Selected Topics in Plasma Astrophysics

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1 Lecture 1

1.1 Astrophysical Plasmas in General

Plasma astrophysics is a very broad subject, encompassing a lot of different sub fields. There are a lot of different techniques to study them as well. For relativistic plasma physics, there are force-free electrodynamics to study e.g. pulsar magnetospheres. There is a family of (GR)(M)HD that study accretion, jets, etc. There is PIC that is used to study shocks, and Dynamic space-time + MHD to study compact object mergers.

For non-relativistic theory we use force-free for e.g. solar corona, MHD for e.g. star formation, disks, etc., and kinetic theory for shocks, reconnection, turbulence, etc. Even within MHD, there is a wide range of fluid models which are useful for different situations.

In the first lecture we focus on fluid models with some kinetic elements.

1.2 Stellar Winds

Stellar winds is a topic that is rich and has influence over many other branches of plasma astrophysics, and it will be our topic today. We will be discussing different kinds of winds:

- Thermally driven winds (from sun-like stars): hydrodynamic theory, kinetic theory
- Magnetocentrifugically driven winds: MHD, rotation as energy source but trapped by B fields.
- Radiation pressure driven outflow: $L > L_{\rm edd}$: continuum driven or line-driven, depending on the spectrum

1.3 Solar Corona and Wind

Solar corona and outflow is an important part of solar dynamics. The solar outflow is very small in terms of mass and energy. However the solar wind is efficiently extracting the angular momentum from the sun. Its spin has slowed down by a factor of 30 to 50 since its birth. The time scale of dJ/dt is on the order of 10^{10} years.

The solar corona is not in thermal equilibrium. We know that $T_i \gg T_p \geq T_e$, and is anisotropic $T_\perp \geq T_\parallel$. The corona is mostly collisionless where $\ell_{mfp} \sim 10^8 \rho_{\rm Larmor}$. It turns out that despite the collisionlessness we can use fluid theory to somewhat describe the solar wind.

Lets start by talking about the Parker model. Consider a time-independent outflow, the continuity equation is

$$\nabla \cdot (\rho \mathbf{v}) = 0 \Longrightarrow r^2 \rho v_r = \text{const} \tag{1.1}$$

From this the mass loss rate is simply $\dot{M} = 4\pi r^2 \rho v_r$ which is also a constant. The radial velocity equation reads

$$v\frac{dv}{dr} = -\frac{1}{\rho}\frac{dP}{dr} - \frac{GM}{r^2} \tag{1.2}$$

If we assume temperature being constant, pressure $P = \rho c_s^2 = \rho kT/m_p$, then we have this equation

$$\frac{1}{v_r}\frac{dv_r}{dr}(v_r^2 - c_s^2) = \frac{2c_s^2}{r} - \frac{GM}{r^2} \tag{1.3}$$

This equation describes spherical wind, or spherically symmetric (Bondi) accretion.

There is a special point in this equation called the sonic point. This is where $v_r = c_s$. This requires that the right hand side is also zero

$$\frac{2c_s^2}{r} = \frac{GM}{r^2} \tag{1.4}$$

In the regime where $v < c_s$, we can ignore the left hand side of equation (1.3). This gives a spatial distribution of density. However when this is true, the pressure is very big. In fact if we require that the flow goes through the sonic point peacefully, we can uniquely obtain a unique outflow solution from this equation. TODO: clean this paragraph up

1.4 Magnetic Field

Around the sun there are magnetic field lines that open up to infinity, and field lines that close back to the star itself. For a non-rotating star, the above theory can be thought of as describing the acceleration of the outflow along the open magnetic field lines. The only non-trivial thing is the structure of the global magnetic field.

However now let's consider when the star is rotating. Lets simplify the actual magnetic field to a split monopole configuration, which can be called a "theorist's monopole". This is a useful toy model where we can calculate structures of the outflow relatively easily. If we have a very powerful outflow that blows up the magnetic field lines, then at large distances it will actually look like a split monopole field.

We will now focus on the equatorial plane. Imagine the magnetic field lines have infinite tension. The outflow will rigidly follow the field lines, and forced to corotate with the star. As they flow out their rotation velocity will increase with radius, as well as their specific angular momentum. Therefore they extract angular momentum from the star. The magnetic field acts as a medium that extracts angular momentum from the star and transfer it to the outflow.

The corotation only can keep up till some point, where the magnetic energy becomes comparable to the kinetic energy of the material

$$\frac{B^2}{8\pi} \sim \rho v_r^2 \tag{1.5}$$

At this point there is so much energy in the gas that you can't treat the magnetic field as rigid. This is called the *Alfven point* r_A. This is also when

$$v_r = v_A \sim \frac{B}{\sqrt{4\pi\rho}} \tag{1.6}$$

Lets look at this outflow in more detail. Let

$$\mathbf{v} = v_r \hat{r} + v_\phi \hat{\phi}, \quad \mathbf{B} = B_r \hat{r} + B_\phi \hat{\phi} \tag{1.7}$$

We want to use the conservation of momentum and energy again like before to write down the equation governing the outflow. We have

$$\dot{M} = 4\pi r^2 \rho v_r = \text{const}, \quad \frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\nabla \times \mathbf{B}) = 0 \tag{1.8}$$

We can show that

$$\frac{B_\phi}{B_r} = \frac{v_\phi - r\Omega}{v_r} \tag{1.9}$$

The way to think about this result is that, if there is perfect corotation, then there is no B_ϕ. If v_ϕ can't keep up with $r\Omega$ the magnetic field will start to lag behind and be swept back, leading to a negative component of B_ϕ.

The next equation which is important is the momentum equation in the ϕ direction. It tells us how does angular momentum change

$$\rho v_r \frac{d}{dr}(r v_\phi) = \frac{B_r}{4\pi} \frac{d}{dr}(r B_\phi) \tag{1.10}$$

If we multiply by r^2 we can easily integrate this equation, giving

$$L = r v_\phi - \frac{r B_\phi B_r}{4\pi \rho v_r} = \text{const} \tag{1.11}$$

There are two contributions to the angular momentum: L_{gas} and L_{mag}. Therefore the angular momentum is carried out both by the gas and the magnetic field, and the total angular momentum is a constant in the problem.

To find what the constant is, we can substitute equation (1.9) into this equation and solve:

$$v_\phi = r\Omega \left(\frac{v_r^2 L / r^2 \Omega - v_{Ar}^2}{v_r^2 - V_{Ar}} \right) \tag{1.12}$$

where $v_{Ar}^2 = B_r^2 / 4\pi\rho$. This has the same structure as before: the denominator is zero at some point. Here is apparent that the magnetic field component that determines the Alfven point is the radial point. Therefore at the Alfven point we have

$$L = r_A^2 \Omega \tag{1.13}$$

This means that the wind solution determines exactly how much angular momentum is extracted from the star. This idea is also why the outflow model is so important in plasma astrophysics. Because the magnetic field can be very strong, we could have $r_A \gg R_*$. The rate of angular momentum loss is

$$\dot{J} = \dot{M} L = \dot{M} r_A^2 \Omega \tag{1.14}$$

so the time scale is

$$t = \frac{J}{\dot{J}} \sim \frac{M R_*^2}{\dot{M} r_A^2} \tag{1.15}$$

Due to the large "level arm" of the Alfven radius, this time scale can be much smaller than the time scale of mass loss. This efficient extraction of angular momentum due to magnetic field will occur in other places in plasma astrophysics as well.

The last thing we want to talk about is the energy carried by the outflow. Consider hydrodynamics, we have the Bernoulli constant which is essentially energy per unit mass:

$$B_l = \frac{1}{2}v^2 + \phi + h, \quad h = \frac{5}{2}\frac{kT}{m_p} \tag{1.16}$$

where h is called the enthalpy $h = \gamma/(\gamma-1)kT/m$. In thermally driven wind, if we heat up the gas it will be able to escape. However in MHD we have an additional loss of energy in the form of Poynting flux, which also taps into the rotational energy of the star. We find that

$$B_l + \frac{S_r}{\rho v_r} = \text{const} \tag{1.17}$$

where S_r is the radial Poynting flux. Given the magnetic field structure it is easy to calculate the Poynting flux

$$\frac{S_r}{\rho v_r} = -\frac{r\Omega B_r B_\phi}{4\pi\rho v_r} \tag{1.18}$$

If we call this constant ϵ, then it is like the angular momentum flux, it has a gas component and also a magnetic field component, where in fact $\epsilon_{\text{mag}} = \Omega L_{\text{mag}}$.

Now the question is which of these is more important. Lets roughly estimate it to get a feeling:

$$\epsilon_{\text{gas}} \sim \frac{kT_{\text{corona}}}{m_p}, \quad \epsilon_{\text{mag}} \sim r_A^2\Omega^2 \tag{1.19}$$

For the sun, the gas energy flux is much larger than the magnetic energy flux, and most of the energy outflow is thermal energy. This is a thermally driven wind. In the other regime most of the energy that goes to infinity is the rotational energy of the central object. This is what we call a magnetically driven outflow. Which regime we have depends no the temperature of the corona, rate of rotation, and Alfven radius.

Lets come back to the question is why do we use a fluid model at all for the sun. The sun is a slowly rotating object. Around the sun the presence of magnetic field means $\rho_{\text{Larmor}} \ll R$, so plasma streams only in one direction. Along the B field pressure is the origin of acceleration, but fluid theory is okay within order of about a few. Kinetic instabilities also helps us by limiting how much our distribution function can deviate from Maxwellian.

The state of the art of solar corona study is to understand how particles are heated in the solar corona. This heating has been measured from the corona all the way to much further away from the Earth. The temperature profile is shallower than adiabatic, so there must be heating going on. One idea is that Alfven waves are launched and the turbulence keep heating the plasma.

The time-reversal of this outflow wind model is the spherical Bondi accretion. There is also the theory of magnetically directed accretion to the central object analogous to the magnetically driven wind models. But it works not as good because in the wind model the outflow is driven by the rotation of the star, and same can't be said for accretion models.

1.5 Radiative Driven Winds

Lets now briefly talk about radiation pressure driven winds. Around RGB and AGB stars, a lot of dust forms in stellar atmosphere which have high κ. They feel the radiation pressure much higher than the gas but are coupled to the gas through collision, therefore driving a wind. Around massive stars $L > L_{\rm edd}$ on metal lines, and the plasma absorption of metal line photons drives the wind.

In thermally driven winds the energy outflow is $\dot{M}$ times the speed of sound squared. However for the radiation pressure driven winds we should think of momentum flux due to photons

$$\dot{P} \sim \dot{M} v_\infty \sim L/c, \quad v_\infty \sim v_{\rm escape} \tag{1.20}$$

We can derive this from the momentum equation directly

$$v\frac{dv}{dr} = -\frac{1}{\rho}\frac{dP}{dr} - \frac{GM}{r^2} + \kappa\frac{F}{c} \tag{1.21}$$

where the right hand side has a new term which is an average radiation pressure force over all wavelengths. If we only focus on the new term, we can write

$$v\frac{dv}{dr} = \frac{\kappa L}{4\pi r^2 c} \tag{1.22}$$

Multiplying both sides by $4\pi r^2 \rho$ we have

$$\dot{M}\frac{dv}{dr} = \rho\kappa\frac{L}{c} \tag{1.23}$$

Integrating this over all radii to infinity we now have

$$\dot{M} v_\infty = \tau\frac{L}{c} \tag{1.24}$$

where τ is the optical depth $\tau = \int \rho\kappa\, dr$. That is exactly we have, and the optical depth is the one uncertain numerical coefficient.

For line-driven winds refer to (Lucy & Solomon 1970; Castor, Abott, Klein 1975).

For applications of this wind theory in other models/fields, refer to the original slides! Mentioned models are thermally driven galactic winds, line driven winds from accreting black holes, magnetized winds from accretion disks.

2 Lecture 2

2.1 Instabilities in Ideal Fluids and Dilute Plasmas

Last time we used the outflow from massive stars to introduce different techniques and methods that show up in different physical contexts. Today we will do the same on another topic.

2.2 Instabilities in Ideal Fluids

Who cares about linear instabilities when we have simulations? We will argue that even in the present days the study of linear instabilities is still very useful. We can't simulate everything, so we need to know what physics to include in the first place. It is very instructive to identify the key physics problems of interest and study of linear instabilities is a way to approach that.

Linear instabilities are also a way to produce turbulent transport of mass, momentum, energy, etc. The physics of linear instabilities is often still there even when the system has already evolved into the nonlinear state. Linear instabilities can also fundamentally change the equilibrium structure and dynamics of the system. For example the FRW universe is unstable to gravitational perturbations and it leads to the large scale structures we know today. Another example is the convection in stars. If we write down the equilibrium theory of stars we will get everything wrong, because the equilibrium theory relies on diffusion but a large portion of the star like the sun is convective.

Ideal single fluid (M)HD is a useful starting point for astrophysical plasmas. It encapsulates the conservation of key quantities like mass, energy, momentum, and does better than expected. However non-ideal effects and multi-fluid effects can be very important in many systems. For example in star formation and planet formation we have dust in addition to multi-fluid MHD. In intracluster plasma in the galaxy the plasma is very dilute and near collisionless so we need to consider anisotropic conduction, etc. In luminous accreting black holes we need to consider radiation pressure effects in addition to MHD. It is because of these diversities that we need to study the system in the linear regime first before we dive into simulations directly.

2.2.1 Buoyancy

The basic setup of considering whether a system is stable under buoyancy is as follows. Suppose we have a gravity field and a downwards entropy gradient, it is a typical system for convection: $ds/dz < 0$. This is called the Schwarzschild criterion. This is however only a specific criterion. The motion is usually slow and adiabatic, with time scale

$$t_{\text{sound}} \sim \text{hr} \ll t_{\text{buoyancy}} \sim \text{month} \ll t_{\text{diffusion}} \sim 10^4\,\text{yr} \tag{2.1}$$

If we have a blob of fluid that flow from region of higher entropy to lower entropy further out. If the motion is adiabatic and in pressure equilibrium, it will decrease due to

$$s(p,\rho) \propto \ln(p/\rho^\gamma) \tag{2.2}$$

and since it is less dense than the surroundings it will continue to go up, thus run away, triggering the instability.

When people model the star, people first ignore convection, and model using radiation, gravity, etc. Afterwards people add in convection and ask whether the model is stable under convection.

The above reasoning hides an effect which is part of everyday life: difference in chemical composition. If we put heavy fluid above light fluid, the heavy one will sink and create fingers into the lighter one, creating the convection instability.

The key thing to realize is that the pressure of an ideal gas depends on the composition

$$p = \sum_j n_j kT = \frac{\rho kT}{\mu m_p} \tag{2.3}$$

where μ is the mean molecular weight, i.e. average mass per particle. $\mu = 1/2$ for ionized H, $\mu = 4/3$ for He, and $\mu = 0.62$ for solar metallicity. Now we can expand out the entropy derivative ds/dz

$$\frac{ds}{dz} = \frac{d\ln p}{dz} - \gamma\frac{d\ln\rho}{dz} = \frac{d\ln T}{dz} - (\gamma-1)\frac{d\ln\rho}{dz} - \frac{d\ln\mu}{dz} \tag{2.4}$$

This means that $d\mu/dz > 0$ has a destabilizing effect, which confirms our everyday experience: heavy on top of light is unstable. This is a continuous version of the Rayleigh-Taylor instability.

This is very relevant in stars. The standard composition gradient in the normal course of stellar evolution is that heavy elements show up towards the center, and it has a stabilizing effect on the stellar structure.

Lets look at our assumption of s being constant. Consider the diffusion time scale

$$t_{\text{diff}} \sim H^2/\ell c \sim \tau H/c, \quad t_{\text{conv}} \gtrsim H/c_s \tag{2.5}$$

where H is the characteristic scale height of system, and ℓ is the photon free path. Therefore $t_{\text{diff}} \lesssim t_{\text{conv}}$ if $\tau \lesssim c/c_s$. The place we see photon come from is where $\tau \sim 1$. Somewhere between that and where $\tau \sim c/c_s$, the assumption of adiabatic will fail, since we have rapid thermal diffusion in the surface layers of the star.

Lets examine our reasoning again. When we thought of the fluid element going from initial to final position, we assumed entropy being constant. However when we have rapid diffusion, in the limit we have constant T because rapid thermal diffusion tries to even out the temperature gradient. This means at the final position the temperature of the fluid element will be about the same as the surroundings, and because it is still in pressure equilibrium, it will tend to wipe out density gradients. This has a suppression effect on Buoyancy.

This effect has been seen in simulations, e.g. (Jiang et. al. 2015). In their simulations the convective flux near the surface of the star is down by nearly two orders of magnitude. The main difference in the simulation between the surface of the star and interior is the optical depth.

Lets think about the microscopic energy transport due to photon transfer. This effect of heat transport in normal stars because its free path is way smaller than electron free path. However in degenerate plasmas like white dwarfs and neutron stars, and in dilute and hot non-degenerate plasmas like solar corona and hot accretion flows, thermal conduction can dominate. In the former case thermal conduction is typically isotropic for lower magnetic fields like in WDs, but anisotropic for NS. For the later case thermal conduction is highly anisotropic because $l_e \gg \rho_e$. The main mechanism that determines free path is Coulomb collision between electron and protons.

Anisotropic heat conduction can induce convection and fundamentally change the way heat is conducted in the plasma. The setup we want to consider is similar to the previous one. Consider an environment where the temperature is hot down below and cold up above. A weak B field points horizontally, which only channels heat flow but weak enough to have now dynamical effect. We also assume the thermal conduction time is much smaller than the buoyancy time. If we move a blob of fluid from the hot region to the cold region along the B field line.

Since thermal conduction is efficient, T is maintained to be constant along the magnetic field line. At the final position the blob has the same pressure as the surroundings, with the same temperature as initially, which is hotter than the surroundings, so it has less density than the surroundings. Therefore this introduces a convective instability. The condition now becomes $dT/dz < 0$, and the growth time is comparable to the dynamic time of the system.

An example simulation is covered in (McCourt et. al. 2011). The instability saturates by generating sustained convection and amplifying the magnetic field.

There is sometimes a disconnect between actually doing the math of linear instability and drawing the picture of the instability. So let us do some math to illustrate the connection between the theory and the picture we discussed above.

Consider the set of MHD including the Braginskii term (refer to Kunz's lecture). The key term in the equations is the anisotropy term $\nabla \cdot \mathbf{Q}$, where

$$\mathbf{Q} = -\chi \hat{b}(\hat{b} \cdot \nabla)T \tag{2.6}$$

which describes the heat flow along the magnetic field line. We've assumed that the fluid element is always in pressure equilibrium with the surroundings. This is actually an approximation, and what this means mathematically is that we keep density perturbations due to buoyancy but not due to sound waves. Sound waves is what we assume to wipe out the pressure differences. This also means $\nabla \cdot \mathbf{v} = 0$.

This looks odd at first since it says incompressible fluid, but we are interested in density variations. This is simply done to get sound waves out of the problem, and only keeping the density perturbations due to gravity.

Lets assume the perturbations vary as $e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)}$, where $\mathbf{k}$ is in the x direction, and B field is in the x direction as well. If we perturb the heat flux (Eulerian, at fixed place)

$$\begin{aligned}\delta Q = &-\chi\delta\hat{b}(\hat{b}\cdot\nabla)T \\ &- \chi\hat{b}(\delta\hat{b}\cdot\nabla)T \\ &- \chi\hat{b}(\hat{b}\cdot\nabla)\delta T\end{aligned} \tag{2.7}$$

The first term is zero due to our assumption that T is constant along the B field direction. Due to our setup there is no first order change in the magnitude of the B field, so $\delta\hat{b} = \delta\mathbf{B}/B_0$. We also have

$$\begin{aligned}\frac{\partial\mathbf{B}}{\partial t} &= \nabla\times(\mathbf{v}\times\mathbf{B}) \\ &= (\mathbf{B}\cdot\nabla)\mathbf{v} - (\mathbf{v}\cdot\nabla)\mathbf{B}\end{aligned} \tag{2.8}$$

the second term is zero, so we have the linearized version as

$$-i\omega\delta\mathbf{B} = ikB_0\delta\mathbf{v} = ikB_0(-i\omega)\delta\boldsymbol{\xi} \tag{2.9}$$

so we can write $\delta\hat{b} = ik\delta\boldsymbol{\xi}$. Plugging this into our equation of heat flux we get

$$\begin{aligned}\delta Q &= -\chi(ik\boldsymbol{\xi}\cdot\nabla)T\hat{b} - \chi ik\hat{b}\delta T \\ &= -ik\chi(\xi_z\frac{\partial T}{\partial z} + \delta T)\hat{b}\end{aligned} \tag{2.10}$$

The divergence of this is simply bringing in another ik, so we have

$$\nabla\cdot\delta Q = k^2\chi(\xi_z\frac{\partial T}{\partial z} + \delta T) \tag{2.11}$$

Since the time scale for thermal conduction is very short, which means

$$t_{\text{cond}} \sim \frac{\lambda^2}{\chi} = \frac{1}{k^2\chi} \to 0 \tag{2.12}$$

If we take this limit, our equation will blow up, which is bad, unless we also take the terms inside the brackets to be zero. What this means is that rapid conduction implies that

$$\delta T + \xi_z\frac{\partial T}{\partial z} = 0 = \Delta T \tag{2.13}$$

which is exactly the Lagrangian change in temperature. Which means that moving with the blob, the temperature doesn't change.

Lastly if we now use pressure equilibrium, which says

$$\frac{\delta\rho}{\rho} + \frac{\delta T}{T} = 0 \tag{2.14}$$

therefore we can replace δT:

$$\frac{\delta\rho}{\rho} = \xi_z \frac{d\ln T}{dz} \tag{2.15}$$

What this means is that if ξ_z is positive and if dT/dz is negative, then ρ will decrease and it corresponds to a buoyancy system.

One thing that we didn't get is the growth rate of the instability. What we did was deriving the condition of the instability. To get the growth rate, we need to keep all the linear terms and do the whole perturbation theory. One will eventually get a growth rate

$$\gamma \approx \left[g \left| \frac{d\ln T}{dz} \right| \right]^{1/2} \tag{2.16}$$

When the magnetic field is too strong, the magnetic force will prevent the bending of magnetic field lines, so it will suppress the instability. This happens roughly when $\beta \sim 1$.

Is this an overly simplified problem because we have only horizontal magnetic field? It turns out it is more generic than our simple setup. Even when there is vertical magnetic field which allows temperature exchange vertically, it doesn't wipe out the instability.

If we revert the temperature gradient, then this argument tells us that this system is stable, but it is wrong. The physics is different, because this instability only happens when there is a component of the magnetic field in the vertical direction. This is called the heat flux-driven buoyancy instability (HBI). This instability operates best when $\mathbf{k}$ is 45 degrees with the $\mathbf{B}$ field. We can derive this using the same algebra as above, which is worked out in the notes.

It turns out that a weakly magnetized plasma with anisotropic heat transport is always buoyantly unstable. This manifests mostly in hot plasma in galaxy clusters where both thermal conduction and viscosity are important.