An Introduction to Cosmic Rays

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1 Lecture 1

Why should we study cosmic rays? There are two parts of motivations. On the collective side we are concerned about the totality of cosmic rays: we want to understand better how energy is partitioned between thermal gas, magnetic fields, and cosmic rays in the ISM/ICM. We want to understand how cosmic rays interact with thermal gas. On the particle side we want understand how cosmic rays are produced and the energy spectrum.

The plan of the lecture follows this bifurcation. We will talk about he particle picture and the fluid picture. Then we will talk about diffusion which will lead to a self-consistent diffusion model with classical cosmic ray hydrodynamics. There is also the generalized cosmic ray hydrodynamics. Both will lead to the discussion of applications to galactic winds.

Some early history of cosmic ray astrophysics. In 1912 Hess showed the source of atmosphere ionization are cosmic. The ionization increases with height, which is an evidence that the source is not terrestrial. In 1927 Clay showed ionizing flux is latitude dependent, suggesting "rays" are charged particles. In 1934 Baad and Zwicky proposed that cosmic rays are from supernovae. In 1949 Hall and Hiltner observed galactic magnetic field, and in the same year Fermi proposed the acceleration mechanism.

One way to detect cosmic rays is through Cherenkov detectors in large water banks. The energy spectrum is a broken power law

$$\begin{aligned} N(E) &\sim E^{-2.7}, \quad E_{\mathrm{PeV}} < 3 \\ &\sim E^{-3.0}, \quad 3 < E_{\mathrm{PeV}} < 100 \end{aligned} \tag{1.1}$$

The energy density is about $1\,\mathrm{eV/cm^3}$ which is in equipartition with the magnetic and thermal turbulent energy density of interstellar gas. Most of the pressure comes from $\sim$ GeV particles.

The composition of cosmic rays is similar to solar system composition, but they are rich in Li, Be, B. This can be explained by collisions of C, N, O cosmic rays with interstellar gas which shatters them to smaller atoms. Cosmic rays are also not enriched in r-process elements which suggests they are not direct ejecta of SNe.

We can use ^{10}Be to date the confinement time of cosmic rays because it is an unstable isotope with half life of about a million years.

One can't talk about cosmic rays without talking about interstellar magnetic field. We compute the galactic magnetic field by measuring the Faraday rotation of the (~ 38000) extragalactic sources. There is a coherent and nearly azmuthal component of the B field nearly tangent to the galactic plane. The total field is about $\sim 5\,\mu$G.

If we separate the spectrum element by element, it becomes really complicated, and every element has different features. This probably represents some joint effect of acceleration and propagation.

The distribution of cosmic rays arriving on earth is highly isotropic to better than 0.1%, and it becomes more anisotropic at energies higher than about 10^{16} eV.

Remote sensor data shows that dense molecular clouds are sources of γ-ray emission. One can get more quantitative by fitting the γ-ray spectrum and radio spectrum of other galaxies. By fitting the radio and γ-ray spectra one can test the assumption of equipartition at various different galaxies. Equipartition holds in the milky way, but it can't keep up in galaxies with higher and higher star formation rates.

There is a tight correlation between far-infrared radio luminosity and synchrotron luminosity, which seems to hold at least to $z \sim 2$.

Lets draw some inferences from the above. From light element abundances we can infer about 2.6×10^7 yr galactic confinement time for cosmic rays. Similar processes occur in other galaxies. Source spectrum is about $E^{-(2.0-2.2)}$, and source power is equivalent to about 10% of SN energy input.

What are we going to do with all these? From particle point of view we want to explain the mechanism of acceleration and understand how propagation affects them. From the collective view we want to understand how they modify the structure and energetics of the ambient medium.

If we look at the particle orbits in galactic magnetic field, one can find that Larmor radius is $r_L \sim 10^{12}$ cm. We can't really put these cosmic ray particles into our simulations because the scale is too tiny. We need to develop a statistical picture for transport and diffusion in phase space, and this will help us developing a fluid picture for global feedback.

The elements of field-particle interaction include many topics, including gyromotion, drifts, etc. For example grad-B drift will move particles along the B field lines, and it is most effective when the B field varies at the same length scale as the particle gyro radius. There is also magnetic mirroring, where we have an adiabatic invariant $\mu = p_\perp^2/B$. There is gyroresonant scattering. Since particle orbits follow field lines and short wavelength fluctuations of the field lines average out. However if the Doppler shifted fluctuation frequency is similar to the gyrofrequency of the particle, its motion will grow in amplitude. Finally there is Landau resonance, which is the condition where $\omega = k_\parallel v_\parallel$. Resonant particles can exchange energy with a wave through parallel electric field, and this is the mechanism for Landau damping. Wave dissipation by this mechanism is sometimes also called transit damping.

An example of cross-field transport is (Desiati & EZ 2014), or (arXiv:1402.1475).

Can we quantify this transport? We define the running diffusion tensor

$$D_{ij}(t) = \frac{1}{2N}\sum_{n=1}^{N}\frac{\left[x_{i,n}(t) - x_{i,n}(0)\right]\left[x_{j,n}(t) - x_{j,n}(0)\right]}{\Delta t} \tag{1.2}$$

the diffusivity in the parallel and perpendicular directions are quite different:

$$\kappa_\parallel = \left\langle \frac{v_{parallel}^2}{\nu} \right\rangle = \frac{v^2}{3\nu}, \quad \kappa_\perp = \kappa_\parallel = \frac{r_g^2 \nu}{3} \tag{1.3}$$

I fell asleep... Details plz refer to the paper.

Next we will talk about cosmic ray spectrum and 2nd order Fermi acceleration, derive a Fokker-Planck equation and a transport equation.

1.1 Analytics

Lets first talk about Fermi's argument. Suppose particles are injected into our system at a constant rate and they can be lost statistically. Lets denote the number of cosmic ray particles that enter our system in a time period dt by $\tilde{n}(t)dt = \tilde{n}_0 e^{-t/\tau_L} dt$, where τ_L is a time scale of particle loss. If we know how much energy is inserted at different times, we can invert that and define

$$n(E)dE = \tilde{n}(t(E))dt, \quad n(E) = \tilde{n}(t(E))\frac{dt}{dE} \tag{1.4}$$

Lets suppose that $dE/dt = E/\tau_{\text{accel}}$, which is some acceleration law. Then this equation is very easy to solve

$$E = E_0 e^{t/\tau_{\text{accel}}}, \quad t(E) = \tau_{\text{accel}} \ln\left(\frac{E}{E_0}\right) \tag{1.5}$$

If we implement this into our distribution equation we get

$$n(E) = \frac{\tilde{n}_0 \tau_{\text{accel}}}{E_0}\left(\frac{E}{E_0}\right)^{-(1+\tau_{\text{accel}}/\tau_L)} \tag{1.6}$$

This is a power law spectrum, and the spectrum steepness is related to the relation between acceleration time scale and loss time scale.

In this model the oldest cosmic rays are the most energetic, however in our data we saw that energetic cosmic rays have less confinement time, so they are younger. This means data is against this kind of distributive model.

To make this kind of spectrum work with observation we need $\tau_{\text{accel}} \sim \tau_L$. There are so many ways to accelerate particles and so many ways to lose particles, how do we make sure this is true?

Fermi considered the so-called "magnetic clouds". These clouds move back and forth at random at velocity v, and particles can be trapped in them, and can be reflected off them. When particles are reflected at a moving mirror they can either gain or lose energy. Consider a head-on collision, the particle with momentum p colliding a cloud with velocity v, then $\Delta E = +2pv$, however if particles are overtaking the cloud, then $\Delta E = -2pv$. If the motions are all random then there is no net acceleration. However if we denote the average separation of the clouds as L, then the frequency of head-on collisions will be

$$\nu = \frac{c+v}{L} \tag{1.7}$$

whereas those overtaking collisions will be $\nu = (c-v)/L$. Therefore if we sum over many collisions the net gain of energy will be

$$\frac{dE}{dt} = 2pv\left(\frac{c+v}{L} - \frac{c-v}{L}\right) = \frac{4pv^2}{L} = \frac{4Ev^2}{c^2 L} \tag{1.8}$$

This has exactly the same form as the one assumed above, which leads to a power law spectrum. The acceleration time scale is about $\tau_{\text{accel}} \sim c^2 L/v^2 c$. What is this number? In galaxies $c^2/v^2 \sim 10^9$, the separation between the clouds is around $L \sim 3 \times 10^{20}\,\text{cm}$. Putting everything together the acceleration time would be $\tau \sim 10^{19}\,\text{s} \sim 3 \times 10^{11}\,\text{yr}$. We also know that the loss time is about $\tau_L \sim 2.6 \times 10^7\,\text{yr}$. This gives extremely steep spectrum which does not work at all. This is called second order Fermi acceleration because v^2 shows up in the time scale.

Lets consider diffusion. We define the pitch angle of scattering

$$\mu = \frac{\mathbf{p} \cdot \mathbf{B}}{pB} \tag{1.9}$$

We are going to derive a diffusion in μ space, and show that it leads to diffusion in x space and apply it to shock acceleration and worry about how to make it self-consistent.

We are going to derive a probability function

$$P(\mu, \Delta\mu) = \text{Probability that a particle with } \mu \text{ is scattered by } \Delta\mu \text{ in time } \Delta t \tag{1.10}$$

And we normalize this function such that integrated over $\Delta\mu$ it gives some number ξ. Integrating this we have a distribution function $f(\mu, t)$ which is our cosmic ray distribution function (which also depends on x and t). We have

$$f(\mu, t + \Delta t) = \int f(\mu - \Delta\mu, t) P(\mu - \Delta\mu, \Delta\mu) d\Delta\mu \tag{1.11}$$

which simply means that particles at $t + \Delta t$ with μ can come from various "kicks" of different $\Delta\mu$. We are going to expand both sides, next time.

2 Lecture 2

Lets refresh our memory about some of the quantities we defined

$$\mu = \frac{\mathbf{p} \cdot \mathbf{B}}{pB} \tag{2.1}$$

We define the probability of scattering, and then we could take the expectation value like the following

$$\frac{\langle \Delta\mu \rangle}{\Delta t} = \frac{1}{\xi \Delta t} \int \Delta\mu P(\mu, \Delta\mu) \, d\Delta\mu \tag{2.2}$$

For a purely random process this expectation value should be zero. Similar expectation value expression for $\Delta\mu\Delta\mu$. We can derive an equation for f by Taylor expanding equation (1.11)

$$\frac{\partial f(\mu, t)}{\partial t} = -\frac{\partial}{\partial \mu} \left[\frac{\langle \Delta\mu \rangle}{\Delta t} f(\mu, t) \right] + \frac{1}{2} \frac{\partial^2}{\partial \mu^2} \left[\frac{\langle \Delta\mu\Delta\mu \rangle}{\Delta t} f(\mu, t) \right] \tag{2.3}$$

Using chain rule we can expand the derivatives and we can see that there are terms with f, ∂f, and $\partial^2 f$.

P preserves isotropy if and only if

$$-\frac{\langle \Delta\mu \rangle}{\Delta t} + \frac{\partial}{\partial \mu} \frac{\langle \Delta\mu\Delta\mu \rangle}{\Delta t} = 0 \tag{2.4}$$

If we assume this condition, then we can write the equation in a much simpler form

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial \mu} \frac{\langle \Delta\mu\Delta\mu \rangle}{2\Delta t} \frac{\partial f}{\partial \mu} \tag{2.5}$$

This is simply a diffusion equation in pitch angle space, which is what we promised. If one does this in cylindrical coordinates, one could write

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial \mu} \frac{\nu(1 - \mu^2)}{2} \frac{\partial f}{\partial \mu} \tag{2.6}$$

Lets consider the whole Vlasov equation with the scattering term, in 1D for simplicity

$$\frac{\partial f}{\partial t} + \mu v \frac{\partial f}{\partial z} + q\left(\frac{\mathbf{v}\times\mathbf{B}}{c}\nabla_p f\right) = \frac{\partial}{\partial \mu}\nu(1-\mu^2)\frac{\partial f}{\partial \mu} \tag{2.7}$$

where we define the one spatial dimension as the dimension of the magnetic field $\mathbf{B} = \hat{z}B$. In this case $\mathbf{v}\times\mathbf{B}$ goes to zero. We solve this equation using perturbation approach.

There are three time scales in this problem. The first is the scattering time scale $1/\nu$. Second is the advective time scale L/c. Finally there is the evolution time τ. We assume the hierarchy to be

$$\nu^{-1} \ll \frac{L}{c} \ll \tau \tag{2.8}$$

If we do that then we can solve the Vlasov equation order by order. To lowest order we simply have

$$\frac{\partial f_0}{\partial \mu} = 0 \tag{2.9}$$

which says to zeroth order the cosmic ray is isotropic. The next order becomes

$$\mu v \frac{\partial f_0}{\partial z} = \frac{\partial}{\partial \mu}\nu(1-\mu^2)\frac{\partial f_1}{\partial \mu} \tag{2.10}$$

One could integrate this one time with respect to μ:

$$\frac{\partial f_1}{\partial \mu} = -\frac{v}{\nu}\frac{\partial f_0}{\partial z} \tag{2.11}$$

This simply says that the cosmic ray angle is simply related to the density anisotropy. The evolution equation for f_0 is

$$\frac{\partial f_0}{\partial t} = \frac{\partial}{\partial z}\frac{v^2}{3\nu}\frac{\partial f}{\partial z} \tag{2.12}$$

where we now label $D = v^2/3\nu$.

Lets now apply these to diffusive shock acceleration (DSA), following Blandford & Ostriker. We assume there is a discontinuity as a shock, where u_1 and u_2 are the velocities upstream and downstream. There is a magnetic field along the flow direction. u_1 and u_2 are related by mass conservation. If there is a compression ratio $\rho_2 = r\rho_1$, then $u_2 = u_1/r$. A cosmic ray starts downstream, undergoes a head-on collision at upstream, gains energy by $2p \cdot u_1$, and at downstream it collides again and loses energy by $2p \cdot u_2$. Because $u_1 > u_2$ the cosmic ray gains net energy, as opposed to the second order Fermi process.

How do we characterize this process in math? Leap of faith:

$$\frac{\partial f}{\partial t} + u\frac{\partial f}{\partial z} - \frac{p}{3}\frac{\partial u}{\partial z}\frac{\partial f}{\partial p} = \frac{\partial}{\partial z}D\frac{\partial f}{\partial z} \tag{2.13}$$

To get a feeling of why this equation is viable, one can integrate over momentum space to get the continuity equation ($n = \int f d^3p$)

$$\frac{\partial n}{\partial t} = -\frac{\partial}{\partial z}(un) \tag{2.14}$$

The second way is that if we multiply the equation by $cp/3$ we can get the pressure equation for a $\gamma = 4/3$ gas.

Now lets look at the equation. We assume steady state $\partial_t = 0$ and idealize the shock as a delta function $\partial u/\partial z = (u_2 - u_1)\delta(z)$. Now the equation simply becomes

$$u\frac{\partial f}{\partial z} = \frac{\partial}{\partial z}D\frac{\partial f}{\partial z} \tag{2.15}$$

Lets apply the jump condition (which is in the notes), and then we can get

$$f_z(p) \propto p^{-3r/(r-1)} \tag{2.16}$$

What is r? For a gas with γ this is

$$\gamma \to \frac{\gamma+1}{\gamma-1}, \quad \text{for } \frac{u_1}{c_s} \gg 1 \tag{2.17}$$

For a gas with $\gamma = 5/3$ we have $r \to 4$ which agrees nicely with observation. We said SNe can't be direct source of cosmic rays but now DSA provides a way to accelerate cosmic ray from SNe.

Two problems remain however. We have no shock layer, and we don't know the amplitude of the spectrum.

Now turn to the slides... notes end here.