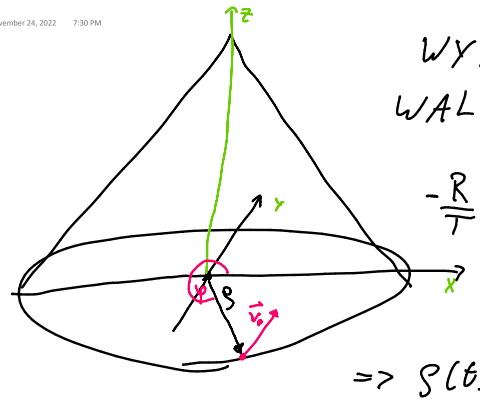
6.3

Thursday, November 24, 2022 7:30 PM



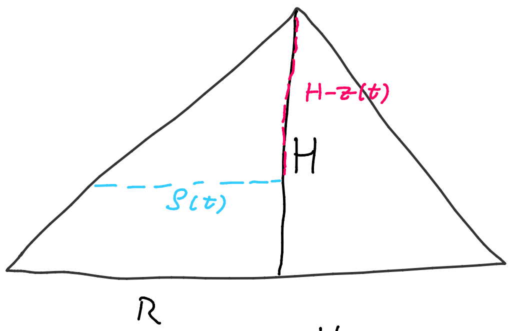
WYBIERAMY UKŁAD WALCOWY

$$-\frac{R}{T} = \frac{d\rho}{dt} \Rightarrow \rho = -\frac{R}{T}t + C$$

$$\rho(0) = R \Rightarrow C = R$$

$$\Rightarrow \rho(t) = -\frac{R}{T}t + R$$

PRZEKRÓJ STOŻKA:



$$\Rightarrow \frac{H - z(t)}{\rho(t)} = \frac{H}{R} \Rightarrow H - z(t) = \frac{H}{R}\left(-\frac{R}{T}t + R\right)$$

$$\Rightarrow z(t) = \frac{H}{T}t$$

$$\vec{r}(t) = \rho(t)\,\hat{e}_\rho + z(t)\,\hat{e}_z$$

$$\Rightarrow \vec{r}(t) = \left(R - \frac{R}{T}t\right)\hat{e}_\rho + \frac{H}{T}t\,\hat{e}_z$$

$$\dot{\hat{e}}_z = 0\,; \quad \dot{\hat{e}}_\rho = \dot{\varphi}\,\hat{e}_\varphi\,; \quad \dot{\hat{e}}_\varphi = -\dot{\varphi}\,\hat{e}_\rho$$

$$\Rightarrow v(t) = \frac{d}{dt}\vec{r}(t) = -\frac{R}{T}\hat{e}_\rho + \dot{\varphi}\left(R - \frac{R}{T}t\right)\hat{e}_\varphi + \frac{H}{T}\hat{e}_z$$

$$|\vec{v}| = v_0$$

$$\Rightarrow v_0^2 = \frac{R^2}{T^2} + \dot{\varphi}^2\left(R - \frac{R}{T}t\right)^2 + \frac{H^2}{T^2}$$

$$\Rightarrow \frac{\sqrt{v_0^2 - \frac{R^2 + H^2}{T^2}}}{R - \frac{R}{T}t} = \dot{\varphi} = \omega(t)$$

$$\Rightarrow \vec{v}(t) = -\frac{R}{T}\hat{e}_\rho + \sqrt{v_0^2 - \frac{R^2 + H^2}{T^2}}\cdot\hat{e}_\varphi + \frac{H}{T}\hat{e}_z$$

$$\Rightarrow \vec{a}(t) = \frac{d}{dt}\vec{v}(t) = -\omega\frac{R}{T}\hat{e}_\varphi - \omega\sqrt{v_0^2 - \frac{R^2 + H^2}{T}}\,\hat{e}_\rho$$

$$\vec{a}(t) = -\omega\left(\sqrt{v_0^2 - \frac{R^2+H^2}{T^2}}\,\hat{e}_\rho + \frac{R}{T}\,\hat{e}_\varphi\right)$$

$$\hat{t} = \frac{\vec{v}}{v_0} = \frac{1}{v_0}\vec{v}$$

$$\dot{\hat{t}} = \frac{1}{v_0}\frac{d}{dt}\vec{v} = \frac{1}{v_0}\vec{a}$$

$$|\vec{a}|^2 = \omega^2\left(v_0^2 - \frac{R^2}{T^2} - \frac{H^2}{T^2} + \frac{R^2}{T^2}\right) = \omega^2\left(v_0^2 - \frac{H^2}{T^2}\right)$$

$$\Rightarrow \hat{n} = \frac{\dot{\hat{t}}}{|\dot{\hat{t}}|} = \frac{\frac{1}{v_0}\vec{a}}{\frac{1}{v_0}\omega\sqrt{v_0^2 - \frac{H^2}{T^2}}} = \frac{1}{\omega\sqrt{v_0^2 - \frac{H^2}{T^2}}}\,\vec{a}$$

$$\Rightarrow a_n = \hat{n}\cdot\vec{a} = \frac{1}{\omega\sqrt{v_0^2 - \frac{H^2}{T^2}}}\,\vec{a}\cdot\vec{a} = \frac{|\vec{a}|^2}{\omega\sqrt{v_0^2 - \frac{H^2}{T^2}}} = \frac{\omega^2\left(v_0^2 - H^2/T^2\right)}{\omega\sqrt{v_0^2 - \frac{H^2}{T^2}}} =$$

$$= \omega\sqrt{v_0^2 - \frac{H^2}{T^2}} = \frac{\sqrt{v_0^2 - \frac{H^2}{T^2}}\,\sqrt{v_0^2 - \frac{R^2+H^2}{T^2}}}{\left(R - \frac{R}{T}t\right)}$$

$$a_n = \frac{v_0^2}{\rho} \Rightarrow \rho = \frac{v_0^2}{a_n} = v_0^2\,\frac{\left(R - \frac{R}{T}t\right)}{\sqrt{\left(v_0^2 - \frac{H^2}{T^2}\right)\left(v_0^2 - \frac{R^2+H^2}{T^2}\right)}}$$

$$\Rightarrow \rho(0) = \frac{v_0^2 R}{\sqrt{\left(v_0^2 - \frac{H^2}{T^2}\right)\left(v_0^2 - \frac{R^2+H^2}{T^2}\right)}}$$