Selected Topics in Plasma Astrophysics

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July 19, 2016

1 Lecture 1

1.1 Astrophysical Plasmas in General

Plasma astrophysics is a very broad subject, encompassing a lot of different sub fields. There are a lot of different techniques to study them as well. For relativistic plasma physics, there are force-free electrodynamics to study e.g. pulsar magnetospheres. There is a family of (GR)(M)HD that study accretion, jets, etc. There is PIC that is used to study shocks, and Dynamic space-time + MHD to study compact object mergers.

For non-relativistic theory we use force-free for e.g. solar corona, MHD for e.g. star formation, disks, etc., and kinetic theory for shocks, reconnection, turbulence, etc. Even within MHD, there is a wide range of fluid models which are useful for different situations.

In the first lecture we focus on fluid models with some kinetic elements.

1.2 Stellar Winds

Stellar winds is a topic that is rich and has influence over many other branches of plasma astrophysics, and it will be our topic today. We will be discussing different kinds of winds:

- Thermally driven winds (from sun-like stars): hydrodynamic theory, kinetic theory
- Magnetocentrifugically driven winds: MHD, rotation as energy source but trapped by B fields.
- Radiation pressure driven outflow: $L > L_{\rm edd}$: continuum driven or line-driven, depending on the spectrum

1.3 Solar Corona and Wind

Solar corona and outflow is an important part of solar dynamics. The solar outflow is very small in terms of mass and energy. However the solar wind is efficiently extracting the angular momentum from the sun. Its spin has slowed down by a factor of 30 to 50 since its birth. The time scale of dJ/dt is on the order of 10^{10} years.

The solar corona is not in thermal equilibrium. We know that $T_i \gg T_p \geq T_e$, and is anisotropic $T_\perp \geq T_\parallel$. The corona is mostly collisionless where $\ell_{mfp} \sim 10^8 \rho_{\rm Larmor}$. It turns out that despite the collisionlessness we can use fluid theory to somewhat describe the solar wind.

Lets start by talking about the Parker model. Consider a time-independent outflow, the continuity equation is

$$\nabla \cdot (\rho \mathbf{v}) = 0 \Longrightarrow r^2 \rho v_r = \text{const} \tag{1.1}$$

From this the mass loss rate is simply $\dot{M} = 4\pi r^2 \rho v_r$ which is also a constant. The radial velocity equation reads

$$v\frac{dv}{dr} = -\frac{1}{\rho}\frac{dP}{dr} - \frac{GM}{r^2} \tag{1.2}$$

If we assume temperature being constant, pressure $P = \rho c_s^2 = \rho kT/m_p$, then we have this equation

$$\frac{1}{v_r}\frac{dv_r}{dr}(v_r^2 - c_s^2) = \frac{2c_s^2}{r} - \frac{GM}{r^2} \tag{1.3}$$

This equation describes spherical wind, or spherically symmetric (Bondi) accretion.

There is a special point in this equation called the sonic point. This is where $v_r = c_s$. This requires that the right hand side is also zero

$$\frac{2c_s^2}{r} = \frac{GM}{r^2} \tag{1.4}$$

In the regime where $v < c_s$, we can ignore the left hand side of equation (1.3). This gives a spatial distribution of density. However when this is true, the pressure is very big. In fact if we require that the flow goes through the sonic point peacefully, we can uniquely obtain a unique outflow solution from this equation. TODO: clean this paragraph up

1.4 Magnetic Field

Around the sun there are magnetic field lines that open up to infinity, and field lines that close back to the star itself. For a non-rotating star, the above theory can be thought of as describing the acceleration of the outflow along the open magnetic field lines. The only non-trivial thing is the structure of the global magnetic field.

However now let's consider when the star is rotating. Lets simplify the actual magnetic field to a split monopole configuration, which can be called a "theorist's monopole". This is a useful toy model where we can calculate structures of the outflow relatively easily. If we have a very powerful outflow that blows up the magnetic field lines, then at large distances it will actually look like a split monopole field.

We will now focus on the equatorial plane. Imagine the magnetic field lines have infinite tension. The outflow will rigidly follow the field lines, and forced to corotate with the star. As they flow out their rotation velocity will increase with radius, as well as their specific angular momentum. Therefore they extract angular momentum from the star. The magnetic field acts as a medium that extracts angular momentum from the star and transfer it to the outflow.

The corotation only can keep up till some point, where the magnetic energy becomes comparable to the kinetic energy of the material

$$\frac{B^2}{8\pi} \sim \rho v_r^2 \tag{1.5}$$

At this point there is so much energy in the gas that you can't treat the magnetic field as rigid. This is called the *Alfven point* r_A. This is also when

$$v_r = v_A \sim \frac{B}{\sqrt{4\pi\rho}} \tag{1.6}$$

Lets look at this outflow in more detail. Let

$$\mathbf{v} = v_r\hat{r} + v_\phi\hat{\phi}, \quad \mathbf{B} = B_r\hat{r} + B_\phi\hat{\phi} \tag{1.7}$$

We want to use the conservation of momentum and energy again like before to write down the equation governing the outflow. We have

$$\dot{M} = 4\pi r^2 \rho v_r = \text{const}, \quad \frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\nabla \times \mathbf{B}) = 0 \tag{1.8}$$

We can show that

$$\frac{B_\phi}{B_r} = \frac{v_\phi - r\Omega}{v_r} \tag{1.9}$$

The way to think about this result is that, if there is perfect corotation, then there is no B_ϕ. If v_ϕ can't keep up with $r\Omega$ the magnetic field will start to lag behind and be swept back, leading to a negative component of B_ϕ.

The next equation which is important is the momentum equation in the ϕ direction. It tells us how does angular momentum change

$$\rho v_r \frac{d}{dr}(r v_\phi) = \frac{B_r}{4\pi}\frac{d}{dr}(r B_\phi) \tag{1.10}$$

If we multiply by r^2 we can easily integrate this equation, giving

$$L = r v_\phi - \frac{r B_\phi B_r}{4\pi\rho v_r} = \text{const} \tag{1.11}$$

There are two contributions to the angular momentum: L_{gas} and L_{mag}. Therefore the angular momentum is carried out both by the gas and the magnetic field, and the total angular momentum is a constant in the problem.

To find what the constant is, we can substitute equation (1.9) into this equation and solve:

$$v_\phi = r\Omega\left(\frac{v_r^2 L/r^2\Omega - v_{Ar}^2}{v_r^2 - V_{Ar}}\right) \tag{1.12}$$

where $v_{Ar}^2 = B_r^2/4\pi\rho$. This has the same structure as before: the denominator is zero at some point. Here is apparent that the magnetic field component that determines the Alfven point is the radial point. Therefore at the Alfven point we have

$$L = r_A^2\Omega \tag{1.13}$$

This means that the wind solution determines exactly how much angular momentum is extracted from the star. This idea is also why the outflow model is so important in plasma astrophysics. Because the magnetic field can be very strong, we could have $r_A \gg R_*$. The rate of angular momentum loss is

$$\dot{J} = \dot{M}L = \dot{M}r_A^2\Omega \tag{1.14}$$

so the time scale is

$$t = \frac{J}{\dot{J}} \sim \frac{MR_*^2}{\dot{M}r_A^2} \tag{1.15}$$

Due to the large "level arm" of the Alfven radius, this time scale can be much smaller than the time scale of mass loss. This efficient extraction of angular momentum due to magnetic field will occur in other places in plasma astrophysics as well.

The last thing we want to talk about is the energy carried by the outflow. Consider hydrodynamics, we have the Bernoulli constant which is essentially energy per unit mass:

$$B_l = \frac{1}{2}v^2 + \phi + h, \quad h = \frac{5}{2}\frac{kT}{m_p} \tag{1.16}$$

where h is called the enthalpy $h = \gamma/(\gamma-1)kT/m$. In thermally driven wind, if we heat up the gas it will be able to escape. However in MHD we have an additional loss of energy in the form of Poynting flux, which also taps into the rotational energy of the star. We find that

$$B_l + \frac{S_r}{\rho v_r} = \text{const} \tag{1.17}$$

where S_r is the radial Poynting flux. Given the magnetic field structure it is easy to calculate the Poynting flux

$$\frac{S_r}{\rho v_r} = -\frac{r\Omega B_r B_\phi}{4\pi\rho v_r} \tag{1.18}$$

If we call this constant ϵ, then it is like the angular momentum flux, it has a gas component and also a magnetic field component, where in fact $\epsilon_{\text{mag}} = \Omega L_{\text{mag}}$.

Now the question is which of these is more important. Lets roughly estimate it to get a feeling:

$$\epsilon_{\text{gas}} \sim \frac{kT_{\text{corona}}}{m_p}, \quad \epsilon_{\text{mag}} \sim r_A^2\Omega^2 \tag{1.19}$$

For the sun, the gas energy flux is much larger than the magnetic energy flux, and most of the energy outflow is thermal energy. This is a thermally driven wind. In the other regime most of the energy that goes to infinity is the rotational energy of the central object. This is what we call a magnetically driven outflow. Which regime we have depends no the temperature of the corona, rate of rotation, and Alfven radius.

Lets come back to the question is why do we use a fluid model at all for the sun. The sun is a slowly rotating object. Around the sun the presence of magnetic field means $\rho_{\text{Larmor}} \ll R$, so plasma streams only in one direction. Along the B field pressure is the origin of acceleration, but fluid theory is okay within order of about a few. Kinetic instabilities also helps us by limiting how much our distribution function can deviate from Maxwellian.

The state of the art of solar corona study is to understand how particles are heated in the solar corona. This heating has been measured from the corona all the way to much further away from the Earth. The temperature profile is shallower than adiabatic, so there must be heating going on. One idea is that Alfven waves are launched and the turbulence keep heating the plasma.

The time-reversal of this outflow wind model is the spherical Bondi accretion. There is also the theory of magnetically directed accretion to the central object analogous to the magnetically driven wind models. But it works not as good because in the wind model the outflow is driven by the rotation of the star, and same can't be said for accretion models.

1.5 Radiative Driven Winds

Lets now briefly talk about radiation pressure driven winds. Around RGB and AGB stars, a lot of dust forms in stellar atmosphere which have high κ. They feel the radiation pressure much higher than the gas but are coupled to the gas through collision, therefore driving a wind. Around massive stars $L > L_{\mathrm{edd}}$ on metal lines, and the plasma absorption of metal line photons drives the wind.

In thermally driven winds the energy outflow is $\dot{M}$ times the speed of sound squared. However for the radiation pressure driven winds we should think of momentum flux due to photons

$$\dot{P} \sim \dot{M} v_\infty \sim L/c, \quad v_\infty \sim v_{\mathrm{escape}} \tag{1.20}$$

We can derive this from the momentum equation directly

$$v\frac{dv}{dr} = -\frac{1}{\rho}\frac{dP}{dr} - \frac{GM}{r^2} + \kappa\frac{F}{c} \tag{1.21}$$

where the right hand side has a new term which is an average radiation pressure force over all wavelengths. If we only focus on the new term, we can write

$$v\frac{dv}{dr} = \frac{\kappa L}{4\pi r^2 c} \tag{1.22}$$

Multiplying both sides by $4\pi r^2 \rho$ we have

$$\dot{M}\frac{dv}{dr} = \rho\kappa\frac{L}{c} \tag{1.23}$$

Integrating this over all radii to infinity we now have

$$\dot{M} v_\infty = \tau\frac{L}{c} \tag{1.24}$$

where τ is the optical depth $\tau = \int \rho\kappa \, dr$. That is exactly we have, and the optical depth is the one uncertain numerical coefficient.

For line-driven winds refer to (Lucy & Solomon 1970; Castor, Abott, Klein 1975).

For applications of this wind theory in other models/fields, refer to the original slides! Mentioned models are thermally driven galactic winds, line driven winds from accreting black holes, magnetized winds from accretion disks.

2 Lecture 2

2.1 Instabilities in Fluids and Plasmas