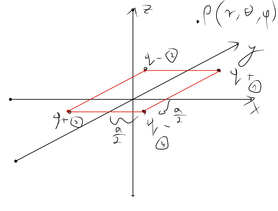

a)

$$x = r\sin\theta\cos\varphi$$

$$y = r\sin\theta\sin\varphi$$

$$z = r\cos\theta$$

$$d = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2}$$

$$r^2 = \left(r_1\sin\theta_1\cos\varphi_1 - r_2\sin\theta_2\cos\varphi_2\right)^2 + \left(r_1\sin\theta_1\sin\varphi_1 - r_2\sin\theta_2\sin\varphi_2\right)^2 + \left(r_1\cos\theta_1 - r_2\cos\theta_2\right)^2$$

$$= r_1^2\sin^2\theta_1\cos^2\varphi_1 + r_2^2\sin^2\theta_2\cos^2\varphi_2 - 2r_1r_2\sin\theta_1\sin\theta_2\cos\varphi_1\cos\varphi_2$$

$$+ r_1^2\sin^2\theta_1\sin^2\varphi_1 + r_2^2\sin^2\theta_2\sin^2\varphi_2 - 2r_1r_2\sin\theta_1\sin\theta_2\sin\varphi_1\sin\varphi_2$$

$$+ r_1^2\cos^2\theta_1 + r_2^2\cos^2\theta_2 - 2r_1r_2\cos\theta_1\cos\theta_2 =$$

$$= r_1^2 + r_2^2 - 2r_1r_2\left(\sin\theta_1\sin\theta_2\cos\varphi_1\cos\varphi_2 + \sin\theta_1\sin\theta_2\sin\varphi_1\sin\varphi_2 + \cos\theta_1\cos\theta_2\right)$$

$r_1 = r_2 = r_3 = r_4 = \frac{a}{\sqrt{2}}$ — odległości między Tedumkami a punktem P.

$\theta_1 = \theta_2 = \theta_3 = \theta_4 = \frac{\pi}{2}$

$\varphi_1 = \frac{\pi}{4}$

$\varphi_2 = \frac{3\pi}{4}$

$\varphi_3 = \frac{5\pi}{4}$

$\varphi_4 = \frac{7\pi}{4}$

$$d_1^2 = r^2 + \frac{a^2}{2} - ar\sin\theta\left(\cos(\varphi) + \sin(\varphi)\right)$$

$$d_2^2 = r^2 + \frac{a^2}{2} - ar\sin\theta\left(-\cos(\varphi) + \sin(\varphi)\right)$$

$$d_3^2 = r^2 + \frac{a^2}{2} - ar\sin\theta\left(-\cos(\varphi) - \sin(\varphi)\right)$$

$$d_4^2 = r^2 + \frac{a^2}{2} - ar\sin\theta\left(\cos(\varphi) - \sin(\varphi)\right)$$

$$\phi = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{d_1} + \frac{1}{d_3} - \frac{1}{d_2} - \frac{1}{d_4}\right)$$

do wyrażeń kwadratowych

$$\frac{1}{d_i} = \frac{1}{r\sqrt{1 + \frac{1}{2}\left(\frac{a}{r}\right)^2 - \left(\frac{a}{r}\right)\sin\theta[;]}} \approx \frac{1}{r}\left[1 - \frac{1}{2}\left(\frac{1}{2}\left(\frac{a}{r}\right)^2 + \left(\frac{a}{r}\right)\sin\theta[;]\right) + \frac{3}{8}\left(\frac{1}{2}\left(\frac{a}{r}\right)^2 - \left(\frac{a}{r}\right)\sin\theta[;]\right)^2\right] \approx$$

$$\approx \frac{1}{r}\left[1 - \frac{1}{4}\frac{a^2}{r^2} + \frac{1}{2}\frac{a}{r}\sin\theta[;] + \frac{3}{8}\frac{a^2}{r^2}\sin^2\theta[;]^2\right]$$

$$\phi = \frac{q}{4\pi\varepsilon_0 r}\left(1 - \frac{1}{4}\frac{a^2}{r^2} + \frac{1}{2}\frac{a}{r}\sin\theta\left(\cos\varphi + \sin\varphi\right) + \frac{3}{8}\frac{a^2}{r^2}\sin^2\theta\left(1 + \sin(2\varphi)\right)\right.$$

$$1 + \frac{1}{4}\frac{a^2}{r^2} + \frac{1}{2}\frac{a}{r}\sin\theta\left(\cos\varphi - \sin\varphi\right) + \frac{3}{8}\frac{a^2}{r^2}\sin^2\theta\left(-1 + \sin(2\varphi)\right)$$

$$1 - \frac{1}{4}\frac{a^2}{r^2} + \frac{1}{2}\frac{a}{r}\sin\theta\left(-\cos\varphi - \sin\varphi\right) + \frac{3}{8}\frac{a^2}{r^2}\sin^2\theta\left(1 + \sin(2\varphi)\right)$$

$$\left. 1 + \frac{1}{4}\frac{a^2}{r^2} - \frac{1}{2}\frac{a}{r}\sin\theta\left(-\cos\varphi + \sin\varphi\right) + \frac{3}{8}\frac{a^2}{r^2}\sin^2\theta\left(-1 + \sin(2\varphi)\right)\right) =$$

$$= \frac{q}{4\pi\varepsilon_0 r}\left(\frac{3}{2}\frac{a^2}{r^2}\sin^2(\theta)\sin(2\varphi)\right) = \frac{3}{8}\frac{q}{\pi\varepsilon_0}\frac{a^2}{r^3}\sin^2(\theta)\sin(2\varphi)$$

$$\bar{E} = -\nabla\phi = -\left[\frac{d\phi}{dr}, \frac{1}{r}\frac{d\phi}{d\theta}, \frac{1}{r\sin\theta}\frac{d\phi}{d\varphi}\right]_{r,\theta,\varphi} = -\frac{3}{8}\frac{q}{\pi\varepsilon_0}a^2\left[-\frac{3\sin^2(\theta)\sin(2\varphi)}{r^4}; \frac{\sin(2\varphi)}{r^4}\sin(2\theta); \frac{\sin\theta}{r^4}2\cos(2\varphi)\right] =$$

$$= \frac{3}{8}\frac{q}{\pi\varepsilon_0}\frac{a^2}{r^4}\left(3\sin^2\theta\sin(2\varphi)\,\bar{e}_r - \sin(2\varphi)\sin(2\theta)\,\bar{e}_\theta - 2\sin(\theta)\cos(2\varphi)\,\bar{e}_\varphi\right)$$

b) $\bar{E} = d\bar{l} = dr\,\bar{e}_r + r\,d\theta\,\bar{e}_\theta + r\sin\theta\,d\varphi\,\bar{e}_\varphi = E_r\bar{e}_r + E_\theta\bar{e}_\theta + E_\varphi\bar{e}_\varphi$

dla $z = 0 \Rightarrow \theta = \frac{\pi}{2}$

$$\frac{1}{d}\bar{E}_r\,dr = \frac{1}{E_\theta}r\,d\theta = \frac{1}{E_\varphi}r\sin\theta\,\bar{e}_\varphi$$

$$\bar{E} = \frac{3}{8}\frac{q}{\pi\varepsilon_0}\frac{a^2}{r^4}\left(3\sin(2\varphi)\bar{e}_r - 2\cos(2\varphi)\bar{e}_\varphi\right)$$

$$\frac{dr}{3\sin(2\varphi)} = \frac{r\,d\varphi}{2\cos(2\varphi)}$$

$$-\frac{2}{3}\frac{dr}{r} = \frac{\sin 2\varphi}{\cos 2\varphi}\,d\varphi$$

$$\frac{4}{3}\ln\left(\frac{r}{r_0}\right) = \ln|\cos 2\varphi|$$

$$r = r_0|\cos 2\varphi|^{\frac{3}{4}}$$

dla przekątnej $x = y \Rightarrow \varphi = \frac{\pi}{4}$

$$\bar{E} = \frac{3}{8}\frac{q}{\pi\varepsilon_0}\frac{a^2}{r^4}\left(3\sin^2\theta\,\bar{e}_r - \sin(2\theta)\,\bar{e}_\theta\right)$$

$$\frac{dr}{3\sin^2\theta} = \frac{-r\,d\theta}{\sin 2\theta}$$

$$-\frac{2}{3}\frac{dr}{r} = \frac{\sin\theta}{\cos\theta}\,d\theta$$

$$\frac{2}{3}\ln\left(\frac{r}{r_0}\right) = \ln|\cos\theta|\,d\theta$$

$$r = r_0|\cos\theta|^{\frac{3}{2}}$$