$$h_* \partial_x = \frac{\partial x}{\partial r}\partial_r + \frac{\partial x}{\partial \theta}\partial_\theta + \frac{\partial x}{\partial \varphi}\partial_\varphi = \sin\theta\cos\varphi\,\partial_r + r\cos\theta\cos\varphi\,\partial_\theta - r\sin\theta\sin\varphi\,\partial_\varphi$$

$$h_* \partial_y = \sin\theta\sin\varphi\,\partial_r + r\cos\theta\sin\varphi\,\partial_\theta + r\sin\theta\cos\varphi\,\partial_\varphi$$

$$h_* \partial_z = \cos\theta\,\partial_r - r\sin\theta\,\partial_\theta$$

$$h_* v_1 = r\sin\theta\sin\varphi(\cos\theta\,\partial_r - r\sin\theta\,\partial_\theta) - r\cos\theta(\sin\theta\sin\varphi\,\partial_r + r\cos\theta\sin\varphi\,\partial_\theta - r\sin\theta\cos\varphi\,\partial_\varphi) =$$

$$= (r\sin\theta\sin\varphi\cos\theta - r\cos\theta\sin\theta\sin\varphi)\partial_r + (-r^2\sin^2\theta\sin\varphi - r^2\cos^2\theta\sin\varphi)\partial_\theta + r^2\cos\theta\sin\theta\cos\varphi\,\partial_\varphi =$$

$$= -r^2\sin\varphi\,\partial_\theta + r^2\cos\theta\sin\theta\cos\varphi\,\partial_\varphi$$

$$h_* v_2 = r\sin\theta\cos\varphi(\cos\theta\,\partial_r - r\sin\theta\,\partial_\theta) - r\cos\theta(\sin\theta\cos\varphi\,\partial_r + r\cos\theta\cos\varphi\,\partial_\theta - r\sin\theta\sin\varphi\,\partial_\varphi) =$$

$$= (r\sin\theta\cos\varphi\cos\theta - r\cos\theta\sin\theta\cos\varphi)\partial_r + (-r^2\sin^2\theta\cos\varphi - r^2\cos^2\theta\cos\varphi)\partial_\theta + r^2\cos\theta\sin\theta\sin\varphi\,\partial_\varphi =$$

$$= -r^2\cos\varphi\,\partial_\theta + r^2\cos\theta\sin\theta\sin\varphi\,\partial_\varphi$$

Aby było $v_3 = x\partial_y - y\partial_x$

$$h_* v_3 = r\sin\theta\cos\varphi(\sin\theta\sin\varphi\,\partial_r + r\cos\theta\sin\varphi\,\partial_\theta + r\sin\theta\cos\varphi\,\partial_\varphi) -$$

$$- r\sin\theta\sin\varphi(\sin\theta\cos\varphi\,\partial_r + r\cos\theta\cos\varphi\,\partial_\theta - r\sin\theta\sin\varphi\,\partial_\varphi) =$$

$$= r^2\sin^2\theta\,\partial_\varphi$$