

# HOMEWORK 9 - FRANCISCO CASTILLO

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## Defined functions

---

```
function phi=EntropyFix(y)
eps=0.1;
phi=abs(y).*(abs(y)>=eps)+((y.^2+eps^2)./(2*eps)).*(abs(y)<eps);
end
```

```
function alpha = HY_alpha(u,E,M)
eps=0.1;
alpha = zeros(M+3,1);
for i=1:M+3
    absDelta=abs(u(i+1)-u(i));
    if absDelta>=eps
        alpha(i) = (E(i+1)-E(i))/(u(i+1)-u(i));
    elseif absDelta<eps
        alpha(i) =(u(i)+u(i+1))/2;
    end
end
end
```

```
function beta = HY_beta(u,G,M)
eps=1e-12;
beta=zeros(M+1,1);
for i=2:M+2
    absDelta=abs(u(i+1)-u(i));
    if absDelta>=eps
        beta(i-1) = (G(i+1)-G(i))/(u(i+1)-u(i));
    elseif absDelta<eps
        beta(i-1)=0;
    end
end
end
end
```

```
function G=HY_G(u,alpha,dt,dx,M)
sigma=sigmaG(alpha,dt,dx);
G=zeros(M+4,1);
for i=3:M+2
    S=sign(u(i+1)-u(i));
    G(i) = S*max(0,min(sigma(i)*abs(u(i+1)-u(i)),...
        S*sigma(i-1)*(u(i)-u(i-1))));
end
% Periodic Boundary Conditions
G(1)=G(M+1);
G(2)=G(M+2);
G(M+3)=G(3);
G(M+4)=G(4);
end
```

```
function Phi = HY_Phi(u,G,alpha,beta,M)
psi=EntropyFix(alpha(2:end-1)+beta);
Phi=zeros(M+1,1);
for i=2:M+2
    Phi(i-1)=G(i+1)+G(i)-psi(i-1)*(u(i+1)-u(i));
end
end
```

```
function u = initialCondition(x,M)
u=zeros(M+4,1);
for i=3:M+2
    if (x(i)<4.5 || x(i)>5.5)
        u(i)=0.25+0.5*sin(pi/4*(x(i)-3));
    elseif (x(i)>=4.5 && x(i)<=5.5)
        u(i)=0.25+0.5*sin(pi/4*(x(i)-3))+(1+cos(2*pi*x(i)))*cos(8*pi*x(i));
    end
end
end
```

```

% Periodic Boundary Conditions
u(1)=u(M+1);
u(2)=u(M+2);
u(M+3)=u(3);
u(M+4)=u(4);
end

function h = numericalFlux(E,Phi,M)
h=zeros(M+1,1);
for i=2:M+2
    h(i-1)=0.5*(E(i+1)+E(i)+Phi(i-1));
end

function sigma = sigmaG(alpha,dt,dx)
sigma = 0.5*(EntropyFix(alpha)-(dt/dx)*alpha.^2);
end

function u = TVD2order(u,h,dt,dx,M)
for i=3:M+2
    u(i)=u(i)- dt/dx*(h(i-1)-h(i-2));
end
% Periodic Boundary Conditions
u(1)=u(M+1);
u(2)=u(M+2);
u(M+3)=u(3);
u(M+4)=u(4);
end

```

## ProblemHomework8.m

```

clear variables
close all
clc
format long

axisSize=14;
linewidth=1.5;
L=4;
CFL=0.1;
M=32;
i=0;
T=nan(3,7);
check=1;
stp=1;
while check>0.01

```

```

    i=i+1;
    M=2*M;
    dx=L/M;
    x=linspace(3-1.5*dx,7+1.5*dx,M+4)'; % Cell centered mesh with two
                                         % ghost cells at each side

```

## Initialization

```

time=0;
u=initialCondition(x,M);
% Plot initial condition
if (M==256 || M==1024)
    figure(1)
    plot(x,u,'linewidth',linewidth)
    grid on
    axis([min(x) max(x) -1 3])
    xlabel('$x$', 'Interpreter','latex')
    ylabel('$u(x,t)$', 'Interpreter','latex')
    set(gca,'fontsize',axisSize)
    if M==256
        txt='Latex/FIGURES/uinitial_M256';
    elseif M==1024
        txt='Latex/FIGURES/uinitial_M1024';
    end
    saveas(gcf,txt,'eps')
end
E=0.5*u.^2;
alpha=HY_alpha(u,E,M);
dt=CFL*dx/(max(abs(alpha)));

outputTime=[0.1 0.5 1 3.0];

```

```
endtime=outputTime(end);
```

```
n=1;  
while time < endtime
```

```
E=0.5*u.^2;  
alpha=HY_alpha(u,E,M);  
if (time < outputTime(n) && time+dt >= outputTime(n))  
    dt=outputTime(n)-time;  
    n=n+1;  
else  
    dt=CFL*dx/(max(abs(alpha)));  
end  
G=HY_G(u,alpha,dt,dx,M);  
beta=HY_beta(u,G,M);  
psi=EntropyFix(alpha(2:end-1)+beta);  
Phi=HY_Phi(u,G,alpha,beta,M);  
h = numericalFlux(E,Phi,M);  
  
u=TVD2order(u,h,dt,dx,M);  
time=time+dt;
```

## Plot required at part 5

```
if (M==256 && ismember(time,outputTime))  
    figure(n)  
    plot(x,u,'linewidth',linewidth)  
    grid on  
    axis([3 7 -1 3])  
    xlabel('$x$', 'Interpreter', 'latex')  
    ylabel('$u(x,t)$', 'Interpreter', 'latex')  
    set(gca, 'fontsize', axisSize)  
    txt=['Latex/FIGURES/u_M256_' num2str(n-1)];  
    saveas(gcf,txt, 'eps')  
end  
if (M==1024 && ismember(time,outputTime))  
    figure(n)  
    plot(x,u,'linewidth',linewidth)  
    grid on  
    axis([3 7 -1 3])  
    xlabel('$x$', 'Interpreter', 'latex')  
    ylabel('$u(x,t)$', 'Interpreter', 'latex')  
    set(gca, 'fontsize', axisSize)  
    txt=['Latex/FIGURES/u_M1024_' num2str(n-1)];  
    saveas(gcf,txt, 'eps')  
end
```

## u at x=6 and t=1

```
if time==1  
    uGCI(i)=(u(find(x<=6,1,'last'))+u(find(x>=6,1)))/2;  
end
```

```
end
```

## GCI analysis

```
if i>=3  
    r=2;  
    Fsec=1.25;  
    p(i)=log(abs(uGCI(i-2)-uGCI(i-1))/abs(uGCI(i-1)-uGCI(i)))/log(r);  
    u_h0(i)=uGCI(i)+(uGCI(i)-uGCI(i-1))/(r^p(i)-1);  
    GCI12(i)=Fsec*abs(1-uGCI(i-1)/uGCI(i))/(r^p(i)-1);  
    GCI23(i)=Fsec*abs(1-uGCI(i-2)/uGCI(i-1))/(r^p(i)-1);  
    coeff(i)=GCI12(i)*r^p(i)/GCI23(i);  
    percent(i)=GCI12(i)*100;  
    check=abs(percent(i));  
    % Include results in a table  
    T(i,2)=p(i);  
    T(i,3)=u_h0(i);  
    T(i,4)=GCI12(i);  
    T(i,5)=GCI23(i);  
    T(i,6)=coeff(i);  
    T(i,7)=percent(i);  
end  
% Include results in a table  
T(i,1)=M;
```

```
end
T=array2table(T,'VariableNames',{'M','p','phi0','GCI12','GCI23','coeff','Check'})
if CFL==0.1
    save('Case1_CFL01');
elseif CFL==0.5
    save('Case2_CFL05');
end
```