a) $\int_0^a \sin\left(\frac{n\pi}{a}x\right)\sin\left(\frac{n'\pi}{a}x\right)dx = \frac{1}{2}\int_0^a \cos\left((n-n')\frac{\pi}{a}x\right) - \cos\left((n+n')\frac{\pi}{a}x\right)dx =$

1) $n = n'$ $\quad = \frac{1}{2}\int_0^a 1 - \cos\left(\frac{2\pi}{a}x\right)dx = \frac{a}{2} - \frac{a}{4\pi}\sin\left(\frac{2\pi}{a}x\right)\Big|_0^a = \frac{a}{2}$

2) $n \neq n'$ $\quad = \frac{a}{4\pi}\left[\frac{1}{n-n'}\sin\left((n-n')\frac{\pi}{a}x\right) - \frac{1}{n+n'}\sin\left((n+n')\frac{\pi}{a}x\right)\right]_0^a = 0$

$\Rightarrow \frac{a}{2}\delta_{nn'}$

c) $\int_{-\infty}^{\infty} \sin(k_1x)\sin(k_2x)dx = \frac{-1}{4}\int_{-\infty}^{\infty}\left(e^{ik_1x} - e^{-ik_1x}\right)\left(e^{ik_2x} - e^{-ik_2x}\right)dx =$

$= \frac{-1}{4}\int_{-\infty}^{\infty} e^{i(k_1+k_2)x} - e^{i(k_1-k_2)x} - e^{-i(k_1-k_2)x} + e^{-i(k_1+k_2)x}dx =$

$= -\frac{1}{4}\cdot 2\pi\left(\delta(k_1+k_2) - \delta(k_1-k_2) - \delta(-(k_1-k_2)) + \delta(-(k_1+k_2)x)\right) =$

$= \pi\delta(k_1-k_2) - \pi\delta(k_1+k_2)$