Digital Logic Design

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Number System

- Positioned number

$$N = (a_{n-1}a_{n-2} \ldots a_1a_0 \,.\, a_{-1}a_{-2} \ldots a_{-m})_r$$

  - . = radix point
  - r = radix or base
  - n = number of integer digits to the left of the radix point
  - m = number of fractional digits to the right of the radix point
  - a_{n-1} = most significant digit (MSD)
  - a_{-m} = least significant digit (LSD)

- Polynomial notation
  - Series representation

$$N = a_{n-1} \times r^{n-1} + a_{n-2} \times r^{n-2} + \ldots + a_0 \times r^0 + a_{-1} \times r^{-1} \ldots + a_{-m} \times r^{-m}$$

$$\sum_{i=-m}^{n-1} a_i r^i$$

Binary Addition, subtraction

- Consider two binary numbers (A, B) and a carry bit C_i
- How can we add these two binary numbers with a carry bit?

$$\begin{array}{rr} & C_i \\ & A \\ + & B \\ \hline & C\ S \end{array}$$

$$\begin{array}{rr} & A \\ - & B \\ \hline & O \end{array}$$

Outline

- Arithmetic Operations
  - Addition
  - Subtraction
  - Multiplication
  - Division
- Signed and Unsigned Numbers

Arithmetic Operations

Single-bit Binary Multiplication

- Consider two binary digits (A, B)
- How can we multiply these two binary numbers?

$$\begin{array}{rr} & A \\ \times & B \\ \hline & O \end{array}$$

Single-bit Binary Multiplication (cont'd)

- Multiply two binary digits (A, B)

0	1	1	0
× 0	× 0	× 1	× 1
0	0	1	0

A
× B
O

Multi-Bit Binary Multiplication (cont'd)

- 10111 * 1010

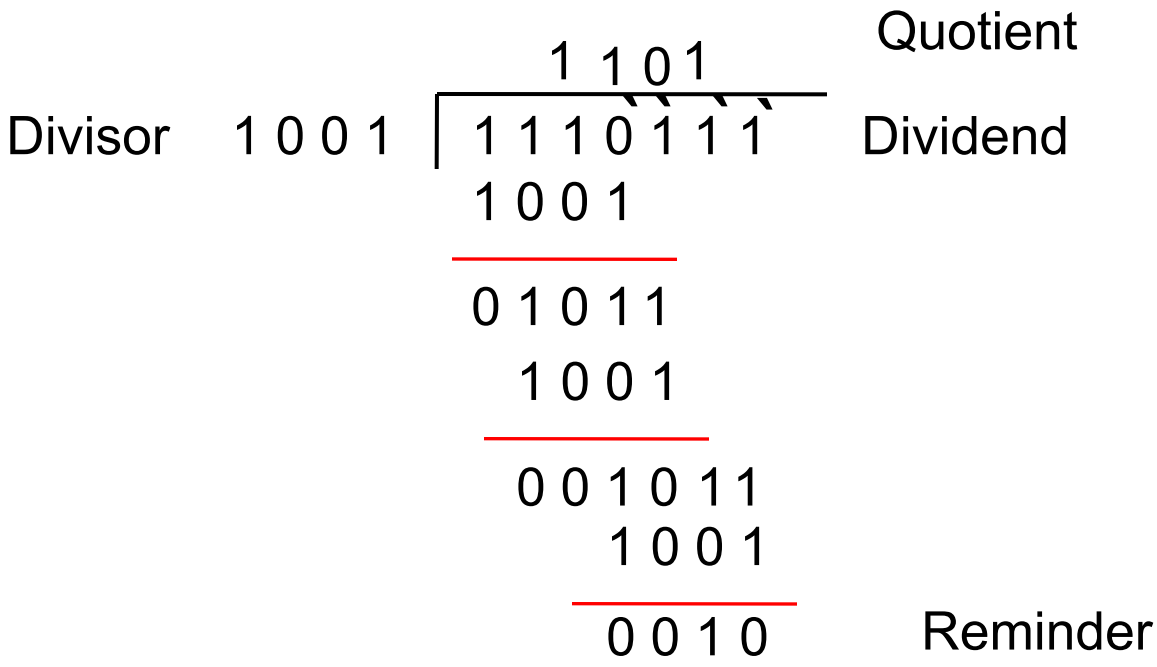
Multi-Bit Binary Multiplication (cont'd)


```
          1 0 1 1 1      Multiplicand
      x       1 0 1 0    Multiplier
    -------------------
        0 0 0 0 0
      1 0 1 1 1
    0 0 0 0 0
  1 0 1 1 1
-------------------------
  1 1 1 0 0 1 1 0
```

Multi-Bit Binary Division

- 1110111 / 1001

Multi-Bit Binary Division (cont'd)



Octal Arithmetic

- Addition
- Subtraction

```
   4163          6204
 + 7520        - 5173
------        ------
```

Octal Arithmetic (cont'd)

```
    4 1 6 3
+   7 5 2 0
-----------
        3
```

```
      1
    4 1 6 3
+   7 5 2 0
-----------
      0 3
```

```
      1
    4 1 6 3
+   7 5 2 0
-----------
    7 0 3
```

```
  1   1
    4 1 6 3
+   7 5 2 0
-----------
    3 7 0 3
```

```
  1   1
    4 1 6 3
+   7 5 2 0
-----------
  1 3 7 0 3
```

Octal Arithmetic (cont'd)

```
      6 2 0 4
    - 5 1 7 3
    ---------
            1
```

```
        1 10
      6 2 0 4
    - 5 1 7 3
    ---------
          1 1
```

```
        1 10
      6 2 0 4
    - 5 1 7 3
    ---------
        0 1 1
```

```
        1 10
      6 2 0 4
    - 5 1 7 3
    ---------
      1 0 1 1
```

Octal Arithmetic (cont'd)

- Multiplication

×	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7
2	0	2	4	6	10	12	14	16
3	0	3	6	11	14	17	22	25
4	0	4	10	14	20	24	30	34
5	0	5	12	17	24	31	36	43
6	0	6	14	22	30	36	44	52
7	0	7	16	25	34	43	52	61

Octal Arithmetic (cont'd)

- Multiplication

$$\begin{array}{r} 4\,1\,6\,3 \\ \times \quad 2\,5 \\ \hline \end{array}$$

Octal Arithmetic (cont'd)

```
      4 1 6 3
x           2 5
-----------------
    2 5 0 7 7
  1 0 3 4 6
-----------------
1 3 0 5 5 7
```

×	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7
2	0	2	4	6	10	12	14	16
3	0	3	6	11	14	17	22	25
4	0	4	10	14	20	24	30	34
5	0	5	12	17	24	31	36	43
6	0	6	14	22	30	36	44	52
7	0	7	16	25	34	43	52	61

Octal Arithmetic (cont'd)

- Division

$$\begin{array}{r} 4\,1\,6\,3 \\ \div \quad 2\,5 \\ \hline \end{array}$$

Octal Arithmetic (cont'd)

×	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7
2	0	2	4	6	10	12	14	16
3	0	3	6	11	14	17	22	25
4	0	4	10	14	20	24	30	34
5	0	5	12	17	24	31	36	43
6	0	6	14	22	30	36	44	52
7	0	7	16	25	34	43	52	61

```
        1 4 7
      ---------
2 5 | 4 1 6 3
      2 5
    -------
      1 4 6
    -
      1 2 4
    -------
        2 2 3
    -   2 2 3
    -----------
            0
```

Hexadecimal Arithmetic

- Addition
- Subtraction

```
   2A58          9F1B
+  71D0       -  4A36
-------       -------
```

Hexadecimal Arithmetic (cont'd)


```
  2 A 5 8
+ 7 1 D 0
---------
        8
```

```
    1
  2 A 5 8
+ 7 1 D 0
---------
      2 8
```

```
    1
  2 A 5 8
+ 7 1 D 0
---------
    C 2 8
```

```
    1
  2 A 5 8
+ 7 1 D 0
---------
  9 C 2 8
```

Hexadecimal Arithmetic (cont'd)

```
  9 F 1 B
- 4 A 3 6
---------
        5
```

```
    E 11
  9 F 1 B
- 4 A 3 6
---------
    E 5
```

```
    E 11
  9 F 1 B
- 4 A 3 6
---------
  4 E 5
```

```
    E 11
  9 F 1 B
- 4 A 3 6
---------
  5 4 E 5
```

Hexadecimal Arithmetic (cont'd)

- Multiplication

x	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F	10
1	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F	10
2	2	4	6	8	A	C	E	10	12	14	16	18	1A	1C	1E	20
3	3	6	9	C	F	12	15	18	1B	1E	21	24	27	2A	2D	30
4	4	8	C	10	14	18	1C	20	24	28	2C	30	34	38	3C	40
5	5	A	F	14	19	1E	23	28	2D	32	37	3C	41	46	4B	50
6	6	C	12	18	1E	24	2A	30	36	3C	42	48	4E	54	5A	60
7	7	E	15	1C	23	2A	31	38	3F	46	4D	54	5B	62	69	70
8	8	10	18	20	28	30	38	40	48	50	58	60	68	70	78	80
9	9	12	1B	24	2D	36	3F	48	51	5A	63	6C	75	7E	87	90
A	A	14	1E	28	32	3C	46	50	5A	64	6E	78	82	8C	96	A0
B	B	16	21	2C	37	42	4D	58	63	6E	79	84	8F	9A	A5	B0
C	C	18	24	30	3C	48	54	60	6C	78	84	90	9C	A8	B4	C0
D	D	1A	27	34	41	4E	5B	68	75	82	8F	9C	A9	B6	C3	D0
E	E	1C	2A	38	46	54	62	70	7E	8C	9A	A8	B6	C4	D2	E0
F	F	1E	2D	3C	4B	5A	69	78	87	96	A5	B4	C3	D2	E1	F0
10	10	20	30	40	50	60	70	80	90	A0	B0	C0	D0	E0	F0	100

Hexadecimal Arithmetic (cont'd)

- Multiplication

```
  5 C 2 A
x     D 5
-----------
```

Hexadecimal Arithmetic (cont'd)

x	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F	10
1	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F	10
2	2	4	6	8	A	C	E	10	12	14	16	18	1A	1C	1E	20
3	3	6	9	C	F	12	15	18	1B	1E	21	24	27	2A	2D	30
4	4	8	C	10	14	18	1C	20	24	28	2C	30	34	38	3C	40
5	5	A	F	14	19	1E	23	28	2D	32	37	3C	41	46	4B	50
6	6	C	12	18	1E	24	2A	30	36	3C	42	48	4E	54	5A	60
7	7	E	15	1C	23	2A	31	38	3F	46	4D	54	5B	62	69	70
8	8	10	18	20	28	30	38	40	48	50	58	60	68	70	78	80
9	9	12	1B	24	2D	36	3F	48	51	5A	63	6C	75	7E	87	90
A	A	14	1E	28	32	3C	46	50	5A	64	6E	78	82	8C	96	A0
B	B	16	21	2C	37	42	4D	58	63	6E	79	84	8F	9A	A5	B0
C	C	18	24	30	3C	48	54	60	6C	78	84	90	9C	A8	B4	C0
D	D	1A	27	34	41	4E	5B	68	75	82	8F	9C	A9	B6	C3	D0
E	E	1C	2A	38	46	54	62	70	7E	8C	9A	A8	B6	C4	D2	E0
F	F	1E	2D	3C	4B	5A	69	78	87	96	A5	B4	C3	D2	E1	F0
10	10	20	30	40	50	60	70	80	90	A0	B0	C0	D0	E0	F0	100

$$
\begin{array}{r}
5\,C\,2\,A \\
\times \quad D\,5 \\
\hline
1 \\
1\,C\,C\,D\,2 \\
+\;4\,A\,E\,2\,2 \\
\hline
4\,C\,A\,E\,F\,2
\end{array}
\qquad\qquad
3\,3\,8\,2\,9
$$

Check Yourself

1 1 0 1 0 0 0 0	1 1 0 1 1 0 0 0	1 1 0 0	1 1 0 0
- 1 0 0 0 1 1 0 1	+ 1 0 0 0 1 1 0 1	x 1 0 1	÷ 1 0 1
$(3\ 5\ 1)_8$	$(3\ 7\ 1)_8$	$(5\ 1)_8$	$(7\ 1)_8$
+ $(7\ 2\ 1)_8$	- $(3\ 5\ 1)_8$	x $(7\ 1)_8$	÷ $(5\ 1)_8$
$(C\ 5\ 8)_{16}$	$(C\ 5\ 8)_{16}$	$(C\ 8)_{16}$	$(C\ 8)_{16}$
+ $(A\ D\ B)_{!6}$	- $(3\ D\ B)_{16}$	x $(1\ B)_{16}$	÷ $(1\ B)_{16}$

Thank You