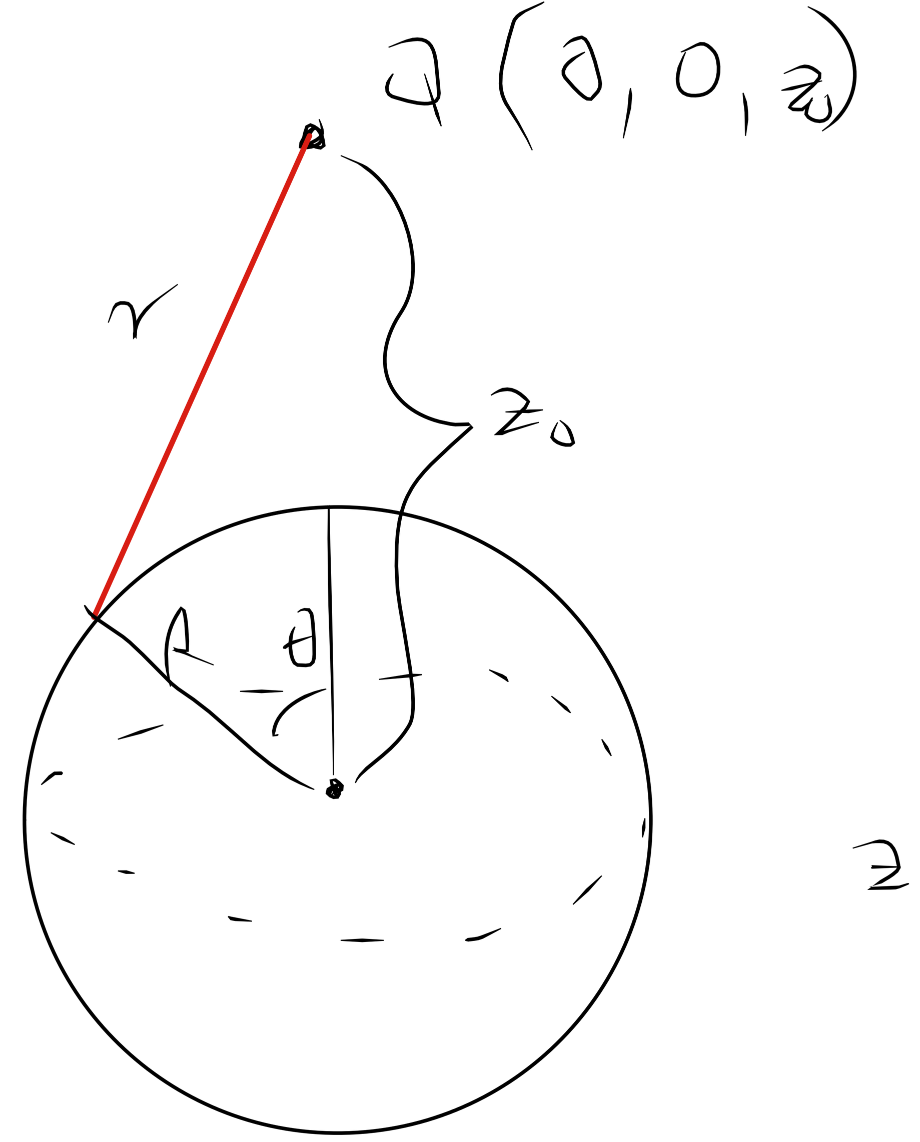

$$dS \longrightarrow \sin\theta \, R^2 \, d\theta \, d\varphi$$

$$r^2 = R^2 + z_0^2 - 2Rz_0\cos\theta$$

$$z_0 > R$$

a) $$\phi = \frac{Q}{4\pi\varepsilon_0}\,\frac{1}{S}\int_S \frac{1}{r}\,dS = \frac{Q}{4\pi\varepsilon_0}\,\frac{1}{4\pi \cancel{R^2}}\int_0^{\pi}\int_0^{2\pi}\frac{\cancel{R^2}\sin\theta}{\sqrt{R^2+z_0^2-2Rz_0\cos\theta}}\,d\varphi\,d\theta =$$

$$= \frac{Q}{4\pi\varepsilon_0}\,\frac{1}{2}\int_0^{\pi}\frac{\sin\theta}{\sqrt{R^2+z_0^2-2Rz_0\cos\theta}}\,d\theta = \frac{Q}{4\pi\varepsilon_0}\,\frac{1}{2Rz_0}\,\frac{1}{2}\int_{(R-z)^2}^{(R+z)^2}\frac{1}{\sqrt{u}}\,du = \frac{Q}{4\pi\varepsilon_0}\cdot\frac{1}{2}\cdot\frac{1}{Rz_0}\Big[\sqrt{u}\Big]_{(R-z_0)^2}^{(R+z_0)^2} = \frac{Q}{4\pi\varepsilon_0}\,\frac{1}{2}\cdot\frac{1}{\cancel{R}z_0}\Big[\overset{1}{\cancel{R}} + \cancel{z_0} - \cancel{z_0} + \overset{1}{\cancel{R}}\Big] = \frac{Q}{4\pi\varepsilon_0 z_0}$$

$$\begin{cases} R^2+z_0^2-2Rz_0\cos\theta = u \\ 2Rz_0\sin\theta\,d\theta = du \end{cases}$$

w środku kuli $r = z_0$

$$\phi = \frac{Q}{4\pi\varepsilon_0 z_0}$$, koniec dowodu.

b) Z prawa Gaussa:

$$SE = \frac{Q}{\varepsilon_0} \qquad \phi = -\int E\,dr$$

$$E = \frac{Q}{S\,\varepsilon_0} \qquad \phi = -\int \frac{Q}{4\pi R^2\varepsilon_0}\,dR$$

$$E = \frac{Q}{4\pi R^2\varepsilon_0} \qquad \phi = \frac{Q}{4\pi R\varepsilon_0}$$