

The Knife Edge Viscometer

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Outline

- 1 Introduction
 - The Knife Edge Viscometer
 - Quick Motivation
 - Governing Equations
 - Boundary Conditions

Outline

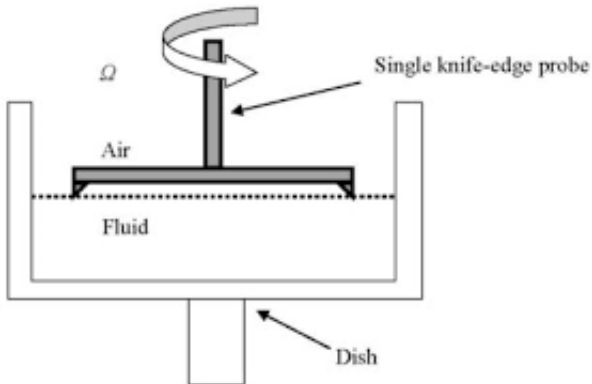
1 Introduction

- The Knife Edge Viscometer
- Quick Motivation
- Governing Equations
- Boundary Conditions

2 Results

- Observables
- Parameter Sweep
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The Knife Edge Viscometer



Quick Motivation

Why do we study this problem?

- How to measure surface shear viscosity remains a controversial issue.
- Monomolecular layers are key in broad areas
 - Pharmaceuticals: interfacial processing.
 - Food processing: surfactants.
 - Natural: Gas absorption into a fluid (lungs, oceans).
- Applications where a high degree of mixing is desired (microbioreactors), although at a low level of shear stress.

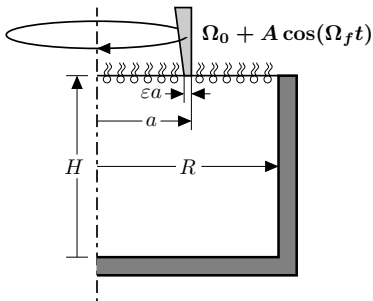
The schematic shows a cross-section of a microfluidic channel. A vertical dashed line indicates the centerline. The channel has a total height H and a radius R . A constriction is located at the top, with a width εa . The pressure drop across the constriction is given as $\Delta P = \Delta P_0 + A \cos(\Omega_f t)$. Arrows indicate the direction of fluid flow.

$$\nabla \times \mathbf{u} = \left(-\frac{1}{r} \frac{\partial \gamma}{\partial z}, \eta, \frac{1}{r} \frac{\partial \gamma}{\partial r} \right)$$

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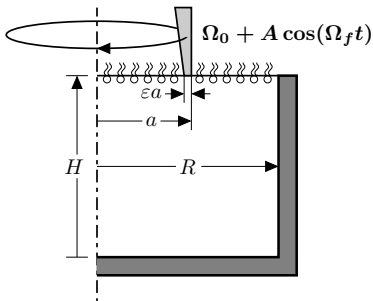
$$\begin{aligned} \frac{\partial \gamma}{\partial t} - \frac{1}{r} \frac{\partial \psi}{\partial z} \frac{\partial \gamma}{\partial r} + \frac{1}{r} \frac{\partial \psi}{\partial r} \frac{\partial \gamma}{\partial z} &= \frac{1}{Re} \left(\frac{\partial^2 \gamma}{\partial z^2} + \frac{\partial^2 \gamma}{\partial r^2} - \frac{1}{r} \frac{\partial \gamma}{\partial r} \right) \\ \frac{\partial \eta}{\partial t} - \frac{1}{r} \frac{\partial \psi}{\partial z} \frac{\partial \eta}{\partial r} + \frac{1}{r} \frac{\partial \psi}{\partial r} \frac{\partial \eta}{\partial z} + \frac{\eta}{r^2} \frac{\partial \psi}{\partial z} - \frac{2\gamma}{r^3} \frac{\partial \gamma}{\partial z} &= \frac{1}{Re} \left(\frac{\partial^2 \eta}{\partial z^2} + \frac{\partial^2 \eta}{\partial r^2} + \frac{1}{r} \frac{\partial \eta}{\partial r} - \frac{\eta}{r^2} \right) \\ \frac{\partial^2 \psi}{\partial z^2} + \frac{\partial^2 \psi}{\partial r^2} - \frac{1}{r} \frac{\partial \psi}{\partial r} &= -r\eta \end{aligned}$$

The diagram illustrates a microfluidic device with a central vertical channel of width $2a$ and two side channels of width εa . The total height of the device is H . A top reservoir is connected to the central channel, and a flow rate $Q_0 + A \cos(\Omega_f t)$ is indicated. A pressure drop ΔP is shown across the central channel. A distance R is marked from the center of the central channel to the right wall of the side channel.



$$\eta = -\frac{1}{r} \frac{\partial^2 \psi}{\partial z^2} \Big|_{z=0}$$

Boundary Conditions



Bottom (no slip)

$$\psi = \gamma = 0 \text{ at } z = 0$$

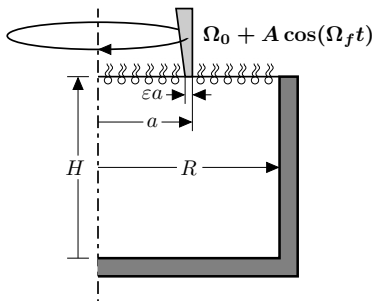
$$\eta = -\frac{1}{r} \frac{\partial^2 \psi}{\partial z^2} \bigg|_{z=0}$$

End Wall (no slip)

$$\psi = \gamma = 0 \text{ at } r = A_R$$

$$\eta = -\frac{1}{A_R} \frac{\partial^2 \psi}{\partial r^2} \bigg|_{r=A_R}$$

Boundary Conditions



Bottom (no slip)

$$\psi = \gamma = 0 \text{ at } z = 0$$

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End Wall (no slip)

$$\psi = \gamma = 0 \text{ at } r = A_R$$

$$\eta = -\frac{1}{A_R} \frac{\partial^2 \psi}{\partial r^2} \bigg|_{r=A_R}$$

Axis (symmetry)

$$\psi = \gamma = \eta = 0 \text{ at } r = 0$$

Are we sure? $\left(-\frac{1}{r} \frac{\partial \psi}{\partial r} \bigg|_{r=0} = 0 \right)$

Diagram illustrating the geometry of the microfluidic device. The device consists of a vertical channel of height H and a horizontal channel of width a . A vertical wall of thickness a is located at a distance R from the center of the vertical channel. A horizontal wall of thickness a is located at a distance εa from the vertical wall. A grey wedge-shaped object is positioned at the top of the vertical wall, with a horizontal arrow indicating its position as $\Omega_0 + A \cos(\Omega_f t)$.

$$v = \frac{\gamma}{r}$$

$$\psi = 0 \quad \text{at } z = A_H$$

$$\eta = -\frac{1}{r} \frac{\partial^2 \psi}{\partial z^2} \Big|_{z=A_H}$$

$$\frac{\partial^2 v}{\partial r^2} + \frac{1}{r} \frac{\partial v}{\partial r} - \frac{v}{r^2} = \frac{1}{Bo} \frac{\partial v}{\partial z} \quad \forall r \neq A_R/2, z = A_H$$

$$v = 1 \quad r = A_R/2, z = A_H$$

[illegible]

$$v = \frac{\gamma}{r}$$

$$\psi = 0 \quad \text{at } z = A_H$$

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$$v = 1 \quad r = A_R/2, z = A_H$$

$$\frac{\partial^2 v}{\partial r^2} + \frac{1}{r} \frac{\partial v}{\partial r} - \frac{v}{r^2} = 0$$

The diagram shows a cross-section of a microfluidic device. A central vertical channel of width $2a$ is flanked by two side channels of width εa . The total height is H . A top reservoir is connected to the left side channel. A grey L-shaped structure is on the right. A wavy arrow indicates flow from the top reservoir into the central channel. A label $\Omega_0 + A \cos(\Omega_f t)$ is at the top right.

$$v = \frac{\gamma}{r}$$

$$\psi = 0 \quad \text{at } z = A_H$$

$$\eta = -\frac{1}{r} \frac{\partial^2 \psi}{\partial z^2} \Big|_{z=A_H}$$

$$\frac{\partial^2 v}{\partial r^2} + \frac{1}{r} \frac{\partial v}{\partial r} - \frac{v}{r^2} = \frac{1}{Bo} \frac{\partial v}{\partial z} \quad \forall r \neq A_R/2, z = A_H$$

$$v = 1 \quad r = A_R/2, z = A_H$$

$$\frac{\partial^2 v}{\partial r^2} + \frac{1}{r} \frac{\partial v}{\partial r} - \frac{v}{r^2} = 0$$

The bulk flow and the monolayer are decoupled!
Analytic solution possible

Observables

Global

- Kinetic Energy

$$E_k = \frac{1}{2} \int \|\mathbf{u}\|^2 dV$$

- Enstrophy

$$E_w = \int \|\nabla \times \mathbf{u}\|^2 dV$$

- Angular Momentum

$$E_\gamma = \int \gamma^2 dV$$

Local

We probe the value of the three velocity components:

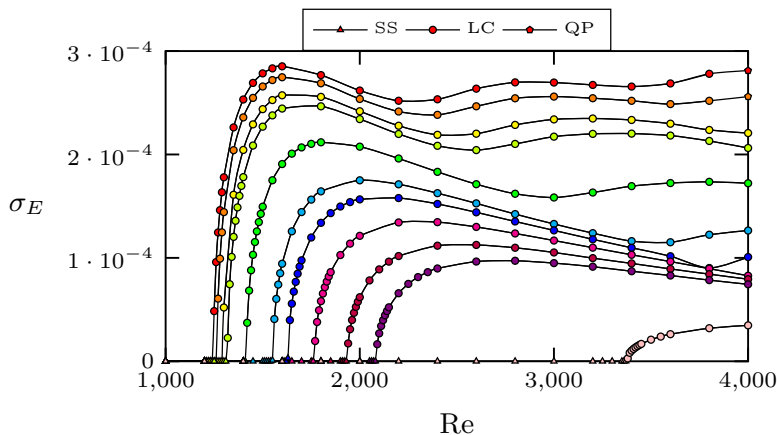
$$u = -\frac{1}{r} \frac{\partial \psi}{\partial z}$$

$$v = \frac{\gamma}{r}$$

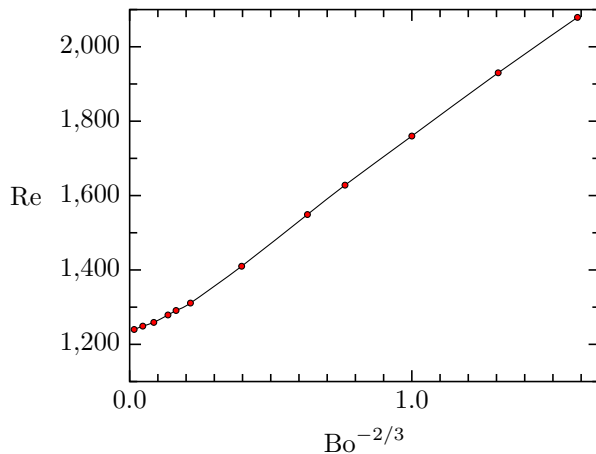
$$w = \frac{1}{r} \frac{\partial \psi}{\partial r}$$

at the point $(\frac{3}{4}A_H, \frac{3}{4}A_R)$.

Parameter Sweep



Scaling



At the Hopf Bifurcation

We now focus on the results for $Bo = 20$ and $Re = 1270, 1300$ (right after the bifurcation).

► [Zero Forcing Frequency Webpage](#)

Thanks to Jason Yalim.

Flow Field Animations

MOVIES IMPOSSIBLE TO EMBED IN THE BEAMER PRESENTATION

Thank you.