HOMEWORK 6 - FRANCISCO CASTILLO

Contents

Defined functions

```
function u=ADI_u(u,M,N,dt,hx,hy,Re)
%% Solve for u(x,y,t)

%%%%%%%%%% STEP 1 %%%%%%%%%%
[a,b,c,d]=uvectors(u,M,N,dt,hx,hy,1/Re,1); %Obtain tridiagonal vectors
d=GaussTriSol(a,b,c,d); %Gaussian elimination
% Correspondance with u, obtain u(n+1/2)
for j=2:N+1
    for i=2:M
        u(i,j)=d((j-2)*(M-1)+i-1);
    end
end
% Update top and bottom boundaries (ghost cells), they depend on the
% interior
u(:,N+2)=-u(:,N+1);
u(:,1)=-u(:,2);

%%%%%%%%%% STEP 2 %%%%%%%%%%
[a,b,c,d]=uvectors(u,M,N,dt,hx,hy,1/Re,2);
d=GaussTriSol(a,b,c,d);
for i=2:M
    for j=2:N+1
        u(i,j)=d((i-2)*N+j-1);
    end
end
% Update top and bottom boundaries (ghost cells), they depend on the
% interior
u(:,N+2)=-u(:,N+1); % Top
u(:,1)=-u(:,2); % Bottom
end



function v=ADI_v(v,M,N,dt,hx,hy,Re)
%% Solve for v(x,y,t)
xiv=linspace(-hx/2,4+hx/2,M+2);
%%%%%%%%%% STEP 1 %%%%%%%%%%
[a,b,c,d]=vvectors(v,M,N,dt,hx,hy,1/Re,1); %Obtain tridiagonal vectors
% keyboard

d=GaussTriSol(a,b,c,d); %Gaussian elimination
% Correspondance with v, obtain v(n+1/2)
for j=2:N
    for i=2:M+1
        v(i,j)=d((j-2)*M+i-1);
    end
end
% Update left and right boundaries (ghost cells), as well as the outlet
% Neumann BC, they depend on the interior.
v(1,:)=-v(2,:); % Left
v(M+2,:)=-v(M+1,:); % Right
for i=1:M+2
    if (xiv(i)>=1.5 && xiv(i)<=2.5) % Outlet
        v(i,1)=(4/3)*v(i,2)-(1/3)*v(i,3);
    end
end

%%%%%%%%%% STEP 2 %%%%%%%%%%
[a,b,c,d]=vvectors(v,M,N,dt,hx,hy,1/Re,2); %Obtain tridiagonal vectors
% keyboard

d=GaussTriSol(a,b,c,d); %Gaussian elimination
% Correspondance with v, obtain v(n+1/2)
```

```
for i=2:M+1
    for j=2:N
        v(i,j)=d((i-2)*(N-1)+j-1);
    end
end
% Update left and right boundaries (ghost cells), as well as the outlet
% Neumann BC, they depend on the interior.
v(1,:)=-v(2,:); % Left
v(M+2,:)=-v(M+1,:); % Right
for i=1:M+2
    if (xiv(i)>=1.5 && xiv(i)<=2.5) % Outlet
        v(i,1)=(4/3)*v(i,2)-(1/3)*v(i,3);
    end
end
end



function Y=ADI_Y(Y,M,N,dt,hx,hy,Re,Sc)
%% Solve for Y(x,y,t)
xiY=linspace(-hx/2,4+hx/2,M+2);
yjY=linspace(-hy/2,2+hy/2,N+2);
%%%%%%%%%% STEP 1 %%%%%%%%%%
[a,b,c,d]=Yvectors(Y,M,N,dt,hx,hy,1/(Re*Sc),1); %Obtain tridiagonal vectors
d=GaussTriSol(a,b,c,d); %Gaussian elimination
% Correspondance with Y, obtain Y(n+1/2)
for j=2:N+1
    for i=2:M+1
        Y(i,j)=d((j-2)*M+i-1);
    end
end
% Update ghost cells.
Y(:,1)=Y(:,2); % Bottom
Y(:,N+2)=Y(:,N+1); % Top, we will overwrite the inlet 3
Y(find(xiY>=0.5,1):find(xiY<=1,1,'last'),N+2)...
    =-Y(find(xiY>=0.5,1):find(xiY<=1,1,'last'),N+1); % inlet 3
Y(1,:)=Y(2,:); % Left
Y(1,find(yjY>=0.5,1):find(yjY<=1,1,'last'))=...
    2-Y(2,find(yjY>=0.5,1):find(yjY<=1,1,'last')); % inlet 1
Y(M+2,:)=Y(M+1,:); % Right
Y(M+2,find(yjY>=1,1):find(yjY<=1.5,1,'last'))=...
    0.5-Y(M+1,find(yjY>=1,1):find(yjY<=1.5,1,'last')); % inlet 2

%%%%%%%%%% STEP 2 %%%%%%%%%%
[a,b,c,d]=Yvectors(Y,M,N,dt,hx,hy,1/(Re*Sc),2); %Obtain tridiagonal vectors
d=GaussTriSol(a,b,c,d); %Gaussian elimination
% Correspondance with Y, obtain Y(n+1)
for i=2:M+1
    for j=2:N+1
        Y(i,j)=d((i-2)*N+j-1);
    end
end
% Update ghost cells.
Y(:,1)=Y(:,2); % Bottom
Y(:,N+2)=Y(:,N+1); % Top, we will overwrite the inlet 3
Y(find(xiY>=0.5,1):find(xiY<=1,1,'last'),N+2)...
    =-Y(find(xiY>=0.5,1):find(xiY<=1,1,'last'),N+1); % inlet 3
Y(1,:)=Y(2,:); % Left
Y(1,find(yjY>=0.5,1):find(yjY<=1,1,'last'))=...
    2-Y(2,find(yjY>=0.5,1):find(yjY<=1,1,'last')); % inlet 1
Y(M+2,:)=Y(M+1,:); % Right
Y(M+2,find(yjY>=1,1):find(yjY<=1.5,1,'last'))=...
    0.5-Y(M+1,find(yjY>=1,1):find(yjY<=1.5,1,'last')); % inlet 2
end



function d=GaussTriSol(a,b,c,d)
    N=length(a);
    for i=2:N
        b(i)=b(i)-c(i-1)*a(i)/b(i-1);
        d(i)=d(i)-d(i-1)*a(i)/b(i-1);
    end
    d(N)=d(N)/b(N);
    for i=N-1:-1:1
        d(i)=(d(i)-c(i)*d(i+1))/b(i);
    end
end



function [u,v,Y]=initialization(M,N,hx,hy)
%% Initialize u
u = zeros(M+1,N+2);
% Impose left and right boundaries, only once, since they do not depend on
% the interior nor they are altered.
yju=linspace(-hy/2,2+hy/2,N+2);
```

```
for j=1:N+2
    if (yju(j)>=0.5 && yju(j)<=1) % Inlet 1
        u(1,j)=2;
    elseif (yju(j)>=1 && yju(j)<=1.5) % Inlet 2
        u(M+1,j)=-1;
    end
end
%% Initialize v
v = zeros(M+2,N+1);
% Impose top and bottom boundaries, only once, since they do not depend on
% the interior except the outlet Neumann BC, which will be updated after
% iterations since for the initial condition it is satisfied.
xiv=linspace(-hx/2,4+hx/2,M+2);
for i=1:M+2
    if (xiv(i)>=0.5 && xiv(i)<=1) % Inlet 3
        v(i,N+1)=-1;
    end
end
%% Initialize Y
Y=zeros(M+2,N+2);
yjY=linspace(-hy/2,2+hy/2,N+2);
% Since Y is initialized to zero, the ghost cells are updated except the
% ones corresponding to the inlets 1 and 2.
% Inlet 1
a=find(yjY>=0.5,1,'first');
b=find(yjY<=1,1,'last');
for j=a:b
    Y(1,j)=2-Y(2,j);
end
% Inlet 2
a=find(yjY>=1,1,'first');
b=find(yjY<=1.5,1,'last');
for j=a:b
    Y(M+2,j)=0.5-Y(M+1,j);
end
end


function [a,b,c,d] = uvectors(u,M,N,dt,hx,hy,alpha,step)
dx=alpha*dt/hx^2;
dy=alpha*dt/hy^2;
d1=dx/2;
d2=dy/2;
if step==1
    a = -d1*ones((M-1)*N,1);
    b = (1+2*d1)*ones((M-1)*N,1);
    c = -d1*ones((M-1)*N,1);
    d = zeros((M-1)*N,1);
    % Case j=2 and i=2, includes BCs
    a(1)=0;
    d(1)=d2*u(2,3)+(1-2*d2)*u(2,2)+d1*u(1,2)+d2*u(2,1);
    %Case j=2 and i between [3,M-1], includes BCs
    for i=3:M-1
        d(i-1)=d2*u(i,3)+(1-2*d2)*u(i,2)+d2*u(i,1);
    end
    % Case j=2 and i=M, includes BCs
    c(M-1)=0;
    d(M-1)=d2*u(M,3)+(1-2*d2)*u(M,2)+d1*u(M+1,2)+d2*u(M,1);
    % Case j between [3,N] and i=2, includes BCs
    for j=3:N
        a((j-2)*(M-1)+1)=0;
        d((j-2)*(M-1)+1)=d2*u(2,j+1)+(1-2*d2)*u(2,j)...
            +d2*u(2,j-1)+d1*u(1,j);
    end
    % Interior of the interior, j in [3,N] and i in [3,M-1], no BCs
    for j=3:N
        for i=3:M-1
            d((j-2)*(M-1)+i-1)=d2*u(i,j+1)+(1-2*d2)*u(i,j)+d2*u(i,j-1);
        end
    end
    % Case j between [3,N] and i=M, includes BCs
    for j=3:N
        c((j-2)*(M-1)+M-1)=0;
        d((j-2)*(M-1)+M-1)=d2*u(M,j+1)+(1-2*d2)*u(M,j)+d2*u(M,j-1)...
            +d1*u(M+1,j);
    end
    % Case j=N+1 and i=2, includes BCs
    a((N-1)*(M-1)+1)=0;
    d((N-1)*(M-1)+1)=(1-2*d2)*u(2,N+1)+d2*u(2,N)...
        +d1*u(1,N+1)+d2*u(2,N+2);
    % Case j=N+1 and i [3,M-1], includes BCs
    for i=3:M-1
        d((N-1)*(M-1)+i-1)=(1-2*d2)*u(i,N+1)+d2*u(i,N)...
            +d2*u(i,N+2);
    end
    % Case j=N+1 and i=M, includes BCs
    c((M-1)*N)=0;
    d((M-1)*N)=(1-2*d2)*u(M,N+1)+d2*u(M,N)...
        +d1*u(M+1,N+1)+d2*u(M,N+2);
elseif step==2
    a = -d2*ones((M-1)*N,1);
    b = (1+2*d2)*ones((M-1)*N,1);
    c = -d2*ones((M-1)*N,1);
    d = zeros((M-1)*N,1);
    % Case i=2 and j=2, includes BCs
    a(1)=0;
    b(1)=1+3*d2;
    d(1)=d1*u(3,2)+(1-2*d1)*u(2,2)+d1*u(1,2);
    %Case i=2 and j between [3,N], includes BCs
    for j=3:N
        d(j-1)=d1*u(3,j)+(1-2*d1)*u(2,j)+d1*u(1,j);
    end
    % Case i=2 and j=N+1, includes BCs
    b(N)=1+3*d2;
    c(N)=0;
    d(N)=d1*u(3,N+1)+(1-2*d1)*u(2,N+1)+d1*u(1,N+1);
    % Case i between [3,M-1] and j=2, includes BCs
    for i=3:M-1
        a((i-2)*N+1)=0;
        b((i-2)*N+1)=1+3*d2;
        d((i-2)*N+1)=d1*u(i+1,2)+(1-2*d1)*u(i,2)+d1*u(i-1,2);
    end
    % Interior of the interior, i in [3,M-1] and j in [3,N], no BCs
    for i=3:M-1
        for j=3:N
            d((i-2)*N+j-1)=d1*u(i+1,j)+(1-2*d1)*u(i,j)+d1*u(i-1,j);
        end
    end
    % Case i between [3,M-1] and j=N+1, includes BCs
    for i=3:M-1
        b((i-1)*N)=1+3*d2;
        c((i-1)*N)=0;
        d((i-1)*N)=d1*u(i+1,N+1)+(1-2*d1)*u(i,N+1)+d1*u(i-1,N+1);
    end
    % Case i=M and j=2, includes BCs
    a((M-2)*N+1)=0;
    b((M-2)*N+1)=1+3*d2;
    d((M-2)*N+1)=(1-2*d1)*u(M,2)+d1*u(M-1,2)...
        +d1*u(M+1,2);
    % Case i=M and j [3,N], includes BCs
    for j=3:N
        d((M-2)*N+j-1)=(1-2*d1)*u(M,j)+d1*u(M-1,j)...
            +d1*u(M+1,j);
    end
    % Case i=M and j=N+1, includes BCs
    b((M-1)*N)=1+3*d2;
    c((M-1)*N)=0;
    d((M-1)*N)=(1-2*d1)*u(M,N+1)+d1*u(M-1,N+1)...
        +d1*u(M+1,N+1);
end
end


function [a,b,c,d] = vvectors(v,M,N,dt,hx,hy,alpha,step)
xi=linspace(-hx/2,4+hx/2,M+2);
yj=linspace(0,2,N+1);
dx=alpha*dt/hx^2;
dy=alpha*dt/hy^2;
d1=dx/2; %debug d1=1/8;
d2=dy/2;
if step==1
    a = -d1*ones(M*(N-1),1);
    b = (1+2*d1)*ones(M*(N-1),1);
    c = -d1*ones(M*(N-1),1);
    d = zeros(M*(N-1),1);
    % Case j=2 and i=2, includes BCs but they are already
    % imposed for the current timestep
    a(1)=0;
    b(1)=1+3*d1;
    d(1)=d2*v(2,3)+(1-2*d2)*v(2,2)+d2*v(2,1);
    % Case j=2 and i between [3,M], includes BCs but they are already
    % imposed for the current timestep
    for i=3:M
        d(i-1)=d2*v(i,3)+(1-2*d2)*v(i,2)+d2*v(i,1);
    end
    % Case j=2 and i=M+1, includes BCs
    b(M)=1+3*d1;
    c(M)=0;
    d(M)=d2*v(M+1,3)+(1-2*d2)*v(M+1,2)+d2*v(M+1,1);
    % Case j between [3,N-1] and i=2, includes BCs
    for j=3:N-1
        a((j-2)*M+1)=0;
        b((j-2)*M+1)=1+3*d1;
        d((j-2)*M+1)=d2*v(2,j+1)+(1-2*d2)*v(2,j)+d2*v(2,j-1);
    end
    % Interior of the interior, j in [3,N-1] and i in [3,M], no BCs
    for j=3:N-1
        for i=3:M
            d((j-2)*M+i-1)=d2*v(i,j+1)+(1-2*d2)*v(i,j)+d2*v(i,j-1);
        end
    end
    % Case j between [3,N-1] and i=M+1, includes BCs
    for j=3:N-1
        b((j-1)*M)=1+3*d1;
        c((j-1)*M)=0;
        d((j-1)*M)=d2*v(M+1,j+1)+(1-2*d2)*v(M+1,j)+d2*v(M+1,j-1);
    end
    % Case j=N and i=2, includes BCs but they are already
    % imposed for the current timestep
    a((N-2)*M+1)=0;
    b((N-2)*M+1)=1+3*d1;
    d((N-2)*M+1)=d2*v(2,N+1)+(1-2*d2)*v(2,N)+d2*v(2,N-1);
    % Case j=N and i [3,M], includes BCs but they are already
    % imposed for the current timestep
    for i=3:M
        d((N-2)*M+i-1)=d2*v(i,N+1)+(1-2*d2)*v(i,N)+d2*v(i,N-1);
    end
    % Case j=N and i=M+1, includes BCs but they are already
    % imposed for the current timestep
    b((N-1)*M)=1+3*d1;
    c((N-1)*M)=0;
    d((N-1)*M)=d2*v(M+1,N+1)+(1-2*d2)*v(M+1,N)+d2*v(M+1,N-1);
elseif step==2
    a = -d2*ones(M*(N-1),1);
    b = (1+2*d2)*ones(M*(N-1),1);
    c = -d2*ones(M*(N-1),1);
    d = zeros(M*(N-1),1);
    % Case j=2 and i=2, includes BCs
    a(1)=0;
    d(1)=d1*v(3,2)+(1-3*d1)*v(2,2);
    %Case i=2 and j between [3,N-1], includes BCs
    for j=3:N-1
        d(j-1)=d1*v(3,j)+(1-3*d1)*v(2,j);
    end
    % Case i=2 and j=N, includes BCs
    c(N-1)=0;
    d(N-1)=d1*v(3,N)+(1-3*d1)*v(2,N)+d2*v(2,N+1);
    % Case i between [3,M] and j=2, includes BCs
    for i=3:M
        a((i-2)*(N-1)+1)=0;
        if (xi(i)>=1.5 && xi(i)<=2.5)
            b((i-2)*(N-1)+1)=1+2*d2/3;
            c((i-2)*(N-1)+1)=-2*d2/3;
        end
        d((i-2)*(N-1)+1)=d1*v(i+1,2)+(1-2*d1)*v(i,2)+d1*v(i-1,2);
    end
    % Interior of the interior, i in [3,M] and j in [3,N-1], no BCs
    for i=3:M
        for j=3:N-1
            d((i-2)*(N-1)+j-1)=d1*v(i+1,j)+(1-2*d1)*v(i,j)+d1*v(i-1,j);
        end
    end
    % Case i between [3,M] and j=N, includes BCs
    for i=3:M
        c((i-1)*(N-1))=0;
        d((i-1)*(N-1))=d1*v(i+1,N)+(1-2*d1)*v(i,N)+d1*v(i-1,N)...
            +d2*v(i,N+1);
    end
    % Case i=M+1 and j=2, includes BCs
    a((M-1)*(N-1)+1)=0;
    d((M-1)*(N-1)+1)=(1-3*d1)*v(M+1,2)+d1*v(M,2);
    % Case i=M+1 and j [3,N-1], includes BCs
    for j=3:N-1
        d((M-1)*(N-1)+j-1)=(1-3*d1)*v(M+1,j)+d1*v(M,j);
    end
    % Case i=M+1 and j=N, includes BCs
    c((N-1)*M)=0;
    d((N-1)*M)=(1-3*d1)*v(M+1,N)+d1*v(M,N)...
        +d2*v(M+1,N+1);
end
end



function [a,b,c,d] = Yvectors(Y,M,N,dt,hx,hy,alpha,step)
xi=linspace(-hx/2,4+hx/2,M+2);
```

```
yj=linspace(-hy/2,2+hy/2,N+2);
dx=alpha*dt/hx^2;
dy=alpha*dt/hy^2;
d1=dx/2;
d2=dy/2;

if step==1
    a = -d1*ones(M*N,1);
    b = (1+2*d1)*ones(M*N,1);
    c = -d1*ones(M*N,1);
    d = zeros(M*N,1);
    % Case j=2 and i=2, includes BCs but they are already
    % imposed for the current timestep
    a(1)=0;
    b(1)=1+d1;
    d(1)=d2*Y(2,3)+(1-2*d2)*Y(2,2)+d2*Y(2,1);
    % Case j=2 and i between [3,M], includes BCs but they are already
    % imposed for the current timestep
    for i=3:M
        d(i-1)=d2*Y(i,3)+(1-2*d2)*Y(i,2)+d2*Y(i,1);
    end
    % Case j=2 and i=M+1, includes BCs
    b(M)=1+d1;
    c(M)=0;
    d(M)=d2*Y(M+1,3)+(1-2*d2)*Y(M+1,2)+d2*Y(M+1,1);
    % Case j between [3,N] and i=2, includes BCs
    for j=3:N
        a((j-2)*M+1)=0;
        if (yj(j)>=0.5 && yj(j)<=1)
            b((j-2)*M+1)=1+3*d1;
            d((j-2)*M+1)=d2*Y(2,j+1)+(1-2*d2)*Y(2,j)+d2*Y(2,j-1)...
                +2*d1;
        else
            b((j-2)*M+1)=1+d1;
            d((j-2)*M+1)=d2*Y(2,j+1)+(1-2*d2)*Y(2,j)+d2*Y(2,j-1);
        end
    end
    % Interior of the interior, j in [3,N] and i in [3,M], no BCs
    for j=3:N
        for i=3:M
            d((j-2)*M+i-1)=d2*Y(i,j+1)+(1-2*d2)*Y(i,j)+d2*Y(i,j-1);
        end
    end
    % Case j between [3,N] and i=M+1, includes BCs
    for j=3:N
        c((j-1)*M)=0;
        if (yj(j)>=1 && yj(j)<=1.5)
            b((j-1)*M)=1+3*d1;
            d((j-1)*M)=d2*Y(M+1,j+1)+(1-2*d2)*Y(M+1,j)+d2*Y(M+1,j-1)...
                +0.5*d1;
        else
            b((j-1)*M)=1+d1;
            d((j-1)*M)=d2*Y(M+1,j+1)+(1-2*d2)*Y(M+1,j)+d2*Y(M+1,j-1);
        end
    end
    % Case j=N+1 and i=2, includes BCs but they are already
    % imposed for the current timestep
    a((N-1)*M+1)=0;
    b((N-1)*M+1)=1+d1;
    d((N-1)*M+1)=d2*Y(2,N+2)+(1-2*d2)*Y(2,N+1)+d2*Y(2,N);
    % Case j=N+1 and i [3,M], includes BCs but they are already
    % imposed for the current timestep
    for i=3:M
        d((N-1)*M+i-1)=d2*Y(i,N+2)+(1-2*d2)*Y(i,N+1)+d2*Y(i,N);
    end
    % Case j=N+1 and i=M+1, includes BCs but they are already
    % imposed for the current timestep
    b(N*M)=1+d1;
    c(N*M)=0;
    d(N*M)=d2*Y(M+1,N+2)+(1-2*d2)*Y(M+1,N+1)+d2*Y(M+1,N);
elseif step==2
    a = -d2*ones(M*N,1);
    b = (1+2*d2)*ones(M*N,1);
    c = -d2*ones(M*N,1);
    d = zeros(M*N,1);
    % Case j=2 and i=2, includes BCs
    a(1)=0;
    b(1)=1+d2;
    d(1)=d1*Y(3,2)+(1-2*d1)*Y(2,2)+d1*Y(1,2);
    %Case i=2 and j between [3,N], includes BCs
    for j=3:N
        d(j-1)=d1*Y(3,j)+(1-2*d1)*Y(2,j)+d1*Y(1,j);
    end
    % Case i=2 and j=N+1, includes BCs
    b(N)=1+d2;
    c(N)=0;
    d(N)=d1*Y(3,N+1)+(1-2*d1)*Y(2,N+1)+d1*Y(1,N+1);
    % Case i between [3,M] and j=2, includes BCs
    for i=3:M
        a((i-2)*N+1)=0;
        b((i-2)*N+1)=1+d2;
        d((i-2)*N+1)=d1*Y(i+1,2)+(1-2*d1)*Y(i,2)+d1*Y(i-1,2);
    end
    % Interior of the interior, i in [3,M] and j in [3,N], no BCs
    for i=3:M
        for j=3:N
            d((i-2)*N+j-1)=d1*Y(i+1,j)+(1-2*d1)*Y(i,j)+d1*Y(i-1,j);
        end
    end
    % Case i between [3,M] and j=N+1, includes BCs
    for i=3:M
        c((i-1)*N)=0;
        if (xi(i)>=0.5 && xi(i)<=1)
            b((i-1)*N)=1+3*d2;
        else
            b((i-1)*N)=1+d2;
        end
        d((i-1)*N)=d1*Y(i+1,N+1)+(1-2*d1)*Y(i,N+1)+d1*Y(i-1,N+1);
    end
    % Case i=M+1 and j=2, includes BCs
    a((M-1)*N+1)=0;
    b((M-1)*N+1)=1+d2;
    d((M-1)*N+1)=d1*Y(M+2,2)+(1-2*d1)*Y(M+1,2)+d1*Y(M,2);
    % Case i=M+1 and j [3,N], includes BCs
    for j=3:N
        d((M-1)*N+j-1)=d1*Y(M+2,j)+(1-2*d1)*Y(M+1,j)+d1*Y(M,j);
    end
    % Case i=M+1 and j=N+1, includes BCs
    b(M*N)=1+d2;
    c(M*N)=0;
    d(M*N)=d1*Y(M+2,N+1)+(1-2*d1)*Y(M+1,N+1)+d1*Y(M,N+1);
end
end
```

```
clear all; close all; format long; clc
axisSize=14;
markersize=16;
linewidth=3.5;
Lx = 4;
Ly = 2;
Mv = [1 0.5 0.25]*128;
Nv = [1 0.5 0.25]*64;
CFL = 2;
Re = 2;
Sc = 0.25;
p1(1)=0;
p2(1)=0;
p3(1)=0;
for i=1:3
```

```
    M=Mv(i);
    N=Nv(i);
    if (M==128 && N==64)
        outputTime=[0.1 0.5 1 10];
    else
        outputTime=[0.1 0.5 1];
    end
    endtime=outputTime(end);
    hx = Lx/M;
    hy = Ly/N;
    if hx~=hy
        error('Cells not square')
    end
    time=0;
    dt = min(CFL*0.25*hx^2*Re,CFL*0.25*hx^2*Re*Sc);
    n=1;
    % Define the points of the different meshes
    xu=linspace(0,4,M+1);
    yu=linspace(-hy/2,2+hy/2,N+2);
    xv=linspace(-hx/2,4+hx/2,M+2);
    yv=linspace(0,2,N+1);
    xY=linspace(-hx/2,4+hx/2,M+2);
    yY=linspace(-hy/2,2+hy/2,N+2);

    [u,v,Y]=initialization(M,N,hx,hy);
    iter=1;
    t=0;
    while time < endtime
```

```
        if (time < outputTime(n) && time+dt >= outputTime(n))
            dt=outputTime(n)-time;
            n=n+1;
        else
            dt = min(CFL*0.25*hx^2*Re,CFL*0.25*hx^2*Re*Sc);
        end
```

Solve for u(x,y,t)

```
        u=ADI_u(u,M,N,dt,hx,hy,Re);
```

Solve for v(x,y,t)

```
        v=ADI_v(v,M,N,dt,hx,hy,Re);
```

Solve for Y(x,y,t)

```
        Y=ADI_Y(Y,M,N,dt,hx,hy,Re,Sc);
```

Next time step

```
        time=time+dt;
```

Filled contour plots, only for M=128 and N=64

```
        if (M==128 && N==64 && ismember(time,outputTime))
            % Plot u
            figure(n-1)
            contourf(xu,yu,u')
            hold on
            plot(1,0.5,'r.','markersize',markersize,'linewidth',linewidth)
            axis([0 4 0 2])
            caxis([-1 2])
            colorbar
            xlabel('$x$','Interpreter','latex')
            ylabel('$y$','Interpreter','latex')
            yticks([0 0.25 0.50 0.75 1.0 1.25 1.50 1.75 2.0]);
            xticks([0 0.50 1.0 1.50 2.0 2.50 3.0 3.50 4.0]);
            set(get(gca,'ylabel'),'rotation',0)
            set(gca,'fontsize',axisSize)
            pbaspect([2 1 1])
            grid on
            txt=['Latex/FIGURES/u_' num2str(n-1)];
            saveas(gcf,txt,'epsc')
            % Plot v
            figure(n+3)
            contourf(xv,yv,v')
            hold on
            plot(1,1.5,'r.','markersize',markersize,'linewidth',linewidth)
            axis([0 4 0 2])
            caxis([-1 0])
            colorbar
            xlabel('$x$','Interpreter','latex')
            ylabel('$y$','Interpreter','latex')
            yticks([0 0.25 0.50 0.75 1.0 1.25 1.50 1.75 2.0]);
            xticks([0 0.50 1.0 1.50 2.0 2.50 3.0 3.50 4.0]);
            set(get(gca,'ylabel'),'rotation',0)
            set(gca,'fontsize',axisSize)
            pbaspect([2 1 1])
            grid on
            txt=['Latex/FIGURES/v_' num2str(n-1)];
            saveas(gcf,txt,'epsc')
            % Plot Y
            figure(n+7)
            contourf(xY,yY,Y')
            hold on
            plot(0.5,0.5,'r.','markersize',markersize,'linewidth',linewidth)
            axis([0 4 0 2])
            caxis([0 1])
            colorbar
            xlabel('$x$','Interpreter','latex')
            ylabel('$y$','Interpreter','latex')
            yticks([0 0.25 0.50 0.75 1.0 1.25 1.50 1.75 2.0]);
            xticks([0 0.50 1.0 1.50 2.0 2.50 3.0 3.50 4.0]);
            set(get(gca,'ylabel'),'rotation',0)
            set(gca,'fontsize',axisSize)
            pbaspect([2 1 1])
            grid on
            txt=['Latex/FIGURES/Y_' num2str(n-1)];
            saveas(gcf,txt,'epsc')
        end
```

Probes, only for M=128 and N=64

```
        if (M==128 && N==64 && time<=2+dt)
            iter=iter+1;
            t=[t;time];
            p1(iter) = (u (find(xu==1),find(yu<=0.5,1,'last'))...
                +u (find(xu==1),find(yu>=0.5,1)))/2;
            p2(iter) = (v (find(xv<=1,1,'last'),find(yv==1.5))...
                +v (find(xv>=1,1),find(yv==1.5)))/2;
            p3(iter) = (Y (find(xY<=0.5,1,'last'),find(yY<=0.5,1,'last'))...
                +Y (find(xY<=0.5,1,'last'),find(yY>=0.5,1))...
                +Y (find(xY>=0.5,1),find(yY<=0.5,1,'last'))...
                +Y (find(xY<=0.5,1),find(yY<=0.5,1)))/4;
        end
```

Probes value for u, v and Y at t=1

```
        if time==1
            p11(i)= (u (find(xu==1),find(yu<=0.5,1,'last'))...
                    +u (find(xu==1),find(yu>=0.5,1)))/2;
            p21(i)= (v (find(xv<=1,1,'last'),find(yv==1.5))...
                    +v (find(xv>=1,1),find(yv==1.5)))/2;
            p31(i)= (Y (find(xY<=0.5,1,'last'),find(yY<=0.5,1,'last'))...
                    +Y (find(xY<=0.5,1,'last'),find(yY>=0.5,1))...
                    +Y (find(xY>=0.5,1),find(yY<=0.5,1,'last'))...
                    +Y (find(xY<=0.5,1),find(yY<=0.5,1)))/4;
        end
```

```
    end
```

Plot the probes, only for M=128 and N=64

```
    if (M==128 && N==64)
        figure
        plot(t,p1,'linewidth',2)
        xlim([0 2])
        xlabel('$t$','Interpreter','latex')
        ylabel('$u(1,0.5,t)$','Interpreter','latex')
        set(gca,'fontsize',axisSize)
        grid on
        txt='Latex/FIGURES/probe1';
        saveas(gcf,txt,'epsc')
        figure
        plot(t,p2,'linewidth',2)
        xlim([0 2])
        xlabel('$t$','Interpreter','latex')
        ylabel('$v(1,1.5,t)$','Interpreter','latex')
        set(gca,'fontsize',axisSize)
        grid on
        txt='Latex/FIGURES/probe2';
        saveas(gcf,txt,'epsc')
        figure
        plot(t,p3,'linewidth',2)
        xlim([0 2])
        xlabel('$t$','Interpreter','latex')
        ylabel('$Y(0.5,0.5,t)$','Interpreter','latex')
        set(gca,'fontsize',axisSize)
        grid on
        txt='Latex/FIGURES/probe3';
        saveas(gcf,txt,'epsc')
    end
```

```
end
```

GCI analysis

```
clc
r=2
Fsec=1.25
```

```
% Probe 1
p_u=log((p11(3)-p11(2))/(p11(2)-p11(1)))/log(r)
u_h0=p11(1)+(p11(1)-p11(2))/(r^p_u-1)
GCI21_u=Fsec*(p11(2)-p11(1))/(p11(1)*(r^p_u-1))
GCI32_u=Fsec*(p11(3)-p11(2))/(p11(2)*(r^p_u-1))
coeff_u=GCI21_u*r^p_u/GCI32_u
percent_u=GCI21_u*100
% pause

% Probe 2
p_v=log((p21(3)-p21(2))/(p21(2)-p21(1)))/log(r)
v_h0=p21(1)+(p21(1)-p21(2))/(r^p_v-1)
GCI12_v=Fsec*(p21(1)-p21(2))/(p21(1)*(r^p_v-1))
GCI23_v=Fsec*(p21(2)-p21(3))/(p21(2)*(r^p_v-1))
coeff_v=GCI12_v*r^p_v/GCI23_v
percent_v=GCI12_v*100
% pause

% Probe 3
p_Y=log((p31(3)-p31(2))/(p31(2)-p31(1)))/log(r)
Y_h0=p31(1)+(p31(1)-p31(2))/(r^p_Y-1)
GCI12_Y=Fsec*(p31(1)-p31(2))/(p31(1)*(r^p_Y-1))
GCI23_Y=Fsec*(p31(2)-p31(3))/(p31(2)*(r^p_Y-1))
coeff_Y=GCI12_Y*r^p_Y/GCI23_Y
percent_Y=GCI12_Y*100
```

Published with MATLAB® R2017a