Homework 7

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Ques 2. Let the random variable X have the pmf

$$f(x) = (|x| + 1)^2/9, \qquad x = -1, 0, 1$$

Compute $E(X), E(X^2) and E(3X^2 - 2X + 4)$

Ans 2. Since

$$E(X) = \sum X f(x)$$

$$\begin{aligned} &= -1 * 4/9 + 0 * 1/9 + 1 * 4/9 \\ &= 0, \end{aligned}$$

$$E(X^2) = \sum X^2 f(x)$$

$$\begin{aligned} &= 1 * 4/9 + 0 * 1/9 + 1 * 4/9 \\ &= 8/9 \\ &= 0.88 \end{aligned}$$

and

$$\begin{aligned} E(3X^2 - 2X + 4) &= E(3X^2) - E(2X) + E(4) \\ &= (3 * E(X^2) - 2E(X) + 4) \\ &= 3 * 8/9 - 2 * 0 + 4 \\ &= 20/3 \\ &= 6.66 \end{aligned}$$

Ques 3. Let the random variable X be the number of days that a certain needs to be in the hospital. Suppose X has the pmf

$$f(x) = (5 - x)/10, \qquad x = 1, 2, 3, 4.$$

If the patient is to recieve \$200 from an insurance company for each of the first two days in the hospital and \$100 for each day after the first two days, what is the expected payment for the hospitalization?

Ans 3. Let P(X) where X=1,2,3,4 be payment recieved for hospitalization and X be number of days.

$$P(X) = \begin{cases} \$200 & X = 1, \\ \$400 & X = 2, \\ \$500 & X = 3, \\ \$600 & X = 4. \end{cases}$$

Expected payment for hospitalization = E(P(X)),   X=1,2,3,4
= $200*4/10 + $400*3/10 + $500*2/10 + $600*1/10
= $80 + $120 + $100 + $60
= $360

Ques 8. Let X be a random variable with support {1,2,3,5,15,25,50}, each point of which has the same probability 1/7. Argue that $c = 5$ is the value that minimizes $h(c) = E(|X - c|)$. Compare c with the value of b that minimizes $g(b) = E[(X - b)^2]$.

Ans 8. For

$$
\begin{aligned}
c = 5, \qquad h(c) &= E(|X-5|) \\
&= |-4|*1/7 + |-3|*1/7 + |-2|*1/7 + 0*1/7 + 10*1/7 + 20*1/7 + 45*1/7 \\
&= 84/7 \\
&= 12
\end{aligned}
$$

now checking for

$$
\begin{aligned}
c = 4, \qquad h(c) &= E(|X-4|) \\
&= |-3|*1/7 + |-2|*1/7 + |-1|*1/7 + 1*1/7 + 11*1/7 + 21*1/7 + 46*1/7 \\
&= 85/7 \\
&= 12.14
\end{aligned}
$$

and

$$
\begin{aligned}
c = 6, \qquad h(c) &= E(|X-6|) \\
&= |-5|*1/7 + |-4|*1/7 + |-3|*1/7 + |-1|*1/7 + 9*1/7 + 19*1/7 + 44*1/7 \\
&= 85/7 \\
&= 12.14.
\end{aligned}
$$

Since for $h(c)$ for $c = 4, 6$ is greater than $h(c)$ for $c = 5$, which indicates $h(5)$ is least in its neighbourhood also $h(c)$ is a function with only one minima, it implies $h(5)$ minimizes $h(c)$.