UMONS

I. Introduct° : Historic timeline on physics.

• GR and the theory of connect° in Diff Geom.

– 1854 Riemann's habilitat° dissertat° "On the hypotheses which lie at the foundat° of geometry" introduce not° of manifold, metric (tensor) and curvature.
M $g_{\mu\nu}$ $R^{\mu}{}_{\nu,\sigma\beta}$

– 1869 Christoffel introduit ses symboles $\Gamma^{\mu}_{\nu\rho}$ pr la dérivat° covariante et calcul de la courbure. $\nabla_{\mu} = \partial_{\mu} + \Gamma^{\alpha}_{\mu\beta}$

– 1887→1896 Ricci-Curbastro développe le calcul diff absolu (nommé plus tard calcul tensoriel) Publie avec son élève Levi-Civita le 1er traité complet du sujet (qu'Einstein utilisera comme référence) en 1900.

C'est l'état de l'art qu'Einstein, via Grossman son ami, dû apprendre pr concevoir la RG (1911-15)
E montre son travail à Hilbert et d'autres à Göttingen au début de l'été 1915.
↳ "Every schoolboy in the streets of Göttingen knows more about 4-dim geometry than Einstein. Yet, inspite of that, Einstein did the work not the mathematicians." H.

La th finale publiée en novembre 1915 suscite un vif intérêt aussi chez les mathématiciens. Les géomètres diff notables de la période furent ts +/- inspirés par la RG.

– 1917 Levi-Civita introduit la notion de transport //, afin de simplifier calcul du tenseur de Riemann-Christoffel et éclairer son sens géométrique concret. $[\nabla, \nabla] = R$
↳ cas variété 2D plongé ds $\mathbb{R}^3$ | Realise that cov prop of ∇ and Riem tensor due to trsf prop of Γ not g !

– 1918 Schouten indépendemment propose la m̂ idée, mais intrinsèque et nD.

Incidemment cette not° éclaire et/ou initie la notion de connexion !

L'idée : $X \in T_xM$ et $Y \in T_yM$. La métrique $g_{\mu\nu}$ permet de comparer $|X|$ et $|Y|$ à distance m̂ qd $x \neq y$. Quid de leurs direct° ? Cela fait-il sens sur une variété courbe ?

Besoin d'un chemin γ entre x et y et d'une prescript° pour trspter // $Y \in T_yM$ à T_xM. Alors on peut comparer X et l'image de Y en T_xM ✓

Rappel déf connexion et // trsp in Riem geom.

linear connect° is a map $\nabla : TM \times TM \to TM$.
$$X, Y \longmapsto \nabla_X Y$$

s.t : lin in 1st arg
$$\begin{cases} \nabla_{X+X'} Y = \nabla_X Y + \nabla_{X'} Y \\ \nabla_{\lambda X} Y = \lambda \nabla_X Y \quad \text{for } \lambda \in C^\infty(M). \end{cases}$$

quasi-lin 2nd arg
$$\begin{cases} \nabla_X (Y+Y') = \nabla_X Y + \nabla_X Y' \\ \nabla_X (f Y) = (X \cdot f) Y + f \nabla_X Y \end{cases}$$

Given coord syst $\{x^\mu\}$ on $U \subset M$, natural basis vect $\{\partial_\mu\}$. Christoffel symb def by $\Gamma^\alpha_{\beta\nu} \partial_\alpha = \nabla_{\partial_\beta} \partial_\nu$

$$\nabla_X Y = X^\mu \nabla_{\partial_\mu} (Y^\nu \partial_\nu) = X^\mu \{ \partial_\mu Y^\nu \partial_\nu + Y^\nu \underbrace{\nabla_{\partial_\mu} \partial_\nu}_{\Gamma^\alpha_{\mu\nu} \partial_\alpha} \} = X^\mu \{ \underbrace{\partial_\mu Y^\nu + \Gamma^\nu_{\mu\alpha} Y^\alpha}_{\nabla_\mu Y^\alpha} \} \partial_\nu$$

ex : given another coord syst $\{y^\nu\}$ on $U \subset M$ and Jacobian btwn $\{x^\mu\} \to \{y^\nu\}$: $G^\mu{}_\nu = G$.
$\partial_\mu = G_\mu{}^\nu \partial'_\nu$ $\Rightarrow$ Find trsf law of Γ.

Soit une courbe $\gamma(t)$ ds $U \subset M$, $\gamma(0) = x$ et $\gamma(1) = y$. $\dot\gamma \in T_\gamma M$ et X vect field on M.
La dér cov de X le long de γ : $\nabla_{\dot\gamma} X$.
Say X is // iff $\nabla_{\dot\gamma} X = 0$. $\rightsquigarrow$ C'est une ED 1er ordre : pr init cond $X(0) \in T_x M$
$\exists !$ $X(1) \in T_y M$ $\Rightarrow$ Map $h_\gamma : T_x M \to T_y M$ lin   c'est le trsprt // ✓
$$X(0) \longmapsto h_\gamma(X(0)) = X(1)$$

- 1918-19 Weyl propose "truly infinitesimal geom" : in Riem geom direct° is local matter needs // trsp & connect° to be defined at distance, while norme is absolute, given by g. Suppose it is not ! and change of gauge from point to point.

Essentially introduces conf class of metric $[g_{\mu\nu}] = \{ \lambda g_{\mu\nu} \mid \lambda \in C^\infty(M) \}$
and a linear form : $\phi_\nu dx^\nu$, $\phi_\nu \in C^\infty(M)$ tq : $(g_{\mu\nu} ; \phi_\mu) \to (\lambda g_{\mu\nu} ; \phi_\mu + \lambda^{-1} \partial_\mu \lambda)$
λ is a change of scale, gauge. ϕ_μ is interpreted as the EM potential !

Weyl connect° :
$$\bar\Gamma^\alpha_{\mu\nu} = \Gamma^\alpha_{\mu\nu} + \frac{1}{2} g^{\alpha\lambda} (g_{\mu\lambda} \phi_\nu + g_{\nu\lambda} \phi_\mu - g_{\mu\nu} \phi_\lambda)$$

$$\hat g_{\mu\nu} = e^{\int_{x_0}^{x} \phi_\rho(y) dy^\rho} g_{\mu\nu}(x) \quad (1) \qquad \hat g = g \text{ iff } \phi = df \quad \text{integr phase factor}$$

Einstein's object° : due to (1) rate of clock would depend on path → no stable spectral lines in atoms.

P Flag point : From here 2 paths : one in dvlpmt of DG, another in Physics. In the rest of this chapter we focus on physics.

- QM and the gauge principle.

- 1925 QM is born via Matrix Mechanics of Heisenberg.
- 1926 Schrödinger gives it the form of Wave Mechanics. Dirac hints at equiv btn the 2.
- 1932 Von Neumann gives proper fondat° of QM in term of operators on Hilbert space (he coined the term) showing that MM and WM are indeed equivalent.

In WM, wave fct° is $\mathbb{C}$ and Born rule (1926) says that proba of event $\sim |\psi|^2$

So ψ and $e^{i\alpha}\psi$ have same phys content : physics given by ray in Hilbert space.

↳ Said otherwise physics is (cst) phase invariant.

Aparté : in sept 1925 Goudsmith et Uhlenbeck découvrent le spin de l'$\bar{e}$.
(1922 Stern et Gerlach avaient découvert le spin ac atom argent)
Pauli, at first distrusting the not° of $\bar{e}$ spin, work out the math theory in 1927.
(↳ in spite of being 1st to propose it ??)

- 1928 Dirac propose son équat° Q-relativiste : Schrö Eq non relat, 1 time deriv → for a classical part! So obviously...
gives Klein-Gordon Eq (which Schrö himself thought of) but describes nothing known at the time. 1-pace der down, $\sqrt{\ }$ of KG eq ⇒ Dirac Eq which is indeed relativistic and cov under Lorentz trsf ! → explains spin, predicts antimatter ✓

NB: Introduces spinors in physics !
(Discovered in 1913 by E. Cartan)

Rmq: It is not a quantum eq ! It is a class eq for a class spin ½ field, like Maxwell eq are for class spin 1 field, and Einstein eq are for class spin 2 field.

$$\mathcal{L}_D = \bar{\psi}(i\not{\partial} - m)\psi \quad \text{avec} \quad \not{\partial} = \gamma^\mu \partial_\mu \quad \text{et} \quad \{\gamma_\mu, \gamma_\nu\} = 2\,\mathrm{id}_4\, \eta_{\mu\nu} \; , \quad \bar{\psi} = \psi^* \gamma^0 .$$

$$\frac{\delta \mathcal{L}_D}{\delta \bar{\psi}} = 0 \quad \rightsquigarrow \quad (i\not{\partial} - m)\psi = 0 .$$

$\mathcal{L}_D$ has same phase inv as QM.
Dirac eq has same phase cov as Schrö eq.

↳ Open door for QFT, QED (2nd quantizat°)

- 1929 Weyl's paper : the gauge principle and EM !

Poses and answer 2 questions :

Q1 : How spinorial matter couples to gravity ?

→ Spinors are rep of $SO(1,3) \subseteq SL(2,\mathbb{C})$: ok for special relativity since Lorentz is sym grp of M^4 ✓

$$\left.\begin{array}{l} x \in M^4 \\ S \in SO(1,3) \\ Sx \in M^4 \\ |x|^2 = x^T \eta x \end{array}\right\} \longrightarrow \left\{\begin{array}{l} \bar{x} \in \mathrm{Herm}(2,\mathbb{C}) \\ \bar{S} \in SL(2,\mathbb{C}) \\ \bar{S}^* \bar{x} \bar{S} \in \mathrm{Herm}(2,\mathbb{C}) \\ |x|^2 = \det(\bar{x}) \end{array}\right.$$

ψ spinors trsf as $\psi' = \bar{S}^{-1}\psi$ (like 4-vect trsf as $v' = S^{-1} v$ if basis $e' = S^{-1} v$)
"square root of 4-vect!"

→ In GR spacetime $\mathcal{M} \neq M^4$: spinors in GR ?
Cartan's moving frame / tetrad : locally at $x \in \mathcal{M}$ on $T_x\mathcal{M}$ one defines 4-vect def a Lorentz frame : $e^a_{\ \mu} = \{e_\mu\}^{a=0,\dots,3}$, so that any vect X on $T_x\mathcal{M}$ is given by coord wrt $e^a_{\ \mu}$. C-à-d $e^a_{\ \mu} : T_x\mathcal{M} \longrightarrow \mathbb{R}^4$

$$X \longmapsto \{X^a\} = e^a_{\ \mu} X^\mu \quad (\text{proj } X^\mu \text{ on } e^a_{\ \mu})$$

NB : $e = e^a_{\ \mu}$ not unique, can be rotated by $S \in SO(1,3)$ at each $x \in \mathcal{M}$; $S = S(x)$ (it's a local Lorentz trsf!) $e' = S^{-1} e$ ie $e'^a_{\ \mu}(x) = S^a_{\ b}(x)\, e^b_{\ \mu}(x)$.

In local frame, norm of X given by $\eta \rightsquigarrow |X| = \eta_{ab} X^a X^b$, which is also given by $|X| = g_{\mu\nu} X^\mu X^\nu$ in term of $g_{\mu\nu}$ on $\mathcal{M}$! $\Rightarrow \boxed{g_{\mu\nu} = \eta_{ab}\, e^a_{\ \mu} e^b_{\ \nu}}$

Tetrad is "square root" of metric !

Rmq : $g_{\mu\nu}$ indep of choice of e/e' → Local Lorentz inv ✓

→ I can def spinors on each tetrad. But how do I compare 2 spinors / 2 tetrads at different points in $\mathcal{M}$?

- Tetrad : a tensor is // transported if $\nabla_\mu T^{\mu_1 \dots \mu_r}_{\ \ \nu_1 \dots \nu_s} = 0$.

  ↳ // transp of tetrad $\nabla_\mu (e^{-1})^\nu_{\ a} = \partial_\mu (e^{-1})^\nu_{\ a} + \Gamma^\nu_{\lambda\mu} (e^{-1})^\lambda_{\ a} - \omega^c_{\ a,\mu} (e^{-1})^\nu_{\ c} = 0$.

  $\times\, e^a_{\ \nu}$ left : $\boxed{\omega^a_{\ b\mu} = e^a_{\ \nu} \partial_\mu (e^{-1})^\nu_{\ b} + e^a_{\ \nu} \Gamma^\nu_{\lambda\mu} (e^{-1})^\lambda_{\ b}}$ spin connect° rotate Lorentz indices

ex : Show that $\nabla g = 0 \Rightarrow \omega^T \eta + \eta \omega = 0$ ie $\omega^a_{\ b} \in so(1,3)$!

- Spinor : def wrt $e^a_{\ \mu}$ on $T_x\mathcal{M}$, acted upon by $\bar{S}(x) \in SO(1,3) \subseteq SL(2,\mathbb{C})$ as $\psi' = \bar{S}^{-1}\psi$ if tetrad is rotated by $e' = S^{-1} e$.

  ↳ $\boxed{\nabla_\mu \psi^A = \partial_\mu \psi^A + \bar{\omega}_\mu{}^A{}_B\, \psi^B}$ + trsf of spin connect° : ...

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Rmq: due to spin/ψ, $\exists$ not° of torsion in add to curvature!
See distant parallelism of Einstein ($R=0$, $T\neq 0$) in 20's
and Cartan's "espaces généralisés" (chap 3), Also | Einstein-Cartan th ($R\neq 0$, $T\neq 0$) (1928)
(1922) | Kibble - Sciama (1960)

Q2: Given wave fct° / spinors, why should the phase factor be cst throughout spacetime?
Isn't it | contrary to idea of curved manifold? (Weyl justif and link to GR)
| at odd with spirit of field theory? (Yang-Mills justif 54)

→ Let the factor be local: $\psi \to e^{i\alpha(x)}\psi$, then L_D and Dirac eq not inv!
Since $\not\partial\psi \to \not\partial(e^{i\alpha(x)}\psi) = \not\partial e^{i\alpha(x)}.\psi + e^{i\alpha(x)}\not\partial\psi = i\not\partial\alpha(x)\, e^{i\alpha(x)}\psi + e^{i\alpha(x)}\not\partial\psi$
$= e^{i\alpha(x)}\big(\underbrace{i\not\partial\alpha(x)\psi}_{\circledast} + \not\partial\psi\big).$
If we are to impose | local (gauge!) phase | trsf, need modif L_D.
→ Proposition: $\partial_\mu \to \partial_\mu - iA_\mu = D_\mu$ where $A_\mu(x)$ is a 4-vect field. To absorb $\circledast$
we must require $A_\mu \to A_\mu + \partial_\mu\alpha$ $\Rightarrow$ Looks like EM potential 4-vect!

If so $L'_D = \bar\psi(\not D - m)\psi$ and $D_\mu\psi = \partial_\mu\psi - iA_\mu\psi$ is min coupling of $\psi(e)$ with an ext EM field A_μ.

→ L_D is non dynamical. To give a dynamics def the gauge inv & sym tensor
$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ ~ The Maxwell-Faraday tensor, Maxwell Lagrangian $L_{Maxw} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$
is gauge-inv and Lorentz / coord-inv ✓
Full theory of dyn EM field and charged fields in interaction: $L = L_{Maxw} + L'_D$
↳ $\frac{\delta L}{\delta\psi} = 0 \Rightarrow$ coupled Dirac eq and $\frac{\delta L}{\delta A_\mu} = 0 \Rightarrow$ Maxwell eq sourced by ψ!

ex: Find these eq.

→ Gauge Principle: impose local sym (point dep version of rigid sym) forces apparit° of interact° (EM), turn free (L_D) theory into interact° th (L)!

Terminology: D_μ = cov derivative is s.t $D_\mu\psi$ has same trsf prop as ψ under local phase trsf.

Rmq: The local phase inv is called a gauge sym

$e^{i\alpha} \in U(1)$ then $e^{i\alpha(x)} \in U(1)$ gauge group

it is the sym of L which is then a $U(1)$-gauge theory.

NB: ψ can have a mass term: $m\bar{\psi}\psi$ because it is gauge-inv under $\psi \to e^{i\alpha(x)}\psi$.

A_μ cannot! Due to $A'_\mu = A_\mu + \partial_\mu \alpha$ a term like $m^2 A_\mu A^\mu$ wouldn't be inv.

⇒ gauge principle "produces" interact° field, and predict it will be massless!

So $U(1)$ gauge sym explains why photon (A_μ) exists and why it has 0-mass ✓

Tremendous success conceptually.

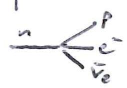
• Yang-Mills Theory. / Yang-Mills-Shaw-Utiyama Theory

→ Nucleus discovered in 1911 by Geiger-Marsden (exp) and Rutherford (th interpr)
know by then that H nucleus = proton (charge $+e$ ~ 1836 times more massive than e^-)
Heavier atoms have nucleus made of many: should ∃ "nuclear force" to hold them together and overcome EM repulsion, hence "strong nuclear force".

→ Chadwick discovers neutron in 1932: neutral part about same mass as proton.

→ Heisenberg, même année, propose that p and n are ≠ quantum states of same basic object; nucleon. Endow new q-number isospin: $\frac{1}{2} = p$, $-\frac{1}{2} = n$.
nucleon is a $\mathbb{C}$-doublet $\begin{pmatrix} p \\ n \end{pmatrix}$ acted upon by $SU(2)$ grp. (like spin for e^-/p in Pauli/Dirac)

→ Around 1936, exp indicates that strong nuclear forces are charge indep: don't care if acting btwn pp, pn, nn. ⇝ $SU(2)$ invariance!

→ Fermi propose in 1934 his theory of "weak nuclear force" that explains β-radioactivity as transmutat° of $n \to p + e^- + \bar{\nu}_e$: 4-point int, no mediating field, .
Why "weak" because energy involved are too small to account for nuclear binding energy, which is therefore explained by a "strong nuclear force".

⇒ 2 new nuclear interact° to describe and explain!

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→ 1936 Yukawa suggest that weak / strong nuclear int is mediated by a field, like EM int. Analogue of the Coulomb potential would be $g^2 \frac{e^{-\lambda r}}{r^2}$

g = cst coupling ~ charge $\bar{e}$ in EM

λ = dim m^{-1}. détermine la portée de l'int et est relié à la masse de la particule associée au chp d'int $(m = \frac{\lambda \hbar}{c})$ ⇒ il était donc clair que les masses des bosons médiateurs des chps de Yukawa devaient être importantes !

→ 1938 Klein (from Klein Gordon eq, Kaluza-Klein th ...) attempt unificat° of Grav, EM and Strong Forces hit upon first hint of SU(2)-gauge sym/th, in a roundabout way (putting, in todays parlance, a connect° in the 5-d metric ...).

1953 Pauli letters to Pais: try describe unified grav/nuclear int by means of a Kaluza-type th. ["Meson-Nucleon Interact° and Diff Geometry"] He identifies strong field with some Christoffel of the 4+2-d spacetime! Never published because he saw he might have pb with masses.

→ If Yukawa is right, then perhaps nuclear int also comes from a gauge structure, like EM. The gauge sym should be more complicated than EM/U(1): 2 states $\mathbb{C}$-funct° $\binom{p}{n}$ grp SU(2) is inv/sym of strong force.

Yang and Mills 1954: this sym is a global one $e^{i\alpha_a \sigma^a}$, α_a = cte, don't fit in the spirit of field th → should be made local $\alpha_a(x)$ like in EM.

Then following Weyl if $\psi^a = \binom{p}{n}$ C = SU(2) matrix, point dep: $C = C(x)$ → $\psi'^a = C^a_b \psi^b$

one introduces $A^a_{b\mu} \left(= A^{\alpha}_{\mu}(x)\, \sigma_\alpha\right)$ so that cov der is: $D_\mu \psi^a = \partial_\mu \psi^a - \varepsilon A^a_{b\mu} \psi^b$

and $A'^a_{b\mu} = C^{-1a}_{\ c} A^c_{d\mu} C^d_b + C^{-1a}_{\ c} \partial_\mu C^c_b$ ensures $D'_\mu \psi'^a = C^{-1a}_{\ b} D_\mu \psi^b$.

YM searched analogue of $F_{\mu\nu}$, found: $F^a_{b\mu\nu} = \partial_\mu A^a_{b\nu} - \partial_\nu A^a_{b\mu} \underbrace{+ \varepsilon A^a_{c\mu} A^c_{b\nu} - \varepsilon A^a_{c\nu} A^c_{b\mu}}$

↳ Means that strong field $A^a_{b\mu}$ is charged and self-interact, contrary to photon A_μ which do not carry $\bar{e}$ charge!

⇒ YM Eq ~ analogue of Maxwell's Eq: $\begin{cases} \partial^\mu F_{\mu\nu} = J_\nu \\ \partial_{[\mu} F_{\nu\rho]} = 0 \end{cases}$ ⟶ $\begin{cases} D^\mu F^a_{b\mu\nu} = J^a_{b\nu} & (1) \\ D_{[\mu} F^a_{b\nu\rho]} = 0 & (2) \end{cases}$

(2) is "cinematic", comes from def of $F^a_{b\mu\nu}$ (as in EM)

(1) comes from YM Lagrangian: $L_{YM}(A) = -\frac{1}{4} F^a_{b\mu\nu} F^{b\ \mu\nu}_{a}$. $\frac{\delta L_{YM}}{\delta A^a_{b\mu}} = 0$ ⇒ (1).

→ 1955 same theory by R. Shaw (at the time a PhD student ~~to~~ under Salam.
inspired by notes of Schwinger on EM rewritten as SO(2) theory
↳ in a flash saw that going from SO(2) to SU(2) should be easy!

→ 1955 also, most comprehensive work by Ryoyu Utiyama: show not only how to treat SU(2) gauge th, but how to "gauge" (ie make local) any Lie grp in a field theory!
Explicitely intended to produce a general framework for non-abelian gauge theories!
↳ showed how EM, YM and GR can be derived from gauge principle applied to U(1), SU(2) and SO(1,3)!

NB: • YM, Shaw and Utiyama work indep and within a few month from one another.
Yang considered the SU(2) th since 1949, Mills was finishing his PhD when collab with Yang began in summer 53. Yang presented | published the theory in February | April 54 | 54 (Pauli was a jerk, again)
• Shaw did complete his work on SU(2) gauge field in January 54, published his thesis in Sept 55.
Salam did promote him referring to "Yang-Mills-Shaw theory" in his Nobel lecture | talk (see below)
• Utiyama found his results in ~~[illegible]~~ March 54, presented them in Kyoto in May June 54
Sept 54 at Princeton, told that Yang had the theory. Only in March 55 did he realized that his scheme was much more general! So he published in 1956 after acceptat° in July 55.

Terminology: general framework of gauge theories & gauge principle usually goes by the name YM theory. It is unjust. Should be "Yang-Mills-Shaw-Utiyama", if not "Utiyama theory | framework". Sometimes YM theories stands for "Non-Abelian gauge theory" to distinguish it from abelian/U(1)/Maxwell gauge theory. ~~[illegible]~~ But this do not account for the fact that gravity is also "non-abelian".
↳ Gravitat° still is ≠ from class of internal non-ab gauge th ⇒ see Chap 3.

⇒ Framework of gauge th didn't catch at first: nuclear forces short range ⇝ massive fields.
but gauge sym ⇒ massless fields...
It thus wasn't obvious that it is the correct way to describe/understand nuclear interact°!

- The EW-unificat° and SSB

From solid state / condensed matter physics have notion of SSB : a theory can have larger sym than its solut°.

Ex : phase transit° ; basic th is SO(3) inv but as T°/E decrease SO(3) broken down to discrete cristallographic groups (subgroups) ✓

Not° also used in Ferromagnetism and superconductivity theories.

→ 1960-61, Nambu apply the idea to field theory. But SSB of rigid grp produces bosonic massless fields.

→ 1961, Goldstone proves that it must be so : SSB rigid grp produces Goldstone bosons.

→ 1963, Anderson hint at the possibility that coupling to long range field might kill Goldstone modes

→ August 1964 Brout - Englert; October 1964 Higgs; November 1964 Guralnik - Hagen - Kibble : Show that if a scalar field is coupled to a gauge field SSB avoid Goldstone bosons but give mass to g.

In U(1)-gauge th : $L(A_\mu) = -\frac{1}{4} F_{\mu\nu}F^{\mu\nu}$, A_μ have no mass term because gauge sym.

$m^2 A_\mu A^\mu$ not U(1)-inv since $A'_\mu = A_\mu + \partial_\mu \alpha$.

In a U(1)-SSB th : One introduces a $\mathbb{C}$-scalar field coupled minimally to A_μ and embedded into a potential.

$$L(A_\mu, \phi) = -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} + D_\mu \bar{\phi} D^\mu \phi - V(\phi) \quad \text{with} \quad \begin{cases} D_\mu \phi = \partial_\mu \phi + A_\mu \phi \\ V(\phi) = \mu^2 \phi^2 + \lambda \phi^4 , \ \lambda > 0 \end{cases}$$

↳ The vacuum of the th is given by $V'(\phi) = 0$, ie the min of potential of ϕ : $V'(\phi) = 2\phi(\mu^2 + 2\lambda \phi^2) = 0$ donc $\phi_0 = 0$ (unique) ou $|\phi_0| = \sqrt{\frac{-\mu^2}{2\lambda}}$ (multiple : all ϕ_0's s.t $|\phi_0| = \sqrt{\ldots}$)

Theory has 2 phases :

- in $\mu^2 > 0$: vacuum is $\phi_0 = 0$, perturbat° around vacuum. $\phi = \phi_0 + H$

↳ $\begin{cases} D_\mu \phi = D_\mu(\phi_0 + H) = \partial_\mu H + A_\mu H \\ V(\phi) = V(H) = \mu^2 H^2 + \lambda H^4 \end{cases}$ ⇒ Theory : $L(A_\mu, H) = -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} + \partial_\mu \bar{H} \partial^\mu H - \mu^2 H^2 + A_\mu \bar{H} A^\mu H - \lambda H^4$

Coupling of massless A_μ with massive H.

- in $\mu^2 < 0$: vacuum is $\phi_0 \neq 0$ donc $\begin{cases} D_\mu \phi = \partial_\mu H + A_\mu H + A_\mu \phi_0 \\ V(\phi) = \mu^2 \phi_0^2 + \mu^2 \phi_0 H + \mu^2 H^2 + \ldots \end{cases}$

Spont Choose one breaks U(1) sym!

⇒ the theory contains term $\phi_0^2 A_\mu A^\mu$: mass for A_μ given by vacuum expectat° value of ϕ!

→ 1967, Weinberg (indep Salam 1968) applied the gauge SSB mechanism to a model of unificat° of EM and Weak Interact° based on the gauge grp $U(1) \times SU(2)$. The gauge potentials couple to a $\mathbb{C}^2$-scalar field with potential. $SU(2)$ is spont broken giving masses to 3 of the 4 gauge fields, those responsible for weak interact°. The unbroken $U(1)$ has gauge field resp for EM.

↳ Predict neutral weak int Z^0 in additi° to charged ones, $W^{\pm}$, and precise relat° btwn those masses!

"A model of leptons" november 67 : 2nd most cited paper / 1st for decades (only deserving in my view)

- Neutral weak current discovered in 1973
- Weinberg-Salam-Glashow awarded Nobel prize 1979.
- January 1983 discovery of $W^{\pm}$ bosons at CERN
- May 1983 ——— Z^0 ———
- July 2012 ——— H / scalar boson at LHC.
- 2013 Englert-Higgs awarded Nobel prize.

- 1971 t'Hooft | Veltmann prove renormalizability
- 1999 both awarded Nobel prize.

⇒ The EW unif theory is a tremendous succes for gauge th and part of the Standard Model of particle physics.

Weak int elucidated, understood why it can be short range despite being gauge interact° thanks to noti° of SSB. Is it same strategy for strong int ??

- <u>QCD and Quarks.</u>

Go back to the time of YM-Shaw-Utiyama.

Zoo of strongly int part found thks to new tech devlpmt in prod and detect° of particles.

→ 1955, Gell-Mann - Pais - Nishijima introduce new q-number, conserved in strong and EM int but not in weak int : strangeness.

→ 1962, names "Mesons and Baryons" adopted for strongly int particles (SIP)

→ 1964, Gell-Mann and Zweig propose mesons and baryons are made up of q and $\bar{q}$ with fractional e^- charges and spin ½.

	Q	Str	Isos	Bar number
u	2/3	0	½	1/3
d	-1/3	0	½	1/3
s	-1/3	-1	0	1/3

Mesons = $q\bar{q}$

Baryons = qqq.

⇒ "Eight fold way" allowing classificat° of SIP via $SU(3)_F$ [Flavor $SU(3)$]

→ 1965, the "color" d.o.f is introduced by Nambu et Han
to explain how wave funct° of baryon can be antisym (fermionic)
Each q can have 3 colors, one impose $SU(3)_c$ gauge sym !

↳ 8 gauges fields = gluons, responsible for strong interact° (one assumes).

↳ Again not immediately seen as a breakthrough: gluons are massless but strong int is short range. So?

→ 1973, Gross and Wilczek, and Politzer indep showed $SU(3)_c$ is asymptotically free.
ie strength of coupling decrease as distance decrease btw q.
Explains deep inelastic scattering of e^- on baryon, and adequacy of Feynman's partons model !

→ 1973-74 see the not° of confinement: coupling increase with distance so that
q and g-bound states are color singlets and q/g never free !

↳ Explains why strong int is short range despite gluons being massless ✓

↳ Birth of QCD, Largely confirmed by jet physics in accelerators in late 70's and 80's.

Thus was constituted the Standard Model of particle physics, a quantum gauge field theory based on the gauge groups $U(1) \times SU(2) \times SU(3)$ and describe all known particles and their int via EM, weak and strong int.

Not unified with gravity, which also has a gauge struct based on local $SO(1,3)$.

All attempts to go beyond SM and or GR are within the framework of gauge theory.
Ex: $SU(5)$, GUT, HS, LQG, Strings/M th, SME ...

The rest of this course is devoted to introducing the geometric picture behind gauge theory.