$$X = x^2 \partial_x + x y^2 \partial_y$$

$$\begin{cases} \dfrac{d}{dt} \varphi_t^1 \Big|_{t=0} = x \\ \dfrac{d}{dt} \varphi_t^2 \Big|_{t=0} = y \end{cases} \qquad \begin{cases} \dfrac{d}{dt} \varphi_t^1 = \left(\varphi_t^1\right)^2 \\ \dfrac{d}{dt} \varphi_t^2 = \varphi_t^1 \left(\varphi_t^2\right)^2 \end{cases}$$

$$\frac{d\varphi_t^1}{\left(\varphi_t^1\right)^2} = dt$$

$$-\frac{1}{\varphi_t^1} = t + C$$

$$\varphi_t^1 = \frac{-1}{t+C}$$

$$\varphi_0^1 = \frac{-1}{C} = x$$

$$C = -\frac{1}{x}$$

$$\varphi_t^1 = \frac{1}{\frac{1}{x} - t}$$

$$\frac{d\varphi_t^2}{\left(\varphi_t^2\right)^2} = \frac{1}{\frac{1}{x} - t}\, dt$$

$$-\frac{1}{\varphi_t^2} = \ln\left|\frac{1}{x} - t\right| - C$$

$$\varphi_t^2 = \frac{1}{C - \ln\left|\frac{1}{x} - t\right|}$$

$$\varphi_0^2 = y = \frac{1}{C + \ln|x|}$$

$$C = \frac{1}{y} - \ln|x|$$

$$\varphi_t^2 = \frac{1}{\frac{1}{y} - \ln|1 - xt|}$$

$$\varphi_t(x,y) = \left(\frac{1}{\frac{1}{x} - t} \; ; \; \frac{1}{\frac{1}{y} - \ln|1-xt|} \right)$$