FRANCISCO CASTILLO APM 505 HOMEWORK 6

Contents

Initialization of the code

```
clear all    % Clear workspace
clc          % Clear command window
format long

tol=1e-12; % Tolerance
```

Function to create the matrix A

```
    type DominantEigenvalueMatrix.m
```

```
function [A,P,D]=DominantEigenvalueMatrix(N,f)
% This function gives a NxN matrix A, an orthogonal matrix P and a diagonal
% matrix D such that A=A = P*D*P'. Needs the dimension N and a factor f.
% The matrix A will have an dominant eigenvalue if f>>1 and will have the two
% larger eigenvalues very similar if f is close to unity.
P = orth(rand(N));
lambdaV = randi([1,100],N,1);
k=randi([1,N],1);
j=find(lambdaV==max(lambdaV));
while k==j
    k=randi([1,N],1);
end
lambdaV(k)=f*max(lambdaV);
D = diag(lambdaV);
A = P*D*P';
end
```

Function for the Power Method Iteration

```
    type powermethod.m
```

```
function [lambda,k,q]=powermethod(A,tol)
% This function uses the powermethod to, given the matrix A and a tolerance,
% obtain the eigenvalue with larger absolute value and its eigenvector.
% It will also provide the number of iterations needed to meet the tolerance.
    N=size(A,1);
    lambdaprev=1;   % Initialize lambdaprev
    lambda=0;       % Initialize lambda
    k=0;            % Start the counter of iterations
    q=rand(N,1);    % The first guess of q is a random vector as the problem specifies
    while norm(lambdaprev-lambda)>tol   % This is the power method algorithm to obtain the dominant eigenvalue
        k=k+1;
        lambdaprev=lambda;
        z=A*q;
        q=z/norm(z);
        lambda=q'*A*q;
    end
end
```

Run different cases of study

```
for i=1:4
```

```
    switch i
        case 1
            N=3;    % Dimension of the matrix A
            f=30;   % The factor f large means that the matrix A is going to have one dominant eigenvalue
```

```
        case 2
            N=3;
            f=1.0001;   % The factor f close to unity means that the matrix A is not going to have any dominant eigenvalue
        case 3
            N=9;
            f=30;
        case 4
            N=9;
            f=1.0001;
    end
```

Create the matrix A

```
    [A,P,D]=DominantEigenvalueMatrix(N,f);
```

Power Method Iteration

```
    [lambda,k,q]=powermethod(A,tol);
```

Results and discussion

```
    fprintf('>>Case %d\n',i)
    A
    P
    D
    lambda
    k
    q
    v=P'*q
```

```
>>Case 1

A =

   1.0e+03 *

   0.580128284192242   0.046439680048687  -1.066230401034395
   0.046439680048687   0.102100781435018  -0.103794325135872
  -1.066230401034395  -0.103794325135872   2.452770934372743


P =

  -0.661224606225391  -0.412065352084474  -0.626884491540169
  -0.674207337631308  -0.040035772924967   0.737456170067350
  -0.328977941519156   0.910274289704922  -0.251344845771740


D =

          97           0           0
           0        2940           0
           0           0          98


lambda =

     2.940000000000002e+03


k =

     7


q =

  -0.412065351343676
  -0.040035772822932
   0.910274290044755


v =

  -0.000000000670424
   1.000000000000000
  -0.000000000474563
```

```
>>Case 2

A =

  74.887638532738137   1.739944581910716  -1.023052419428474
   1.739944581910713  49.023414014787832  15.211559370014635
  -1.023052419428474  15.211559370014632  66.096447452474195


P =

  -0.640357468083793   0.057758367208964  -0.765902267973316
  -0.419614448045632  -0.861513599435976   0.285863661521065
  -0.643324201364036   0.504438587941312   0.575912912630982


D =

  75.000000000000000                   0                   0
                   0  40.000000000000000                   0
                   0                   0  75.007499999999993


lambda =

  75.007499994765965


k =

       78027


q =

  -0.765367055957023
   0.286214101221276
   0.576450134805931


v =

  -0.000835384599672
  -0.000000000000000
   0.999999651066224



>>Case 3

A =

   1.0e+03 *

  Columns 1 through 3

   0.443970111010125   0.510976164082971  -0.543671649876610
   0.510976164082971   0.666204059248030  -0.694334971099687
  -0.543671649876610  -0.694334971099687   0.847520151636893
  -0.083936469354642  -0.093883800645051   0.124996525001864
   0.032184535261895   0.067968338216192  -0.062582397111010
   0.152928488794021   0.161292045410396  -0.203862732325698
  -0.596361455739444  -0.778499755162343   0.896579745760358
   0.101291493232316   0.121609257194190  -0.140582920477222
   0.163165963455063   0.213774091043083  -0.253493015736820

  Columns 4 through 6

  -0.083936469354642   0.032184535261895   0.152928488794021
  -0.093883800645051   0.067968338216192   0.161292045410396
   0.124996525001864  -0.062582397111010  -0.203862732325698
   0.066609719598916  -0.016420088942217  -0.014689568229925
  -0.016420088942217   0.083004573317740   0.007717862604530
  -0.014689568229925   0.007717862604530   0.118982864698969
   0.121001595177795  -0.056852768856697  -0.217953437445107
  -0.022821342397491   0.017090901033227   0.059558616639926
  -0.017843411058920   0.025785427684795   0.059021909312473

  Columns 7 through 9

  -0.596361455739444   0.101291493232316   0.163165963455063
  -0.778499755162343   0.121609257194190   0.213774091043083
   0.896579745760358  -0.140582920477222  -0.253493015736820
   0.121001595177795  -0.022821342397491  -0.017843411058920
```

```
  -0.056852768856697   0.017090901033227   0.025785427684795
  -0.217953437445107   0.059558616639926   0.059021909312473
   1.011036248770395  -0.175307351494070  -0.272547369934421
  -0.175307351494070   0.084173368753364   0.048036301810715
  -0.272547369934421   0.048036301810715   0.154498902965565


P =

  Columns 1 through 3

  -0.313126832446919   0.286880519378418  -0.195581287592358
  -0.254757755662804  -0.300445938909296   0.267671928540534
  -0.355522453173574  -0.424632959743121  -0.065930211615566
  -0.291846512287872  -0.382940618273510  -0.172650721863202
  -0.493808489260941   0.051044941855103   0.550616392607344
  -0.305471644512459   0.264230837177965  -0.652661958407240
  -0.292598225286998   0.450700118685200   0.049412954372303
  -0.239163431420720  -0.387479119740511  -0.320095024121810
  -0.381453605631020   0.273461059655093   0.147973395461173

  Columns 4 through 6

  -0.431490228629131  -0.236068123811833  -0.278498868764144
   0.593816537877668   0.099374399525087   0.198387319394036
  -0.244699205345149  -0.231725832118027   0.474832020303060
  -0.114058184892477   0.536724881188420  -0.474828144090100
  -0.280044584993832   0.251034340544430  -0.020482020674051
   0.119680746833721   0.372572927422272   0.419278779939618
   0.492051928941130  -0.013281068478147  -0.306836592728813
   0.230510844853649  -0.510803919165350  -0.363989022783417
   0.017672865816613  -0.360139213758088   0.171180389802118

  Columns 7 through 9

   0.549695180765594  -0.361067760376492  -0.171437448129230
   0.359114968650791  -0.458274563804354  -0.179867855439712
   0.243707725673938   0.520970636760620  -0.138202649009392
  -0.205544260544209   0.073459086459673  -0.405196877959256
  -0.090054358817229  -0.039768836742349   0.546639833191155
  -0.089326654110403  -0.130851309433392   0.238477311494795
   0.206723018880378   0.573149937154274  -0.033743181966806
  -0.196828603018061  -0.097376702677061   0.442077297586852
  -0.608000661809337  -0.161620408970832  -0.450238602473552


D =

  Columns 1 through 6

          83           0           0           0           0           0
           0          54           0           0           0           0
           0           0         100           0           0           0
           0           0           0           8           0           0
           0           0           0           0          45           0
           0           0           0           0           0          11
           0           0           0           0           0           0
           0           0           0           0           0           0
           0           0           0           0           0           0

  Columns 7 through 9

           0           0           0
           0           0           0
           0           0           0
           0           0           0
           0           0           0
           0           0           0
          97           0           0
           0        3000           0
           0           0          78


lambda =

     2.999999999999998e+03


k =

     8


q =
```

```
  -0.361067760374693
  -0.458274563803179
   0.520970636761827
   0.073459086459806
  -0.039768836741593
  -0.130851309432796
   0.573149937155288
  -0.097376702676880
  -0.161620408971415


v =

  -0.000000000002003
   0.000000000000023
  -0.000000000000207
   0.000000000000000
                   0
                   0
   0.000000000002085
   1.000000000000000
   0.000000000000123



>>Case 4

A =

  Columns 1 through 3

  37.997441302207015  -6.343648439051272  -3.008479883561845
  -6.343648439051274  50.835678304928507   8.179822958428350
  -3.008479883561845   8.179822958428350  40.564534481031146
  -6.918505689818491   3.026399529607823   4.439185174191890
  -2.946812620759903   8.040768368197924 -17.651056133310924
  -4.427438960848171 -13.202368335568481   3.088896020741021
  -1.315081993400808   9.975998870386299  -5.347765183897969
   5.150927563705187 -15.797347529267604  -4.945816718338830
   0.483911656188302 -20.724919283832989 -15.497929398880235

  Columns 4 through 6

  -6.918505689818492  -2.946812620759903  -4.427438960848171
   3.026399529607823   8.040768368197931 -13.202368335568481
   4.439185174191893 -17.651056133310924   3.088896020741021
  34.935133013634747 -19.307913965304735   2.432046290254544
 -19.307913965304735  72.265110657279095  -6.204386918183552
   2.432046290254543  -6.204386918183552  31.349604549274371
   1.047590760902260   7.033239792433132 -10.956162896107896
  -1.368163106501495  -3.412180772989069   5.723392643603385
  -5.937781670977831   7.065955777679497   8.238960268787361

  Columns 7 through 9

  -1.315081993400808   5.150927563705188   0.483911656188303
   9.975998870386301 -15.797347529267604 -20.724919283832989
  -5.347765183897969  -4.945816718338831 -15.497929398880233
   1.047590760902260  -1.368163106501494  -5.937781670977830
   7.033239792433132  -3.412180772989073   7.065955777679495
 -10.956162896107896   5.723392643603386   8.238960268787361
  38.446094828372289 -13.532912505435482  -2.970430953765988
 -13.532912505435483  40.366153116889997   9.464908245011923
  -2.970430953765987   9.464908245011923  43.249549746382932


P =

  Columns 1 through 3

  -0.365529111285548  -0.315562580148483   0.031011499994501
  -0.316756913719916  -0.396023362283070  -0.229085684366749
  -0.408690907719279   0.371620118283125   0.357431001594002
  -0.324997014566044  -0.155917591067038   0.400736844516781
  -0.190473927267043   0.078976147695020   0.210041166611882
  -0.355593967904948  -0.222523070725888  -0.630590003175989
  -0.341758391877649   0.522423658905549  -0.299974176657449
  -0.253707472664749   0.447467887445704  -0.182620997855437
  -0.386663344268026  -0.228618040473023   0.305147697576678

  Columns 4 through 6

   0.250811561888935  -0.819728546807853   0.112565062388468
  -0.425122314931425   0.027040052825711  -0.311987480410201
  -0.107986704968881   0.061781936837600   0.418427843058715
```

```
   0.481547641063636   0.269517070714118  -0.445318770195648
   0.331818382856577   0.180990261784203  -0.022812169633759
   0.295206154724789   0.394583026458633   0.262905014942224
   0.034585240499419  -0.107611632875390   0.129827462786298
  -0.167426962444924  -0.126189852405340  -0.637447797802963
  -0.535104923452685   0.186815695153236   0.154168559955964

  Columns 7 through 9

   0.089880864394435   0.046095960253124   0.090063216939943
  -0.519376262991865   0.163016536040222  -0.339452315842357
   0.064426128443395   0.435934789903237  -0.427040332572865
   0.130020705851759  -0.310396080521814  -0.305596346351545
  -0.568597531385897   0.319442342844903   0.587058163395245
   0.293956277845989   0.155493682477986   0.061988683360665
  -0.333593502052755  -0.614361977028177  -0.043970212892343
   0.352377794648048   0.281723728188036   0.219592459908298
   0.236527877688598  -0.319194783853941   0.449748882746995


D =

  Columns 1 through 3

  20.000000000000000                   0                   0
                   0  23.000000000000000                   0
                   0                   0  18.000000000000000
                   0                   0                   0
                   0                   0                   0
                   0                   0                   0
                   0                   0                   0
                   0                   0                   0
                   0                   0                   0

  Columns 4 through 6

                   0                   0                   0
                   0                   0                   0
                   0                   0                   0
  23.000000000000000                   0                   0
                   0  44.000000000000000                   0
                   0                   0  32.000000000000000
                   0                   0                   0
                   0                   0                   0
                   0                   0                   0

  Columns 7 through 9

                   0                   0                   0
                   0                   0                   0
                   0                   0                   0
                   0                   0                   0
                   0                   0                   0
                   0                   0                   0
  93.000000000000000                   0                   0
                   0  44.000000000000000                   0
                   0                   0  93.009299999999996


lambda =

  93.009299994701465


k =

       58318


q =

   0.090131033752469
  -0.339844246612371
  -0.426991581801404
  -0.305498119102155
   0.586628816266948
   0.062210545182583
  -0.044221998196759
   0.219858373650069
   0.449927286902806


v =

                   0
```

```
   0.000000000000000
  -0.000000000000000
  -0.000000000000000
  -0.000000000000000
                   0
   0.000754804360502
  -0.000000000000001
   0.999999715135148
```

```
end
```

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In the first case we have a 3x3 matrix A which has a dominant eigenvalue as we see from its diagonal form D. The function powermethod obtains the value of that eigenvalue in 7 iterations and gives us the eigenvector q which coincides with some small error with the second column (because the dominant eigenvalue is on the second column of D) of the matrix P, since the latter has the eigenvector as columns. This is better seen in the basis where A is diagonal. In that basis we have that the eigenvector is v, which has a 1 in the second component and a little and similar error in the first and third. This indicates that the larger eigenvalue is on the second column of D as we knew and that the other two eigenvalues have similar values, as we can check.

In the second case we have a 3x3 matrix A which has a dominant eigenvalue with similar value to the second biggest eigenvalue as we see from its diagonal form D. The powermethod function obtains the value of the larger eigenvalue in 78027 iterations against the 7 iterations needed when the matrix has a largely dominant eigenvalue. The function gives us the eigenvector q as well, it coincides with the third column with some small error, bigger than in the previous case, of the matrix P. This is again better seen in the basis where A is diagonal. In that basis we have that the eigenvector is v, which has a value close to 1 in the third component and a appreciable error in the first. This indicates that the larger eigenvalue is on the third column of D as we knew and that the second larger eigenvalue is in the first column of D, as we can check. In this case the error is bigger because the two larger eigenvalues are very similar. In the previous case this error was much smaller since the eigenvalue dominance was much bigger.

The discussion for the next two cases is the same but with higher dimensions.