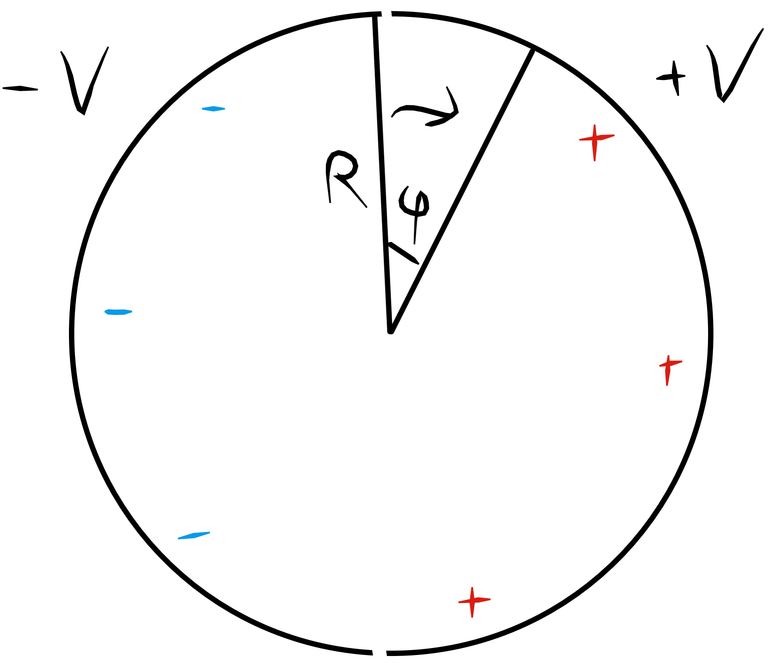

$$\Phi(\rho,\varphi) = R'(\rho)\psi(\varphi)$$

$$\Delta\Phi(\rho,\varphi) = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\phi}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2\phi}{\partial\varphi^2} = 0$$

$$\underbrace{\frac{\rho}{R}\frac{\partial}{\partial\rho}\left(\rho\frac{dR}{d\rho}\right)}_{m^2} + \underbrace{\frac{1}{\psi}\frac{d^2\psi}{d\varphi^2}}_{-m^2} = 0$$

dla $m = 0$

$$R'(\rho) = a + b\ln(\rho)$$

$$\psi(\varphi) = a + b\varphi$$

$$\frac{\partial}{\partial\rho}\left(\rho\frac{dR'}{d\rho}\right) = m^2\frac{R'}{\rho}$$

Zgadując $R' = \rho^\beta$

$$\frac{\partial}{\partial\rho}\left(\beta\rho^\beta\right) = m^2\rho^{\beta-1}$$

$$\beta^2\rho^{\beta-1} = m^2\rho^{\beta-1} \qquad \beta = \pm m$$

$$R'(\rho) = a_1\rho^m + a_2\rho^{-m}$$

$$\frac{d^2\psi}{d\varphi^2} = -m^2\psi \qquad \psi(\rho) = b_1\sin(m\varphi) + b_2\cos(m\varphi)$$

$$\Phi(\rho,\varphi) = \sum_{m=1}^{\infty}\left(A_m\rho^m + B_m\rho^{-m}\right)\left(C_m\sin(m\varphi) + D_m\cos(m\varphi)\right) + \left(A_0 + B_0\ln(\rho)\right)\left(C_0 + D_0\varphi\right)$$

$B_m = 0$, bo inaczej wybucha w 0

$D_{m>0} = 0$, bo musi być nieparzyste → rysunek

$D_0 = 0$, bo $\Phi(\rho,\varphi) = \Phi(\rho,\varphi+2\pi)$

$$\Phi(\rho,\varphi) = \sum_{m=1}^{\infty} a_m\rho^m\sin(m\varphi) + a_0$$

$$\Phi(R,\varphi\in\,]0;\pi[\,) = V \qquad \Phi(R,\varphi\in\,]\pi;2\pi[\,) = -V$$

$$\int_0^{2\pi}\Phi(R,\varphi)\sin(m'\varphi)\,d\varphi = \int_0^{2\pi}\left(\sum_{m=1}^{\infty}a_m R^m\sin(m\varphi) + a_0\right)\sin(m'\varphi)\,d\varphi$$

dla m-parzystych

$$0 = \int_0^{2\pi} a_m R^m \sin\left(\frac{m'\pi}{\pi}\varphi\right)\sin\left(\frac{m\pi}{\pi}\varphi\right)d\varphi = \pi a_m R^m\delta_{mm'}$$

$\Rightarrow a_m = 0$ dla m-parzystych

dla m-nieparzystych

$$V\int_0^{\pi}\sin(m'\varphi)\,d\varphi - V\int_{\pi}^{2\pi}\sin(m'\varphi)\,d\varphi = \pi a_m R^m\delta_{mm'} + \int_0^{2\pi}a_0\sin(m'\varphi)\,d\varphi$$

$$4\frac{V}{m} = \pi a_m R^m$$

dla $m = 0$ $\quad \int_0^{2\pi}\Phi\,d\varphi = \int_0^{2\pi}a_0\,d\varphi$

$$a_m = \frac{4V}{\pi m}R^{-m}$$

$$0 = V - V = 2\pi a_0$$

$$a_0 = 0$$

$$\Phi(\rho,\varphi) = \sum_{k=0}^{\infty}\frac{4V}{\pi(2k+1)}\left(\frac{\rho}{R}\right)^{2k+1}\sin\left((2k+1)\varphi\right)$$