

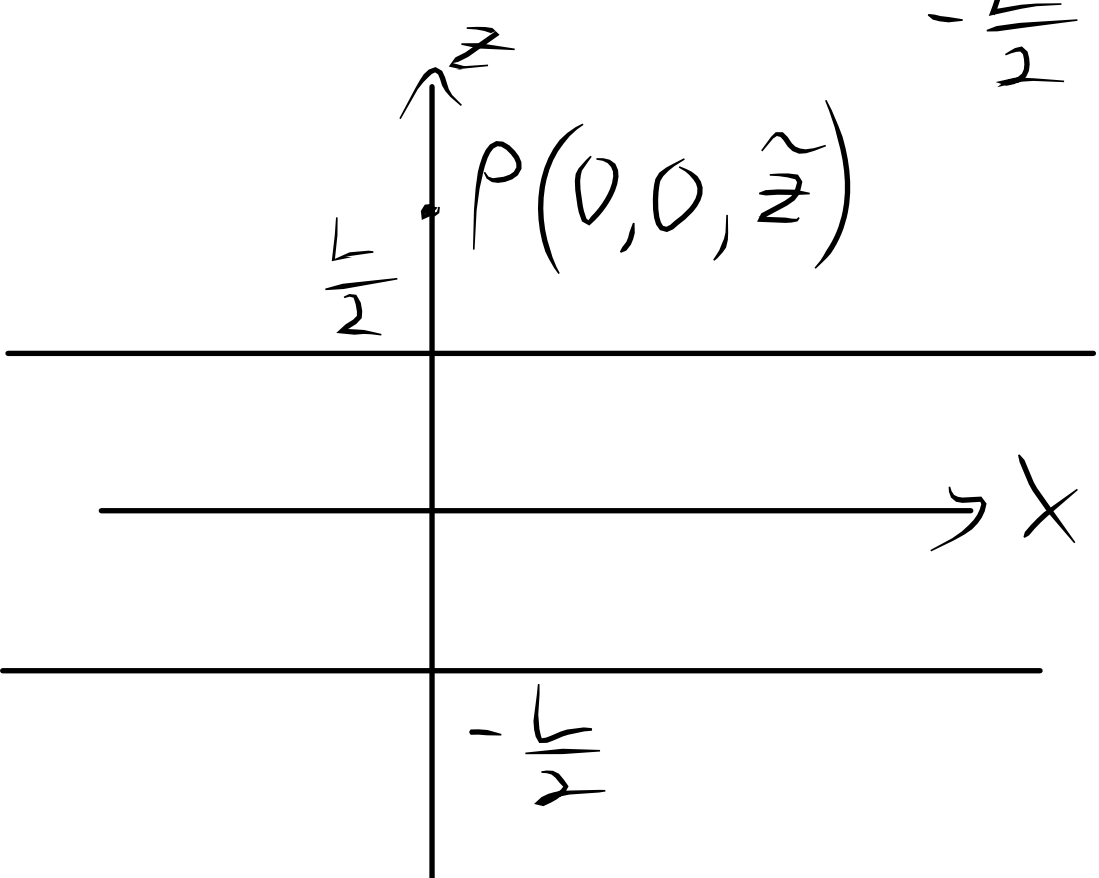
jedna warstwa $Q = S d\sigma = S \rho dz$

$$dE = \operatorname{sgn}(z) \frac{d\sigma}{2\epsilon_0} \hat{e}_z \quad d\sigma = \rho dz$$

$$\rho = \rho_0 \cos^2\left(\frac{\sqrt{\epsilon_0}}{L} z\right)$$

Dla $|\tilde{z}| \geq \frac{L}{2}$

$$E_z = \operatorname{sgn}(z) \frac{\rho_0 \hat{e}_z}{2\epsilon_0} \int_{-\frac{L}{2}}^{\frac{L}{2}} \cos^2\left(\frac{\sqrt{\epsilon_0}}{L} z\right) dz = \operatorname{sgn}(z) \frac{\rho_0 \hat{e}_z}{2\epsilon_0} \frac{L}{\sqrt{\epsilon_0}} \int_{-\frac{\sqrt{\epsilon_0}}{2}}^{\frac{\sqrt{\epsilon_0}}{2}} \cos^2(u) du =$$



$$\int_a^b \cos^2(x) dx = I = \cos x \sin x \Big|_a^b + \int_a^b \sin^2(x) dx =$$

$$= \cos x \sin x \Big|_a^b + \int_a^b 1 dx - \int_a^b \cos^2(x) dx =$$

$$+ \cos x \cos x - \sin x \sin x \Rightarrow I = \left[\frac{1}{2} \cos x \sin x + \frac{1}{2} x \right]_a^b$$

$$= \operatorname{sgn}(z) \frac{\rho_0 \hat{e}_z}{2\epsilon_0} \frac{L}{\sqrt{\epsilon_0}} \underbrace{\left(\frac{1}{2} \cos x \sin x + \frac{1}{2} x \right)}_{\frac{\sqrt{\epsilon_0}}{2}} \Big|_{-\frac{\sqrt{\epsilon_0}}{2}}^{\frac{\sqrt{\epsilon_0}}{2}} = \operatorname{sgn}(z) \frac{\rho_0 L}{4\epsilon_0} \hat{e}_z$$

Dla $|\tilde{z}| < \frac{L}{2}$ Niech $\tilde{z} = La$ $a \in]-\frac{1}{2}, \frac{1}{2}[$

$$E = \frac{\rho_0 \hat{e}_z}{2\epsilon_0} \left(\int_{-\frac{L}{2}}^{La} \cos^2\left(\frac{\sqrt{\epsilon_0}}{L} z\right) dz - \int_{La}^{\frac{L}{2}} \cos^2\left(\frac{\sqrt{\epsilon_0}}{L} z\right) dz \right) =$$

$$= \frac{\rho_0 \hat{e}_z}{2\epsilon_0} \frac{L}{\sqrt{\epsilon_0}} \left(\int_{-\frac{\sqrt{\epsilon_0}}{2}}^{a\sqrt{\epsilon_0}} \cos^2(x) dx - \int_{a\sqrt{\epsilon_0}}^{\frac{\sqrt{\epsilon_0}}{2}} \cos^2(x) dx \right) =$$

$$= \frac{\rho_0 \hat{e}_z}{2\epsilon_0} \frac{L}{\sqrt{\epsilon_0}} \frac{1}{2} \left(\left[\sin x \cos x + x \right]_{-\frac{\sqrt{\epsilon_0}}{2}}^{a\sqrt{\epsilon_0}} - \left[\sin x \cos x + x \right]_{a\sqrt{\epsilon_0}}^{\frac{\sqrt{\epsilon_0}}{2}} \right) =$$

$$= \frac{\rho_0 \hat{e}_z}{4\epsilon_0} \frac{L}{\sqrt{\epsilon_0}} \left(\frac{1}{2} \sin(2a\sqrt{\epsilon_0}) + a\sqrt{\epsilon_0} + \frac{\sqrt{\epsilon_0}}{2} + \frac{1}{2} \sin(2a\sqrt{\epsilon_0}) + a\sqrt{\epsilon_0} - \frac{\sqrt{\epsilon_0}}{2} \right) =$$

$$= \frac{\rho_0 \hat{e}_z}{4\epsilon_0} \frac{L}{\sqrt{\epsilon_0}} \left(\sin(2a\sqrt{\epsilon_0}) + 2a\sqrt{\epsilon_0} \right) = \frac{\rho_0 \hat{e}_z}{4\epsilon_0} \frac{L}{\sqrt{\epsilon_0}} \left(\sin\left(\frac{2\sqrt{\epsilon_0}}{L} \tilde{z}\right) + \frac{2\sqrt{\epsilon_0}}{L} \tilde{z} \right)$$

Dla $\tilde{z} > 0$, bo symetria. Niech $\phi\left(\frac{L}{2}\right) = 0$

$$\tilde{z} > \frac{L}{2}$$

$$\phi = - \int_{\frac{L}{2}}^{\tilde{z}} \frac{\rho_0 L}{4\epsilon_0} dz = - \frac{\rho_0 L}{4\epsilon_0} \left(\tilde{z} - \frac{L}{2} \right)$$

$$\tilde{z} < \frac{L}{2}$$

$$\phi = - \int_{\frac{L}{2}}^{\tilde{z}} \frac{\rho_0 L}{4\sqrt{\epsilon_0}} \left(\sin\left(\frac{2\sqrt{\epsilon_0}}{L} z\right) + \frac{2\sqrt{\epsilon_0}}{L} z \right) dz = \frac{\rho_0 L}{4\sqrt{\epsilon_0}} \left(\frac{L}{2\sqrt{\epsilon_0}} \cos\left(\frac{2\sqrt{\epsilon_0}}{L} \tilde{z}\right) + \frac{L}{2\sqrt{\epsilon_0}} - \frac{\sqrt{\epsilon_0}}{L} \tilde{z}^2 + \frac{\pi L}{4} \right)$$

niech bierzemy bregami $\Delta\phi = 0$

niech bierzemy i skrajem $|\phi(0) - \phi(\frac{L}{2})| = |\phi(0)| = \frac{\rho_0 L}{4\sqrt{\epsilon_0}} \left(\frac{L}{2\sqrt{\epsilon_0}} + \frac{L}{2\sqrt{\epsilon_0}} + \frac{\pi L}{4} \right) =$

$$= \frac{\rho_0 L^2}{4\sqrt{\epsilon_0}} \left(\frac{1}{\sqrt{\epsilon_0}} + \frac{\sqrt{\epsilon_0}}{4} \right)$$