1 Big O

Time Complexity

The time complexity of an algorithm estimates how much time the algorithm will use for a given input. The time complexity is denoted by $O(\cdots)$ where the three dots represent some function based on the input size, usually denoted by n.

Common Time Complexities

$O(1)$ *Constantant time.* The running time does not depend on the input size. A typical constant-time is a direct formula that calculates the answer.

$O(\log n)$ The *logarithmic* often halves the input size at each step. The running time of such an algorithm is logarithmic, because $\log_2 n$ equals the number of times n must be divided by 2 to get 1.

$O(\sqrt{n})$ A *square root* algorithm is slower than $O(\log n)$ but faster than $O(n)$. A special property of square roots is that $\sqrt{n} = n/\sqrt{n}$, so n elements can be divided into $O(\sqrt{n})$ blocks of $O(\sqrt{n})$ elements.

$O(n)$ A linear algorithm goes through the input a constant number of times. This is often the best possible time complexity, because it is usually necessary to access each input element at least once before reporting the answer.

$O(n \log n)$ This time complexity often indicates that the algorithm sorts the input, because the time complexity of efficient sorting algorithms is $O(n \log n)$. Another possibility is that the algorithm uses a data structure where each operation takes $O(\log n)$ time.

$O(n^2)$ A quadratic algorithm often contains two nested loops. It is possible to go through all pairs of the input elements in $O(n^2)$ time.

$O(n^3)$ A cubic algorithm often contains three nested loops. It is possible to go through all triplets of the input elements in $O(n^3)$ time.

$O(2^n)$ This time complexity often indicates that the algorithm iterates through all subsets of the input elements. For example, the subsets of {1, 2, 3} are $\emptyset$, {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, and {1, 2, 3}.

$O(n!)$ This time complexity often indicates that the algorithm iterates through all permutations of the input elements. For example, the permutations of {1, 2, 3} are (1,2,3), (1,3,2), (2,1,3), (2,3,1), (3,1,2), and (3,2,1).

2 Bit Manipultation

Bit Operations

AND Operation The AND operation $x\&y$ produces a number that has one bits in positions where both x and y have one bits. For example, 22 & 26 = 18:

$$10110\ (22)\ \mathbf{\&}\ 11010\ (26) = 10010\ (18)$$

OR Operation The or operation $x|y$ produces a number that has one bits in positions where at least one of x and y have one bits. For example, 22 | 26 = 30:

$$10110\ (22)\ |\ 11010\ (26) = 11110\ (30)$$

XOR Operation The xor operation $x \wedge y$ produces a number that has one bits in positions where exactly one of x and y have one bits. For example, $22 \wedge 26 = 12$:

$$10110\ (22) \wedge 11010\ (26) = 01100\ (12)$$

NOT Operation The not operation $\sim x$ produces a number where all the bits of x have been inverted. The formula $\sim x = -x - 1$ holds, for example, $\sim 29 = -30$. The result of the not operation at the bit level depends on the length of the bit representation because the operation inverts all bits. For example, if the numbers are 32-bit int numbers, the result is as follows:

$$\begin{aligned} x &= \ \ 29\ 00000000000000000000000000011101 \\ \hat{}x &= -30\ 11111111111111111111111111100010 \end{aligned}$$