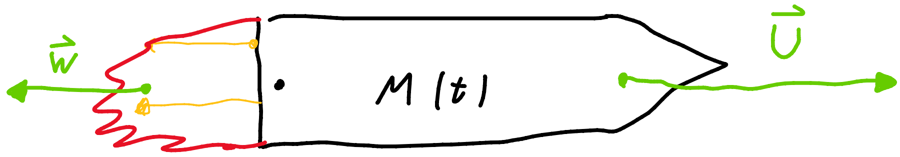

W – pr. względem rakiety

u – pr. w LAB

$$dp = dp'$$

Z.Z.P

$$d\left[\frac{VM}{\sqrt{1-\frac{V^2}{c^2}}}\right] = u\frac{dm}{\sqrt{1-\frac{w^2}{c^2}}} \qquad (1)$$

Z.Z. M-E

$$dE = dE'$$

$$d\left[\frac{M}{\sqrt{1-\frac{V^2}{c^2}}}\cdot c^2\right] = -\frac{dm\, c^2}{\sqrt{1-\frac{w^2}{c^2}}}$$

$$\Rightarrow dm = -d\left[\frac{M}{\sqrt{1-\frac{V^2}{c^2}}}\right]\sqrt{1-\frac{w^2}{c^2}} \qquad (2)$$

$\Rightarrow$ (2) $\rightarrow$ (1)

$$d\left[\frac{\vec{V} M}{\sqrt{1-\frac{V^2}{c^2}}}\right] = -u\, d\left[\frac{M}{\sqrt{1-\frac{V^2}{c^2}}}\right]$$

$$u = \frac{w-V}{1-\frac{Vw}{c^2}}$$

$$\Rightarrow d\left[\frac{VM}{\sqrt{1-\frac{V^2}{c^2}}}\right] = \frac{V-w}{1-\frac{Vw}{c^2}}\, d\left[\frac{M}{\sqrt{1-\frac{V^2}{c^2}}}\right]$$

$$d\left(\frac{1}{\sqrt{1-\frac{V^2}{c^2}}}\right) = \frac{V\,dV}{(c^2-V^2)\left(1-\frac{V^2}{c^2}\right)^{1/2}}$$

$$\Rightarrow \frac{V}{\sqrt{1-\frac{V^2}{c^2}}}dM + \frac{M}{\sqrt{1-\frac{V^2}{c^2}}}dV + \frac{V^2 M\,dV}{(c^2-V^2)\left(1-\frac{V^2}{c^2}\right)} = \left(\frac{V-w}{1-\frac{Vw}{c^2}}\right)\left(\frac{dM}{\sqrt{1-\frac{V^2}{c^2}}} + \frac{MV\,dV}{\sqrt{1-\frac{V^2}{c^2}}\,(c^2-V^2)}\right)$$

$$\Rightarrow V\,dM + M\,dV + \frac{V^2 M\,dV}{c^2-V^2} = \frac{V+w}{\left(1+\frac{Vw}{c^2}\right)}\left(dM + \frac{MV\,dV}{c^2-V^2}\right)$$

$$\Rightarrow dV\left(M + \frac{V^2 M}{c^2-V^2} - \frac{(V-W)}{1-\frac{Vw}{c^2}}\cdot\frac{MV}{c^2-V^2}\right) = -dM\left(V - \frac{V-W}{\left(1-\frac{Vw}{c^2}\right)}\right)$$

$$dV\left(M + \frac{MV}{c^2-V^2}\left(V - \frac{V-w}{1-\frac{Vw}{c^2}}\right)\right) = -dM\left(V - \frac{V-W}{1-\frac{Vw}{c^2}}\right)$$

$$dU \cdot M\left(1+\frac{V}{c^2-V^2}\left(\frac{U\left(1-\frac{Vw}{c^2}\right)-U+w}{1-\frac{Vw}{c^2}}\right)\right) = -dM\left(\frac{V\left(1-\frac{Uw}{c^2}\right)-V+w}{1-\frac{Vv}{c^2}}\right)$$

$$dU \cdot M\left(1+\frac{V}{c^2-U^2}\left(\frac{U-\frac{V^2w}{c^2}-U+w}{1-\frac{Vw}{c^2}}\right)\right) = -dM\left(\frac{-\frac{V^2w}{c^2}+w}{1-\frac{Uw}{c^2}}\right)$$

$$M \cdot dV\left(1+\frac{V}{c^2-V^2}\left(\frac{wc^2-V^2w}{c^2-Vw}\right)\right) = -dM\left(\frac{wc^2-V^2w}{c^2-Vw}\right)$$

$$M \cdot dU\left(1+\frac{Vw}{\cancel{c^2-V^2}}\,\frac{\cancel{c^2-V^2}}{c^2-Vw}\right) = -dMw\,\frac{c^2-V^2}{c^2-Vw}$$

$$M \cdot dV\left(\frac{c^2-Vw+Vw}{c^2-Vw}\right) = -dMw\,\frac{c^2-V^2}{c^2-Vw} \quad /\cdot(c^2-Vw)$$

$$M \cdot dV \cdot c^2 = -dMw(c^2-V^2)$$

$$\Rightarrow \quad \frac{dM}{M} = -c^2\frac{dU}{w(c^2-V^2)} = -\frac{dV}{w\left(1-\frac{V^2}{c^2}\right)}$$

$$\int\frac{dM}{M} = -\int\frac{dV}{w\left(1-\frac{V^2}{c^2}\right)} = \frac{1}{w}\int\frac{1}{\left(1-\frac{V}{c}\right)\left(1+\frac{V}{c}\right)}\,dV = \frac{1}{w}\int\frac{1/2}{\left(1-\frac{V}{c}\right)}+\frac{1/2}{\left(1+\frac{V}{c}\right)} =$$

$$\shortparallel$$

$$\ln\frac{M}{M_0}$$

$$= \frac{c}{2w}\left[-\ln\left|1-\frac{V}{c}\right| + \ln\left|1+\frac{U}{c}\right|\right] +$$

$$\Rightarrow \quad \ln\frac{M}{M_0} = \ln\left|\left(\frac{1+\frac{V}{c}}{1-\frac{V}{c}}\right)^{\frac{c}{2w}}\right| \qquad v > c$$

$$\Rightarrow \quad \ln M_0 - \ln \left| \left(1 - \frac{v}{c} \right) \qquad \right|$$

$$\Rightarrow \quad \frac{M}{M_0} = \left(\frac{1 + \frac{v}{c}}{1 - \frac{v}{c}} \right)^{\frac{c}{2w}}$$

$w = c \Rightarrow$

$$\frac{M}{M_0} = \sqrt{\frac{1 + \frac{v}{c}}{1 - \frac{v}{c}}}$$

$$\Rightarrow M^2 - M^2 \frac{v}{c} = M_0^2 + M_0^2 \frac{v}{c}$$

$$M^2 - M_0^2 = \frac{v}{c} \left(M^2 + M_0^2 \right)$$

$$\Rightarrow v = \frac{M^2 - M_0^2}{M^2 + M_0^2} c \quad \blacksquare$$