Markov chain Monte Carlo (MCMC)

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What is Monte Carlo?

Monte Carlo

From Wikipedia, the free encyclopedia

Coordinates: 43°44′23″N 7°25′38″E

For other uses, see Monte Carlo (disambiguation).

Monte Carlo (/ˈmɒnti ˈkɑːrloʊ/; Italian: [ˈmonte ˈkarlo]; French: *Monte-Carlo* [mɔ̃te kaʁlo], or colloquially *Monte-Carl* [mɔ̃te kaʁl]; Monégasque: *Munte Carlu*) officially refers to an administrative area of the Principality of Monaco, specifically the ward of Monte Carlo/Spélugues, where the Monte Carlo Casino is located. Informally the name also refers to a larger district, the Monte Carlo Quarter (corresponding to the former municipality of Monte Carlo), which besides Monte Carlo/Spélugues also includes the wards of La Rousse/Saint Roman, Larvotto/Bas Moulins, and Saint Michel. The permanent population of the ward of Monte Carlo is about 3,500, while that of the quarter is about 15,000. Monaco has four traditional quarters. From west to east they are: Fontvieille (the newest), Monaco-Ville (the oldest), La Condamine, and Monte Carlo.

Monte Carlo (literally "Mount Charles") is situated on a prominent escarpment at the base of the Maritime Alps along the French Riviera. Near the quarter's western end is the world-famous Place du Casino, the gambling center which has made Monte Carlo "an international byword for the extravagant display and reckless dispersal of wealth".[1] It is also the location of the Hôtel de Paris, the Café de Paris, and the Salle Garnier (the casino theatre which is the home of the Opéra de Monte-Carlo).

The quarter's eastern part includes the community of Larvotto with Monaco's only public beach, as well as its new convention center (the Grimaldi Forum), and the Monte-Carlo Bay Hotel & Resort. At the quarter's eastern border, one crosses into the French town of Beausoleil (sometimes referred to as Monte-Carlo-Supérieur), and just 8 kilometres (5 mi) to its east is the western border of Italy.

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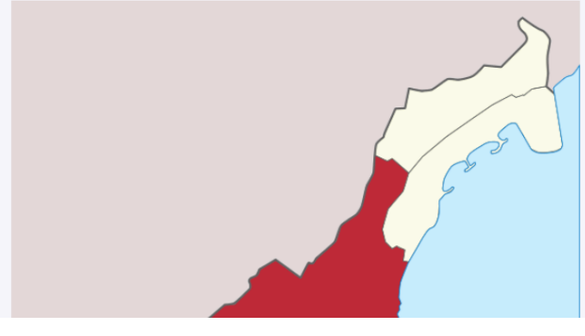
Monte Carlo
Quarter and ward of Monaco



The Monte Carlo method

https://github.com/fjdu/seminar-pmo-sf/blob/master/2019-01-04-MCMC-qzyan/MCMCPI.ipynb

What is Markov chain?

Markov Chain

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A Markov chain is collection of random variables $\{X_t\}$ (where the index t runs through 0, 1, ...) having the property that, given the present, the future is conditionally independent of the past.

In other words,

$$P(X_t = j \,|\, X_0 = i_0, X_1 = i_1, \ldots, X_{t-1} = i_{t-1}) = P(X_t = j \,|\, X_{t-1} = i_{t-1}).$$

If a Markov sequence of random variates X_n take the discrete values a_1, ..., a_N, then

$$P\left(x_n = a_{i_n} \,|\, x_{n-1} = a_{i_{n-1}}, \ldots, x_1 = a_{i_1}\right) = P\left(x_n = a_{i_n} \,|\, x_{n-1} = a_{i_{n-1}}\right),$$

and the sequence x_n is called a Markov chain (Papoulis 1984, p. 532).

A simple random walk is an example of a Markov chain.

The Season 1 episode "Man Hunt" (2005) of the television crime drama *NUMB3RS* features Markov chains.

Markov chain examples

Ming: 26 A-Z,

1. B->M->B->P-A……

Hong: 26 A-Z,

2. A-C-M-A

ACM CMA

Li: C

3. C-C-C-C

Bayesian inference

Simply put, take each parameter as a distribution.

https://github.com/fjdu/seminar-pmo-sf/blob/master/2019-01-04-MCMC-qzyan/distanceInference.ipynb

What is the likelihood?

A kind of PDF.

What is the prior?

Prior probability distributions, such as normal, exponential, or uniform.

Age: U(0,150) year

Age: N(30,50) year

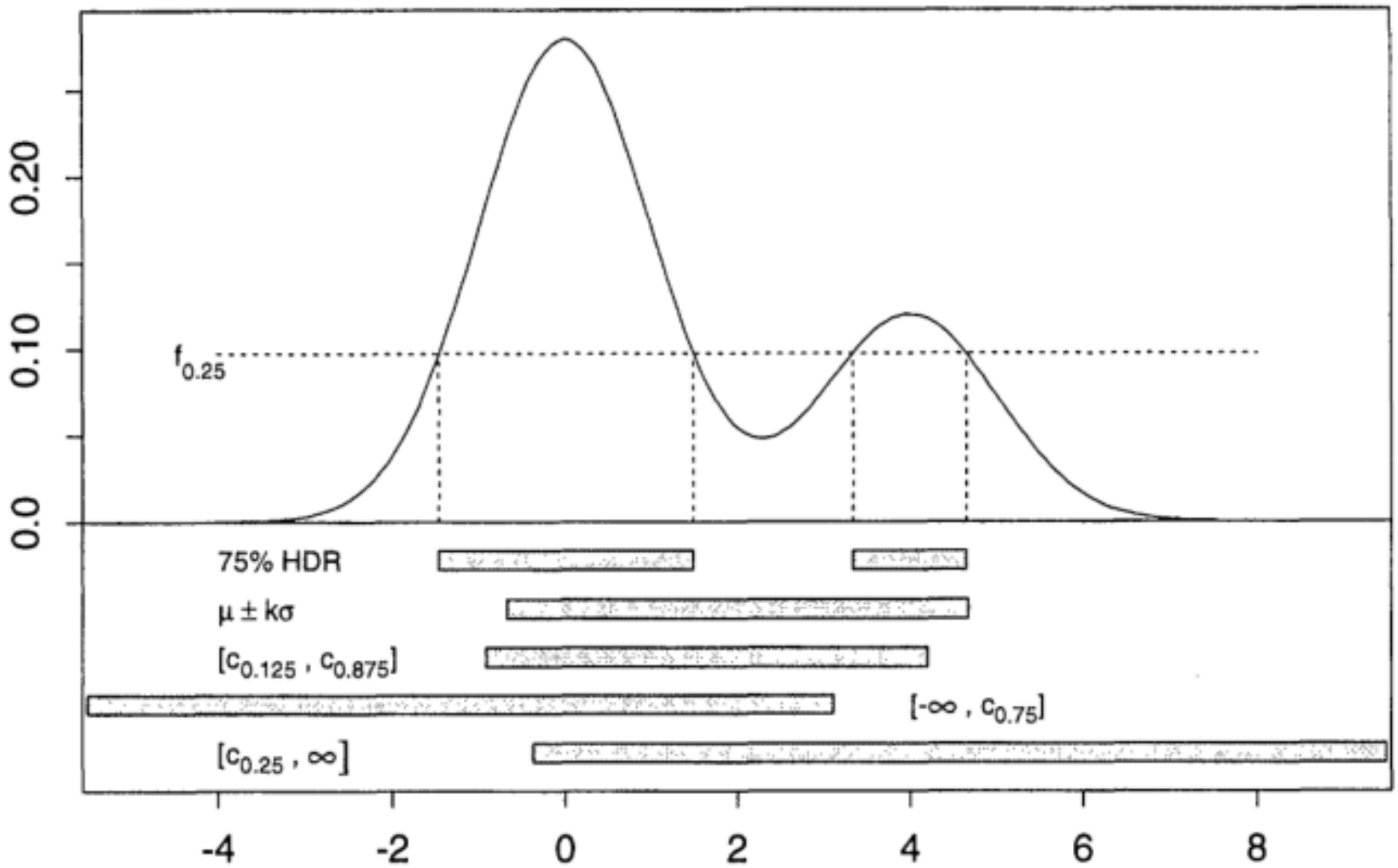
What are posteriors?

The conditional probability given the data.

Mu: U(0,100)
Std: U(0,10)

Mu: N(30,4)
Std: N(2,1)

What is Highest Posterior Density?



MCMC examples

How to draw a sample from a distribution?

Algorithm 1 Metropolis-Hastings algorithm

Initialize $x^{(0)} \sim q(x)$

for iteration $i = 1, 2, \ldots$ **do**

Propose: $x^{cand} \sim q(x^{(i)}|x^{(i-1)})$

Acceptance Probability:

$\alpha(x^{cand}|x^{(i-1)}) = \min\{1, \frac{q(x^{(i-1)}|x^{cand})\pi(x^{cand})}{q(x^{cand}|x^{(i-1)})\pi(x^{(i-1)})}\}$

$u \sim \text{Uniform}\ (u; 0, 1)$

if $u < \alpha$ **then**

Accept the proposal: $x^{(i)} \leftarrow x^{cand}$

else

Reject the proposal: $x^{(i)} \leftarrow x^{(i-1)}$

end if

end for

Sampling a Gaussian distribution

https://github.com/fjdu/seminar-pmo-sf/blob/master/2019-01-04-MCMC-qzyan/sampleGaussian.ipynb

Fitting a Gaussian distribution

https://github.com/fjdu/seminar-pmo-sf/blob/master/2019-01-04-MCMC-qzyan/MCMC_gaussianFitting.ipynb