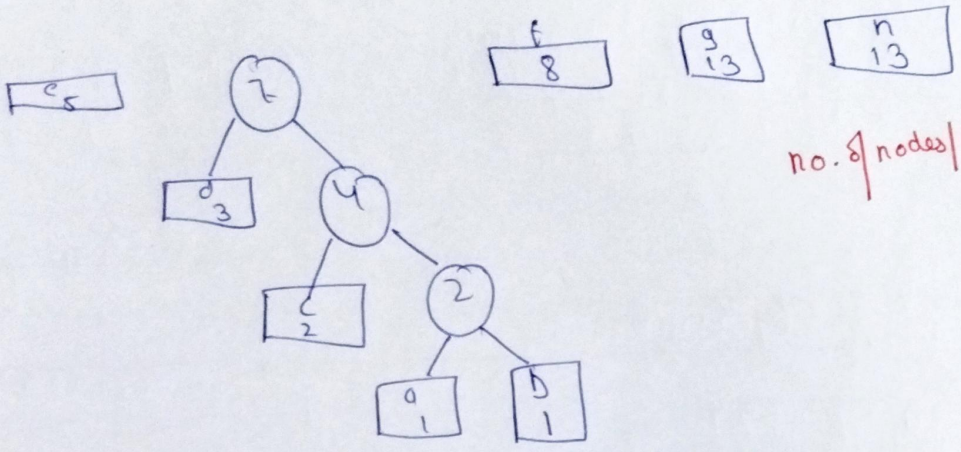
• again at joining 2 & c as they have min frequencies

3)



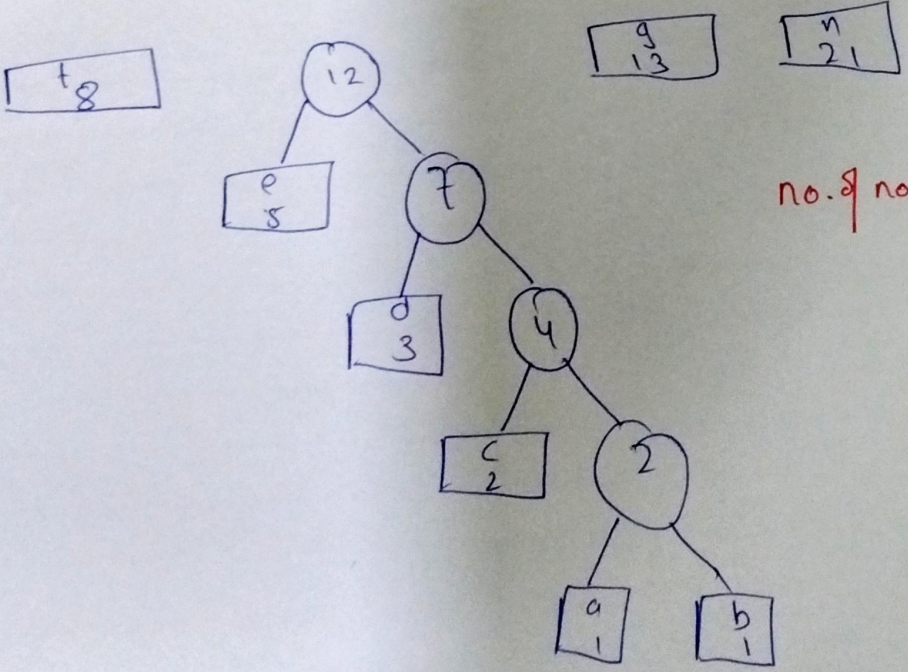
no. of nodes/trees = 6

- joining d & 4 as they have min frequencies
  → 3+4=7
  → this tree will be put after e & before f



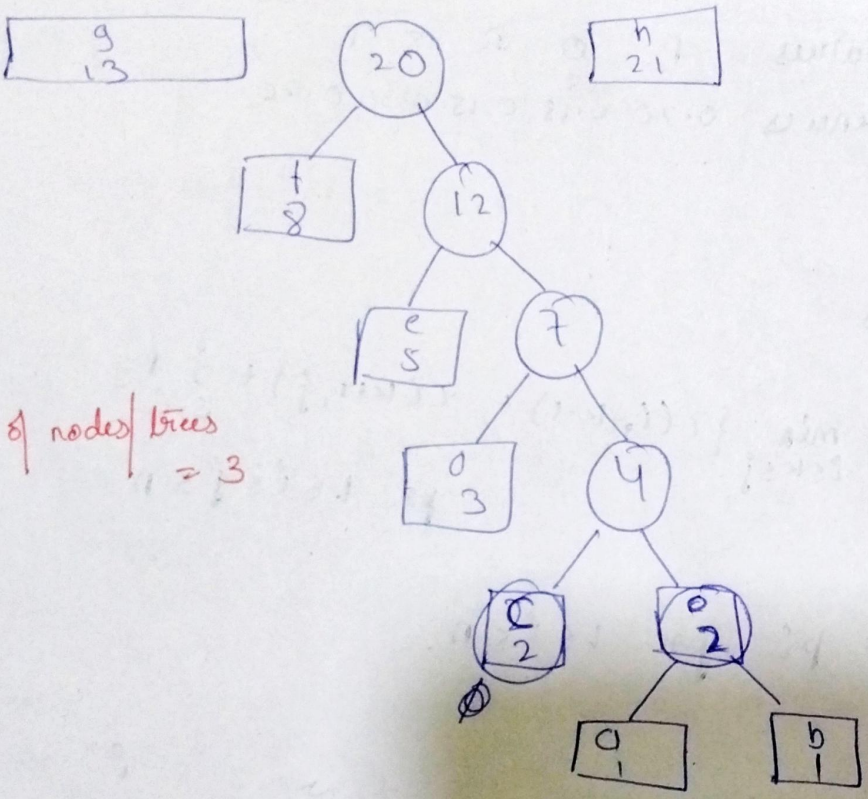
no. of nodes/trees = 5

- Now joining 7 & e as they have min frequencies & 7+5=12, so putting it after f & before g.



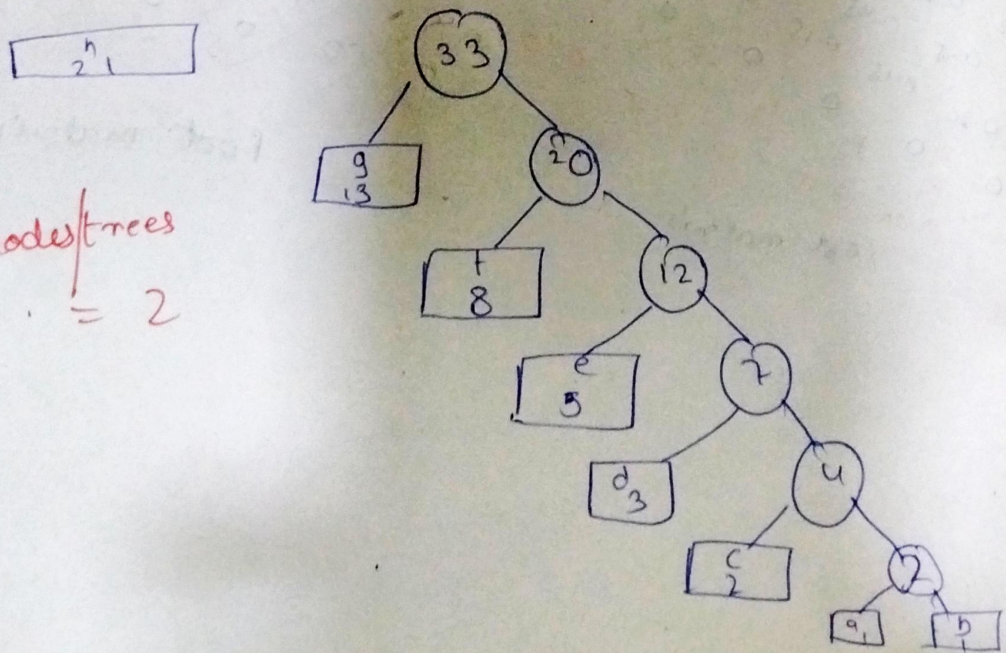
no. of nodes/trees = 4.

• now joining 12 & f as they ~~have~~ are minimum.
12+8 = 20, so putting it after g & before 4



no. of nodes/trees = 3

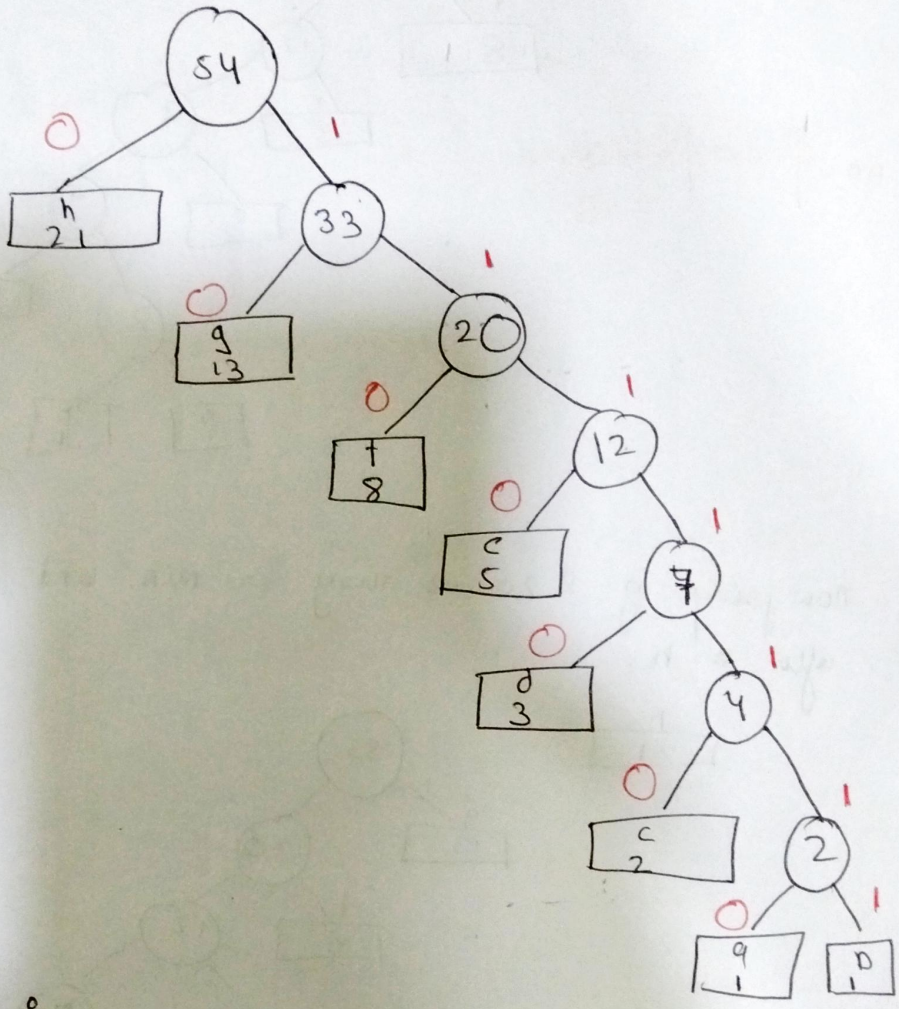
• Now joining g & 20, as they are minimum & putting it after h.



no. of nodes/trees = 2

finally joining / combining h & 33

Huffman tree



For huffman codes :

Put Right edges as 1

& left edges as 0

Huffman codes

Character	code
a →	1111110
b →	1111111
c →	111110
d →	11110
e →	1110
f →	110
g →	10
h →	0

← Predicted Huffman codes

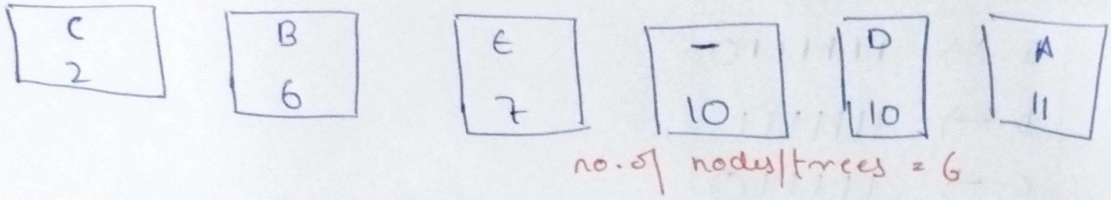
3

String: A_DEAD_DAD_CEDED_A_BAD_BABE_A_BEADED_ABACA_BED

Character	frequency
A	11
B	6
C	2
D	10
E	7
space(_)	10

Arranging frequencies in ascending order:

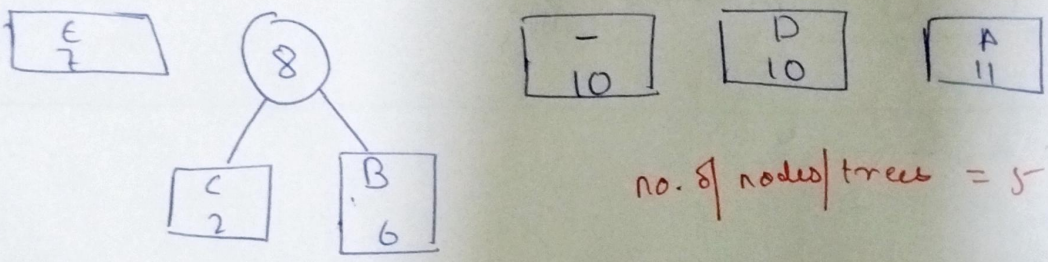
(1)



no. of nodes/trees = 6

- Choosing 2 min frequencies & joining them; here C & B have least frequency so join them & put them after E(7) & before ~~th~~ space.

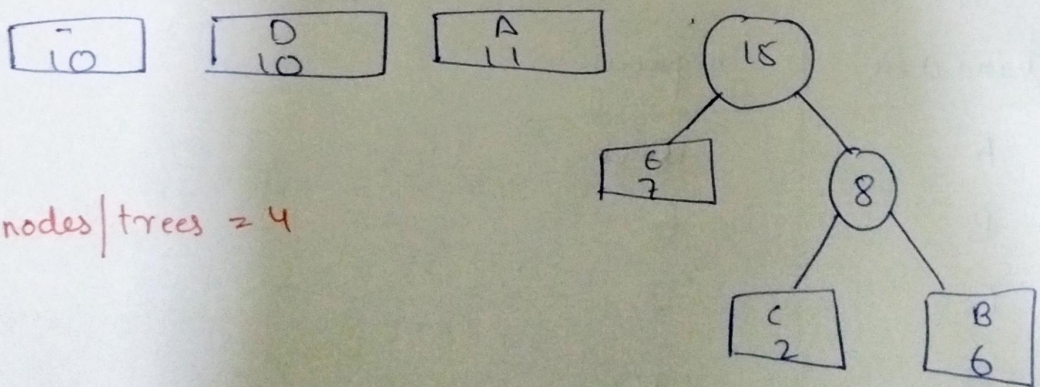
(2)



no. of nodes/trees = 5

- Now joining E & 8 as they are minimum & putting it after A

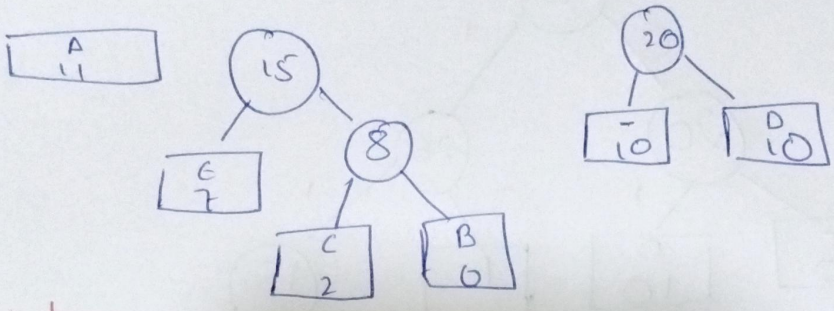
(3)



no. of nodes/trees = 4

- Now joining (—) & D as they are minimum & putting it after 15

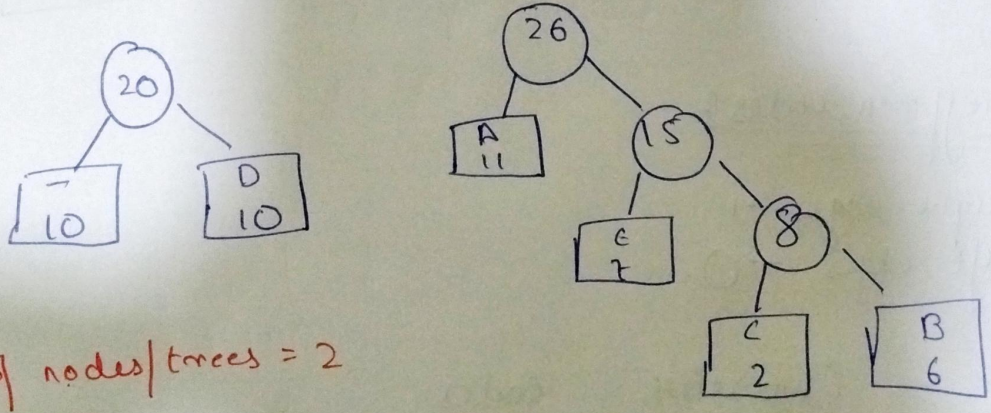
④



no. of nodes/trees = 3

- now joining A & 15, as they have minimum frequency. and putting it after 20

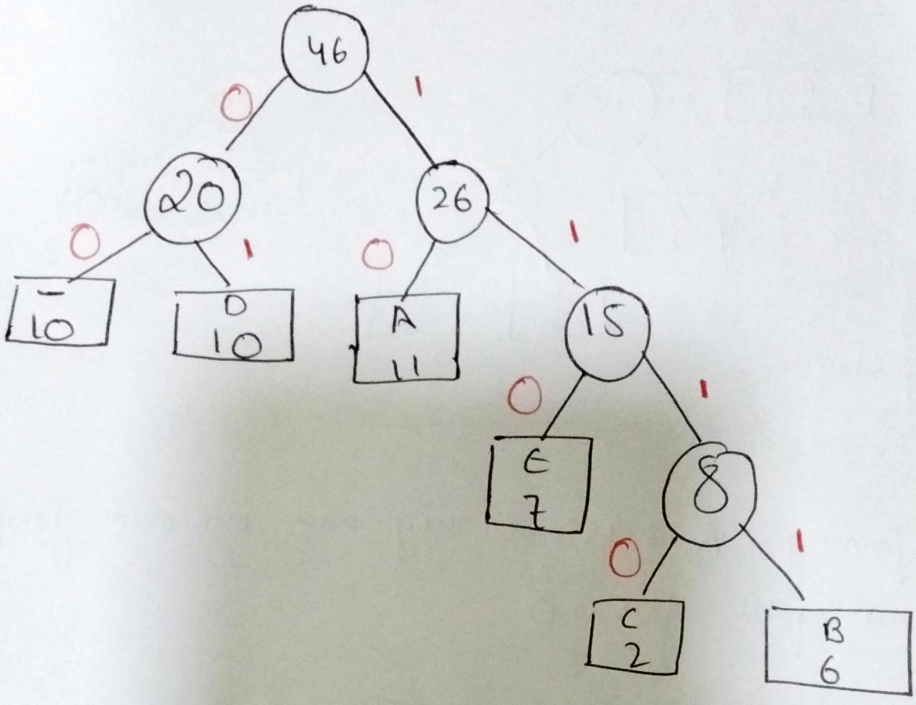
⑤



no. of nodes/trees = 2

• Now finally joining the last ~~tre~~ two trees.

Huffman Tree



For Huffman codes:

Put Right edges – 1
Put left edges – 0

Character	Codes
A	10
B	1111
C	1110
D	01
E	110
–	00

⇒

(–) → 00
D → 01
A → 10
E → 110
C → 1110
B → 1111