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APM 505 HOMEWORK 6

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Initialization of the code

```
clear all       % Clear workspace
clc             % Clear command window
format long

tol=1e-12;    % Tolerance
```

Function to create the matrix A

```
   type DominantEigenvalueMatrix.m

function [A,P,D]=DominantEigenvalueMatrix(N,f)
% This function gives a NxN matrix A, an orthogonal matrix P and a diagonal
% matrix D such that A=A = P*D*P'. Needs the dimension N and a factor f.
% The matrix A will have an dominant eigenvalue if f>>1 and will have the two
% larger eigenvalues very similar if f is close to unity.
    P = orth(rand(N));
    lambdaV = randi([1,100],N,1);
    k=randi([1,N],1);
    j=find(lambdaV==max(lambdaV));
    while k==j
        k=randi([1,N],1);
    end
    lambdaV(k)=f*max(lambdaV);
    D = diag(lambdaV);
    A = P*D*P';
end
```

Function for the Power Method Iteration

```
type powermethod.m
```

```
function [lambda,k,q]=powermethod(A,tol)
% This function uses the powermethod to, given the matrix A and a tolerance,
% obtain the eigenvalue with larger absolute value and its eigenvector.
% It will also provide the number of iterations needed to meet the tolerance.
  N=size(A,1);
  lambdaprev=1;     % Initialize lambdaprev
  lambda=0;         % Initialize lambda
  k=0;              % Start the counter of iterations
  q=rand(N,1);      % The first guess of q is a random vector as the problem specifies
  while norm(lambdaprev-lambda)>tol   % This is the power method algorithm to
obtain the dominant eigenvalue
    k=k+1;
    lambdaprev=lambda;
    z=A*q;
    q=z/norm(z);
    lambda=q'*A*q;
  end
end
```

Run different cases of study

```
for i=1:4

  switch i
    case 1
      N=3;   % Dimension of the matrix A
      f=30;   % The factor f large means that the matrix A is going to have one
dominant eigenvalue
    case 2
      N=3;
      f=1.0001;   % The factor f close to unity means that the matrix A is not going to
have any dominant eigenvalue
    case 3
      N=9;
      f=30;
    case 4
      N=9;
      f=1.0001;
  end
```

Create the matrix A

```
[A,P,D]=DominantEigenvalueMatrix(N,f);
```

Power Method Iteration

```
[lambda,k,q]=powermethod(A,tol);
```

Results

```
fprintf('>>Case %d\n',i)
A
P
D
lambda
k
q
v=P'*q
```

```
>>Case 1


A =

   1.0e+02 *

   4.244400126989851   4.430312881081557   4.593522529237088
   4.430312881081557   5.734330623065364   5.491528954683146
   4.593522529237088   5.491528954683145   5.711269249944783


P =

  -0.510247581094400   0.314757900790437   0.800359213027070
  -0.605782215577903   0.529042718064529  -0.594257275725272
  -0.610471386124559  -0.788061714812293  -0.079268028676552


D =

        1500           0           0
           0          19           0
           0           0          50
```

```
lambda =

   1.500000000000000e+03
k =

   6



q =

  0.510247580467174
  0.605782216043702
  0.610471386186589



v =

 -1.000000000000000
  0.000000000000120
 -0.000000000783727
```

“ In this case we have a 3x3 matrix A which has a dominant eigenvalue as we see from its diagonal form D. The powermethod function obtains the value of that eigenvalue in 6 iterations and gives us the eigenvector q which coincides with some small error with the first column (because the dominant eigenvalue is on the first column of D) of the matrix P. This is better seen in the basis where A is diagonal. In that basis we have that the eigenvector is v, which has a 1 in the first component and a little bit of error in the third. This indicates that the larger eigenvalue is on the first column of D as we knew and that the second larger eigenvalue is in the third column of D, as we can check.”

>>Case 2

A =

```
  65.778181610333306  18.745394160193712   8.551846981232508
  18.745394160193712  88.427661887601872  -4.830455561442018
   8.551846981232506  -4.830455561442013  96.804056502064824
```

P =

```
  -0.506964658168641   0.849834050322119  -0.144114268138363
  -0.630178516585746  -0.479493175124557  -0.610705601941524
  -0.588100223262788  -0.218788441015685   0.778633254797267
```

D =

```
  99.000000000000000                   0                   0
                   0  53.000000000000000                   0
                   0                   0  99.009900000000002
```

lambda =

```
  99.009899994823670
```

k =

```
       82002
```

q =

```
  -0.143747650799676
  -0.610249768257143
   0.779058298994162
```

v =

```
  -0.000723087215910
   0.000000000000000
   0.999999738572405
```

“ In this case we have a 3x3 matrix A which does not have a dominant eigenvalue as we see from its diagonal form D. The powermethod function obtains the value of the larger eigenvalue in 82002 iterations against the 6 iterations needed when the matrix has a dominant eigenvalue. The function gives us the eigenvector q as well, it coincides with the third column with some small error, bigger than in the previous case, of the matrix P. This is again better seen in the basis where A is diagonal. In that basis we have that the eigenvector is v, which has a value close to 1 in the third component and a appreciable error in the first. This indicates that the larger eigenvalue is on the third column of D as we knew and that the second larger eigenvalue is in the first column of D, as we can check. In this case the error is bigger because the two larger eigenvalues are very similar. In the previous case this error was much smaller since the eigenvalue dominance was much bigger.”

The discussion for the next two cases is the same but with higher dimensions.

>>Case 3

```
A =

   1.0e+03 *

  Columns 1 through 3

   0.159770893583506   0.129147730794881   0.150935223484620
   0.129147730794881   0.234830266896458   0.200203618167385
   0.150935223484620   0.200203618167385   0.265293940959512
   0.124747996371854   0.150799243836747   0.176540141835334
   0.029414017268002   0.046168250897279   0.039248263262767
  -0.420646058435207  -0.563412650681473  -0.614279905030066
  -0.071125839802573  -0.060507505115308  -0.074629657681418
  -0.020714999695953  -0.050021355680658  -0.052325921189849
  -0.069777838377471  -0.080653658655024  -0.083097583574760

  Columns 4 through 6

   0.124747996371854   0.029414017268002  -0.420646058435207
   0.150799243836747   0.046168250897279  -0.563412650681473
   0.176540141835334   0.039248263262767  -0.614279905030066
   0.179742887017126   0.034324468889425  -0.489371728994714
   0.034324468889425   0.034888190396923  -0.099581103562088
  -0.489371728994714  -0.099581103562088   1.882393375403074
  -0.045304164642929   0.002181395586064   0.220328899811667
  -0.054765311417360  -0.000882844519325   0.167133737648765
  -0.052294972798042  -0.010119390186758   0.260322210676756

  Columns 7 through 9

  -0.071125839802573  -0.020714999695953  -0.069777838377471
  -0.060507505115308  -0.050021355680658  -0.080653658655024
  -0.074629657681418  -0.052325921189849  -0.083097583574761
  -0.045304164642929  -0.054765311417360  -0.052294972798042
   0.002181395586064  -0.000882844519325  -0.010119390186758
   0.220328899811667   0.167133737648765   0.260322210676756
   0.062685172897367   0.024461561836225   0.038628331298822
   0.024461561836225   0.072604572408457   0.023305917365969
   0.038628331298822   0.023305917365969   0.085790700437577
```

P =

Columns 1 through 3

```
-0.285985325861685  -0.045841822673435   0.660632527514076
-0.361059040691761  -0.359857910053400  -0.170834383953848
-0.444187360072406  -0.328046315556672   0.015217220476342
-0.366390146754152   0.048598420864836  -0.241158150967046
-0.341666285798385   0.137187573637101  -0.066840064967388
-0.356208316609314  -0.354707286692061   0.097472533051270
-0.224789891210753   0.310816995255430  -0.389524512900342
-0.299488064664406   0.637309061417285   0.424475274944598
-0.269185438499242   0.333191966151706  -0.360713688538134
```

Columns 4 through 6

```
-0.091215413791133  -0.363183185822673   0.460124915523551
-0.338744287662819  -0.264590339263331   0.030071238556585
 0.170095498033361   0.744778151089664   0.070687077523720
 0.220948574891607  -0.331999131569354  -0.398564340573450
 0.176899102720398  -0.227678627121840  -0.355940809581921
 0.043055528109062  -0.044087746688880  -0.064599065178326
 0.499565945500597  -0.089710588455810   0.634813760273075
-0.009192523888783   0.238153281176257  -0.243331596333669
-0.718598355983013   0.120936952700870   0.174542003731039
```

Columns 7 through 9

```
 0.115707535740811  -0.273220173859514   0.199250774106761
-0.258596655460033   0.623558212175572   0.263812761888316
 0.086443910718613  -0.123695794227658   0.288885453548695
-0.408611272178892  -0.517889691608634   0.228409863974275
 0.783515436312477   0.184182091028161   0.048290094127253
-0.073290335528655  -0.047306280135505  -0.849833852405019
-0.067536533914579   0.160165585547371  -0.101568312458336
-0.315728190792722   0.319979399179660  -0.076693809100334
 0.147615813547937  -0.298018041318999  -0.119701763210098
```

D =

Columns 1 through 6

85	0	0	0	0	0
0	56	0	0	0	0
0	0	86	0	0	0
0	0	0	35	0	0
0	0	0	0	45	0
0	0	0	0	0	6
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

Columns 7 through 9

0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
0	0	0
18	0	0
0	67	0
0	0	2580

lambda =

2.579999999999999e+03

k =

7

q =

0.199250774126381
0.263812762025903
0.288885453685193
0.228409864109691
0.048290094244377
-0.849833852303685
-0.101568312344931
-0.076693809044853
-0.119701763090340

v =

```
  -0.000000000315992
   0.000000000002025
  -0.000000000102896
  -0.000000000000091
  -0.000000000000439
  -0.000000000000000
   0.000000000000001
   0.000000000010423
   1.000000000000000
```

>>Case 4

A =

Columns 1 through 3

68.743687634112050	7.216133128096675	-2.023355694125127
7.216133128096675	58.962683663068454	21.172273875264608
-2.023355694125127	21.172273875264608	36.625513406494363
16.037710987292503	0.579567681758949	11.389376182306835
-6.295376191315462	15.633833070506201	16.589518650001335
4.551567227742286	6.072907991140039	-6.455026072344011
-4.435155145239763	-2.614738104228238	14.188964736029245
9.963145002762150	-6.194700174678191	4.458438359732682
7.070898685476714	-1.679069962997798	-15.617158797487098

Columns 4 through 6

16.037710987292503	-6.295376191315460	4.551567227742288
0.579567681758950	15.633833070506201	6.072907991140040
11.389376182306835	16.589518650001335	-6.455026072344011
37.206544126738088	7.891034015346418	9.936592682470517
7.891034015346419	37.555651203319570	5.213525511383073
9.936592682470517	5.213525511383071	55.281304654036823
16.434502048783251	5.128897848895898	10.201535854057877
4.404803510331437	17.207403712162954	-8.690645119653523
-12.137352734607234	6.498519284906872	22.076627994031018

Columns 7 through 9

-4.435155145239765	9.963145002762152	7.070898685476712
-2.614738104228240	-6.194700174678188	-1.679069962997798
14.188964736029245	4.458438359732682	-15.617158797487102
16.434502048783251	4.404803510331437	-12.137352734607230
5.128897848895896	17.207403712162954	6.498519284906873
10.201535854057877	-8.690645119653524	22.076627994031018
58.498248370136288	0.481584909251263	-2.471065162659516
0.481584909251264	50.605819356402066	17.445363836283885
-2.471065162659518	17.445363836283885	65.530047585692301

P =

Columns 1 through 3

-0.375982532111804	0.308208096379842	0.087258692731289
-0.393966726790053	-0.388543775874343	-0.290203609542183
-0.299188167642597	-0.184131504277446	0.258207045928799
-0.341942686075590	0.338908710868010	0.254873531677058
-0.367694656139252	-0.379303779401582	0.363699891628779
-0.349759004288032	0.338712532016251	-0.347631105156090
-0.332968750596865	0.422403816987890	-0.117871890837968
-0.265684144478991	-0.358512511126181	-0.583122680830150
-0.240836082794325	-0.199309399508423	0.411686227415506

Columns 4 through 6

-0.750489958029435	0.310686273566923	-0.092072827902104
-0.067210975491183	-0.188252067485142	0.316313577798320
0.006014128784260	-0.185580712688828	-0.766632300401704
-0.111803762871185	-0.573604976734461	0.348250893693428
0.164333643027203	0.596548614255719	0.241990250928247
0.382646587232888	0.110413806769151	-0.307784549591089
0.400935869240347	0.173918287104744	0.161308899060668
-0.157934597394937	-0.132035054995190	-0.056150702754186
0.246159280312420	-0.298043694808881	0.000687777432975

Columns 7 through 9

0.226140716082581	-0.174154178134955	-0.079517173362222
-0.251519988950244	-0.255968530502155	-0.583792076284771
-0.422780793293976	-0.076633249453554	0.055618720962315
-0.187138142737550	0.424194978205097	0.159662357182294
-0.096975514297966	0.348197318566442	0.128728286060159
0.278320259641229	0.420125157842243	-0.367133232914957
-0.238361456954258	-0.557720043680917	0.334555690186820
0.204197872794342	0.112507827472001	0.600803110113495
0.697462841538800	-0.311255875285476	-0.005352670745087

D =

Columns 1 through 3

95.000000000000000	0	0
0	69.000000000000000	0

```
                 0                   0  14.000000000000000
                 0                   0                   0
                 0                   0                   0
                 0                   0                   0
                 0                   0                   0
                 0                   0                   0
                 0                   0                   0

  Columns 4 through 6

                 0                   0                   0
                 0                   0                   0
                 0                   0                   0
  73.000000000000000                   0                   0
                 0  12.000000000000000                   0
                 0                   0  12.000000000000000
                 0                   0                   0
                 0                   0                   0
                 0                   0                   0

  Columns 7 through 9

                 0                   0                   0
                 0                   0                   0
                 0                   0                   0
                 0                   0                   0
                 0                   0                   0
                 0                   0                   0
  95.009500000000003                   0                   0
                 0  33.000000000000000                   0
                 0                   0  66.000000000000000


lambda =

  95.009499994725957


k =

       95221
```

```
q =

   0.226420794633503
  -0.251226377942537
  -0.422557753429854
  -0.186883312262444
  -0.096701521280747
   0.278580784785018
  -0.238113298662035
   0.204395775064336
   0.697642092812802


v =

  -0.000745091325438
  -0.000000000000000
   0.000000000000000
  -0.000000000000000
   0.000000000000000
   0.000000000000000
   0.999999722419420
  -0.000000000000000
   0.000000000000001

end
```