> **with(Physics):with(DifferentialGeometry):with(plots):with(PDEtools):with(Tensor):**

> #*The sign convention for the Ricci tensor of the DifferentialGeometry and Physics packages is the same followed in MTW.*

>

> #*Manifold definition:*

> **DGsetup([t, r, u, v], M, verbose)**

The following coordinates have been protected:

$$[t, r, u, v]$$

The following vector fields have been defined and protected:

$$[`*`(D\_t), `*`(D\_r), `*`(D\_u), `*`(D\_v)]$$

The following differential 1-forms have been defined and protected:

$$[`*`(dt), `*`(dr), `*`(du), `*`(dv)]$$

frame name: M (1)

M >

M > **g1 := evalDG(dt &t dt + (1/f(r))*((1 - u*chi*cos(psi + v))^2)*dr &t dr + (1/f(r))* du &t du + (1/f(r))*(u^2)*dv &t dv)**

$$g1 := (`*`(dt)) \, dt + \left(\frac{(u\chi\cos(\psi+v)-1)^2}{f(r)} \, dr \right) dr + \left(\frac{1}{f(r)} \, du \right) du \qquad (2)$$

$$+ \left(\frac{u^2}{f(r)} \, dv \right) dv$$

M > **g1i:= InverseMetric(g1)**

$$g1i := (`*`(D\_t)) \, D\_t + \left(\frac{f(r)}{\cos(\psi+v)^2 \chi^2 u^2 - 2u\chi\cos(\psi+v)+1} \, D\_r \right) D\_r \qquad (3)$$

$$+ (f(r) \, D\_u) \, D\_u + \left(\frac{f(r)}{u^2} \, D\_v \right) D\_v$$

M > #*Christoffel symbols of the second kind:*

M > **C1 := Christoffel(g1, "SecondKind")**

$$C1 := - \left(\left(\left(\frac{\frac{d}{dr} f(r)}{2 f(r)} \, D\_r \right) dr \right) dr \right) + \left(\left(\frac{\chi\cos(\psi+v)}{u\chi\cos(\psi+v)-1} \, D\_r \right) dr \right) du \qquad (4)$$

$$- \left(\left(\left(\frac{u\chi\sin(\psi+v)}{u\chi\cos(\psi+v)-1} \, D\_r \right) dr \right) dv \right) + \left(\left(\frac{\chi\cos(\psi+v)}{u\chi\cos(\psi+v)-1} \, D\_r \right) du \right) dr$$

$$+ \left(\left(\frac{\frac{d}{dr} f(r)}{2 f(r) \left(\cos(\psi+v)^2 \chi^2 u^2 - 2u\chi\cos(\psi+v)+1 \right)} \, D\_r \right) du \right) du$$

$$- \left(\left(\left(\frac{u\chi\sin(\psi+v)}{u\chi\cos(\psi+v)-1} \, D\_r \right) dv \right) dr \right)$$

$$+\left(\left(\frac{u^2\left(\frac{d}{dr}f(r)\right)}{2\,f(r)\,(\cos(\psi+v)^2\,\chi^2\,u^2-2\,u\chi\cos(\psi+v)+1)}\,D\_r\right)dv\right)dv$$

$$-\left(\left(\left((u\chi\cos(\psi+v)-1)\,\chi\cos(\psi+v)\,D\_u\right)dr\right)dr\right)$$

$$-\left(\left(\left(\frac{\frac{d}{dr}f(r)}{2\,f(r)}\,D\_u\right)dr\right)du\right)-\left(\left(\left(\frac{\frac{d}{dr}f(r)}{2\,f(r)}\,D\_u\right)du\right)dr\right)$$

$$-\left(\left((u\,D\_u)\,dv\right)dv\right)+\left(\left(\frac{(u\chi\cos(\psi+v)-1)\,\chi\sin(\psi+v)}{u}\,D\_v\right)dr\right)dr$$

$$-\left(\left(\left(\frac{\frac{d}{dr}f(r)}{2\,f(r)}\,D\_v\right)dr\right)dv\right)+\left(\left(\frac{1}{u}\,D\_v\right)du\right)dv$$

$$-\left(\left(\left(\frac{\frac{d}{dr}f(r)}{2\,f(r)}\,D\_v\right)dv\right)dr\right)+\left(\left(\frac{1}{u}\,D\_v\right)dv\right)du$$

M > *#Ricci tensor:*

M > R1:=RicciTensor(C1)

$$R1:=-\left(\left(\frac{\left(\frac{d}{dr}f(r)\right)^2-\left(\frac{d^2}{dr^2}f(r)\right)f(r)}{f(r)^2}\,dr\right)dr\right)\qquad\qquad\textbf{(5)}$$

$$-\left(\left(\frac{\left(\frac{d}{dr}f(r)\right)\chi\cos(\psi+v)}{2\,f(r)\,(u\chi\cos(\psi+v)-1)}\,dr\right)du\right)$$

$$+\left(\frac{\left(\frac{d}{dr}f(r)\right)u\chi\sin(\psi+v)}{2\,f(r)\,(u\chi\cos(\psi+v)-1)}\,dr\right)dv$$

$$-\left(\left(\frac{\left(\frac{d}{dr}f(r)\right)\chi\cos(\psi+v)}{2\,f(r)\,(u\chi\cos(\psi+v)-1)}\,du\right)dr\right)$$

$$-\left(\left(\frac{3\left(\frac{d}{dr}f(r)\right)^2-2\left(\frac{d^2}{dr^2}f(r)\right)f(r)}{4\,f(r)^2\,(\cos(\psi+v)^2\,\chi^2\,u^2-2\,u\chi\cos(\psi+v)+1)}\,du\right)du\right)$$

$$+\left(\frac{\left(\frac{d}{dr}f(r)\right)u\chi\sin(\psi+v)}{2\,f(r)\,(u\chi\cos(\psi+v)-1)}\,dv\right)dr$$

$$-\left(\left(\frac{u^2\left(3\left(\frac{d}{dr}f(r)\right)^2-2\left(\frac{d^2}{dr^2}f(r)\right)f(r)\right)}{4\,f(r)^2\,(\cos(\psi+v)^2\,\chi^2\,u^2-2\,u\chi\cos(\psi+v)+1)}\,dv\right)dv\right)$$

M >
M >
M > #4-potential vector:
M > A := DGzip(([At, Ar, Au, Av])(t, r, u, v), [D_t, D_r, D_u, D_v], "plus")

$$A := At(t, r, u, v)\, D\_t + Ar(t, r, u, v)\, D\_r + Au(t, r, u, v)\, D\_u + Av(t, r, u, v)\, D\_v \qquad \textbf{(6)}$$

M > #Covariant derivative of the 4-potential:
M > Dc:=CovariantDerivative(A, C1):
M > #The second covariant derivative of the 4-potential:
M > Dc2:=CovariantDerivative(Dc, C1):
M > #Contraction of the indices of the covariant derivatives to get the Laplacian:
M > L:=ContractIndices(g1i, Dc2, [[1, 2], [2,3]]):
M >
M > ####"Mass term" of the Maxwell equations, i.e, contraction of the Ricci tensor with the 4-potential:
M > #Rising one index of the Ricci tensor:
M > R1up:=ContractIndices(g1i, R1, [[1, 1]]):
M > #Contraction of the Ricci tensor with the four-potential:
M > Mass:=ContractIndices(A, R1up, [[1,2]]):

M > #Maxwell equations!!!!!!!!!!!!!!!!!:
M > Me:= L &minus Mass

$$
\begin{aligned}
Me := &\left(2\left(\frac{\partial^2}{\partial u^2} At(t, r, u, v)\right) f(r)\cos(\psi+v)^2\chi^2 u^4 + 4\left(\frac{\partial}{\partial u} At(t, r, u, v)\right) f(r)\cos(\psi+v)^2\chi^2 u^3 + 2\left(\frac{\partial^2}{\partial t^2} At(t, r, u, v)\right)\cos(\psi+v)^2\chi^2 u^4 \right. \\
&+ 2 f(r)\cos(\psi+v)^2\left(\frac{\partial^2}{\partial v^2} At(t, r, u, v)\right)\chi^2 u^2 - 2\left(\frac{\partial}{\partial v} At(t, r, u, v)\right) f(r)\cos(\psi+v)\sin(\psi+v)\chi^2 u^2 - 4\left(\frac{\partial^2}{\partial u^2} At(t, r, u, v)\right) f(r)\cos(\psi+v)\chi u^3 \\
&- 6\left(\frac{\partial}{\partial u} At(t, r, u, v)\right) f(r)\cos(\psi+v)\chi u^2 - 4\left(\frac{\partial^2}{\partial t^2} At(t, r, u, v)\right)\cos(\psi+v)\chi u^3 - 4 f(r)\cos(\psi+v)\left(\frac{\partial^2}{\partial v^2} At(t, r, u, v)\right)\chi u + 2\left(\frac{\partial}{\partial v} At(t, r, u, v)\right) f(r)\sin(\psi+v)\chi u \\
&- u^2\left(\frac{d}{dr} f(r)\right)\left(\frac{\partial}{\partial r} At(t, r, u, v)\right) + 2\left(\frac{\partial^2}{\partial u^2} At(t, r, u, v)\right) f(r) u^2 + 2\left(\frac{\partial^2}{\partial r^2} At(t, r, u, v)\right) f(r) u^2 + 2 u\left(\frac{\partial}{\partial u} At(t, r, u, v)\right) f(r) \\
&\left. + 2\left(\frac{\partial^2}{\partial t^2} At(t, r, u, v)\right) u^2 + 2\left(\frac{\partial^2}{\partial v^2} At(t, r, u, v)\right) f(r)\right) \Big/
\end{aligned}
\qquad \textbf{(7)}
$$

$$\left(2\,u^2\left(\cos(\psi+v)^2\,\chi^2\,u^2-2\,u\,\chi\,\cos(\psi+v)+1\right)\right)D\_t-\left[\left(2\,f(r)^2\left(\frac{\partial}{\partial u}\right.\right.\right.$$

$$\left.Ar(t,r,u,v)\right)u-3\left(\frac{d}{dr}f(r)\right)\left(\frac{\partial}{\partial r}Ar(t,r,u,v)\right)f(r)\,u^2+2\left(\frac{\partial}{\partial u}Au(t,r,u,\right.$$

$$\left.v)\right)\left(\frac{d}{dr}f(r)\right)f(r)\,u^2+6\left(\frac{\partial}{\partial v}Ar(t,r,u,v)\right)f(r)^2\cos(\psi+v)^2\sin(\psi$$

$$+v)\,\chi^3\,u^3-12\left(\frac{\partial}{\partial v}Ar(t,r,u,v)\right)f(r)^2\cos(\psi+v)\sin(\psi+v)\,\chi^2\,u^2-2\left(\frac{d}{dr}\right.$$

$$\left.f(r)\right)f(r)\cos(\psi+v)\left(\frac{\partial}{\partial v}Av(t,r,u,v)\right)\chi\,u^3-2\left(\frac{d}{dr}f(r)\right)f(r)\,Av(t,r,u,$$

$$v)\sin(\psi+v)\,\chi\,u^3-3\left(\frac{d}{dr}f(r)\right)^2\cos(\psi+v)\,Ar(t,r,u,v)\,\chi\,u^3+6\left(\frac{\partial}{\partial v}Ar(t,\right.$$

$$\left.r,u,v)\right)f(r)^2\sin(\psi+v)\,\chi\,u+3\left(\frac{d^2}{dr^2}f(r)\right)f(r)\cos(\psi+v)\,Ar(t,r,u,v)\,\chi\,u^3$$

$$-3\left(\frac{d^2}{dr^2}f(r)\right)f(r)\,Ar(t,r,u,v)\,u^2+2\left(\frac{\partial^2}{\partial u^2}Ar(t,r,u,v)\right)f(r)^2\,u^2$$

$$+2\left(\frac{\partial^2}{\partial r^2}Ar(t,r,u,v)\right)f(r)^2\,u^2+2\left(\frac{\partial^2}{\partial t^2}Ar(t,r,u,v)\right)u^2\,f(r)+2\left(\frac{d}{dr}\right.$$

$$\left.f(r)\right)f(r)\left(\frac{\partial}{\partial v}Av(t,r,u,v)\right)u^2+2\,Au(t,r,u,v)\left(\frac{d}{dr}f(r)\right)f(r)\,u$$

$$+2\,f(r)^2\left(\frac{\partial^2}{\partial v^2}Ar(t,r,u,v)\right)+3\left(\frac{d}{dr}f(r)\right)\left(\frac{\partial}{\partial r}Ar(t,r,u,v)\right)f(r)\cos(\psi$$

$$+v)\,\chi\,u^3-2\left(\frac{\partial}{\partial u}Au(t,r,u,v)\right)\left(\frac{d}{dr}f(r)\right)f(r)\cos(\psi+v)\,\chi\,u^3+3\left(\frac{d}{dr}\right.$$

$$\left.f(r)\right)^2 Ar(t,r,u,v)\,u^2+6\left(\frac{\partial^2}{\partial u^2}Ar(t,r,u,v)\right)f(r)^2\cos(\psi+v)^2\,\chi^2\,u^4$$

$$-6\left(\frac{\partial^2}{\partial u^2}Ar(t,r,u,v)\right)f(r)^2\cos(\psi+v)\,\chi\,u^3-2\left(\frac{\partial^2}{\partial r^2}Ar(t,r,u,\right.$$

$$\left.v)\right)f(r)^2\cos(\psi+v)\,\chi\,u^3-2\left(\frac{\partial^2}{\partial t^2}Ar(t,r,u,v)\right)f(r)\cos(\psi+v)^3\,\chi^3\,u^5$$

$$+6\left(\frac{\partial^2}{\partial t^2}Ar(t,r,u,v)\right)f(r)\cos(\psi+v)^2\,\chi^2\,u^4-6\left(\frac{\partial^2}{\partial t^2}Ar(t,r,u,\right.$$

$$\left.v)\right)f(r)\cos(\psi+v)\,\chi\,u^3-8\,f(r)^2\cos(\psi+v)^3\left(\frac{\partial}{\partial u}Ar(t,r,u,v)\right)\chi^3\,u^4$$

$$+18\,f(r)^2\cos(\psi+v)^2\left(\frac{\partial}{\partial u}Ar(t,r,u,v)\right)\chi^2\,u^3-12\,f(r)^2\cos(\psi+v)\left(\frac{\partial}{\partial u}\right.$$

$$Ar(t, r, u, v) \Big) \chi\, u^2 + 4\, f(r)^2 \left(\frac{\partial}{\partial r}\, Av(t, r, u, v) \right) \sin(\psi + v)\, \chi\, u^3 - 4 \left(\frac{\partial}{\partial r}\, Au(t,\right.$$

$$\left. r, u, v) \right) f(r)^2 \cos(\psi + v)\, \chi\, u^2 - 2\, f(r)^2 \cos(\psi + v)^3 \left(\frac{\partial^2}{\partial v^2}\, Ar(t, r, u, v) \right) \chi^3\, u^3$$

$$+ 6\, f(r)^2 \cos(\psi + v)^2 \left(\frac{\partial^2}{\partial v^2}\, Ar(t, r, u, v) \right) \chi^2\, u^2 - 6\, f(r)^2 \cos(\psi + v) \left(\frac{\partial^2}{\partial v^2}\right.$$

$$\left. Ar(t, r, u, v) \right) \chi\, u - 2 \left(\frac{\partial^2}{\partial u^2}\, Ar(t, r, u, v) \right) f(r)^2 \cos(\psi + v)^3 \chi^3\, u^5 \Big) \Big/$$

$$(2\, u^2\, f(r) \left(\cos(\psi + v)^3 \chi^3\, u^3 - 3 \cos(\psi + v)^2 \chi^2\, u^2 + 3\, u \chi \cos(\psi + v)\right.$$

$$\left.- 1) \right) D\_r) + \left(-2 \left(\frac{d}{dr}\, f(r) \right) f(r) \cos(\psi + v)^2 \left(\frac{\partial}{\partial u}\, Ar(t, r, u, v) \right) \chi^2\, u^4\right.$$

$$+ 4 \left(\frac{d}{dr}\, f(r) \right) f(r) \cos(\psi + v) \left(\frac{\partial}{\partial u}\, Ar(t, r, u, v) \right) \chi\, u^3 + 2 \left(\frac{d}{dr}\right.$$

$$f(r) \right) f(r) \cos(\psi + v)^2\, Ar(t, r, u, v)\, \chi^2\, u^3 - 2 \left(\frac{d}{dr}\, f(r) \right) f(r) \cos(\psi$$

$$+ v)\, Ar(t, r, u, v)\, \chi\, u^2 - 4\, f(r)^2 \cos(\psi + v)^2 \left(\frac{\partial}{\partial v}\, Av(t, r, u, v) \right) \chi^2\, u^3$$

$$- 4\, Au(t, r, u, v)\, f(r)^2 \cos(\psi + v)^2 \chi^2\, u^2 + 8\, f(r)^2 \cos(\psi + v) \left(\frac{\partial}{\partial v}\, Av(t, r, u,\right.$$

$$\left. v) \right) \chi\, u^2 + 4\, Au(t, r, u, v)\, f(r)^2 \cos(\psi + v)\, \chi\, u - 3 \left(\frac{\partial}{\partial r}\, Au(t, r, u, v) \right) \left(\frac{d}{dr}\right.$$

$$f(r) \right) f(r)\, u^2 + 2 \left(\frac{\partial}{\partial u}\, Au(t, r, u, v) \right) f(r)^2\, u - 2 \left(\frac{d}{dr}\, f(r) \right) f(r) \left(\frac{\partial}{\partial u}\, Ar(t, r,\right.$$

$$\left. u, v) \right) u^2 + 4\, f(r)^2 \cos(\psi + v)\, Av(t, r, u, v) \sin(\psi + v)\, \chi^2\, u^3 - 2 \left(\frac{\partial}{\partial v}\, Au(t, r,\right.$$

$$\left. u, v) \right) f(r)^2 \cos(\psi + v) \sin(\psi + v)\, \chi^2\, u^2 - 2\, Au(t, r, u, v)\, f(r)^2 + 3\, Au(t, r, u,$$

$$v) \left(\frac{d}{dr}\, f(r) \right)^2 u^2 - 4\, f(r)^2 \left(\frac{\partial}{\partial v}\, Av(t, r, u, v) \right) u - 2\, Au(t, r, u, v) \left(\frac{d^2}{dr^2}\right.$$

$$f(r) \right) f(r)\, u^2 + 2 \left(\frac{\partial^2}{\partial v^2}\, Au(t, r, u, v) \right) f(r)^2 + 4 \left(\frac{\partial}{\partial u}\, Au(t, r, u,\right.$$

$$\left. v) \right) f(r)^2 \cos(\psi + v)^2 \chi^2\, u^3 - 6 \left(\frac{\partial}{\partial u}\, Au(t, r, u, v) \right) f(r)^2 \cos(\psi + v)\, \chi\, u^2$$

$$+ 2 \left(\frac{\partial^2}{\partial v^2}\, Au(t, r, u, v) \right) f(r)^2 \cos(\psi + v)^2 \chi^2\, u^2 - 4 \left(\frac{\partial^2}{\partial v^2}\, Au(t, r, u,\right.$$

$$v) \Big) f(r)^2 \cos(\psi + v)\, \chi\, u - 4 \left(\frac{\partial^2}{\partial t^2}\, Au(t, r, u, v) \right) f(r) \cos(\psi + v)\, \chi\, u^3$$

$$+ 2 \left(\frac{\partial^2}{\partial u^2}\, Au(t, r, u, v) \right) f(r)^2 \cos(\psi + v)^2 \chi^2 u^4 + 2 \left(\frac{\partial^2}{\partial t^2}\, Au(t, r, u, \right.$$

$$\left. v) \right) f(r) \cos(\psi + v)^2 \chi^2 u^4 - 4 \left(\frac{\partial^2}{\partial u^2}\, Au(t, r, u, v) \right) f(r)^2 \cos(\psi + v)\, \chi\, u^3$$

$$+ 2 \left(\frac{\partial^2}{\partial u^2}\, Au(t, r, u, v) \right) f(r)^2 u^2 + 2 \left(\frac{\partial^2}{\partial r^2}\, Au(t, r, u, v) \right) f(r)^2 u^2 + 2 \left(\frac{\partial^2}{\partial t^2} \right.$$

$$\left. Au(t, r, u, v) \right) u^2 f(r) + 4 \left(\frac{\partial}{\partial r}\, Ar(t, r, u, v) \right) f(r)^2 \cos(\psi + v)\, \chi\, u^2$$

$$- 2 f(r)^2 Av(t, r, u, v) \sin(\psi + v)\, \chi\, u^2 + 2 \left(\frac{\partial}{\partial v}\, Au(t, r, u, v) \right) f(r)^2 \sin(\psi$$

$$+ v)\, \chi\, u - 4 \left(\frac{\partial}{\partial r}\, Ar(t, r, u, v) \right) f(r)^2 \cos(\psi + v)^2 \chi^2 u^3 \Bigg) \Bigg/$$

$$\left(2 f(r)\, u^2 \left(\cos(\psi + v)^2 \chi^2 u^2 - 2\, u\, \chi \cos(\psi + v) + 1 \right) \right) D\_u$$

$$+ \left(8 f(r)^2 \cos(\psi + v)^2 \left(\frac{\partial}{\partial u}\, Av(t, r, u, v) \right) \chi^2 u^4 + 4 \left(\frac{\partial}{\partial v}\, Au(t, r, u, \right.\right.$$

$$\left. v) \right) f(r)^2 \cos(\psi + v)^2 \chi^2 u^2 - 14 f(r)^2 \cos(\psi + v) \left(\frac{\partial}{\partial u}\, Av(t, r, u, v) \right) \chi\, u^3$$

$$- 8 \left(\frac{\partial}{\partial v}\, Au(t, r, u, v) \right) f(r)^2 \cos(\psi + v)\, \chi\, u - 2 \left(\frac{\partial}{\partial v}\, Ar(t, r, u, v) \right) \left(\frac{\mathrm{d}}{\mathrm{d}r} \right.$$

$$\left. f(r) \right) f(r)\, u + 4 \left(\frac{\partial}{\partial r}\, Ar(t, r, u, v) \right) f(r)^2 \cos(\psi + v) \sin(\psi + v)\, \chi^2 u^3$$

$$- 2 f(r)^2 \cos(\psi + v) \left(\frac{\partial}{\partial v}\, Av(t, r, u, v) \right) \sin(\psi + v)\, \chi^2 u^3 + 3 Av(t, r, u, \right.$$

$$\left. v) \left(\frac{\mathrm{d}}{\mathrm{d}r}\, f(r) \right)^2 u^3 + 6 f(r)^2 \left(\frac{\partial}{\partial u}\, Av(t, r, u, v) \right) u^2 - 2 \left(\frac{\mathrm{d}}{\mathrm{d}r}\, f(r) \right) f(r) \cos(\psi \right.$$

$$\left. + v)\, Ar(t, r, u, v) \sin(\psi + v)\, \chi^2 u^3 - 2 \left(\frac{\partial}{\partial v}\, Ar(t, r, u, v) \right) \left(\frac{\mathrm{d}}{\mathrm{d}r} \right.\right.$$

$$\left. f(r) \right) f(r) \cos(\psi + v)^2 \chi^2 u^3 + 4 \left(\frac{\partial}{\partial v}\, Ar(t, r, u, v) \right) \left(\frac{\mathrm{d}}{\mathrm{d}r}\, f(r) \right) f(r) \cos(\psi$$

$$+ v)\, \chi\, u^2 + 2 \left(\frac{\mathrm{d}}{\mathrm{d}r}\, f(r) \right) f(r)\, Ar(t, r, u, v) \sin(\psi + v)\, \chi\, u^2 + 4 f(r)^2 \cos(\psi$$

$$+ v)^2 Av(t, r, u, v)\, \chi^2 u^3 - 2 f(r)^2 \cos(\psi + v)\, Av(t, r, u, v)\, \chi\, u^2 - 2 \left(\frac{\mathrm{d}^2}{\mathrm{d}r^2} \right.$$

$$\left. f(r) \right) f(r)\, Av(t, r, u, v)\, u^3 + 4 \left(\frac{\partial}{\partial v}\, Au(t, r, u, v) \right) f(r)^2 + 2 \left(\frac{\partial^2}{\partial v^2}\, Av(t, r, u, \right.$$

$$v) \Big) f(r)^2 \cos(\psi + v)^2 \chi^2 u^3 - 4 \left(\frac{\partial^2}{\partial v^2} Av(t, r, u, v) \right) f(r)^2 \cos(\psi + v) \chi u^2$$

$$+ 2 \left(\frac{\partial^2}{\partial u^2} Av(t, r, u, v) \right) f(r)^2 u^3 + 2 \left(\frac{\partial^2}{\partial r^2} Av(t, r, u, v) \right) f(r)^2 u^3 + 2 \left(\frac{\partial^2}{\partial t^2} \right.$$

$$\left. Av(t, r, u, v) \right) u^3 f(r) + 2 \left(\frac{\partial^2}{\partial v^2} Av(t, r, u, v) \right) f(r)^2 u - 4 \left(\frac{\partial^2}{\partial u^2} Av(t, r, u, \right.$$

$$v) \Big) f(r)^2 \cos(\psi + v) \chi u^4 - 4 \left(\frac{\partial^2}{\partial t^2} Av(t, r, u, v) \right) f(r) \cos(\psi + v) \chi u^4$$

$$+ 2 \left(\frac{\partial^2}{\partial u^2} Av(t, r, u, v) \right) f(r)^2 \cos(\psi + v)^2 \chi^2 u^5 + 2 \left(\frac{\partial^2}{\partial t^2} Av(t, r, u, \right.$$

$$\left. v) \right) f(r) \cos(\psi + v)^2 \chi^2 u^5 - 3 \left(\frac{d}{dr} f(r) \right) f(r) \left(\frac{\partial}{\partial r} Av(t, r, u, v) \right) u^3$$

$$- 2 f(r)^2 Av(t, r, u, v) \chi^2 u^3 - 4 \left(\frac{\partial}{\partial r} Ar(t, r, u, v) \right) f(r)^2 \sin(\psi + v) \chi u^2$$

$$+ 2 f(r)^2 \left(\frac{\partial}{\partial v} Av(t, r, u, v) \right) \sin(\psi + v) \chi u^2 + 2 Au(t, r, u, v) f(r)^2 \sin(\psi$$

$$+ v) \chi u \Big) \Big/ \left(2 u^3 \left(\cos(\psi + v)^2 \chi^2 u^2 - 2 u \chi \cos(\psi + v) + 1 \right) f(r) \right) D\_v$$

M >

M >