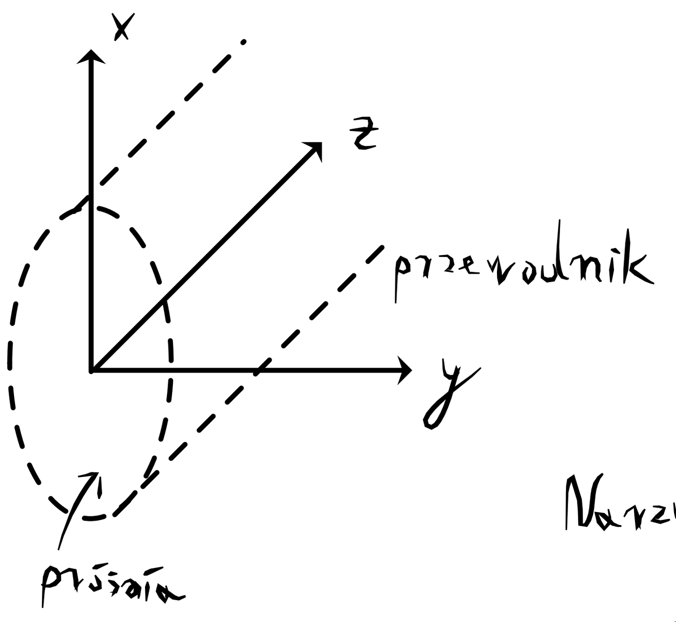


$\bar\nabla\cdot\bar E=0 \quad \bar\nabla\cdot\bar B=0 \quad \bar\nabla\times\bar E=-\frac{\partial\bar B}{\partial t} \quad \bar\nabla\times\bar B=\frac{1}{c^2}\frac{\partial\bar E}{\partial t}$

Na powierzchni przewodnika $E_{\parallel}=0 \quad B_{\perp}=0$

Narzucam $\bar E=\bar E_0(x,y)\,e^{i(kz-\omega t)} \quad \bar B=\bar B_0(x,y)\,e^{i(kz-\omega t)}$

$$\bar\nabla\times\bar E=e^{i(kz-\omega t)}\begin{bmatrix}\partial_y E_z-ikE_y\\ ikE_x-\partial_x E_z\\ \partial_x E_y-\partial_y E_x\end{bmatrix}$$ Podobnie dla $\bar\nabla\times\bar B$

$$\begin{bmatrix}\partial_y E_z-ikE_y\\ ikE_x-\partial_x E_z\\ \partial_x E_y-\partial_y E_x\\ \partial_y B_z-ikB_y\\ ikB_x-\partial_x B_z\\ \partial_x B_y-\partial_y B_x\end{bmatrix}=\begin{bmatrix}i\omega B_x\\ i\omega B_y\\ i\omega B_z\\ -i\frac{\omega}{c^2}E_x\\ -i\frac{\omega}{c^2}E_y\\ -i\frac{\omega}{c^2}E_z\end{bmatrix}$$

$(2)k-(4)\omega$

$$ik^2E_x-k\partial_xE_z-\omega\partial_yB_z+\cancel{ik\omega B_y}=\cancel{i\omega kB_y}+i\frac{\omega^2}{c^2}E_x$$

$$E_x\,i\left(k^2-\left(\tfrac{\omega}{c}\right)^2\right)=k\partial_xE_z+\omega\partial_yB_z$$

$(1)k+(5)\omega$

$$k\partial_yE_z-ik^2E_y+\cancel{ik\omega B_x}-\omega\partial_xB_z=\cancel{i\omega kB_x}-i\frac{\omega^2}{c^2}E_y$$

$$E_y\,i\left(k^2-\left(\tfrac{\omega}{c}\right)^2\right)=k\partial_yE_z-\omega\partial_xB_z$$

$(2)\frac{\omega}{c^2}-(4)k$

$$\cancel{\frac{ik\omega}{c^2}E_x}-\frac{\omega}{c^2}\partial_xE_z-k\partial_yB_z+ik^2B_y=\frac{i\omega^2}{c^2}B_y+\cancel{i\frac{\omega k}{c^2}E_x}$$

$$B_y\,i\left(k^2-\left(\tfrac{\omega}{c}\right)^2\right)=k\partial_yB_z+\frac{\omega}{c^2}\partial_xE_z$$

$(1)\frac{\omega}{c^2}+(5)k$

$$\frac{\omega}{c^2}\partial_yE_z-\cancel{i\frac{k\omega}{c^2}E_y}+ik^2B_x-k\partial_xB_z=i\frac{\omega^2}{c^2}B_x-\cancel{i\frac{\omega k}{c^2}E_y}$$

$$B_x\,i\left(k^2-\left(\tfrac{\omega}{c}\right)^2\right)=k\partial_xB_z-\frac{\omega}{c^2}\partial_yE_z$$

$$\begin{bmatrix}E_x\\ E_y\\ B_x\\ B_y\end{bmatrix}=\frac{i}{\left(\frac{\omega}{c}\right)^2-k^2}\begin{bmatrix}k\partial_xE_z+\omega\partial_yB_z\\ k\partial_yE_z-\omega\partial_xB_z\\ k\partial_xB_z-\frac{\omega}{c^2}\partial_yE_z\\ k\partial_yB_z+\frac{\omega}{c^2}\partial_xE_z\end{bmatrix}$$

$\bar\nabla\cdot\bar E=0 \Rightarrow \partial_xE_x+\partial_yE_y+\partial_zE_z=0$

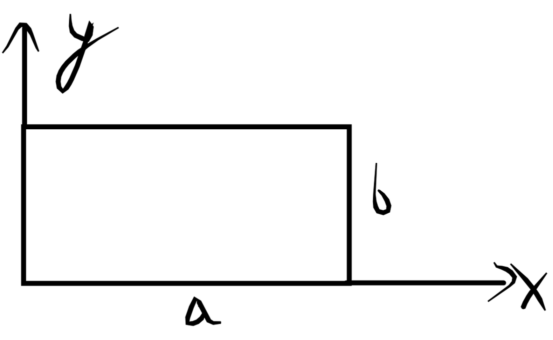
$$\frac{i}{\left(\frac{\omega}{c}\right)^2-k^2}\left(k\partial_x^2E_z+\omega\partial_x\partial_yB_z+k\partial_y^2E_z-\omega\partial_x\partial_yB_z\right)+ikE_z=0$$

$$\left(\partial_x^2+\partial_y^2+\left(\tfrac{\omega}{c}\right)^2-k^2\right)E_z=0$$

Podobnie dla B: $\left(\partial_x^2+\partial_y^2+\left(\tfrac{\omega}{c}\right)^2-k^2\right)B_z=0$

$$\boxed{\begin{aligned}&\left(\partial_x^2+\partial_y^2+\left(\tfrac{\omega}{c}\right)^2-k^2\right)E_z=0\\ &\left(\partial_x^2+\partial_y^2+\left(\tfrac{\omega}{c}\right)^2-k^2\right)B_z=0\end{aligned}}$$

Zadanie:



TE $\quad E_z=0$

$$B_z(x,y)=X(x)\,Y(y) \qquad \underbrace{\frac{\partial_x^2X}{X}}_{-k_x^2}+\underbrace{\frac{\partial_y^2Y}{Y}}_{-k_y^2}=k^2-\left(\tfrac{\omega}{c}\right)^2 \quad (*)$$

$X=A_x\sin(k_xx)+B_x\cos(k_xx) \qquad B_y(x,0)=B_y(x,b)=0$

$Y=A_y\sin(k_yy)+B_y\cos(k_yy) \qquad B_x(0,y)=B_x(a,y)=0$

$\Big\}$ bo $B_\perp=0$

dla $E_z=0$

$B_y\sim\partial_yB_z=X k_y\left(A_y\cos(k_yy)-B_y\sin(k_yy)\right)$

$B\sim\partial_xB_z=Y k_x\left(A_x\cos(k_xx)-B\sin(k_xx)\right)$

$\Rightarrow$ dla $x=0\Rightarrow A_x=0 \quad y=0\Rightarrow A_y=0$

dla $x=a\Rightarrow k_xa=n\pi \quad y=b\Rightarrow k_yb=m\pi$

$$B_{z(n,m)}=\cos\left(\frac{n\pi}{a}x\right)\cos\left(\frac{m\pi}{b}y\right)$$

$(*)\quad k_x^2+k_y^2=\left(\tfrac{\omega}{c}\right)^2-k^2$

$k^2=\left(\tfrac{\omega}{c}\right)^2-\pi^2\left(\left(\tfrac{n}{a}\right)^2+\left(\tfrac{m}{b}\right)^2\right)>0$

Propagacja dla $\omega>c\pi\sqrt{\left(\tfrac{n}{a}\right)^2+\left(\tfrac{m}{b}\right)^2}=\omega_{m,n}$

$\min(\omega_{m,n})=\dfrac{c\pi}{\max(a,b)}$

$\omega(k)=c\sqrt{k^2+\left(\frac{\omega_{m,n}}{c}\right)^2}$

$v_\varphi=\dfrac{\omega}{k}=\dfrac{c}{\sqrt{1-\left(\frac{\omega_{m,n}}{\omega}\right)^2}}>c \qquad v_{gr}=\dfrac{d\omega}{dk}=c\sqrt{1-\left(\frac{\omega_{m,n}}{\omega}\right)^2}<c$