Fourier Analysis and Wavelets
Homework 2

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Problem 10

Consider the function $f(x) = \pi - x$, $0 \leq x \leq \pi$.

(a) Sketch the even, 2π-periodic extension of f. Find the Fourier cosine series for f.

Solution: In the following figure we can see the even extension of f.
Since the extended function is even, the Fourier series is going to be only a seri0es of cosines. We now calculate the coefficients:

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(t)dt = \frac{1}{\pi}\int_0^{\pi}(\pi - t)dt = \frac{1}{\pi}\left(\pi^2 - \frac{1}{2}\pi^2\right) = \frac{\pi}{2},$$

and

$$\begin{aligned} a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\cos(nt)dt &= \frac{2}{\pi}\int_0^{\pi}(\pi - t)\cos(nt)dt \\ &= 2\cancelto{0}{\int_0^{\pi}\cos(nt)dt} - \frac{2}{\pi}\int_0^{\pi} t\cos(nt)dt \\ &= \frac{2}{n\pi}\int_0^{\pi}\sin(nt)dt \\ &= -\frac{2}{n^2\pi}\cos(nt)\Big]_0^{\pi}, \end{aligned}$$

where we have used intergration by parts and that $\sin(n\pi) = \sin(0) = 0$. Thus,

$$a_n = \frac{2}{n^2\pi}\left[1 - \cos(n\pi)\right] = \begin{cases} 0 & n \text{ even} \\ \frac{4}{n^2\pi} & n \text{ odd} \end{cases}$$

(b) Sketch the odd, 2π-periodic extension of f. Find the Fourier sine series for f.

Solution: In the following figure we can see the odd extension of f.

Since the extended function is odd, the Fourier series is going to be only a series of sines. We now calculate the coefficients:

$$\begin{aligned}
b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\sin(nt)dt &= \frac{2}{\pi}\int_0^{\pi}(\pi - t)\sin(nt)dt \\
&= 2\int_0^{\pi}\sin(nt)dt - \frac{2}{\pi}\int_0^{\pi} t\sin(nt)dt \\
&= -\frac{2}{n}\cos(nt)\Big]_0^{\pi} + \frac{2}{n\pi}\, t\cos(nt)\Big]_0^{\pi} - \cancelto{0}{\frac{2}{n\pi}\int_0^{\pi}\cos(nt)dt} \\
&= \frac{2}{n}\left[1 - \cos(n\pi)\right] + \frac{2}{n\pi}\left[\pi\cos(n\pi)\right] \\
&= \frac{2}{n}
\end{aligned}$$

where we have used intergration by parts and that $\sin(n\pi) = \sin(0) = 0$. Thus,

$$a_n = \frac{2}{n^2\pi}\left[1 - \cos(n\pi)\right] = \begin{cases} 0 & n \text{ even} \\ \frac{4}{n^2\pi} & n \text{ odd} \end{cases}$$

(c) Sketch the π-periodic extension of f. Find the Fourier series for f.

Solution: a3

Problem 18

Let f and g be 2π-periodic, piecewise smooth functions having Fourier series $f(x) = \sum_n \alpha_n e^{inx}$ and $g(x) = \sum_n \beta_n e^{inx}$, and define the convolution of f and g to be $f * g(x) = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(t)g(x-t)dt$. Show that the complex form of the Fourier series for $f * g$ is

$$f * g(x) = \sum_{n=-\infty}^{\infty} \alpha_n \beta_n e^{inx} .$$

Solution: To prove the result is rather simple, it suffices with plugging in the Fourier series of the different functions in the convolution definition and operate:

$$\begin{aligned}
f * g(x) = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(t)g(x-t)dt &= \frac{1}{2\pi}\int_{-\pi}^{\pi}\sum_n \alpha_n e^{int}\sum_m \beta_m e^{im(x-t)}dt \\
&= \frac{1}{2\pi}\sum_n\sum_m \alpha_n\beta_m e^{imx}\int_{-\pi}^{\pi} e^{int}e^{-imt}dt \\
&= \frac{1}{2\pi}\sum_n\sum_m \alpha_n\beta_m e^{imx}\int_{-\pi}^{\pi} e^{i(n-m)t}dt.
\end{aligned}$$

We know that the set of complex exponentials is orthogonal. Hence, the integral is zero if $n \neq m$ and 2π if $n = m$. Therefore,

$$\begin{aligned} f * g(x) &= \frac{1}{2\pi} \sum_n \sum_m \alpha_n \beta_m e^{imx} \int_{-\pi}^{\pi} e^{i(n-m)t} dt \\ &= \sum_n \alpha_n \beta_n e^{inx}, \end{aligned}$$

and the result is proved.

Problem 23

Sketch two periods of the pointwise limit of the Fourier series for each of the following functions. State wether or not each function's Fourier series converges uniformly.

(e) $f(x) = \cos(x) + |\cos(x)|$, $-\pi \leq x \leq \pi$.

Solution: As we can see in the next figure, the function f is continuous, piecewise smooth and 2π-periodic. Hence, by *Theorem 1.30*, the Fourier Series for this function converges uniformly to it.

Problem 34

Consider the function

$$f(x) = e^{-x^2/10} \left(\cos 2x + 2\sin 4x + 0.4\cos 2x \cos 40x\right) \ .$$

For what values of n would you expect the Fourier coefficients $a(n)$ and $b(n)$ to be significant. Why? Compute the $a(n)$ and $b(n)$ through $n = 50$ and see if you are right. Plot the partial Fourier series through $n = 6$ and compare with the plot of the original $f(x0$.

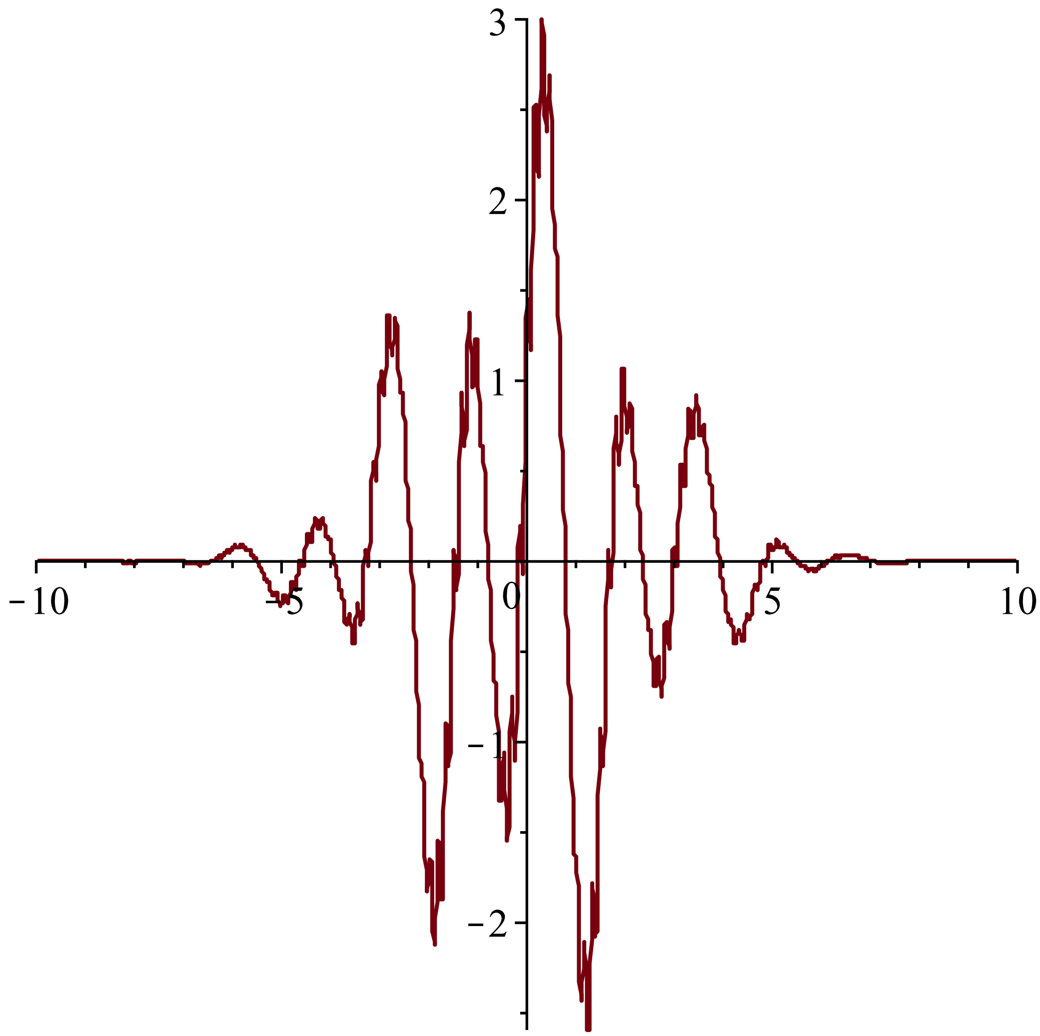
Solution:

restart;

$f := x \rightarrow (\cos(2\cdot x) + 2\cdot\sin(4\cdot x) + 0.4\cdot\cos(2\cdot x)\cdot\cos(40\cdot x))\cdot\exp\left(-\frac{x^2}{10}\right)$

$$x \rightarrow (\cos(2\,x) + 2\,\sin(4\,x) + 0.4\,\cos(2\,x)\,\cos(40\,x))\,e^{-\frac{1}{10}\,x^2} \quad \textbf{(1)}$$

plot(*f*, −10 ..10, *numpoints* = 2000)



$a0 := value(\ (1/(2\cdot \mathrm{Pi}))*Int(f(x), x=-\mathrm{Pi}..\mathrm{Pi})\ )$; # *the value of a_0*

$$-0.01939716410 - 7.321127380\ 10^{-15}\,\mathrm{I} \quad \textbf{(2)}$$

$a := n \rightarrow value(\ (1/(\mathrm{Pi}))*Int(f(x)*\cos(n*x), x=-\mathrm{Pi}..\mathrm{Pi})\ )$; # *the value of a_n*

$$n \rightarrow value\left(\frac{\int_{-\pi}^{\pi} f(x)\,\cos(n\,x)\,\mathrm{d}x}{\pi}\right) \quad \textbf{(3)}$$

$b := n \rightarrow value(\ (1/(\mathrm{Pi}))*Int(f(x)*\sin(n*x), x=-\mathrm{Pi}..\mathrm{Pi})\ )$; # *the value of b_n*

$$n \rightarrow value\left(\frac{\int_{-\pi}^{\pi} f(x)\,\sin(n\,x)\,\mathrm{d}x}{\pi}\right) \tag{4}$$

abs($a0$)

$$0.01939716410 \tag{5}$$

for j **from** 1 **by** 1 **to** 50 **do** $print$([j], abs(a(j))) **end do**

[1], 0.1476872214

[2], 0.7445477843

[3], 0.1422293659

[4], 0.02149944693

[5], 0.01003136555

[6], 0.005926364799

[7], 0.003969783887

[8], 0.002871757925

[9], 0.002188597981

[10], 0.001732390166

[11], 0.001411692307

[12], 0.001177360422

[13], 0.001000940386

[14], 0.0008649783387

[15], 0.0007582674793

[16], 0.0006733448947

[17], 0.0006050957368

[18], 0.0005499380238

[19], 0.0005053288305

[20], 0.0004694573254

[21], 0.0004410519426

[22], 0.0004192615147

[23], 0.0004035885961

[24], 0.0003938648562

[25], 0.0003902675324

[26], 0.0003933856890

[27], 0.0004043593937

[28], 0.0004251411185

[29], 0.0004589829907

[30], 0.0005113772273

[31], 0.0005919861954

[32], 0.0007189579587

$[33], 0.0009297561837$

$[34], 0.001313016999$

$[35], 0.002126626146$

$[36], 0.004413440527$

$[37], 0.02855314628$

$[38], 0.1488080948$

$[39], 0.02963351236$

$[40], 0.007849913064$

$[41], 0.02962380552$

$[42], 0.1488275819$

$[43], 0.02852373085$

$[44], 0.004373870827$

$[45], 0.002076594105$

$[46], 0.001252126110$

$[47], 0.0008575132591$

$[48], 0.0006347625537$

$[49], 0.0004951172555$

$[50], 0.0004009763730$ **(6)**

for j **from** 1 **by** 1 **to** 50 **do** $print([j], \mathrm{abs}(b(j)))$ **end do**

$[1], 0.01091687016$

$[2], 0.03459209603$

$[3], 0.2753152502$

$[4], 1.500943508$

$[5], 0.2765252713$

$[6], 0.03726288640$

$[7], 0.01569455126$

$[8], 0.008398605247$

$[9], 0.005128640916$

$[10], 0.003403251820$

$[11], 0.002392010828$

$[12], 0.001754049513$

$[13], 0.001328960626$

$[14], 0.001033460984$

$[15], 0.0008209713323$

$[16], 0.0006638742324$

$[17], 0.0005450045873$

$[18], 0.0004532703805$

$[19], 0.0003812656244$

$[20], 0.0003239054464$

$[21], 0.0002776133758$

$[22], 0.0002398203080$

$[23], 0.0002086461556$

$[24], 0.0001826921482$

$[25], 0.0001609021128$

$[26], 0.0001424678719$

$[27], 0.0001267635071$

$[28], 0.0001132989032$

$[29], 0.0001016864089$

$[30], 0.00009161657464$

$[31], 0.00008284026687$

$[32], 0.00007515532835$

$[33], 0.00006839651869$

$[34], 0.00006242785255$

$[35], 0.00005713670894$

$[36], 0.00005242926239$

$[37], 0.00004822691080$

$[38], 0.00004446346069$

$[39], 0.00004108289374$

$[40], 0.00003803758254$

$[41], 0.00003528685600$

$[42], 0.00003279583906$

$[43], 0.00003053450916$

$[44], 0.00002847692439$

$[45], 0.00002660058933$

$[46], 0.00002488593129$

$[47], 0.00002331586568$

$[48], 0.00002187543397$

$[49], 0.00002055150067$

$[50], 0.00001933249879$ **(7)**

$N := 6:$

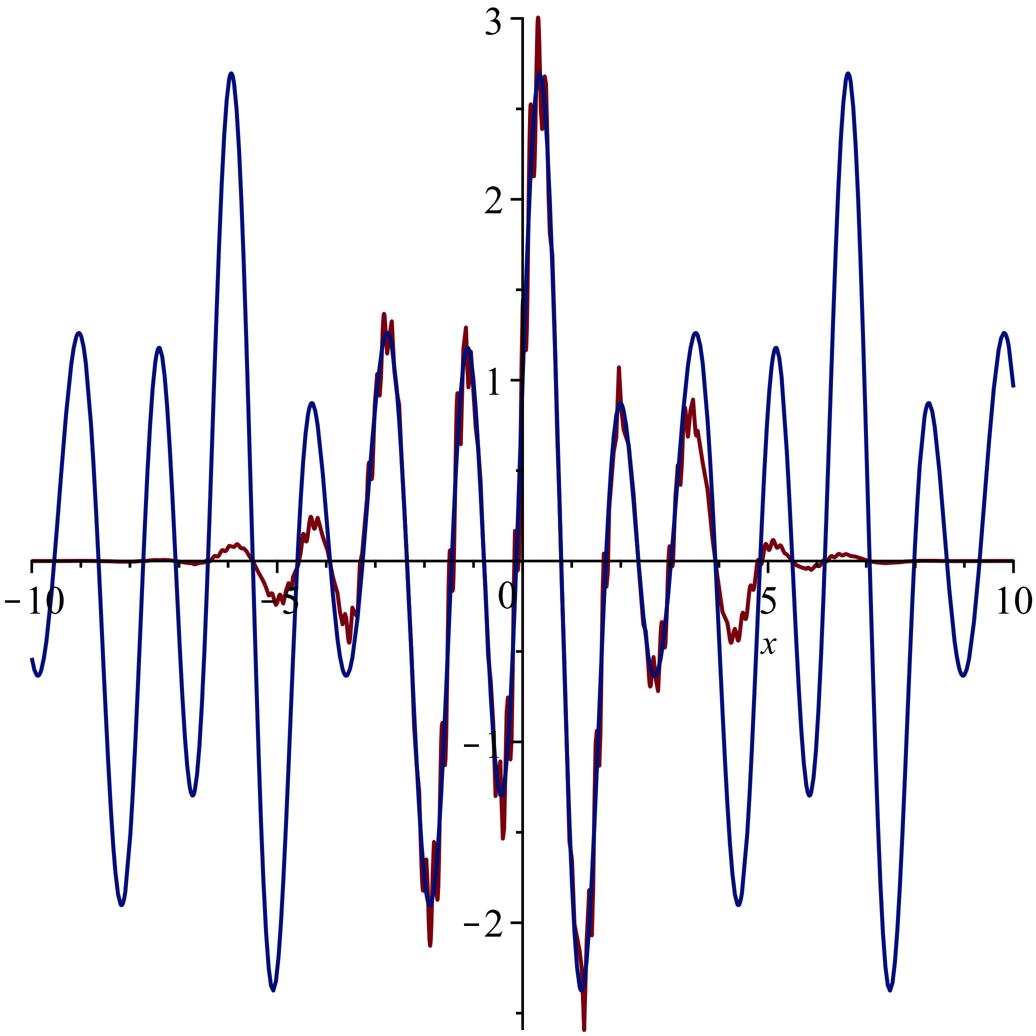
$a0 + Sum(a(n)\cdot\cos(n\cdot x), n = 1..N) + Sum(b(n)\cdot\sin(n\cdot x), n = 1..N);$

$$-0.01939716410 - 7.321127380\ 10^{-15}\ \mathrm{I} + \sum_{n=1}^{6} 0.08920620578\ \mathrm{e}^{-2.500000000\,n^2 - 210.\,n} \big(\mathrm{e}^{420.\,n - 4410.}\ \mathrm{erf}(0.9934588266 + 66.40783086\ \mathrm{I} - 1.581138830\ \mathrm{I}\,n) + \mathrm{e}^{-3610. + 20.\,n}\ \mathrm{erf}(0.9934588266 + 60.08327554\ \mathrm{I} + 1.581138830\ \mathrm{I}\,n) \tag{8}$$

$$+ e^{400. n - 3610.} \operatorname{erf}(0.9934588266 + 60.08327554\,I - 1.581138830\,I\,n)$$
$$+ 5.\, e^{-10. + 200. n} \operatorname{erf}(0.9934588266 + 3.162277660\,I + 1.581138830\,I\,n)$$
$$+ 5.\, e^{220. n - 10.} \operatorname{erf}(0.9934588266 + 3.162277660\,I - 1.581138830\,I\,n)$$
$$+ 5.\, e^{220. n - 10.} \operatorname{erf}(0.9934588266 - 3.162277660\,I + 1.581138830\,I\,n)$$
$$+ 5.\, e^{-10. + 200. n} \operatorname{erf}(0.9934588266 - 3.162277660\,I - 1.581138830\,I\,n)$$
$$+ e^{400. n - 3610.} \operatorname{erf}(0.9934588266 - 60.08327554\,I + 1.581138830\,I\,n)$$
$$+ e^{-3610. + 20. n} \operatorname{erf}(0.9934588266 - 60.08327554\,I - 1.581138830\,I\,n)$$
$$+ e^{420. n - 4410.} \operatorname{erf}(0.9934588266 - 66.40783086\,I + 1.581138830\,I\,n)$$
$$+ 5.772112761\; 10^{-1916} \operatorname{erf}(0.9934588266 + 66.40783086\,I + 1.581138830\,I\,n)$$
$$+ 5.772112761\; 10^{-1916} \operatorname{erf}(0.9934588266 - 66.40783086\,I - 1.581138830\,I\,n)\Big) \cos(n\,x)$$
$$+ \sum_{n=1}^{6} \Big(-0.8920620578\, e^{-2.500000000\, n^2 - 20. n - 40.} \big(-1.\, e^{40. n} \operatorname{erf}(0.9934588266$$
$$+ 6.324555320\,I - 1.581138830\,I\,n) - 1.\, e^{40. n} \operatorname{erf}(0.9934588266 - 6.324555320\,I$$
$$+ 1.581138830\,I\,n) + \operatorname{erf}(0.9934588266 + 6.324555320\,I + 1.581138830\,I\,n)$$
$$+ \operatorname{erf}(0.9934588266 - 6.324555320\,I - 1.581138830\,I\,n)\big) \sin(n\,x)\Big)$$

$S := value(\%)$:

$plot(\{f(x), S\}, x = -10 .. 10)$



3
2
1
−1
−2
−10
−5
0
5
10
x

Problem 35

Consider the function

$$g(x) = e^{-x^2/8}\left(\cos 2x + 2\sin 4x + 0.4\cos 2x\cos 10x\right) \ .$$

Compute the partial Fourier series through $N = 25$. Throw away any coefficients that are smaller than 0.01 in absolute value. Plot the resulting series and compare with the original function $g(x)$. Try experimenting with different tolerances.

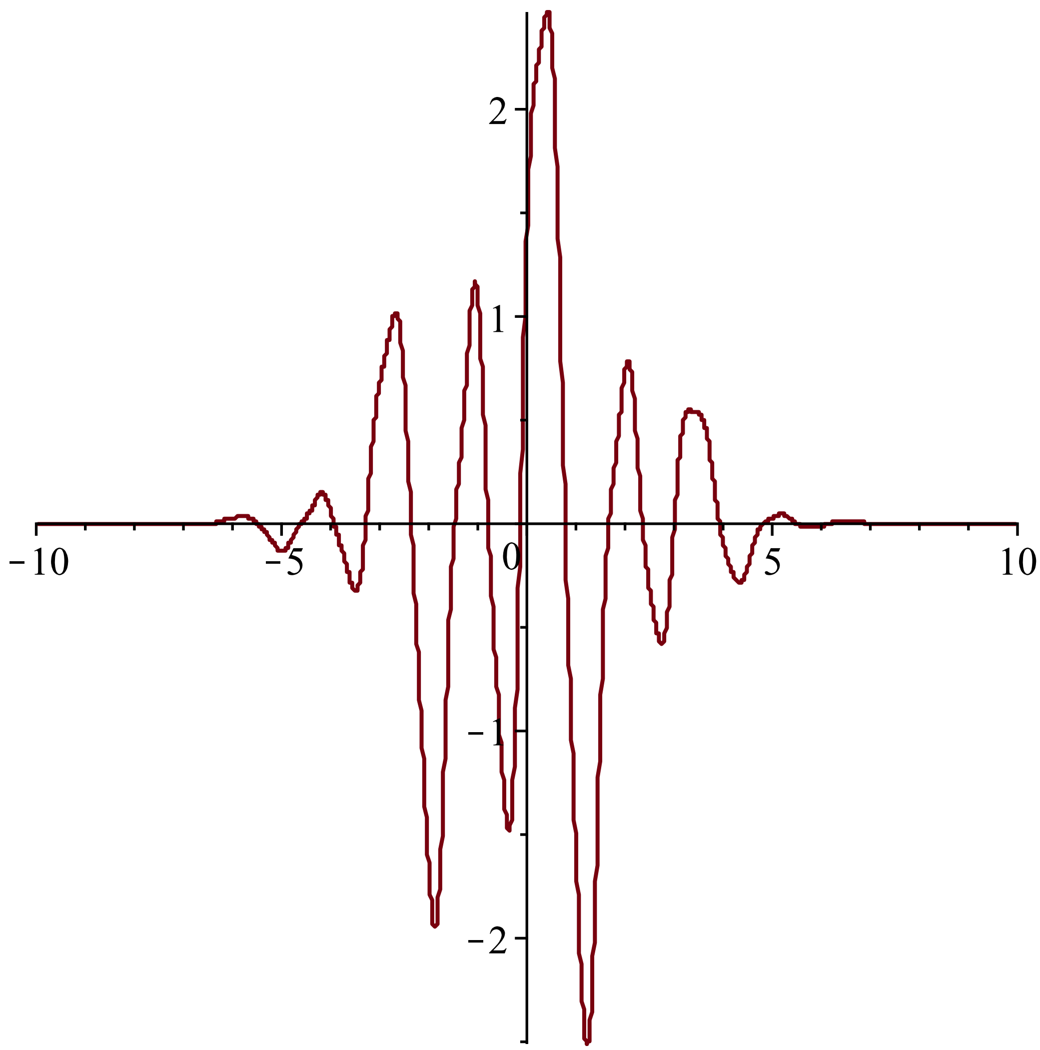
Solution:

restart;

$f := x \rightarrow (\cos(2 \cdot x) + 2 \cdot \sin(4 \cdot x) + 0.4 \cdot \cos(2 \cdot x) \cdot \cos(10 \cdot x)) \cdot \exp\left(-\frac{x^2}{8}\right)$

$$x \rightarrow (\cos(2\,x) + 2\,\sin(4\,x) + 0.4\,\cos(2\,x)\,\cos(10\,x))\,\mathrm{e}^{-\frac{1}{8}\,x^2} \qquad \textbf{(1)}$$

plot(*f*, −10 ..10, *numpoints* = 2000)



$a0 := value(\ (1/(2 \cdot \mathrm{Pi}))*Int(f(x), x = -\mathrm{Pi}..\mathrm{Pi})\ )$; # *the value of a_0*

$$-0.01831626531 - 3.819718633\ 10^{-15}\ \mathrm{I} \qquad \textbf{(2)}$$

$a := n \rightarrow value(\ (1/(\mathrm{Pi}))*Int(f(x)*\cos(n*x), x = -\mathrm{Pi}..\mathrm{Pi})\ )$; # *the value of a_n*

$$n \rightarrow value\left(\frac{\int_{-\pi}^{\pi} f(x)\,\cos(n\,x)\,\mathrm{d}x}{\pi}\right) \qquad \textbf{(3)}$$

$b := n \rightarrow value(\ (1/(\mathrm{Pi}))*Int(f(x)*\sin(n*x), x = -\mathrm{Pi}..\mathrm{Pi})\ )$; # *the value of b_n*

$$n \to value\left(\frac{\int_{-\pi}^{\pi} f(x)\,\sin(n\,x)\,\mathrm{d}x}{\pi}\right) \tag{4}$$

abs($a0$)

$$0.01831626531 \tag{5}$$

for j **from** 1 **by** 1 **to** 25 **do** $print$([j], abs(a(j))) **end do**

[1], 0.1689352965

[2], 0.6997913956

[3], 0.1639504845

[4], 0.02131923542

[5], 0.01173682840

[6], 0.009845704196

[7], 0.03653179993

[8], 0.1372609244

[9], 0.03582556008

[10], 0.008915777563

[11], 0.03504938027

[12], 0.1389502637

[13], 0.03358240613

[14], 0.004845510193

[15], 0.002664029127

[16], 0.001784969863

[17], 0.001331466062

[18], 0.001056006945

[19], 0.0008706973172

[20], 0.0007372293492

[21], 0.0006363674129

[22], 0.0005574061177

[23], 0.0004939059027

[24], 0.0004417517322

[25], 0.0003981843462 **(6)**

for j **from** 1 **by** 1 **to** 25 **do** $print$([j], abs(b(j))) **end do**

[1], 0.01050139527

[2], 0.03191534929

[3], 0.3171675791

[4], 1.412572635

[5], 0.3183464179

$[6], 0.03451620402$
$[7], 0.01514965726$
$[8], 0.008154429497$
$[9], 0.004990677877$
$[10], 0.003315434140$
$[11], 0.002331820945$
$[12], 0.001710634903$
$[13], 0.001296442641$
$[14], 0.001008383427$
$[15], 0.0008011744704$
$[16], 0.0006479430091$
$[17], 0.0005319759580$
$[18], 0.0004424681294$
$[19], 0.0003722023342$
$[20], 0.0003162218538$
$[21], 0.0002710395211$
$[22], 0.0002341498905$
$[23], 0.0002037191620$
$[24], 0.0001783828166$
$[25], 0.0001571104239$ **(7)**

 # With tolerance of 0.01 we obtain

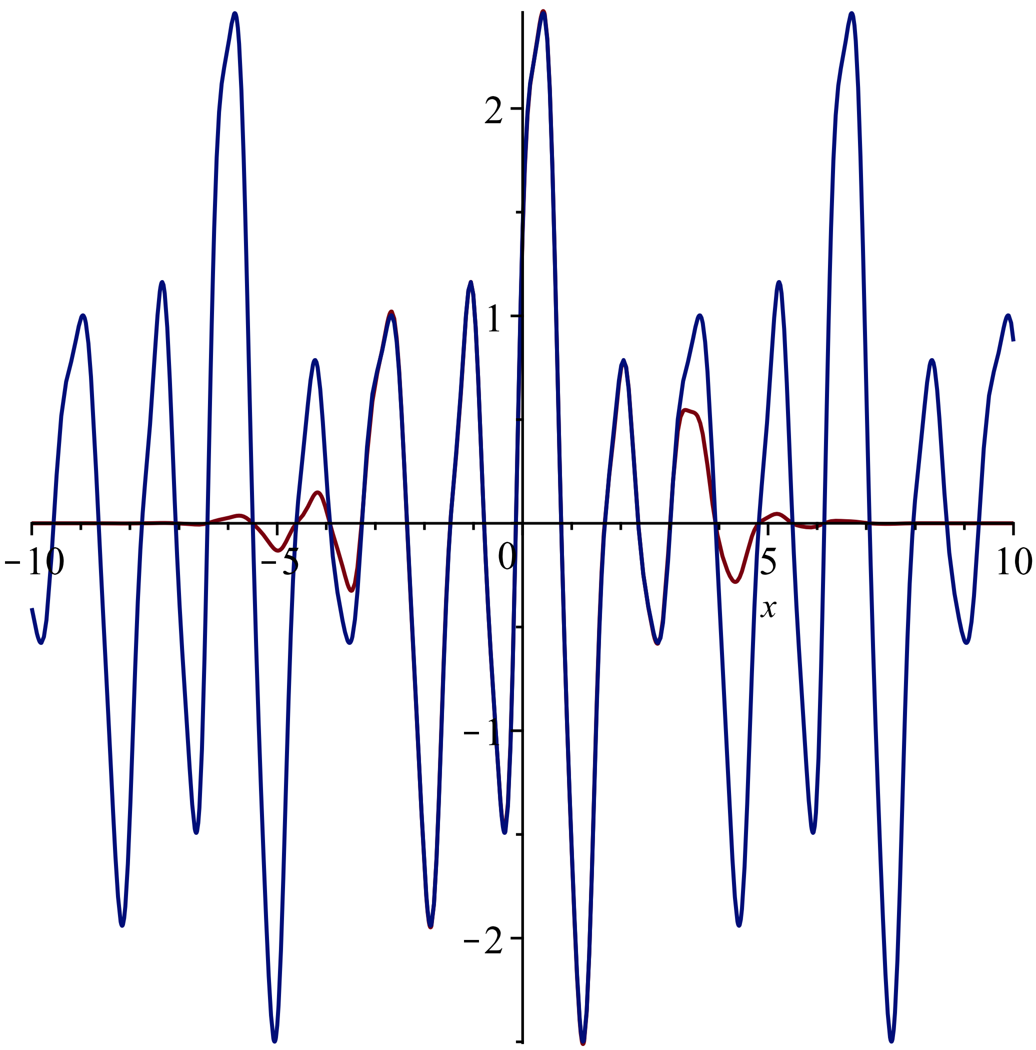
$a0 + Sum(a(n) \cdot \cos(n \cdot x), n = 1..5) + Sum(a(n) \cdot \cos(n \cdot x), n = 7..9) + Sum(a(n) \cdot \cos(n \cdot x), n = 11..13) + Sum(b(n) \cdot \sin(n \cdot x), n = 1..7);$

$$-0.01831626531 - 3.819718633\ 10^{-15}\,\mathrm{I} + \sum_{n=1}^{5} 0.07978845607\,\big(8.378942534\ 10^{-126}\,\mathrm{erf}(1.110720735 + 16.97056275\,\mathrm{I} + 1.414213562\,\mathrm{I}\,n) + 8.378942534\ 10^{-126}\,\mathrm{erf}(1.110720735 - 16.97056275\,\mathrm{I} - 1.414213562\,\mathrm{I}\,n) + \mathrm{e}^{96.\,n - 288.}\,\mathrm{erf}(1.110720735 + 16.97056275\,\mathrm{I} - 1.414213562\,\mathrm{I}\,n) + \mathrm{e}^{-128. + 16.\,n}\,\mathrm{erf}(1.110720735 + 11.31370850\,\mathrm{I} + 1.414213562\,\mathrm{I}\,n) + \mathrm{e}^{80.\,n - 128.}\,\mathrm{erf}(1.110720735 + 11.31370850\,\mathrm{I} - 1.414213562\,\mathrm{I}\,n) + 5.\,\mathrm{e}^{-8. + 40.\,n}\,\mathrm{erf}(1.110720735 + 2.828427125\,\mathrm{I} + 1.414213562\,\mathrm{I}\,n) + 5.\,\mathrm{e}^{56.\,n - 8.}\,\mathrm{erf}(1.110720735 + 2.828427125\,\mathrm{I} - 1.414213562\,\mathrm{I}\,n) + 5.\,\mathrm{e}^{56.\,n - 8.}\,\mathrm{erf}(1.110720735 - 2.828427125\,\mathrm{I} + 1.414213562\,\mathrm{I}\,n) + 5.\,\mathrm{e}^{-8. + 40.\,n}\,\mathrm{erf}(1.110720735 - 2.828427125\,\mathrm{I} - 1.414213562\,\mathrm{I}\,n) + \mathrm{e}^{80.\,n - 128.}\,\mathrm{erf}(1.110720735 - 11.31370850\,\mathrm{I} + 1.414213562\,\mathrm{I}\,n) + \mathrm{e}^{-128. + 16.\,n}\,\mathrm{erf}(1.110720735 - 11.31370850\,\mathrm{I} - 1.414213562\,\mathrm{I}\,n) + \mathrm{e}^{96.\,n - 288.}\,\mathrm{erf}(1.110720735 - 16.97056275\,\mathrm{I} + 1.414213562\,\mathrm{I}\,n)\big)\,\mathrm{e}^{-2.\,n^2 - 48.\,n}\cos(n\,x) \tag{8}$$

$$+\sum_{n=7}^{9} 0.07978845607\, \big(8.378942534\; 10^{-126}\, \mathrm{erf}(1.110720735+16.97056275\, \mathrm{I}$$
$$+1.414213562\, \mathrm{I}\, n)+8.378942534\; 10^{-126}\, \mathrm{erf}(1.110720735-16.97056275\, \mathrm{I}$$
$$-1.414213562\, \mathrm{I}\, n)+\mathrm{e}^{96.\, n-288.}\, \mathrm{erf}(1.110720735+16.97056275\, \mathrm{I}-1.414213562\, \mathrm{I}\, n)$$
$$+\mathrm{e}^{-128.+16.\, n}\, \mathrm{erf}(1.110720735+11.31370850\, \mathrm{I}+1.414213562\, \mathrm{I}\, n)$$
$$+\mathrm{e}^{80.\, n-128.}\, \mathrm{erf}(1.110720735+11.31370850\, \mathrm{I}-1.414213562\, \mathrm{I}\, n)$$
$$+5.\, \mathrm{e}^{-8.+40.\, n}\, \mathrm{erf}(1.110720735+2.828427125\, \mathrm{I}+1.414213562\, \mathrm{I}\, n)$$
$$+5.\, \mathrm{e}^{56.\, n-8.}\, \mathrm{erf}(1.110720735+2.828427125\, \mathrm{I}-1.414213562\, \mathrm{I}\, n)$$
$$+5.\, \mathrm{e}^{56.\, n-8.}\, \mathrm{erf}(1.110720735-2.828427125\, \mathrm{I}+1.414213562\, \mathrm{I}\, n)$$
$$+5.\, \mathrm{e}^{-8.+40.\, n}\, \mathrm{erf}(1.110720735-2.828427125\, \mathrm{I}-1.414213562\, \mathrm{I}\, n)$$
$$+\mathrm{e}^{80.\, n-128.}\, \mathrm{erf}(1.110720735-11.31370850\, \mathrm{I}+1.414213562\, \mathrm{I}\, n)$$
$$+\mathrm{e}^{-128.+16.\, n}\, \mathrm{erf}(1.110720735-11.31370850\, \mathrm{I}-1.414213562\, \mathrm{I}\, n)$$
$$+\mathrm{e}^{96.\, n-288.}\, \mathrm{erf}(1.110720735-16.97056275\, \mathrm{I}+1.414213562\, \mathrm{I}\, n)\big)\, \mathrm{e}^{-2.\, n^2-48.\, n}\cos(n\, x)$$
$$+\sum_{n=11}^{13} 0.07978845607\, \big(8.378942534\; 10^{-126}\, \mathrm{erf}(1.110720735+16.97056275\, \mathrm{I}$$
$$+1.414213562\, \mathrm{I}\, n)+8.378942534\; 10^{-126}\, \mathrm{erf}(1.110720735-16.97056275\, \mathrm{I}$$
$$-1.414213562\, \mathrm{I}\, n)+\mathrm{e}^{96.\, n-288.}\, \mathrm{erf}(1.110720735+16.97056275\, \mathrm{I}-1.414213562\, \mathrm{I}\, n)$$
$$+\mathrm{e}^{-128.+16.\, n}\, \mathrm{erf}(1.110720735+11.31370850\, \mathrm{I}+1.414213562\, \mathrm{I}\, n)$$
$$+\mathrm{e}^{80.\, n-128.}\, \mathrm{erf}(1.110720735+11.31370850\, \mathrm{I}-1.414213562\, \mathrm{I}\, n)$$
$$+5.\, \mathrm{e}^{-8.+40.\, n}\, \mathrm{erf}(1.110720735+2.828427125\, \mathrm{I}+1.414213562\, \mathrm{I}\, n)$$
$$+5.\, \mathrm{e}^{56.\, n-8.}\, \mathrm{erf}(1.110720735+2.828427125\, \mathrm{I}-1.414213562\, \mathrm{I}\, n)$$
$$+5.\, \mathrm{e}^{56.\, n-8.}\, \mathrm{erf}(1.110720735-2.828427125\, \mathrm{I}+1.414213562\, \mathrm{I}\, n)$$
$$+5.\, \mathrm{e}^{-8.+40.\, n}\, \mathrm{erf}(1.110720735-2.828427125\, \mathrm{I}-1.414213562\, \mathrm{I}\, n)$$
$$+\mathrm{e}^{80.\, n-128.}\, \mathrm{erf}(1.110720735-11.31370850\, \mathrm{I}+1.414213562\, \mathrm{I}\, n)$$
$$+\mathrm{e}^{-128.+16.\, n}\, \mathrm{erf}(1.110720735-11.31370850\, \mathrm{I}-1.414213562\, \mathrm{I}\, n)$$
$$+\mathrm{e}^{96.\, n-288.}\, \mathrm{erf}(1.110720735-16.97056275\, \mathrm{I}+1.414213562\, \mathrm{I}\, n)\big)\, \mathrm{e}^{-2.\, n^2-48.\, n}\cos(n\, x)$$
$$+\sum_{n=1}^{7}\Big(-0.7978845607\, \mathrm{e}^{-2.\, n^2-16.\, n-32.}\, \big(-1.\, \mathrm{e}^{32.\, n}\, \mathrm{erf}(1.110720735-5.656854249\, \mathrm{I}$$
$$+1.414213562\, \mathrm{I}\, n)-1.\, \mathrm{e}^{32.\, n}\, \mathrm{erf}(1.110720735+5.656854249\, \mathrm{I}-1.414213562\, \mathrm{I}\, n)$$
$$+\mathrm{erf}(1.110720735-5.656854249\, \mathrm{I}-1.414213562\, \mathrm{I}\, n)+\mathrm{erf}(1.110720735$$
$$+5.656854249\, \mathrm{I}+1.414213562\, \mathrm{I}\, n)\big)\sin(n\, x)\Big)$$

S := *value*(%) :

plot({ *f*(*x*), *S* }, *x* =-10..10)



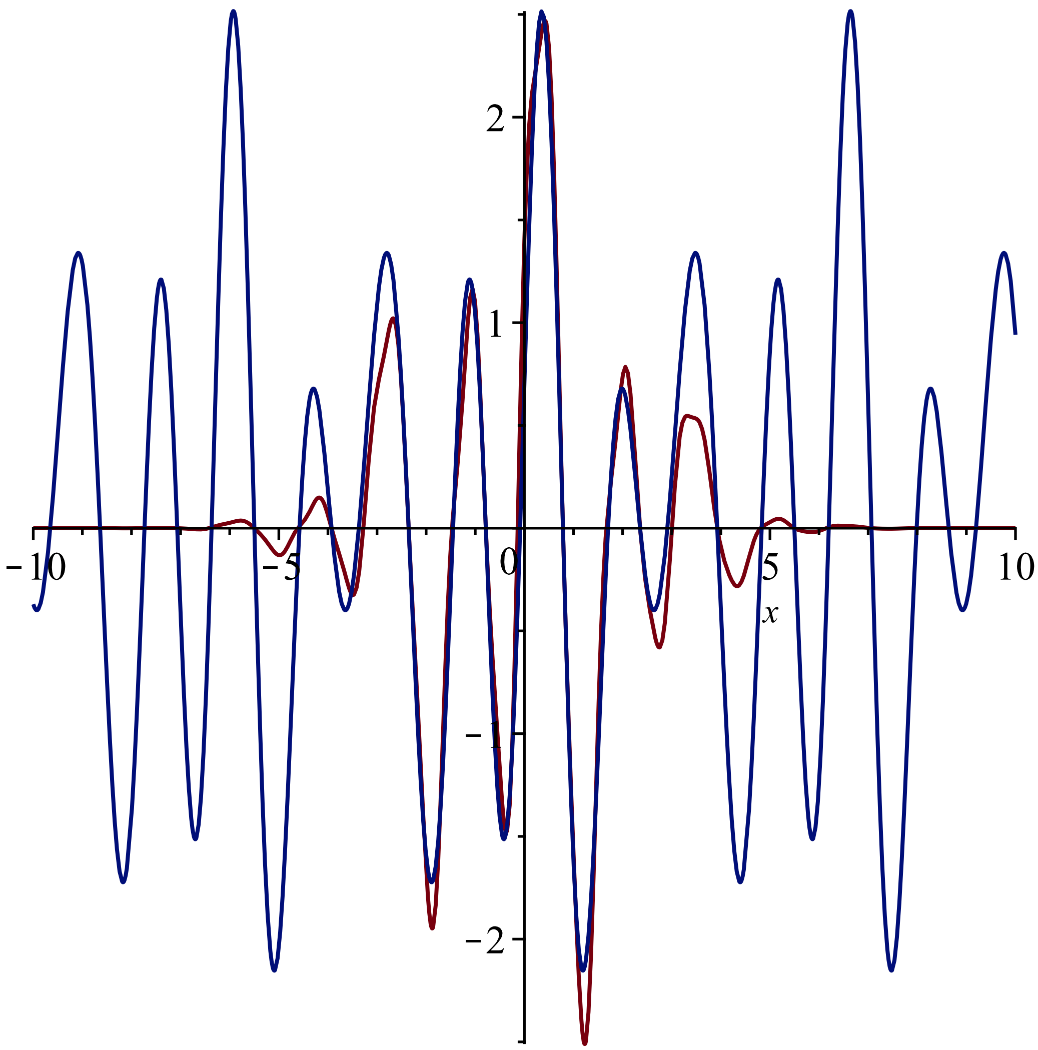
 # With tolerance of 0.3 we obtain

$a(2)\cdot\cos(2\cdot x) + Sum(b(n)\cdot\sin(n\cdot x), n = 3..5);$

$$(0.6997913956 + 0.\,\mathrm{I})\cos(2\,x) + \sum_{n=3}^{5}\Big(-0.7978845607\,\mathrm{e}^{-2.\,n^2 - 16.\,n - 32.}\big(-1.\,\mathrm{e}^{32.\,n}\,\mathrm{erf}(1.110720735 - 5.656854249\,\mathrm{I} + 1.414213562\,\mathrm{I}\,n) - 1.\,\mathrm{e}^{32.\,n}\,\mathrm{erf}(1.110720735 + 5.656854249\,\mathrm{I} - 1.414213562\,\mathrm{I}\,n) + \mathrm{erf}(1.110720735 - 5.656854249\,\mathrm{I} - 1.414213562\,\mathrm{I}\,n) + \mathrm{erf}(1.110720735 + 5.656854249\,\mathrm{I} + 1.414213562\,\mathrm{I}\,n)\big)\sin(n\,x)\Big) \tag{9}$$

$S := value(\%)$:

$plot(\{f(x), S\}, x = -10..10)$



2
1
−1
−2
−10
−5
0
5
10
x