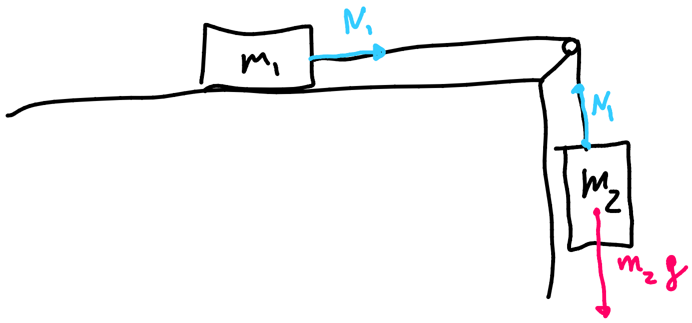
5.2

Saturday, November 26, 2022 6:57 PM

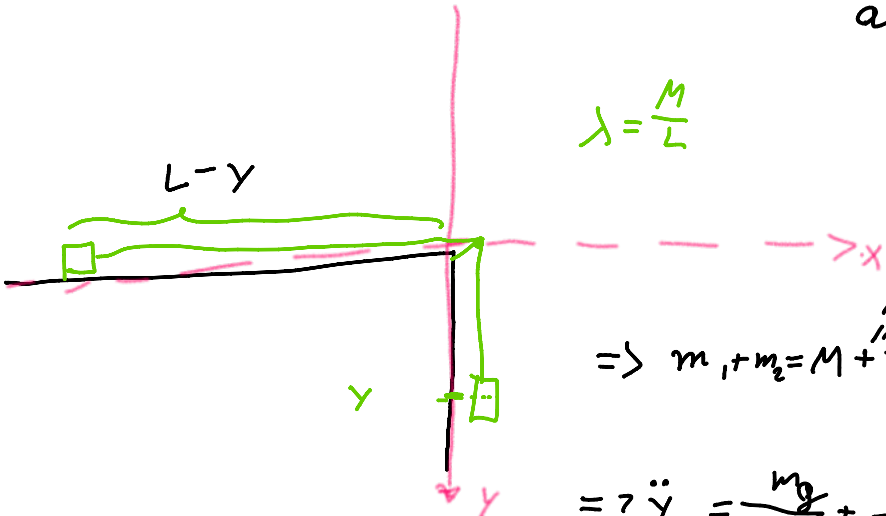
Zadanie 2
Wiotka lina o długości L i masie M leży na stole i sięga kantu. W pewnej chwili do końca liny doczepiono masę m i całość zaczęła się zsuwać. Znajdź ruch końca liny pomijając tarcie.

Odpowiedź: $y(t) = \frac{m}{M}L[\cosh\left(\sqrt{\frac{Mg}{(M+m)L}}t\right) - 1]$



$$a m_1 = N_1$$
$$a m_2 = m_2 g - N_1$$
$$\Rightarrow a(m_1+m_2) = m_2 g$$
$$\Rightarrow a = \frac{m_2 g}{m_1+m_2}$$

$$\lambda = \frac{M}{L}$$



$$\Rightarrow m_1 + m_2 = \overset{\tilde{M}}{M + m}, \quad m_2 = m + \lambda y$$

$$\Rightarrow \ddot{y} = \frac{mg}{M+m} + \frac{\lambda g}{m+M} y$$

$$\Rightarrow \ddot{y} - A\lambda y = Am$$

RORJ: $y = e^{bt} \Rightarrow b^2 - A\lambda = 0$

$$b = \pm\sqrt{A\lambda} = \pm\sqrt{\frac{M}{(m+M)L}}$$

$\Rightarrow$ RSRN:

$$y = C \Rightarrow -A\lambda C = Am$$
$$\Rightarrow C = -\frac{m}{\lambda} = -\frac{m}{M}L$$

RORN:

$$\sqrt{A\lambda}\,t \qquad -\sqrt{A\lambda}\,t$$

КОКИ.

$$y = -\frac{m}{M}L + C_1 e^{\sqrt{A\lambda}\,t} + C_2 e^{-\sqrt{A\lambda}\,t}$$

$$y = \frac{m}{M}L\left(\tilde{C}_1 e^{\sqrt{A\lambda}\,t} + \tilde{C}_2 e^{-\sqrt{A\lambda}\,t} - 1\right)$$

$$y(0) = 0 \Rightarrow \tilde{C}_1 + \tilde{C}_2 - 1 = 0 \Rightarrow \tilde{C}_1 + \tilde{C}_2 = 1 \Rightarrow \tilde{C}_1 = 1 - \tilde{C}_2$$

$$\dot{y}(t) = \frac{m}{M}L\sqrt{A\lambda}\left(\tilde{C}_1 e^{\sqrt{A\lambda}\,t} - \tilde{C}_2 e^{-\sqrt{A\lambda}\,t}\right)$$

$\Rightarrow$

$$\dot{y}(t) = 0 \Rightarrow \begin{cases} \tilde{C}_1 - \tilde{C}_2 = 1 \\ \tilde{C}_1 + C_2 = 1 \end{cases}$$

$$\tilde{C}_1 = \tfrac{1}{2} \Rightarrow \tilde{C}_2 = \tfrac{1}{2}$$

$$y = \frac{m}{M}L\left(\frac{e^{\sqrt{A\lambda}\,t} + e^{-\sqrt{A\lambda}\,t}}{2} - 1\right)$$

$$y = \frac{m}{M}L\left(\cosh\left(\sqrt{\frac{Mg}{(m+M)L}}\,t\right) - 1\right)$$