a) $$\rho(r) = \begin{cases} \dfrac{Q}{\frac{4}{3}\pi R^3} & \text{dla } r \leq R \\ 0 & \text{dla } r > 0 \end{cases}$$

$$\phi(r) = \frac{Q}{8\pi \varepsilon_0 R}\left(3 - \frac{r^2}{R^2}\right)$$

$$W = \frac{1}{2}\rho \int_0^R \int_0^\pi \int_0^{2\pi} \frac{Q}{8\pi\varepsilon_0 R}\left(3 - \frac{r^2}{R^2}\right) r^2 \sin\theta \, d\varphi \, d\theta \, dr =$$

$$= \frac{1}{4}\rho \frac{Q}{\varepsilon_0 R}\int_0^R \left(3r^2 - \frac{r^4}{R^2}\right) dr = \frac{1}{4}\rho \frac{Q}{\varepsilon_0 R}\left(R^3 - \frac{R^5}{5R^2}\right) = \frac{1}{5}\rho \cdot \frac{Q}{\varepsilon_0 R} R^3 \cdot \frac{4}{3} \cdot \frac{3}{4} \cdot \frac{\pi}{\pi} = \frac{3}{5}\frac{Q^2}{4\pi\varepsilon_0 R}$$

Q

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r^2} \quad \text{dla } r > R$$

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\rho \frac{4}{3}\pi r^3}{r^2} = \frac{\rho}{3\varepsilon_0} r = \frac{Q r}{4\pi\varepsilon_0 R^3} \quad \text{dla } r < R$$

b) $$W = \frac{\varepsilon_0}{2}\int_{\mathbb{R}^3} E^2 dV = \frac{\varepsilon_0}{2}\left[\int_0^\pi \int_0^{2\pi}\left(\int_0^R \left(\frac{Q}{4\pi\varepsilon_0 R^3}\right)^2 r^4 dr + \int_R^\infty \left(\frac{Q}{4\pi\varepsilon_0}\right)^2 \frac{1}{r^2} dr\right)\sin\theta \, d\varphi \, d\theta\right]$$

$$= 2\pi\varepsilon_0\left[\left(\frac{Q}{4\pi\varepsilon_0}\right)^2 \frac{1}{5R} + \left(\frac{Q^2}{4\pi\varepsilon_0}\right)^2 \frac{1}{R}\right] = \frac{\overset{1}{\cancel{2\pi}}\cancel{\varepsilon_0} Q^2}{\underset{4}{\cancel{16}}\cancel{\pi^2}\cancel{\varepsilon_0^2}} \frac{\overset{3}{\cancel{6}}}{5R} = \frac{3}{5}\frac{Q^2}{4\pi\varepsilon_0 R}$$