ME 533 - Final Project
Controlling a Flexible Drive with Uncertain Parameters

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I. Introduction

In many industrial applications, motion delivery systems are required to operate with varying load masses. Even though accurate models, and therefore model based linear controller design approaches, of such systems are available through advanced system identification techniques and realistic analytical representations, variations from the modelled parameters due to variable load masses alters the performance of the closed loop control performance. Conventionally, this problem has been overcome by tuning the controllers with high gains in order to achieve high robustness against parameter variations. This solution, however, increases the power requirement of driver circuits as a trade off by requiring control signals to have high values during operation.

In order to overcome this problem, several applications of parameter adaptation strategies in order to capture the variations in the load mass are proposed in this project. Comparison of these control strategies with each other, as well performance of the state-of-the-art P-PI control strategy that is usually applied to such systems against parameter variations will also be presented.

II. Modeling of Flexible Drives

Most commonly used motion delivery systems include ball screw mechanisms and series elastic actuators. Motion transfer using such mechanisms is carried out by materials which have limited stiffness. This implies that such systems can be modelled more realistically by taking their flexibility properties into account compared to modeling them as rigid bodies. A two mass drive model can be used to capture the dynamics of a ball screw system or a series elastic actuator driving a load. A general representation of a two mass system can be seen in Figure 1.

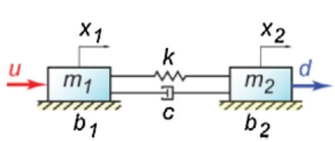

Fig. 1: General representation of a two mass system

Figure 1 represents the most general case. Here; m_1, b_1, k, c, m_2 and b_2 represents the equivalent mass of actuator inertia, viscous friction due to actuator bearing, flexibility of transmission element, material damping of transmission element, equivalent mass of load inertia and viscous friction due to bearings on the load side respectively. However, $c = 0$ assumption is going to hold for the sake of simplicity throughout the paper. The equations of motion of the system can be written using Lagrange's method or Newton's second law of motion as follows:

$$m_1\ddot{x}_1 = u - k(x_1 - x_2) - b_1\dot{x}_1 \tag{1}$$

$$m_2\ddot{x}_2 = k(x_1 - x_2) - b_2\dot{x}_2 \tag{2}$$

By applying a Laplace transform to the equations and rewriting x_1 in terms of x_2, it is possible to obtain an equation purely in terms of x_2 (the load). Transfer functions from input u and outputs x_1 and x_2 are [1]

$$\frac{X_1(s)}{U(s)} = \frac{(m_2 s^2 + b_2 s + k)/(m_1 m_2)}{s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0} \tag{3}$$

$$\frac{X_2(s)}{U(s)} = \frac{k/(m_1 m_2)}{s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0} \tag{4}$$

where

$$a_0 = 0, \qquad a_1 = \frac{(b_1 + b_2)k}{m_1 m_2},$$

$$a_2 = \frac{b_1 b_2 + (m_1 + m_2)k}{m_1 m_2}, \qquad a_3 = \frac{m_1 b_2 + m_2 b_1}{m_1 m_2}$$

Using the obtained transfer function, a state space model of the system can be obtained in the companion form as follows

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u \tag{5}$$

where

$$\mathbf{x} = \begin{bmatrix} x_2 \\ \dot{x}_2 \\ \ddot{x}_2 \\ \dddot{x}_2 \end{bmatrix}, \qquad \mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -a_0 & -a_1 & -a_2 & -a_3 \end{bmatrix},$$

$$\mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ k/(m_1 m_2) \end{bmatrix}$$

III. Designing Controllers

Since the flexible drives can be modelled as fourth order linear systems, it is common practice to design and implement linear controllers using root locus, frequency response or state space techniques. In this section a conventional linear controller and several adaptive controllers are described. All introduced controllers in this section designed by using the system model with the following control signal normalized values:

$$k/(m_1 m_2) = 5.7498(10^6) \quad [\frac{mm}{Vs^4}],$$

$$a_1 = 1.2851(10^4) \quad [\frac{1}{s^3}],$$

$$a_2 = 4.134(10^3) \quad [\frac{1}{s^2}], \quad a_3 = 11.3483 \quad [\frac{1}{s}] \tag{6}$$

A. P-PI Control

Most commonly used linear controllers for such systems are P-PI controllers which contain a velocity control loop where the velocity is controlled by means of a PI controller and a position control loop where the position is controlled by means of a proportional controller. An example block diagram of a P-PI controller is shown at figure 2.

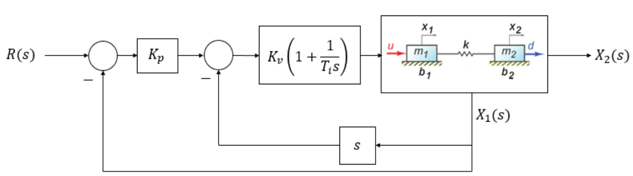

Fig. 2: Block diagram of a P-PI controller

As it can be seen from Figure 2, position measurement for feedback is obtained from m_1. The rationale behind this selection is the fact that closed loop systems based on measurements from m_2 have the risk to be unstable for high controller gains. With several simple block diagram reduction operations, it can be shown that P-PI controllers increase the order of the system to be controlled by one due to its integral action and add two zeros to the system. This behavior of P-PI controllers is analogous to PID controllers. However, one of the zeros of the controller does not appear in the closed loop transfer function. Since the zeros of a transfer function attenuate the performance as they get closer to the dominant pole region in the complex plane, eliminating the effect of one of the zeros in the closed loop transfer function is an advantage of P-PI controllers over PID controllers.

Since P-PI controllers have 3 free parameters and the closed loop transfer function (for flexible drives) have 5 poles, only 3 of the closed loop poles can be placed arbitrarily. Table I shows the closed loop poles and zeros of the P-PI controller specifically tuned for the system defined by values at equation set (6). The controller is designed using the procedure defined by Zirn.

Closed Loop Zeros	-3.1191 (effective) -14.8865 (ineffective)
Closed Loop Poles	-3.1191 $-19.8897 \pm 52.6017i$ $-21.4721 \pm 17.0394i$

TABLE I: Closed Loop Poles and Zeros for a P-PI Controller

B. Single Output Feedback Adaptive Control

In this section, and adaptive controller which uses a control scheme to cancel the behavior of the controlled system and to impose a desired behavior is introduced. Since the model of the system is not known, this controller tries to find the ideal control parameters to achieve perfect tracking by means of updating the controller parameters during operation. Figure 3 shows the block diagram of a Single Output Feedback Adaptive Controller.

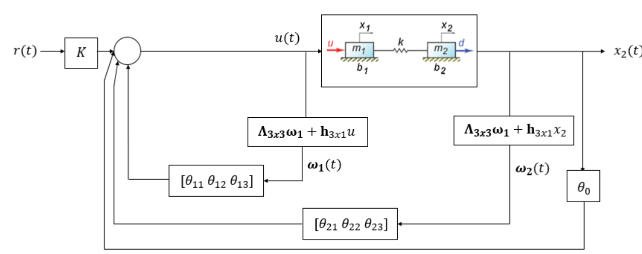

Fig. 3: Block diagram of a Single Output Feedback Adaptive Controller

Here, $\mathbf{\Lambda}$ and $\boldsymbol{h}$ define a state space equation which generates the state output vector of $\boldsymbol{\omega_1}$ when subjected to signal u and state output vector of $\boldsymbol{\omega_2}$ when subjected to signal x_2. Vectors $\boldsymbol{\theta_1}$ and $\boldsymbol{\theta_2}$ ($\boldsymbol{\theta_1} = [\theta_{11} \ \theta_{12} \ \theta_{13}]$ and $\boldsymbol{\theta_2} = [\theta_{21} \ \theta_{22} \ \theta_{23}]$) generates scalar values out of these values. In addition, K and θ_0 are scalar gains. Static gain K compensates the difference between static gain of the open loop system and the desired closed loop behavior while $\boldsymbol{\theta_1}$ cancels the zeros of the open loop system and $\boldsymbol{\theta_2}$ and θ_0 together moves the poles of open loop system to desired closed loop locations [2]. With some algebra, it can be shown that the eigenvalues of $\mathbf{\Lambda}$ constitute the zeros of closed loop transfer function [2]. Therefore, they should lie further than closed loop poles in the left half side of the complex plane in order to be non-dominant. In order to control a system with relative degree n, $\boldsymbol{\theta_2}$ has to have $n-1$ eigenvalues [2]. Therefore, 3 closed loop zeros must be assigned in order to control the flexible drive model described by equation set (6). However, this effect on closed loop zeros is an indirect effect and the desired closed loop behavior must be assigned with the same number of relative degree with the system to be controlled [2]. Let us define the desired closed loop behavior $G_d(s)$ with the same poles shown in table I as follows

$$G_d(s) = \frac{1}{\begin{array}{r}\big(4.208(10^{-7})s^4 + 3.481(10^{-5})s^3 \\ + 0.002366s^2 + 0.06973s + 1\big)\end{array}} \quad (7)$$

$G_d(s)$ has the closed loop poles shown in table I except -3.1191. Since this pole was cancelled by the P-PI controller zero, the closed loop behavior of two systems can be assumed as identical.
Using above definitions, the control law for the given closed loop system in Figure 3 becomes

$$u(t) = K(t)r(t) + \boldsymbol{\theta_1}(t)\boldsymbol{\omega_1}(t) + \boldsymbol{\theta_2}(t)\boldsymbol{\omega_2}(t) + \theta_0(t)x_2(t) \quad (8)$$

In order to write the control law in a more compact form, let us define the following vectors

$$\boldsymbol{\theta}(t) = [k(t)\ \ \boldsymbol{\theta_1}(t)\ \ \boldsymbol{\theta_2}(t)\ \ \theta_0(t)]^T$$
$$\boldsymbol{\omega}(t) = [r(t)\ \ \boldsymbol{\omega_1^T}(t)\ \ \boldsymbol{\omega_2^T}(t)\ \ x_2(t)]^T$$

Using these vectors, the control law can be written in a more compact form as

$$u(t) = \boldsymbol{\theta}^T(t)\boldsymbol{\omega}(t) \quad (9)$$

In equation (9), $\boldsymbol{\theta}$ consists of all unknown parameters while $\boldsymbol{\omega}$ can be constructed using the reference input and measurement of x_2. Therefore, an adaptation law can be used to determine the ideal control parameters during operation. The error between the desired closed loop performance and the behavior of the system shown on Figure 3 $e(t)$ cannot directly be used for adaptation [2]. A so called augmented error is defined in order to construct the adaptation law. The definition of augmented error $\varepsilon(t)$ is

$$\varepsilon(t) = e(t) + \alpha(t)\eta(t) \quad (10)$$

where

$$\eta(t) = \boldsymbol{\phi}^T L^{-1}\left[G_d(s)L[\boldsymbol{\omega}]\right] + L^{-1}\left[G_d(s)L[\boldsymbol{\phi}^T\boldsymbol{\omega}]\right],$$

$$\boldsymbol{\phi}(t) = \boldsymbol{\theta}(t) - \boldsymbol{\theta}_{ideal}$$

Here, $\alpha(t)$ is an adaptation parameter and updated at each sampling time and $L[.]$ is the Laplace transform operator. Defining the augmented error allows us to gradient method with normalization to update the adaptation parameters [2]. Finally the adaptation laws are

$$\dot{\boldsymbol{\theta}} = -\frac{sgn\big(k/(m_1m_2)\big)\gamma\varepsilon\ \underline{\boldsymbol{\omega}}}{1 + \underline{\boldsymbol{\omega}}^T\underline{\boldsymbol{\omega}}} \quad (11)$$

$$\dot{\alpha} = -\frac{\gamma\varepsilon\ \eta}{1 + \underline{\boldsymbol{\omega}}^T\underline{\boldsymbol{\omega}}} \quad (12)$$

C. Sliding Variable Adaptive Control

For sliding variable adaptive control, we use the transfer function derived in Eq. 4 to rewrite our dynamic equation as [1]

$$x_2^{(4)}\frac{m_1m_2}{k} = -a_1'x_2^{(3)} - a_2'x_2^{(2)} - a_3'\dot{x}_2 + u \quad (13)$$

where $a_1' = m_1b_2 + m_2b_1/k$, $a_2' = b_1b_2 + (m_1 + m_2)$ and $a_3' = (b_1 + b_2)$.
Consider the Lyapunov function

$$V = \frac{1}{2}Js^2 \quad (14)$$

where $J = \frac{k}{m_1m_2}$ and $s = \mathbf{c}^T(\mathbf{x} - \mathbf{x_{des}})$ with $\mathbf{x}$ being the state vector. c^T is chosen such that the coefficients form a Hurwitz polynomial. A simple way to do this is to use

$$s = (\frac{d}{dt} + \lambda)^{n-1}(x - x_{des}) \quad (15)$$

and substitute an appropriate value of λ. The chosen Lyapunov function is clearly convex and thus has a lower bound, and is positive definite. Since s is a scalar, it is easy to show

$$\dot{V} = J\dot{s} \quad (16)$$

$$= -c_4\begin{bmatrix}\frac{c_{(1:3)}}{c_4}(\dot{x}_{(1:3)} - \dot{x}_{(1:3)des}) & x_2^{(3)} & x_2^{(2)} & \dot{x}_2\end{bmatrix}\begin{bmatrix}J \\ a_1' \\ a_2' \\ a_3'\end{bmatrix} + c_4u \quad (17)$$

If we choose the control law

$$u = \mathbf{Ya} - \alpha s \quad (18)$$

where $\mathbf{Y} = \begin{bmatrix}\frac{c_{(1:3)}}{c_4}(\dot{x}_{(1:3)} - \dot{x}_{(1:3)des}) & x_2^{(3)} & x_2^{(2)} & \dot{x}_2\end{bmatrix}$ and $\mathbf{a} = \begin{bmatrix}J & a_1' & a_2' & a_3'\end{bmatrix}^T$ and α is a positive value, it can be shown by substitution that the derivative of the Lyapunov function $\dot{V}$ is negative definite. Thus, the controller makes the system stable. However, since all the parameters of the system are not known, $\mathbf{a}$ may not contain the true values of the system. Due to an error in the estimated value, the conditions for stability may not hold. Instead, we can use the Lyapunov function

$$V = \frac{1}{2}Js^2 + \frac{1}{2}\tilde{\mathbf{a}}^T\mathbf{P}^{-1}\tilde{\mathbf{a}} \quad (19)$$

where $\tilde{\mathbf{a}} = \hat{\mathbf{a}} - \mathbf{a}$ with $\hat{\mathbf{a}}$ being the estimated value of $\mathbf{a}$ at any given time. The derivative of the Lyapunov function becomes

$$\dot{V} = sJ\dot{s} + \dot{\hat{\mathbf{a}}}\mathbf{P}^{-1}\hat{\mathbf{a}} \quad (20)$$

By substituting $u = \mathbf{YA} - \alpha s$,

$$\dot{V} = -\alpha s^2 + s\mathbf{Y}\hat{\mathbf{a}} + \dot{\hat{\mathbf{a}}}\mathbf{P}^{-1}\hat{\mathbf{a}} \quad (21)$$

Thus, to ensure $\dot{V}$ remains negative definite even in the presence of unknown parameters, we use the adaptation law given in Eq. 22

$$\dot{\hat{\mathbf{a}}} = -\mathbf{P}\mathbf{Y}^T s \quad (22)$$

where $\mathbf{P}$ is a constant, symmetric, positive definite matrix. The adaptation cancels out the terms arising due to error in the value of $\mathbf{a}$ in the derivative of the Lyapunov function and ensures stability.

D. State Feedback Sliding Variable Adaptive Control

In this section, an adaptive controller that uses the state space model (5) and sliding variable control is introduced. Let us partition equation (5) in the following way in order to be able to implement the controller [3]

$$\left[\begin{array}{c}\dot{x}_2\\ \ddot{x}_2\\ \dddot{x}_2\\ \hline x_2^{iv}\end{array}\right]=\left[\begin{array}{ccc|c}0&1&0&0\\0&0&1&0\\0&0&0&1\\ \hline -a_0&-a_1&-a_2&-a_3\end{array}\right]\left[\begin{array}{c}x_2\\ \dot{x}_2\\ \ddot{x}_2\\ \hline \dddot{x}_2\end{array}\right]+\left[\begin{array}{c}0\\0\\0\\ \hline k/(m_1m_2)\end{array}\right]u \tag{23}$$

or in more compact form

$$\left[\begin{array}{c}\dot{z}_1\\ \hline \dot{z}_2\end{array}\right]=\left[\begin{array}{c|c}\mathbf{A_{11}}&\mathbf{A_{12}}\\ \hline \mathbf{A_{21}}&\mathbf{A_{22}}\end{array}\right]\left[\begin{array}{c}z_1\\ \hline z_2\end{array}\right]+\left[\begin{array}{c}\mathbf{B_1}\\ \hline \mathbf{B_2}\end{array}\right]u$$

Now, let us define the sliding condition as follows;

$$\begin{aligned}\sigma&=\left[\begin{array}{ccc|c}s_1&s_2&s_3&s_4\end{array}\right]\left[\begin{array}{c}x_2\\ \dot{x}_2\\ \ddot{x}_2\\ \dddot{x}_2\end{array}\right]\\ &=\mathbf{Sz}=\mathbf{S_1z_1}+\mathbf{S_2z_2}=0\end{aligned} \tag{24}$$

where σ is the sliding variable. When condition defined by equation 24 is satisfied, $\mathbf{z_2}$ can be written as: $\mathbf{z_2}=-\mathbf{S_2^{-1}S_1z_1}$. Therefore, dynamics of $\mathbf{z_1}$ can be written as

$$\dot{\mathbf{z}}_1=\mathbf{A_{11}z_1}+\mathbf{A_{12}z_2}=(\mathbf{A_{11}}-\mathbf{A_{12}S_2^{-1}S_1})\mathbf{z_1} \tag{25}$$

Since $\mathbf{S_1}$ and $\mathbf{S_2}$ are control parameters, dynamic behavior of $\mathbf{z_1}$ can be arbitrarily selected by placing the eigenvalues of matrix $(\mathbf{A_{11}}-\mathbf{A_{12}S_2^{-1}S_1})$ to desired locations. In order to derive a control law which will guarantee tracking convergence for time varying trajectories, sliding variable should be modified as follows

$$\sigma=\mathbf{S}(\mathbf{z_d}-\mathbf{z}) \tag{26}$$

where $\mathbf{z}_d$ is a vector containing the desired trajectories for each state. The derivation of the control law will be carried out by defining a Lyapunov function candidate and making sure that its derivative always decreases for nonzero tracking errors. Let us define a Lyapuonv function candidate as follows

$$V=\frac{1}{2}\sigma^2 \tag{27}$$

Derivative of this equation is going to be

$$\dot{V}=\sigma\dot{\sigma}=\sigma\mathbf{S}(\mathbf{Az}+\mathbf{B}u-\dot{\mathbf{z}}_\mathbf{d}) \tag{28}$$

Equation 28 does not guaranteed to be negative for all nonzero $\mathbf{z}$ values. Let us equate equation (28) to a value that is negative for all nonzero $\mathbf{z}$ values and then solve the equation for u as follows

$$\dot{V}=-K_s\sigma^2 \quad\rightarrow\quad \mathbf{SAz}+\mathbf{SB}u-\mathbf{S}\dot{\mathbf{z}}_\mathbf{d}=-K_s\sigma$$

$$u=(\mathbf{SB})^{-1}(\mathbf{S}\dot{\mathbf{z}}_\mathbf{d}-K_s\sigma-\mathbf{SAz}) \tag{29}$$

In this adaptive controller, the adaptation parameters are contained in the vector $\mathbf{SA}$. In order to use the same adaptation law with section III-C, let us rephrase the derivative of sliding variable in the following way

$$\dot{\sigma}=(\mathbf{SB})u-\mathbf{S}\dot{\mathbf{z}}_\mathbf{d}-\mathbf{z}^T\left[-\mathbf{SA}^T\right] \tag{30}$$

It can be shown that with such manipulation, adaptation law defined in (22) can be used where $\mathbf{Y}=\mathbf{z}^T$, $s=\sigma$ and $\mathbf{a}=-\mathbf{SA}^T$.

E. Sliding Variable Robust Adaptive Control

Robust adaptive control can be used when external disturbances are applied to the system. For simplicity, we assume that this disturbance appears in the equation as follows

$$x_2^{(4)}\frac{m_1m_2}{k}=-a_1x_2^{(3)}-a_2x_2^{(2)}-a_3\dot{x}_2+u+w \tag{31}$$

The approach used is similar to adaptive control with additional terms to swamp uncertainties. In order to ensure that no drastic change in parameters occurs due to the disturbance while near the desired state, the adaptation law is also modified. Here, we define our Lyapunov function as

$$V=\frac{1}{2}Js_\Delta^2+\frac{1}{2}\tilde{\mathbf{a}}^T\mathbf{P}^{-1}\tilde{\mathbf{a}} \tag{32}$$

where s_Δ is defined as follows

$$\begin{aligned}s_\Delta&=s-c \text{ if s } <-\phi\\ s_\Delta&=s+c \text{ if s } >\phi\\ s_\Delta&=0 \text{ if} -\phi<s<\phi\end{aligned} \tag{33}$$

The control law used is

$$u=-\mathbf{Y}\hat{\mathbf{a}}-\alpha sat(\frac{s}{\phi})-\hat{f} \tag{34}$$

where $\hat{f}$ is an estimate of the disturbance and α is a positive value. It can be shown that when the adaptation law is

$$\dot{\hat{\mathbf{a}}}=-\mathbf{P}\mathbf{Y}^Ts_\Delta \tag{35}$$

the Lyapunov function is positive definite and its derivative is negative definite, leading to stability.

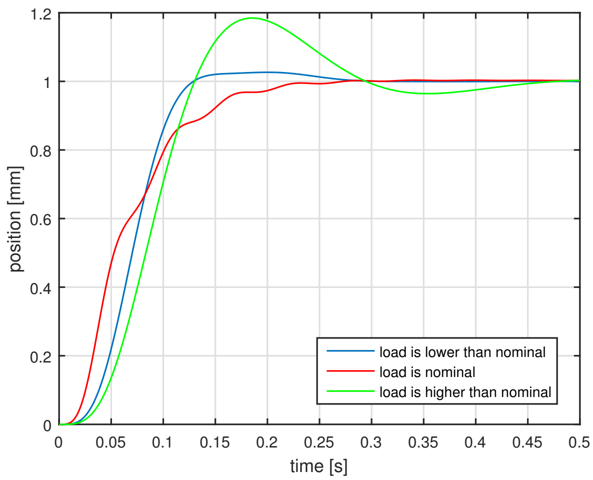

Fig. 4: Performance of a PPI controller with nominal and various loads

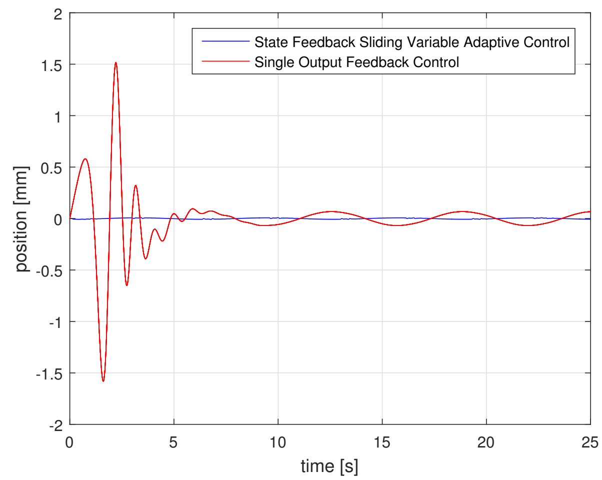

Fig. 6: Tracking error using Single Output Feedback Adaptive Control and State Feedback Sliding Variable Adaptive Control

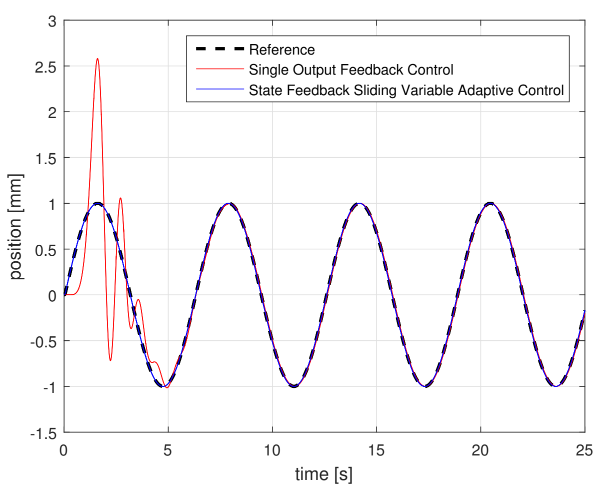

Fig. 5: Desired and Actual Trajectories using Single Output Feedback Adaptive Control and State Feedback Sliding Variable Adaptive Control

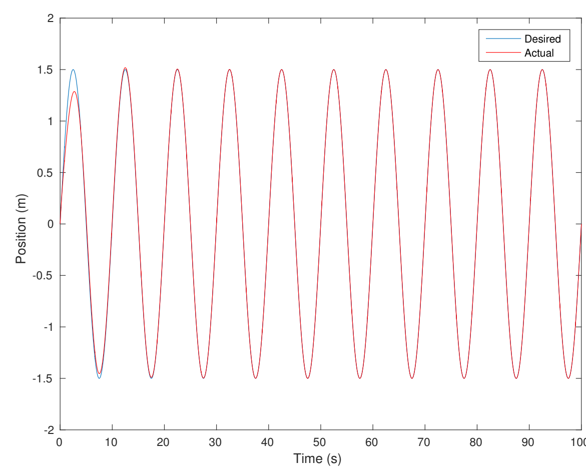

Fig. 7: Desired and Actual Trajectories using Sliding Variable Adaptive Control for k=1 N/m

IV. Results

Here, we present the results obtained by using the different control strategies detailed in this work.

Figure 4 shows the unit step response of the P-PI controller in the presence of nominal load, 20 % of the nominal load and 180 % of the nominal load. As it can be seen, variations in the load value alters the step response of the controller. Since the controller is linear, it can be concluded that the performance will be affected for all trajectories. However, using the introduced adaptive controllers, the variations of the load mass can be ”learned” by the controller and the tracking performance can be kept unaffected after parameter adaptation is complete.

Figures 5 and 6 show the tracking performances of Single Output Feedback Adaptive Control and State Feedback Sliding Variable Adaptive Control approaches. As it can be seen from the figure, State Feedback Sliding Variable Adaptive Control outperforms Single Output Feedback Adaptive Control for a trajectory that contains one sinusoidal component. Since Single Output Feedback Adaptive Control needs to adapt 8 parameters, its convergence is expected to be slow. However, on the other hand, State Feedback Sliding Variable Adaptive Control needs measurements or ”observation” of all states in order to operate properly. In the simulations shown on both figures 5 and 6, initial values for the parameters to be determined by the adaptation law are zeros.

Fig. 7 shows the results for a sliding variable adaptive controller that uses a random initial guess with all parameters set to unit values. Fig. 8 shows the results when the initial guess remains the same, but the stiffness is increased to

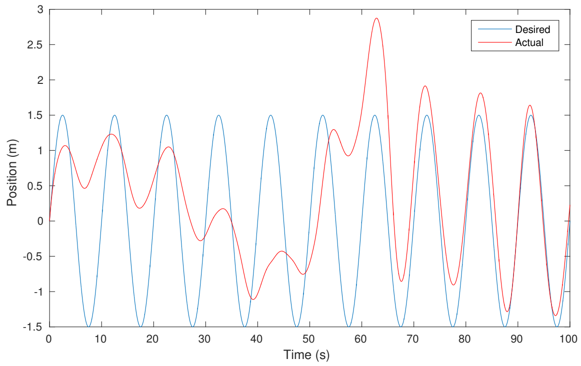

Fig. 8: Desired and Actual Trajectories using Sliding Variable Adaptive Control for k =100 N/m

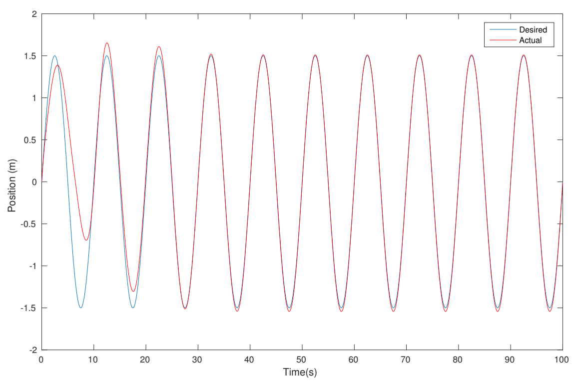

Fig. 9: Desired and Actual Trajectories using Sliding Variable Robust Adaptive Control with Snap Disturbance of Amplitude 15 m/s^4

100 N/m. The larger difference in the initial guess and true paraeter may be a possible reason for the longer settling time. Finally, Fig. 9 shows the results when a sliding variable robust adaptive control strategy is used in the presence of incorrectly modelled external disturbances.

References

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