CRYSTALLIZATION OF ELECTRONS ON THE SURFACE OF LIQUID HELIUM

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A system of electrons on the surface of liquid helium is shown to fulfill the requirements for crystalline ordering.

The problem of a collection of electrons moving in a uniform background of positive charge was analyzed by Wigner [1]. In a dilute system the quantum mechanical kinetic energy of the electrons is small compared to their potential energy and they form a regular lattice to minimize coulomb repulsion energy. There have been many subsequent discussions of this effect [2-10] which has received the name "Wigner crystallization". To date, however, no clear example has been found of a physical system which behaves in this way. For crystallization the average distance between electrons must be about ten Bohr radii or greater [1]. This is difficult to achieve in a solid together with the constraint that the electrons move in a uniform background of positive charge.

We have recently found a system which seems to satisfy these conditions. In addition it offers considerable flexibility for doing experiments and the possibility of making refined calculations. Figure 1a shows our system [11] consisting of electrons deposited on the surface of liquid helium from a negative corona discharge. Because of the negative electron affinity of liquid helium, they do not enter the liquid but are bound by the electrostatic image force in shallow surface states localized outside the liquid [12]. An isolated electron is free to move laterally along the surface [13]. In the experiments, however, the collection of electrons is confined to a limited area by a positively biased metal plate located 1 to 10 mm below the surface. Electron densities between 10^7 and 10^9 cm^{-2} are readily obtained experimentally so that the average spacing between electrons is of order 10^{-4} cm. Since their separation from the positively charged plate is much greater than this, there are no significant lateral variations of the field except near the edges of the plate. Therefore the conditions of a dilute electron gas in a uniform positive potential are fulfilled.
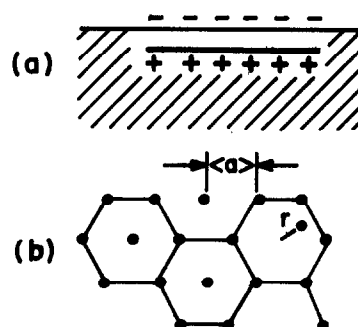

Fig. 1. (a) Electrons on the surface of liquid helium confined to a limited area by a metal plate below the surface bearing an equal amount of positive charge. (b) Close-packed hexagonal lattice.

We have estimated the effects of lateral coulomb repulsion forces among the electrons, assuming them to lie at the points of a close-packed hexagonal lattice of lattice spacing, a (fig. 1b). An electron in its lattice site is at a minimum of potential energy. Under a small displacement, r, in the plane of the lattice, it experiences a quadratic potential of the form, $U(r) = Br^2$. The constant, B, is evaluated by summing the potentials due to all the remaining electrons on their lattice sites, giving $B \approx 2.7\, e^2/a^3$. As an example we take $a = 4.5 \times 10^{-5}$ cm. For the quantum mechanical harmonic oscillator the frequency, ν, is 2.1×10^{10} Hz and the zero point energy $h\nu/k$, is 1.0 K. In the ground state, the average displacement from equilibrium, r, is 3.0×10^{-6} cm. This is only 7% of the mean distance, a, between electrons. To a good approximation, therefore, the electrons are localized on their lattice sites and the state of minimum energy for the array is one of crystalline order.

We note that, in an externally applied electric field, one electron cannot move independently of

all the others. This will have a strong effect on experiments designed to measure the lateral mobility of electrons on the surface. In a field, the whole crystal should move as a unit as suggested by Kohn [5].

Finally, a distribution of free electrons separated from their corresponding positive charges may be produced in a solid by means of a field electrode. In analogy with the present experiment, conditions for crystallization of such electrons may eventually be found.

We are indebted to Jules Levine for acquainting us with the concept of crystallized electron systems and their possible relation to our experimental system.

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ELECTROOPTIC EFFECT AND DIELECTRIC PROPERTIES OF CADMIUM-MERCURY-THIOCYANATE CRYSTALS

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The electrooptic half-wave field-distance product of Cd [Hg (SCN)$_4$] single crystals, belonging to the tetragonal crystal class I $\bar{4}$, has been measured. The low electrooptic coefficient r_{13} in comparison with the high d_{31}-coefficient for SHG can be explained by the Miller-rule.

Cd [Hg (SCN)$_4$] crystals belong to the tetragonal symmetry class I $\bar{4}$. They have been grown in our laboratories in sizes of several millimeters [1]. Stürmer and Deserno [1] showed that the SHG-coefficient is comparable to that of LiNbO$_3$. So it was of great interest for technical applications to know its electrooptic and dielectric properties.

The nonvanishing electrooptic coefficients in crystal class I $\bar{4}$ are: $r_{13} = -r_{23}$, $r_{41} = r_{52}$, $r_{51} = -r_{42}$ and r_{63} [2]. The equation of the indicatrix for an applied electrical field E_3 can be written as:

$$\left(\frac{1}{n_0^2} + r_{13} E_3\right) x_1^2 + \left(\frac{1}{n_0^2} - r_{13} E_3\right) x_2^2 + \frac{1}{n_e^2} x_3^2 + 2 r_{63} E_3 x_1 x_2 = 1 \qquad (1)$$

The half-wave field-distance product $[E \cdot l]_{\lambda/2}$ for light propagation in x_1- or x_2-direction is:

$$[E \cdot l]_{\lambda/2} = \lambda / n_0^3 r_{13} \qquad (2)$$

The measurements of the transverse electrooptic effect have been achieved by placing the crystal between crossed polarizers and analyzing the amplitude modulation of light by means of a photomultiplier. Because the crystals were of small size, only several millimeters, we could not achieve full modulation without electrical breakdown. Therefore the light intensity, transmitted by the modulation device, was adjusted to be $\frac{1}{2}(I_{max} + I_{min})$ by means of optical bias, where I_{max} and I_{min} are the light intensities at the on- and off-position of the light modulator. The half-wave field-distance product and the electrooptic coefficient were derived