Reg. No.:

Name :

VIT BHOPAL
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TERM END EXAMINATIONS (TEE) – January 2021

Programme	:	**B.Tech.**	**Semester**	:	**Fall 2020-21**
Course Name	:	**Calculus and Laplace Transform**	**Course Code**	:	**MAT1001**
Faculty Name	:	**Dr. Reena Jain**	**Slot / Class No**	:	**E11+E12+E13/1508**
Time	:	**1½ hours**	**Max. Marks**	:	**50**

Answer ALL the Questions

Q. No.		Question Description	Marks
		PART - A (30 Marks)	
1	(a)	Show that the only possible maxima and minima of z on the surface $z = x^3 + y^3 - 9xy + 27$ occurs at (0,0) and (3, 3). Show that neither a maximum nor a minimum occurs at (0, 0). Determine whether z has a maximum or a minimum at (3, 3).	10
		OR	
	(b)	By converting into polar coordinates, integrate the function $f(x,y) = 1/(1 - x^2 - y^2)$ over the disk $x^2 + y^2 \leq 3/4$. Does the integral of $f(x,y)$ over the disk $x^2 + y^2 \leq 1$ exist? Give reasons for your answer.	10
2	(a)	Verify Stoke's theorem for $\vec{F} = (x^2 + y^2)i - 2xyj$ taken round the rectangle bounded by $x = \pm a, y = 0, y = b$.	10
		OR	
	(b)	Solve by using the method of undetermined coefficients. $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = x^2 - 4 + 2e^{-x}$.	10
3	(a)	A cold drink is poured out at 50^0F. After 2 minutes of sitting in a 70^0F room, it's temperature has risen to 56^0F. Find the drink's temperature at any time t. What will the temperature be after 10 minutes? When will the drink have warmed to 66^0 F.	10
		OR	
	(b)	Find the inverse Laplace transform of $f(s) = \frac{s}{(s^2 - 2s + 2)(s^2 + 2s + 2)}$	10
		PART - B (20 Marks)	
4		Prove that the function is not continuous at $(0,0)$ $f(x,y) = \begin{cases} \frac{x^3 - y^3}{x^3 + y^3} & (x,y) \neq (0,0) \\ 5, & (x,y) = (0,0) \end{cases}$	10

5 Using Laplace Transform, solve the ODE 10

$$y'' - 6y' + 15y = 2\sin 3t\,, \quad if\ y(0) = -1, y'(0) = -4$$

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