```
> with(Physics):with(DifferentialGeometry):with(plots):with
  (PDEtools):with(Tensor):
```

> #The sign convention for the Ricci tensor of the DifferentialGeometry and Physics packages is the same followed in MTW.

>

> #Manifold definition:

```
> DGsetup([t, r, u, v], M, verbose)
```

The following coordinates have been protected:

$$[t, r, u, v]$$

The following vector fields have been defined and protected:

$$[\ \text{`*`}(D\_t),\ \text{`*`}(D\_r),\ \text{`*`}(D\_u),\ \text{`*`}(D\_v)\ ]$$

The following differential 1-forms have been defined and protected:

$$[\ \text{`*`}(dt),\ \text{`*`}(dr),\ \text{`*`}(du),\ \text{`*`}(dv)\ ]$$

frame name: M **(1)**

```
M > g1 := evalDG( -f(r)*dt &t dt +  dr &t dr +  du &t du + dv &t
    dv)
```

$$g1 := -\left(\left(f(r)\ dt\right)\ dt\right) + \left(\ \text{`*`}(dr)\ \right)\ dr + \left(\ \text{`*`}(du)\ \right)\ du + \left(\ \text{`*`}(dv)\ \right)\ dv \qquad \textbf{(2)}$$

```
M > g1i:= InverseMetric(g1)
```

$$g1i := -\left(\left(\frac{1}{f(r)}\ D\_t\right)\ D\_t\right) + \left(\ \text{`*`}(D\_r)\ \right)\ D\_r + \left(\ \text{`*`}(D\_u)\ \right)\ D\_u + \left(\ \text{`*`}(D\_v)\ \right)\ D\_v \qquad \textbf{(3)}$$

M > #Christoffel symbols of the second kind:

```
M > C1 := Christoffel(g1, "SecondKind")
```

$$C1 := \left(\left(\frac{\frac{\mathrm{d}}{\mathrm{d}r}\ f(r)}{2\ f(r)}\ D\_t\right)\ dt\right)\ dr + \left(\left(\frac{\frac{\mathrm{d}}{\mathrm{d}r}\ f(r)}{2\ f(r)}\ D\_t\right)\ dr\right)\ dt + \left(\left(\frac{\left(\frac{\mathrm{d}}{\mathrm{d}r}\ f(r)\right)}{2}\ D\_r\right)\ dt\right)\ dt \qquad \textbf{(4)}$$

M > #Ricci tensor:

```
M > R1:=RicciTensor(C1)
```

$$R1 := \left(\left(-\frac{\left(\frac{\mathrm{d}}{\mathrm{d}r}\ f(r)\right)^2}{4\ f(r)} + \frac{\left(\frac{\mathrm{d}^2}{\mathrm{d}r^2}\ f(r)\right)}{2}\right)\ dt\right)\ dt - \left(\left(\frac{2\left(\frac{\mathrm{d}^2}{\mathrm{d}r^2}\ f(r)\right)\ f(r) - \left(\frac{\mathrm{d}}{\mathrm{d}r}\ f(r)\right)^2}{4\ f(r)^2}\ dr\right)\ dr\right) \qquad \textbf{(5)}$$

M > #4−potential vector ansatzs:

```
M > xi(r):=exp(I*k*(omega*int(sqrt((1)/(f(r))),r)))
```

(6)

$$\xi(r) := \mathrm{e}^{\mathrm{I}\,k\,\omega\left(\int \sqrt{\frac{1}{f(r)}}\,\mathrm{d}r\right)} \quad \textbf{(6)}$$

```
M > xi2(t):=exp(-I*(omega*t))
```

$$\xi 2(t) := \mathrm{e}^{-\mathrm{I}\,\omega\,t} \quad \textbf{(7)}$$

```
M > xi3(u,v):=exp(I*(k_u*u + k_v*v))
```

$$\xi 3(u, v) := \mathrm{e}^{\mathrm{I}\,(k\_u\,u + k\_v\,v)} \quad \textbf{(8)}$$

```
M > xi5(u,v):=c*cos(k_u*u)*sin(k_v*v)
```

$$\xi 5(u, v) := c\cos(k\_u\,u)\,\sin(k\_v\,v) \quad \textbf{(9)}$$

M > *#Complete case:*

```
M > A1 := DGzip(([At, Ar, Au, Av])(t, r, u, v), [D_t, D_r, D_u,
    D_v], "plus"):
```

M > *#Stationary propagating wave, temporal gauge:*

```
M > A2:= DGzip(([0*Bt(u,v)*xi(t,r), 0*Br(u,v)*xi(t,r), Bu(u,v)*xi
    (t,r), Bv(u,v)*xi(t,r)]), [D_t, D_r, D_u, D_v], "plus"):
```

M > *#Stationary propagating wave, , variable amplitude, temporal gauge:*

```
M > A3:= DGzip(([0,Cr(r,u,v)*xi(t,r), Cu(r,u,v)*xi(t,r), Cv(r,u,
    v)*xi(t,r)]), [D_t, D_r, D_u, D_v], "plus"):
```

M > *#Stationary propagating wave, transversally flat, temporal gauge:*

```
M > A4:= DGzip(([0,0*Cr(r)*xi(t,r), A4u(u,v)*Cu(r)*xi(t,r), A4v
    (u,v)*Cv(r)*xi(t,r)]), [D_t, D_r, D_u, D_v], "plus"):
```

M > *#Stationary wave, temporal gauge, separated variables:*

```
M > A5:= DGzip(([0,Cr(r,u,v)*xi2(t), Tu(u,v)*Lu(r)*xi2(t), Tv(u,
    v)*Lv(r)*xi2(t)]), [D_t, D_r, D_u, D_v], "plus"):
```

M > *#Stationary wave, temporal gauge, separated variables, transversal plane wave:*

```
M > A6:= DGzip(([0,0*Cr(r,u,v)*xi2(t), xi2(t)*xi3(u,v)*L(r), xi2
    (t)*xi3(u,v)*L(r)]), [D_t, D_r, D_u, D_v], "plus")
```

$$A6 := \mathrm{e}^{-\mathrm{I}\,\omega\,t}\,\mathrm{e}^{\mathrm{I}\,(k\_u\,u + k\_v\,v)}\,L(r)\,D\_u + \mathrm{e}^{-\mathrm{I}\,\omega\,t}\,\mathrm{e}^{\mathrm{I}\,(k\_u\,u + k\_v\,v)}\,L(r)\,D\_v \quad \textbf{(10)}$$

```
M > xi4(r):=(1/(f(r))^(a))
```

$$\xi 4(r) := \frac{1}{f(r)^{a}} \quad \textbf{(11)}$$

M > *#Ansatz*

```
M > A7:= DGzip(([0,0*Cr(r,u,v)*xi2(t), xi2(t)*xi5(u,v)*xi(r), xi2
    (t)*xi5(u,v)*xi(r)]), [D_t, D_r, D_u, D_v], "plus")
```

$$A7 := \mathrm{e}^{-\mathrm{I}\,\omega\,t}\,c\cos(k\_u\,u)\,\sin(k\_v\,v)\,\mathrm{e}^{\mathrm{I}\,k\,\omega\left(\int \sqrt{\frac{1}{f(r)}}\,\mathrm{d}r\right)}\,D\_u + \mathrm{e}^{-\mathrm{I}\,\omega\,t}\,c\cos(k\_u\,u)\,\sin(k\_v\,v)\,\mathrm{e}^{\mathrm{I}\,k\,\omega\left(\int \sqrt{\frac{1}{f(r)}}\,\mathrm{d}r\right)}\,D\_v \quad \textbf{(12)}$$

```
> 

> #Covariant derivative of the 4-potential:
> Dc:=CovariantDerivative(A6, C1):

> #Lorentz condition, i.e. contraction of the indices of the covariant derivative
      of the 4-potential:
> Lorentz:=ContractIndices(Dc, [[1,2]])
```

$$Lorentz := \mathrm{I}\,\mathrm{e}^{\mathrm{I}\,(k\_u\,u + k\_v\,v - \omega\,t)}\,L(r)\,(k\_u + k\_v) \tag{13}$$

```
> 
> #The second covariant derivative of the 4-potential:
> Dc2:=CovariantDerivative(Dc, C1):
> 
> 
> #Contraction of the indices of the covariant derivatives to get the Laplacian:
> Lap:=ContractIndices(g1i, Dc2, [[1, 2], [2,3]]):
> 
> 
> ####"Mass term" of the Maxwell equations, i.e, contraction of the Ricci tensor
      with the 4-potential:
> #Rising one index of the Ricci tensor:
> R1up:=ContractIndices(g1i, R1, [[1, 1]]):
> 
> #Contraction of the Ricci tensor with the four-potential to give the mass:
> Mass:=ContractIndices(A6, R1up, [[1,2]])
```

$$Mass := 0\,D\_t \tag{14}$$

```
> 
> 
> 
> #Maxwell equations!!!!!!!!!!!!!!!!:
> Me:= simplify(Lap &minus Mass)
```

$$Me := \frac{1}{2\,f(r)}\left(\mathrm{e}^{\mathrm{I}\,(k\_u\,u + k\_v\,v - \omega\,t)}\left(-2\,k\_u^2\,L(r)\,f(r) - 2\,k\_v^2\,L(r)\,f(r) + 2\,\omega^2\,L(r) + 2\left(\frac{\mathrm{d}^2}{\mathrm{d}r^2}\,L(r)\right)f(r) + \left(\frac{\mathrm{d}}{\mathrm{d}r}\,f(r)\right)\left(\frac{\mathrm{d}}{\mathrm{d}r}\,L(r)\right)\right)\right)D\_u + \frac{1}{2\,f(r)}\left(\mathrm{e}^{\mathrm{I}\,(k\_u\,u + k\_v\,v - \omega\,t)}\left(-2\,k\_u^2\,L(r)\,f(r) - 2\,k\_v^2\,L(r)\,f(r) + 2\,\omega^2\,L(r) + 2\left(\frac{\mathrm{d}^2}{\mathrm{d}r^2}\,L(r)\right)f(r) + \left(\frac{\mathrm{d}}{\mathrm{d}r}\,f(r)\right)\left(\frac{\mathrm{d}}{\mathrm{d}r}\,L(r)\right)\right)\right)D\_v \tag{15}$$

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