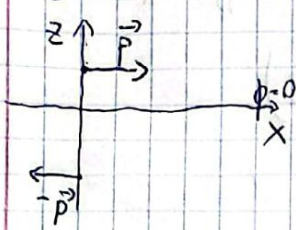


5.1



a) Potencjał:

$$\phi = \frac{1}{4\pi\epsilon_0} \frac{p \cos\theta}{r^2}$$

$$r = \sqrt{x^2 + y^2 + z^2}, \quad r \cos\theta = z$$

$$\phi = \frac{p}{4\pi\epsilon_0} \left(\frac{z}{(x^2 + y^2 + z^2)^{3/2}} \right)$$

$$\vec{F} = p \frac{\partial \vec{E}}{\partial x} \quad \vec{p} = p \hat{e}_x$$

$$\vec{E} = -\left(\frac{\partial \phi}{\partial x}, \frac{\partial \phi}{\partial y}, \frac{\partial \phi}{\partial z}\right) = -\frac{p}{4\pi\epsilon_0} \left(\frac{\partial}{\partial x} \frac{z}{(x^2 + y^2 + z^2)^{3/2}}, 0, \frac{\partial}{\partial z} \frac{z}{(x^2 + y^2 + z^2)^{3/2}} \right) =$$

$$= -\frac{p}{4\pi\epsilon_0} \left(\frac{1}{(x^2 + y^2 + z^2)^{3/2}} - \frac{3xz}{(x^2 + y^2 + z^2)^{5/2}}, 0, \frac{-3xz}{(x^2 + y^2 + z^2)^{5/2}} \right)$$

$$\frac{\partial E_x}{\partial x} = \frac{p}{4\pi\epsilon_0} \frac{\partial}{\partial x} \left(\frac{3xz}{(x^2 + y^2 + z^2)^{5/2}} - \frac{1}{(x^2 + y^2 + z^2)^{3/2}} \right) =$$

$$= \frac{p}{4\pi\epsilon_0} \left(\frac{3x}{(x^2 + y^2 + z^2)^{5/2}} + \frac{6xz}{(x^2 + y^2 + z^2)^{7/2}} - \frac{3x}{(x^2 + y^2 + z^2)^{5/2}} \right) =$$

$$\left. \frac{\partial E_x}{\partial x} \right|_{\substack{z=h \\ x=0}} = 0$$

$$\frac{\partial E_z}{\partial x} = \frac{p}{4\pi\epsilon_0} \frac{\partial}{\partial x} \left(\frac{3xz}{(x^2 + y^2 + z^2)^{5/2}} \right) = \frac{p}{4\pi\epsilon_0} \left(\frac{3z}{(x^2 + y^2 + z^2)^{5/2}} - \frac{5}{2} \frac{3xz}{(x^2 + y^2 + z^2)^{7/2}} \cdot 2x \right)$$

$$\left. \frac{\partial E_z}{\partial x} \right|_{\substack{z=h \\ x=0}} = \frac{p}{4\pi\epsilon_0} \cdot 3 \cdot 2h \cdot (2h)^{-5} = \frac{3p}{64\pi\epsilon_0 h^4}$$

$$\vec{F} = p \cdot \left(0, 0, \frac{-3p}{64\pi\epsilon_0 h^4} \right) = -\frac{3p^2}{64\pi\epsilon_0 h^4} \hat{e}_z$$

"b" od ładunku obrazowego

$$b) \sigma(x, y) = \epsilon_0 E_z(x, y, z=0)$$

$$E_z = -\frac{\partial \phi}{\partial z} = -\frac{\partial}{\partial z} \frac{p}{4\pi\epsilon_0} \left(\frac{z-h}{\sqrt{x^2 + y^2 + (z-h)^2}} - \frac{z+h}{\sqrt{x^2 + y^2 + (z+h)^2}} \right) =$$

$$= \frac{px}{4\pi\epsilon_0} \left(\frac{z-h}{(x^2 + y^2 + (z-h)^2)^{3/2}} - \frac{z+h}{(x^2 + y^2 + (z+h)^2)^{3/2}} \right)$$

$$\sigma(x, y) = \epsilon_0 E_z(x, y, z=0) = \frac{px}{4\pi\epsilon_0} \left(\frac{-h}{(x^2 + y^2 + h^2)^{3/2}} - \frac{h}{(x^2 + y^2 + h^2)^{3/2}} \right) =$$

$$= \frac{-px}{2\pi\epsilon_0} \frac{h}{(x^2 + y^2 + h^2)^{3/2}}$$

$$\text{sgn } \sigma = -\text{sgn } x, \quad \sigma(0, y) = 0$$