Zu b) $\varphi_t : \mathbb{R}^2 \to \mathbb{R}^2$

$$X = \underbrace{x^2 \partial x + x y^2 \partial y}_{\text{pole wektorowe}}$$

$$\begin{cases} \frac{d}{dt} q^1_t = (q^1_t)^2 \\ \frac{d}{dt} q^2_t = q^1_t \, q^2_t \end{cases}$$

$$\frac{d}{dt} q^1_t = (q^1_t)^2 \qquad q^1_t = p$$

$$\frac{\dot{p}}{p^2} = 1$$

$$\int \frac{\dot{p}}{p^2} \, dt = \int dt$$

$$\int \frac{dp}{p^2} = \int dt$$

$$-\frac{1}{p} = t + C$$

p

$$-1 = p(t+C)$$

$$p = \frac{-1}{t+C}$$

$$q^1_t = \frac{-1}{t+C}$$

$$q^1_0 = \frac{-1}{C} = x$$

$$C = -\frac{1}{x} \qquad q^1_t = \frac{-1}{t-\frac{1}{x}} = \frac{-x}{tx-1}$$

$$q^2_t = k$$

$$\frac{d}{dt}k = -\frac{x}{tx-1} \cdot k$$

$$\frac{\dot{k}}{k} = \frac{-x}{tx-1}$$

$$tx-1$$

$$\int \frac{dk}{k} = \int \frac{-x}{tx-1}\, dt$$

$$\ln|k| + k = \int \frac{1}{t-\frac{1}{x}}\, dt$$

$$\ln|k| = \ln\left|t-\frac{1}{x}\right| + C_\beta$$

$$\ln|k| = \ln\left|C_1\left(t-\frac{1}{x}\right)\right|$$

$$k = C_1\left(t-\frac{1}{x}\right)$$

$$q_t^2 = C_1\left(t-\frac{1}{x}\right)$$

$$q_0^2 = -\frac{C_1}{x} = y$$

$$-C_1 = yx$$

$$C_1 = -yx$$

$$C_1 = \; yx$$

$$q^2_t = -yx\left(t - \frac{1}{x}\right) =$$

$$= -yxt + y$$

$$\begin{cases} q^1_t = -\dfrac{1}{t-\frac{1}{x}} \\[2ex] q^2_t = -yxt + y \end{cases}$$

$$\varphi_t(x,y) = \left(-\frac{1}{t-\frac{1}{x}},\; -yxt+y\right)$$

$$\varphi_0(x,y) = (x,\; y) \checkmark$$

$$\varphi_t \circ \varphi_s(x,y) = \left(-\frac{1}{t - \frac{1}{\frac{-1}{s-\frac{1}{x}}}},\; -(-yxs+y)\left(-\frac{1}{s-\frac{1}{x}}\right)t + (-yxs+y)\right)$$

1 2

1 $\dfrac{-1}{t+s-\frac{1}{x}}$ ✓

2 $-(-yxs+y)\left(\dfrac{-1}{s-\frac{1}{x}}\right)t+(-yxs+y)=$

$=(-yxs+y)\left(1+\dfrac{t}{s-\frac{1}{x}}\right)=$

$=y(-xs+1)\left(\dfrac{sx-1+tx}{sx-1}\right)=$

$=-y\,\dfrac{\cancel{(xs-1)}^{1}\,(sx-1+tx)}{\cancel{(xs-1)}}=$

$=-yx(s+t)+y$ ✓