$$\bar{B} = \frac{\mu_0 \, n \, \bar{I}_n}{2} \left(\cos\alpha - \cos\beta \right) \hat{e}_z$$

$$\cos\alpha = \frac{z}{\sqrt{R^2 + z^2}}$$

$$\cos\beta = \frac{z+L}{\sqrt{R^2 + (z+L)^2}}$$

$$\bar{B} = \frac{\mu_0 \, n \, I_{in}}{2} \left(\frac{z}{\sqrt{R^2 + z^2}} - \frac{z+L}{\sqrt{R^2 + (z+L)^2}} \right)$$

$$\bar{B} = \frac{\mu_0 \, n \, I_{in}}{2} \left(\underbrace{\frac{sgn(z)}{\sqrt{\left(\frac{R}{z}\right)^2 + 1}}}_{\downarrow \atop 0} - \underbrace{\frac{sgn(z+L)}{\sqrt{\left(\frac{R}{z+L}\right)^2 + 1}}}_{\downarrow \atop 0} \right) \overset{z>0}{=} \frac{-\mu_0 \, n \, I_{in}}{2} L$$