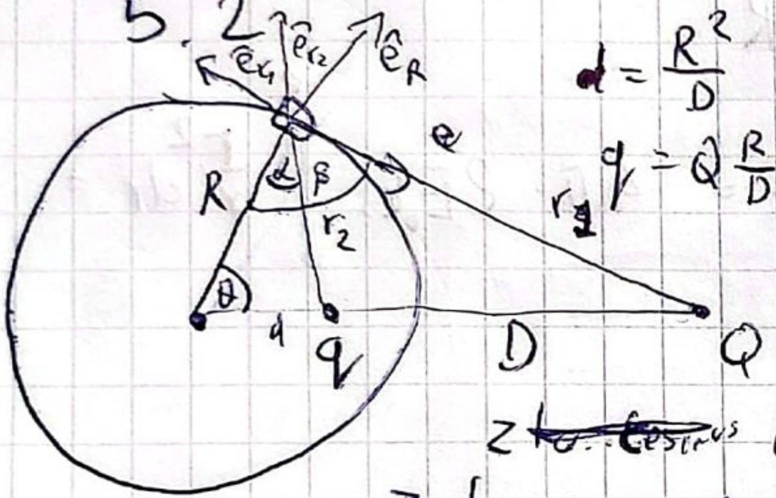
5.2



$$d = \frac{R^2}{D}$$

$$q = Q\frac{R}{D}$$

$$\bar{E} = \bar{E}_Q + \bar{E}_q = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{R^2+D^2-2RD\cos\theta}\hat{e}_{r_1} - \frac{q}{R^2+d^2-2Rd\cos\theta}\hat{e}_{r_2}\right)$$

z ~~tw. cosinus~~ $\cos\alpha = \dfrac{R^2-Rd\cos\theta}{R\sqrt{R^2+d^2-2Rd\cos\theta}}$

z tw. cosinusów

$$\cos\beta = \frac{R-D\cos\theta}{\sqrt{R^2+D^2-2RD\cos\theta}}$$

$$E\hat{e}_R = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{(R^2+D^2-2RD\cos\theta)^{3/2}}(R-D\cos\theta) - \frac{Q\frac{R}{D}\left(R-\frac{R^2}{D}\cos\theta\right)}{\left(R^2+\frac{R^4}{D^2}-2R\frac{R^2}{D}\cos\theta\right)^{3/2}}(R-d\cos\theta)\right)$$

$$\beta = \frac{R}{D}$$

$$\frac{Q(\beta-\cos\theta)}{D^2(\beta^2+1-2\beta\cos\theta)^{3/2}} - \frac{Q\cancel{\beta}(\beta-\beta^2\cos\theta)}{R^2\cancel{\beta}(1+\beta^2-2\beta\cos\theta)^{3/2}} =$$

$$= \frac{QR^2(\beta - \cos\theta) - QD^2(\beta - \beta^2\cos\theta)}{R^2D^2(1+\beta^2 - 2\beta\cos\theta)^{3/2}} = \frac{Q\frac{R^3}{D} - \cancel{QR^2\cos\theta} - QRD + \cancel{QR^2\cos\theta}}{R^2D^2(1+\beta^2-2\beta\cos\theta)^{3/2}} =$$

$$= \frac{Q}{R^2}\frac{(\beta^3 - \beta)}{(1+\beta^2-2\beta\cos\theta)^{3/2}}$$

$$\overline{E} = -\frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}\frac{\beta(1-\beta^2)}{(1+\beta^2-2\beta\cos\theta)^{3/2}}\hat{e}_R$$

$$\sigma(\theta) = E\,\varepsilon_0 = -\frac{Q}{4\pi R^2}\frac{\beta(1-\beta^2)}{(1+\beta^2-2\beta\cos\theta)^{3/2}}$$

a) $$\sigma_{Max} = \sigma(0) = -\frac{Q}{4\pi R^2}\frac{\frac{1}{2}\left(1-\frac{1}{4}\right)}{\left(1+\frac{1}{4}-1\right)^{3/2}} = \frac{-3Q}{4\pi R^2}$$

$$\sigma_{Min} = \sigma(\pi) = -\frac{Q}{4\pi R^3}\frac{\frac{1}{2}\left(1-\frac{1}{4}\right)}{\left(1+\frac{1}{4}\right)^{3/2}} = -\frac{3Q}{4\pi R^2}\cdot\frac{1}{5^{3/2}}$$