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> with(Physics):with(DifferentialGeometry):with(plots):with
(PDEtools):with(Tensor):
> #The sign convention for the Ricci tensor of the DifferentialGeometry and Physics
packages is the same followed in MTW.
>
> #Manifold definition:
> DGsetup([t, r, u, v], M, verbose)
    The following coordinates have been protected:
                [t, r, u, v]
    The following vector fields have been defined and protected:
                [ '*'(D_t), '*'(D_r), '*'(D_u), '*'(D_v) ]
    The following differential 1-forms have been defined and protected:
                [ '*'(dt), '*'(dr), '*'(du), '*'(dv) ]
                frame name: M
(1)

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M >
M > g1 := evalDG( dt &t dt + ((1 - u*chi*cos(int(-tau(r), r) +
v))^2)*dr &t dr + du &t du + (u^2)*dv &t dv)
g1:= ('*(dt)) dt + ( (u*chi*cos(- (int(tau(r) dr) + v) - 1)^2 dr) dr + ('*(du)) du
+ (u^2 dv) dv
(2)

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M > g1i:= InverseMetric(g1)
g1i:= ('*(D_t)) D_t
+ ( 1 / (cos(- (int(tau(r) dr) + v)^2 chi^2 u^2 - 2 u*chi*cos(- (int(tau(r) dr) + v) + 1) ) D_r) D_r
+ ('*(D_u)) D_u + ( 1 / u^2 D_v) D_v
(3)

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M > #Christoffel symbols of the second kind:
M > C1 := Christoffel(g1, "SecondKind")
C1:= ( ( ( u*chi*tau(r) sin(- (int(tau(r) dr) + v) ) / (u*chi*cos(- (int(tau(r) dr) + v) - 1) D_r) dr ) dr
+ ( ( chi*cos(- (int(tau(r) dr) + v) ) / (u*chi*cos(- (int(tau(r) dr) + v) - 1) D_r) dr ) du
- ( ( ( u*chi*sin(- (int(tau(r) dr) + v) ) / (u*chi*cos(- (int(tau(r) dr) + v) - 1) D_r) dr ) dv )
(4)

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$$\begin{aligned}
& + \left( \left( \frac{\chi \cos\left(-\left(\int \tau(r) dr\right) + v\right)}{u \chi \cos\left(-\left(\int \tau(r) dr\right) + v\right) - 1} D_{-r} \right) du \right) dr \\
& - \left( \left( \left( \frac{u \chi \sin\left(-\left(\int \tau(r) dr\right) + v\right)}{u \chi \cos\left(-\left(\int \tau(r) dr\right) + v\right) - 1} D_{-r} \right) dv \right) dr \right) - \left( \left( \left( \left( u \chi \cos\left(-\left(\int \tau(r) dr\right) + v\right) - 1 \right) \chi \cos\left(-\left(\int \tau(r) dr\right) + v\right) D_{-u} \right) dr \right) dv \right) \\
& + \left( \left( \frac{\left( u \chi \cos\left(-\left(\int \tau(r) dr\right) + v\right) - 1 \right) \chi \sin\left(-\left(\int \tau(r) dr\right) + v\right)}{u} D_{-v} \right) dr \right) dr \\
& + \left( \left( \frac{1}{u} D_{-v} \right) du \right) dv + \left( \left( \frac{1}{u} D_{-v} \right) dv \right) du
\end{aligned}$$

**M** > #Ricci tensor:

**M** > **R1:=RicciTensor(C1)**

$$R1 := (0 \, dt) \, dt$$

(5)

**M** >

**M** >

**M** > #4-potential vector:

**M** > **A := DGzip([At, Ar, Au, Av])(t, r, u, v), [D\_t, D\_r, D\_u, D\_v], "plus")**

$$A := At(t, r, u, v) D_t + Ar(t, r, u, v) D_r + Au(t, r, u, v) D_u + Av(t, r, u, v) D_v$$

(6)

**M** > #Covariant derivative of the 4-potential:

**M** > **Dc:=CovariantDerivative(A, C1):**

**M** > #The second covariant derivative of the 4-potential:

**M** > **Dc2:=CovariantDerivative(Dc, C1):**

**M** > #Contraction of the indices of the covariant derivatives to get the Laplacian:

**M** > **L:=ContractIndices(g1i, Dc2, [[1, 2], [2,3]]):**

**M** >

**M** > ##### "Mass term" of the Maxwell equations, i.e, contraction of the Ricci tensor with the 4-potential:

**M** > #Rising one index of the Ricci tensor:

**M** > **R1up:=ContractIndices(g1i, R1, [[1, 1]]):**

**M** > #Contraction of the Ricci tensor with the four-potential:

**M** > **Mass:=ContractIndices(A, R1up, [[1,2]]):**

**M** > #Maxwell equations!!!!!!!!!!!!!!:

**M** > **Me:= L &minus; Mass**

$$Me := \left( \left( \frac{\partial^2}{\partial u^2} At(t, r, u, v) \right) \cos\left(-\left(\int \tau(r) dr\right) + v\right)^3 \chi^3 u^5 + \left( \frac{\partial^2}{\partial t^2} At(t, r, u, v) \right) \right)$$

(7)

$$\begin{aligned}
& \nu) \Big) \cos\Big(-\Big(\int \tau(r) \, dr\Big) + \nu\Big)^3 \chi^3 u^5 + 2 \cos\Big(-\Big(\int \tau(r) \, dr\Big) + \nu\Big)^3 \Big(\frac{\partial}{\partial u} At(t, r, u, \\
& \nu) \Big) \chi^3 u^4 - \Big(\frac{\partial}{\partial \nu} At(t, r, u, \nu) \Big) \sin\Big(-\Big(\int \tau(r) \, dr\Big) + \nu\Big) \cos\Big(-\Big(\int \tau(r) \, dr\Big) \\
& + \nu\Big)^2 \chi^3 u^3 + \cos\Big(-\Big(\int \tau(r) \, dr\Big) + \nu\Big)^3 \Big(\frac{\partial^2}{\partial \nu^2} At(t, r, u, \nu) \Big) \chi^3 u^3 - 3 \Big(\frac{\partial^2}{\partial u^2} At(t, \\
& r, u, \nu) \Big) \cos\Big(-\Big(\int \tau(r) \, dr\Big) + \nu\Big)^2 \chi^2 u^4 - 3 \Big(\frac{\partial^2}{\partial t^2} At(t, r, u, \nu) \Big) \cos\Big(-\Big(\int \tau(r) \\
& dr\Big) + \nu\Big)^2 \chi^2 u^4 - 5 \cos\Big(-\Big(\int \tau(r) \, dr\Big) + \nu\Big)^2 \Big(\frac{\partial}{\partial u} At(t, r, u, \nu) \Big) \chi^2 u^3 + 2 \Big(\frac{\partial}{\partial \nu} \\
& At(t, r, u, \nu) \Big) \sin\Big(-\Big(\int \tau(r) \, dr\Big) + \nu\Big) \cos\Big(-\Big(\int \tau(r) \, dr\Big) + \nu\Big) \chi^2 u^2 - \tau(r) \Big(\frac{\partial}{\partial r} \\
& At(t, r, u, \nu) \Big) \sin\Big(-\Big(\int \tau(r) \, dr\Big) + \nu\Big) \chi u^3 - 3 \cos\Big(-\Big(\int \tau(r) \, dr\Big) + \nu\Big)^2 \Big(\frac{\partial^2}{\partial \nu^2} \\
& At(t, r, u, \nu) \Big) \chi^2 u^2 + 3 \Big(\frac{\partial^2}{\partial u^2} At(t, r, u, \nu) \Big) \cos\Big(-\Big(\int \tau(r) \, dr\Big) + \nu\Big) \chi u^3 + \Big(\frac{\partial^2}{\partial r^2} \\
& At(t, r, u, \nu) \Big) \cos\Big(-\Big(\int \tau(r) \, dr\Big) + \nu\Big) \chi u^3 + 3 \Big(\frac{\partial^2}{\partial t^2} At(t, r, u, \nu) \Big) \cos\Big(-\Big(\int \tau(r) \\
& dr\Big) + \nu\Big) \chi u^3 + 4 \cos\Big(-\Big(\int \tau(r) \, dr\Big) + \nu\Big) \Big(\frac{\partial}{\partial u} At(t, r, u, \nu) \Big) \chi u^2 - \Big(\frac{\partial}{\partial \nu} At(t, \\
& r, u, \nu) \Big) \sin\Big(-\Big(\int \tau(r) \, dr\Big) + \nu\Big) \chi u + 3 \cos\Big(-\Big(\int \tau(r) \, dr\Big) + \nu\Big) \Big(\frac{\partial^2}{\partial \nu^2} At(t, r, u, \\
& \nu) \Big) \chi u - \Big(\frac{\partial^2}{\partial u^2} At(t, r, u, \nu) \Big) u^2 - \Big(\frac{\partial^2}{\partial r^2} At(t, r, u, \nu) \Big) u^2 - \Big(\frac{\partial^2}{\partial t^2} At(t, r, u, \\
& \nu) \Big) u^2 - u \Big(\frac{\partial}{\partial u} At(t, r, u, \nu) \Big) - \Big(\frac{\partial^2}{\partial \nu^2} At(t, r, u, \nu) \Big) \Big) \Big/ \Big(u^2 \Big(\cos\Big(-\Big(\int \tau(r) \\
& dr\Big) + \nu\Big)^3 \chi^3 u^3 - 3 \cos\Big(-\Big(\int \tau(r) \, dr\Big) + \nu\Big)^2 \chi^2 u^2 + 3 u \chi \cos\Big(-\Big(\int \tau(r) \, dr\Big) + \nu\Big) \\
& - 1\Big) \Big) D_t + \Big(3 u \chi \sin\Big(-\Big(\int \tau(r) \, dr\Big) + \nu\Big) \Big(\frac{\partial}{\partial \nu} Ar(t, r, u, \nu) \Big) - 4 \cos\Big(-\Big(\int \tau(r) \, dr\Big) + \nu\Big) \Big(\frac{\partial^2}{\partial \nu^2} Ar(t, r, u, \nu) \Big) \chi u + 6 \Big(\frac{\partial^2}{\partial \nu^2} Ar(t, r, u, \nu) \Big) \cos\Big(-\Big(\int \tau(r) \, dr\Big) + \nu\Big)^2 \chi^2 u^2 + \Big(\frac{\partial^2}{\partial u^2} Ar(t, r, u, \nu) \Big) \cos\Big(-\Big(\int \tau(r) \, dr\Big) + \nu\Big)^4 \chi^4 u^6 \\
& + \Big(\frac{\partial^2}{\partial t^2} Ar(t, r, u, \nu) \Big) \cos\Big(-\Big(\int \tau(r) \, dr\Big) + \nu\Big)^4 \chi^4 u^6 + \cos\Big(-\Big(\int \tau(r) \, dr\Big) \\
& + \nu\Big)^4 \Big(\frac{\partial^2}{\partial \nu^2} Ar(t, r, u, \nu) \Big) \chi^4 u^4 - 4 \Big(\frac{\partial^2}{\partial u^2} Ar(t, r, u, \nu) \Big) \cos\Big(-\Big(\int \tau(r) \, dr\Big)
\end{aligned}$$

$$\begin{aligned}
& + v)^3 \chi^3 u^5 - 4 \left( \frac{\partial^2}{\partial t^2} Ar(t, r, u, v) \right) \cos \left( - \left( \int \tau(r) dr \right) + v \right)^3 \chi^3 u^5 - 4 \cos \left( \right. \\
& - \left. \left( \int \tau(r) dr \right) + v \right)^3 \left( \frac{\partial^2}{\partial v^2} Ar(t, r, u, v) \right) \chi^3 u^3 + 6 \left( \frac{\partial^2}{\partial u^2} Ar(t, r, u, v) \right) \cos \left( \right. \\
& - \left. \left( \int \tau(r) dr \right) + v \right)^2 \chi^2 u^4 + \left( \frac{\partial^2}{\partial r^2} Ar(t, r, u, v) \right) \cos \left( - \left( \int \tau(r) dr \right) + v \right)^2 \chi^2 u^4 \\
& + 6 \left( \frac{\partial^2}{\partial t^2} Ar(t, r, u, v) \right) \cos \left( - \left( \int \tau(r) dr \right) + v \right)^2 \chi^2 u^4 - 4 \left( \frac{\partial^2}{\partial u^2} Ar(t, r, u, \right. \\
& v) \left. \right) \cos \left( - \left( \int \tau(r) dr \right) + v \right) \chi u^3 - 2 \left( \frac{\partial^2}{\partial r^2} Ar(t, r, u, v) \right) \cos \left( - \left( \int \tau(r) dr \right) \right. \\
& + v) \chi u^3 - 4 \left( \frac{\partial^2}{\partial t^2} Ar(t, r, u, v) \right) \cos \left( - \left( \int \tau(r) dr \right) + v \right) \chi u^3 - 3 \left( \frac{\partial}{\partial v} Ar(t, r, \right. \\
& u, v) \left. \right) \sin \left( - \left( \int \tau(r) dr \right) + v \right) \cos \left( - \left( \int \tau(r) dr \right) + v \right)^3 \chi^4 u^4 + 9 \left( \frac{\partial}{\partial v} Ar(t, r, u, \right. \\
& v) \left. \right) \sin \left( - \left( \int \tau(r) dr \right) + v \right) \cos \left( - \left( \int \tau(r) dr \right) + v \right)^2 \chi^3 u^3 - 2 \left( \frac{\partial}{\partial r} Av(t, r, u, \right. \\
& v) \left. \right) \sin \left( - \left( \int \tau(r) dr \right) + v \right) \cos \left( - \left( \int \tau(r) dr \right) + v \right) \chi^2 u^4 - 7 \cos \left( - \left( \int \tau(r) dr \right) \right. \\
& + v) \left( \frac{\partial}{\partial u} Ar(t, r, u, v) \right) \chi u^2 - 9 \left( \frac{\partial}{\partial v} Ar(t, r, u, v) \right) \sin \left( - \left( \int \tau(r) dr \right) \right. \\
& + v) \cos \left( - \left( \int \tau(r) dr \right) + v \right) \chi^2 u^2 + \left( \frac{d}{dr} \tau(r) \right) Ar(t, r, u, v) \sin \left( - \left( \int \tau(r) dr \right) \right. \\
& + v) \cos \left( - \left( \int \tau(r) dr \right) + v \right) \chi^2 u^4 + \tau(r) \left( \frac{\partial}{\partial r} Ar(t, r, u, v) \right) \sin \left( - \left( \int \tau(r) dr \right) \right. \\
& + v) \cos \left( - \left( \int \tau(r) dr \right) + v \right) \chi^2 u^4 + \left( \frac{\partial^2}{\partial u^2} Ar(t, r, u, v) \right) u^2 + \left( \frac{\partial^2}{\partial r^2} Ar(t, r, u, \right. \\
& v) \left. \right) u^2 + \left( \frac{\partial^2}{\partial t^2} Ar(t, r, u, v) \right) u^2 + Ar(t, r, u, v) \tau(r)^2 \cos \left( - \left( \int \tau(r) dr \right) \right. \\
& + v) \chi u^3 - \left( \frac{d}{dr} \tau(r) \right) Ar(t, r, u, v) \sin \left( - \left( \int \tau(r) dr \right) + v \right) \chi u^3 - \tau(r) \left( \frac{\partial}{\partial r} \right. \\
& Ar(t, r, u, v) \left. \right) \sin \left( - \left( \int \tau(r) dr \right) + v \right) \chi u^3 - \tau(r) Av(t, r, u, v) \cos \left( - \left( \int \tau(r) dr \right) \right. \\
& + v) \chi u^3 - \tau(r) Au(t, r, u, v) \sin \left( - \left( \int \tau(r) dr \right) + v \right) \chi u^2 + 4 \left( \frac{\partial}{\partial u} Ar(t, r, u, \right. \\
& v) \left. \right) \cos \left( - \left( \int \tau(r) dr \right) + v \right)^4 \chi^4 u^5 - 13 \left( \frac{\partial}{\partial u} Ar(t, r, u, v) \right) \cos \left( - \left( \int \tau(r) dr \right) \right. \\
& + v) \left. \right)^3 \chi^3 u^4 - Ar(t, r, u, v) \tau(r)^2 \chi^2 u^4 + 2 \left( \frac{\partial}{\partial r} Au(t, r, u, v) \right) \cos \left( - \left( \int \tau(r) \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \mathrm{d}r) + \nu)^2 \chi^2 u^3 + \tau(r) A\nu(t, r, u, \nu) \chi^2 u^4 + 15 \left( \frac{\partial}{\partial u} A r(t, r, u, \nu) \right) \cos \left( - \left( \int \tau(r) \mathrm{d}r \right) + \nu \right)^2 \chi^2 u^3 \\
& + 2 \left( \frac{\partial}{\partial r} A\nu(t, r, u, \nu) \right) \sin \left( - \left( \int \tau(r) \mathrm{d}r \right) + \nu \right) \chi u^3 \\
& - 2 \left( \frac{\partial}{\partial r} A u(t, r, u, \nu) \right) \cos \left( - \left( \int \tau(r) \mathrm{d}r \right) + \nu \right) \chi u^2 + \frac{\partial^2}{\partial \nu^2} A r(t, r, u, \nu) + \left( \frac{\partial}{\partial u} A r(t, r, u, \nu) \right) u \Bigg/ \left( u^2 \left( \cos \left( - \left( \int \tau(r) \mathrm{d}r \right) + \nu \right)^4 \chi^4 u^4 - 4 \cos \left( - \left( \int \tau(r) \mathrm{d}r \right) + \nu \right)^3 \chi^3 u^3 \right. \right. \\
& + 6 \cos \left( - \left( \int \tau(r) \mathrm{d}r \right) + \nu \right)^2 \chi^2 u^2 - 4 u \chi \cos \left( - \left( \int \tau(r) \mathrm{d}r \right) + \nu \right) + 1 \Bigg) D_- r + \left( A u(t, r, u, \nu) - 2 A r(t, r, u, \nu) \tau(r) \sin \left( - \left( \int \tau(r) \mathrm{d}r \right) + \nu \right) \cos \left( - \left( \int \tau(r) \mathrm{d}r \right) + \nu \right)^2 \chi^3 u^4 \right. \\
& + 3 A r(t, r, u, \nu) \tau(r) \sin \left( - \left( \int \tau(r) \mathrm{d}r \right) + \nu \right) \cos \left( - \left( \int \tau(r) \mathrm{d}r \right) + \nu \right) \chi^2 u^3 + \left( \frac{\partial^2}{\partial u^2} A u(t, r, u, \nu) \right) \cos \left( - \left( \int \tau(r) \mathrm{d}r \right) + \nu \right)^3 \chi^3 u^5 \\
& + \left( \frac{\partial^2}{\partial t^2} A u(t, r, u, \nu) \right) \cos \left( - \left( \int \tau(r) \mathrm{d}r \right) + \nu \right)^3 \chi^3 u^5 + \cos \left( - \left( \int \tau(r) \mathrm{d}r \right) + \nu \right)^3 \left( \frac{\partial^2}{\partial \nu^2} A u(t, r, u, \nu) \right) \chi^3 u^3 - 3 \left( \frac{\partial^2}{\partial u^2} A u(t, r, u, \nu) \right) \cos \left( - \left( \int \tau(r) \mathrm{d}r \right) + \nu \right)^2 \chi^2 u^4 \\
& - 3 \cos \left( - \left( \int \tau(r) \mathrm{d}r \right) + \nu \right)^2 \left( \frac{\partial^2}{\partial t^2} A u(t, r, u, \nu) \right) \chi^2 u^2 + 3 \left( \frac{\partial^2}{\partial u^2} A u(t, r, u, \nu) \right) \cos \left( - \left( \int \tau(r) \mathrm{d}r \right) + \nu \right) \chi u^3 + \left( \frac{\partial^2}{\partial r^2} A u(t, r, u, \nu) \right) \cos \left( - \left( \int \tau(r) \mathrm{d}r \right) + \nu \right) \chi u^3 \\
& + 3 \left( \frac{\partial^2}{\partial t^2} A u(t, r, u, \nu) \right) \cos \left( - \left( \int \tau(r) \mathrm{d}r \right) + \nu \right) \chi u^3 + 3 \cos \left( - \left( \int \tau(r) \mathrm{d}r \right) + \nu \right) \left( \frac{\partial^2}{\partial \nu^2} A u(t, r, u, \nu) \right) \chi u + 2 \sin \left( - \left( \int \tau(r) \mathrm{d}r \right) + \nu \right) \cos \left( - \left( \int \tau(r) \mathrm{d}r \right) + \nu \right)^2 A\nu(t, r, u, \nu) \chi^3 u^4 \\
& - \sin \left( - \left( \int \tau(r) \mathrm{d}r \right) + \nu \right) \cos \left( - \left( \int \tau(r) \mathrm{d}r \right) + \nu \right)^2 \left( \frac{\partial}{\partial \nu} A u(t, r, u, \nu) \right) \chi^3 u^3 - 3 \sin \left( - \left( \int \tau(r) \mathrm{d}r \right) + \nu \right) \cos \left( - \left( \int \tau(r) \mathrm{d}r \right) + \nu \right) A\nu(t, r, u, \nu) \chi^2 u^3 \\
& - \left( \frac{\partial}{\partial r} A u(t, r, u, \nu) \right) \tau(r) \sin \left( - \left( \int \tau(r) \mathrm{d}r \right) + \nu \right) \chi u^3 + 2 \sin \left( - \left( \int \tau(r) \mathrm{d}r \right) + \nu \right) \cos \left( - \left( \int \tau(r) \mathrm{d}r \right) + \nu \right) \left( \frac{\partial}{\partial \nu} A u(t, r, u, \nu) \right) \chi^2 u^2
\end{aligned}$$

$$-Ar(t,r,u,v)\tau(r)\sin\left(-\left(\int \tau(r)dr\right)+v\right)\chi u^2-\left(\frac{\partial^2}{\partial u^2}Au(t,r,u,v)\right)u^2$$
$$-\left(\frac{\partial^2}{\partial r^2}Au(t,r,u,v)\right)u^2-\left(\frac{\partial^2}{\partial t^2}Au(t,r,u,v)\right)u^2+2\left(\frac{\partial}{\partial v}Av(t,r,u,v)\right)u$$
$$-\left(\frac{\partial}{\partial u}Au(t,r,u,v)\right)u-\left(\frac{\partial^2}{\partial v^2}Au(t,r,u,v)\right)-2\left(\frac{\partial}{\partial r}Ar(t,r,u,$$
$$v)\right)\cos\left(-\left(\int \tau(r)dr\right)+v\right)^3\chi^3u^4+2\cos\left(-\left(\int \tau(r)dr\right)+v\right)^3\left(\frac{\partial}{\partial u}Au(t,r,u,$$
$$v)\right)\chi^3u^4-2\cos\left(-\left(\int \tau(r)dr\right)+v\right)^3\left(\frac{\partial}{\partial v}Av(t,r,u,v)\right)\chi^3u^4-2\cos\left(-\left(\int \tau(r)$$
$$\int \tau(r)dr\right)+v\right)^3Au(t,r,u,v)\chi^3u^3+4\left(\frac{\partial}{\partial r}Ar(t,r,u,v)\right)\cos\left(-\left(\int \tau(r)dr\right)$$
$$+v\right)^2\chi^2u^3-5\cos\left(-\left(\int \tau(r)dr\right)+v\right)^2\left(\frac{\partial}{\partial u}Au(t,r,u,v)\right)\chi^2u^3+6\cos\left(-\left(\int \tau(r)dr\right)+v\right)^2$$
$$\left(\frac{\partial}{\partial v}Av(t,r,u,v)\right)\chi^2u^3+4\cos\left(-\left(\int \tau(r)dr\right)+v\right)^2Au(t,r,u,v)\chi^2u^2-2\left(\frac{\partial}{\partial r}Ar(t,r,u,v)\right)\cos\left(-\left(\int \tau(r)dr\right)+v\right)\chi u^2+\sin\left(-\left(\int \tau(r)$$
$$dr\right)+v\right)Av(t,r,u,v)\chi u^2+4\cos\left(-\left(\int \tau(r)dr\right)+v\right)\left(\frac{\partial}{\partial u}Au(t,r,u,v)\right)\chi u^2$$
$$-6\cos\left(-\left(\int \tau(r)dr\right)+v\right)\left(\frac{\partial}{\partial v}Av(t,r,u,v)\right)\chi u^2-\sin\left(-\left(\int \tau(r)dr\right)+v\right)\left(\frac{\partial}{\partial v}Au(t,r,u,v)\right)\chi u-3\cos\left(-\left(\int \tau(r)dr\right)+v\right)Au(t,r,u,v)\chi u\Bigg/\nonumber\\(u^2\left(\cos\left(-\left(\int \tau(r)dr\right)+v\right)^3\chi^3u^3-3\cos\left(-\left(\int \tau(r)dr\right)+v\right)^2\chi^2u^2\right.$$
$$+3u\chi\cos\left(-\left(\int \tau(r)dr\right)+v-1\right))D_-u-\left((-3\cos\left(-\left(\int \tau(r)dr\right)+v\right)\left(\frac{\partial^2}{\partial v^2}Av(t,r,u,v)\right)\chi u^2-\cos\left(-\left(\int \tau(r)dr\right)+v\right)^3\left(\frac{\partial^2}{\partial u^2}Av(t,r,u,$$
$$v)\right)\chi^3u^6-\cos\left(-\left(\int \tau(r)dr\right)+v\right)^3\left(\frac{\partial^2}{\partial t^2}Av(t,r,u,v)\right)\chi^3u^6-\cos\left(-\left(\int \tau(r)dr\right)+v\right)^3\left(\frac{\partial^2}{\partial v^2}Av(t,r,u,v)\right)\chi^3u^4+3\cos\left(-\left(\int \tau(r)dr\right)+v\right)^2\left(\frac{\partial^2}{\partial u^2}Av(t,r,$$
$$u,v)\right)\chi^2u^5+3\cos\left(-\left(\int \tau(r)dr\right)+v\right)^2\left(\frac{\partial^2}{\partial t^2}Av(t,r,u,v)\right)\chi^2u^5+3\cos\left(-\left(\int \tau(r)dr\right)+v\right)^2\left(\frac{\partial^2}{\partial v^2}Av(t,r,u,v)\right)\chi^2u^3-3\cos\left(-\left(\int \tau(r)dr\right)+v\right)\left(\frac{\partial^2}{\partial u^2}$$
$$Av(t,r,u,v)\right)\chi u^4-\cos\left(-\left(\int \tau(r)dr\right)+v\right)\left(\frac{\partial^2}{\partial r^2}Av(t,r,u,v)\right)\chi u^4-3\cos\left(-\left(\int \tau(r)dr\right)+v\right)\left(\frac{\partial^2}{\partial t^2}Av(t,r,u,v)\right)\chi u^4-$$

$$\begin{aligned}
& - \left( \int \tau(r) \, dr \right) + \nu \left( \frac{\partial^2}{\partial t^2} Av(t, r, u, \nu) \right) \chi u^4 - Av(t, r, u, \nu) \chi^2 u^3 + 3 \left( \frac{\partial}{\partial u} Av(t, \right. \\
& r, u, \nu) \left. \right) u^2 - 4 \cos \left( - \left( \int \tau(r) \, dr \right) + \nu \right)^3 \left( \frac{\partial}{\partial u} Av(t, r, u, \nu) \right) \chi^3 u^5 - 2 \cos \left( \right. \\
& - \left( \int \tau(r) \, dr \right) + \nu \left. \right)^3 Av(t, r, u, \nu) \chi^3 u^4 - 2 \cos \left( - \left( \int \tau(r) \, dr \right) + \nu \right)^3 \left( \frac{\partial}{\partial \nu} Au(t, r, \right. \\
& u, \nu) \left. \right) \chi^3 u^3 + 11 \cos \left( - \left( \int \tau(r) \, dr \right) + \nu \right)^2 \left( \frac{\partial}{\partial u} Av(t, r, u, \nu) \right) \chi^2 u^4 + \cos \left( - \left( \right. \right. \\
& \left. \left. \int \tau(r) \, dr \right) + \nu \right) Av(t, r, u, \nu) \chi^3 u^4 + 3 \cos \left( - \left( \int \tau(r) \, dr \right) + \nu \right)^2 Av(t, r, u, \nu) \chi^2 u^3 \\
& + Ar(t, r, u, \nu) \tau(r) \chi^2 u^3 + 6 \cos \left( - \left( \int \tau(r) \, dr \right) + \nu \right)^2 \left( \frac{\partial}{\partial \nu} Au(t, r, u, \nu) \right) \chi^2 u^2 \\
& - 10 \cos \left( - \left( \int \tau(r) \, dr \right) + \nu \right) \left( \frac{\partial}{\partial u} Av(t, r, u, \nu) \right) \chi u^3 - 2 \left( \frac{\partial}{\partial r} Ar(t, r, u, \right. \\
& \left. \nu) \right) \sin \left( - \left( \int \tau(r) \, dr \right) + \nu \right) \chi u^2 + \sin \left( - \left( \int \tau(r) \, dr \right) + \nu \right) \left( \frac{\partial}{\partial \nu} Av(t, r, u, \nu) \right) \chi u^2 \\
& - \cos \left( - \left( \int \tau(r) \, dr \right) + \nu \right) Av(t, r, u, \nu) \chi u^2 + \sin \left( - \left( \int \tau(r) \, dr \right) + \nu \right) Au(t, r, u, \right. \\
& \left. \nu) \chi u - 6 \cos \left( - \left( \int \tau(r) \, dr \right) + \nu \right) \left( \frac{\partial}{\partial \nu} Au(t, r, u, \nu) \right) \chi u + \left( \frac{\partial^2}{\partial u^2} Av(t, r, u, \right. \right. \\
& \left. \left. \nu) \right) u^3 + \left( \frac{\partial^2}{\partial r^2} Av(t, r, u, \nu) \right) u^3 + \left( \frac{\partial^2}{\partial t^2} Av(t, r, u, \nu) \right) u^3 + \left( \frac{\partial^2}{\partial \nu^2} Av(t, r, u, \right. \right. \\
& \left. \left. \nu) \right) u + 2 Ar(t, r, u, \nu) \tau(r) \cos \left( - \left( \int \tau(r) \, dr \right) + \nu \right)^3 \chi^3 u^4 - 2 \left( \frac{\partial}{\partial r} Ar(t, r, u, \right. \right. \\
& \left. \left. \nu) \right) \sin \left( - \left( \int \tau(r) \, dr \right) + \nu \right) \cos \left( - \left( \int \tau(r) \, dr \right) + \nu \right)^2 \chi^3 u^4 + \sin \left( - \left( \int \tau(r) \, dr \right) \right. \\
& \left. + \nu \right) \cos \left( - \left( \int \tau(r) \, dr \right) + \nu \right)^2 \left( \frac{\partial}{\partial \nu} Av(t, r, u, \nu) \right) \chi^3 u^4 - Ar(t, r, u, \nu) \tau(r) \cos \left( \right. \\
& - \left( \int \tau(r) \, dr \right) + \nu \left. \right) \chi^3 u^4 - 3 Ar(t, r, u, \nu) \tau(r) \cos \left( - \left( \int \tau(r) \, dr \right) + \nu \right)^2 \chi^2 u^3 \\
& + \left( \frac{\partial}{\partial r} Av(t, r, u, \nu) \right) \tau(r) \sin \left( - \left( \int \tau(r) \, dr \right) + \nu \right) \chi u^4 + 4 \left( \frac{\partial}{\partial r} Ar(t, r, u, \right. \\
& \left. \nu) \right) \sin \left( - \left( \int \tau(r) \, dr \right) + \nu \right) \cos \left( - \left( \int \tau(r) \, dr \right) + \nu \right) \chi^2 u^3 - 2 \sin \left( - \left( \int \tau(r) \, dr \right) \right. \\
& \left. + \nu \right) \cos \left( - \left( \int \tau(r) \, dr \right) + \nu \right) \left( \frac{\partial}{\partial \nu} Av(t, r, u, \nu) \right) \chi^2 u^3 - \sin \left( - \left( \int \tau(r) \, dr \right) \right. \\
& \left. + \nu \right) \cos \left( - \left( \int \tau(r) \, dr \right) + \nu \right) Au(t, r, u, \nu) \chi^2 u^2 + Ar(t, r, u, \nu) \tau(r) \cos \left( - \left( \right. \right.
\end{aligned}$$

$$\left[ \int \tau(r) \, \mathrm{d}r \right) + \nu) \, \chi \, u^2 + 2 \left( \frac{\partial}{\partial \nu} \, Au(t, \, r, \, u, \, \nu) \right) \Bigg) \Bigg/ \left( u^3 \left( \cos \Big( - \left( \int \tau(r) \, \mathrm{d}r \right) \right. \right. \right. \\ \left. \left. \left. + \nu \right)^3 \chi^3 \, u^3 - 3 \cos \Big( - \left( \int \tau(r) \, \mathrm{d}r \right) + \nu \right)^2 \chi^2 \, u^2 + 3 \, u \chi \cos \Big( - \left( \int \tau(r) \, \mathrm{d}r \right) + \nu \right) \right. \right. \\ \left. \left. - 1 \right) \right) \, D_\nu \Big)$$

$$[\mathbf{M} >$$