a) $\bar{A} \cdot (\bar{B} \times \bar{C}) = A_i \bar{e}_i \cdot (B_j \bar{e}_j \times C_k \bar{e}_k) = A_i B_j C_k \bar{e}_i \cdot (\bar{e}_j \times \bar{e}_k) =$

$= A_i B_j C_k \bar{e}_i \cdot \varepsilon_{jkl} \bar{e}_l = A_i B_j C_k \varepsilon_{jkl} \delta_{il} = A_i B_j C_k \varepsilon_{jki} =$

$= B_i C_j A_k \varepsilon_{ijk} = \bar{B} \cdot (\bar{C} \times \bar{A}) = C_i A_j B_k \varepsilon_{kij} = \bar{C} \cdot (\bar{A} \times \bar{B})$

b) $\bar{A} \times (\bar{B} \times \bar{C}) = A_i \bar{e}_i \times (B_j \bar{e}_j \times C_k \bar{e}_k) = A_i B_j C_k \bar{e}_i \times (\bar{e}_j \times \bar{e}_k) =$

$= A_i B_j C_k \bar{e}_i \times \varepsilon_{jkl} \bar{e}_l = A_i B_j C_k \varepsilon_{jkl} \varepsilon_{ilm} \bar{e}_m = A_i B_j C_k \varepsilon_{ljk} \varepsilon_{lmi} \bar{e}_m =$

$= A_i B_j C_k (\delta_{jm} \delta_{ki} - \delta_{ji} \delta_{km}) \bar{e}_m = A_i B_j C_k (\bar{e}_j \delta_{ik} - \bar{e}_k \delta_{ji}) = L$

$\bar{B}(\bar{A} \cdot \bar{C}) - \bar{C}(\bar{A} \cdot \bar{B}) = B_j \bar{e}_j (A_i \bar{e}_i \cdot C_k \bar{e}_k) - C_k \bar{e}_k (A_i \bar{e}_i \cdot B_j \bar{e}_j) =$

$= A_i B_j C_k (\bar{e}_j \delta_{ik} - \bar{e}_k \delta_{ij}) = P \qquad L = P$

c) $(\bar{A} \times \bar{B}) \cdot (\bar{C} \times \bar{D}) = (A_i \bar{e}_i \times B_j \bar{e}_j) \cdot (C_k \bar{e}_k \times D_l \bar{e}_l) = A_i B_j C_k D_l \varepsilon_{ijm} \bar{e}_m \cdot \varepsilon_{klm} \bar{e}_m =$

$= A_i B_j C_k D_l \varepsilon_{ijm} \varepsilon_{klm} \delta_{mm} = A_i B_j C_k D_l \varepsilon_{mij} \varepsilon_{mkl} = A_i B_j C_k D_l (\delta_{ik} \delta_{jl} - \delta_{il} \delta_{jk}) =$

$= (A_i C_k \delta_{ik})(B_j D_l \delta_{jl}) - (A_i D_l \delta_{il})(B_j C_k \delta_{jk}) = (\bar{A} \cdot \bar{C})(\bar{B} \cdot \bar{D}) - (\bar{A} \cdot \bar{D})(\bar{B} \cdot \bar{C})$

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