$\bar a = \text{const}$ $\quad \bar b = \text{const}$

a) $\bar\nabla\left((\bar a\cdot\bar r)\bar b\right) = \bar\nabla\left(a_i r_i b_j \bar e_j\right) = a_i b_j\, \partial_j r_i = a_i b_j\, \delta_{ij} = a_i b_i = \bar a\cdot\bar b$

$\bar\nabla\times\left((\bar a\cdot\bar r)\bar b\right) = \bar\nabla\times\left(a_i r_i b_j \bar e_j\right) = \varepsilon_{kjl}\, \partial_k\left(a_i r_i b_j\right)\bar e_l =$

$= \varepsilon_{kjl}\, a_i b_j\, \delta_{ki}\, \bar e_l = \varepsilon_{ijl}\, a_i b_j\, \bar e_l = \bar a\times\bar b$

b) $\bar\nabla\cdot\left((\bar a\cdot\bar r)\bar r\right) = \bar\nabla\cdot\left(a_i r_i r_j \bar e_j\right) = a_i\, \partial_j\left(r_i r_j\right) = a_i\left(\partial_j r_i\, r_j + r_i\, \partial_j r_j\right) =$

$= a_j r_j + 3 a_i r_i = 4\, \bar a\cdot\bar r$

$\bar\nabla\times\left((\bar a\cdot\bar r)\bar r\right) = \bar\nabla\times\left(a_i r_i r_j \bar e_j\right) = \varepsilon_{kjl}\, \partial_k\left(a_i r_i r_j\right)\bar e_l = \varepsilon_{kjl}\, a_i\left(r_j\delta_{ki} + r_i\delta_{kj}\right)\bar e_l =$

$= \varepsilon_{ijl}\, a_i r_j \bar e_l + \cancelto{0}{\varepsilon_{jjl}}\, a_i r_i \bar e_l = \bar a\times\bar r$

c) $\bar\nabla\cdot\left(\bar a\times\bar r\right) = \bar\nabla\cdot\left(\varepsilon_{ijk}\, a_i r_j \bar e_k\right) = \varepsilon_{ijk}\, \partial_k\left(a_i r_j\right) = \varepsilon_{ijj}\, a_i r_j = 0$

$\bar\nabla\times\left(\bar a\times\bar r\right) = \bar\nabla\times\left(\varepsilon_{ijk}\, a_i r_j \bar e_k\right) = \varepsilon_{lkm}\varepsilon_{ijk}\, a_i\, \partial_l r_j\, \bar e_m = \underset{kml}{\varepsilon_{lkm}}\, \underset{kil}{\varepsilon_{ilk}}\, a_i \bar e_m =$

$= \left(\delta_{mi}\delta_{ll} - \delta_{ml}\delta_{il}\right) a_i \bar e_m = 3\, a_i \bar e_i - a_i \bar e_i = 2\, \bar a$