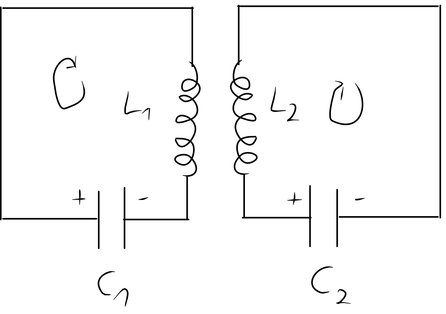

$$\phi_1 = L_1 I_1 + M I_2$$

$$\phi_2 = L_2 I_2 + M I_1$$

$$\varepsilon_1 = -L_1 \dot{I}_1 - M \dot{I}_2 = \frac{Q_1}{C_1}$$

$$\varepsilon_2 = -L_2 \dot{I}_2 - M \dot{I}_1 = \frac{Q_2}{C_2}$$

$$\dot{Q}_1 = I_1 \qquad \dot{Q}_2 = I_2$$

$$Q_2 = -L_2 C_2 \ddot{Q}_2 - M C_2 \ddot{Q}_1$$

$$Q_1 = -L_1 C_1 \ddot{Q}_1 - M C_1 \ddot{Q}_2$$

$$\begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} = \begin{bmatrix} -L_1 C_1 & -M C_1 \\ -M C_2 & -L_2 C_2 \end{bmatrix} \begin{bmatrix} \ddot{Q}_1 \\ \ddot{Q}_2 \end{bmatrix} \qquad Q_1 = A e^{i\omega t} \qquad Q_2 = B e^{i\omega t}$$

$$\begin{bmatrix} 1 - L_1 C_1 \omega^2 & -M C_1 \omega^2 \\ -M C_2 \omega^2 & 1 - L_2 C_2 \omega^2 \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} = 0$$

$$\det = 0$$

$$\left(1 - L_1 C_1 \omega^2\right)\left(1 - L_2 C_2 \omega^2\right) - M^2 C_1 C_2 \omega^4 = 0$$

$$\omega^4 \left(L_1 L_2 C_1 C_2 - M^2 C_1 C_2\right) + \omega^2 \left(-L_1 C_1 - L_2 C_2\right) + 1 = 0$$

$$\omega^4 C_1 C_2 \left(1 - \frac{M^2}{L_1 L_2}\right) - \omega^2 \left(\frac{C_1}{L_2} + \frac{C_2}{L_1}\right) + \frac{1}{L_1 L_2} = 0$$

a) $\frac{M^2}{L_1 L_2} = 1$ $\qquad \omega^2 = \frac{\frac{1}{L_1 L_2}}{\frac{C_1}{L_2} + \frac{C_2}{L_1}} = \frac{1}{C_1 L_1 + C_2 L_2}$

b) $\frac{M^2}{L_1 L_2} = 0$ $\qquad \omega^4 C_1 C_2 - \omega^2 \left(\frac{C_1}{L_2} + \frac{C_2}{L_1}\right) + \frac{1}{L_1 L_2} = 0$

$$\omega^2 = \frac{\frac{C_1}{L_2} + \frac{C_2}{L_1} \pm \sqrt{\frac{C_1^2}{L_2^2} + \frac{C_2^2}{L_1^2} - \frac{2 C_1 C_2}{L_1 L_2}}}{2 C_1 C_2}$$

$$\omega^2 = \frac{\frac{C_1}{L_2} + \frac{C_2}{L_1} \pm \left(\frac{C_1}{L_2} - \frac{C_1}{L_1}\right)}{2 C_1 C_2}$$

$$\omega_1 = \frac{1}{\sqrt{L_1 C_1}} \qquad \omega_2 = \frac{1}{\sqrt{L_2 C_2}}$$

c) $L_1 = L_2 = L \qquad C_1 = C_2 = C$

$$\omega^4 \left(L^2 C^2 - M^2 C^2\right) + \omega^2 \left(-2 L C\right) + 1 = 0$$

$$\omega^2 = \frac{2 L C \pm \sqrt{4 L^2 C^2 - 4 L^2 C^2 + 4 M^2 C^2}}{2 C^2 \left(L^2 - M^2\right)} = \frac{L \pm M}{\left(L^2 - M^2\right) C}$$

$$\omega_1 = \frac{1}{(L - M) C} \qquad \omega_2 = \frac{1}{(L + M) C}$$

d) $L_1 C_1 = L_2 C_2 = X$

$$\omega^4 \left(X^2 - M^2 C_1 C_2\right) + \omega^2 \left(-2 X\right) + 1 = 0$$

$$\omega^2 = \frac{2 X \pm \sqrt{4 X^2 - 4 X^2 + 4 M^2 C_1 C_2}}{2\left(X^2 - M^2 C_1 C_2\right)} = \frac{X \pm M\sqrt{C_1 C_2}}{X^2 - M^2 C_1 C_2}$$

$$\omega_1 = \frac{1}{X - M\sqrt{C_1 C_2}} \qquad \omega_2 = \frac{1}{X + M\sqrt{C_1 C_2}}$$