$|\bar{r}'| = R$

$$\int_{S(R)} \frac{\cos\Theta}{|\bar{r}-\bar{r}'|}\, dS = \int_0^{2\pi}\int_0^{\pi} \frac{\cos\Theta}{|\bar{r}-\bar{r}'|} \sin\Theta\, d\Theta\, d\varphi =$$

$$= 2\pi \int_0^{\pi} d\Theta \cos\Theta \sin\Theta \begin{cases} \frac{1}{R}\sum_{l=0}^{\infty}\left(\frac{r}{R}\right)^l P_l(\cos\Theta) & \text{dla } r<R \\ \frac{1}{r}\sum_{l=0}^{\infty}\left(\frac{R}{r}\right)^l P_l(\cos\Theta) & \text{dla } r>R \end{cases} =$$

$$= \left\{ \begin{array}{l} \cos\Theta = x \\ -\sin\Theta\, d\Theta = dx \end{array} \right\} = 2\pi \int_{-1}^{1} dx\, P_1(x) \begin{cases} \frac{1}{R}\sum_{l=0}^{\infty}\left(\frac{r}{R}\right)^l P_l(x) & \text{dla } r<R \\ \frac{1}{r}\sum_{l=0}^{\infty}\left(\frac{R}{r}\right)^l P_l(x) & \text{dla } r>R \end{cases} =$$

$$= \left\{ \int_{-1}^{1} P_l(x) P_{l'}(x)\, dx = \frac{2\delta_{ll'}}{2l+1} \right\} = \frac{4}{3}\pi \begin{cases} \frac{r}{R^2} & \text{dla } r<R \\ \frac{R}{r^2} & \text{dla } r>R \end{cases}$$