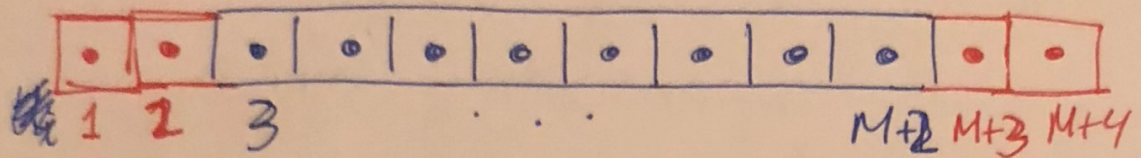

$$u_i^{n+1} = u_i^n - \frac{\Delta t}{\Delta x}\left(h^n_{i+\frac{1}{2}} - h^n_{i-\frac{1}{2}}\right),$$

where
$$\begin{cases} h^n_{i+\frac{1}{2}} = \frac{1}{2}\left[\left(E^n_{i+1} + E^n_i\right) + \Phi^n_{i+\frac{1}{2}}\right] \\ h^n_{i-\frac{1}{2}} = \frac{1}{2}\left[\left(E^n_i + E^n_{i-1}\right) + \Phi^n_{i-\frac{1}{2}}\right] \end{cases}$$

$$\boxed{E_i^n = \tfrac{1}{2}(u_i^n)^2}$$

and $$\Phi^n_{i+\frac{1}{2}} = G^n_{i+1} + G^n_i - \psi\left(\alpha^n_{i+\frac{1}{2}} + \beta^n_{i+\frac{1}{2}}\right)\Delta u^n_{i+\frac{1}{2}}$$

with $$\psi(y) = \begin{cases} |y| & |y| > \varepsilon \\ \frac{y^2+\varepsilon^2}{2\varepsilon} & |y| < \varepsilon \end{cases} \quad \text{for } 0 \le \varepsilon \le \frac{1}{8}$$

↳ choos $\varepsilon = 0.1$

$$\alpha^n_{i+\frac{1}{2}} = \begin{cases} \frac{E^n_{i+1} - E^n_i}{u^n_{i+1} - u^n_i} & |\Delta u^n_{i+1/2}| \ge \varepsilon' \\ (u^n_i + u^n_{i+1})/2 & |\Delta u^n_{i+1/2}| < \varepsilon' \end{cases}$$

$\varepsilon' = 10^{-12}$

$$\beta^n_{i+\frac{1}{2}} = \begin{cases} \frac{G^n_{i+1} - G^n_i}{u^n_{i+1} - u^n_i} & |\Delta u^n_{i+1/2}| \ge \varepsilon' \\ 0 & |\Delta u^n_{i+\frac{1}{2}}| < \varepsilon' \end{cases}$$

$$\alpha^n_{i-\frac{1}{2}} = \begin{cases} \frac{E^n_i - E^n_{i-1}}{u^n_i - u^n_{i-1}} & |\Delta u^n_{i-1/2}| \ge \varepsilon' \\ (u^n_{i-1} + u^n_i)/2 & |\Delta u^n_{i-\frac{1}{2}}| < \varepsilon' \end{cases}$$

$$\beta_{i-1/2}^n \begin{cases} \dfrac{G_i^n - G_{i-1}^n}{u_i^n - u_{i-1}^n} & |\Delta u_{i-1/2}^n| \geq \varepsilon' \\ 0 & |\Delta u_{i-1/2}^n| < \varepsilon' \end{cases}$$

Recall that $\Delta u_{i+1/2}^n = u_{i+1}^n - u_i^n$ and $\Delta u_{i-1/2}^n = u_i^n - u_{i-1}^n$

Then $G_i^n = S \cdot \max\left[0, \min\left(\sigma_{i+1/2}^n |\Delta u_{i+1/2}^n|, S \cdot \sigma_{i-1/2}^n \Delta u_{i-1/2}^n\right)\right]$

with $S = \text{sign}(\Delta u_{i+1/2}^n)$ and

$$\sigma_{i+1/2}^n = \frac{1}{2}\left[\psi(\alpha_{i+1/2}^n) - \frac{\Delta t}{\Delta x}(\alpha_{i+1/2}^n)^2\right].$$

I used a CFL = 0.1. Hence,

$$\Delta t = CFL \cdot \frac{\Delta x}{\max|\alpha|} \quad \text{with } \Delta x = \frac{L}{M}$$

and $L = 7 - 3 = 4$

Ghost cells BCs: periodic

left $\begin{cases} u_2 = u_{M+2} \\ u_1 = u_{M+1} \end{cases}$ right $\begin{cases} u_{M+3} = u_3 \\ u_{M+4} = u_4 \end{cases}$