Computational Fluid Dynamics

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Homework 10

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1 Introduction

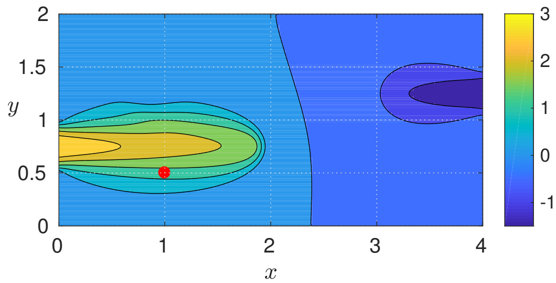
In this assignment we will study the solution to the following system

$$\begin{aligned}
\frac{\partial u}{\partial t}+\frac{\partial u^2}{\partial x}+\frac{\partial uv}{\partial y}&=\frac{1}{Re}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)\\
\frac{\partial v}{\partial t}+\frac{\partial uv}{\partial x}+\frac{\partial v^2}{\partial y}&=\frac{1}{Re}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)\\
\frac{\partial Y}{\partial t}+u\frac{\partial Y}{\partial x}+v\frac{\partial Y}{\partial y}&=\frac{1}{ReSc}\left(\frac{\partial^2 Y}{\partial x^2}+\frac{\partial^2 Y}{\partial y^2}\right)
\end{aligned}$$

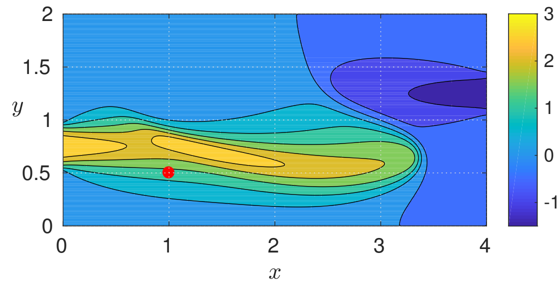
and periodic boundary conditions. We will use an `Adams-Bashforth Crank-Nicolson` method for u and v on a staggered mesh and a `WENO-5 TVD-RK-3` method with Crank-Nicolson for Y on a cell centered mesh.

2 Results

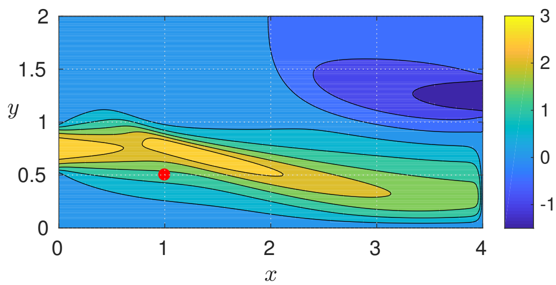
In the figures below we can see the surface plots of the horizontal velocity u, vertical velocity v and mass fraction Y, for different values of t. They are all obtained using $M = 256$, $N = 128$ and $CFL = 0.8$. Below the contour plots, and using the same parameters, we have the curve plots of the probe measurements together with the performance parameter R *vs* time. To finish, we have the GCI-analysis of the quantities required by the problem. At the very end, the same GCI-analysis will be performed to the steady state solution.



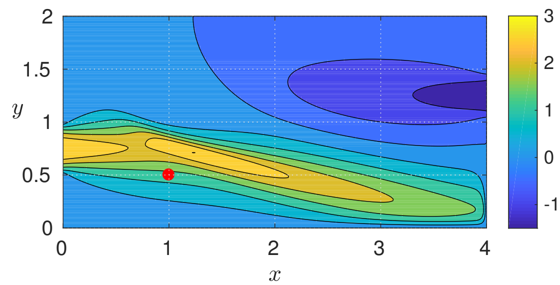
(a) $t = 1$ s.



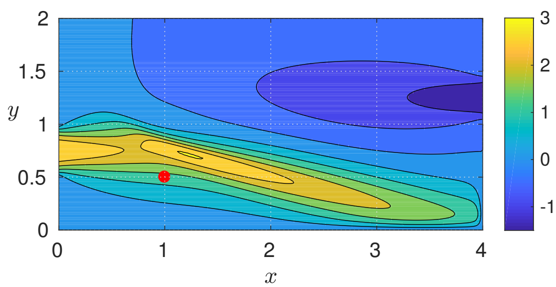
(b) $t = 2$ s.



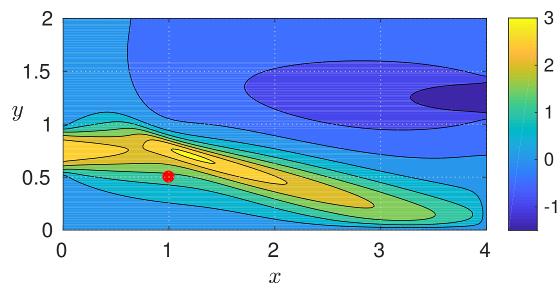
(c) $t = 3$ s.



(d) $t = 5$ s.

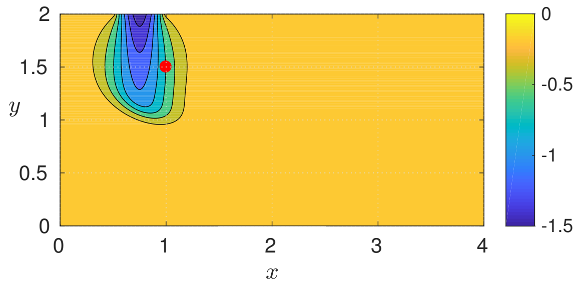


(e) $t = 7$ s.

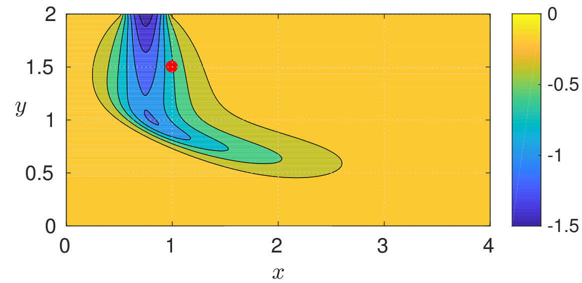


(f) $t = 10$ s.

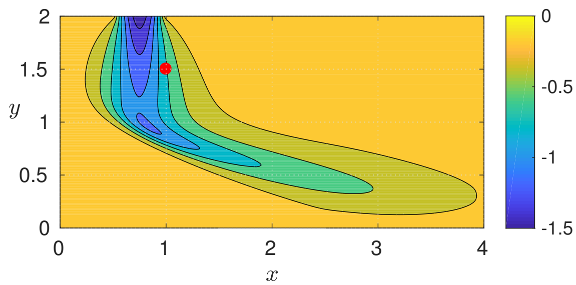
Figure 1: Profiles of horizontal velocity u for $M = 256$, $N = 128$ and $CFL = 0.8$.



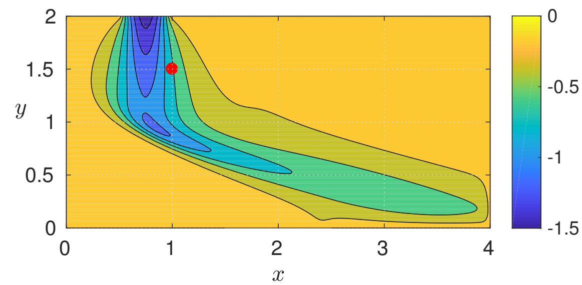
(a) $t = 1$ s.



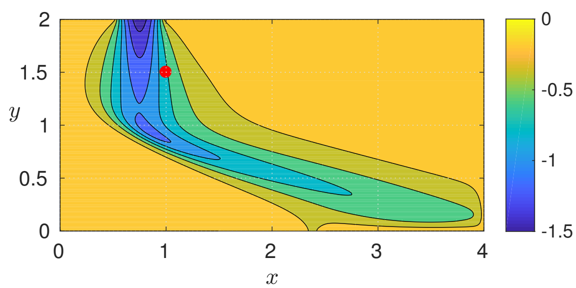
(b) $t = 2$ s.



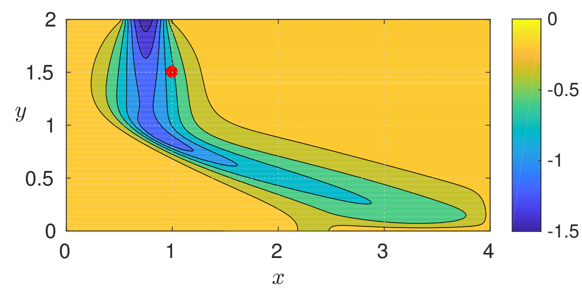
(c) $t = 3$ s.



(d) $t = 5$ s.

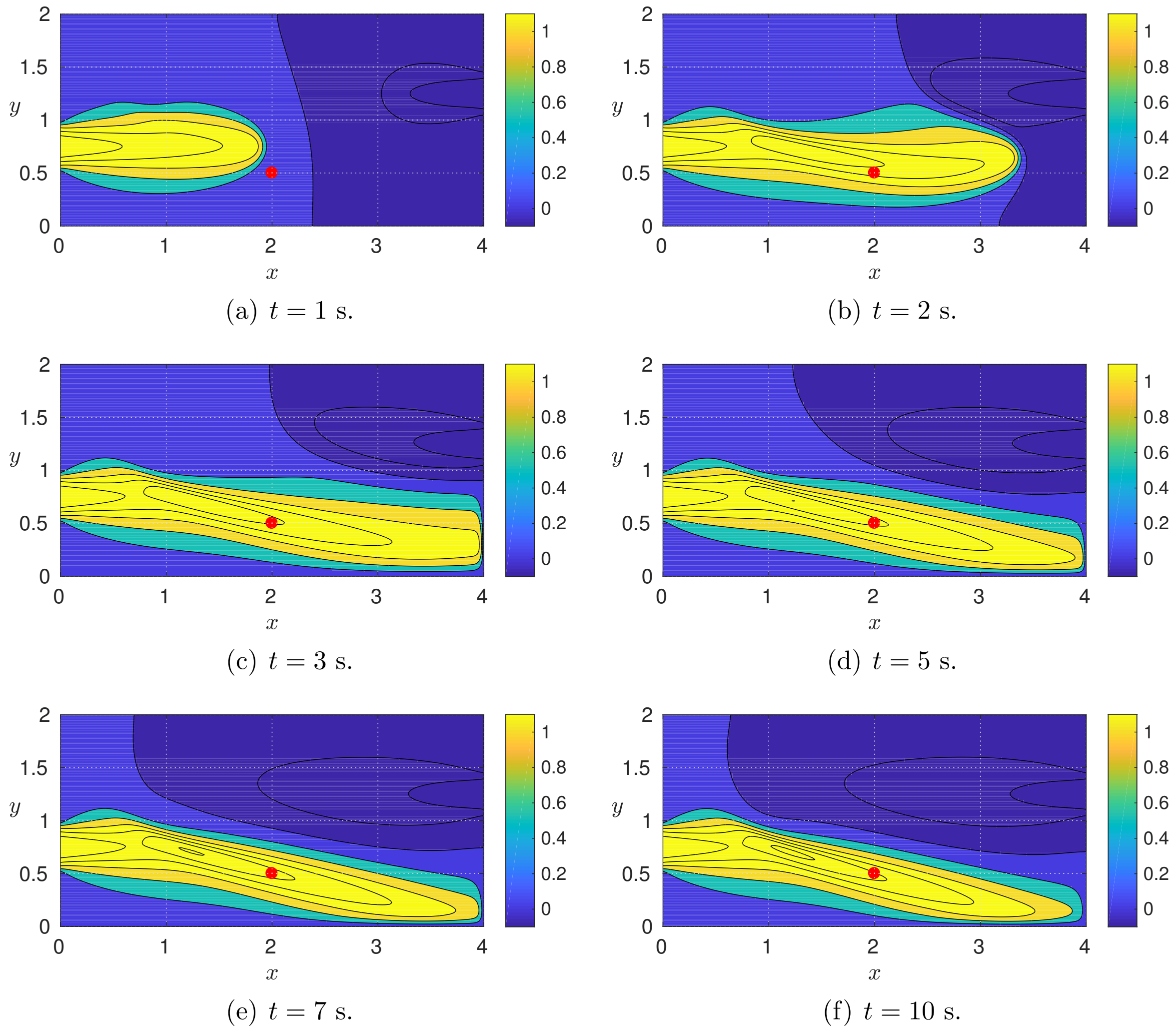


(e) $t = 7$ s.



(f) $t = 10$ s.

Figure 2: Profiles of the vertical velocity v for $M = 256$, $N = 128$ and $CFL = 0.8$.

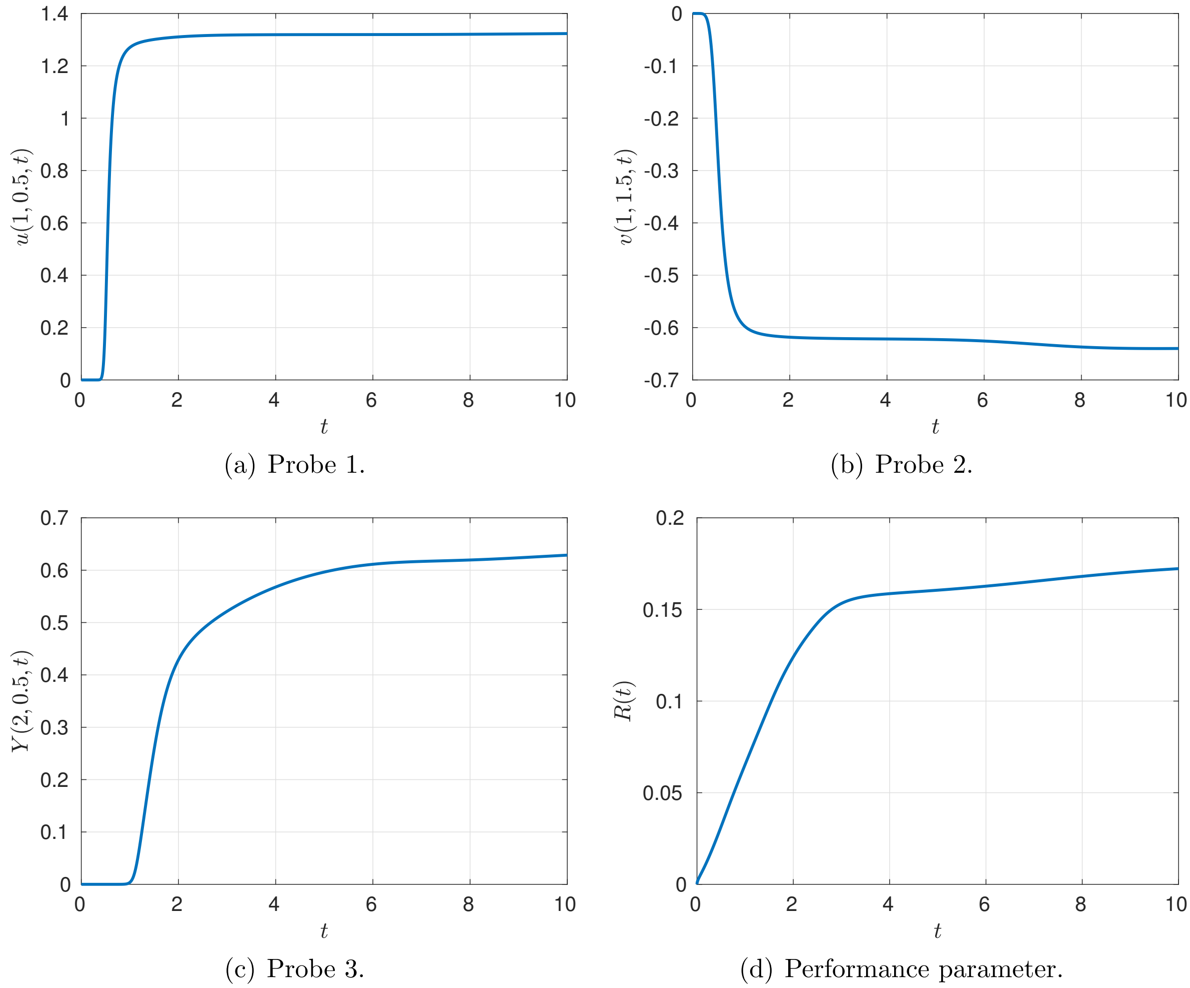


(a) $t = 1$ s. (b) $t = 2$ s.

(c) $t = 3$ s. (d) $t = 5$ s.

(e) $t = 7$ s. (f) $t = 10$ s.

Figure 3: Profiles of the mass fraction Y for $M = 256$, $N = 128$ and $CFL = 0.8$.



(a) Probe 1. (b) Probe 2. (c) Probe 3. (d) Performance parameter.

Figure 4: Probe curves and performance parameter with time for $M = 256$, $N = 128$ and $CFL = 0.8$.

GCI analysis for u at t=5

The GCI analysis details are shown in the tables below. Note that

$$\beta = \frac{GCI_{12}}{GCI_{23}} r^p,$$

and $u_{h=0}$ is obtained by Richardson extrapolation. We can see that $\beta \in [0.95, 1.05]$ which implies that we are in the asymptotic range of convergence, and for the last mesh we have a GCI_{12} value less than 0.02%, the requested accuracy.

M	N	$u(1, 0.5)$
16	8	1.543690071443089
32	16	1.335416609160283
64	32	1.322019032777453
128	64	1.319745850662477
256	128	1.319205904441201
512	256	1.319067860936907

Table 1: GCI analysis data.

M	N	p	u_h0	GCI12	GCI23	beta
16	8	NaN	NaN	NaN	NaN	NaN
32	16	NaN	NaN	NaN	NaN	NaN
64	32	3.9584350799233	1.32109795907916	0.0870896783116511	1.34027931647915	1.01013417814015
128	64	2.5591868664554	1.31928134384948	0.0439958584412932	0.258854908675024	1.00172243929686
256	128	2.07382564275753	1.31903769772281	0.0159382547694397	0.0670728514988665	1.00040929639546
512	256	1.96769271899691	1.31902044645506	0.00449318068173795	0.0175728809198171	1.0001046523139

Figure 5: GCI analysis results for the probe 1 measurement.

GCI analysis for v at t=5

We can see that $\beta \in [0.95, 1.05]$ which implies that we are in the asymptotic range of convergence, and for the last mesh we have a GCI_{12} value less than 0.02%, the requested accuracy.

M	N	$v(1, 1.5)$
16	8	-0.757061737618705
32	16	-0.625718980721275
64	32	-0.622907528189143
128	64	-0.622804506934971
256	128	-0.622781023934629
512	256	-0.622773391772338

Table 2: GCI analysis data.

M	N	p	v_h0	GCI12	GCI23	beta
16	8	NaN	NaN	NaN	NaN	NaN
32	16	NaN	NaN	NaN	NaN	NaN
64	32	5.54587714377773	-0.622846031378078	0.0123406782470828	0.573929637687545	1.00451343482764
128	64	4.77030177533616	-0.622800588290663	0.000786491640724292	0.0214598262263602	1.00016541507425
256	128	2.13325335506559	-0.622774090776325	0.00139157224559648	0.00610467616774274	1.000037706673
512	256	1.62145299393937	-0.62276971689401	0.000737603431890711	0.00226946554749653	1.00001225512323

Figure 6: GCI analysis results for the probe 2 measurement.

GCI analysis for Y at t=5

We can see that $\beta \in [0.95, 1.05]$ which implies that we are in the asymptotic range of convergence, and for the last mesh we have a GCI_{12} value less than 0.4%, the requested accuracy.

M	N	$Y(2, 0.5)$
16	8	0.582656635075406
32	16	0.588540399303644
64	32	0.591682314985302
128	64	0.594958713346013
256	128	0.596429634107325
512	256	0.597084423776656

Table 3: GCI analysis data.

M	N	p	Y_h0	GCI12	GCI23	beta
16	8	NaN	NaN	NaN	NaN	NaN
32	16	NaN	NaN	NaN	NaN	NaN
64	32	0.90509497255469	0.595282672183354	0.76061872791951	1.431990243934	0.994689860416509
128	64	-0.0604663135207332	0.515135896804494	-16.7706629785703	-16.1713507912412	0.994493066010762
256	128	1.1553912471178	0.597627991753401	0.251152352588463	0.56081168901127	0.997533789944036
512	256	1.16761606493148	0.597609767701518	0.109981081389411	0.247332947322324	0.998903354964158

Figure 7: GCI analysis results for the probe 3 measurement.

GCI analysis for R at t=5

We can see that $\beta \in [0.95, 1.05]$ which implies that we are in the asymptotic range of convergence, and for the last mesh we have a GCI_{12} value less than 0.5%, the requested accuracy.

M	N	$v(1, 1.5)$
16	8	0.168352427538196
32	16	0.162648932748133
64	32	0.161109373859052
128	64	0.160690350976423
256	128	0.160521007689289
512	256	0.160443010433551

Table 4: GCI analysis data.

M	N	p	R_h0	GCI12	GCI23	beta
16	8	NaN	NaN	NaN	NaN	NaN
32	16	NaN	NaN	NaN	NaN	NaN
64	32	1.88932914252264	0.160540142816762	0.441649536472937	1.62066075053525	1.00955598580148
128	64	1.87741611637481	0.160533657951695	0.12189050538481	0.4466809515543	1.00260764184087
256	128	1.30707823223154	0.160406151892441	0.0894398484824594	0.221076656325998	1.00105496027943
512	256	1.11845552861879	0.16037641124134	0.0518869535290025	0.112599318019901	1.00048613682521

Figure 8: GCI analysis results for the parameter R.

Steady state results

I ran the simulation using $M = 256$ and $N = 128$ up to a $t = 30$ to reach the steady state solution of the problem. In the following tables are the results obtained, with the GCI analysis.

M	N	p	u_h0	GCI12	GCI23	beta
16	8	NaN	NaN	NaN	NaN	NaN
32	16	NaN	NaN	NaN	NaN	NaN
64	32	2.42025580560709	1.32325394491015	0.931314723997211	4.82842657100011	1.03242956496118
128	64	3.03635881705907	1.32718554467362	0.0688575707540411	0.562686755373776	1.00396849764547
256	128	2.18484741417542	1.32643123629202	0.0307875868946674	0.13986241874637	1.00087357609845

Figure 9: GCI analysis results for the probe 1 measurement.

M	N	p	v_h0	GCI12	GCI23	beta
16	8	NaN	NaN	NaN	NaN	NaN
32	16	NaN	NaN	NaN	NaN	NaN
64	32	10.0848745651173	-0.6357220573214	1.61430363000829e-05	0.0175345987016561	0.999859872011669
128	64	-2.99897524107977	-0.635620156381226	-0.159869715629154	-0.0200203163251724	0.998881025587533
256	128	2.03872828578847	-0.636663203035808	0.0109470301413326	0.0449917553978741	0.999727740193495

Figure 10: GCI analysis results for the probe 2 measurement.

M	N	p	Y_h0	GCI12	GCI23	beta
16	8	NaN	NaN	NaN	NaN	NaN
32	16	NaN	NaN	NaN	NaN	NaN
64	32	3.09041984891174	0.645196690168092	0.0851668530235064	0.72170703862935	1.0051218935378
128	64	7.97747394230764	0.645649757906483	1.01190659983891e-05	0.00255039941721193	0.999979678170884
256	128	-5.06035733276536	0.645636179483926	-0.0873197317099373	-0.00261871016576605	0.999322377632678

Figure 11: GCI analysis results for the probe 3 measurement.

M	N	p	R_h0	GCI12	GCI23	beta
16	8	NaN	NaN	NaN	NaN	NaN
32	16	NaN	NaN	NaN	NaN	NaN
64	32	1.0603013686232	0.175980285661211	1.13828471535011	2.35050986276912	1.00988365419262
128	64	1.42644495586338	0.176557559780812	0.27333458615699	0.731976915731664	1.00369075099285
256	128	1.25704037320375	0.176474670900958	0.139078136022136	0.331890297942506	1.00154660371752

Figure 12: GCI analysis results for the parameter R.