zad 3

$$[X,Y] = \left(X^k Y^i{}_{,k} - Y^k X^i{}_{,k}\right)\partial_i$$

$$[X,Y](f) = X^k \frac{\partial}{\partial x^k}\left(Y^i \frac{\partial f}{\partial y^i}\right) - Y^i \frac{\partial}{\partial y^i}\left(X^k \frac{\partial f}{\partial x^k}\right)$$

$$= X^k\left(\frac{Y^i}{\partial x^k}\frac{\partial f}{\partial y^i} + \underline{Y^i \frac{\partial^2 f}{\partial x^k \partial y^i}}\right) +$$

$$- Y^i\left(\frac{\partial X^k}{\partial y^i}\frac{\partial f}{\partial x^k} + \underline{X^k \frac{\partial^2 f}{\partial y^i \partial x^k}}\right) =$$

(druga pochodna jest symetryczna, stąd podkreślone wyrazy się skracają)

$$= X^k \frac{\partial Y^i}{\partial x^k}\frac{\partial f}{\partial y^i} - Y^i \frac{\partial X^k}{\partial y^i}\frac{\partial f}{\partial x^k} \text{ stąd}$$

$$= \left(X^k \frac{\partial Y^i}{\partial x^k} - Y^k \frac{\partial X^i}{\partial x^k}\right)\frac{\partial}{\partial x^i}(f)$$