Computational Fluid Dynamics

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Homework 7

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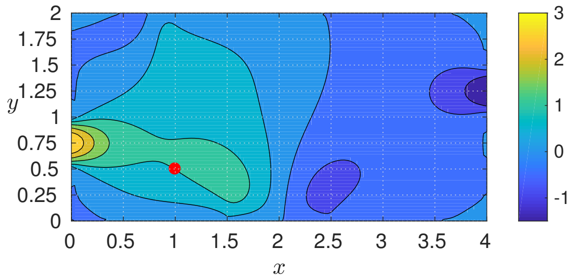
1 Introduction

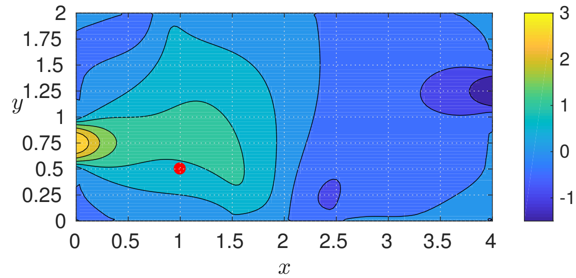
In this assignment we will study a two dimentional mixing chamber. Unlike the previous assignment, in this case we include the continuity equation. Thus, we will have an actual outlet flow through the outlet imposing such equation. In addition, we have included the pressure term as well. First we will show the results requested from the simulation and the code. At the end of the document we will present the equations in sub-index form for the Poisson equation and the projection step of the fractional step method, and the boundary conditions for the Lagrange multiplier. To finish, the calculation of the coefficients that characterize the fully developed velocity profiles given information of the inlets and the average velocities.

2 Results

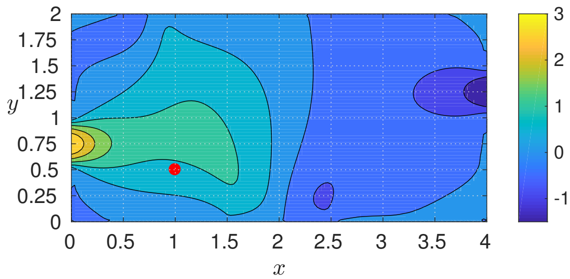
In figures 1 and 2 we can see the velocity contours at different values of time. In the first figure we can see the horizontal velocity contours. The red dot represents the probe located to meassure such quantity. As we can see we have inputs through inlets 1 (positive u) and 2 (negative u). Figure 2 shows the vertical velocity contours at different values of time. As we can see we only introduce fluid vertically through the inlet 3 and, to satisfy the conservation of mass, there is a fluid going out through the outlet where also have zero Neumann boundary conditions. For both u and v the contours don't change much with time, and their solution is not close to zero (initial condition), indicating that the steady state is reached very quickly.



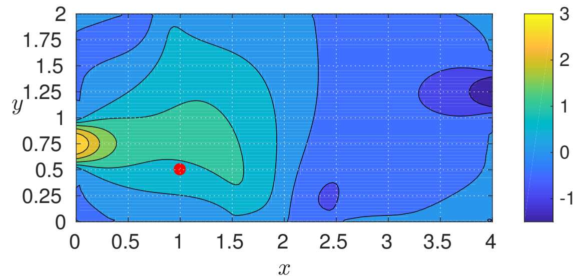
(a) $t = 0.1$ s.



(b) $t = 0.5$ s.

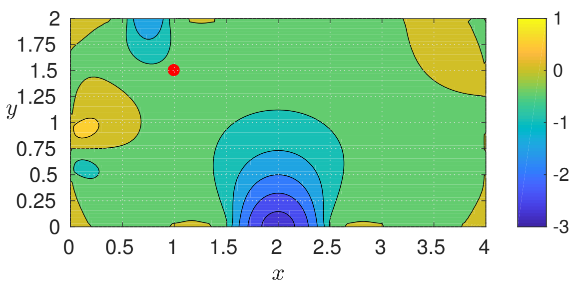


(c) $t = 1$ s.

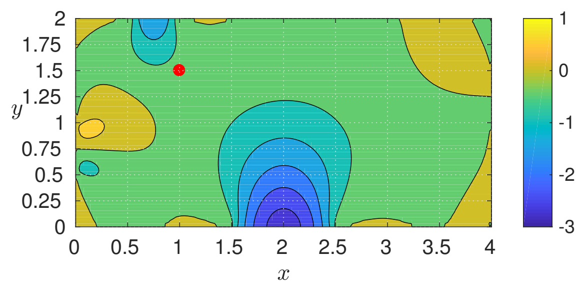


(d) $t = 2$ s.

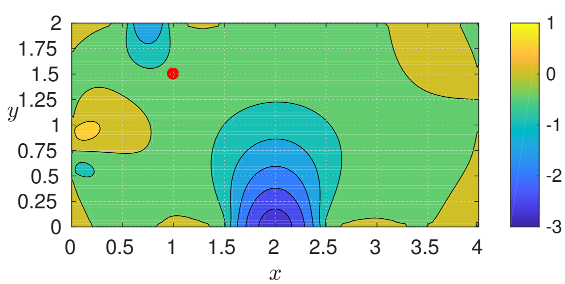
Figure 1: Horizontal velocity contours.



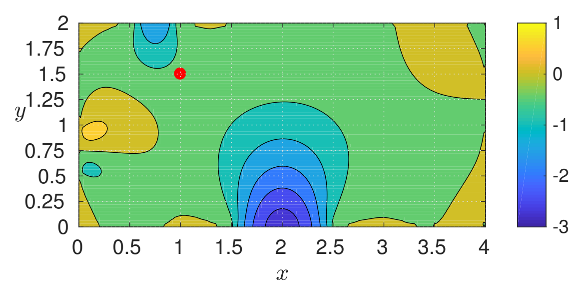
(a) $t = 0.1$ s.



(b) $t = 0.5$ s.



(c) $t = 1$ s.

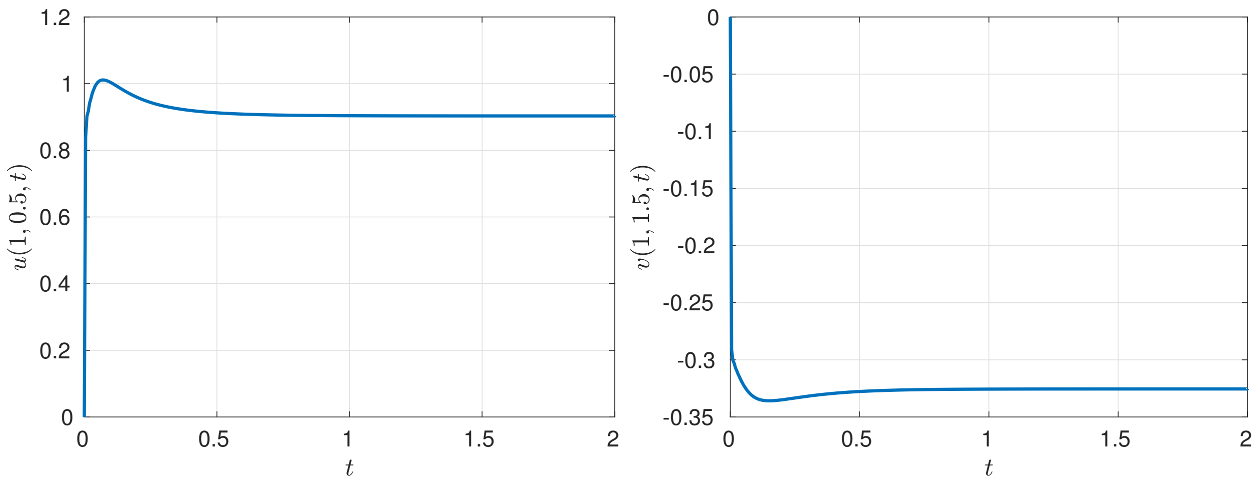


(d) $t = 2$ s.

Figure 2: Vertical velocity contours.

In the next figure we can see the probes meassurements with time during the first 2 seconds. As commented above, the probes show us how for both u and v the fluid reaches the steady state very quickly.

To finish with the results, we proceed with the GCI analysis. In the following table we have the data needed to perform it obtained by doing simulations with different meshes.



(a) Probe 1. (b) Probe 2.

Figure 3: Meassures of the probes.

Grid	M	N	$u(1, 0.5, 1)$	$v(1, 1.5, 1)$
1	128	64	0.903783280349676	-0.325610242364466
2	64	32	0.908404354402535	-0.326805053418745
3	32	16	0.925282713909409	-0.331680997541251

Table 1: GCI analysis data.

Taking the data from u, we can calculate an order of convergence $p = 1.868874573994991$, close to the theoretical value two. Using Richardson extrapolation with the two finest grids we estimate the solution at $h = 0$,

$$u_{h=0} = 0.902041106237625.$$

We obtain the following GCI values

$$GCI_{21} = 0.002409557343461, \qquad GCI_{32} = 0.008756078906480,$$

which give us the following value

$$\frac{GCI_{21}}{GCI_{32}} r^p = 1.005113033349180,$$

where $r = 2$. The previous value tells us that we are in the asymptotic range of convergence. Thus, we can say that the value meassured by the probe is

$$u(1, 0.5, 1) = 0.902041106237625 \pm 0.2409557343461\%$$

Now doing the same for v, we can calculate an order of convergence $p = 2.028899102510403$, close to the theoretical value two. Using Richardson extrapolation with the two finest grids we estimate the solution at $h = 0$,

$$v_{h=0} = -0.325222434200792.$$

We obtain the following GCI values

$$GCI_{21} = 0.001488774434957, \qquad GCI_{32} = 0.006053376475505,$$

which give us the following value

$$\frac{GCI_{12}}{GCI_{23}} r^p = 1.003669451690472.$$

The previous value tells us that we are in the asymptotic range of convergence. Thus, we can say that the value meassured by the probe is

$$v(1, 1.5, 1) = -0.325222434200792 \pm 0.1488774434957\%$$

These values have been obtained using a tolerance of $eps = 10^{-12}$ for the multigrid function.