Zadanie 4
Znajdź ruch cząstki o masie m, energii E i momencie pędu L w polu o energii potencjalnej $E_p(r) = -\frac{k_1}{r} + \frac{k_2}{r^2}$, gdzie k_1 i k_2 to pewne stałe dodatnie.

Odpowiedź: $r(t) = \frac{A}{1+\epsilon\cos(\omega\varphi+\varphi_0)}$, $A = \frac{2k_2m+L^2}{k_1m}$, $\epsilon = \sqrt{1+\frac{2E}{k_1}\frac{2k_2m+L^2}{k_1m}}$ $\omega = \sqrt{1+\frac{2k_2m}{L^2}}$

m, E, L

$$E_p(r) = -\frac{k_1}{r} + \frac{k_2}{r^2}$$

$$\vec{F} = \vec{\nabla}\vec{E}_p(r) \quad ; \quad \vec{\nabla} = \left(\frac{\partial}{\partial r} ; \frac{1}{r}\frac{\partial}{\partial\varphi}\right)$$

$$\Rightarrow \vec{F} = \frac{k_1}{r^2} - 2\frac{k_2}{r^3}$$

$$\ddot{\omega} + \omega = -\frac{m}{L^2\omega^2}\left(k_1\omega^2 - 2k_2\omega^3\right)$$

$$\Rightarrow \ddot{\omega} + \omega = -K_1 + K_2\omega \Rightarrow \ddot{\omega} + (1-K_2)\omega = -K_1$$

$$K_1 = \frac{mk_1}{L^2\omega^2} \quad ; \quad K_2 = \frac{2k_2m}{L^2\omega^2}$$

RORJ:

$$\ddot{\omega} + (1-k_2)\omega = 0 \Rightarrow \omega = A\cos\left(\sqrt{1-K_2}\cdot\varphi\right)$$

$$\text{RSRN: } \omega = \frac{-K_1}{1-K_2} = B$$

$$\Rightarrow \omega = A\cos\left(\sqrt{1-K_2}\cdot\varphi\right) + B$$

$$\Rightarrow r = \frac{1}{A\cos\left(\sqrt{1-K}\cdot\varphi\right) + B}$$

$$\Rightarrow \quad r = \frac{}{A\cos\left(\sqrt{1-K_2}\cdot\varphi\right)+B}$$

$$E = \frac{1}{2}mv^2 + \frac{1}{2}mr^2\omega^2 - \frac{k_1}{r} + \frac{k_2}{r^2}$$

z tego wyznaczymy A