NNSE 784
Advanced Analytics Methods

Instructor: F Doyle (CESTM L210)

MW 4:30 – 5:50, NFN 203

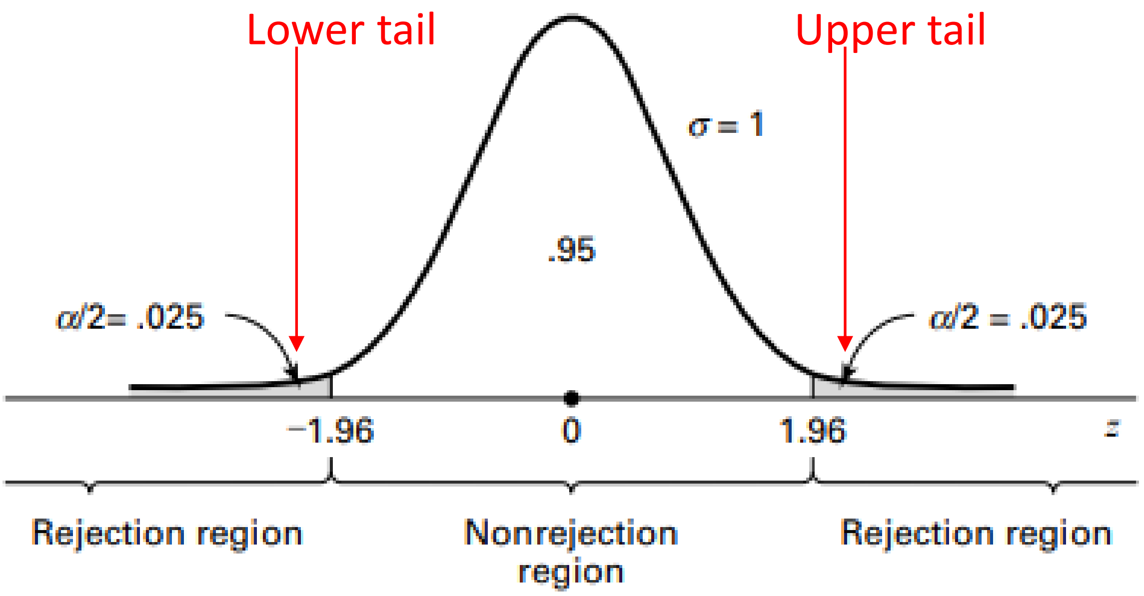
Slide Set #11
Inferential Statistics: Hypothesis Testing continued

Lecture Outline

- Hypothesis testing recap
- Hypothesis t-test for a single population mean
- Hypothesis t-test for one sided, two sample
- Hypothesis t-test for paired samples

Hypothesis Testing - recap

1) Examine and understand your data
2) Define assumptions (assuming normal distribution?)
3) State hypotheses (null and alternative)
4) Determine test statistic (e.g., z? t?)
5) Distribution of test statistic
6) Decision rule (e.g., let a = .05 with a two sided test [see figure at right])
7) Calculate the test statistic
8) Statistical decision (reject null hypothesis?)
9) Conclusion
10) p-value



Hypothesis Test – Single Population Mean

A protocol produces gold nanoparticles for a new biological application. The following are 15 sample measurements. Given these measurements, can we say that the protocol produces an average particle diameter different from 34.5nm, with a significance level of .05?

Diameters:
33.38, 32.15, 33.99, 34.10, 33.97,
34.34, 33.95, 33.85, 34.23, 32.73,
33.46, 34.13, 34.45, 34.19, 34.05

$\mathbf{H_O} = \mathbf{34.5}$, $\mathbf{H_A} \neq \mathbf{34.5}$

Test statistic for $H_0\colon \mu = \mu_0$ is $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$

First, import the libraries

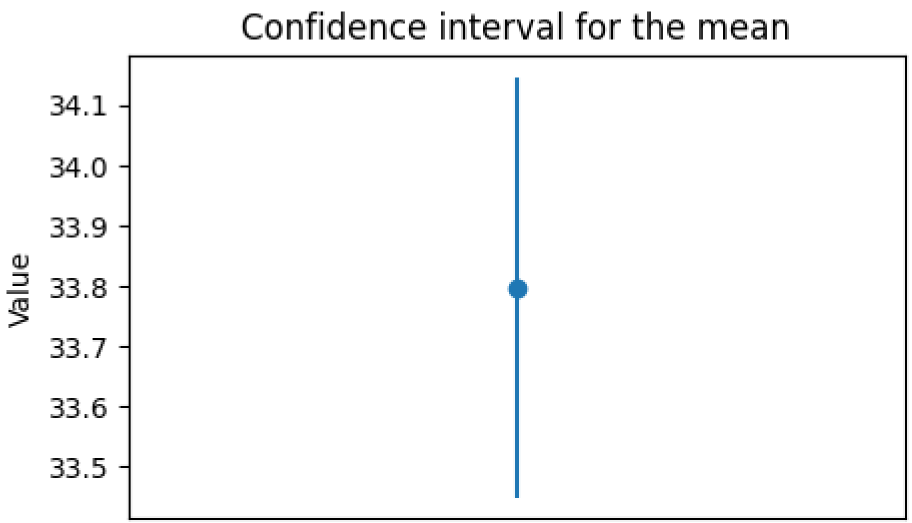
```
import matplotlib.pyplot as plt
import numpy as np
from scipy import stats
```

Enter the data and make a couple of preliminary calcs:

```
diameters = [33.38,32.15,33.99,34.10,33.97,34.34,33.95,33.85,
             34.23,32.73,33.46,34.13,34.45,34.19,34.05]
x_bar = np.mean(diameters)
sem = stats.sem(diameters)
print("The sample's average diameter was: {}".format(x_bar))
```

Let’s do a quick confidence interval:

```
interval = stats.t.interval(confidence=.95, df=len(diameters)-1,loc=x_bar,scale=sem)
plt.figure(figsize=(5, 3))
plt.errorbar(x=0, y=x_bar, yerr=(interval[1]-x_bar), fmt='o')
plt.xticks([])
plt.ylabel('Value')
plt.title('Confidence interval for the mean')
plt.show()
```



Now, calculate the test statistic and p-value

```
#use ttest_1samp to calculate the t statistic
t_stat, p_val = stats.ttest_1samp(a=diameters, popmean = 34.5, alternative='two-sided')
print("t_stat {}, p_val {}".format(t_stat, p_val))
```

```
t_stat -4.313576593274865, p_val 0.0007145337906520521
```

Note, we are specifying the type of test here

Hypothesis Test – Two Population Means

Two deposition tools using the same protocol for depositing an oxide layer yield sample thicknesses as shown on following slides.
An engineer believes that tool 1 has been, on average, depositing thicker oxide using the same process. Is this correct?
Research Hypothesis is that tool 1's average thickness is greater than tool 2.
Null hypothesis is that tool 1's average thickness is equal to or less than tool 2's.

Test statistic for $H_0\colon \mu_1 - \mu_2 \leq 0, \quad H_A\colon \mu_1 - \mu_2 > 0$ is $t' = \dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)_0}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$

Enter the data:

```
deposition_tool_1 = [94.0, 84.9, 82.6, 69.5, 80.1, 79.6, 81.4, 77.8,
                     81.7, 78.8, 73.2, 87.9, 87.9, 93.5, 82.3, 79.3,
                     78.3, 71.6, 88.6, 74.6, 74.1, 80.6]
deposition_tool_2 = [77.1, 71.7, 91.0 , 72.2, 74.8, 85.1, 87.6, 69.9,
                     75.3, 71.7, 65.7, 72.6, 71.5, 78.2]
```

Quick calc of sample means…

```
d1_avg = np.mean(deposition_tool_1)
d2_avg = np.mean(deposition_tool_2)
print("The sample average deposition thickness with tool 1 was {}.".format(d1_avg))
print("The sample average deposition thickness with tool 2 was {}.".format(d2_avg))
```

```
The sample average deposition thickness with tool 1 was 81.0136363636363634.
The sample average deposition thickness with tool 2 was 76.0285714285714144.
```

Calculate the test statistic and p-value…

```
#use ttest_ind to calculate this t statistic
tstat,p_value = stats.ttest_ind(deposition_tool_1,deposition_tool_2,alternative='greater')
print("t_stat {}, p_val {}".format(tstat,p_value))
```

```
t_stat 2.143447322719482, p_val 0.019658411013040247
```

H_A was population 1 (deposition_tool_1) was greater (not just different)

The Paired t-test
(aka “Dependent Sample t-test”)

- Compares the means of two measurements from the same individual, object or related units
- A measurement taken at two different times (e.g., pre-test and post-test score with an intervention administered between the two time points)
- A measurement taken under two different conditions (e.g., completing a test under a "control" condition and an "experimental" condition)
- Measurements taken from two halves or sides of a subject or experimental unit (e.g., measuring hearing loss in a subject's left and right ears).

Hypothesis Test - Paired t-test

Values for days before required maintenance of a set of vacuum pumps pre and post introduction of new lubricant are shown below.

Research hypothesis is that they are different.

Null hypothesis is that they are the same.

Test statistic for paired t-test: $$t = \frac{\bar{d} - \mu_{d_0}}{s_{\bar{d}}}$$

$\bar{d}$ is the sample mean difference,

μ_{d_0} is the hypothesized population mean

$s_{\bar{d}} = s_d/\sqrt{n}$, n is the number of sample differences,

s_d is the standard deviation of the sample differences.

Enter the data, and perform test...

```
# pre holds the days before required maintenance
#pre introduction of new oil
pre = [30, 31, 34, 40, 36, 35, 34, 30, 28, 29]
# post holds the days before required maintenance
# post application of the new oil
post = [30, 31, 32, 38, 32, 31, 32, 29, 28, 30]
# Performing the paired sample t-test (ttest_rel is ttest "related")
tstat, p_val = stats.ttest_rel(pre, post)
```

We didn't specify "two-sided", but it is the default value for the 'alternative' parameter

Results…

```
print("tstat {}, p_val {}".format(tstat,p_val))
```

```
tstat 2.584921310565987, p_val 0.029457853822895275
```