

A new optimal algorithm for Level-Synchronous Parallel BFS in the Cilk model

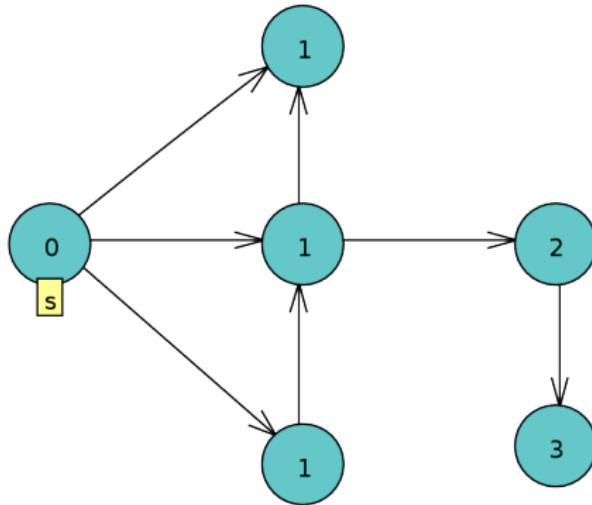
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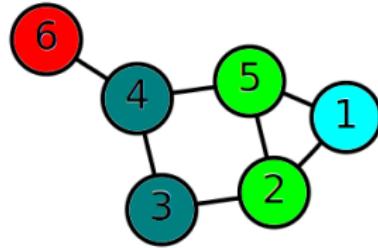
BFS

- ▶ Given
 - ▶ Graph $G = (\mathbb{V}, \mathbb{E})$ with diameter D and max out-degree Δ
 - ▶ Source vertex $s \in \mathbb{V}$
- ▶ Calculate
 - ▶ $Dist_{u \in \mathbb{V}} = \text{length of the shortest path from } s \text{ to } u \text{ in } G$



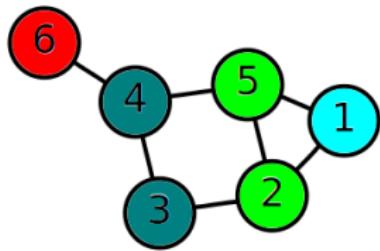
Parallel BFS

- ▶ The runtime for serial BFS is $T_s = \Theta(|V| + |E|)$
- ▶ For large graphs, calculating $Dist$ can be slow
- ▶ We can speed up BFS by processing edges and/or nodes in parallel
- ▶ Our new TBN-BFS achieves near-perfect linear speedup when $p \ll \frac{|V|+|E|}{D \log(|V|+|E|)}$



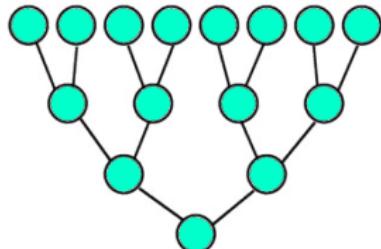
Level-Synchronous BFS

- ▶ All nodes at distance d from s are processed before any nodes at distance $d' > d$
- ▶ Level-Synchronous BFS has lower bounds of
 - ▶ $T_p = \Omega\left(\frac{|\mathbb{V}|+|\mathbb{E}|}{p} + D\right)$
 - ▶ $T_1, W_p = \Omega(T_s) = \Omega(|\mathbb{V}| + |\mathbb{E}|)$
- ▶ Non Level-Synchronous BFS algorithms
 - ▶ Distinguished-BFS [Ullman and Yannakakis, 1990]



Cilk Model

- ▶ Large shared memory
- ▶ Consistent caches between cores
- ▶ Synchronizing (or launching) n tasks takes $T_\infty = \Theta(\log n)$ time
- ▶ Cilk+ has this model with the addition of randomized work stealing [Frigo et al., 2009]
- ▶ A parallel algorithm has the following properties
 - ▶ T_p = running time using p cores
 - ▶ T_1 = running time using 1 core
 - ▶ W_p = work done using p cores



Non Cilk Model

- ▶ Level Synchronous BFS algorithms not using the Cilk Model
 - ▶ Cray-BFS uses the Cray MTA-2 hardware model
[Bader and Madduri, 2006]
 - ▶ Block-Queue-BFS uses the Intel Mic hardware model
[Saule and Catalyurek, 2012]
 - ▶ Blue-BFS uses the BlueGene/L hardware model
[Yoo et al., 2005]



Level-Synchronous Parallel BFS in the Cilk Model

- ▶ In the Cilk Model, Level-Synchronous BFS has a lower bound of $T_p = \Omega\left(\frac{|\mathbb{V}|+|\mathbb{E}|}{p} + D \log p\right)$
- ▶ MIT-Bag-BFS uses penants, bags, and reducer hyperobjects [Leiserson and Schardl, 2010]
- ▶ Our new TBN-BFS algorithm
 - ▶ Runs on standard hardware implemented using Cilk+
 - ▶ Can do its own scheduling deterministically
 - ▶ Achieves the lower bound for T_p, T_1, W_p in the worst case independently of Δ (scale-free)



Serial-BFS

1. for each vertex $u \in \mathbb{V}$
 2. $Dist_u \leftarrow \infty$
 3. $Dist_s \leftarrow 0$
 4. $Q \leftarrow \emptyset$
 5. ENQUEUE(Q, s)
 6. while $Q \neq \emptyset$ do
 7. $u \leftarrow \text{DEQUEUE}(Q)$
 8. for each vertex v in $\Gamma(u)$
 9. if $Dist_v = \infty$ then
 10. $Dist_v \leftarrow Dist_u + 1$
 11. ENQUEUE(Q, v)

{Potential source of parallelism: lines 7–8}

{Potential issues for
parallelism: lines 9–11}

TBN-BFS - Summary

- ▶ $T_1 = \mathcal{O}(|\mathbb{V}| + |\mathbb{E}|)$ (optimal)
- ▶ $W_p = \mathcal{O}(|\mathbb{V}| + |\mathbb{E}|)$ (optimal)
- ▶ $T_p = \mathcal{O}\left(\frac{|\mathbb{V}|+|\mathbb{E}|}{p} + D \log p\right)$ (optimal)
- ▶ Scale-free
- ▶ Runs on standard hardware implemented using Cilk+
- ▶ Can do its own scheduling deterministically



TBN-BFS - High Level

- ▶ To get optimal T_1, T_p the algorithm is
 1. For each level $\ell \in [0 \dots D]$
 - 1.1 Prepare to split work
 - 1.2 Split work and process edges
 - 1.3 Deduplicate vertexes and combine queues
- ▶ To get optimal W_p modify the algorithm to
 1. Reduce search space
 2. Dynamically choose number of cores



TBN-BFS - Prepare to Split Work

1. Receive input $Q_{in} \subseteq \mathbb{V}$ where
 - 1.1 Each vertex in Q_{in} is unique
 - 1.2 $\forall u \in Q_{in} : Dist_u = \ell$
 - 1.3 $\forall u \in Q_{in} : |\Gamma(u)| > 0$
2. Generate $OutDegrees[0 \leq i < |Q_{in}|] = |\Gamma(Q_{in}[i])|$ in parallel
3. Perform a parallel prefix sum on $OutDegrees$

$Q_{in} =$	1	3	2	4
$OutDegrees_{before} =$	5	7	1	2
$OutDegrees_{after} =$	5	12	13	15	...	m_ℓ



TBN-BFS - Split Work and Process Edges

1. Each core i in parallel

1.1 searches $OutDegrees$ for $1 + \left\lfloor \frac{i m_\ell}{p} \right\rfloor$ to find starting edge

$$\left\{ \mathcal{O} \left(\log \frac{n_\ell}{p} + \log p \right) \text{ work} \right\}$$

1.2 processes $\left\lfloor \frac{m_\ell}{p} \right\rfloor$ consecutive edges

1.2.1 $Q_i \leftarrow \emptyset$

1.2.2 for each edge (u, v)

1.2.3 if $Dist_v = \infty$ then

{Benign race condition}

1.2.4 $Dist_v \leftarrow Dist_u + 1$

{Maintains $Q_i n$ invariant 2}

1.2.5 $Owner_v \leftarrow i$

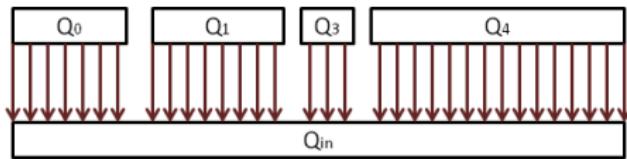
1.2.6 if $|\Gamma(v)| > 0$

{Maintains $Q_i n$ invariant 3}

1.2.7 ENQUEUE(Q_i, v)

TBN-BFS - Deduplicate Vertices and Combine Queues

1. Each core i in parallel
 - 1.1 $Q_i \leftarrow \{u \in Q_i : Owner_u = i\}$ {Remove duplicate vertexes}
 - 1.2 $Size_i \leftarrow |Q_i|$
 2. Perform a parallel prefix sum on $Size$
 3. Each core i copies its queue back into Q_{in} at offset $Size_{i-1}$
 $\{Size_{-1}$ is treated as 0 $\}$

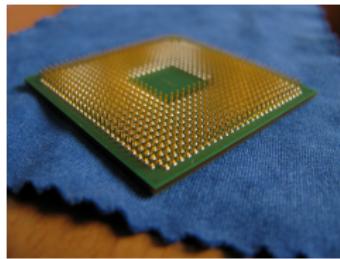


TBN-BFS - Reduce Search Space

1. $N \equiv |\text{OutDegrees}|$
2. $\text{OutDegree}_k \equiv \text{OutDegree}[\left\lfloor \frac{(k+1)N}{p} \right\rfloor : \left\lfloor \frac{kN}{p} \right\rfloor]$
3. Invariant: each core i first edge is in OutDegree_k for exactly one k
 - 3.1 For each $0 \leq k < p$ in parallel
 - 3.1.1 Find cores $[f_k : \ell_k)$ whose first degree is in OutDegree_k
 {Each range $[f_k : \ell_k)$ is pairwise disjoint, $\sum_k ([f_k : \ell_k]) = [0 : p)$ }
 - 3.1.2 Assign k in parallel to $\text{SubList}_{i \in [f:\ell)}$
 4. Core i searches for first degree in $\text{OutDegree}_{\text{SubList}_i}$ instead of OutDegree
 5. W_p reduces from $\mathcal{O}(|\mathbb{V}| + |\mathbb{E}| + Dp \log p)$ to $\mathcal{O}(|\mathbb{V}| + |\mathbb{E}| + Dp)$

TBN-BFS - Dynamically Choose Number of Cores

- ▶ In “Prepare to Split Work”, right after the parallel prefix sum,
 $p_\ell \leftarrow \text{MIN}(m_\ell, p)$
- ▶ Use at most p_ℓ cores until next time m_ℓ is calculated
- ▶ This ensures the $\mathcal{O}(p_\ell)$ work every level is $\mathcal{O}(m_\ell)$
- ▶ W_p reduces from $\mathcal{O}(|\mathbb{V}| + |\mathbb{E}| + Dp)$ to
 $\mathcal{O}(|\mathbb{V}| + |\mathbb{E}| + D) = \mathcal{O}(|\mathbb{V}| + |\mathbb{E}|)$ (optimal)



Conclusion

- ▶ BFS is a hot topic recently
- ▶ Our new TBN-BFS algorithm
 - ▶ $T_1 = \mathcal{O}(|V| + |E|)$ (optimal)
 - ▶ $W_p = \mathcal{O}(|V| + |E|)$ (optimal)
 - ▶ $T_p = \mathcal{O}\left(\frac{|V|+|E|}{p} + D \log p\right)$ (optimal)
 - ▶ Scale-free
 - ▶ Runs on standard hardware implemented using Cilk+
 - ▶ Can do its own scheduling deterministically
 - ▶ Preliminary experiments show merit



Future Work

- ▶ Optimize TBN-BFS for the PRAM model
 - ▶ TBN-BFS runs in same time for PRAM but is not asymptotically optimal
 - ▶ Try using approximate parallel prefix sum [Goldberg and Zwick, 1995]
- ▶ Modify TBN-BFS to remove false sharing
 - ▶ Examine using an $\mathcal{O}(n)$ sort algorithm for levels where $m_\ell = \Omega\left(n^{\frac{1}{c}}\right)$
 - ▶ Try to distribute work by cacheline
- ▶ Examine non level-synchronous approaches
- ▶ Implement and optimize TBN-BFS
- ▶ Run experiments comparing TBN-BFS to existing algorithms

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TBN-BFS - Reduce Search Space

1. $N \leftarrow |OutDegrees|$
2. Each core i
 - 2.1 $FirstDegree \leftarrow OutDegrees[\left\lfloor \frac{iN}{p} \right\rfloor - 1]$
 - 2.2 $FirstDegreeNext \leftarrow OutDegrees[\left\lfloor \frac{(i+1)N}{p} \right\rfloor - 1]$
 - 2.3 $FirstCore \leftarrow \left\lceil \frac{p \ FirstDegree}{m_\ell} \right\rceil$
 - 2.4 $LastCore \leftarrow \left\lceil \frac{p \ FirstDegreeNext}{m_\ell} \right\rceil$
 - 2.5 parallel for $j \leftarrow FirstCore$ to $LastCore$
 - 2.6 $SubList_i \leftarrow i$
3. Using $SubList_i$, core i can search only n_ℓ/p indexes
4. W_p reduces from $\mathcal{O}(|\mathbb{V}| + |\mathbb{E}| + Dp \log p)$ to $\mathcal{O}(|\mathbb{V}| + |\mathbb{E}| + Dp)$