

$$\bar{\Phi}(r) = \frac{q}{4\pi\epsilon_0} \frac{e^{-dr}}{r} \left(1 + \frac{dr}{2}\right) \quad \Delta \bar{\Phi}(r) = \frac{-\rho}{\epsilon_0} \quad \Delta f(r) = \frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{df}{dr} \right)$$

$$\begin{aligned} \frac{d}{dr} \bar{\Phi}(r) &= \frac{q}{4\pi\epsilon_0} \left(\frac{\cancel{d} e^{-dr}}{\cancel{2} r} - \frac{e^{-dr}}{r^2} - \frac{d e^{-dr}}{r} - \frac{d^2 e^{-dr}}{2} - \frac{\cancel{d} e^{-dr}}{\cancel{2} r} \right) = \\ &= \frac{-q}{4\pi\epsilon_0} \frac{e^{-dr}}{r^2} \left(1 + dr + \frac{1}{2} d^2 r^2 \right) \end{aligned}$$

$$\Delta \bar{\Phi}(r) = \frac{-q}{4\pi\epsilon_0} \frac{e^{-dr}}{r^2} \left(\cancel{-d} - \cancel{d^2} r - \frac{1}{2} d^3 r^2 + \cancel{d} + \cancel{d^2} r \right) = \frac{q d^3}{8\pi\epsilon_0} e^{-dr}$$

$$\bar{\Phi}(r \rightarrow 0) = \frac{q}{4\pi\epsilon_0 r} (1 + 0(r)) \Rightarrow \frac{q}{4\pi\epsilon_0} \Delta \frac{1}{r} = \frac{q}{4\pi\epsilon_0} (-4\pi \delta^3(0)) = \frac{-\rho}{\epsilon_0}$$

$$\rho(0) = q \delta^3(0)$$

$$\rho(r) = \frac{-q d^3}{8\pi} e^{-dr} + q \delta^3(r)$$