$$\frac{d^2x}{dt^2} = \frac{d}{dt}\left(\frac{dx}{d\tau}\frac{d\tau}{dt}\right) = \frac{d^2\tau}{dt^2}\frac{dx}{d\tau} + \frac{d}{d\tau}\left(\frac{dx}{d\tau}\right)\frac{d\tau}{dt}\frac{d\tau}{dt} =$$

$$= \frac{d^2\tau}{dt^2}\frac{dx}{d\tau} + \frac{d^2x}{d\tau^2}\left(\frac{d\tau}{dt}\right)^2$$

$$\boxed{\frac{d^2x}{dt^2} = \frac{d^2\tau}{dt^2}\frac{dx}{d\tau} + \frac{d^2x}{d\tau^2}\left(\frac{d\tau}{dt}\right)^2} \qquad \boxed{\frac{dx}{dt} = \frac{dx}{d\tau}\frac{d\tau}{dt}}$$

$$\frac{d^2\tau}{dt^2}\frac{dx^i}{d\tau} + \frac{d^2x^i}{d\tau^2}\left(\frac{d\tau}{dt}\right)^2 + \Gamma^i_{jk}\frac{dx^j}{d\tau}\frac{dx^k}{d\tau}\left(\frac{d\tau}{dt}\right)^2 = f(t)\frac{dx^i}{d\tau}\frac{d\tau}{dt}$$

$$\left(\frac{dt}{d\tau}\right)^2\frac{d^2\tau}{dt^2}\frac{dx^i}{d\tau} + \frac{d^2x^i}{d\tau^2} + \Gamma^i_{jk}\frac{dx^j}{d\tau}\frac{dx^k}{d\tau} = f(t)\frac{dx^i}{d\tau}\left(\frac{dt}{d\tau}\right)$$

$$\frac{d^2x^i}{d\tau^2} + \Gamma^i_{jk}\frac{dx^j}{d\tau}\frac{dx^k}{d\tau} = f(t)\frac{dx^i}{d\tau}\left(\frac{dt}{d\tau}\right) - \left(\frac{dt}{d\tau}\right)^2\frac{d^2\tau}{dt^2}\frac{dx^i}{d\tau}$$

$$\Rightarrow \quad f(t)\frac{dt}{d\tau} - \left(\frac{dt}{d\tau}\right)^2\frac{d^2\tau}{dt^2} = 0 \qquad f(t) = \frac{dt}{d\tau}\cdot\frac{d^2\tau}{}$$

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