```
> with(Physics):with(DifferentialGeometry):with(plots):with
  (PDEtools):with(Tensor):
> #The sign convention for the Ricci tensor of the DifferentialGeometry and Physics
      packages is the same followed in MTW.
>
> #Manifold definition:
> DGsetup([t, r, u, v], M, verbose)
```

```
                The following coordinates have been protected:
                                [t, r, u, v]
         The following vector fields have been defined and protected:
              [ `*`(D_t), `*`(D_r), `*`(D_u), `*`(D_v) ]
      The following differential 1-forms have been defined and protected:
                [ `*`(dt), `*`(dr), `*`(du), `*`(dv) ]
                               frame name: M                              (1)
```

```
M > g1 := evalDG( -f(r)*dt &t dt +  dr &t dr +  du &t du + dv &t
    dv)
```

$$g1 := -((f(r)\,dt)\,dt) + (\text{`*`}(dr))\,dr + (\text{`*`}(du))\,du + (\text{`*`}(dv))\,dv \tag{2}$$

```
M > g1i:= InverseMetric(g1)
```

$$g1i := -\left(\left(\frac{1}{f(r)}\,D\_t\right)D\_t\right) + (\text{`*`}(D\_r))\,D\_r + (\text{`*`}(D\_u))\,D\_u + (\text{`*`}(D\_v))\,D\_v \tag{3}$$

```
M > #Christoffel symbols of the second kind:
M > C1 := Christoffel(g1, "SecondKind")
```

$$C1 := \left(\left(\left(\frac{\frac{\mathrm{d}}{\mathrm{d}r}\,f(r)}{2\,f(r)}\,D\_t\right)dt\right)dr + \left(\left(\frac{\frac{\mathrm{d}}{\mathrm{d}r}\,f(r)}{2\,f(r)}\,D\_t\right)dr\right)dt + \left(\left(\frac{\left(\frac{\mathrm{d}}{\mathrm{d}r}\,f(r)\right)}{2}\,D\_r\right)dt\right)dt \tag{4}$$

```
M > #Ricci tensor:
M > R1:=RicciTensor(C1)
```

$$R1 := \left(\left(-\frac{\left(\frac{\mathrm{d}}{\mathrm{d}r}\,f(r)\right)^2}{4\,f(r)} + \frac{\left(\frac{\mathrm{d}^2}{\mathrm{d}r^2}\,f(r)\right)}{2}\right)dt\right)dt - \left(\left(\frac{2\left(\frac{\mathrm{d}^2}{\mathrm{d}r^2}\,f(r)\right)f(r) - \left(\frac{\mathrm{d}}{\mathrm{d}r}\,f(r)\right)^2}{4\,f(r)^2}\,dr\right)dr\right) \tag{5}$$

```
M > #4-potential vector ansatzs:
M > xi(r):=exp(I*k*(omega*int(sqrt((1)/(f(r))),r)))
```

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$$\xi(r) := \mathrm{e}^{\mathrm{I}\,k\,\omega\left(\int\sqrt{\frac{1}{f(r)}}\,\mathrm{d}r\right)} \tag{6}$$

```
M > xi2(t):=exp(-I*(omega*t))
```

$$\xi 2(t) := \mathrm{e}^{-\mathrm{I}\,\omega\,t} \tag{7}$$

```
M > xi3(u,v):=exp(I*(k_u*u + k_v*v))
```

$$\xi 3(u, v) := \mathrm{e}^{\mathrm{I}\,(k\_u\,u + k\_v\,v)} \tag{8}$$

```
M > xi5(u,v):=c*cos(k_u*u)*sin(k_v*v)
```

$$\xi 5(u, v) := c\cos(k\_u\,u)\,\sin(k\_v\,v) \tag{9}$$

```
M > #Complete case:
M > A1 := DGzip(([At, Ar, Au, Av])(t, r, u, v), [D_t, D_r, D_u,
    D_v], "plus"):
M > #Stationary propagating wave, temporal gauge:
M > A2:= DGzip(([0*Bt(u,v)*xi(t,r), 0*Br(u,v)*xi(t,r), Bu(u,v)*xi
    (t,r), Bv(u,v)*xi(t,r)]), [D_t, D_r, D_u, D_v], "plus"):
M > #Stationary propagating wave, , variable amplitude, temporal gauge:
M > A3:= DGzip(([0,Cr(r,u,v)*xi(t,r), Cu(r,u,v)*xi(t,r), Cv(r,u,
    v)*xi(t,r)]), [D_t, D_r, D_u, D_v], "plus"):
M > #Stationary propagating wave, transversally flat, temporal gauge:
M > A4:= DGzip(([0,0*Cr(r)*xi(t,r), A4u(u,v)*Cu(r)*xi(t,r), A4v
    (u,v)*Cv(r)*xi(t,r)]), [D_t, D_r, D_u, D_v], "plus"):
M > #Stationary wave, temporal gauge, separated variables:
M > A5:= DGzip(([0,Cr(r,u,v)*xi2(t), Tu(u,v)*Lu(r)*xi2(t), Tv(u,
    v)*Lv(r)*xi2(t)]), [D_t, D_r, D_u, D_v], "plus"):
M > #Stationary wave, temporal gauge, separated variables, transversal plane
      wave:
M > A6:= DGzip(([0,0*Cr(r,u,v)*xi2(t), xi2(t)*xi3(u,v)*xi(r)*L
    (r), xi2(t)*xi3(u,v)*xi(r)*L(r)]), [D_t, D_r, D_u, D_v],
    "plus")
```

$$\begin{aligned} A6 := {} & \mathrm{e}^{-\mathrm{I}\,\omega\,t}\,\mathrm{e}^{\mathrm{I}\,(k\_u\,u + k\_v\,v)}\,\mathrm{e}^{\mathrm{I}\,k\,\omega\left(\int\sqrt{\frac{1}{f(r)}}\,\mathrm{d}r\right)}\,L(r)\;D\_u \\ & + \mathrm{e}^{-\mathrm{I}\,\omega\,t}\,\mathrm{e}^{\mathrm{I}\,(k\_u\,u + k\_v\,v)}\,\mathrm{e}^{\mathrm{I}\,k\,\omega\left(\int\sqrt{\frac{1}{f(r)}}\,\mathrm{d}r\right)}\,L(r)\;D\_v \end{aligned} \tag{10}$$

```
M > xi4(r):=(1/(f(r))^(a))
```

$$\xi 4(r) := \frac{1}{f(r)^{a}} \tag{11}$$

```
M > #Ansatz
M > A7:= DGzip(([0,0*Cr(r,u,v)*xi2(t), xi2(t)*xi5(u,v)*xi(r), xi2
    (t)*xi5(u,v)*xi(r)]), [D_t, D_r, D_u, D_v], "plus")
```

$$A7 := e^{-I\omega t}\, c\cos(k\_u\, u)\,\sin(k\_v\, v)\, e^{I k \omega \left(\int \sqrt{\frac{1}{f(r)}}\, dr\right)} D\_u + e^{-I\omega t}\, c\cos(k\_u\, u)\,\sin(k\_v\, v)\, e^{I k \omega \left(\int \sqrt{\frac{1}{f(r)}}\, dr\right)} D\_v \tag{12}$$

```
M > 

M > #Covariant derivative of the 4-potential:
M > Dc:=CovariantDerivative(A6, C1):

M > #Lorentz condition, i.e. contraction of the indices of the covariant derivative
        of the 4-potential:
M > Lorentz:=ContractIndices(Dc, [[1,2]])
```

$$Lorentz := I\, e^{I\left(k\omega\left(\int \sqrt{\frac{1}{f(r)}}\, dr\right) + k\_u\, u + k\_v\, v - \omega t\right)} L(r)\, (k\_u + k\_v) \tag{13}$$

```
M > 
M > #The second covariant derivative of the 4-potential:
M > Dc2:=CovariantDerivative(Dc, C1):
M > 
M > 
M > #Contraction of the indices of the covariant derivatives to get the Laplacian:
M > Lap:=ContractIndices(g1i, Dc2, [[1, 2], [2,3]]):
M > 
M > 
M > ####"Mass term" of the Maxwell equations, i.e, contraction of the Ricci tensor
        with the 4-potential:
M > #Rising one index of the Ricci tensor:
M > R1up:=ContractIndices(g1i, R1, [[1, 1]]):
M > 
M > #Contraction of the Ricci tensor with the four-potential to give the mass:
M > Mass:=ContractIndices(A6, R1up, [[1,2]])
```

$$Mass := 0\, D\_t \tag{14}$$

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M > 
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M > #Maxwell equations!!!!!!!!!!!!!!!!:
M > Me:= simplify(Lap &minus Mass)
```

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$$
\begin{aligned}
Me := {} & \frac{1}{2\, f(r) \sqrt{\frac{1}{f(r)}}} \Bigg(\mathrm{e}^{\mathrm{I} \left(k\, \omega \left(\int \sqrt{\frac{1}{f(r)}}\, \mathrm{d}r \right) + k\_u\, u + k\_v\, v - \omega\, t \right)} \Bigg(-2 \sqrt{\frac{1}{f(r)}}\, L(r)\, k^{2}\, \omega^{2} \\
& - 2 \sqrt{\frac{1}{f(r)}}\, L(r)\, f(r)\, k\_u^{2} - 2 \sqrt{\frac{1}{f(r)}}\, L(r)\, f(r)\, k\_v^{2} + 2 \sqrt{\frac{1}{f(r)}}\, L(r)\, \omega^{2} \\
& + 4\, \mathrm{I} \left(\frac{\mathrm{d}}{\mathrm{d}r} L(r) \right) k\, \omega + \left(\frac{\mathrm{d}}{\mathrm{d}r} f(r) \right) \sqrt{\frac{1}{f(r)}} \left(\frac{\mathrm{d}}{\mathrm{d}r} L(r) \right) + 2 \sqrt{\frac{1}{f(r)}} \left(\frac{\mathrm{d}^{2}}{\mathrm{d}r^{2}} \right. \\
& \left. L(r) \right) f(r) \Bigg) \Bigg) D\_u + \frac{1}{2\, f(r) \sqrt{\frac{1}{f(r)}}} \Bigg(\mathrm{e}^{\mathrm{I} \left(k\, \omega \left(\int \sqrt{\frac{1}{f(r)}}\, \mathrm{d}r \right) + k\_u\, u + k\_v\, v - \omega\, t \right)} \Bigg(\\
& -2 \sqrt{\frac{1}{f(r)}}\, L(r)\, k^{2}\, \omega^{2} - 2 \sqrt{\frac{1}{f(r)}}\, L(r)\, f(r)\, k\_u^{2} - 2 \sqrt{\frac{1}{f(r)}}\, L(r)\, f(r)\, k\_v^{2} \\
& + 2 \sqrt{\frac{1}{f(r)}}\, L(r)\, \omega^{2} + 4\, \mathrm{I} \left(\frac{\mathrm{d}}{\mathrm{d}r} L(r) \right) k\, \omega + \left(\frac{\mathrm{d}}{\mathrm{d}r} f(r) \right) \sqrt{\frac{1}{f(r)}} \left(\frac{\mathrm{d}}{\mathrm{d}r} L(r) \right) \\
& + 2 \sqrt{\frac{1}{f(r)}} \left(\frac{\mathrm{d}^{2}}{\mathrm{d}r^{2}} L(r) \right) f(r) \Bigg) \Bigg) D\_v
\end{aligned}
\tag{15}
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