

Relativistic Astrophysics

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1 Lecture 1 — Accretion Disks

1.1 Child's Garden of Astrophysical Disks

Here is a (incomplete) list of astrophysical disks

- Galactic disk: spiral/elliptical
- Supermassive BH: quasar/Seyfert/LINER/LLAGN/TDE
- Stellar mass BH: microquasar/GRB
- Neutron star: LMXB/HMXB/GRB
- White dwarf: dwarf nova/nova
- Protostar: protoplanetary/debris
- Planet: protolunar disk/planetary rings

Consider the galaxy NGC 4258. If we zoom in to ~ 0.1 pc, we see maser spots around the central engine. The velocities of these spots fall nicely on a Keplerian orbit. The combination of the acceleration, velocity, and angular position of these maser spots allows us to measure the mass of the central object.

Here let us introduce the first dimensionless parameter of a thin disk

$$\frac{H}{R} \ll 1, \quad H = \frac{c_s}{\Omega} \propto T^{1/2} \quad (1.1)$$

For this system this parameter is $\sim 10^{-3}$. This is an example of a thin disk. The disk of NGC 4258 is not a flat disk, since one can see warps following the maser spots. The disk is actually made of rings at the positions of the masers.

This particular disk is relatively far away from the object, and the disk is heated due to the radiation from the central object.

The next example is the center of our galaxy. We know that the accretion flow near $\sim 10r_g$ emits synchrotron radiation which is the target of the Event Horizon Telescope (EHT). The mean free path of Coulomb scattering in Sgr A* is very large, so the plasma is almost collisionless. This allows us to introduce the second number

$$\kappa_n = \frac{\lambda_{mfp}}{R} \quad (1.2)$$

which is about 10^5 for this system.

The next example is the HL Tau. This is a cold disk of protoplanetary disk. At 50 au the temperature is less than 100 K. Here we introduce the third number which is the magnetic Reynolds number

$$\text{Re}_M = \frac{c_s H}{\eta} \quad (1.3)$$

In regions of this system that we can measure with ALMA, this number is less than 1.

The last number we introduce is

$$Q = \frac{c_s \Omega}{\pi G \Sigma} \quad (1.4)$$

where Σ is the surface density of the disk. When this number is less than one, self gravity becomes important.

The last example is the moon. When people look at the moon most people do not see a disk. The theory of the formation of the moon involves an impact on earth, throwing some mass from the earth which forms a disk around it. This is a system where the magnetic Reynolds number might play a role.

One more system we want to mention is the SS Cyg which is a Cataclysmic variable, formed by a massive star overflowing its Roche lobe, feeding a disk around its companion which is a WD. We have observed this system since 1896 and it alternates between a quiescent state and an active state. This system may have a low magnetic Reynolds number at quiescent state, but has a high magnetic turbulence during its outbursts.

1.2 Disk Evolution

Disks are a class of special objects in astrophysics. The angular momentum is conserved, but its kinetic energy is easily converted to thermal energy due to various instabilities in the disk, and eventually radiated away. Therefore understanding the disk evolution is all about understanding the evolution of angular momentum.

In a thin disk, dynamical equilibrium is reached in a dynamic timescale $\Delta t \sim \Omega^{-1}$, where

$$\Omega = \left(\frac{GM}{R^3} \right)^{1/2} + O\left(\frac{H}{R} \right)^2 \quad (1.5)$$

However it is possible for there to be long living excitations which live in circular orbits, in addition to tilts and warps in the disk.

Thermal equilibrium is reached when $Q^+ \sim Q^-$, when heating rate balances the cooling rate. The time scale is

$$\Delta t \sim \Sigma c_s^2 / Q^+ \sim (\alpha \Omega)^{-1} \quad (1.6)$$

where the α parameter is very important and it describes the intensity of turbulence in the disk, and the above equation can be taken as the definition. It relates the thermal time scale to the dynamical time scale. Its value, usually at 10^{-2} means that the thermal time scale is much longer than the dynamical time scale.

The last time scale is the inflow equilibrium $\dot{M} \sim \text{const.}$ The time scale is

$$\Delta t \sim \frac{M_{\text{disk}}}{\dot{M}} \sim (\alpha \Omega)^{-1} \left(\frac{R}{H} \right)^2 \quad (1.7)$$

which is again longer by a factor of the scale height squared.

Lets write down the disk evolution equation

$$\partial_t \Sigma = \frac{2}{r} \partial_r \left(\frac{\Omega}{r \kappa^2} \partial_r (r^2 W_{r\phi}) - \frac{\Omega}{\kappa^2} \tau \right) + \dot{\Sigma}_{\text{ext}} \quad (1.8)$$

where Σ is the surface density, Ω the orbital frequency, κ the epicyclic frequency which is similar to Ω , $W_{r\phi}$ is the shear stress, τ is connected to external torques per area, and finally $\dot{\Sigma}_{\text{ext}}$ is the mass gain/loss from infall or from wind loss. The full derivation of this equation is left as an exercise. The hint is to start from the angular momentum conservation equation together with conservation of mass. Another hint is that

$$\frac{d}{dr} j = \frac{r \kappa^2}{2\Omega} \quad (1.9)$$

where j is the specific angular momentum.

The standard model for disks is the α disk model. It was introduced by (Shakura & Sunyaev 1973) and (Linden-Bell & Pringle 1974). It adopts simple scaling argument for diffusion of angular momentum by turbulence, by just modeling the turbulent viscosity

$$\nu \sim \alpha c_s H \quad (1.10)$$

In classical α disk models one ignores external torques, infall/winds, etc. Thus one throws away the last two terms in the disk equation, with only the term with $W_{r\phi}$ surviving.

In the α disk model we assume the disk is thin and Keplerian $\Omega \approx (GM/r^3)^{1/2}$. We also assume vertical equilibrium $H \sim c_s/\Omega$. The opacity is assumed to be related to temperature and density $\kappa \sim \kappa_0 \rho^a T^b$. We have a prescription of vertical integration of the density $\Sigma \sim 2\rho H$, which is a crude integral. We also estimate the optical depth to be $\tau \sim \Sigma \kappa / 2$. The surface temperature is determined by the fact that heating by viscous processes $F = \sigma T_{\text{eff}}^4 \sim (9/8) \Sigma \nu \Omega^2$, where $T_{\text{eff}}^4 \sim \sigma T^4 / \tau$. Finally in the steady state $\dot{M} = 3\pi \Sigma \nu$.

As an example, consider a steady state disk around a stellar mass BH. In the inner zone, radiation pressure is much larger than the gas pressure, and electron scattering dominates the opacity. We can find that

$$T \sim 4.3 \times 10^7 \alpha^{-1/4} m^{-1/4} x^{-3/8} K \quad (1.11)$$

$$\Sigma \sim 0.4 x^{3/2} \alpha^{-1} \dot{m}^{-1} \text{ g/cm}^2 \quad (1.12)$$

$$H/r \sim 10 \dot{m} x^{-1} \quad (1.13)$$

Now lets come back to the disk evolution equation and ask what are $W_{r\phi}$, τ , and Σ_{ext} . Part of the answer only lies in the global analysis of turbulence in the disks.

1.3 Turbulence in Disks

In α disk model the turbulent diffusion of angular momentum is simply postulated. Possible sources of turbulence include magnetorotational instability (MRI), gravitational instability, zombie vortex instability, subcritical baroclinic instability, and vertical shear instability. The last three are related to instabilities in fluids. The zombie vortex instability is generated due to vortices generated in the disk sharing angular momentum through sound waves. The subcritical baroclinic instability is loosely related to the baroclinic instability that drives weather. The vertical shear instability is when shearing motion is faster than orbital motion and driving the flow of angular momentum at sound speed.

We will focus on the MRI instability (Balbus & Hawley 1991). This is due to local, linear instability of weakly magnetized disks driven by exchange of angular momentum. The condition of the instability does not involve boundary conditions and occur in a localized region of the disk, and can be treated using the WKB approximation.

We will start with a simple mechanical analogy where 2 masses connected by a spring are in orbit around a central body. The orbital frequency is given by Keplerian motion $\Omega^2 = GM/R^3$. The masses can be thought as fluid elements, and spring can be a model of magnetic field line tension which provides restoring force. We denote the frequency of the spring as γ .

Lets setup x - y coordinates for the orbital motion where x is in radial direction, and y points opposite to the orbital motion. We know that

$$\ddot{x} = -2\Omega\dot{y} + 3\Omega^2x - \gamma^2x \quad (1.14)$$

The first term is the Coriolis force, and the second combines the centrifugal force and the gravitational force in an expansion, and the third term is the spring force. The second equation is

$$\ddot{y} = 2\Omega\dot{x} - \gamma^2y \quad (1.15)$$

As a first go, lets set $\gamma = 0$ and assume x and y scale with $e^{-i\omega t}$, the equations become

$$-\omega^2x = -2\Omega(-i\omega)y + 3\Omega^2x \quad (1.16)$$

$$-\omega^2y = 2\Omega(-i\omega)x \quad (1.17)$$

This simply gives $\omega^2 = \Omega^2$ which means that the natural modes oscillate at the orbital frequency.

Now lets add the γ term, and we get

$$-\omega^2x = -2\Omega(-i\omega)y + 3\Omega^2x - \gamma^2x \quad (1.18)$$

$$-\omega^2y = 2\Omega(-i\omega)x - \gamma^2y \quad (1.19)$$

And solving the dispersion relation gives

$$\omega^4 - \omega^2(\Omega^2 + 2\gamma^2) + \gamma^2(\gamma^2 - 3\Omega^2) = 0 \quad (1.20)$$

Notice that there is a zero solution condition when $\gamma^2 = 3\Omega^2$, and it is a transition from positive to negative frequency square. If we plot ω^2/Ω^2 vs γ^2/Ω^2 , there are two sets of roots. One is simply spring motion. The other starts with $\omega^2 = 0$, and between $\gamma^2 = 0$ and $3\Omega^2$ we have $\omega^2 < 0$ where there is an instability. The fastest growing mode of the instability is at $\gamma^2/\Omega^2 = 15/16$ and $\omega^2/\Omega^2 = -9/16$.

This is directly analogous to an MHD problem. Lets zoom in on the disk and assume it is penetrated by a weak vertical magnetic field. This instability is related to the magnetic instability where the field is bent, and the fluid element above and below are coupled by Alfven waves. In this case we have

$$\gamma^2 \rightarrow (\mathbf{k} \cdot \mathbf{v}_A)^2 \quad (1.21)$$

Lets list some facts about MRI linear theory. The ideal fluid instability only requires $d\Omega^2/dr < 0$. The maximum growth rate is $(3/4)\Omega$ for Keplerian motion. The fastest growing mode is given above. We know that there is a local instability when there is vertical field, but a local instability also occurs even when there is only toroidal field.

However linear theory doesn't tell us what happens in the regime when linear growth saturates. To study this regime we need simulations. In simulations people have used local or global setups. There are models with explicit small dissipation terms, or ILES which self-consistently evolves turbulence cascade from the scale where energy is injected down to the scale of dissipation. There are also isothermal models with equation of state $P = c_s^2 \rho$, or one can use energetically self-consistent equation of state.

Some simulations were shown...

Simulations of MRI told us a few things. In 2D MRI leads to turbulence and α . We know that in 2D MRI does not converge, so α depends on resolution. In 3D MRI also leads to turbulence and α , but sometimes it does converge. We know that it converges when explicit dissipation is used. There are issues with ILES models with unstratified shearing boxes. We also learned that α depends on many parameters. It depends on the local height z , and it increases away from the central plane. It also depends on the magnitude of the vertical magnetic field that threads the disk $\langle B_z \rangle$. When the vertical magnetic field increases the viscosity also increases. It depends on the magnetic Reynolds number and $\text{Pr}_M = \nu/\eta$.

1.4 Current Problems in Disk Theory

Ran out of time...