$u^i u_i = g_{ij} u^i u^j$ $\qquad \gamma(v) = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}$ $\qquad \bar{s} = \begin{bmatrix} ct \\ x^1(t) \\ x^2(t) \\ x^3(t) \end{bmatrix}$

$dt = \gamma(v)\, d\tau$ $\qquad \bar{u} = \frac{d}{d\tau}\bar{s}(t) = \frac{d\bar{s}}{dt}\frac{dt}{d\tau} = \gamma(u) \begin{bmatrix} c \\ u^1 \\ u^2 \\ u^3 \end{bmatrix}$

$u^2 = \gamma^2(v)\left(c^2 - \underbrace{\left[(u^1)^2 + (u^2)^2 + (u^3)^2\right]}_{v^2}\right) = \frac{1}{1 - \frac{v^2}{c^2}}\, c^2\left(1 - \frac{v^2}{c^2}\right) = c^2 \equiv 1$