$$\Phi(r) = \frac{q}{4\pi\varepsilon_0} \frac{e^{-\alpha r}}{r}\left(1+\frac{\alpha r}{2}\right) \qquad \Delta\Phi(r) = \frac{-\rho}{\varepsilon_0} \qquad \Delta f(r) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial f}{\partial r}\right)$$

$$\frac{\partial}{\partial r}\Phi(r) = \frac{q}{4\pi\varepsilon_0}\left(\cancel{\frac{\alpha e^{-\alpha r}}{2r}} - \frac{e^{-\alpha r}}{r^2} - \frac{\alpha e^{-\alpha r}}{r} - \frac{\alpha^2 e^{-\alpha r}}{2} - \cancel{\frac{\alpha e^{-\alpha r}}{2r}}\right) =$$

$$= \frac{-q}{4\pi\varepsilon_0}\frac{e^{-\alpha r}}{r^2}\left(1+\alpha r+\frac{1}{2}\alpha^2 r^2\right)$$

$$\Delta\Phi(r) = \frac{-q}{4\pi\varepsilon_0}\frac{e^{-\alpha r}}{r^2}\left(\cancel{-\alpha} \cancel{-\alpha^2 r} - \frac{1}{2}\alpha^3 r^2 + \cancel{\alpha} + \cancel{\alpha^2 r}\right) = \frac{q\alpha^3}{8\pi\varepsilon_0} e^{-\alpha r}$$

$$\Phi(r\to 0) = \frac{q}{4\pi\varepsilon_0 r}\left(1+O(r)\right) \Rightarrow \frac{q}{4\pi\varepsilon_0}\Delta\frac{1}{r} = \frac{q}{4\pi\varepsilon_0}\left(-4\pi\delta^3(0)\right) = \frac{-\rho}{\varepsilon_0}$$

$$\rho(0) = q\,\delta^3(0)$$

$$\underline{\rho(r) = \frac{-q\alpha^3}{8\pi}e^{-\alpha r} + q\,\delta^3(r)}$$