ROB 541 Project Presentation

Four Link Space Robot

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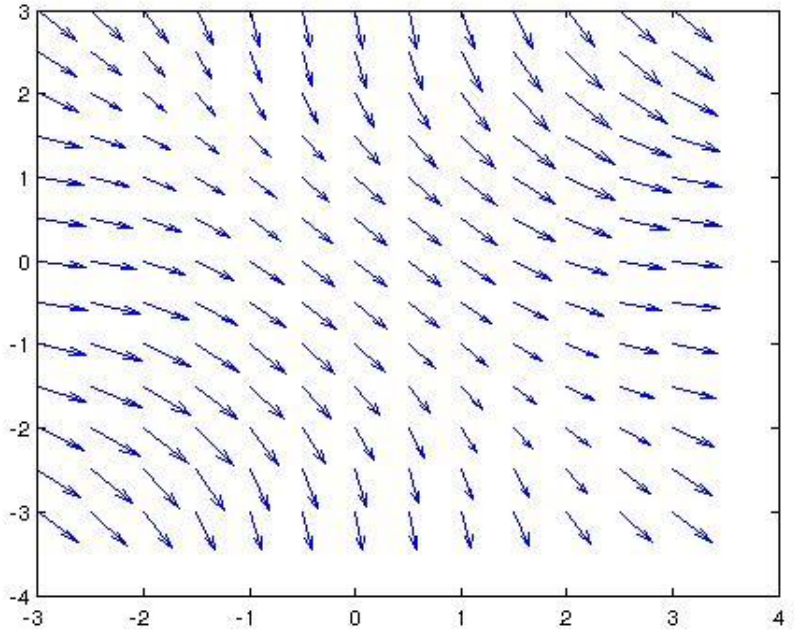
Objective: Extend to higher dimensions

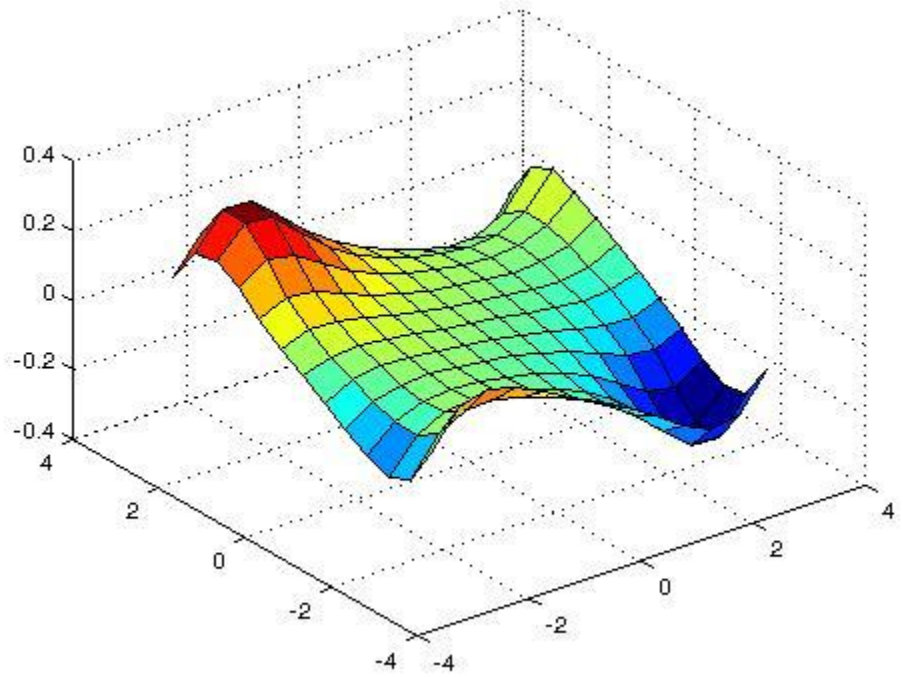
- Why Space Robot?
- Local Connection and Curl for 4 Link Robot
- Transformations in 3D

3 Link Space Robot

Local Connection



Curl of the Local Connection



Floating Snake Local Connection

Lagrangian (KE) Method

$$Kinetic\ Energy = [\overset{\circ}{g}\ \dot{\alpha}]\mathbb{M}(\alpha)\begin{bmatrix}\overset{\circ}{g}\\ \dot{\alpha}\end{bmatrix}$$

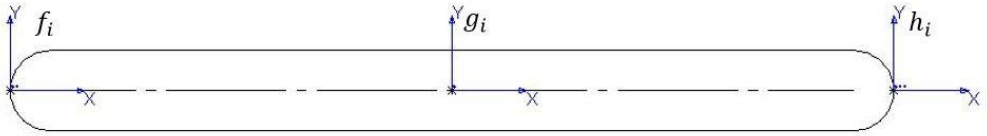
$$\mathbb{M}(\alpha) = \begin{bmatrix}\mathbb{I}(\alpha) & \mathbb{I}(\alpha)\boldsymbol{A}(\alpha)\\ (\mathbb{I}(\alpha)\boldsymbol{A}(\alpha))^T & m(\alpha)\end{bmatrix}$$

Momentum Conservation Method

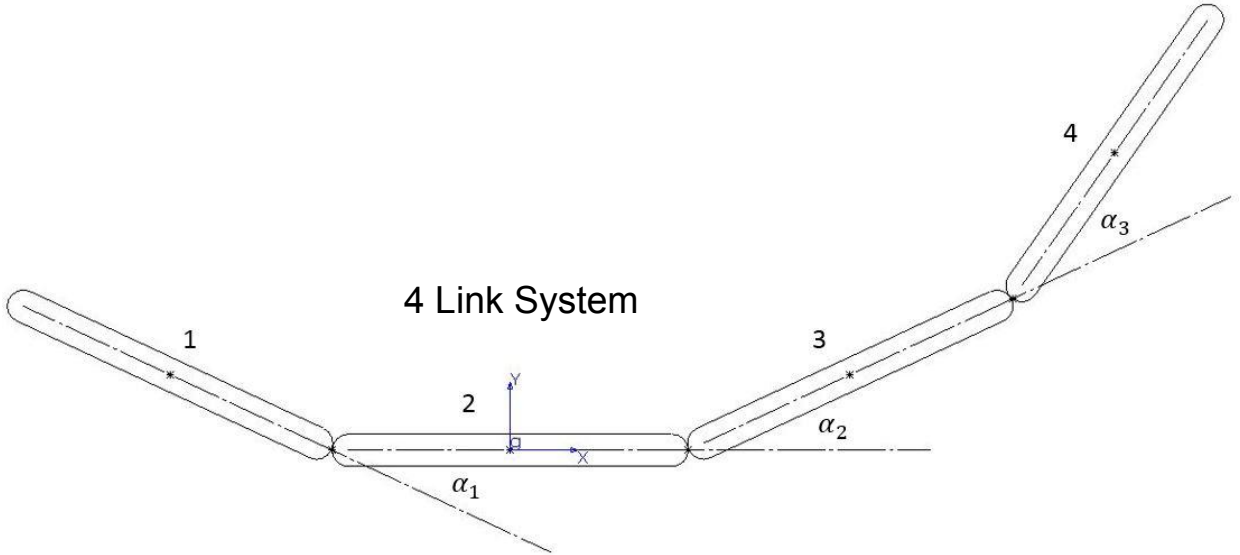
$$\begin{bmatrix}0\\0\\0\end{bmatrix} = \begin{bmatrix}J_x\\J_y\\J_\theta\end{bmatrix} = \omega_g\omega_r\begin{bmatrix}\overset{\circ}{g}\\ \dot{\alpha}\end{bmatrix}$$

$$\overset{\circ}{g} = -\boldsymbol{A}(\alpha)\dot{\alpha} = -\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ a_{31} & a_{32} & a_{33}\end{bmatrix}\begin{bmatrix}\dot{\alpha}_1\\ \dot{\alpha}_2\\ \dot{\alpha}_3\end{bmatrix}$$

System Diagram



Link Notation



4 Link System

Body Velocities

$$\overset{\circ}{\underset{g_1}{\rightarrow}} = \begin{bmatrix} \overset{\circ}{\underset{g^x}{\rightarrow}} \cos(\alpha_1) - \left(\overset{\circ}{\underset{g^y}{\rightarrow}} - \overset{\circ}{\underset{g^\theta}{\rightarrow}} (l_2/2) \right) \sin(\alpha_1) \\ \overset{\circ}{\underset{g^x}{\rightarrow}} \sin(\alpha_1) + \left(\overset{\circ}{\underset{g^y}{\rightarrow}} - \overset{\circ}{\underset{g^\theta}{\rightarrow}} (l_2/2) \right) \cos(\alpha_1) - \left(\overset{\circ}{\underset{g^\theta}{\rightarrow}} - \dot{\alpha}_1 \right)(l_1/2) \\ \overset{\circ}{\underset{g^\theta}{\rightarrow}} - \dot{\alpha}_1 \end{bmatrix}$$

Equation from Book

$$\overset{\circ}{\underset{g_1}{\rightarrow}} = \begin{bmatrix} \cos(\alpha_1) & -\sin(\alpha_1) & (l_2/2)\sin(\alpha_1) & 0 & 0 & 0 \\ \sin(\alpha_1) & \cos(\alpha_1) & -l_1/2 - (l_2/2)\cos(\alpha_1) & l_1/2 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \overset{\circ}{\underset{g^x}{\rightarrow}} \\ \overset{\circ}{\underset{g^y}{\rightarrow}} \\ \overset{\circ}{\underset{g^\theta}{\rightarrow}} \\ \dot{\alpha}_1 \\ \dot{\alpha}_1 \\ \dot{\alpha}_1 \end{bmatrix}$$

Equation For Code

Local Connection

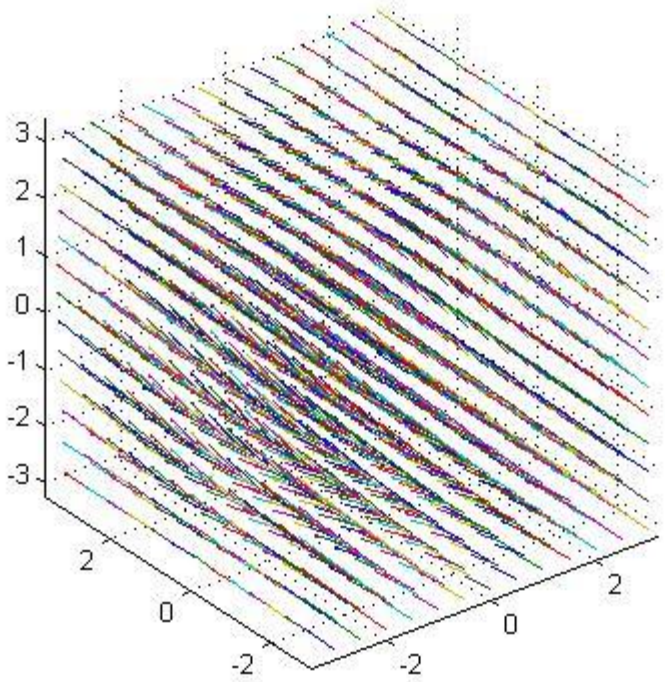
$$\overset{\circ}{\rightarrow}_{g_i} = link_i \begin{bmatrix} \overset{\circ}{\rightarrow}_{g} \\ \dot{\alpha} \end{bmatrix} \longrightarrow M_i = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & J + m * {d_2}^2 \end{bmatrix}$$

$$\mathbb{M}(\alpha) = \sum (link_i)^T M_i link_i$$

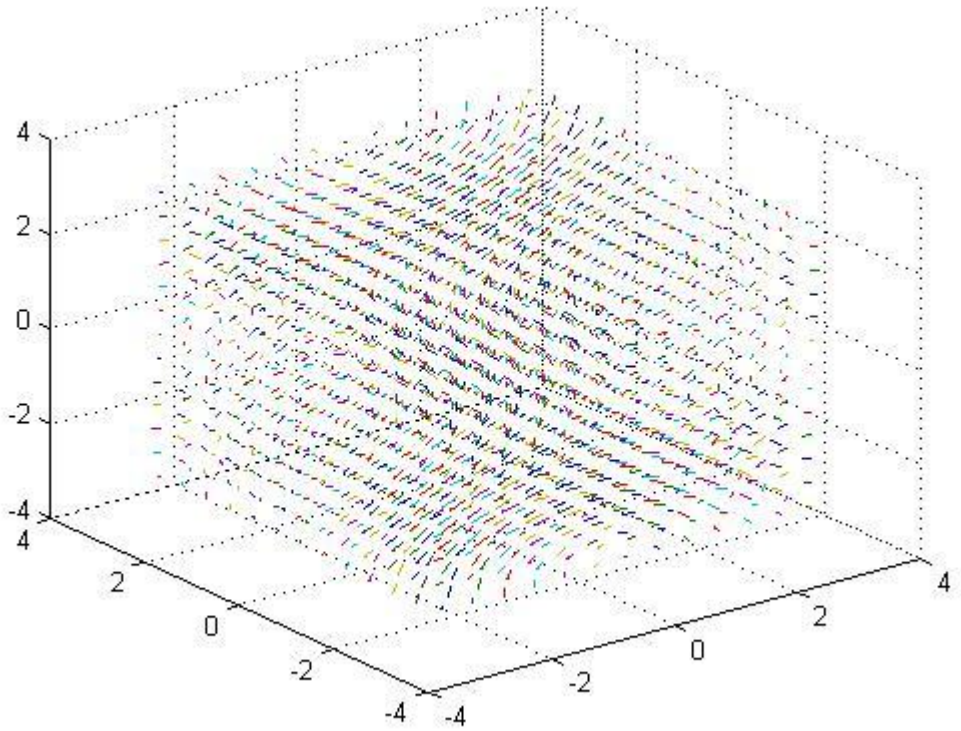
Mass Matrix

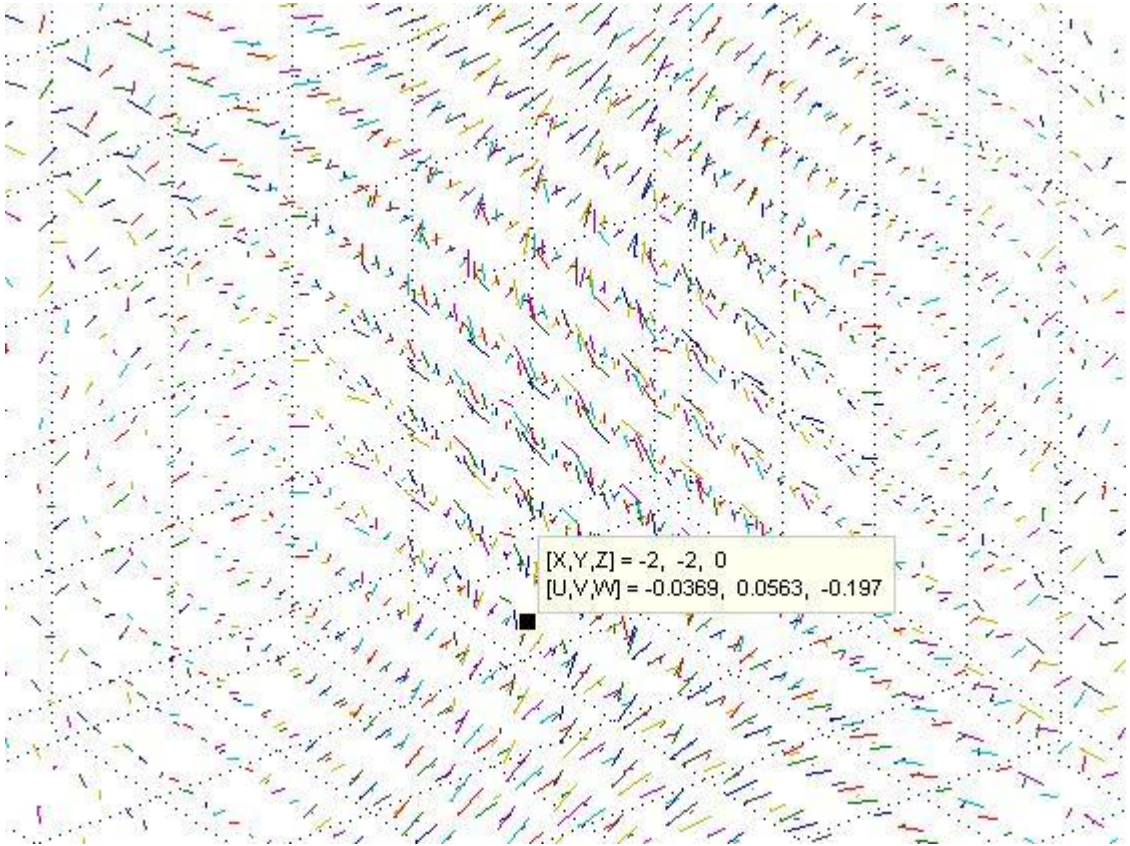
-(9*cos(a1 + a2) + 39*cos(a1) + 3*cos(a1 + a2 + a3) + 37)/(18*cos(a1 + a2) + 42*cos(a2 + a3) + 54*cos(a1) + 126*cos(a2) + 78*cos(a3) + 6*cos(a1 + a2 + a3) + 220)

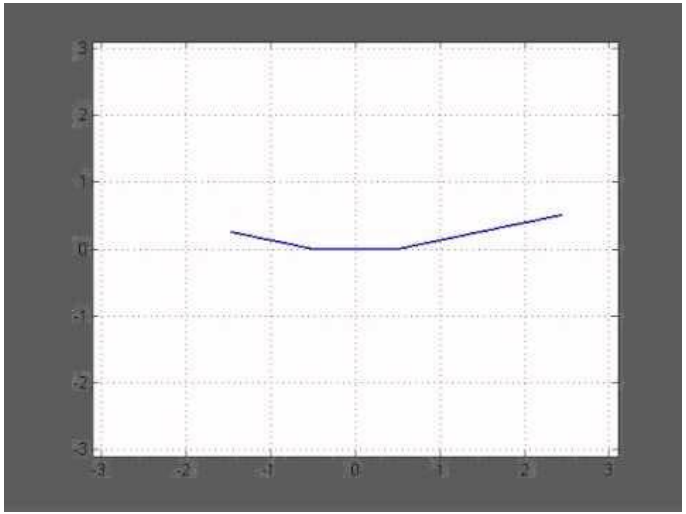
Vector Field

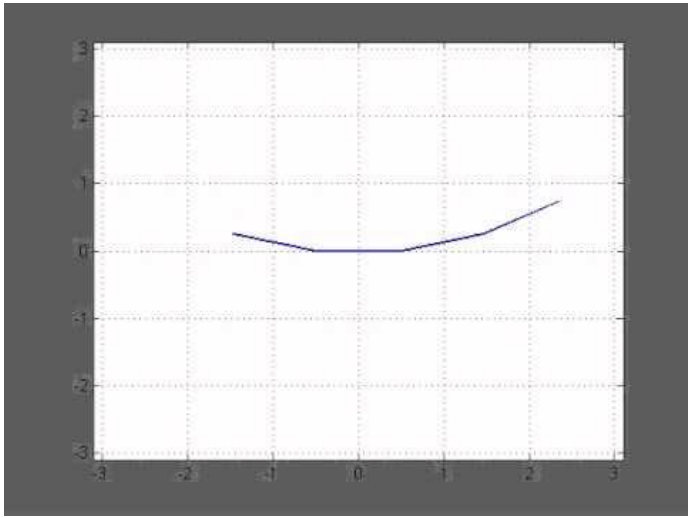


‘Curl’




[X,Y,Z] = -2, -2, 0
[U,V,W] = -0.0369, 0.0563, -0.197


3
2
1
0
-1
-2
-3
-3
-2
-1
0
1
2
3


3
2
1
0
-1
-2
-3
-3
-2
-1
0
1
2
3

Transforms

SO(3) Rotation matrix:

θ: Rotation about X axis (fixed)

ψ: Rotation about Y axis (fixed)

ϕ: Rotation about Z axis (fixed)

$$\begin{bmatrix} cos\phi cos\psi & sin\theta sin\phi cos\psi - cos\theta sin\psi & cos\theta sin\phi cos\psi + sin\theta sin\psi \\ cos\phi sin\psi & sin\theta sin\phi sin\psi + cos\theta cos\psi & cos\theta sin\phi sin\psi - sin\theta cos\psi \\ -sin\phi & sin\theta cos\phi & cos\theta cos\phi \end{bmatrix}$$

Infinitesimal Rotation About (0,0,-0.5)

Infinitesimal Rotation About (0,0,-2)

Rotation:

$$\begin{bmatrix} 0.6897 & -0.4463 & 0.1217 \\ -0.4463 & 0.2888 & -0.0788 \\ 0.1217 & -0.0788 & 0.0215 \end{bmatrix}$$

Scaling factor: 3.0552

Merging Infinitesimal Motions

Thank You