MAT-2001 20BCE10077
Tutorial 3 Fiza Siddiqui

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A11 + A12
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Q1

$$f(x) = \begin{cases} a - |x| & |x| < a \\ 0 & |x| > a \end{cases}$$

Solution - we know that fourier transform is given by $\mathcal{F}\{f(x)\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(x) e^{-iwx}\, dx$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} f(x) e^{-iwx}\, dx$$

breaking the integral in form of limits -

$$\frac{1}{2\pi}\left[\underbrace{\int_{-\infty}^{a} f(x) e^{-iwx}\, dx}_{=0} + \int_{-a}^{a} f(x) e^{-iwx}\, dx + \underbrace{\int_{a}^{\infty} f(x) e^{-iwx}\, dx}_{=0}\right]$$

$$\frac{1}{2\pi}\int_{-a}^{a} (a - |x|) e^{-iwx}\, dx$$

(0 to a)

$$\frac{1}{2\pi}\int_{-a}^{0}(a+x)e^{-iwx}\,dx + \frac{1}{2\pi}\int_{0}^{a}(a-x)e^{-iwx}\,dx$$

Integrating the above terms,

$$\frac{1}{2\pi}\left\{\left[(a+x)\frac{e^{-iwx}}{-iw} - \frac{e^{-iwx}}{(-iw)^2}\right]_{-a}^{0} + \right.$$

$$\left[(a-x)\frac{e^{-iwx}}{-iw} + \frac{e^{-iwx}}{(-iw)^2}\right]_{0}^{a}$$

$$\Rightarrow \frac{1}{2\pi}\left\{\frac{a}{-iw} - \frac{1}{w^2} - 0 + \frac{e^{iaw}}{w^2} + 0 + \frac{e^{-iwa}}{(-iw)^2}\right.$$

$$- \left\{\frac{a}{(-iw)} + \frac{1}{w^2}\right\}$$

$$\Rightarrow \frac{1}{2\pi}\left\{\cancel{-\frac{a}{iw}} - \frac{1}{w^2} + \frac{e^{iaw}}{w^2} + \frac{e^{-iwa}}{(-iw)^2} + \cancel{\frac{a}{iw}} - \frac{1}{w^2}\right\}$$

$$\Rightarrow \frac{1}{2\pi}\left\{\frac{1}{w^2}\left\{-2 + e^{iaw} + e^{-iwa}\right\}\right.$$

$$\frac{1}{2\pi}\left\{\frac{1}{w^2}\left\{-2+e^{iaw}+e^{-iwa}\right\}\right\}$$

$$\frac{1}{2\pi w^2}\left\{-2+e^{iaw}+e^{-iwa}\right\}$$

$$=\frac{1}{2\pi w^2}\left\{-2+\cos aw+i\sin aw+\cos aw\right\}-i\sin aw$$

$$\frac{1}{\pi w^2}\left\{-1+\cos aw\right\}$$

$$\Rightarrow \frac{\cos aw-1}{\pi w^2}$$

$$\boxed{F(w)=\frac{\cos aw-1}{\pi w^2}}$$ — Ans.

Q 2

$f(x) = k$

fourier transform of $f(x) = k$ = const

for fourier cosine transform,

$$F_c[f(x)] = \int_0^\infty f(x) \cos \alpha x \, dx$$

$$= \int_0^\infty k \cos \alpha x \, dx$$

$$= k \left[\frac{\sin \alpha x}{\alpha} \right]_0^\infty$$

$\therefore \lim_{x \to \infty} \sin \alpha x$ does not exist, ~~so this~~

$\therefore$ fourier cosine transform doesnt exist