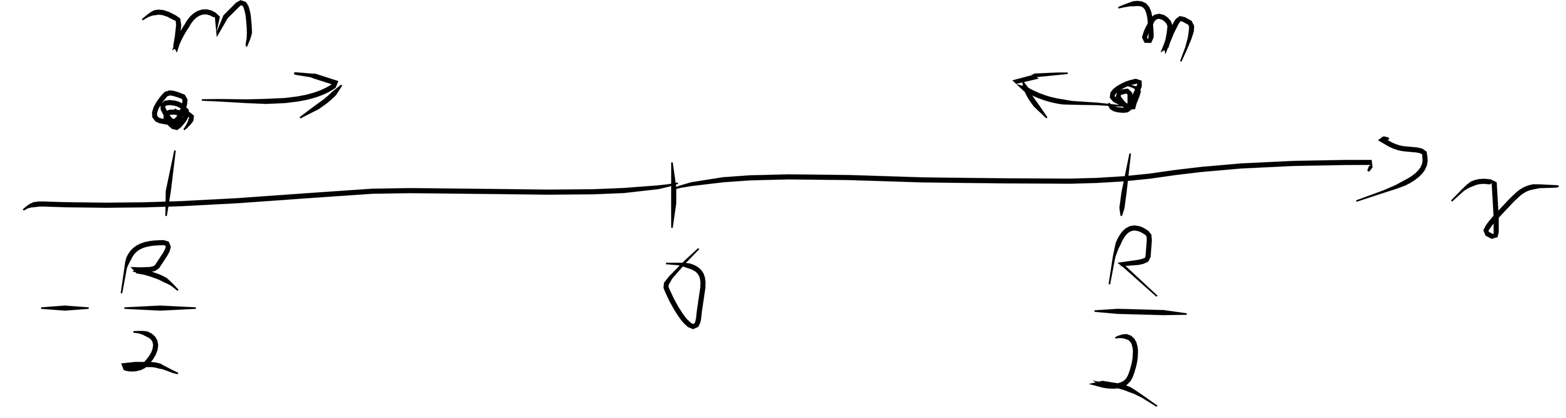
8.



$$E = \frac{-Gm^2}{R} = \frac{2m\dot{r}^2}{2} - \frac{Gm^2}{2r}$$

$$-\frac{Gm}{R} = \dot{r}^2 - \frac{Gm}{2r}$$

$$\dot{r}(t) = \sqrt{GM}\sqrt{\frac{1}{2r} - \frac{1}{R}}$$

$$\frac{dr}{\sqrt{\frac{1}{2r} - \frac{1}{R}}} = \sqrt{GM}\,dt$$

$$\int_0^{\frac{R}{2}} \sqrt{\frac{2rR}{R-2r}}\,dr = \sqrt{GM}\,t$$

$$R\int_0^{\frac{R}{2}} \sqrt{\frac{r}{\frac{R}{2} - r}}\,dr = \sqrt{GM}\,t$$

Zapierdala to podstawienie

$$\left\{ \begin{array}{ll} r = \frac{R}{2}\sin^2\theta & \frac{R}{2} - r = \frac{R}{2}\left(1 - \sin^2\theta\right) = \frac{R}{2}\cos^2\theta \\ r = 0 \rightarrow \theta = 0 & dr = \frac{R}{2}\cdot 2\sin\theta\cos\theta\,d\theta \\ r = \frac{R}{2} \rightarrow \theta = \frac{\pi}{2} & \end{array} \right\}$$

$$R^{\frac{1}{2}}\int_0^{\frac{\pi}{2}} \sqrt{\frac{\cancel{\frac{R}{2}}\sin^{\cancel{2}}\theta}{\cancel{\frac{R}{2}}\cancel{\cos^2\theta}}}\,R\sin\theta\cancel{\cos\theta}\,d\theta = R^{\frac{3}{2}}\int_0^{\frac{\pi}{2}}\sin^2\theta\,d\theta =$$

$$= \frac{1}{2}R^{\frac{3}{2}}\int_0^{\frac{\pi}{2}}(1 - \cos 2\theta)\,d\theta = \frac{1}{2}R^{\frac{3}{2}}\left[\theta - \frac{1}{2}\sin(2\theta)\right]_0^{\frac{\pi}{2}} =$$

$$= \frac{1}{2}R^{\frac{3}{2}}\left(\frac{\pi}{2}\right) = \sqrt{GM}\,t$$

$$t = \frac{\pi}{4}\sqrt{\frac{R^3}{GM}}$$

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