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Melting temperature of two-dimensional electron crystals trapped on thin-film liquid He

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The Kosterlitz-Thouless melting criterion is applied to the solid-liquid transition of the two-dimensional electron system on a thin film of liquid He supported by a dielectric substrate, in which the electron-electron interaction is modified from the pure Coulombic form. Good agreement with recent experiments by Jiang and Dahm is obtained.

Investigation of two-dimensional electrons trapped on a thin film of liquid He supported by a dielectric substrate has attracted much attention recently.[1,2] The characteristic features which differentiate it from the conventional two-dimensional electron system bound on the surface of bulk liquid He are that: (i) the image potential becomes stronger because of the large dielectric constant of the substrate, (ii) the He film becomes stiffer than the bulk He and can support more electrons, and (iii) the electron-electron interaction takes the form

$$V(R)=e^2\left[\frac{1}{R}-\frac{\kappa}{(R^2+4d^2)^{1/2}}\right], \tag{1}$$

where $-e$ is the electronic charge, d the thickness of the film, and κ is given by

$$\kappa=(\epsilon-1)/(\epsilon+1), \tag{2}$$

with ϵ the dielectric constant of the dielectric substrate. In Eq. (1) the second term represents the correction due to the image charge induced in the substrate. This system is of particular interest because the electron mutual interaction is not Coulombic but modified, and varies with the thickness of liquid He. The melting properties of the electron crystal, which were found for the case of bulk liquid ($d=\infty$) (Ref. 3), are expected to be modified.

A theoretical determination of the melting temperature T_m for this system was tried by Peeters[4] in which the ratio of the average mutual interaction $V[(\pi n)^{1/2}]$ to the kinetic energy was calculated, where n is the electron surface density, and this ratio was set to some number $\Gamma^*(d)$. In their phenomenological approach there remains the freedom of the choice of the parameter $\Gamma^*(d)$, and Peeters employed the value

$$\frac{V[(\pi n)^{1/2}]}{k_B T_m}\equiv\Gamma^*(d)=\frac{\Gamma(\infty)s_t^2(\infty)}{s_t^2(d)}, \tag{3}$$

where $\Gamma(\infty)\sim 130$ is the critical plasma parameter for the bulk case and $s_t(d)$ is the velocity of the transverse sound mode of the crystal for thickness d. Their predicted melting temperature becomes very small for small d as compared to the bulk case. The recent experiment by Jiang and Dahm[2,5] suggests that T_m is much higher than the value predicted by Peeters.

In this Rapid Communication it is pointed out that the straightforward application of the melting criterion, due to Kosterlitz and Thouless,[6,7] reproduces the experimental data quite well. According to Kosterlitz and Thouless, the melting mediated by dislocations occurs at the temperature T_m given by

$$k_B T_m=\frac{nma^2}{4\pi}s_t^2(d)\left[1-\frac{s_t^2(d)}{s_l^2(d)}\right], \tag{4}$$

where m is the electron mass, a is the lattice constant of the crystal, and $s_l(d)$ is the velocity of the longitudinal sound mode of the crystal for film thickness d. It is known that the triangular lattice is stable for sufficiently low temperatures;[8] this is called the Wigner crystal. The longitudinal sound velocity $s_l(d)$ is essentially given by the velocity of the two-dimensional plasmons and is very large as compared to $s_t(d)$, so we set $s_t(d)/s_l(d)=0$ in Eq. (4). The numerical evaluation of $s_t(d)$ was performed by Meissner, Namaizawa, and Voss[8] and Peeters,[4] but we present here an approximate analytic calculation which is very accurate and convenient to handle.

The angular frequency $\omega_t(\mathbf{k})$ of the transverse Wigner phonon with wave vector $\mathbf{k}$ is given, to within the harmonic approximation, by

$$\omega_t^2(\mathbf{k})=\frac{1}{m}\sum_{\mathbf{R}\neq 0}\sum_{\alpha,\beta}\epsilon_t^\alpha(\mathbf{k})\epsilon_t^\beta(\mathbf{k})\frac{\partial^2 V(R)}{\partial R_\alpha \partial R_\beta}(1-\cos\mathbf{k}\cdot\mathbf{R}), \tag{5}$$

where $\mathbf{R}$ is the two-dimensional position vector of the lattice point and $\epsilon_t^\alpha(\mathbf{k})$ is the αth component of the polarization vector of the transverse mode. Since

$$\frac{\partial^2 V(R)}{\partial R_\alpha \partial R_\beta}=\delta_{\alpha\beta}\frac{V'(R)}{R}+\frac{R_\alpha R_\beta}{R}\frac{d}{dR}\frac{V'(R)}{R}, \tag{6}$$

where primes denote the differentiation with respect to R, we have

$$\omega_t^2(\mathbf{k})=\frac{1}{m}\sum_{\mathbf{R}\neq 0}\left[\frac{V'(R)}{R}+\sin^2\theta R\frac{d}{dR}\frac{V'(R)}{R}\right]\times[1-\cos(kR\cos\theta)], \tag{7}$$

where θ is the angle between $\mathbf{k}$ and $\mathbf{R}$. For the approximate evaluation of Eq. (7), we replace the discrete sum over the lattice points by the continuous integration

$$\sum_{\mathbf{R}\neq 0}=n\int_{R_c}^{\infty}dR\,R\int_0^{2\pi}d\theta, \tag{8}$$

where R_c is the cutoff length of order of half the lattice constant in order to exclude the origin, viz.,

$$R_c = c(\pi n)^{-1/2}, \tag{9}$$

with c a constant of order of unity. Performing the θ integration, we have

$$\omega_t^2(k) = \frac{\pi n}{m}\int_{R_c}^{\infty} dR\left[\left(2V'(R)+R^2\frac{d}{dR}\frac{V'(R)}{R}\right)[1-J_0(kR)]-R^2\frac{d}{dR}\frac{V'(R)}{R}J_2(kR)\right], \tag{10}$$

where $J_n(x)$ is the nth order Bessel function of the first kind. Inserting the explicit expression for $V(R)$, given in Eq. (1), and noting the formulas for the Hankel transform,[9] we can convert Eq. (10) to an integration in the region $0 < R < R_c$, and we get

$$\omega_t^2(k) = (\pi n e^2/m)[A(0) - \kappa A(d)], \tag{11}$$

where

$$A(d) = \int_0^{R_c} \frac{dRR}{(R^2+4d^2)^{5/2}} \times\{(R^2-8d^2)[1-J_0(kR)]-3R^2J_2(kR)\}. \tag{12}$$

Expanding the Bessel functions up to k^2 and performing the integration, we arrive at the expression for $s_t(d)$ as

$$s_t^2(d) = \left[\frac{e^2(\pi n)^{1/2}c}{8m}\right]f(d), \tag{13}$$

where

$$f(d) = 1-\kappa(1+4\pi n d^2/c^2)^{-3/2}. \tag{14}$$

The choice of the constant $c=1$ is already very good, but in order to be more accurate c is chosen in such a way that the factor in large parentheses in Eq. (13) reproduces the exact numerical value for $s_t^2(\infty)$ obtained by Bonsall and Maradudin:[10]

$$s_t^2(\infty) = 0.138\,26\,e^2(\pi n)^{1/2}/m, \tag{15}$$

which leads to

$$c = 1.1061. \tag{16}$$

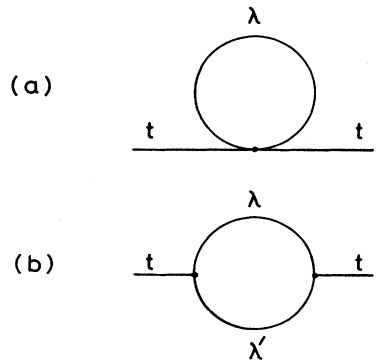

FIG. 1. Diagrams of the lowest-order anharmonic correction where λ and λ′ represent either the longitudinal or transverse phonon modes.

Our analytic formula in Eq. (13) with Eqs. (14) and (16) reproduces the numerical result of Peeters[4] very accurately (cf. Fig. 6 in his paper).

Next we consider the correction due to the anharmonicity which arises from the higher derivatives of $V(R)$. The anharmonic correction comes from two types of diagrams, as shown in Fig. 1. Writing out the explicit forms of the anharmonic correction, we see immediately from the dimensional analysis that $\Delta s_t^2(d)$ has the form

$$\Delta s_t^2(d) = -\gamma(d)k_BT/m, \tag{17}$$

where $\gamma(d)$ is a dimensionless constant, which does not depend on the coupling constant e^2. The melting temperature expected from Eq. (4) is, therefore, given by

$$k_BT_m(d) = e^2(\pi n)^{1/2}f(d)/\Gamma(d), \tag{18}$$

where

$$\Gamma(d) = [2\pi\sqrt{3}+\gamma(d)]/0.138\,26. \tag{19}$$

For the bulk case ($d=\infty$), $\gamma(\infty)$ was calculated by Fisher[11] and Chang and Maki:[12] $\gamma(\infty)\sim 3.5$, which gives $\Gamma(\infty)\sim 105$. For the case $d=0$, it is easy to see from Eq. (1) that the mutual potential $V(R)$ reduces to the pure Coulombic form as in the bulk case and the only difference is that the coupling constant is modified to $e^2(1-\kappa)$. Since γ is independent of the coupling constant, we should have $\gamma(0)=\gamma(\infty)$. Since at both ends γ has the same value, we expect that the d dependence of γ is weak and that $\gamma(d)$ in Eq. (19) may be replaced by the

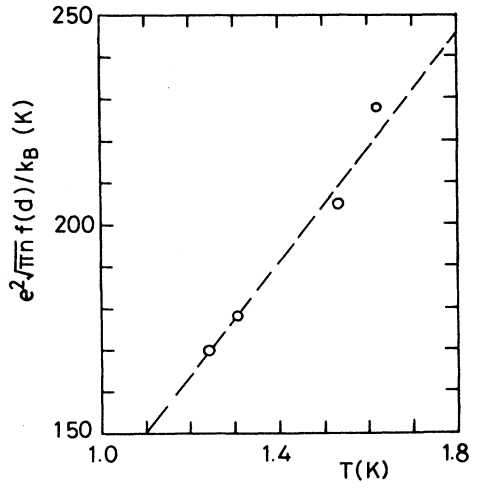

FIG. 2. $e^2(\pi n)^{1/2}f(d)$ against the melting temperature T_m, where $f(d)$ is defined by Eq. (14) in the text. The slope of the dashed line is 137.

value for the bulk He, and $\Gamma(d) \approx \Gamma(\infty)$. From these arguments the difference between Eq. (18) and Eq. (3) is clear. Our criterion gives $T_m \propto f(d)$ for the thickness dependence, while Peeters[4] yielded $T_m \propto f(d)[1-\kappa(1+4\pi nd)^{-1/2}]$, a double counting from our point of view.

In Fig. 2, $e^2(\pi n)^{1/2} f(d)$, as derived from the data by Jiang and Dahm,[2,13] are plotted against the observed melting temperature. The summary of the data is given in Table I. The linearity of the data is clear. Note that without the correction factor $f(d)$ in Eq. (18), no good correlation between $T_m(d)$ and $(\pi n)^{1/2}/\Gamma(\infty)$ is expected. The Γ value derived from the slope is 137, exactly what is expected from the bulk experimental value.[3] These facts strongly suggest that the Kosterlitz-Thouless melting mechanism is operating in this system.

In conclusion, we have shown that the straightforward application of this dislocation-mediated melting criterion reproduces the right order of the magnitudes of the melting temperature, which are observed in the two-dimensional electrons on thin liquid He films.

TABLE I. Data of melting temperature T_m for different film thicknesses d and electron densities n. The dielectric constant of the substrate is $\epsilon = 7.3$.

Sample No.	n (10^{10} cm)	d (nm)	T_m (K)
1	0.75	30.5	1.24
2	0.9	26.0	1.31
3	1.0	28.5	1.54
4	1.3	24.0	>1.62

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