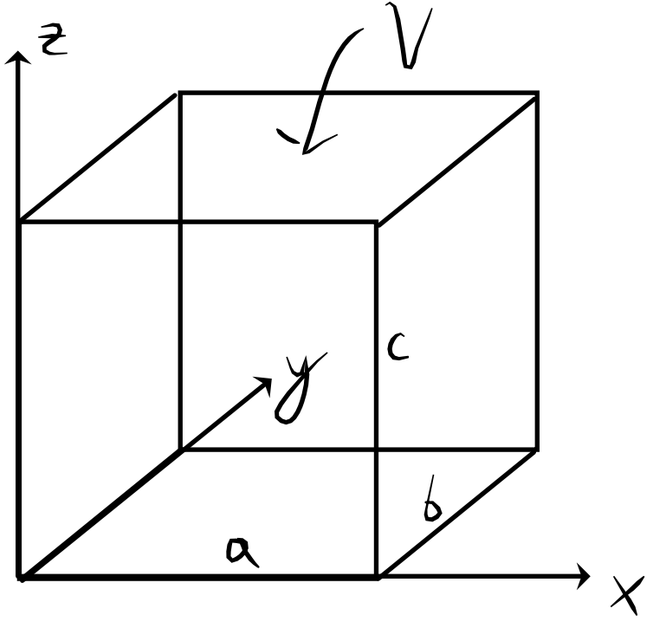

$$\Phi(x,y,z) = A(x)\,B(y)\,C(z)$$

$$A(0)=0 \qquad B(0)=0 \qquad C(0)=0$$

$$A(a)=0 \qquad B(b)=0 \qquad C(c)=0$$

$$\Delta\Phi = 0 \qquad \frac{1}{A}\frac{d^2}{dx^2}A + \frac{1}{B}\frac{d^2}{dy^2}B + \frac{1}{C}\frac{d}{dz^2}C = 0$$

α β γ

$$\frac{d^2}{dx^2}A - \alpha A = 0$$

$\alpha < 0 \quad A = a_1 \sin(\sqrt{|\alpha|}\,x) + a_2 \cos(\sqrt{|\alpha|}\,x)$

$\alpha = 0 \quad A = a_1 + a_2 x$

$\alpha > 0 \quad A = a_1 \sinh(\sqrt{\alpha}\,x) + a_2 \cosh(\sqrt{\alpha}\,x)$

dla $\alpha = 0 \quad A(0) = A(a) = 0 \Rightarrow A = 0$, ale wtedy $\phi(x,y,c) = 0 \neq V$

dla $\alpha > 0 \quad A(0) = 0 \Rightarrow a_2 = 0 \quad A(a) = 0 = a_1 \sinh(\sqrt{\alpha}\,a)$ sprzeczne

więc $\alpha < 0 \quad A(0) = 0 \Rightarrow a_2 = 0$

$$A(a) = 0 = a_1 \sin(\sqrt{|\alpha|}\,a) = 0 \Rightarrow \sqrt{|\alpha|}\,a = n\pi \qquad \alpha = -\frac{n^2\pi^2}{a^2}$$

Podobnie dla β: $0 > \beta = -\frac{k^2\pi^2}{b^2}$

$$\alpha + \beta + \gamma = 0 \qquad \gamma = -\alpha - \beta = \frac{n^2\pi^2}{a^2} + \frac{k^2\pi^2}{b^2} > 0$$

$$C = c_1 \sinh(\sqrt{\gamma}\,z) + c_2 \cosh(\sqrt{\gamma}\,z) \qquad C(0) = 0 \Rightarrow c_2 = 0$$

$$\begin{cases} A = a_n \sin\left(\frac{n\pi}{a}x\right) \\ B = b_k \sin\left(\frac{k\pi}{b}y\right) \\ C = c_{nk} \sinh\left(\sqrt{\left(\frac{n}{a}\right)^2 + \left(\frac{k}{b}\right)^2}\,\pi z\right) \end{cases}$$

$$\Phi(x,y,z) = \sum_{n,k} a_n \sin\left(\frac{n\pi}{a}x\right) b_k \sin\left(\frac{k\pi}{b}y\right) c_{nk} \sinh\left(\sqrt{\left(\frac{n}{a}\right)^2 + \left(\frac{k}{b}\right)^2}\,\pi z\right)$$

$$\phi(x,y,c) = V = \sum_{n,k} D_{nk} \sin\left(\frac{n\pi}{a}x\right) \sin\left(\frac{k\pi}{b}y\right) \sinh\left(\sqrt{\left(\frac{n}{a}\right)^2 + \left(\frac{k}{b}\right)^2}\,\pi c\right)$$

$$\int_0^a dx \int_0^b dy\; V \sin\left(\frac{n'\pi}{a}x\right) \sin\left(\frac{k'\pi}{b}y\right) = \int_0^a dx \int_0^b dy \;\underline{\quad} \sin\left(\frac{n'\pi}{a}x\right) \sin\left(\frac{k'\pi}{b}y\right)$$

0^{**} dla n,k - nieparzystych

$$V\frac{2a}{\pi n'}\frac{2b}{\pi k'} = \sum_{n,k} D_{nk}\frac{a}{2}\delta_{nn'}\frac{b}{2}\delta_{kk'} \sinh\left(\sqrt{\left(\frac{n}{a}\right)^2 + \left(\frac{k}{b}\right)^2}\,\pi c\right)$$

$$D_{nk} = \frac{16\,V}{\pi^2 nk}\,\mathrm{csch}\left(\sqrt{\left(\frac{n}{a}\right)^2 + \left(\frac{k}{b}\right)^2}\,\pi c\right)$$

$$\Phi(x,y,z) = \sum_{n,k} \frac{16\,V}{\pi^2 nk}\,\mathrm{csch}\left(\sqrt{\left(\frac{n}{a}\right)^2 + \left(\frac{k}{b}\right)^2}\,\pi c\right) \sin\left(\frac{n\pi}{a}x\right) \sin\left(\frac{k\pi}{b}y\right) \sinh\left(\sqrt{\left(\frac{n}{a}\right)^2 + \left(\frac{k}{b}\right)^2}\,\pi c\right)$$

nieparzyste