NNSE 784
Advanced Analytics Methods

Instructor: F Doyle (CESTM L210)

MW 4:30 – 5:50, NFN 203

Slide Set #12
Inferential Statistics: Statistical Power

Lecture Outline

- Revisit Type I, Type II errors
- Discuss “power” of tests
  - How is it used?
  - How can we use Python to do power analysis?

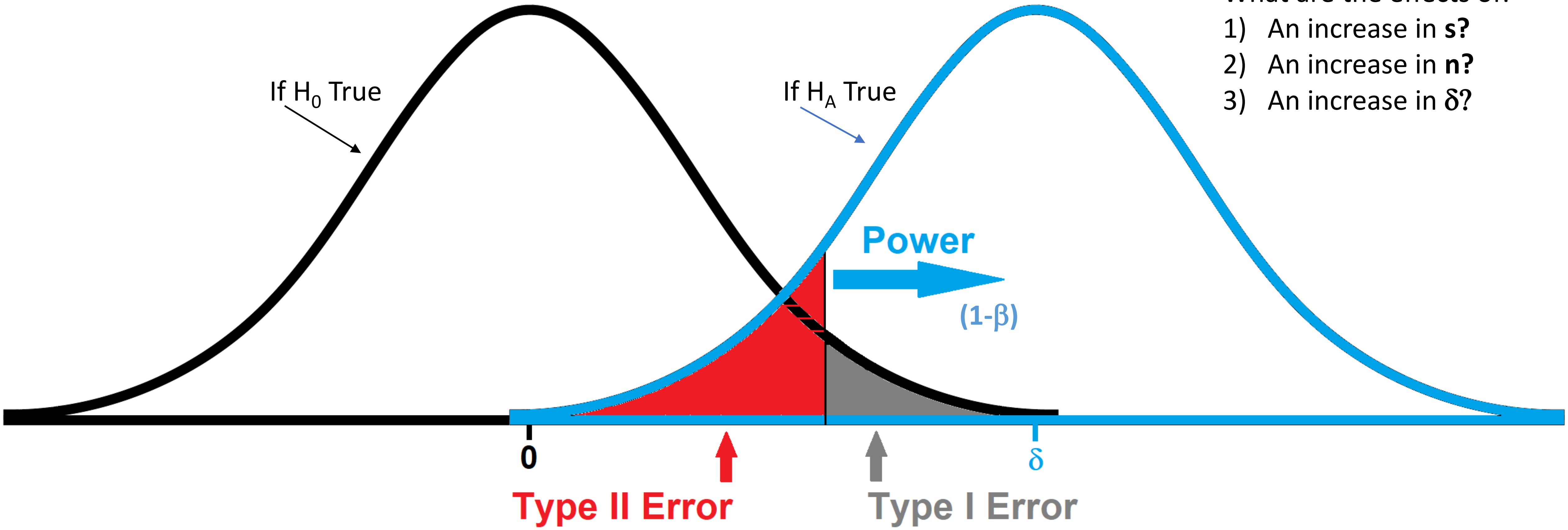
Type I and Type II Errors

		Condition of Null Hypothesis	
		True	False
Possible Action	Fail to reject H_0	$1 - \alpha$ Correct action **True Negative**	β Type II error **False Negative**
	Reject H_0	α Type I error **False Positive**	$1-\beta$ Correct action **True Positive**

Level of significance

Power of a statistical test

Type I , Type II Errors and Power



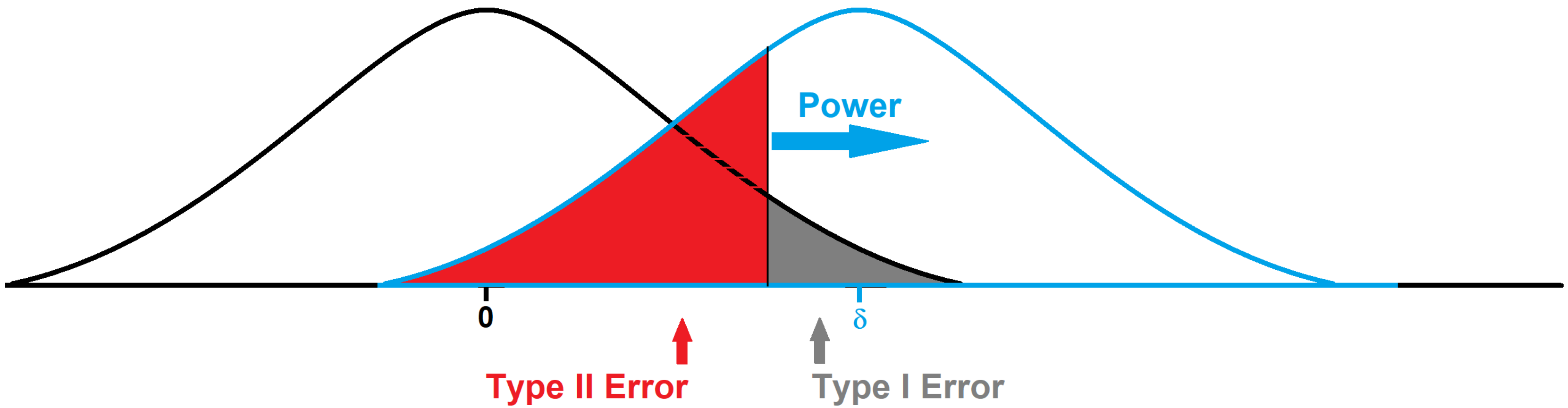
What are the effects of:

1) An increase in **s?**
2) An increase in **n?**
3) An increase in δ?

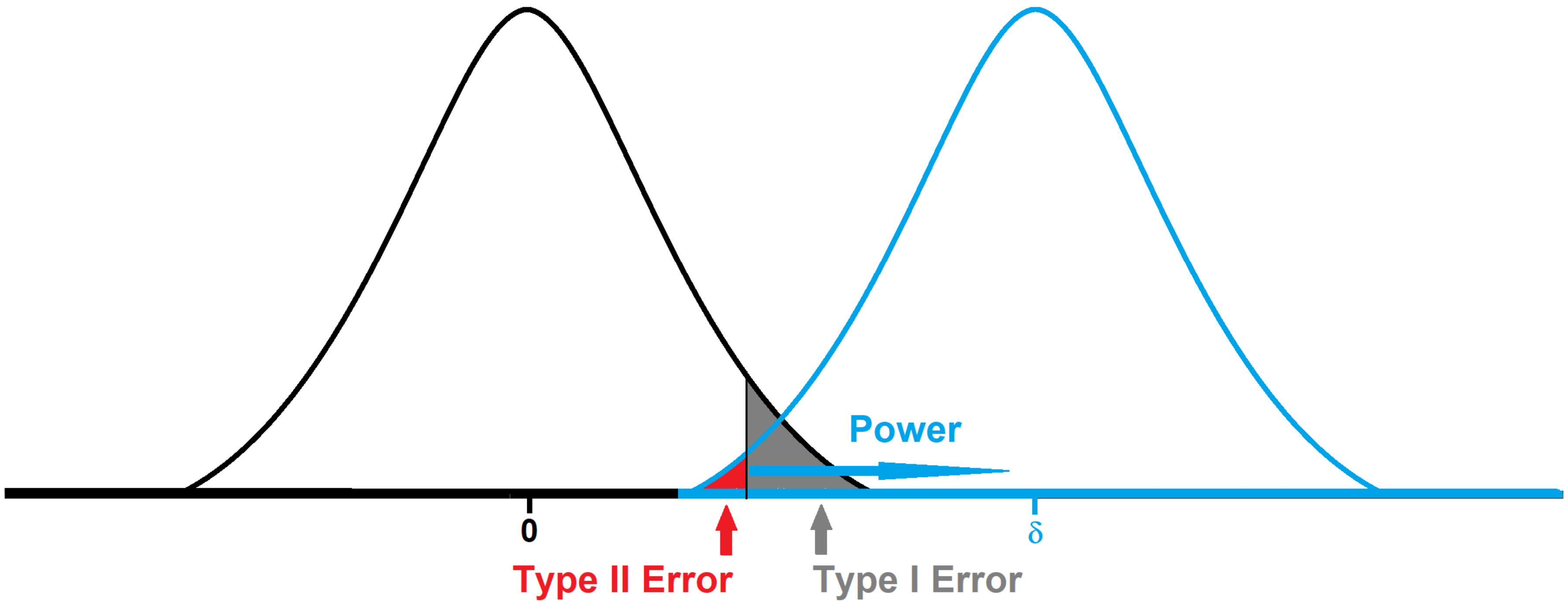
β – Probability of failing to reject the null hypothesis when it is false

α – Probability of rejecting the null hypothesis when it is true

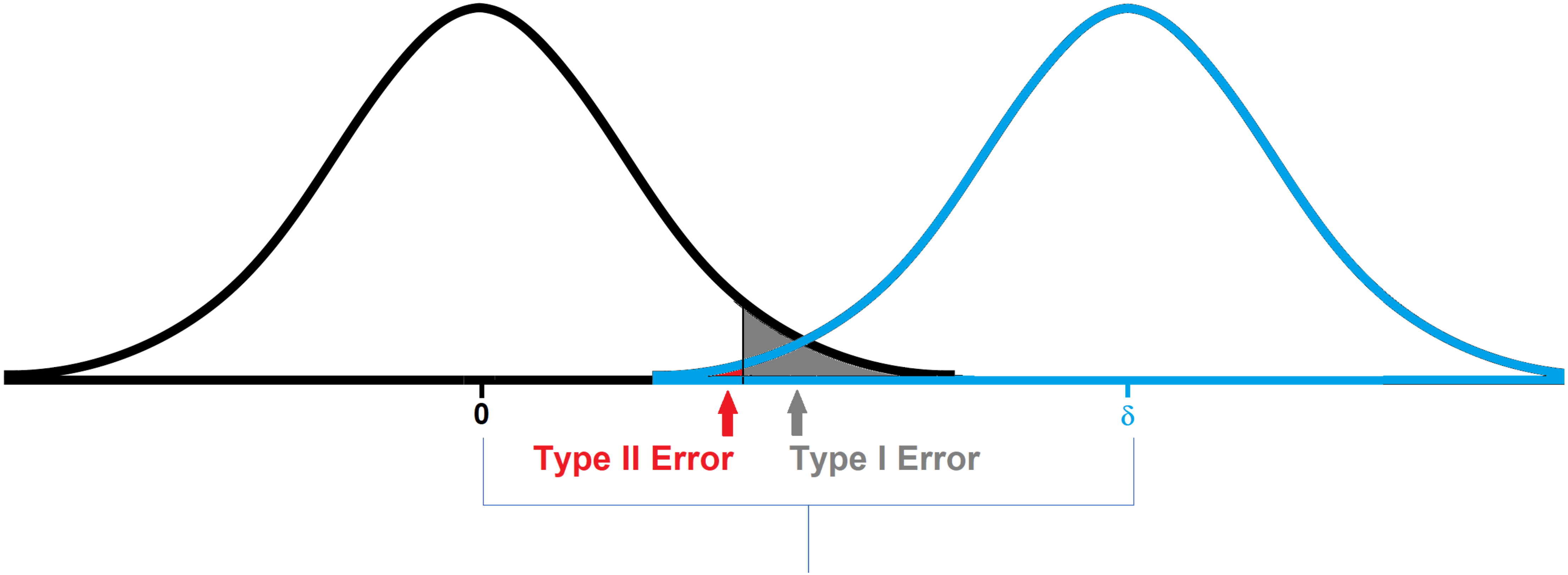
Increase in **s** (sample standard deviation)



Increase in n (sample size)



Increase in δ (difference in population means)



δ = the difference in population means

Power Analysis

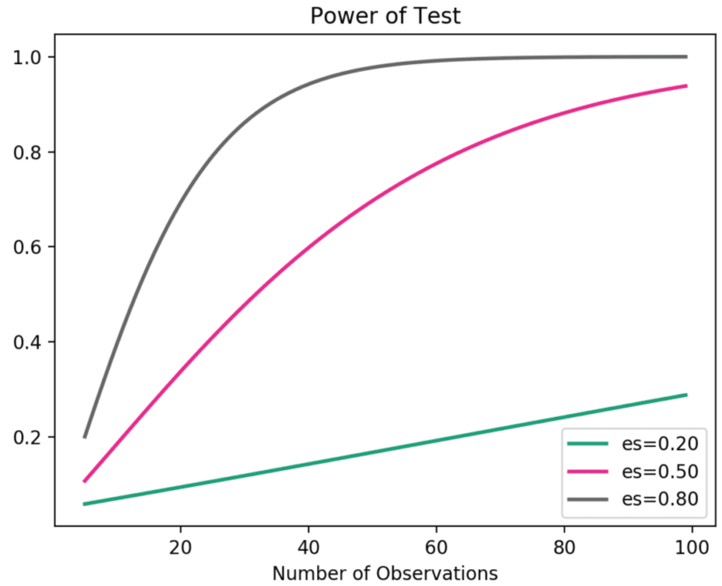
Calculated using 4 interrelated variables:

- Significance level (α)
- Power ($1 - \beta$)
- Sample Size (n)
- **<u>Effect size</u> –** standardized common scale. For example *Cohen's d*:
  - (mean of treatment group – mean of control group)/pooled standard deviation
  - < 0.1 – trivial effect
  - 0.20 - small effect
  - 0.50 - medium effect
  - 0.80 – large effect
  - 1.3 – very large effect

We can calculate any 4^{th} variable via the other three

How Can We Use Power Analysis?

1. Using pilot study data, determine what sample sizes you will need to be able to detect an effect of some specified size with a given significance level, and power
2. Determine the power of a specific proposed sample size
3. Plot power curves (line plots showing how the change in effect and sample size impact the statistical test)



Case 1 - How many samples needed?

Hypothetical pilot study

- Two sample groups
- Both groups size n=4
- Both groups sample standard deviation s=5
- Mean of group 1 = 90
- Mean of group 2 = 85

What was the measured effect size?

How many samples needed to detect this with statistical significance (α) of .05 and power ($1-\beta$) of .80

Case 1 - continued

```python
# import required modules
from math import sqrt
from statsmodels.stats.power import TTestIndPower

#calculation of effect size
# size of samples in pilot study
n1, n2 = 4, 4

# variance of samples in pilot study
s1, s2 = 5**2, 5**2

# calculate the pooled standard deviation
# (for Cohen's d)
s = sqrt(((n1 - 1) * s1 + (n2 - 1) * s2) / (n1 + n2 - 2))

# means of the samples
u1, u2 = 90, 85

# calculate the effect size
d = (u1 - u2) / s
print(f'Effect size: {d}')
```

Pooled variance estimate

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

Case 1 - continued

```
# factors for power analysis
alpha = 0.05
power = 0.8

# perform power analysis to find sample size
# for given effect
obj = TTestIndPower()
n = obj.solve_power(effect_size=d, alpha=alpha, power=power,
                    ratio=1, alternative='two-sided')

print("Sample size/Number needed in each group: {:.3f}".format(n))
```

```
Effect size: 1.0
Sample size/Number needed in each group: 16.715
```

Case 2 – Calculate power of a specific proposed sample size

```
from statsmodels.stats.power import TTestPower

power = TTestPower()
n_test = power.solve_power(nobs=40, effect_size = 0.5,
                           power = None, alpha = 0.05)
print('Power: {:.3f}'.format(n_test))
```

```
Power: 0.869
```

Case 3 - Generate Power Curves

```
# import required libraries
import numpy as np
import matplotlib.pyplot as plt
from statsmodels.stats.power import TTestIndPower


# power analysis varying parameters
effect_sizes = np.array([0.2, 0.5, 0.8,1.3])
sample_sizes = np.array(range(5, 100))

# plot power curves
obj = TTestIndPower()
obj.plot_power(dep_var='nobs', nobs=sample_sizes,
               effect_size=effect_sizes)

plt.show()
```