



$$\phi_1 = \phi_2$$

$$\frac{1}{4\pi\epsilon_0} \frac{q_1}{R_1} = \frac{1}{4\pi\epsilon_0} \frac{q_2}{R_2}$$

$$\frac{q_1}{R_1} = \frac{q_2}{R_2}$$

$$q_1 + q_2 = Q_1 + Q_2$$

$$q_1 = q_2 \frac{R_1}{R_2}$$

$$q_2 \left(1 + \frac{R_1}{R_2}\right) = Q_1 + Q_2$$

$$q_2 = \frac{Q_1 + Q_2}{1 + \frac{R_1}{R_2}}$$

$$\Delta\phi = |\phi_1 - \phi_2| = \frac{Q_1 + Q_2 \frac{R_1}{R_2} - Q_1 - Q_2}{1 + \frac{R_1}{R_2}} = \frac{R_1 Q_2 - R_2 Q_1}{R_1 + R_2}$$

$$U = \frac{Q}{4\pi\epsilon_0 R}$$

$$q_1 = \frac{Q_1 + Q_2}{1 + \frac{R_2}{R_1}}$$

$$q_1 = R_1 \frac{Q_1 + Q_2}{R_1 + R_2}$$

$$q_2 = R_2 \frac{Q_1 + Q_2}{R_1 + R_2}$$

$$C = 4\pi\epsilon_0 R$$

$$W = \frac{Q^2}{2C} = \frac{1}{2} QU = \frac{1}{2} U^2 C$$

$$\Delta W = \frac{1}{2} \left(\frac{Q_1^2}{4\pi\epsilon_0 R_1} + \frac{Q_2^2}{4\pi\epsilon_0 R_2} - \frac{(Q_1 + Q_2)^2 R_1}{4\pi\epsilon_0 (R_1 + R_2)^2} - \frac{(Q_1 + Q_2)^2 R_2}{4\pi\epsilon_0 (R_1 + R_2)^2} \right) =$$

$$= \frac{1}{8\pi\epsilon_0} \left(\frac{Q_1^2}{R_1} + \frac{Q_2^2}{R_2} - \frac{(Q_1 + Q_2)^2}{R_1 + R_2} \right) = \frac{1}{8\pi\epsilon_0} \frac{(Q_1 R_2 - Q_2 R_1)^2}{R_1 R_2 (R_1 + R_2)} = \frac{(Q_1 R_2 - Q_2 R_1)^2}{2R_1 R_2 (C + C_1)}$$