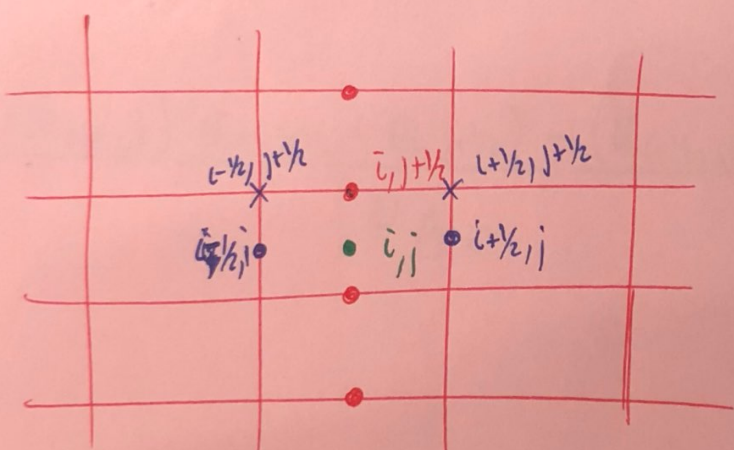
Hyperbolic part for the v equation:

$$\frac{v^{n+1}_{i,j+1/2} - v^{n}_{i,j+1/2}}{\Delta t} = \frac{3}{2}\left[-\frac{\partial uv}{\partial x}\Big|^{n}_{i,j+1/2} - \frac{\partial vv}{\partial y}\Big|^{n}_{i,j+1/2}\right]$$

$$-\frac{1}{2}\left[-\frac{\partial uv}{\partial x}\Big|^{n-1}_{i,j+1/2} - \frac{\partial vv}{\partial y}\Big|^{n-1}_{i,j+1/2}\right]$$

$$+\frac{\nu}{2}\left[\frac{\partial^2 v}{\partial x^2}\Big|^{n}_{i,j+1/2} + \frac{\partial^2 v}{\partial y^2}\Big|^{n}_{i,j+1/2} + \frac{\partial^2 v}{\partial x^2}\Big|^{n+1}_{i,j+1/2} + \frac{\partial^2 v}{\partial y^2}\Big|^{n+1}_{i,j+1/2}\right]$$



$$\left.\begin{aligned}
\frac{\partial vv}{\partial y}\Big|_{i,j+1/2} &= \frac{(v_{i,j+1})^2 - (v_{i,j})^2}{\Delta y} \\
\frac{\partial uv}{\partial x}\Big|_{i,j+1/2} &= \frac{(uv)_{i+1/2,j+1/2} - (uv)_{i-1/2,j+1/2}}{\Delta x}
\end{aligned}\right\} \text{Convective terms}$$

Note that:

$$v_{i,j} = \frac{1}{2}\left(v_{i,j+1/2} + v_{i,j-1/2}\right)$$

$$v_{i+1/2,j+1/2} = \frac{1}{2}\left(v_{i+1,j+1/2} + v_{i,j+1/2}\right)$$

$$u_{i+1/2,j+1/2} = \frac{1}{2}\left(u_{i+1/2,j+1} + u_{i+1/2,j}\right)$$

Then:

$$\left.\frac{\partial vv}{\partial y}\right|_{i,j+1/2} = \frac{(v_{i,j+1})^2 - (v_{i,j})^2}{\Delta y} =$$

$$= \frac{v_{i,j+3/2}^2 + \cancel{v_{i,j+1/2}^2} + 2v_{i,j+3/2}v_{i,j+1/2} - \cancel{v_{i,j+1/2}^2} - v_{i,j-1/2}^2 - 2v_{i,j+1/2}v_{i,j-1/2}}{4\Delta y}$$

$$= \frac{v_{i,j+3/2}^2 + 2v_{i,j+1/2}(v_{i,j+3/2} - v_{i,j-1/2}) - v_{i,j-1/2}^2}{4\Delta y}$$

$$\left.\frac{\partial uv}{\partial x}\right|_{i,j+1/2} = \frac{(uv)_{i+1/2,j+1/2} - (uv)_{i-1/2,j+1/2}}{\Delta x} =$$

$$= \frac{(u_{i+1/2,j+1} + u_{i+1/2,j})(v_{i+1,j+1/2} + v_{i,j+1/2}) - (u_{i-1/2,j+1} + u_{i-1/2,j})(v_{i,j+1/2} + v_{i-1,j+1/2})}{4\Delta x}$$

$$\Delta t_u = \frac{\min(\Delta x, \Delta y)}{\max(|2u|) + \max(|v|)} \; ; \; \Delta t_v = \frac{\min(\Delta x, \Delta y)}{\max(|u|) + \max(|2v|)}$$

$$\Delta t = CFL \cdot \min(\Delta t_u, \Delta t_v)$$

This ~~satisifie~~ is stable for the Y equation as well