HOMEWORK 8 - FRANCISCO CASTILLO

Contents

Defined functions

```
function a = velocity(x,t)
a = 2.3+1.7*sin(2*pi*x)+1.5*sin(5*pi*t);
end


function phi = updateGhostCells(phi,t,M)
% Left boundary, Dirichlet BC
phi(3)=2*phi_left(t)-phi(4);
phi(2)=2*phi_left(t)-phi(5);
phi(1)=2*phi_left(t)-phi(6);
% Right boundary, zero Neumann BC
phi(M+4)=phi(M+3);
phi(M+5)=phi(M+2);
phi(M+6)=phi(M+1);
end


function phileft = phi_left(t)
if 0<=t && t<=0.25
    phileft=1;
elseif 0.25<t && t<=0.5
    phileft=0;
elseif 0.5<t && t<=1
    phileft=(1-cos(4*pi*t))/2;
elseif t>1
    phileft=0;
else
    error('t negative')
end
end


function DphiDx = WENO5(phi,i,hx,a)
if a>0
    DphiDx = (1/(12*hx))*(phi(i-2)-8*phi(i-1)+8*phi(i+1)-phi(i+2))...
        -psiWENO((phi(i-1)-2*phi(i-2)+phi(i-3))/hx,...
        (phi(i)-2*phi(i-1)+phi(i-2))/hx,...
        (phi(i+1)-2*phi(i)+phi(i-1))/hx,...
        (phi(i+2)-2*phi(i+1)+phi(i))/hx);
elseif a<0
    DphiDx = (1/(12*hx))*(phi(i-2)-8*phi(i-1)+8*phi(i+1)-phi(i+2))...
        +psiWENO((phi(i+3)-2*phi(i+2)+phi(i+1))/hx,...
        (phi(i+2)-2*phi(i+1)+phi(i))/hx,...
        (phi(i+1)-2*phi(i)+phi(i-1))/hx,...
        (phi(i)-2*phi(i-1)+phi(i-2))/hx);
end
end


function psi = psiWENO(a,b,c,d)
eps=1e-6;
IS0=13*(a-b)^2+3*(a-3*b)^2;
IS1=13*(b-c)^2+3*(b+c)^2;
IS2=13*(c-d)^2+3*(3*c-d)^2;

a0=(eps+IS0)^(-2);
a1=6*(eps+IS1)^(-2);
a2=3*(eps+IS2)^(-2);

w0=a0/(a0+a1+a2);
w2=a2/(a0+a1+a2);

psi = (a-2*b+c)*w0/3+(w2-0.5)*(b-2*c+d)/6;
end
```

```
function phi3 = TVDRK3(phi0,M,hx,dt,t,a)
% Constants and prealocation
a10=1;
a20=-3/4; a21=1/4;
a30=-1/12; a31=-1/12; a32=2/3;
phi1=zeros(M+6,1);
phi2=zeros(M+6,1);
phi3=zeros(M+6,1);

%%% STEP 1 %%%
for i=4:M+3
    phi1(i)=phi0(i)-a10*a(i)*dt*WENO5(phi0,i,hx,a(i));
end
% Update ghost cells
phi1=updateGhostCells(phi1,t,M);

%%% STEP 2 %%%
for i=4:M+3
    phi2(i)=phi1(i)-a20*a(i)*dt*WENO5(phi0,i,hx,a(i))...
        -a21*a(i)*dt*WENO5(phi1,i,hx,a(i));
end
% Update ghost cells
phi2=updateGhostCells(phi2,t,M);

%%% STEP 3 %%%
for i=4:M+3
    phi3(i)=phi2(i)-a30*a(i)*dt*WENO5(phi0,i,hx,a(i))...
        -a31*a(i)*dt*WENO5(phi1,i,hx,a(i))...
        -a32*a(i)*dt*WENO5(phi2,i,hx,a(i));
end
% Update ghost cells
phi3=updateGhostCells(phi3,t+dt,M);
end
```

Problem

```
clear variables
close all
clc
format long

axisSize=14;
linewidth=1.5;
L=4;
CFL=0.8;
M=64;
i=0;
T=nan(3,7);
check=1;
while check>0.1
```

```
    i=i+1;
    M=2*M
    hx=L/M;
    x=linspace(-1-2.5*hx,3+2.5*hx,M+6)'; % Cell centered mesh with three
                                         % ghost cells at each side
```

Initialization

```
    time=0;
    a=velocity(x,time);
    phi = zeros(M+6,1);
    phi(3)=2*phi_left(time);
    phi(2)=2*phi_left(time);  % Since phi(4:6)=0 it is not necessary
    phi(1)=2*phi_left(time);  % to include them

    dt=CFL*hx/(max(abs(a)));
    outputTime=[0.25 0.5 1 1.25 1.5 2.1];
    endtime=outputTime(end);

    n=1;
    step=0;
    while time < endtime
```

```
        a=velocity(x,time);
        if (time < outputTime(n) && time+dt >= outputTime(n))
            dt=outputTime(n)-time;
            n=n+1;
        else
```

```
        dt=CFL*hx/(max(abs(a)));
    end
    phi = TVDRK3(phi,M,hx,dt,time,a);
    time=time+dt;
```

Plot required at part 5

```
    if (M==256 && CFL==0.8 && ismember(time,outputTime))
        figure(n-1)
        plot(x,phi,'linewidth',linewidth)
        grid on
        axis([-1 3 -0.1 1.1])
        xlabel('$x$','Interpreter','latex')
        ylabel('$\phi(x,t)$','Interpreter','latex')
        set(gca,'fontsize',axisSize)
        txt=['Latex/FIGURES/phi_' num2str(n-1)];
        saveas(gcf,txt,'epsc')
    end
```

Plot required at part 6

For CFL=0.8

```
    if (M==4096 && CFL==0.8 && ismember(time,outputTime))
        figure(n+6)
        plot(x,phi,'linewidth',linewidth)
        grid on
        axis([-1 3 -0.1 1.1])
        xlabel('$x$','Interpreter','latex')
        ylabel('$\phi(x,t)$','Interpreter','latex')
        set(gca,'fontsize',axisSize)
        txt=['Latex/FIGURES/phi08_' num2str(n-1)];
        saveas(gcf,txt,'epsc')
    end
    % For CFL=0.5
    if (M==1024 && CFL==0.5 && ismember(time,outputTime))
        figure(n+6)
        plot(x,phi,'linewidth',linewidth)
        grid on
        axis([-1 3 -0.1 1.1])
        xlabel('$x$','Interpreter','latex')
        ylabel('$\phi(x,t)$','Interpreter','latex')
        set(gca,'fontsize',axisSize)
        txt=['Latex/FIGURES/phi05_' num2str(n-1)];
        saveas(gcf,txt,'epsc')
    end
```

Phi at x=0 and t=1.25

```
    if time==1.25
        phiGCI(i)=(phi(find(x<=0,1,'last'))+phi(find(x>=0,1)))/2;
    end
```

```
end
```

GCI analysis

```
if i>=3
    r=2;
    Fsec=1.25;
    p(i)=log(abs(phiGCI(i-2)-phiGCI(i-1))/abs(phiGCI(i-1)-phiGCI(i)))/log(r);
    phi_h0(i)=phiGCI(i)+(phiGCI(i)-phiGCI(i-1))/(r^p(i)-1);
    GCI12(i)=Fsec*abs(1-phiGCI(i-1)/phiGCI(i))/(r^p(i)-1);
    GCI23(i)=Fsec*abs(1-phiGCI(i-2)/phiGCI(i-1))/(r^p(i)-1);
    coeff(i)=GCI12(i)*r^p(i)/GCI23(i);
    percent(i)=GCI12(i)*100;
    check=abs(percent(i));
    % Include results in a table
    T(i,2)=p(i);
    T(i,3)=phi_h0(i);
    T(i,4)=GCI12(i);
    T(i,5)=GCI23(i);
    T(i,6)=coeff(i);
    T(i,7)=percent(i);
end
% Include results in a table
T(i,1)=M;
```

```
end
T=array2table(T,'VariableNames',{'M','p','phi0','GCI12','GCI23','coeff','Check'})
if CFL==0.8
    save('Case1_CFL08');
elseif CFL==0.5
    save('Case2_CFL05');
end
```

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