Tabla A1.8 Funciones de Bessel de primera clase

β	$J_0(\beta)$	$J_1(\beta)$	$J_2(\beta)$	$J_3(\beta)$	$J_4(\beta)$	$J_5(\beta)$	$J_6(\beta)$	$J_7(\beta)$	$J_8(\beta)$	$J_9(\beta)$	$J_{10}(\beta)$	$J_{11}(\beta)$	$J_{12}(\beta)$	$J_{13}(\beta)$	$J_{14}(\beta)$
0.00	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.25	0.984	0.124	0.008	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.50	0.938	0.242	0.031	0.003	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.75	0.864	0.349	0.067	0.008	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1.00	0.765	0.440	0.115	0.020	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1.25	0.646	0.511	0.171	0.037	0.006	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1.50	0.512	0.558	0.232	0.061	0.012	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1.75	0.369	0.580	0.294	0.092	0.021	0.004	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2.00	0.224	0.577	0.353	0.129	0.034	0.007	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2.25	0.083	0.548	0.405	0.171	0.052	0.012	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2.50	-0.048	0.497	0.446	0.217	0.074	0.020	0.004	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2.75	-0.164	0.426	0.474	0.263	0.101	0.030	0.007	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3.00	-0.260	0.339	0.486	0.309	0.132	0.043	0.011	0.003	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3.25	-0.333	0.241	0.481	0.351	0.167	0.060	0.017	0.004	0.001	0.000	0.000	0.000	0.000	0.000	0.000
3.50	-0.380	0.137	0.459	0.387	0.204	0.080	0.025	0.007	0.002	0.000	0.000	0.000	0.000	0.000	0.000
3.75	-0.401	0.033	0.419	0.414	0.243	0.105	0.036	0.010	0.003	0.001	0.000	0.000	0.000	0.000	0.000
4.00	-0.397	-0.066	0.364	0.430	0.281	0.132	0.049	0.015	0.004	0.001	0.000	0.000	0.000	0.000	0.000
4.25	-0.369	-0.156	0.296	0.434	0.317	0.162	0.065	0.022	0.006	0.002	0.000	0.000	0.000	0.000	0.000
4.50	-0.321	-0.231	0.218	0.425	0.348	0.195	0.084	0.030	0.009	0.002	0.001	0.000	0.000	0.000	0.000
4.75	-0.255	-0.289	0.133	0.401	0.374	0.228	0.106	0.041	0.013	0.004	0.001	0.000	0.000	0.000	0.000
5.00	-0.178	-0.328	0.047	0.365	0.391	0.261	0.131	0.053	0.018	0.006	0.001	0.000	0.000	0.000	0.000
5.25	-0.093	-0.345	-0.038	0.316	0.399	0.293	0.158	0.069	0.025	0.008	0.002	0.001	0.000	0.000	0.000
5.50	-0.007	-0.341	-0.117	0.256	0.397	0.321	0.187	0.087	0.034	0.011	0.003	0.001	0.000	0.000	0.000
5.75	0.076	-0.318	-0.187	0.188	0.383	0.345	0.216	0.107	0.044	0.016	0.005	0.001	0.000	0.000	0.000
6.00	0.151	-0.277	-0.243	0.115	0.358	0.362	0.246	0.130	0.057	0.021	0.007	0.002	0.001	0.000	0.000
6.25	0.213	-0.221	-0.284	0.039	0.321	0.372	0.274	0.154	0.071	0.028	0.010	0.003	0.001	0.000	0.000
6.50	0.260	-0.154	-0.307	-0.035	0.275	0.374	0.300	0.180	0.088	0.037	0.013	0.004	0.001	0.000	0.000
6.75	0.289	-0.080	-0.313	-0.105	0.220	0.366	0.322	0.207	0.107	0.047	0.018	0.006	0.002	0.001	0.000
7.00	0.300	-0.005	-0.301	-0.168	0.158	0.348	0.339	0.234	0.128	0.059	0.024	0.008	0.003	0.001	0.000
7.25	0.292	0.069	-0.273	-0.219	0.092	0.320	0.350	0.259	0.151	0.073	0.031	0.011	0.004	0.001	0.000
7.50	0.266	0.135	-0.230	-0.258	0.024	0.283	0.354	0.283	0.174	0.089	0.039	0.015	0.005	0.002	0.000
7.75	0.225	0.192	-0.176	-0.282	-0.043	0.238	0.350	0.304	0.199	0.107	0.049	0.020	0.007	0.002	0.001
8.00	0.172	0.235	-0.113	-0.291	-0.105	0.186	0.338	0.321	0.223	0.126	0.061	0.026	0.010	0.003	0.001
8.25	0.109	0.262	-0.046	-0.284	-0.161	0.128	0.316	0.332	0.247	0.147	0.074	0.033	0.013	0.005	0.001
8.50	0.042	0.273	0.022	-0.263	-0.208	0.067	0.287	0.338	0.269	0.169	0.089	0.041	0.017	0.006	0.002

Funciones Exponenciales y Logarítmicas

$$\exp(\alpha)\exp(\beta)=\exp(\alpha+\beta)$$

$$\frac{\exp(\alpha)}{\exp(\beta)}=\exp(\alpha-\beta)$$

$$\ln(\alpha\cdot\beta)=\ln(\alpha)+\ln(\beta)$$

$$\ln\left(\frac{\alpha}{\beta}\right)=\ln(\alpha)-\ln(\beta)$$

$$\ln(\alpha)^{\beta}=\beta\cdot\ln(\alpha)$$

$$\log_b(N)=\log_a(N)\cdot\log_b(a)=\frac{\log_a(N)}{\log_a(b)}$$

Fórmulas de Euler

$$\exp(\pm j\theta)=\cos\theta\pm j\sin\theta$$

$$\cos\theta=\tfrac{1}{2}\left[\exp(j\theta)+\exp(-j\theta)\right]$$

$$\sin\theta=\tfrac{1}{2j}\left[\exp(j\theta)-\exp(-j\theta)\right]$$

Tabla A1.4 Sumatorias

$$\sum_{k=1}^{K}k=\frac{K(K+1)}{2}$$

$$\sum_{k=1}^{K}k^2=\frac{K(K+1)(2K+1)}{6}$$

$$\sum_{k=1}^{K}k^3=\frac{K^2(K+1)^2}{4}$$

$$\sum_{k=1}^{K}x^k=\frac{(x^K-1)}{x-1}$$

series y transformadas de Fourier

$$a_0=\frac{1}{T_0}\int_{t_1}^{t_1+T_0}x(t)dt \qquad a_n\cos n\omega_0 t+b_n\sin n\omega_0 t=C_n\cos(n\omega_0 t+\theta_n)$$

$$a_n=\frac{2}{T_0}\int_{t_1}^{t_1+T_0}x(t)\cos n\omega_0 t\,dt \qquad D_n=\frac{1}{T_0}\int_{T_0}x(t)\exp(-jn\omega_0 t)dt$$

$$b_n=\frac{2}{T_0}\int_{t_1}^{t_1+T_0}x(t)\sin n\omega_0 t\,dt \qquad x(t)=\int_{-\infty}^{\infty}X(f)\exp(j2\pi ft)df\leftrightarrow X(f)=\int_{-\infty}^{\infty}x(t)\exp(-j2\pi ft)dt$$

$$s_{DSB}(t)=A_c m(t)\cos(2\pi f_c t) \qquad s_{AM}(t)=\left[A_c+m(t)\right]\cos(\omega_c t)$$

$$s_{SSB}(t)=m(t)\cos 2\pi f_c t\mp m_h(t)\sin 2\pi f_c t \qquad s_{SSB+C}(t)=A_c\cos\omega_c t+\left[m(t)\cos\omega_c t\mp m_h(t)\sin\omega_c t\right]$$

$$S_{VSB}(f)=\left[M(f+f_c)+M(f-f_c)\right]H_i(f), \qquad H_o(f)=\frac{1}{H_i(f+f_c)+H_i(f-f_c)} \qquad |f|\le B$$

$$s_{QAM}(t)=A_c m_1(t)\cos\omega_c t+A_c m_2(t)\sin\omega_c t \qquad s_{PM}(t)=A_c\cos\left[\omega_c t+k_p m(t)\right]$$

$$s_{FM}(t)=A_c\cos\left(\omega_c t+2\pi k_f\int_0^t m(\tau)d\tau\right) \qquad s_{FM}(t)\approx A_c\cos\omega_c t-A_c\beta\sin\omega_m t\sin\omega_c t$$ de banda angosta

$$BW=2(\Delta f+f_m)=2f_m(1+\beta)$$ regla de Carson

$$\beta=\begin{cases}k_p a & para\ PM\\ \dfrac{k_f a}{f_m} & para\ FM\end{cases}\ ; \qquad \beta=\frac{\Delta f}{f_m}\ ; \qquad \Delta f=\left|f_i-f_c\right|_{\max}$$

Identidades Trigonométricas

$\exp(\pm j\theta) = \cos\theta \pm j\sin\theta$

$\cos\theta = \frac{1}{2}\left[\exp(j\theta) + \exp(-j\theta)\right]$

$\sin\theta = \frac{1}{2j}\left[\exp(j\theta) - \exp(-j\theta)\right]$

$\sin^2\theta + \cos^2\theta = 1$

$\cos^2\theta - \sin^2\theta = \cos 2\theta$

$\cos^2\theta = \frac{1}{2}\left[1 + \cos 2\theta\right]$

$\sin^2\theta = \frac{1}{2}\left[1 - \cos 2\theta\right]$

$2\sin\theta\cos\theta = \sin 2\theta$

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

$\tan(\alpha \pm \beta) = \dfrac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$

$\sin\alpha\sin\beta = \frac{1}{2}\left[\cos(\alpha - \beta) - \cos(\alpha + \beta)\right]$

$\cos\alpha\cos\beta = \frac{1}{2}\left[\cos(\alpha - \beta) + \cos(\alpha + \beta)\right]$

$\sin\alpha\cos\beta = \frac{1}{2}\left[\sin(\alpha - \beta) + \sin(\alpha + \beta)\right]$

Integrales

Integrales definidas

$$\int_0^\infty \frac{x\sin(ax)}{b^2 + x^2}\,dx = \frac{\pi}{2}\exp(-ab), \qquad a > 0, b > 0$$

$$\int_0^\infty \frac{\cos(ax)}{b^2 + x^2}\,dx = \frac{\pi}{2b}\exp(-ab), \qquad a > 0, b > 0$$

$$\int_0^\infty \frac{\cos(ax)}{\left(b^2 - x^2\right)^2}\,dx = \frac{\pi}{4b^3}\left[\sin(ab) - ab\cos(ab)\right], \qquad a > 0, b > 0$$

$$\int_0^\infty \mathrm{sinc}(x)\,dx = \int_0^\infty \mathrm{sinc}^2(x)\,dx = \frac{1}{2}$$

$$\int_0^\infty \exp(-ax)^2\,dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}, \qquad a > 0$$

$$\int_0^\infty x^2\exp(-ax)^2\,dx = \frac{1}{4a}\sqrt{\frac{\pi}{a}}, \qquad a > 0$$

Algunas transformadas de Fourier útiles.

$x(t)$	$X(f)$	
1	$\delta(f)$	
$\delta(t)$	1	
$\delta(t - t_0)$	$\exp(-j2\pi f t_0)$	
$\exp(j2\pi f_0 t)$	$\delta(f - f_0)$	
$\exp(-at)\ u(t)$	$\dfrac{1}{a + j2\pi f}$	$a > 0$
$\exp(-a\lvert t\rvert)$	$\dfrac{2a}{a^2 + (2\pi f)^2}$	$a > 0$
$\exp\left[-\pi\left(\frac{t}{T}\right)^2\right]$	$T\exp\left(-\pi(fT)^2\right)$	
$t\ \exp(-at)\ u(t)$	$\dfrac{1}{(a + j2\pi f)^2}$	$a > 0$
$u(t)$	$\dfrac{1}{2}\delta(f) + \dfrac{1}{j2\pi f}$	
$\cos(2\pi f_0 t)$	$\dfrac{1}{2}\left[\delta(f - f_0) + \delta(f + f_0)\right]$	
$\sin(2\pi f_0 t)$	$\dfrac{1}{2j}\left[\delta(f - f_0) - \delta(f + f_0)\right]$	
$rect\left(\frac{t}{T}\right)$	$T\ sinc(fT)$	
$sinc(2Wt)$	$\dfrac{1}{2W}rect\left(\dfrac{f}{2W}\right)$	
$\cos(2\pi f_0 t)\ rect\left(\frac{t}{T}\right)$	$\dfrac{T}{2}\left[sinc(f - f_0)T + sinc(f + f_0)T\right]$	
$\Delta\left(\frac{t}{T}\right)$	$T\ sinc^2(fT)$	
$\sum_{m=-\infty}^{\infty}\delta(t - mT_0)$	$\dfrac{1}{T_0}\sum_{n=-\infty}^{\infty}\delta\left(f - \dfrac{n}{T_0}\right)$	
$\mathrm{sgn}(t)$	$\dfrac{1}{j\pi f}$	
$\dfrac{1}{\pi t}$	$-j\,\mathrm{sgn}(f)$	

Integrales indefinidas

$$\int x\sin(ax)\,dx = \frac{1}{a^2}\left[\sin(ax) - ax\cos(ax)\right]$$

$$\int x\cos(ax)\,dx = \frac{1}{a^2}\left[\cos(ax) + ax\sin(ax)\right]$$

$$\int x\exp(ax)\,dx = \frac{1}{a^2}\exp(ax)(ax - 1)$$

$$\int x\exp(ax^2)\,dx = \frac{1}{2a}\exp(ax^2)$$

$$\int \exp(ax^2)\sin(bx)\,dx = \frac{1}{a^2 + b^2}\exp(ax)\left[a\sin(bx) - b\cos(bx)\right]$$

$$\int \exp(ax^2)\cos(bx)\,dx = \frac{1}{a^2 + b^2}\exp(ax)\left[a\cos(bx) + b\sin(bx)\right]$$

$$\int \frac{dx}{a^2 + b^2x^2} = \frac{1}{ab}\tan^{-1}\left(\frac{bx}{a}\right)$$

$$\int \frac{x^2\,dx}{a^2 + b^2x^2} = \frac{x}{b^2} - \frac{a}{b^3}\tan^{-1}\left(\frac{bx}{a}\right)$$

Fourier Transform Properties

Property Name	Property	
Linearity	$ax(t) + bv(t)$	$aX(\omega) + bV(\omega)$
Time Shift	$x(t - c)$	$e^{-j\omega c}X(\omega)$
Time Scaling	$x(at), \quad a \neq 0$	$\frac{1}{a}X(\omega/a), \quad a \neq 0$
Time Reversal	$x(-t)$	$X(-\omega)$ $\overline{X(\omega)}$ if $x(t)$ is real
Multiply by t^n	$t^n x(t), \quad n = 1, 2, 3, \ldots$	$j^n\dfrac{d^n}{d\omega^n}X(\omega), \quad n = 1, 2, 3, \ldots$
Multiply by Complex Exponential	$e^{j\omega_o t}x(t), \quad \omega_o$ real	$X(\omega - \omega_o), \quad \omega_o$ real
Multiply by Sine	$\sin(\omega_o t)x(t)$	$\dfrac{j}{2}\left[X(\omega + \omega_o) - X(\omega - \omega_o)\right]$
Multiply by Cosine	$\cos(\omega_o t)x(t)$	$\dfrac{1}{2}\left[X(\omega + \omega_o) + X(\omega - \omega_o)\right]$
Time Differentiation	$\dfrac{d^n}{dt^n}x(t), \quad n = 1, 2, 3, \ldots$	$(j\omega)^n X(\omega), \quad n = 1, 2, 3, \ldots$
Time Integration	$\int_{-\infty}^{t} x(\lambda)\,d\lambda$	$\dfrac{1}{j\omega}X(\omega) + \pi X(0)\delta(\omega)$
Convolution in Time	$x(t) * h(t)$	$X(\omega)H(\omega)$
Multiplication in Time	$x(t)w(t)$	$\dfrac{1}{2\pi}X(\omega) * W(\omega)$
Parseval's Theorem (General)	$\int_{-\infty}^{\infty} x(t)\overline{v(t)}\,dt = \dfrac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\overline{V(\omega)}\,d\omega$	
Parseval's Theorem (Energy)	$\int_{-\infty}^{\infty} x^2(t)\,dt = \dfrac{1}{2\pi}\int_{-\infty}^{\infty}\lvert X(\omega)\rvert^2\,d\omega$ if $x(t)$ is real $\int_{-\infty}^{\infty}\lvert x(t)\rvert^2\,dt = \dfrac{1}{2\pi}\int_{-\infty}^{\infty}\lvert X(\omega)\rvert^2\,d\omega$	
Duality: If $x(t) \leftrightarrow X(\omega)$	$X(t)$	$2\pi x(-\omega)$