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Wigner crystals in flat band 2D electron systems

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Wigner crystals — ordered arrays of electrons — have been recently found in various 2D materials, but the first studies of these crystals in 2D electron systems (2DESs) date back from the 1980s. Mansour Shayegan gives a brief history of Wigner crystals and highlights future prospects.

Wigner crystals occur if the Coulomb energy (E_C) dominates the kinetic energy of electrons, causing the electrons to arrange themselves into a periodic lattice to minimize E_C. Realizing Wigner crystals in experiments is a challenge because of competing energies, including disorder, which disturb the periodic arrangement of electrons and, if sufficiently strong, eventually destroy the Wigner crystal phase. Despite this difficulty, in the past year, there have been multiple reports of Wigner crystals in atomic-layer thin 2D materials[1–3].

However, such materials are not the first to host Wigner crystals. As early as 1979, a classical Wigner crystal was realized in a very low-density 2D electron system (2DES) confined to the surface of liquid helium at low temperatures so that E_C was much larger than the kinetic (thermal) energy[4]. At higher electron densities, the Fermi energy is large and plays the role of the kinetic energy; in this case a quantum Wigner crystal can be stabilized if E_C is much larger than the Fermi energy and temperature is sufficiently low. This can be achieved in 2DESs in modulation-doped semiconductor quantum wells with relatively flat energy bands, such as in systems with large electron effective mass, or via the application of magnetic fields.

Here I present a very brief summary of the platforms where quantum Wigner crystals have been realized and highlight some of the unsolved puzzled. Future improvements of the quality and minimization of disorder in host systems, as well as new measurement techniques are likely to shed more light on the physics of this exotics phase of matter.

Quantum Wigner crystals

One way to facilitate the formation of a quantum Wigner crystal is to apply a strong, quantizing, perpendicular magnetic field, which quenches the kinetic energy by forcing the electrons into the lowest Landau level, essentially a flat band[4–8]. Observation of such a magnetic-field-induced Wigner crystal at very small Landau level filling factors (ν) led to the discovery of the fractional quantum Hall (FQH) effect at $\nu = 1/3$ in modulation-doped GaAs quantum wells in 1982.

Around 1990, magneto-transport and microwave resonance studies on higher mobility (lower-disorder) GaAs 2D electron systems revealed that the ground state is a FQH liquid even at the small filling $\nu = 1/5$, but this state is flanked by reentrant insulating phases at nearby fillings[4]. The strongly non-linear current–voltage characteristics of these insulating phases, as well as the observed microwave resonances, are generally interpreted to signal the formation of a Wigner crystal pinned by the small but non-zero disorder potential[4]. The fact that the Wigner crystal is 'reentrant' around the $\nu = 1/5$ FQH state is a consequence of the close competition of the ground-state energies at very low fillings. This competition depends on the parameters of the 2D carrier systems, such as the effective mass and the thickness of the electron layer. Signatures of Wigner crystals have been seen even near integer quantum Hall states where there are very few quasi-particles so that the effective filling factor is small[8].

Realizing a quantum Wigner crystal at zero magnetic field is more challenging. Theoretically, when the ratio of E_C to Fermi energy or, equivalently, when the parameter r_s (the average inter-electron distance measured in units of the effective Bohr radius), exceeds ~35 the 2D electrons should solidify into a quantum Wigner crystal. If disorder can be kept at a low level, since $r_s \sim m^*/n^{1/2}$, where m^* is the effective mass and n is the 2DES density, large r_s can be achieved in a very dilute system with a large m^*. Despite their high mobility, m^* of 2D electrons in GaAs is too small ($m^* = 0.067$ in units of the free electron mass). However, a quantum Wigner crystal was achieved in very dilute GaAs 2D hole systems which have a much larger m^* (~0.4) in 1999 (REF.[9]). Over 20 years later, similar phenomena were reported at very large r_s for 2D electrons in another host system, AlAs, where the electrons have a large m^* (~0.46) (REF.[10]). Recent work on 2D materials with flat bands indicates that these may also be useful platforms to harbour zero-field Wigner crystals.

Bilayer probes

Most experimental studies of the magnetic-field-induced Wigner crystal involve measurements that probe the bulk properties. However, it is also possible to directly probe

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https://doi.org/10.1038/s42254-022-00444-4

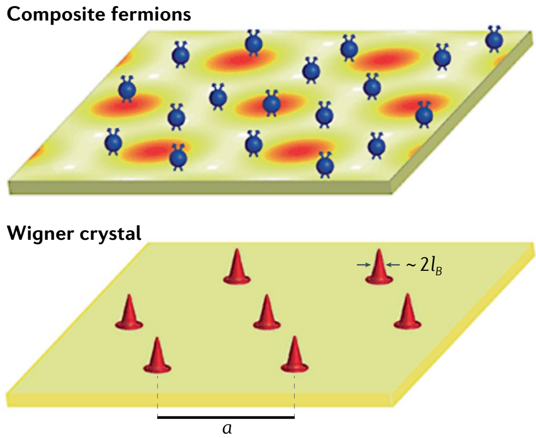

Fig. 1 | **Bilayer system used for probing a Wigner crystal using composite fermions.** The top layer feels a periodic potential modulation from the bottom Wigner crystal layer's charge density. Commensurability oscillations of composite fermions induced by the periodic potential of a Wigner crystal. a, lattice constant; l_B, magnetic length. Reproduced with permission from H. Deng et al., *Phys. Rev. Lett.* **117**, 096601 (2016); APS.

the microscopic structure of the Wigner crystal using a bilayer system. For example, using a GaAs/AlGaAs double quantum well structure, it is possible to create two parallel layers of 2DESs in close proximity, which have different densities[5] (FIG. 1). The bottom layer has a very low density so that its filling factor becomes very small at high magnetic fields (≥10 T), and the 2D electrons condense into a Wigner crystal. The top layer has much larger density so that, at high fields, this layer attains filling factor 1/2 and composite fermions are formed.

Composite fermions are quasi-particles, formed by pairing an even number of magnetic flux quanta with each electron. The pairing makes even a ~10 T magnetic field essentially invisible to the dressed electrons so they behave as if they are non-interacting and are moving near zero magnetic field. These composite fermions can feel the periodic electrostatic potential induced by the Wigner crystal in the bottom layer. Thus, the resistance of the bilayer system as a function of magnetic field near $\nu = 1/2$ of the top layer shows oscillations induced by the periodic potential of the Wigner crystal electrons[5]. These oscillations can provide insight into the symmetry of the Wigner crystal, its lattice constant, and melting[5]. This bilayer experiment demonstrated a method of probing an exotic many-body state of 2D electrons using quasi-particles of another many-body state.

A similar bilayer setup can be used to probe signatures of Wigner crystals at zero magnetic field. In recent work, Wigner crystal formation was attained in a WSe_2/WS_2 moiré heterostructure, and a nearby graphene sensing layer was used to probe the effect of the Wigner crystal's periodic structure, using scanning tunnelling microscopy[3].

Open questions

Among the outstanding questions to address in all studies of Wigner crystals is the amount and role of disorder. How ordered are the observed Wigner crystal phases? Data such as non-linear current–voltage characteristics or microwave resonances in GaAs-based samples have been analysed to extract Wigner crystal 'domain' sizes, which, in the highest quality GaAs samples, are estimated to contain thousands of electrons, but these are yet to be directly measured. A related question is how the disorder affects other correlated electron states. If a perpendicular magnetic field is applied to the 2DES in an AlAs sample, there are clear signs of integer and FQH states even when r_s is as large as 35 (REF.[10]). But no hints of such states have been reported in 2D material samples at large r_s. Why?

Another puzzle is that the reported thermal melting temperatures for the zero-field Wigner crystal states are far larger than those expected theoretically for quantum Wigner crystals in ideal 2D systems (with no disorder)[1,2,10]. The explanation likely lies in disorder breaking up the Wigner crystal into relatively small domains, thus raising the melting temperatures. Future experiments on 2DESs with less disorder, or controlled disorder, could solve this puzzle.

A further question concerns the shape of the Wigner crystal lattice. It is generally assumed that the Wigner crystal in an isotropic 2DES attains a triangular lattice, and the available experimental data are consistent with this conjecture[2,5]. What would happen if the 2D system had an intrinsic in-plane anisotropy, such that the electrons have an anisotropic effective mass or energy versus wavevector dispersion, as is the case for AlAs 2D electrons?[10]

There is also the important and long-standing question regarding the competition between the magnetic-field-induced Wigner crystal and the FQH liquid at very small filling factors. In ultra-high-quality GaAs 2DESs with world-record high mobility that have become available very recently, there is a reasonably deep resistance minimum at $\nu = 1/7$ at intermediate temperatures (~0.1 K), suggesting a developing FQH state. But the sample shows an insulating behaviour at nearby fillings, and even at $\nu = 1/7$, as temperature is lowered, implying a pinned Wigner crystal. The data imply that at these very small ν, even a miniscule amount of disorder and density inhomogeneity tilt the competition in favour of the Wigner crystal.

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Competing interests
The author declares no competing interests.