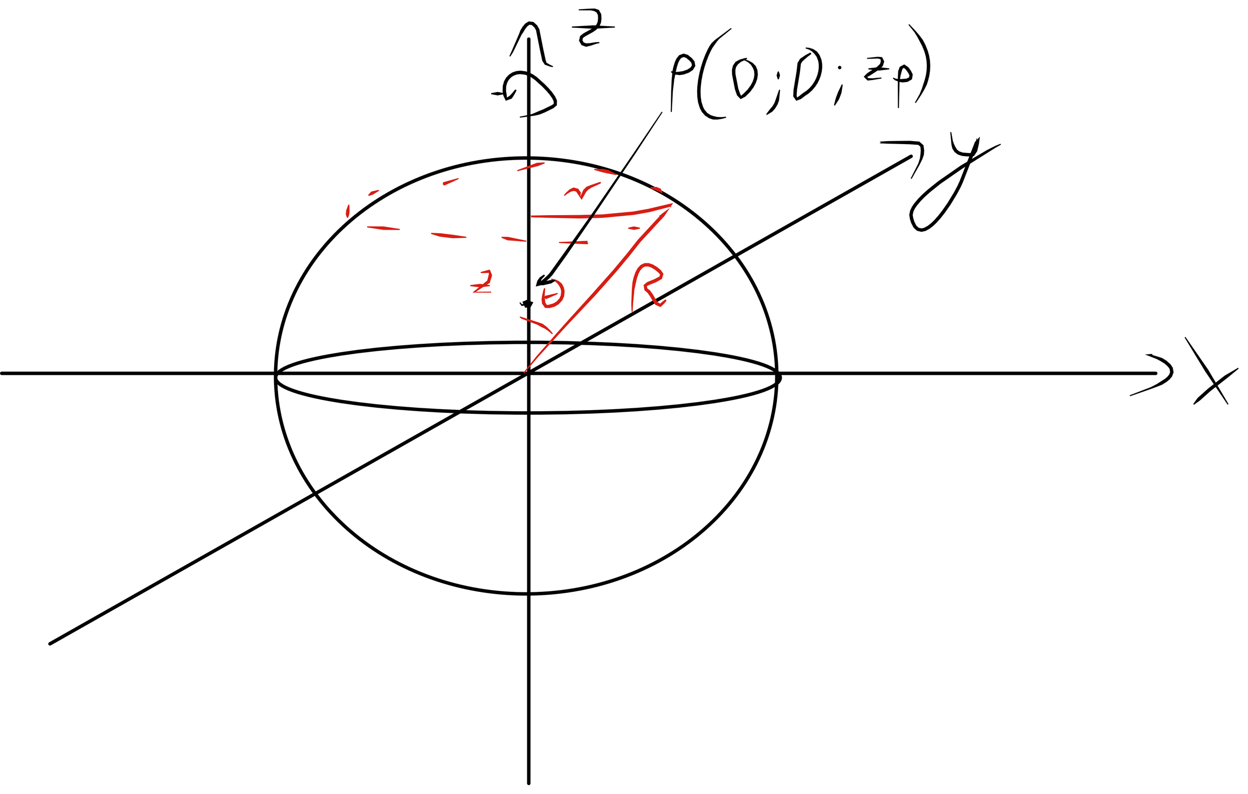

$$d\vec{B} = \frac{\mu_0 \, dI \, r^2}{2\sqrt{r^2+(z-z_p)^2}^{\,3}}\,\hat{e}_z$$

$$z = R\cos\theta$$

$$r = R\sin\theta$$

$$dI = \frac{dQ}{dt} = \frac{\sigma R^2 \sin\theta \, d\theta \, d\varphi}{dt} = \sigma \omega R^2 \sin\theta \, d\theta$$

$$dB = \frac{\mu_0 \sigma \omega R^2 \sin\theta \cdot R^2 \sin^2\theta \, d\theta}{2\sqrt{R^2\sin^2\theta + R^2\cos^2\theta + z_p^2 - 2z_pR\cos\theta}^{\,3}} = \underbrace{\frac{\mu_0 \sigma R^4 \omega}{2}}_{A} \cdot \frac{\sin^3\theta}{\sqrt{z_p^2+R^2-2z_pR\cos\theta}^{\,3}}\,d\theta$$

$$B = A\int_0^{\pi} \frac{\sin^3\theta}{\sqrt{z_p^2+R^2-2z_pR\cos\theta}^{\,3}}\,d\theta \;\underset{a\in[-1;1]}{\overset{\text{niech } z_p = Ra}{=}}\; \frac{A}{R^3}\int_0^{\pi} \frac{\sin^3\theta}{\sqrt{a^2+1-2\cos\theta\, a}^{\,3}}\,d\theta = \frac{A}{R^3}\underbrace{\int_0^{\pi} \frac{\sin\theta\,(1-\cos^2\theta)\,d\theta}{\sqrt{a^2+1-2\cos\theta\, a}^{\,3}}}_{I} =$$

$$= \left\{ \begin{array}{l} u = a^2+1-2\cos\theta\, a \\ du = 2a\sin\theta\, d\theta \\ 0 \to (a-1)^2 \\ \pi \to (a+1)^2 \end{array} \right\} = \frac{A}{R^3}\,\frac{1}{2a}\int_{(a-1)^2}^{(a+1)^2} \frac{1-\frac{1}{4}\left(a+\frac{1}{a}-\frac{u}{a}\right)^2}{u^{\frac{3}{2}}}\,du = ✳$$

$$\int_{(a-1)^2}^{(a+1)^2} \frac{1}{u^{\frac{3}{2}}}\,du = \left[\frac{-2}{\sqrt{u}}\right]_{(a-1)^2}^{(a+1)^2} = -2\left(\frac{1}{|a+1|} - \frac{1}{|a-1|}\right) = \frac{-4a}{a^2-1}$$

$$\int_{(a-1)^2}^{(a+1)^2} \frac{u}{u^{\frac{3}{2}}}\,du = \int_{(a-1)^2}^{(a+1)^2} \frac{1}{\sqrt{u}}\,du = \left[2\sqrt{u}\right]_{(a-1)^2}^{(a+1)^2} = 2\left(|a+1| - |a-1|\right) = 4a$$

$$\int_{(a-1)^2}^{(a+1)^2} \frac{u^2}{u^{\frac{3}{2}}}\,du = \int_{(a-1)^2}^{(a+1)^2} \sqrt{u}\,du = \left[\frac{2}{3}u^{\frac{3}{2}}\right]_{(a-1)^2}^{(a+1)^2} = \frac{2}{3}\left(|a+1|^3 - |a-1|^3\right) = \frac{2}{3}\left(2a^3+6a\right) = \frac{4}{3}a\left(a^2+3\right)$$

$$✳ = \frac{A}{R^3}\,\frac{1}{8a^3}\left[(a^2-1)^2\frac{4a}{a^2-1} + 8a(a^2+1) - \frac{4}{3}a(a^2+3)\right] = \frac{A}{R^3}\,\frac{1}{2a^3}\left[a^3 \cancel{-a} + 2a^3 + \cancel{2}a - \frac{a^3}{3} \cancel{-a}\right] = \frac{A}{R^3}\cdot\frac{4}{3} =$$

$$= \frac{2\mu_0 \sigma R \omega}{3}$$ Nie zależy od „a", jeśli $a \in (-1;1)$