Implementation of Eight Neighbor Grids

Eight neighbor grids are represented by a model that uses a Graph class and a Vertex class.

Vertex Class
The Vertex class represents nodes on the eight neighbor grid and stores their g, h, and f-values. Vertex objects are identified by a unique key in the form “x|y”. The class also maintains a neighbors dictionary with neighboring vertices as the key and that edges weight as the value.

Graph Class
The graph class maintains a list of all of the participating vertices, the starting and goal vertices and a dictionary that keeps track of which cells are blocked and unblocked.

These grids interact with the front end visualization by providing a Graph object that represents the eight neighbor grid with a start and goal node. The path is generated later by the A* or Theta* algorithm class

Implementation of Algorithms

A* and Theta* are implemented as classes with a main method closely following their respective pseudocode.

AStar Class
The AStar class is interacted with through its main method. It accepts a graph object as a parameter and then when the main method is run [*AStar(graph).main()*] it returns the shortest path as a list of nodes from start node to goal node. AStar implements *main, g, h, shortest_path, UpdateVertex.*

shortest_path(end_node)

- *Given the final node after running A* traces the path to that node through its parents and returns a reversed list.*

Note: All other methods are implemented exactly as described in the project description

Problem 1g - Optimization of A*/Theta*

Removing Redundant Code

The first step in optimization was removing redundant code. Initially we used an implementation that traced the path from a node to the start through parents each time g was called. After realizing the pseudocode properly updated a Vertex's g-val we adapted our code to take advantage of this. This provided significant reductions in runtime of the code but is not discussed in detail here because the implementation represented a misunderstanding of the pseudocode and not a true optimization.

G-Attribute Access

In the initial implementation of A*/Theta* g-values are set through a custom setter method called *vertex*.setG(). This method was used in order to update the f-value automatically whenever the g-value was set in the code. F-values however do not need to be stored updated and only need to be updated before being added to the heap. Switching to dot attribute access, not including the time it takes to update f-value, saves about 20% of the time. This setting g-value operation is done twice per loop so this saves a fair amount of time each loop. Additionally, by unwrapping the updating of f-values we save unnecessary updates and only update when necessary. This is a small runtime optimization and has the downside of complicating the codebase slightly. It is less robust in terms of maintaining the codebase.

Average setG Runtime	Average dotG Runtime	Percent Change
0.01023563121	0.00857510101	19.36%

Note: Average runtimes were calculated using the timeit module and are the result of 1,000,000 tests

Closed Data Structure

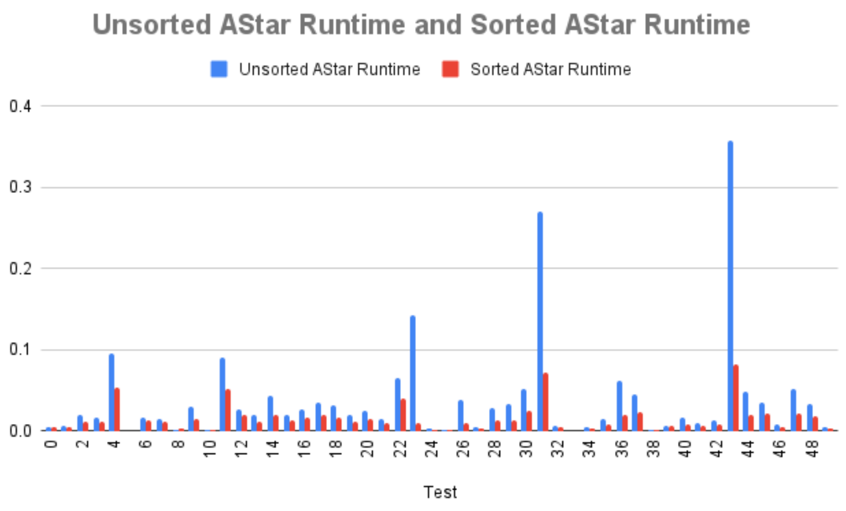
The initial implementation of the closed data structure is a Python list. The operations performed on this list are append and search. Append is performed in O(1) time to a list which is optimal but searching the list is Average Case O(n). There are ways to maintain the closed list that can perform better than this. Python dictionaries are implemented as hash tables so they have an Amortized Worst Case lookup of O(1).

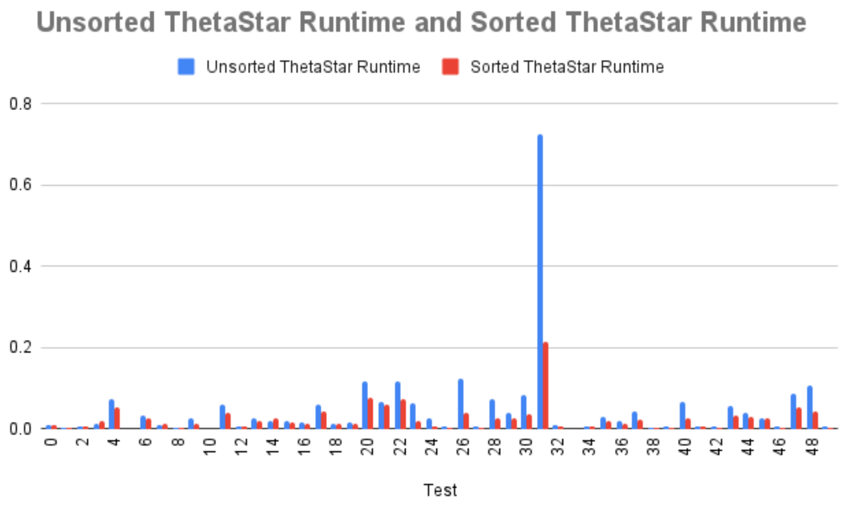
Dictionary(Hashtable) Implementation

One possible implementation of the closed list is with a dictionary. We can search a dictionary for the key of a vertex with an average case O(1) but a worse case of O(n) so this is only slightly better. By initializing a dictionary with a key for every vertex and setting values to 0 or 1 (in closed or not in closed) we can lower the search time for within the algorithm to O(1) but this would require a setup or tear down where all vertex key values are set to 0. The added memory usage and setup/tear down costs made this not an optimal solution for this implementation.

Sorted List w Binary Search Implementation

A better implementation is to store the list as vertex keys in sorted order and search the closed list using binary search. Binary Search is implemented in its iterative version which has O(1) memory complexity and O(logn) time complexity for search. Keys are added to a sorted list in O(log n) using the python bisect library with a python list. The runtime improvements of the algorithm are significant and are shown in the trials below.





Problem 1h

Experimental Design

Data for the analysis of A*/Theta* was collected by generating 200 random eight neighbor grids and running the A* algorithm and Theta* algorithm on each of the test grids. For each grid we observed the A* Runtime, Theta* Runtime, A* Path Length and Theta* Path Length. Grids are generated using the generatetests.py file and randomness is enforced with the built in random module.

Analysis of Path Lengths/Runtimes

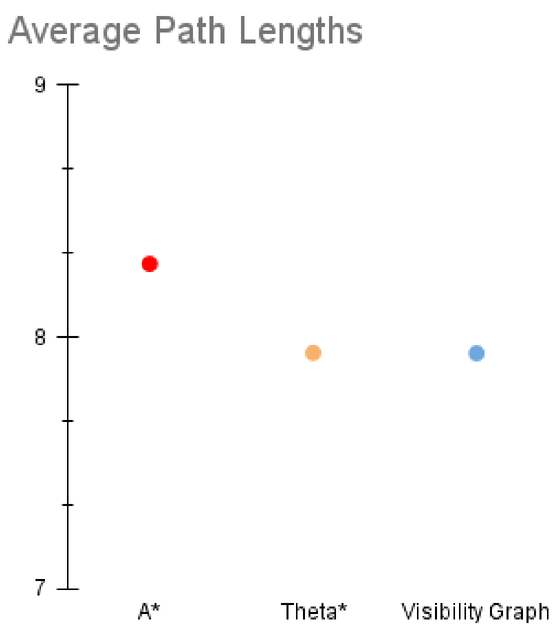
Averages			
AStar Runtime	AStar Path Length	ThetaStar Runtime	ThetaStar Path Length
0.01740716242	41.77888273	0.03091763402	39.86143948

A* outperformed Theta* by 43.7% in terms of runtime. In terms of path lengths Theta* averaged about 4% shorter path lengths. The increase in runtime due to the amount of LineOfSight checks that are performed in Theta*. Every time that Theta* runs UpdateVertex and has the initial overhead of performing a LineOfSight check to choose its path. Optimization of Theta* would have to be largely around how LineOfSight checks are performed. The LineOfSight function used is relatively simple and cannot likely be optimized much however. The current LineOfSight implementation runs through conditional logic with relatively few jumps, does simple calculations with numbers generated through array access, and calls the cell is blocked method. Outside of micro optimizations the only thing that could be changed is the implementation of cellisblocked. CellIsBlocked mainly reads dictionary values (O(1) operation) and performs simple conditional checks. In conclusion there doesn't appear to be much room for optimization in the current LineOfSight implementation in order to make significant progress towards closing the gap between A* and Theta* a different LineOfSight method would be needed. The benefit that Theta* provides is a marginal decrease in path length so any discussion around using it would have to be around whether the 4% increase is significant enough at scale or whether it is important to have paths appear smooth (such as in video game AI or robotics).

Use of Different H-Values

H-Values used in Theta* and A* in terms of runtime are essentially the same. They are both values that can be calculated in O(1) time. The differences in the h-values appear when looking at their outputs. A* Equation 1 consistently outputs a value larger than Theta* c() over 1,000,000 tests with random vertex coordinates. On average the distance between these two h-value equations is about 2.54. This is significant because A* Equation 1 is consistent and admissible meaning it is never greater than the actual distance. If A* Equation 1 is consistent and admissible and always generates an output that is 2.54 higher than Theta* c() it will always be closer to the true distance. A heuristic that is closer to the true distance is always a better heuristic as long as it never over estimates. Therefore the use of different heuristics results in a significant advantage for the A* algorithm.

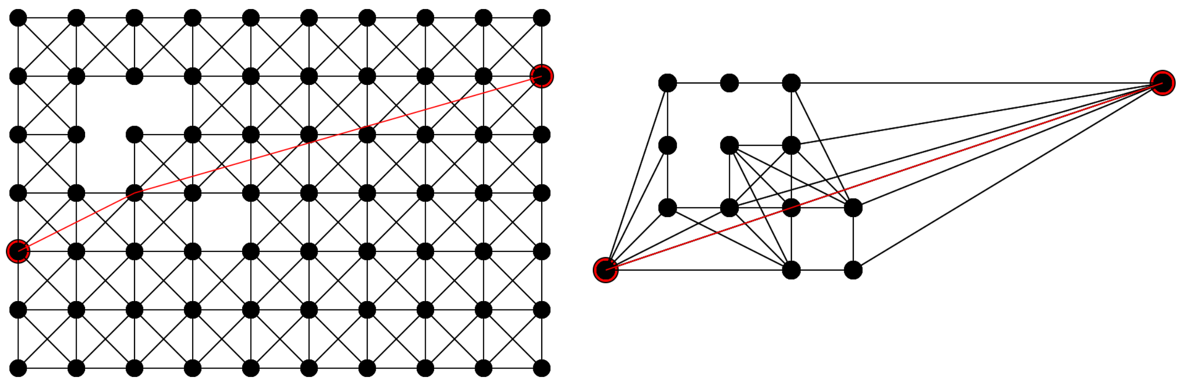
Problem 1i



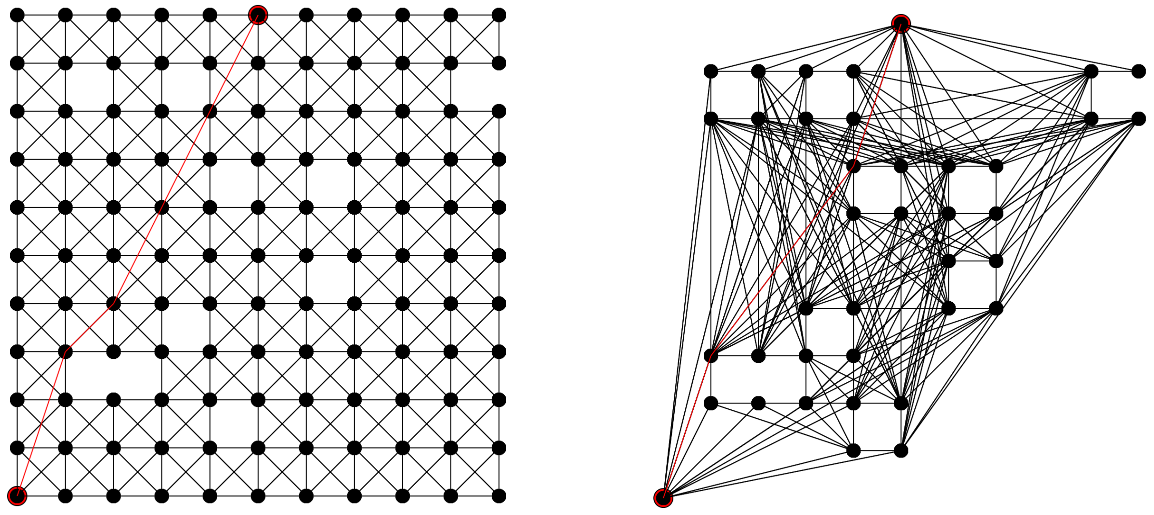
True Shortest Paths

Theta* requires vertex parents to be a neighbor that passes the LineOfSight test or the parent of a neighbor that passes the line of sight test. These are not always the shortest paths. Visibility graphs are guaranteed to generate the shortest paths so they can be used in test cases as the Expected answer while Theta* is the Actual result. By implementing a series of random walks on grids 10x10 we can quickly locate cases where the true shortest path is not found by Theta*. The graph pictured about shows the experimental results of 50 cases where the true path length is shorter than the Theta* path length. The average path lengths in the case where Theta* is not the shortest path are listed below.

A*	Theta*	Visibility Graph
8.290336213	7.93662637	7.935061797



Above is a manually generated example of when Theta* does not find the true shortest path.



Pictured above are two randomly generated cases where Theta* fails to find the shortest path.

Implementation of Visibility Graphs

Visibility graphs are implemented in the program using a simplified version of the Graph class. They share a simplified set of methods and attributes with the Graph class. The vis.py file can be called from the command line with a graph filename and it will display the VisibilityGraph for the graph it is given. More details on this are given in the readme.

Problem 2: Adversarial Search

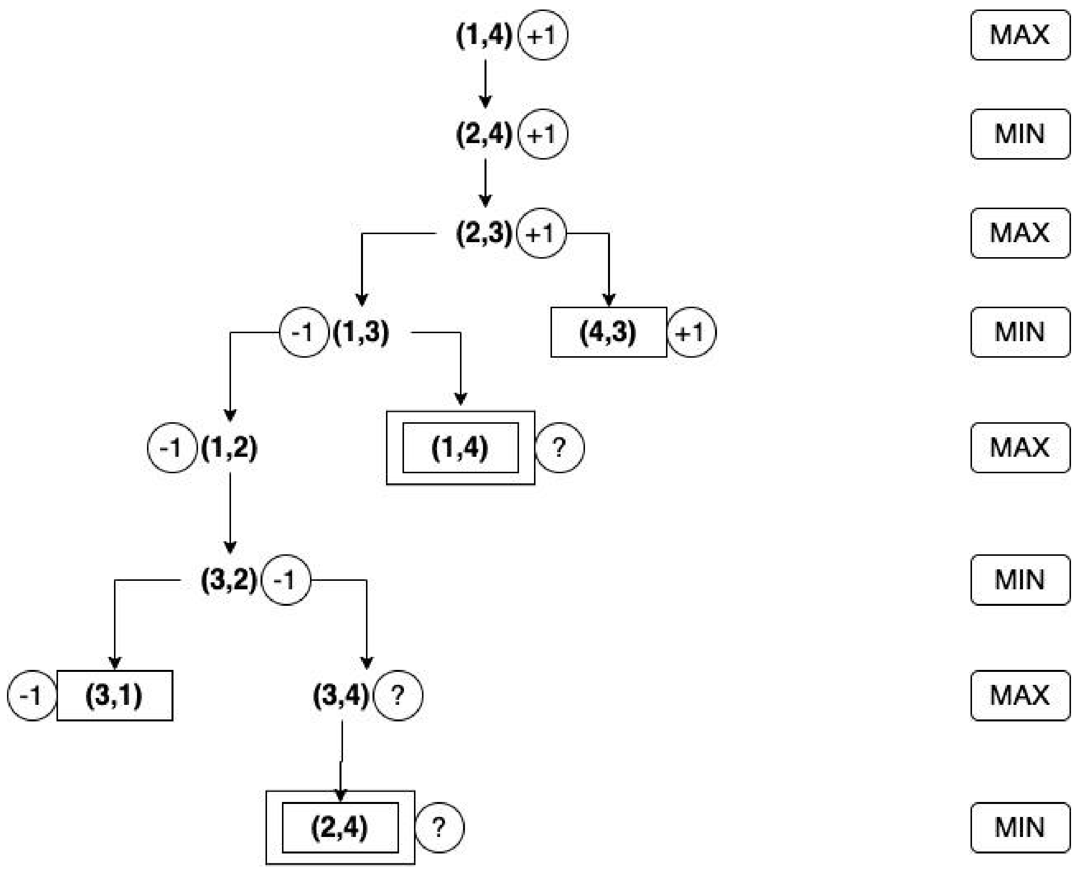


Figure 1: The game tree for the game described in Problem 2. Loop states are denoted with a double square. Terminal states are denoted with single squares. States are represented in tuples in the form (S_A, S_B). Minimax values are assigned as explained in part b.

B. In both loop states, the player whose turn it is stays the same. For example the state (2,4) will always be played by the MIN player. Therefore there is no utility benefit to choosing a loop state. The minimax algorithm should instead ignore the loop states and always pick other states instead.

C. The standard minimax algorithm would fail because the game tree is **infinitely deep.** A modified algorithm could solve this problem by recognizing when a loop state is about to start and treating it as a terminal state.

This can be achieved by modifying the *terminal-state, utility, and min/max procedures.*

Terminal-State: Maintain a list of states that have been visited. Modify the Terminal Test to consider already visited states as terminals instead of allowing them to loop.
Utility Function: If a state is a repeated state, give it a value of “?” (or any distinct value from +1 | -1).

Min/Max Functions: Modify min/max functions to recognize “?”’s and only select them when they are the only option. In other words “?”s are treated as the lowest possible value by the max function and the highest possible value by the min function.

D. For a game over 3 squares, A will move first. B will have no choice but to step over A, and so will win the game.
For a game over 5 squares, A moves first and B moves second. A will be in the 3rd square from its start, and B will be 2nd from its own start. When A moves again, B’s only next option is to step over A once more, leaving B one square away from its goal, while A needs two moves to win. In the state just before this one, the game’s state was that of the 3-square state. It is essentially the same, since in either case B will be closer to its goal than A is.
In an odd-n game, after one move of A and one move of B, the odd-n game is essentially reduced to the beginning state of one odd-n game down from it. This can go down all the way to the size three game, which B is already confirmed to win. Even in a larger size-n game, if n is odd, at the stage where B crosses over A, B will have one less square to traverse to reach its goal than A does. No matter how many moves A will make, B will still win, in any odd-n game.
A will win in even-n games, because after A’s initial move, B will essentially be starting in an odd-n game. As seen before, the one who starts an odd-n game will also lose it.

Problem 3 - Local Search

A. Hill climbing is effective at problems looking to quickly find local minima or maxima. If finding local maxima or minima is sufficient then hill climbing will find solutions faster than simulated annealing. Hill climbing by definition never makes downhill moves so it is impossible for it to be complete because without the randomness component it will always terminate if it finds local maxima or minima.

B. Random search can be useful for problems where the objective function is not smooth. If the objective function isn't smooth then it can be hard to calculate the gradient. The gradient is used by hill climbing to converge on a solution.

C. Simulated annealing is useful when value functions are smooth and contain a large amount of local maxima and minima. Simulated annealing is useful for quickly finding global maxima or minima by incorporating enough exploration early on in the search to avoid getting stuck in locals while not overshooting global maxima or minima.

D. Since simulated annealing terminates when the temperature schedule completes there is no guarantee that it terminates at a maxima or minima. After the termination of simulated annealing we can pass the current state to a hill climbing algorithm to ensure that the answer is a maxima or minima.

E. In the process for simulated annealing, after picking a random move the algorithm evaluates the move to see if it improves the situation. Currently the test to see if the move improves the situation is just a measure of whether that specific move is greater than the current position. Instead, if there is more memory, simulated annealing can be modified to use iterative depth first search to evaluate whether or not the randomly selected move is good or bad. By using the extra memory to look a few moves ahead the algorithm can better decide if a move is good or bad and converge on a solution faster.