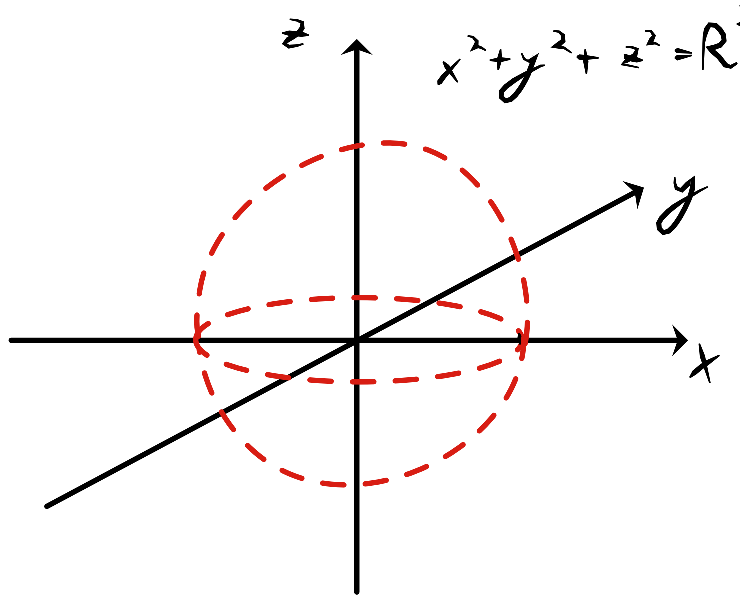



$$\begin{cases} x = R\cos\varphi\sin\theta \\ y = R\sin\varphi\sin\theta \\ z = R\cos\theta \end{cases} \qquad \begin{cases} dx = R\cos\varphi\cos\theta\,d\theta - R\sin\varphi\sin\theta\,d\varphi \\ dy = R\sin\varphi\cos\theta\,d\theta + R\cos\varphi\sin\theta\,d\varphi \\ dz = -R\sin\theta\,d\theta \end{cases}$$

$$g = dx^2+dy^2+dz^2 = R^2\big[\cos^2\varphi\cos^2\theta\,d\theta^2 + \sin^2\varphi\sin^2\theta\,d\varphi^2 - 2\cos\varphi\sin\varphi\cos\theta\sin\theta\,d\varphi\,d\theta$$
$$+\sin^2\varphi\cos^2\theta\,d\theta^2 + \cos^2\varphi\sin^2\theta\,d\varphi^2 + 2\cos\varphi\sin\varphi\cos\theta\sin\theta\,d\varphi\,d\theta$$
$$+\sin^2\theta\,d\theta^2\big] = R^2\,d\theta^2 + R^2\sin^2\theta\,d\varphi^2$$

$$g_{ij} = \begin{bmatrix} R^2 & 0 \\ 0 & R^2\sin^2\theta \end{bmatrix} \qquad g^{ij} = \begin{bmatrix} \frac{1}{R^2} & 0 \\ 0 & \frac{1}{R^2\sin^2\theta} \end{bmatrix}$$

$$\Gamma^i_{kl} = \frac{1}{2} g^{im}\left(g_{km,l} + g_{lm,k} - g_{lk,m}\right)$$

$$\Gamma^1_{11} = \frac{1}{2} g^{11}\left(g_{11,1} + \cancel{g_{11,1}} - \cancel{g_{11,1}}\right) = 0$$

$$\Gamma^1_{22} = \frac{1}{2} g^{11}\left(-g_{22,1}\right) = -\frac{1}{2}\frac{1}{R^2}\,2R^2\sin\theta\cos\theta = -\frac{1}{2}\sin(2\theta)$$

$$\Gamma^1_{21} = \Gamma^1_{12} = \frac{1}{2} g^{11}\left(g_{11,2}\right) = 0$$

$$\Gamma^2_{11} = \frac{1}{2} g^{22}\left(-g_{11,2}\right) = 0$$

$$\Gamma^2_{22} = \frac{1}{2} g^{22}\left(g_{22,2}\right) = 0$$

$$\Gamma^2_{12} = \Gamma^2_{21} = \frac{1}{2} g^{22}\left(g_{22,1}\right) = \frac{1}{2}\,\frac{1}{R^2\sin^2\theta}\,R^2\,2\sin\theta\cos\theta = \cot\theta$$

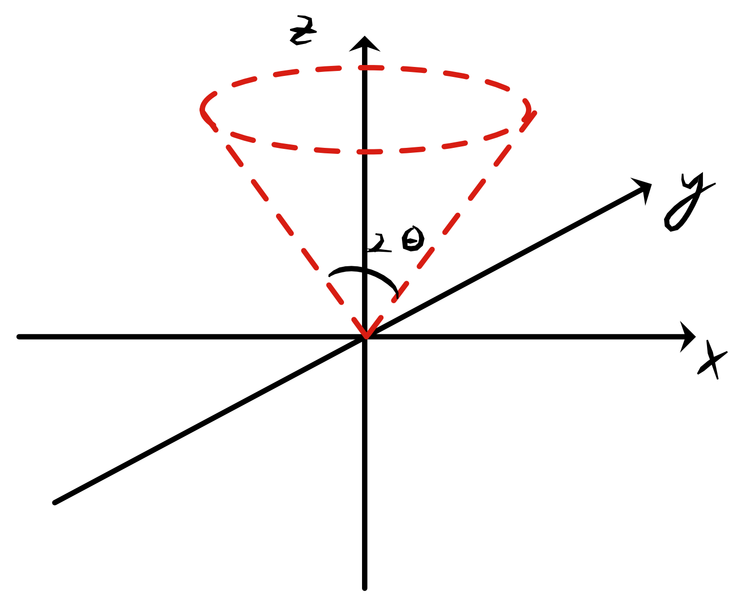
$$\frac{d^2x^k}{d\lambda^2} + \Gamma^k_{ij}\frac{dx^i}{d\lambda}\frac{dx^j}{d\lambda} = 0$$

$$(1)\ \frac{d^2\theta}{d\lambda^2} - \frac{1}{2}\sin(2\theta)\left(\frac{d\varphi}{d\lambda}\right)^2 = 0 \qquad (2)\ \frac{d^2\varphi}{d\lambda^2} + 2\cot(\theta)\frac{d\varphi}{d\lambda}\frac{d\theta}{d\lambda} = 0$$

ciężkie równania, sprawdzę tylko czy koło wielkie na równiku działa: $\theta = \frac{\pi}{2}$ $\quad \varphi = \lambda$

$$1)\ \cancelto{0}{\frac{d^2}{d\lambda}\left(\frac{\pi}{2}\right)} - \frac{1}{2}\cancelto{0}{\sin(\pi)}\left(\frac{d\lambda}{d\lambda}\right)^2 = 0 \ \checkmark$$

$$2)\ \cancelto{0}{\frac{d^2}{d\lambda^2}\lambda} + 2\cot\left(\frac{\pi}{2}\right)\frac{d\lambda}{d\lambda}\cancelto{0}{\frac{d\left(\frac{\pi}{2}\right)}{d\lambda}} = 0 \ \checkmark$$




$$x^2+y^2 = r^2 = z^2\tan^2\theta \qquad \theta \in \left]0;\frac{\pi}{2}\right[$$

$$\begin{cases} x = r\cos\varphi \\ y = r\sin\varphi \\ z = r\cot\theta \end{cases} \qquad \begin{cases} dx = \cos\varphi\,dr - r\sin\varphi\,d\varphi \\ dy = \sin\varphi\,dr + r\cos\varphi\,d\varphi \\ dz = \cot\theta\,dr \end{cases}$$

$$g = dx^2+dy^2+dz^2 = \cos^2\varphi\,dr^2 - 2r\cos\varphi\sin\varphi\,dr\,d\varphi + r^2\sin^2\varphi\,d\varphi^2$$
$$+\sin^2\varphi\,dr^2 + 2r\cos\varphi\sin\varphi\,dr\,d\varphi + r^2\cos^2\varphi\,d\varphi^2$$
$$+\cot^2\theta\,dr^2 = r^2\,d\varphi^2 + \csc^2\theta\,dr^2$$

$$g_{ij} = \begin{bmatrix} \csc^2\theta & 0 \\ 0 & r^2 \end{bmatrix} \qquad g^{ij} = \begin{bmatrix} \sec^2\theta & 0 \\ 0 & \frac{1}{r^2} \end{bmatrix} \qquad \int\sqrt{\csc^2\theta + r^2\left(\frac{dr}{d\theta}\right)^2}\,dt$$

$$\Gamma^1_{11} = 0 \qquad \Gamma^2_{22} = 0$$

$$\Gamma^1_{22} = -r\sec^2\theta \qquad \Gamma^2_{11} = 0$$

$$\Gamma^1_{12} = \Gamma^1_{21} = 0 \qquad \Gamma^2_{12} = \Gamma^2_{21} = \frac{1}{r}$$

$$\frac{d^2x^k}{d\lambda^2} + \Gamma^k_{ij}\frac{dx^i}{d\lambda}\frac{dx^j}{d\lambda} = 0$$

$$(1)\ \frac{d^2r}{d\lambda^2} - r\sec^2\theta\left(\frac{d\varphi}{d\lambda}\right)^2 = 0 \qquad 2)\ \frac{d^2\varphi}{d\lambda^2} + \frac{2}{r}\left(\frac{d\varphi}{d\lambda}\right)\left(\frac{dr}{d\lambda}\right) = 0$$

$$\ddot{r} = r\sec^2\theta\,\dot{\varphi}^2 \qquad \ddot{\varphi} = \frac{2}{r}\dot{\varphi}\dot{r}$$

$$\ddot{r} = \frac{1}{r}\underbrace{\sec^2\theta A^2}_{\omega^2} r^4 \qquad \frac{\ddot{\varphi}}{\dot{\varphi}} = 2\frac{\dot{r}}{r}$$

$$\ddot{r} = \omega^2 r^3 \qquad \ln\dot{\varphi} = 2\ln r + \ln A$$

$$\ddot{r}\dot{r} = \omega^2 r^3\dot{r} \qquad \dot{\varphi} = A r^2$$

$$\frac{\dot{r}^2}{2} = \frac{\omega^2}{4}r^4 + B'$$

$$\dot{r} = \sqrt{B + \tfrac{1}{2}\omega^2 r^4}$$ – zostawiam