$$\rho(r) = \frac{-e_0}{\pi a_0^3} e^{-\frac{2r}{a_0}}$$

Symetria, więc $\bar{E} = E_r \hat{e}_r$

$$\bar{\nabla}\cdot\bar{E} = \frac{\rho(r)}{\varepsilon_0}$$

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2 E_r\right) = \frac{-e_0}{\pi a_0^3 \varepsilon_0} e^{-\frac{2r}{a_0}}$$

$$\frac{d}{dr} r^2 E_r = \frac{-e_0}{\pi a_0^3 \varepsilon_0} e^{-\frac{2r}{a_0}} r^2$$

$$r^2 E_r = \frac{-e_0}{\pi a_0^3 \varepsilon_0} \int_0^r \tilde{r}^2 e^{-\frac{2\tilde{r}}{a_0}} \, d\tilde{r}$$

D	I
$+\ r^2$	$e^{-\frac{2r}{a_0}}$
$-2r$	$-\frac{a_0}{2} e^{-\frac{2r}{a_0}}$
$+\ 2$	$\frac{a_0^2}{4} e^{-\frac{2r}{a_0}}$
$-\ 0$	$-\frac{a_0^3}{8} e^{-\frac{2r}{a_0}}$

$$r^2 E_r = \frac{-e_0}{\pi a_0^3 \varepsilon_0}\left[e^{-\frac{2r}{a_0}} \cdot \frac{a_0}{2}\left(-r^2 - a_0 r - \frac{a_0^2}{2}\right) + \frac{a_0^3}{4}\right]$$

$$E_r = \frac{e_0}{2\pi a_0^2 \varepsilon_0}\left[e^{-\frac{2r}{a_0}}\left(1 + \frac{a_0}{r} + \frac{a_0^2}{2r^2}\right) - \frac{a_0^2}{2r^2}\right]$$

$r << a_0$

$$E_r \approx \frac{e_0}{2\pi a_0^2 \varepsilon_0} e^{-\frac{2r}{a_0}}\left(1 + \frac{a_0}{r} + \frac{a_0^2}{2r^2} - \frac{a_0^2}{2r^2}\left(1 + \frac{2r}{a_0} + \frac{2r^2}{a_0^2} + \frac{4r^3}{3a_0^3}\right)\right) =$$

$$= \frac{e_0}{2\pi a_0^2 \varepsilon_0} e^{-\frac{2r}{a_0}}\left(\cancel{1} + \cancel{\frac{a_0}{r}} + \cancel{\frac{a_0^2}{2r^2}} - \cancel{\frac{a_0^2}{2r^2}} - \cancel{\frac{a_0}{r}} - \cancel{1} - \frac{2r}{3a_0}\right) = \frac{-e_0}{3\pi a_0^3 \varepsilon_0} e^{-\frac{2r}{a_0}} r$$

$\phi(a_0) = 0$

$$\phi = \frac{-e_0}{2\pi a_0^2 \varepsilon_0} \int_{a_0}^{r} e^{-\frac{2\tilde{r}}{a_0}} - \frac{a_0^2}{2\tilde{r}^2} + \underbrace{e^{-\frac{2\tilde{r}}{a_0}}\left(\frac{a_0}{\tilde{r}} + \frac{a_0^2}{2\tilde{r}^2}\right)}_{X} d\tilde{r} =$$

$$= \frac{-e_0}{2\pi a_0^2 \varepsilon_0}\left[-\frac{a_0}{2}\left(e^{-\frac{2r}{a_0}} - e^{-2}\right) + \frac{a_0^2}{2r} - \frac{a_0}{2} - \frac{a_0}{2}\left(e^{-\frac{2r}{a_0}}\frac{a_0}{r} - e^{-2}\right)\right] =$$

$$\boxed{\frac{d}{dr}\left(e^{-\frac{2r}{a_0}} \cdot \frac{a_0}{r}\right) = \frac{-2}{r} e^{-\frac{2r}{a_0}} - e^{-\frac{2r}{a_0}} \frac{a_0}{r^2} = \frac{-2}{a_0} e^{-\frac{2r}{a_0}} \underbrace{\left(\frac{a_0}{r} + \frac{a_0^2}{2r^2}\right)}_{X}}$$

$$= \frac{-e_0}{2\pi a_0 \varepsilon_0}\left[e^{-2} - \frac{1}{2} + \frac{a_0}{2r} - \frac{1}{2} e^{-\frac{2r}{a_0}}\left(\frac{a_0}{r} + 1\right)\right]$$

$$\left(1 - \frac{2r}{a_0} + \frac{2r^2}{a_0^2} + \dots\right)$$

$r << a_0$

$$\phi \approx \frac{-e_0}{2\pi a_0 \varepsilon_0}\left[e^{-2} - \cancel{\frac{1}{2}} + \cancel{\frac{a_0}{2r}} - \cancel{\frac{a_0}{2r}} - \cancel{\frac{1}{2}} + \cancel{1} + \cancel{\frac{r}{a_0}} - \frac{r^2}{a^2} - \cancel{\frac{r}{a_0}}\right] = \frac{e_0}{2\pi a_0 \varepsilon_0}\left(\frac{r^2}{a^2} - e^{-2}\right)$$