



Iran University of Science & Technology
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Digital Logic Design

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Number System

- Positioned number

$$N = (a_{n-1}a_{n-2} \dots a_1a_0 . a_{-1}a_{-2} \dots a_{-m})_r$$

- . = radix point
- r = radix or base
- n = number of integer digits to the left of the radix point
- m = number of fractional digits to the right of the radix point
- a_{n-1} = most significant digit (MSD)
- a_{-m} = least significant digit (LSD)

- Polynomial notation

- Series representation

$$N = a_{n-1} \times r^{n-1} + a_{n-2} \times r^{n-2} + \dots + a_0 \times r^0 + a_{-1} \times r^{-1} \dots + a_{-m} \times r^{-m}$$

$$\sum_{i=-m}^{n-1} a_i r^i$$

Outline

- Arithmetic Operations
 - Addition
 - Subtraction
 - Multiplication
 - Division
- Signed and Unsigned Numbers



Arithmetic Operations

Single-bit Binary Subtraction

- Consider two binary digits (A, B)
- How can we subtract these two binary numbers?

$$\begin{array}{r} A \\ - B \\ \hline 0 \end{array}$$

Single-bit Binary Subtraction (cont'd)

- Subtract two binary digits (A, B)

$$\begin{array}{r}
 & & A \\
 & - & B \\
 \hline
 & & 0
 \end{array}$$

$$\begin{array}{r}
 0 & 1 & 1 \\
 - 0 & - 0 & - 1 \\
 \hline
 0 & 1 & 0
 \end{array}$$

$$\begin{array}{r}
 0 & 1 0 \\
 - 1 & - 1 \\
 \hline
 & 1
 \end{array}$$

Borrow of 1

Borrow rule:

To subtract 1 from 0, we must **borrow 1** from the next significant column.

Multi-Bit Binary Subtraction

Multi-Bit Binary Subtraction (cont'd)



Multi-Bit Binary Subtraction (cont'd)



Multi-Bit Binary Subtraction (cont'd)

Multi-Bit Binary Subtraction (cont'd)



Multi-Bit Binary Subtraction (cont'd)



Multi-Bit Binary Subtraction (cont'd)



Multi-Bit Binary Subtraction (cont'd)



Multi-Bit Binary Subtraction (cont'd)

Multi-Bit Binary Subtraction (cont'd)

Column		a_6	a_5	a_4	a_3	a_2	a_1	a_0
Borrows						10		
Borrows					0	0	10	
Minuend		1	0	0	1	1	0	1
Subtracted	-			1	0	1	1	1
Difference						1	1	0

Multi-Bit Binary Subtraction (cont'd)

Column		a_6	a_5	a_4	a_3	a_2	a_1	a_0
Borrows						10		
Borrows					0	0	10	
Minuend		1	0	0	1	1	0	1
Subtracted	-			1	0	1	1	1
Difference					0	1	1	0

Multi-Bit Binary Subtraction (cont'd)

Column		a_6	a_5	a_4	a_3	a_2	a_1	a_0
Borrows						10		
Borrows					0	0	10	
Minuend		1	0	0	1	1	0	1
Subtracted	-			1	0	1	1	1
Difference					0	1	1	0

Multi-Bit Binary Subtraction (cont'd)

Column		a_6	a_5	a_4	a_3	a_2	a_1	a_0
Borrows						10		
Borrows					0	0	10	
Minuend		1	0	0	1	1	0	1
Subtracted	-			1	0	1	1	1
Difference					0	1	1	0

Multi-Bit Binary Subtraction (cont'd)

Column		a_6	a_5	a_4	a_3	a_2	a_1	a_0
Borrows						10		
Borrows					0	0	10	
Minuend		1	0	0	1	1	0	1
Subtracted	-			1	0	1	1	1
Difference					0	1	1	0

Multi-Bit Binary Subtraction (cont'd)

Column		a_6	a_5	a_4	a_3	a_2	a_1	a_0
Borrows						10		
Borrows		0	10		0	0	10	
Minuend		1	0	0	1	1	0	1
Subtracted	-			1	0	1	1	1
Difference					0	1	1	0

Multi-Bit Binary Subtraction

(cont'd)

Column		a_6	a_5	a_4	a_3	a_2	a_1	a_0
Borrows						10		
Borrows		0	10		0	0	10	
Minuend		1	0	0	1	1	0	1
Subtracted	-			1	0	1	1	1
Difference					0	1	1	0

Multi-Bit Binary Subtraction (cont'd)

Column		a_6	a_5	a_4	a_3	a_2	a_1	a_0
Borrows			1			10		
Borrows		0	10	10	0	0	10	
Minuend		1	0	0	1	1	0	1
Subtracted	-			1	0	1	1	1
Difference					0	1	1	0

Multi-Bit Binary Subtraction (cont'd)

Column		a_6	a_5	a_4	a_3	a_2	a_1	a_0
Borrows			1			10		
Borrows		0	10	10	0	0	10	
Minuend		1	0	0	1	1	0	1
Subtracted	-			1	0	1	1	1
Difference				1	0	1	1	0

Multi-Bit Binary Subtraction (cont'd)

Column		a_6	a_5	a_4	a_3	a_2	a_1	a_0
Borrows			1			10		
Borrows		0	10	10	0	0	10	
Minuend		1	0	0	1	1	0	1
Subtracted	-			1	0	1	1	1
Difference			1	1	0	1	1	0

Multi-Bit Binary Subtraction (cont'd)

Column		a_6	a_5	a_4	a_3	a_2	a_1	a_0
Borrows			1			10		
Borrows		0	10	10	0	0	10	
Minuend		1	0	0	1	1	0	1
Subtracted	-			1	0	1	1	1
Difference		0	1	1	0	1	1	0

Multi-Bit Binary Subtraction (cont'd)

- Subtract two binary numbers (A, B)

$$\begin{array}{r} 0 \\ 11010110 \\ - 01101011 \\ \hline \end{array}$$



Multi-Bit Binary Subtraction (cont'd)



Multi-Bit Binary Subtraction (cont'd)



Multi-Bit Binary Subtraction (cont'd)



Multi-Bit Binary Subtraction (cont'd)

Column		a_7	a_6	a_5	a_4	a_3	a_2	a_1	a_0
Borrows								10	
Borrows							0	0	10
Minuend		1	1	0	1	0	1	1	0
Subtracted	-	0	1	1	0	1	0	1	1
Difference							0	1	1

Multi-Bit Binary Subtraction (cont'd)

Column		a_7	a_6	a_5	a_4	a_3	a_2	a_1	a_0
Borrows								10	
Borrows							0	0	10
Minuend		1	1	0	1	0	1	1	0
Subtracted	-	0	1	1	0	1	0	1	1
Difference							0	1	1

Multi-Bit Binary Subtraction (cont'd)

Column		a_7	a_6	a_5	a_4	a_3	a_2	a_1	a_0
Borrows								10	
Borrows							0	0	10
Minuend		1	1	0	1	0	1	1	0
Subtracted	-	0	1	1	0	1	0	1	1
Difference							0	1	1

Multi-Bit Binary Subtraction (cont'd)

Column		a_7	a_6	a_5	a_4	a_3	a_2	a_1	a_0
Borrows								10	
Borrows					0	10	0	0	10
Minuend		1	1	0	1	0	1	1	0
Subtracted	-	0	1	1	0	1	0	1	1
Difference							0	1	1

Multi-Bit Binary Subtraction (cont'd)

Column		a_7	a_6	a_5	a_4	a_3	a_2	a_1	a_0
Borrows								10	
Borrows					0	10	0	0	10
Minuend		1	1	0	1	0	1	1	0
Subtracted	-	0	1	1	0	1	0	1	1
Difference							1	0	1

Multi-Bit Binary Subtraction (cont'd)

Column		a_7	a_6	a_5	a_4	a_3	a_2	a_1	a_0
Borrows								10	
Borrows					0	10	0	0	10
Minuend		1	1	0	1	0	1	1	0
Subtracted	-	0	1	1	0	1	0	1	1
Difference					0	1	0	1	1

Multi-Bit Binary Subtraction (cont'd)

Column		a_7	a_6	a_5	a_4	a_3	a_2	a_1	a_0
Borrows								10	
Borrows					0	10	0	0	10
Minuend		1	1	0	1	0	1	1	0
Subtracted	-	0	1	1	0	1	0	1	1
Difference					0	1	0	1	1

Multi-Bit Binary Subtraction (cont'd)

Column		a_7	a_6	a_5	a_4	a_3	a_2	a_1	a_0
Borrows								10	
Borrows					0	10	0	0	10
Minuend		1	1	0	1	0	1	1	0
Subtracted	-	0	1	1	0	1	0	1	1
Difference					0	1	0	1	1

Multi-Bit Binary Subtraction (cont'd)

Column		a_7	a_6	a_5	a_4	a_3	a_2	a_1	a_0
Borrows								10	
Borrows			0	10	0	10	0	0	10
Minuend		1	1	0	1	0	1	1	0
Subtracted	-	0	1	1	0	1	0	1	1
Difference					0	1	0	1	1

Multi-Bit Binary Subtraction (cont'd)

Column		a_7	a_6	a_5	a_4	a_3	a_2	a_1	a_0
Borrows								10	
Borrows			0	10	0	10	0	0	10
Minuend		1	1	0	1	0	1	1	0
Subtracted	-	0	1	1	0	1	0	1	1
Difference				1	0	1	0	1	1

Multi-Bit Binary Subtraction (cont'd)

Column		a_7	a_6	a_5	a_4	a_3	a_2	a_1	a_0
Borrows								10	
Borrows			0	10	0	10	0	0	10
Minuend		1	1	0	1	0	1	1	0
Subtracted	-	0	1	1	0	1	0	1	1
Difference				1	0	1	0	1	1

Multi-Bit Binary Subtraction (cont'd)

Column		a_7	a_6	a_5	a_4	a_3	a_2	a_1	a_0
Borrows								10	
Borrows			0	10	0	10	0	0	10
Minuend		1	1	0	1	0	1	1	0
Subtracted	-	0	1	1	0	1	0	1	1
Difference				1	0	1	0	1	1

Multi-Bit Binary Subtraction (cont'd)

Column		a_7	a_6	a_5	a_4	a_3	a_2	a_1	a_0
Borrows			10					10	
Borrows		0	0	10	0	10	0	0	10
Minuend		1	1	0	1	0	1	1	0
Subtracted	-	0	1	1	0	1	0	1	1
Difference				1	0	1	0	1	1

Multi-Bit Binary Subtraction (cont'd)

Column		a_7	a_6	a_5	a_4	a_3	a_2	a_1	a_0
Borrows			10					10	
Borrows		0	0	10	0	10	0	0	10
Minuend		1	1	0	1	0	1	1	0
Subtracted	-	0	1	1	0	1	0	1	1
Difference			1	1	0	1	0	1	1

Multi-Bit Binary Subtraction (cont'd)

Column		a_7	a_6	a_5	a_4	a_3	a_2	a_1	a_0
Borrows			10					10	
Borrows		0	0	10	0	10	0	0	10
Minuend		1	1	0	1	0	1	1	0
Subtracted	-	0	1	1	0	1	0	1	1
Difference		0	1	1	0	1	0	1	1

Thank You

