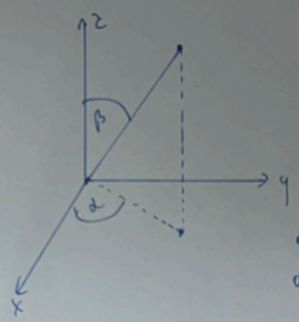
1.



$\alpha = \omega t$ - z treści zadania

$$z = \cos\beta \cdot R$$

$$x = \sin\beta \cdot \cos\alpha \cdot R = \sin\beta \cdot \cos(\omega t) \cdot R$$

$$y = \sin\beta \sin\alpha \cdot R = \sin\beta \cdot \sin(\omega t) \cdot R$$

$$dz = -\sin\beta \, d\beta \cdot R$$

$$dx = \cos\beta \cdot \cos(\omega t) \cdot R \cdot d\beta - \sin\beta \cdot \omega \cdot \sin(\omega t) \cdot R \cdot dt$$

$$dy = \cos\beta \cdot \sin(\omega t) \cdot R \cdot d\beta + \sin\beta \cdot \omega \cdot \cos(\omega t) \cdot R \cdot dt$$

$\frac{ds}{dt} = V = \omega \cdot R$ ← tak mrówa idzie

$$\frac{(ds)^2}{(dt)^2} = \omega^2 \cdot R^2 \qquad ds^2 = (dx)^2 + (dy)^2 + (dz)^2$$

$$\frac{(\sin\beta \, d\beta)^2}{dt^2} + \frac{(\cos\beta \cos(\omega t)\, d\beta)^2}{dt^2} \cancel{- \frac{2 \sin\beta \cos\beta \sin(\omega t) \cdot \cos(\omega t) \cdot \omega \cdot dt \cdot d\beta}{dt^2}}$$

$$+ \sin^2(\beta)\, \omega^2 \sin^2(\omega t) + \left(\frac{\cos\beta \sin(\omega t) \cdot d\beta}{dt}\right)^2 \cancel{+ \frac{2 \cos\beta \cdot \sin\beta \cdot \omega \cdot \sin(\omega t) \cos(\omega t) \cdot d\beta \cdot dt}{dt^2}}$$

$+ (\sin(\beta) \cdot \omega \cdot \cos(\omega t))^2 = \omega^2$ (podzieliłem przez R^2 już)

$$\Downarrow$$

$$\omega^2 = \left(\frac{\sin\beta \, d\beta}{dt}\right)^2 + \left(\frac{\cos\beta \, d\beta}{dt}\right)^2 \left(\cos^2(\omega t) + \sin^2(\omega t)\right) + \sin^2\beta \cdot \omega^2$$

$$\omega^2 = \sin^2\beta\, \omega^2 + \left(\frac{d\beta}{dt}\right)^2 \Rightarrow \omega^2 \cos^2\beta = \left(\frac{d\beta}{dt}\right)^2 \Rightarrow \omega \cos\beta = \frac{d\beta}{dt}$$

$$\frac{d\beta}{\cos\beta} = \omega \, dt \Rightarrow \left[\ln \left| \frac{\sin\frac{\beta}{2} + \cos\frac{\beta}{2}}{\cos\frac{\beta}{2} - \sin\frac{\beta}{2}} \right| \right]_0^\beta = \omega t$$

$$\cos\frac{\beta}{2}\left(e^{\omega t} - 1\right) = \sin\frac{\beta}{2}\left(1 + e^{\omega t}\right) \Rightarrow \operatorname{tg}\frac{\beta}{2} = \left(\frac{e^{\frac{\omega t}{2}} - e^{-\frac{\omega t}{2}}}{e^{\frac{\omega t}{2}} + e^{-\frac{\omega t}{2}}}\right)$$

odp. czyli $\beta = 2 \operatorname{arctg}\left(\operatorname{tgh}\frac{\omega t}{2}\right)$

odp. kiedy $\beta \to \frac{\pi}{2}$ to $t \to \infty$, zatem mrówa nigdy nie dojdzie.