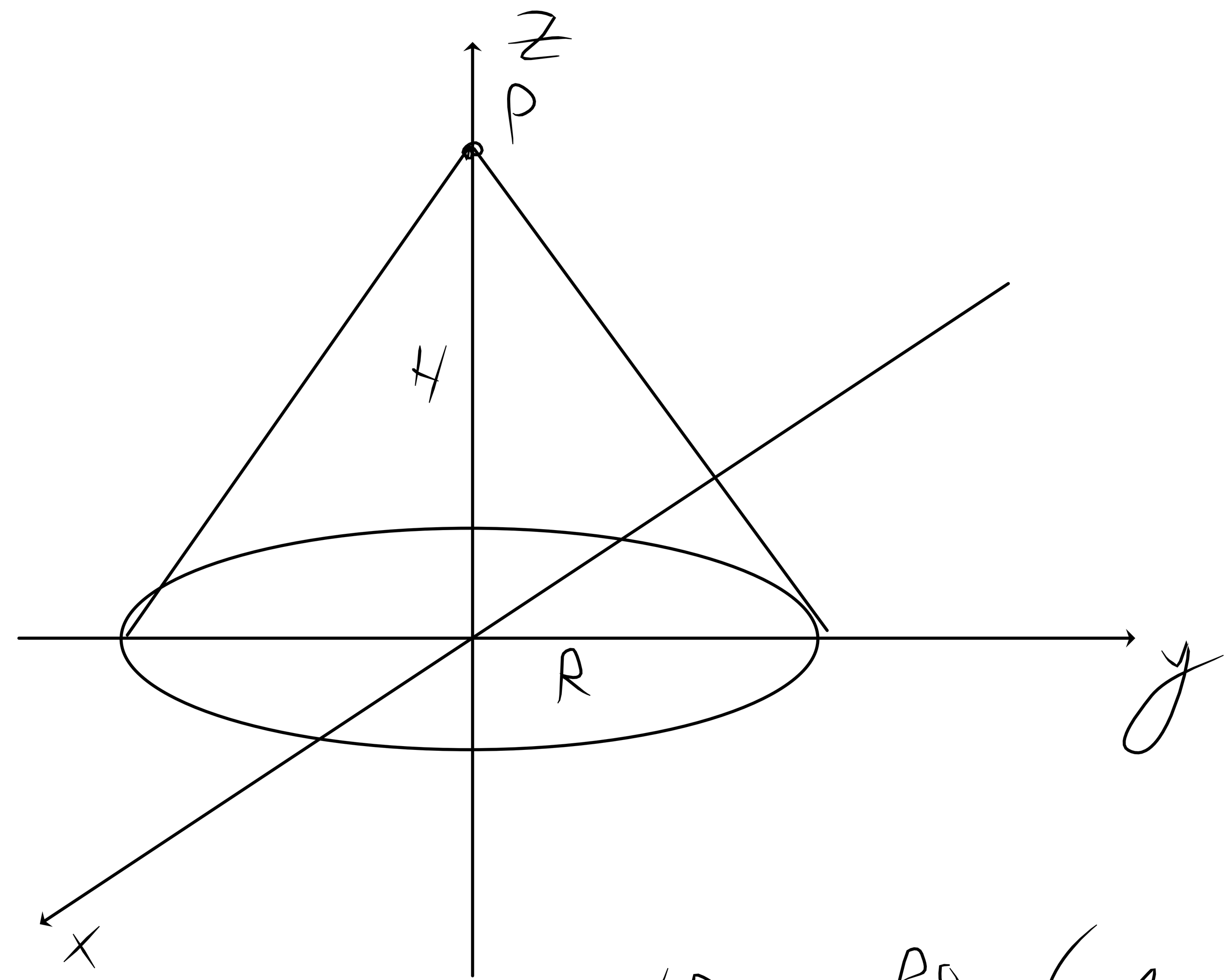


$$E_{z\,\text{krążek}} = \frac{Q}{2\pi\varepsilon_0}\left(\frac{1}{r^2} - \frac{z}{r^2\sqrt{z^2+r^2}}\right)$$

$$Q \to dq \qquad dq = \rho_0 \cdot \pi r^2\, dz$$

$$\rho_0 = \frac{Q}{\frac{1}{3}\pi R^2 H}$$

$$z = H - \frac{H}{R} r \qquad r = R - \frac{R}{H} z$$

$$dE_z = \frac{\rho_0}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2+r^2}}\right)dz = \frac{\rho_0}{2\varepsilon_0}\left(1-\frac{z}{\sqrt{z^2+\left(R-\frac{R}{H}z\right)^2}}\right)dz$$

$$E_z = \int_0^E dE_z = \frac{\rho_0}{2\varepsilon_0}\int_0^H \left(1 - \underbrace{\frac{z}{\sqrt{z^2+\left(R-\frac{R}{H}z\right)^2}}}_{I}\right)dz$$

$$I = \frac{H}{\sqrt{R^2+H^2}}\int_0^H \frac{z\,dz}{\sqrt{\left(z-\frac{R^2H}{H^2+R^2}\right)^2 + \underbrace{\frac{R^2H^2}{R^2+H^2} - \frac{R^4H^2}{(R^2+H^2)^2}}_{C}}} = \left\{\begin{matrix} x = z - \frac{R^2H}{R^2+H^2} & A = \frac{-R^2H}{R^2+H^2} \\ dx = dz & B = \frac{-R^2H}{R^2+H^2}+H \end{matrix}\right\} = \frac{H}{\sqrt{R^2+H^2}}\int_A^B \frac{x+\frac{R^2H}{R^2+H^2}}{\sqrt{x^2+C}}\,dx =$$

$$= \frac{H}{\sqrt{R^2+H^2}}\left(\underbrace{\int_A^B \frac{x}{\sqrt{x^2+C}}\,dx}_{I_1} + \frac{R^2H}{R^2+H^2}\underbrace{\int_A^B\frac{1}{\sqrt{x^2+C}}\,dx}_{I_2}\right)$$

$$I_1 = \left\{\begin{matrix} u = x^2+C \\ du = 2x \end{matrix}\right\} = \frac12\int_{A^2+C}^{B^2+C}\frac{du}{\sqrt u} = \frac12\left[2\sqrt u\right]_{A^2+C}^{B^2+C} = \sqrt{B^2+C}-\sqrt{A^2+C}$$

$$I_2 = \left\{\begin{matrix} \sqrt C\tan\theta = x \\ \sqrt C\sec^2\theta = dx \end{matrix}\right\} = \int_{\hat A}^{\hat B}\frac{\cancel{\sqrt C}\sec^2\theta}{\cancel{\sqrt C}\sqrt{1+\tan^2\theta}}\,d\theta = \int_{\hat A}^{\hat B}\sec\theta\,d\theta = \int_{\hat A}^{\hat B} \frac{\sec^2\theta+\sec\theta\tan\theta}{\sec\theta+\tan\theta}\,d\theta = \left[\ln\left|\sec\theta+\tan\theta\right|\right]_{\hat A}^{\hat B} =$$

$$= \left[\ln\left(\frac{x}{\sqrt C}+\sqrt{1+\frac{x^2}{C}}\right)\right]_A^B = \ln\left(\frac{B+\sqrt{C+B^2}}{A+\sqrt{C+A^2}}\right)$$

$$E_z = \frac{\rho H}{2\varepsilon_0} - \frac{\rho H}{2\varepsilon_0\sqrt{R^2+H^2}}\left(\sqrt{B^2+C}-\sqrt{A^2+C}+\frac{R^2H}{H^2+R^2}\ln\left(\frac{B+\sqrt{B^2+C}}{A+\sqrt{A^2+C}}\right)\right)$$

$$A = \frac{-R^2H}{R^2+H^2} \qquad B = H+A \qquad C = \frac{R^2H^2}{R^2+H^2} - \frac{R^4H^2}{(H^2+R^2)^2} = \frac{R^2H^2}{R^2+H^2}\left(1-\frac{R^2}{H^2+R^2}\right)$$

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