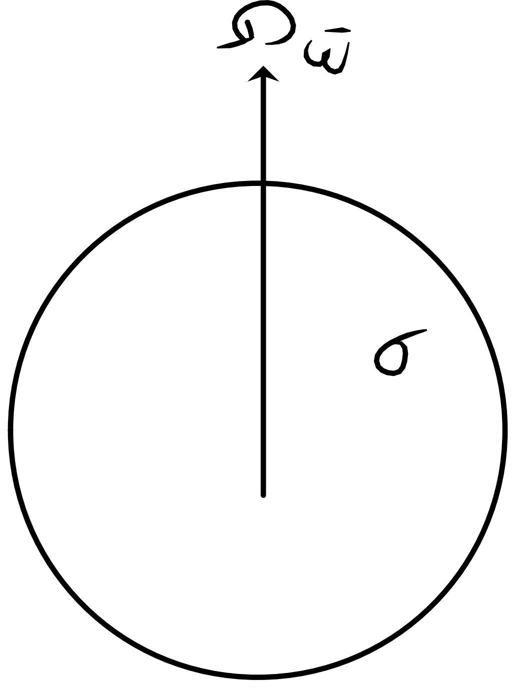


$$\bar B = \bar\nabla \times \bar A \Rightarrow \bar\nabla \cdot \bar A = 0$$

$$\bar\nabla \times \bar B = \mu_0 \bar j$$

$$\bar\nabla \times (\bar\nabla \times \bar A) = \bar\nabla(\cancelto{0}{\bar\nabla \cdot \bar A}) - \nabla^2 \bar A = \mu_0 \bar j$$

Analogia:

$$\nabla^2 \bar\Phi = -\frac{\rho}{\varepsilon_0} \qquad \Big| \qquad \nabla^2 \bar A = -\mu_0 \bar j$$

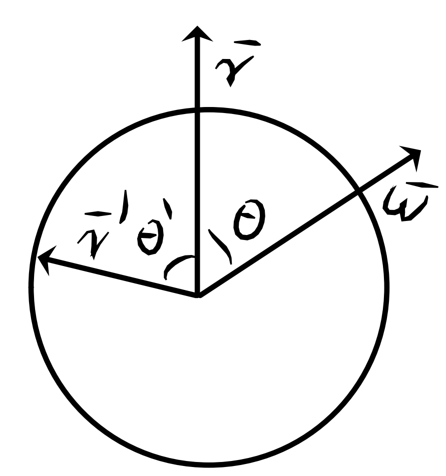
$$\Phi(r) = \frac{1}{4\pi\varepsilon_0}\int_V \frac{\rho(\bar r')\, d^3 r'}{|\bar r - \bar r'|} \qquad \Big| \qquad \bar A(\bar r) = \frac{\mu_0}{4\pi}\int_V \frac{\bar j(\bar r')\, d^3 r'}{|\bar r - \bar r'|}$$

Wektor prądu powierzchniowego: $\bar K(\theta) = \frac{dQ}{dt\,dl}\,\bar e_\varphi = \frac{\sigma\, dS}{dt\,dl}\,\bar e_\varphi =$

$$= \frac{\sigma r^2 \sin\theta\, d\varphi\, \cancel{d\theta}}{dt\, \cancel{r\, d\theta}}\,\bar e_\varphi = \sigma \omega r \sin\theta\, \bar e_\varphi = \sigma\, \bar\omega \times \bar r$$

$$\bar A(\bar r) = \frac{\mu_0}{4\pi}\int_{S(R)} \frac{\bar K(\theta)\, dS}{|\bar r - \bar r'|} = \frac{\mu_0 \sigma}{4\pi}\int_{S(R)} \frac{\bar\omega \times \bar r}{|\bar r - \bar r'|}\, dS = \left\{ \begin{matrix} \text{zmieniam układ współrzędnych} \\ \text{tak, aby } \bar r = r\,\bar e_z \end{matrix} \right\} = *$$

$$\bar r' = R\begin{bmatrix} \sin\theta' \cos\varphi' \\ \sin\theta' \sin\varphi' \\ \cos\theta' \end{bmatrix} \quad \bar\omega = \omega\begin{bmatrix} \sin\theta \\ 0 \\ \cos\theta \end{bmatrix} \quad \bar\omega \times \bar r' = \omega R \begin{bmatrix} \cos\theta \sin\theta' \sin\varphi' \\ \cos\theta' \sin\theta - \sin\theta' \cos\varphi' \cos\theta \\ -\sin\theta \sin\theta' \sin\varphi' \end{bmatrix}$$



$$* = \frac{\mu_0 \sigma \omega}{4\pi} R^3 \int_0^{2\pi}\int_0^{\pi} \begin{bmatrix} \cos\theta \sin\theta' \sin\varphi' \\ \cos\theta' \sin\theta - \sin\theta' \cos\varphi' \cos\theta \\ -\sin\theta \sin\theta' \sin\varphi' \end{bmatrix} \frac{\sin\theta'}{|\bar r - \bar r'|}\, d\theta'\, d\varphi = \frac{2\pi \mu_0 \sigma \omega R^3}{4\pi}\int_0^{\pi} \begin{bmatrix} 0 \\ \cos\theta' \sin\theta \\ 0 \end{bmatrix} \frac{\sin\theta'}{|\bar r - \bar r'|}\, d\theta' =$$

$$= \frac{\omega \mu_0 \sigma R^3 \sin\theta\, \bar e_y}{2}\int_0^{\pi} \frac{\cos\theta'}{|\bar r - \bar r'|}\, d\theta = \frac{2}{3}\,\frac{\omega \mu_0 \sigma R^3 \sin\theta\, \bar e_y}{2}\begin{cases} \frac{r}{R^2} & r < R \\ \frac{R}{r^2} & r > R \end{cases} = \frac{\mu_0 \sigma \omega \sin\theta\, \bar e_y}{3}\begin{cases} R\, r & r < R \\ \frac{R^4}{r^2} & r > R \end{cases}$$

$$= \frac{\mu_0 \sigma}{3}\, \bar\omega \times \bar r \begin{cases} R & r < R \\ \frac{R^4}{r^3} & r > R \end{cases}$$

dla $r < R$

$$\bar B = \bar\nabla \times \bar A = \frac{\mu_0 \sigma R}{3}\bar\nabla \times (\bar\omega \times \bar r) = \frac{\mu_0 \sigma R}{3}\left(\bar\omega(\bar\nabla \cdot \bar r) - \bar r\,(\cancelto{0}{\bar\nabla \cdot \bar\omega}) + \cancelto{0}{(\bar r \cdot \bar\nabla)\bar\omega} - (\bar\omega \cdot \bar\nabla)\bar r\right) = \frac{\mu_0 \sigma R}{3}\left(\bar\omega(\bar\nabla \cdot \bar r) - (\bar\omega \cdot \bar\nabla)\bar r\right)$$

dla $r > R$

$$\bar B = \bar\nabla \times \bar A = \frac{\mu_0 \sigma R^4}{3}\left(\bar\omega\left(\bar\nabla \cdot \frac{\bar r}{r^3}\right) - (\bar\omega \cdot \bar\nabla)\frac{\bar r}{r^3}\right)$$