a) $\bar{\nabla}\cdot(f\bar{A}) = \partial_i (f A_i) = \partial_i f A_i + f \partial_i A_i = \bar{\nabla} f \cdot \bar{A} + f \bar{\nabla}\cdot\bar{A}$

b) $\bar{\nabla}\cdot(\bar{\nabla}\times\bar{A}) = \bar{\nabla}\cdot(\varepsilon_{ijk}\partial_i A_j \bar{e}_k) = \varepsilon_{ijk}\partial_k \partial_i A_j \bar{e}_k \overset{i\Leftrightarrow k}{=} -\varepsilon_{ijk}\partial_k\partial_i A_j \bar{e}_k \Rightarrow 0$

c) $\bar{\nabla}\times(\bar{\nabla} f) = \bar{\nabla}\times(\partial_i f \bar{e}_i) = \varepsilon_{jik}\partial_j\partial_i f \bar{e}_k \overset{j\Leftrightarrow i}{=} -\varepsilon_{jik}\partial_j\partial_i f \bar{e}_k \Rightarrow 0$

d) $\bar{\nabla}\times(\bar{\nabla}\times\bar{A}) = \bar{\nabla}\times(\varepsilon_{ijk}\partial_i A_j \bar{e}_k) = \varepsilon_{lkm}\varepsilon_{ijk}\partial_l\partial_i A_j \bar{e}_m =$

$= \varepsilon_{kml}\varepsilon_{kij}\partial_l\partial_i A_j \bar{e}_m = (\delta_{mi}\delta_{lj} - \delta_{mj}\delta_{li})\partial_l\partial_i A_j \bar{e}_m =$

$= \partial_i \underbrace{\partial_j A_j}_{\bar{\nabla}\cdot\bar{A}} \bar{e}_i - \partial_i\partial_i A_j \bar{e}_j = \bar{\nabla}(\bar{\nabla}\cdot\bar{A}) - \bar{\nabla}^2\bar{A}$

e) $\bar{\nabla}\cdot(\bar{A}\times\bar{B}) = \bar{\nabla}\cdot(\varepsilon_{ijk}A_i B_j \bar{e}_k) = \varepsilon_{ijk}\partial_k(A_i B_j) = \varepsilon_{ijk}\partial_k A_i B_j + \varepsilon_{ijk}A_i\partial_k B_k = L$

$\bar{B}\cdot(\bar{\nabla}\times\bar{A}) - \bar{A}\cdot(\bar{\nabla}\times\bar{B}) = B_j\bar{e}_j\cdot(\varepsilon_{kil}\partial_k A_i \bar{e}_l) - A_i\bar{e}_i(\varepsilon_{kjl}\partial_k B_j \bar{e}_l) =$

$= \varepsilon_{kij}\partial_k A_i B_j + \varepsilon_{jki}A_i\partial_k B_j = P \qquad L = P$

f) $\bar{\nabla}\times(\bar{A}\times\bar{B}) = \bar{\nabla}\times(\varepsilon_{ijk}A_i B_j \bar{e}_k) = \underset{kml}{\varepsilon_{lkm}}\,\underset{kij}{\varepsilon_{ijk}}\,\partial_l(A_i B_j)\bar{e}_m =$

$= (\delta_{mi}\delta_{lj} - \delta_{mj}\delta_{il})(\partial_l A_i B_j - A_i\partial_l B_j)\bar{e}_m =$

$= \partial_j A_i B_j \bar{e}_i - \partial_i A_i B_j \bar{e}_j - A_i\partial_j B_j \bar{e}_i + A_i\partial_i B_j \bar{e}_j =$

$= (B_j\partial_j)(A_i\bar{e}_i) - (\partial_i A_i)(B_j\bar{e}_j) - (\partial_j B_j)(A_i\bar{e}_i) + (A_i\partial_i)(B_j\bar{e}_j) =$

$= (\bar{B}\cdot\bar{\nabla})\bar{A} - (\bar{\nabla}\cdot\bar{A})\bar{B} - (\bar{\nabla}\cdot\bar{B})\bar{A} + (\bar{\nabla}\cdot\bar{A})\bar{B}$