FRANCISCO CASTILLO APM 505 HOMEWORK 6

Contents

Initialization of the code

```
clear all    % Clear workspace
clc          % Clear command window
format long

tol=1e-12; % Tolerance
```

Function to create the matrix A

```
    type DominantEigenvalueMatrix.m
```

```
function [A,P,D]=DominantEigenvalueMatrix(N,f)
% This function gives a NxN matrix A, an orthogonal matrix P and a diagonal
% matrix D such that A=A = P*D*P'. Needs the dimension N and a factor f.
% The matrix A will have an dominant eigenvalue if f>>1 and will have the two
% larger eigenvalues very similar if f is close to unity.
P = orth(rand(N));
lambdaV = randi([1,100],N,1);
k=randi([1,N],1);
j=find(lambdaV==max(lambdaV));
while k==j
    k=randi([1,N],1);
end
lambdaV(k)=f*max(lambdaV);
D = diag(lambdaV);
A = P*D*P';
end
```

Function for the Power Method Iteration

```
    type powermethod.m
```

```
function [lambda,k,q]=powermethod(A,tol)
% This function uses the powermethod to, given the matrix A and a tolerance,
% obtain the eigenvalue with larger absolute value and its eigenvector.
% It will also provide the number of iterations needed to meet the tolerance.
    N=size(A,1);
    lambdaprev=1;   % Initialize lambdaprev
    lambda=0;       % Initialize lambda
    k=0;            % Start the counter of iterations
    q=rand(N,1);    % The first guess of q is a random vector as the problem specifies
    while norm(lambdaprev-lambda)>tol   % This is the power method algorithm to obtain the dominant eigenvalue
        k=k+1;
        lambdaprev=lambda;
        z=A*q;
        q=z/norm(z);
        lambda=q'*A*q;
    end
end
```

Run different cases of study

```
for i=1:4
```

```
    switch i
        case 1
            N=3;    % Dimension of the matrix A
            f=30;   % The factor f large means that the matrix A is going to have one dominant eigenvalue
```

```
        case 2
            N=3;
            f=1.0001;   % The factor f close to unity means that the matrix A is not going to have any dominant eigenvalue
        case 3
            N=9;
            f=30;
        case 4
            N=9;
            f=1.0001;
    end
```

Create the matrix A

```
    [A,P,D]=DominantEigenvalueMatrix(N,f);
```

Power Method Iteration

```
    [lambda,k,q]=powermethod(A,tol);
```

Results and discussion

```
    fprintf('>>Case %d\n',i)
    A
    P
    D
    lambda
    k
    q
    v=P'*q
```

```
>>Case 1

A =

   1.0e+03 *

   0.580128284192242   0.046439680048687  -1.066230401034395
   0.046439680048687   0.102100781435018  -0.103794325135872
  -1.066230401034395  -0.103794325135872   2.452770934372743


P =

  -0.661224606225391  -0.412065352084474  -0.626884491540169
  -0.674207337631308  -0.040035772924967   0.737456170067350
  -0.328977941519156   0.910274289704922  -0.251344845771740


D =

          97           0           0
           0        2940           0
           0           0          98


lambda =

     2.940000000000002e+03


k =

     7


q =

  -0.412065351343676
  -0.040035772822932
   0.910274290044755


v =

  -0.000000000670424
   1.000000000000000
  -0.000000000474563
```

```
>>Case 2

A =

  74.887638532738137   1.739944581910716  -1.023052419428474
   1.739944581910713  49.023414014787832  15.211559370014635
  -1.023052419428474  15.211559370014632  66.096447452474195


P =

  -0.640357468083793   0.057758367208964  -0.765902267973316
  -0.419614448045632  -0.861513599435976   0.285863661521065
  -0.643324201364036   0.504438587941312   0.575912912630982


D =

  75.000000000000000                   0                   0
                   0  40.000000000000000                   0
                   0                   0  75.007499999999993


lambda =

  75.007499994765965


k =

       78027


q =

  -0.765367055957023
   0.286214101221276
   0.576450134805931


v =

  -0.000835384599672
  -0.000000000000000
   0.999999651066224



>>Case 3

A =

   1.0e+03 *

  Columns 1 through 3

   0.443970111010125   0.510976164082971  -0.543671649876610
   0.510976164082971   0.666204059248030  -0.694334971099687
  -0.543671649876610  -0.694334971099687   0.847520151636893
  -0.083936469354642  -0.093883800645051   0.124996525001864
   0.032184535261895   0.067968338216192  -0.062582397111010
   0.152928488794021   0.161292045410396  -0.203862732325698
  -0.596361455739444  -0.778499755162343   0.896579745760358
   0.101291493232316   0.121609257194190  -0.140582920477222
   0.163165963455063   0.213774091043083  -0.253493015736820

  Columns 4 through 6

  -0.083936469354642   0.032184535261895   0.152928488794021
  -0.093883800645051   0.067968338216192   0.161292045410396
   0.124996525001864  -0.062582397111010  -0.203862732325698
   0.066609719598916  -0.016420088942217  -0.014689568229925
  -0.016420088942217   0.083004573317740   0.007717862604530
  -0.014689568229925   0.007717862604530   0.118982864698969
   0.121001595177795  -0.056852768856697  -0.217953437445107
  -0.022821342397491   0.017090901033227   0.059558616639926
  -0.017843411058920   0.025785427684795   0.059021909312473

  Columns 7 through 9

  -0.596361455739444   0.101291493232316   0.163165963455063
  -0.778499755162343   0.121609257194190   0.213774091043083
   0.896579745760358  -0.140582920477222  -0.253493015736820
   0.121001595177795  -0.022821342397491  -0.017843411058920
```

```
  -0.056852768856697   0.017090901033227   0.025785427684795
  -0.217953437445107   0.059558616639926   0.059021909312473
   1.011036248770395  -0.175307351494070  -0.272547369934421
  -0.175307351494070   0.084173368753364   0.048036301810715
  -0.272547369934421   0.048036301810715   0.154498902965565


P =

  Columns 1 through 3

  -0.313126832446919   0.286880519378418  -0.195581287592358
  -0.254757755662804  -0.300445938909296   0.267671928540534
  -0.355522453173574  -0.424632959743121  -0.065930211615566
  -0.291846512287872  -0.382940618273510  -0.172650721863202
  -0.493808489260941   0.051044941855103   0.550616392607344
  -0.305471644512459   0.264230837177965  -0.652661958407240
  -0.292598225286998   0.450700118685200   0.049412954372303
  -0.239163431420720  -0.387479119740511  -0.320095024121810
  -0.381453605631020   0.273461059655093   0.147973395461173

  Columns 4 through 6

  -0.431490228629131  -0.236068123811833  -0.278498868764144
   0.593816537877668   0.099374399525087   0.198387319394036
  -0.244699205345149  -0.231725832118027   0.474832020303060
  -0.114058184892477   0.536724881188420  -0.474828144090100
  -0.280044584993832   0.251034340544430  -0.020482020674051
   0.119680746833721   0.372572927422272   0.419278779939618
   0.492051928941130  -0.013281068478147  -0.306836592728813
   0.230510844853649  -0.510803919165350  -0.363989022783417
   0.017672865816613  -0.360139213758088   0.171180389802118

  Columns 7 through 9

   0.549695180765594  -0.361067760376492  -0.171437448129230
   0.359114968650791  -0.458274563804354  -0.179867855439712
   0.243707725673938   0.520970636760620  -0.138202649009392
  -0.205544260544209   0.073459086459673  -0.405196877959256
  -0.090054358817229  -0.039768836742349   0.546639833191155
  -0.089326654110403  -0.130851309433392   0.238477311494795
   0.206723018880378   0.573149937154274  -0.033743181966806
  -0.196828603018061  -0.097376702677061   0.442077297586852
  -0.608000661809337  -0.161620408970832  -0.450238602473552


D =

  Columns 1 through 6

          83           0           0           0           0           0
           0          54           0           0           0           0
           0           0         100           0           0           0
           0           0           0           8           0           0
           0           0           0           0          45           0
           0           0           0           0           0          11
           0           0           0           0           0           0
           0           0           0           0           0           0
           0           0           0           0           0           0

  Columns 7 through 9

           0           0           0
           0           0           0
           0           0           0
           0           0           0
           0           0           0
           0           0           0
          97           0           0
           0        3000           0
           0           0          78


lambda =

     2.999999999999998e+03


k =

     8


q =
```

```
  -0.361067760374693
  -0.458274563803179
   0.520970636761827
   0.073459086459806
  -0.039768836741593
  -0.130851309432796
   0.573149937155288
  -0.097376702676880
  -0.161620408971415


v =

  -0.000000000002003
   0.000000000000023
  -0.000000000000207
   0.000000000000000
                   0
                   0
   0.000000000002085
   1.000000000000000
   0.000000000000123



>>Case 4

A =

  Columns 1 through 3

  37.997441302207015  -6.343648439051272  -3.008479883561845
  -6.343648439051274  50.835678304928507   8.179822958428350
  -3.008479883561845   8.179822958428350  40.564534481031146
  -6.918505689818491   3.026399529607823   4.439185174191890
  -2.946812620759903   8.040768368197924 -17.651056133310924
  -4.427438960848171 -13.202368335568481   3.088896020741021
  -1.315081993400808   9.975998870386299  -5.347765183897969
   5.150927563705187 -15.797347529267604  -4.945816718338830
   0.483911656188302 -20.724919283832989 -15.497929398880235

  Columns 4 through 6

  -6.918505689818492  -2.946812620759903  -4.427438960848171
   3.026399529607823   8.040768368197931 -13.202368335568481
   4.439185174191893 -17.651056133310924   3.088896020741021
  34.935133013634747 -19.307913965304735   2.432046290254544
 -19.307913965304735  72.265110657279095  -6.204386918183552
   2.432046290254543  -6.204386918183552  31.349604549274371
   1.047590760902260   7.033239792433132 -10.956162896107896
  -1.368163106501495  -3.412180772989069   5.723392643603385
  -5.937781670977831   7.065955777679497   8.238960268787361

  Columns 7 through 9

  -1.315081993400808   5.150927563705188   0.483911656188303
   9.975998870386301 -15.797347529267604 -20.724919283832989
  -5.347765183897969  -4.945816718338831 -15.497929398880233
   1.047590760902260  -1.368163106501494  -5.937781670977830
   7.033239792433132  -3.412180772989073   7.065955777679495
 -10.956162896107896   5.723392643603386   8.238960268787361
  38.446094828372289 -13.532912505435482  -2.970430953765988
 -13.532912505435483  40.366153116889997   9.464908245011923
  -2.970430953765987   9.464908245011923  43.249549746382932


P =

  Columns 1 through 3

  -0.365529111285548  -0.315562580148483   0.031011499994501
  -0.316756913719916  -0.396023362283070  -0.229085684366749
  -0.408690907719279   0.371620118283125   0.357431001594002
  -0.324997014566044  -0.155917591067038   0.400736844516781
  -0.190473927267043   0.078976147695020   0.210041166611882
  -0.355593967904948  -0.222523070725888  -0.630590003175989
  -0.341758391877649   0.522423658905549  -0.299974176657449
  -0.253707472664749   0.447467887445704  -0.182620997855437
  -0.386663344268026  -0.228618040473023   0.305147697576678

  Columns 4 through 6

   0.250811561888935  -0.819728546807853   0.112565062388468
  -0.425122314931425   0.027040052825711  -0.311987480410201
  -0.107986704968881   0.061781936837600   0.418427843058715
```

```
   0.481547641063636   0.269517070714118  -0.445318770195648
   0.331818382856577   0.180990261784203  -0.022812169633759
   0.295206154724789   0.394583026458633   0.262905014942224
   0.034585240499419  -0.107611632875390   0.129827462786298
  -0.167426962444924  -0.126189852405340  -0.637447797802963
  -0.535104923452685   0.186815695153236   0.154168559955964

  Columns 7 through 9

   0.089880864394435   0.046095960253124   0.090063216939943
  -0.519376262991865   0.163016536040222  -0.339452315842357
   0.064426128443395   0.435934789903237  -0.427040332572865
   0.130020705851759  -0.310396080521814  -0.305596346351545
  -0.568597531385897   0.319442342844903   0.587058163395245
   0.293956277845989   0.155493682477986   0.061988683360665
  -0.333593502052755  -0.614361977028177  -0.043970212892343
   0.352377794648048   0.281723728188036   0.219592459908298
   0.236527877688598  -0.319194783853941   0.449748882746995


D =

  Columns 1 through 3

  20.000000000000000                   0                   0
                   0  23.000000000000000                   0
                   0                   0  18.000000000000000
                   0                   0                   0
                   0                   0                   0
                   0                   0                   0
                   0                   0                   0
                   0                   0                   0
                   0                   0                   0

  Columns 4 through 6

                   0                   0                   0
                   0                   0                   0
                   0                   0                   0
  23.000000000000000                   0                   0
                   0  44.000000000000000                   0
                   0                   0  32.000000000000000
                   0                   0                   0
                   0                   0                   0
                   0                   0                   0

  Columns 7 through 9

                   0                   0                   0
                   0                   0                   0
                   0                   0                   0
                   0                   0                   0
                   0                   0                   0
                   0                   0                   0
  93.000000000000000                   0                   0
                   0  44.000000000000000                   0
                   0                   0  93.009299999999996


lambda =

  93.009299994701465


k =

       58318


q =

   0.090131033752469
  -0.339844246612371
  -0.426991581801404
  -0.305498119102155
   0.586628816266948
   0.062210545182583
  -0.044221998196759
   0.219858373650069
   0.449927286902806


v =

                   0
```

```
    0.000000000000000
   -0.000000000000000
   -0.000000000000000
   -0.000000000000000
                    0
    0.000754804360502
   -0.000000000000001
    0.999999715135148
```

```
end
```

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