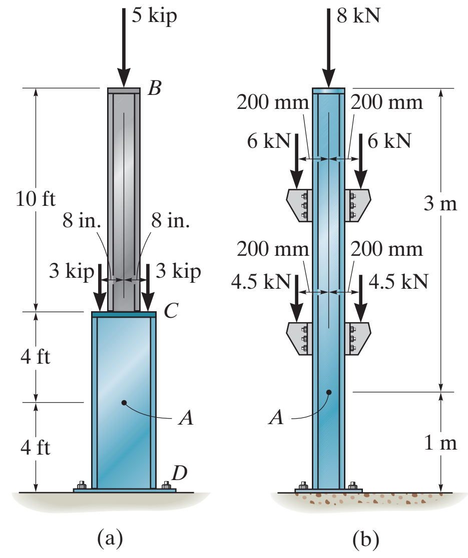
CONTENTS

1–1. Determine the resultant internal normal force acting on the cross section through point A in each column. In (a), segment BC weighs 180 lb/ft and segment CD weighs 250 lb/ft. In (b), the column has a mass of 200 kg/m.

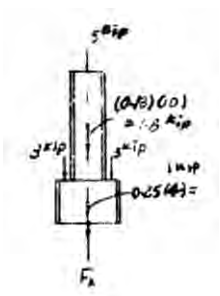


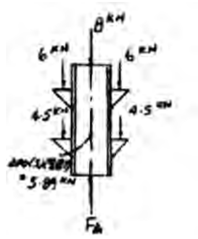
(a) $+\uparrow\ \Sigma F_y = 0;\qquad F_A - 1.0 - 3 - 3 - 1.8 - 5 = 0$

$$F_A = 13.8 \text{ kip}$$ **Ans.**

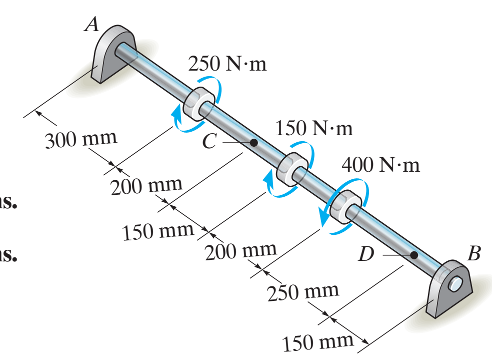
(b) $+\uparrow\ \Sigma F_y = 0;\qquad F_A - 4.5 - 4.5 - 5.89 - 6 - 6 - 8 = 0$

$$F_A = 34.9 \text{ kN}$$ **Ans.**





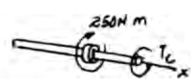
1–2. Determine the resultant internal torque acting on the cross sections through points C and D. The support bearings at A and B allow free turning of the shaft.



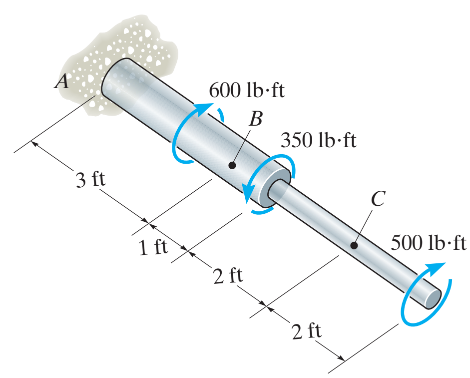
$\Sigma M_x = 0;\qquad T_C - 250 = 0$

$$T_C = 250 \text{ N} \cdot \text{m}$$ **Ans.**

$\Sigma M_x = 0;\qquad T_D = 0$ **Ans.**



1–3. Determine the resultant internal torque acting on the cross sections through points B and C.

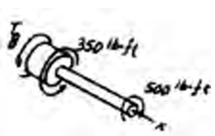


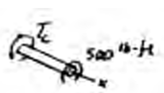
$\Sigma M_x = 0;\qquad T_B + 350 - 500 = 0$

$$T_B = 150 \text{ lb} \cdot \text{ft}$$ **Ans.**

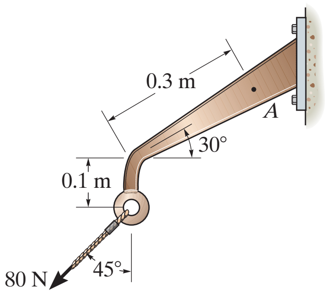
$\Sigma M_x = 0;\qquad T_C - 500 = 0$

$$T_C = 500 \text{ lb} \cdot \text{ft}$$ **Ans.**





***1–4.** A force of 80 N is supported by the bracket as shown. Determine the resultant internal loadings acting on the section through point A.



Equations of Equilibrium:

$\overset{+}{\nearrow}\Sigma F_{x'} = 0; \qquad N_A - 80 \cos 15° = 0$

$N_A = 77.3\text{ N}$ **Ans.**

$\nwarrow^{+}\ \Sigma F_{y'} = 0; \qquad V_A - 80 \sin 15° = 0$

$V_A = 20.7\text{ N}$ **Ans.**

$\circlearrowleft + \quad \Sigma M_A = 0; \qquad M_A + 80 \cos 45°(0.3 \cos 30°)$

$- 80 \sin 45°(0.1 + 0.3 \sin 30°) = 0$

$M_A = -0.555\text{ N}\cdot\text{m}$ **Ans.**

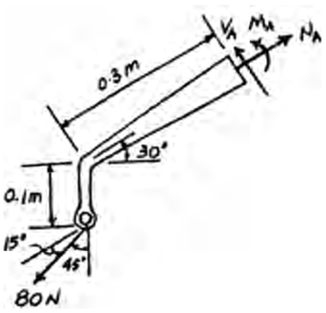
or

$\circlearrowleft + \quad \Sigma M_A = 0; \qquad M_A + 80 \sin 15°(0.3 + 0.1 \sin 30°)$

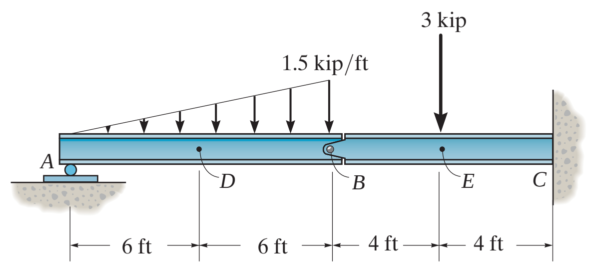
$-80 \cos 15°(0.1 \cos 30°) = 0$

$M_A = -0.555\text{ N}\cdot\text{m}$ **Ans.**

Negative sign indicates that M_A acts in the opposite direction to that shown on FBD.



•1–5. Determine the resultant internal loadings in the beam at cross sections through points D and E. Point E is just to the right of the 3-kip load.



Support Reactions: For member AB

$\zeta + \Sigma M_B = 0; \qquad 9.00(4) - A_y(12) = 0 \qquad A_y = 3.00 \text{ kip}$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad B_x = 0$

$+\uparrow \Sigma F_y = 0; \qquad B_y + 3.00 - 9.00 = 0 \qquad B_y = 6.00 \text{ kip}$

Equations of Equilibrium: For point D

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad N_D = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad 3.00 - 2.25 - V_D = 0$

$V_D = 0.750 \text{ kip}$ **Ans.**

$\zeta + \Sigma M_D = 0; \qquad M_D + 2.25(2) - 3.00(6) = 0$

$M_D = 13.5 \text{ kip} \cdot \text{ft}$ **Ans.**

Equations of Equilibrium: For point E

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad N_E = 0$ **Ans.**

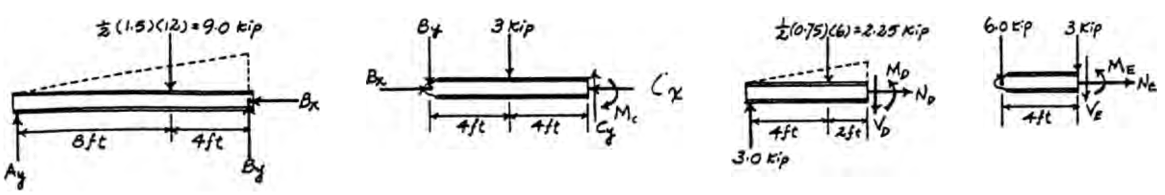
$+\uparrow \Sigma F_y = 0; \qquad -6.00 - 3 - V_E = 0$

$V_E = -9.00 \text{ kip}$ **Ans.**

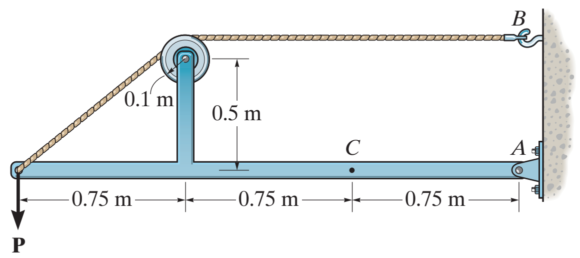
$\zeta + \Sigma M_E = 0; \qquad M_E + 6.00(4) = 0$

$M_E = -24.0 \text{ kip} \cdot \text{ft}$ **Ans.**

Negative signs indicate that M_E and V_E act in the opposite direction to that shown on FBD.



1–6. Determine the normal force, shear force, and moment at a section through point C. Take $P = 8$ kN.

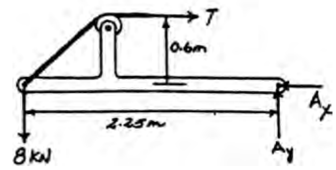


Support Reactions:

$\circlearrowleft + \Sigma M_A = 0;\quad 8(2.25) - T(0.6) = 0 \qquad T = 30.0\text{ kN}$

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad 30.0 - A_x = 0 \qquad A_x = 30.0\text{ kN}$

$+\uparrow \Sigma F_y = 0;\quad A_y - 8 = 0 \qquad A_y = 8.00\text{ kN}$



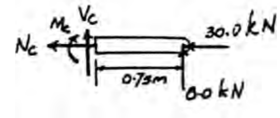
Equations of Equilibrium: For point C

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad -N_C - 30.0 = 0$

$N_C = -30.0\text{ kN}$ **Ans.**

$+\uparrow \Sigma F_y = 0;\quad V_C + 8.00 = 0$

$V_C = -8.00\text{ kN}$ **Ans.**

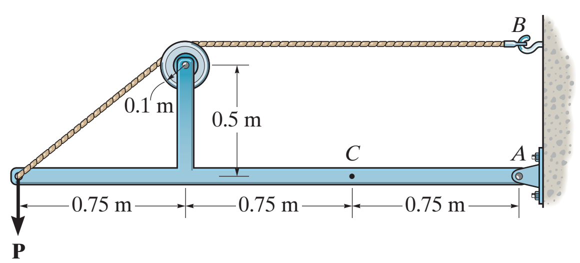


$\circlearrowleft + \Sigma M_C = 0;\quad 8.00(0.75) - M_C = 0$

$M_C = 6.00\text{ kN}\cdot\text{m}$ **Ans.**

Negative signs indicate that N_C and V_C act in the opposite direction to that shown on FBD.

1–7. The cable will fail when subjected to a tension of 2 kN. Determine the largest vertical load P the frame will support and calculate the internal normal force, shear force, and moment at the cross section through point C for this loading.



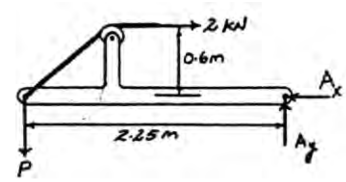
Support Reactions:

$\circlearrowleft + \Sigma M_A = 0;\quad P(2.25) - 2(0.6) = 0$

$P = 0.5333\text{ kN} = 0.533\text{ kN}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad 2 - A_x = 0 \qquad A_x = 2.00\text{ kN}$

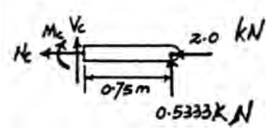
$+\uparrow \Sigma F_y = 0;\quad A_y - 0.5333 = 0 \qquad A_y = 0.5333\text{ kN}$



Equations of Equilibrium: For point C

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad -N_C - 2.00 = 0$

$N_C = -2.00\text{ kN}$ **Ans.**



$+\uparrow \Sigma F_y = 0;\quad V_C + 0.5333 = 0$

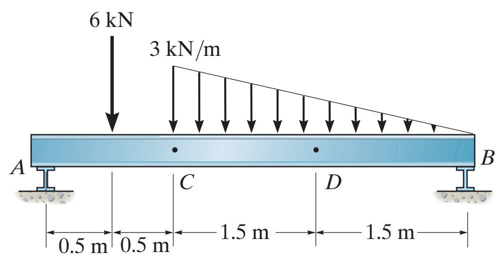
$V_C = -0.533\text{ kN}$ **Ans.**

$\circlearrowleft + \Sigma M_C = 0;\quad 0.5333(0.75) - M_C = 0$

$M_C = 0.400\text{ kN}\cdot\text{m}$ **Ans.**

Negative signs indicate that N_C and V_C act in the opposite direction to that shown on FBD.

***1–8.** Determine the resultant internal loadings on the cross section through point C. Assume the reactions at the supports A and B are vertical.



Referring to the FBD of the entire beam, Fig. *a*,

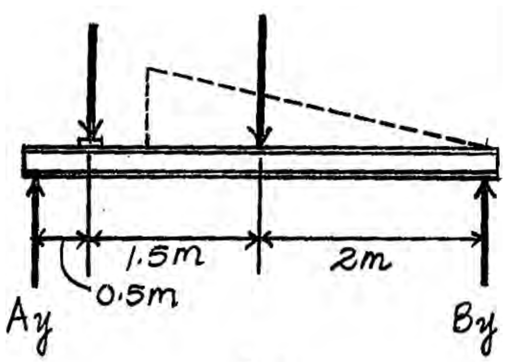
$\circlearrowleft + \Sigma M_B = 0; \quad -A_y(4) + 6(3.5) + \frac{1}{2}(3)(3)(2) = 0 \quad A_y = 7.50 \text{ kN}$

Referring to the FBD of this segment, Fig. *b*,

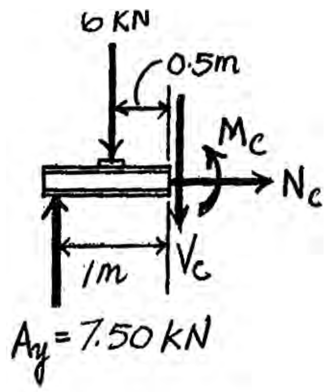
$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad N_C = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0; \quad 7.50 - 6 - V_C = 0 \quad V_C = 1.50 \text{ kN}$ **Ans.**

$\circlearrowleft + \Sigma M_C = 0; \quad M_C + 6(0.5) - 7.5(1) = 0 \quad M_C = 4.50 \text{ kN} \cdot \text{m}$ **Ans.**

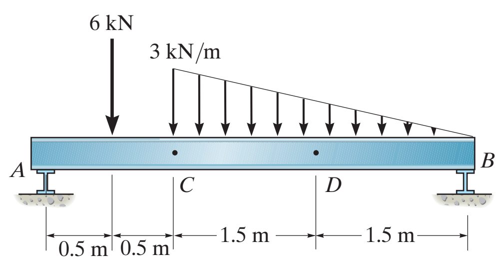


(a)



(b)

•1–9. Determine the resultant internal loadings on the cross section through point D. Assume the reactions at the supports A and B are vertical.



Referring to the FBD of the entire beam, Fig. *a*,

$\circlearrowleft + \Sigma M_A = 0; \quad B_y(4) - 6(0.5) - \frac{1}{2}(3)(3)(2) = 0 \quad B_y = 3.00 \text{ kN}$

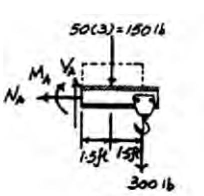
Referring to the FBD of this segment, Fig. *b*,

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad N_D = 0$ **Ans.**

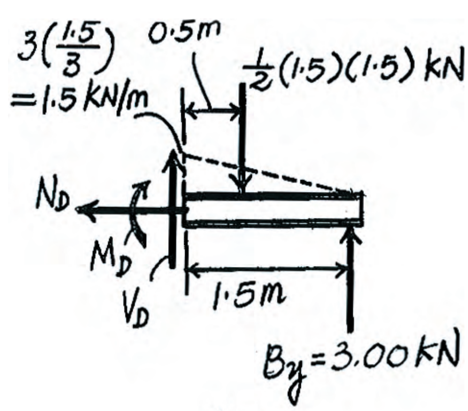
$+\uparrow \Sigma F_y = 0; \quad V_D - \frac{1}{2}(1.5)(1.5) + 3.00 = 0 \quad V_D = -1.875 \text{ kN}$ **Ans.**

$\circlearrowleft + \Sigma M_D = 0; \quad 3.00(1.5) - \frac{1}{2}(1.5)(1.5)(0.5) - M_D = 0 \quad M_D = 3.9375 \text{ kN} \cdot \text{m}$

$= 3.94 \text{ kN} \cdot \text{m}$ **Ans.**

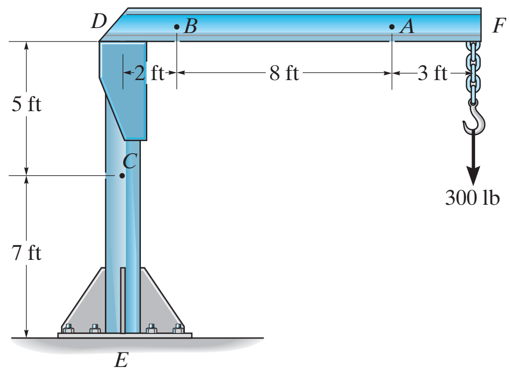


(a)

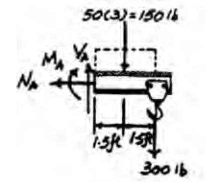


(b)

1–10. The boom DF of the jib crane and the column DE have a uniform weight of 50 lb/ft. If the hoist and load weigh 300 lb, determine the resultant internal loadings in the crane on cross sections through points A, B, and C.



Equations of Equilibrium: For point A



$\overset{+}{\leftarrow} \Sigma F_x = 0; \qquad N_A = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad V_A - 150 - 300 = 0$

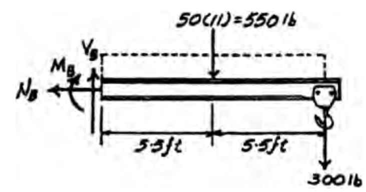
$V_A = 450 \text{ lb}$ **Ans.**

$\zeta + \Sigma M_A = 0; \qquad -M_A - 150(1.5) - 300(3) = 0$

$M_A = -1125 \text{ lb} \cdot \text{ft} = -1.125 \text{ kip} \cdot \text{ft}$ **Ans.**

Negative sign indicates that M_A acts in the opposite direction to that shown on FBD.

Equations of Equilibrium: For point B



$\overset{+}{\leftarrow} \Sigma F_x = 0; \qquad N_B = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad V_B - 550 - 300 = 0$

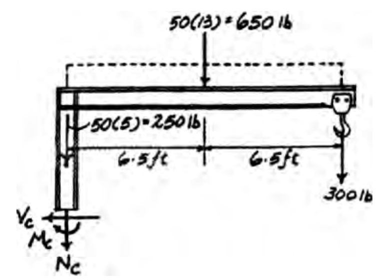
$V_B = 850 \text{ lb}$ **Ans.**

$\zeta + \Sigma M_B = 0; \qquad -M_B - 550(5.5) - 300(11) = 0$

$M_B = -6325 \text{ lb} \cdot \text{ft} = -6.325 \text{ kip} \cdot \text{ft}$ **Ans.**

Negative sign indicates that M_B acts in the opposite direction to that shown on FBD.

Equations of Equilibrium: For point C



$\overset{+}{\leftarrow} \Sigma F_x = 0; \qquad V_C = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad -N_C - 250 - 650 - 300 = 0$

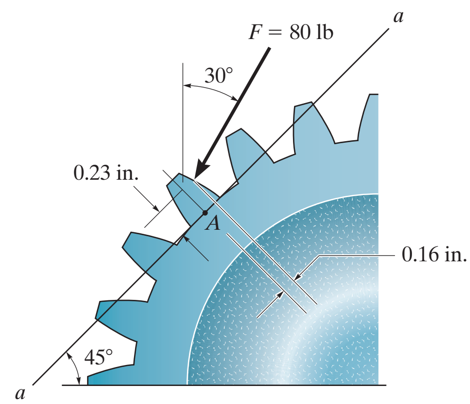
$N_C = -1200 \text{ lb} = -1.20 \text{ kip}$ **Ans.**

$\zeta + \Sigma M_C = 0; \qquad -M_C - 650(6.5) - 300(13) = 0$

$M_C = -8125 \text{ lb} \cdot \text{ft} = -8.125 \text{ kip} \cdot \text{ft}$ **Ans.**

Negative signs indicate that N_C and M_C act in the opposite direction to that shown on FBD.

1–11. The force $F = 80$ lb acts on the gear tooth. Determine the resultant internal loadings on the root of the tooth, i.e., at the centroid point A of section a–a.



Equations of Equilibrium: For section a–a

$$+\nearrow \Sigma F_{x'} = 0; \qquad V_A - 80\cos 15° = 0$$

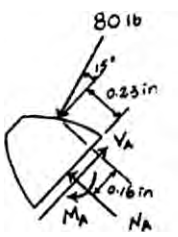
$$V_A = 77.3 \text{ lb} \qquad \textbf{Ans.}$$

$$\nwarrow^{+} \Sigma F_{y'} = 0; \qquad N_A - 80\sin 15° = 0$$

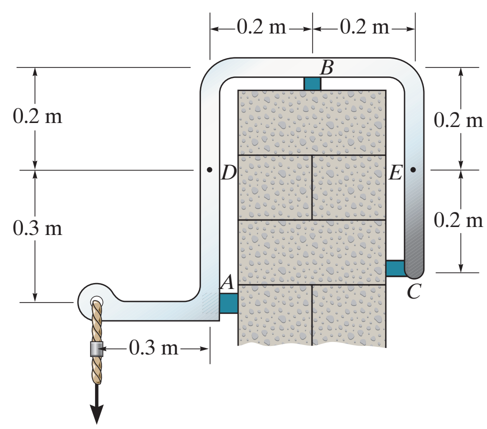
$$N_A = 20.7 \text{ lb} \qquad \textbf{Ans.}$$

$$\circlearrowleft + \Sigma M_A = 0; \qquad -M_A - 80\sin 15°(0.16) + 80\cos 15°(0.23) = 0$$

$$M_A = 14.5 \text{ lb} \cdot \text{in.} \qquad \textbf{Ans.}$$



***1–12.** The sky hook is used to support the cable of a scaffold over the side of a building. If it consists of a smooth rod that contacts the parapet of a wall at points A, B, and C, determine the normal force, shear force, and moment on the cross section at points D and E.



Support Reactions:

$$+\uparrow \Sigma F_y = 0; \qquad N_B - 18 = 0 \qquad N_B = 18.0 \text{ kN}$$

$$\circlearrowleft + \Sigma M_C = 0; \qquad 18(0.7) - 18.0(0.2) - N_A(0.1) = 0$$

$$N_A = 90.0 \text{ kN}$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad N_C - 90.0 = 0 \qquad N_C = 90.0 \text{ kN}$$

Equations of Equilibrium: For point D

$$\xrightarrow{+} \Sigma F_x = 0; \qquad V_D - 90.0 = 0$$

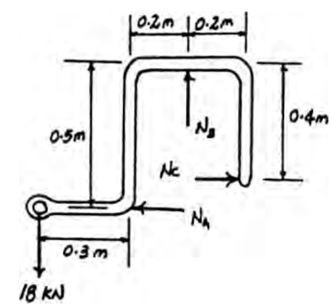
$$V_D = 90.0 \text{ kN} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad N_D - 18 = 0$$

$$N_D = 18.0 \text{ kN} \qquad \textbf{Ans.}$$

$$\circlearrowleft + \Sigma M_D = 0; \qquad M_D + 18(0.3) - 90.0(0.3) = 0$$

$$M_D = 21.6 \text{ kN} \cdot \text{m} \qquad \textbf{Ans.}$$



Equations of Equilibrium: For point E

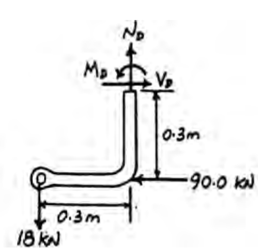
$$\xrightarrow{+} \Sigma F_x = 0; \qquad 90.0 - V_E = 0$$

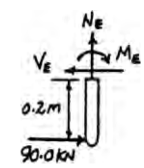
$$V_E = 90.0 \text{ kN} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad N_E = 0 \qquad \textbf{Ans.}$$

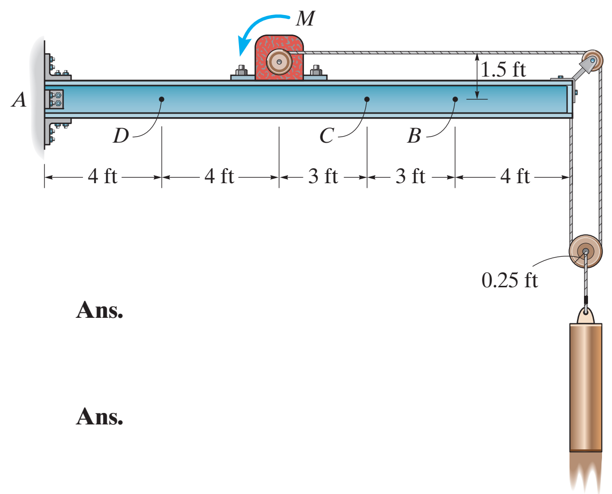
$$\circlearrowleft + \Sigma M_E = 0; \qquad 90.0(0.2) - M_E = 0$$

$$M_E = 18.0 \text{ kN} \cdot \text{m} \qquad \textbf{Ans.}$$





•1–13. The 800-lb load is being hoisted at a constant speed using the motor M, which has a weight of 90 lb. Determine the resultant internal loadings acting on the cross section through point B in the beam. The beam has a weight of 40 lb/ft and is fixed to the wall at A.



$\overset{+}{\rightarrow}\ \Sigma F_x = 0; \qquad -N_B - 0.4 = 0$

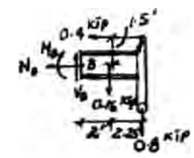
$N_B = -0.4\,\text{kip}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad V_B - 0.8 - 0.16 = 0$

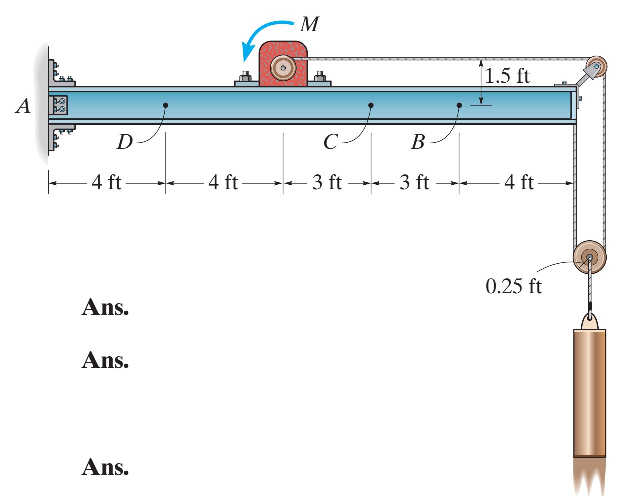
$V_B = 0.960\ \text{kip}$ **Ans.**

$\circlearrowleft + \ \Sigma M_B = 0; \qquad -M_B - 0.16(2) - 0.8(4.25) + 0.4(1.5) = 0$

$M_B = -3.12\ \text{kip} \cdot \text{ft}$ **Ans.**



1–14. Determine the resultant internal loadings acting on the cross section through points C and D of the beam in Prob. 1–13.



For point C:

$\overset{+}{\leftarrow}\ \Sigma F_x = 0; \qquad N_C + 0.4 = 0; \qquad N_C = -0.4\text{kip}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad V_C - 0.8 - 0.04\,(7) = 0; \qquad V_C = 1.08\ \text{kip}$ **Ans.**

$\circlearrowleft + \ \Sigma M_C = 0; \qquad -M_C - 0.8(7.25) - 0.04(7)(3.5) + 0.4(1.5) = 0$

$M_C = -6.18\ \text{kip} \cdot \text{ft}$ **Ans.**

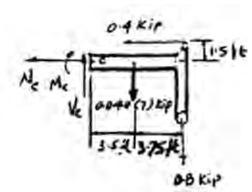
For point D:

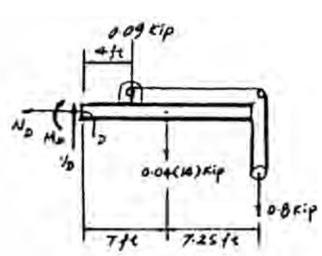
$\overset{+}{\leftarrow} \Sigma F_x = 0; \qquad N_D = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad V_D - 0.09 - 0.04(14) - 0.8 = 0; \qquad V_D = 1.45\ \text{kip}$ **Ans.**

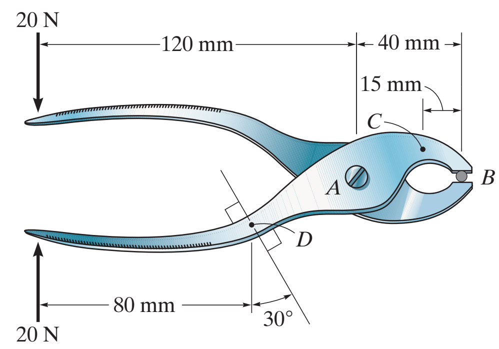
$\circlearrowleft + \Sigma M_D = 0; \qquad -M_D - 0.09(4) - 0.04(14)(7) - 0.8(14.25) = 0$

$M_D = -15.7\ \text{kip} \cdot \text{ft}$ **Ans.**





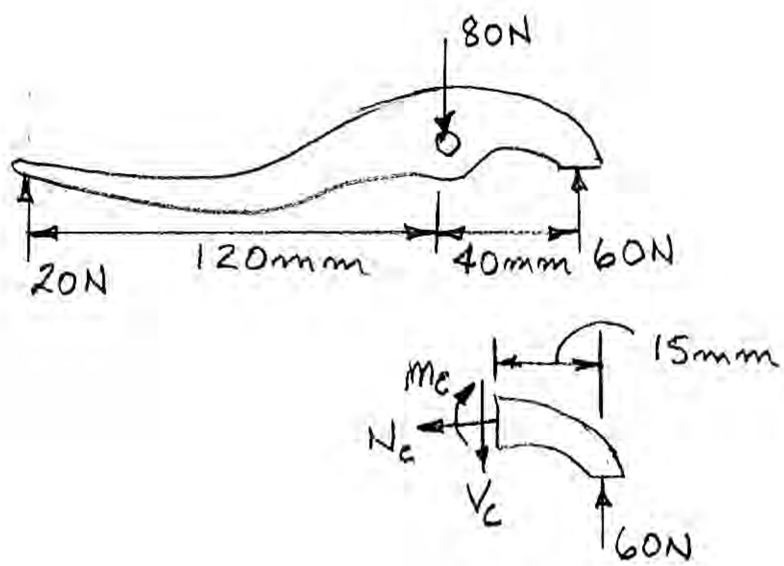
1–15. Determine the resultant internal loading on the cross section through point C of the pliers. There is a pin at A, and the jaws at B are smooth.



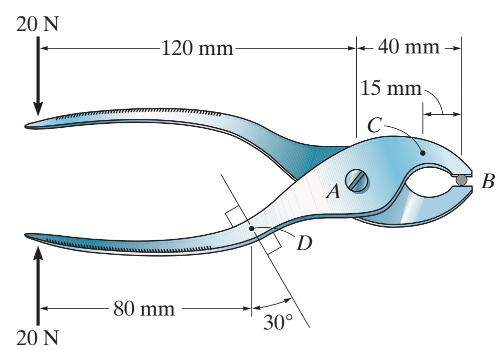
$+\uparrow \Sigma F_y = 0; \quad -V_C + 60 = 0; \quad V_C = 60\text{ N}$ **Ans.**

$\xrightarrow{+} \Sigma F_x = 0; \quad N_C = 0$ **Ans.**

$+\circlearrowleft \Sigma M_C = 0; \quad -M_C + 60(0.015) = 0; \quad M_C = 0.9\text{ N.m}$ **Ans.**



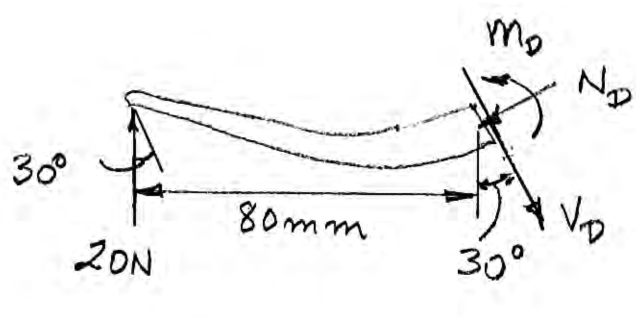
***1–16.** Determine the resultant internal loading on the cross section through point D of the pliers.



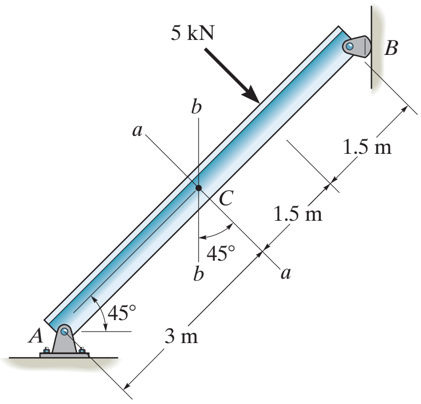
$\searrow + \Sigma F_y = 0; \quad V_D - 20\cos 30^\circ = 0; \quad V_D = 17.3\text{ N}$ **Ans.**

$+\swarrow \Sigma F_x = 0; \quad N_D - 20\sin 30^\circ = 0; \quad N_D = 10\text{ N}$ **Ans.**

$+\circlearrowleft \Sigma M_D = 0; \quad M_D - 20(0.08) = 0; \quad M_D = 1.60\text{ N.m}$ **Ans.**



•1–17. Determine resultant internal loadings acting on section a–a and section b–b. Each section passes through the centerline at point C.



Referring to the FBD of the entire beam, Fig. *a*,

$\circlearrowleft + \Sigma M_A = 0;\quad N_B \sin 45°(6) - 5(4.5) = 0 \qquad N_B = 5.303 \text{ kN}$

Referring to the FBD of this segment (section a–a), Fig. *b*,

$+\swarrow \Sigma F_{x'} = 0;\quad N_{a-a} + 5.303 \cos 45° = 0 \qquad N_{a-a} = -3.75 \text{ kN}$ **Ans.**

$+\nwarrow \Sigma F_{y'} = 0;\quad V_{a-a} + 5.303 \sin 45° - 5 = 0 \qquad V_{a-a} = 1.25 \text{ kN}$ **Ans.**

$\circlearrowleft + \Sigma M_C = 0;\quad 5.303 \sin 45°(3) - 5(1.5) - M_{a-a} = 0 \quad M_{a-a} = 3.75 \text{ kN} \cdot \text{m}$ **Ans.**

Referring to the FBD (section b–b) in Fig. *c*,

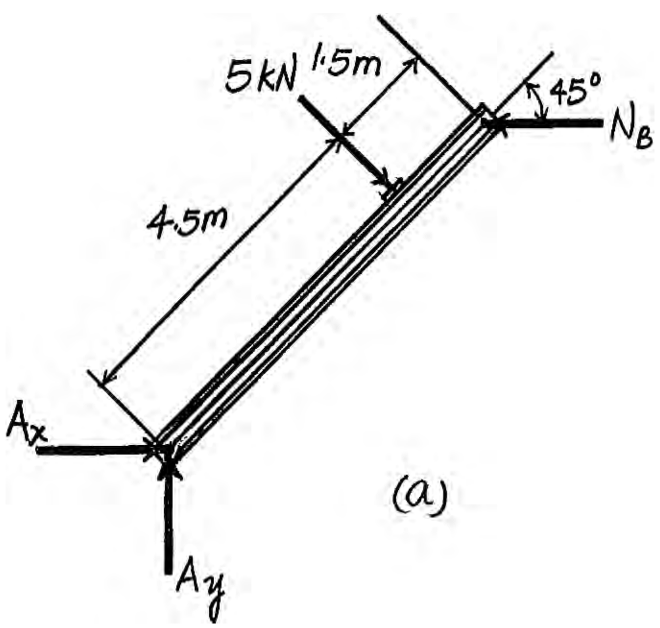
$\overset{+}{\leftarrow} \Sigma F_x = 0;\quad N_{b-b} - 5 \cos 45° + 5.303 = 0 \quad N_{b-b} = -1.768 \text{ kN}$

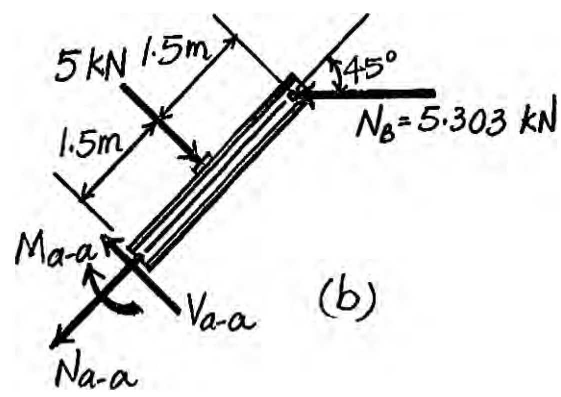
$= -1.77 \text{ kN}$ **Ans.**

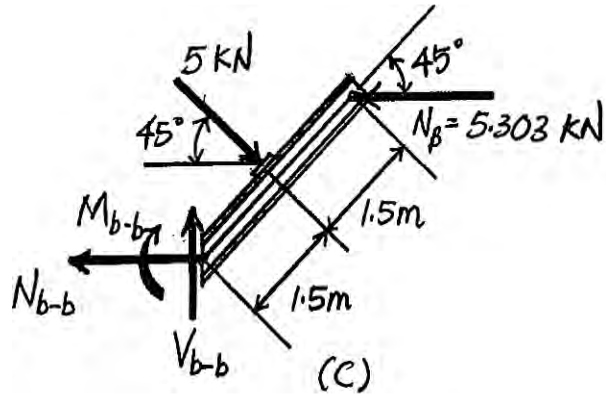
$+\uparrow \Sigma F_y = 0;\quad V_{b-b} - 5 \sin 45° = 0 \qquad V_{b-b} = 3.536 \text{ kN} = 3.54 \text{ kN}$ **Ans.**

$\circlearrowleft + \Sigma M_C = 0;\quad 5.303 \sin 45° (3) - 5(1.5) - M_{b-b} = 0$

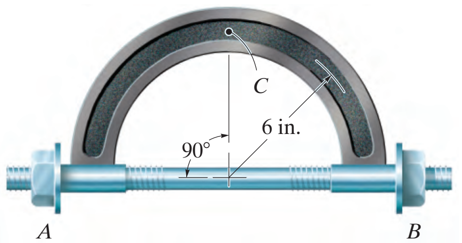
$M_{b-b} = 3.75 \text{ kN} \cdot \text{m}$ **Ans.**



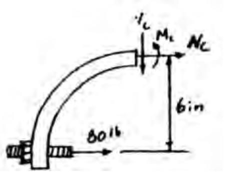




1–18. The bolt shank is subjected to a tension of 80 lb. Determine the resultant internal loadings acting on the cross section at point C.



Segment AC:

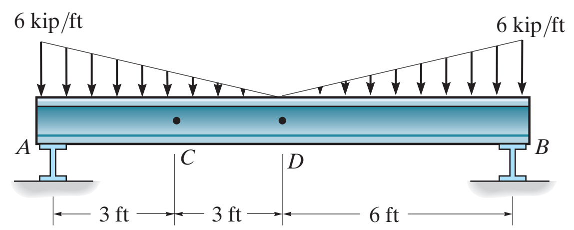


$\xrightarrow{+} \Sigma F_x = 0; \quad N_C + 80 = 0; \quad N_C = -80 \text{ lb}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \quad V_C = 0$ **Ans.**

$\circlearrowleft + \Sigma M_C = 0; \quad M_C + 80(6) = 0; \quad M_C = -480 \text{ lb} \cdot \text{in.}$ **Ans.**

1–19. Determine the resultant internal loadings acting on the cross section through point C. Assume the reactions at the supports A and B are vertical.



Referring to the FBD of the entire beam, Fig. *a*,

$$\circlearrowleft + \Sigma M_B = 0; \quad \frac{1}{2}(6)(6)(2) + \frac{1}{2}(6)(6)(10) - A_y(12) = 0 \quad A_y = 18.0 \text{ kip}$$

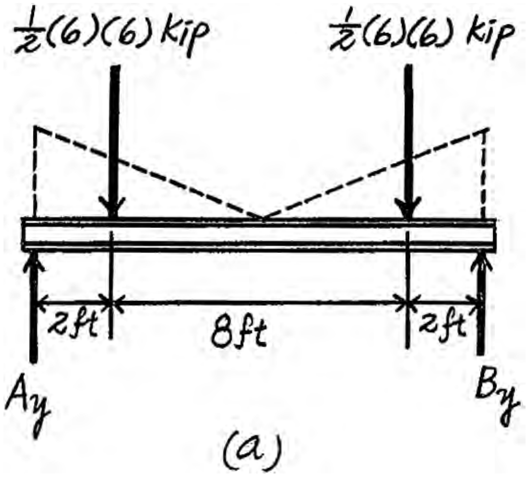
Referring to the FBD of this segment, Fig. *b*,

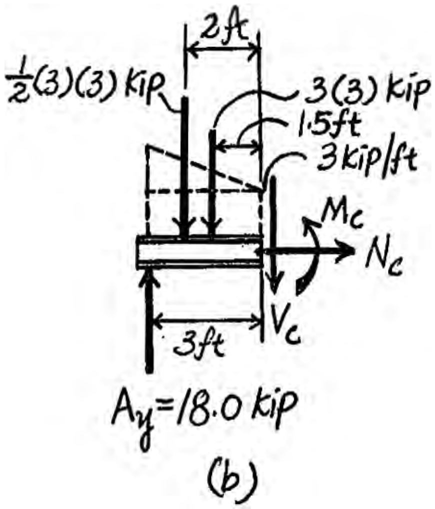
$\xrightarrow{+} \Sigma F_x = 0; \quad N_C = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0; \quad 18.0 - \frac{1}{2}(3)(3) - (3)(3) - V_C = 0 \quad V_C = 4.50 \text{ kip}$ **Ans.**

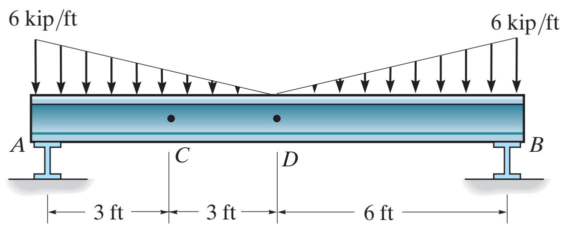
$$\circlearrowleft + \Sigma M_C = 0; \quad M_C + (3)(3)(1.5) + \frac{1}{2}(3)(3)(2) - 18.0(3) = 0$$

$M_C = 31.5 \text{ kip} \cdot \text{ft}$ **Ans.**





***1–20.** Determine the resultant internal loadings acting on the cross section through point D. Assume the reactions at the supports A and B are vertical.



Referring to the FBD of the entire beam, Fig. a,

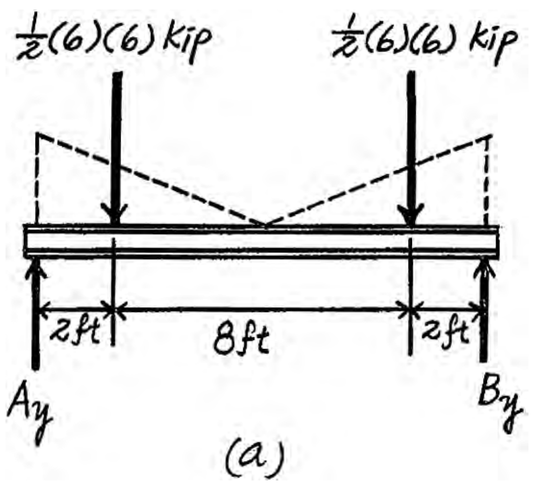
$\circlearrowleft + \Sigma M_B = 0;\quad \frac{1}{2}(6)(6)(2) + \frac{1}{2}(6)(6)(10) - A_y(12) = 0 \quad A_y = 18.0 \text{ kip}$

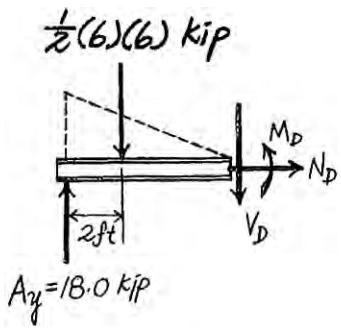
Referring to the FBD of this segment, Fig. b,

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad N_D = 0$ **Ans.**

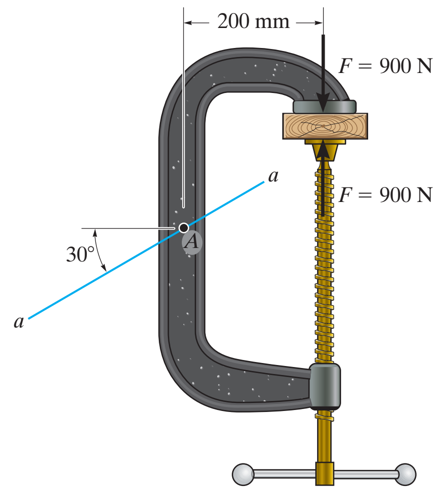
$+\uparrow \Sigma F_y = 0;\quad 18.0 - \frac{1}{2}(6)(6) - V_D = 0 \quad V_D = 0$ **Ans.**

$\circlearrowleft + \Sigma M_A = 0;\quad M_D - 18.0\,(2) = 0 \quad M_D = 36.0 \text{ kip} \cdot \text{ft}$ **Ans.**





•1–21. The forged steel clamp exerts a force of $F = 900$ N on the wooden block. Determine the resultant internal loadings acting on section a–a passing through point A.

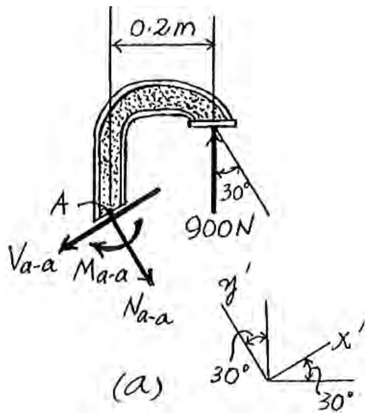


Internal Loadings: Referring to the free-body diagram of the section of the clamp shown in Fig. a,

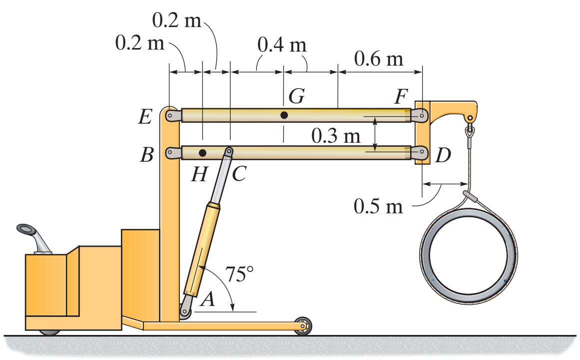
$\Sigma F_{y'} = 0;\quad 900 \cos 30° - N_{a-a} = 0 \quad N_{a-a} = 779 \text{ N}$ **Ans.**

$\Sigma F_{x'} = 0;\quad V_{a-a} - 900 \sin 30° = 0 \quad V_{a-a} = 450 \text{ N}$ **Ans.**

$\circlearrowleft + \Sigma M_A = 0;\quad 900(0.2) - M_{a-a} = 0 \quad M_{a-a} = 180 \text{ N} \cdot \text{m}$ **Ans.**



1–22. The floor crane is used to lift a 600-kg concrete pipe. Determine the resultant internal loadings acting on the cross section at G.



Support Reactions: We will only need to compute $\mathbf{F}_{EF}$ by writing the moment equation of equilibrium about D with reference to the free-body diagram of the hook, Fig. *a*.

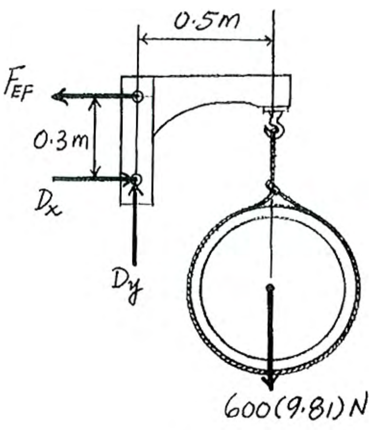
$\circlearrowleft + \Sigma M_D = 0; \qquad F_{EF}(0.3) - 600(9.81)(0.5) = 0 \qquad F_{EF} = 9810 \text{ N}$

Internal Loadings: Using the result for $\mathbf{F}_{EF}$, section FG of member EF will be considered. Referring to the free-body diagram, Fig. *b*,

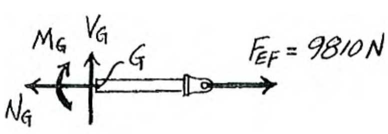
$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 9810 - N_G = 0 \qquad N_G = 9810 \text{ N} = 9.81 \text{ kN}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad V_G = 0$ **Ans.**

$\circlearrowleft + \Sigma M_G = 0; \qquad M_G = 0$ **Ans.**

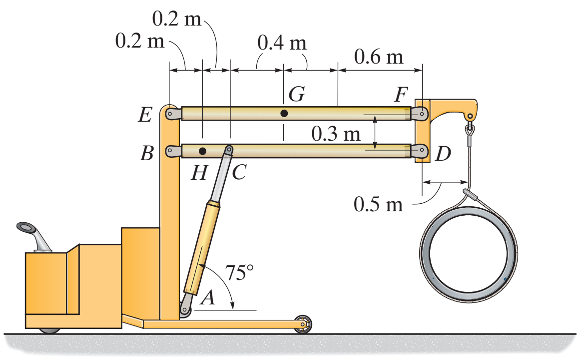


(a)



(b)

1–23. The floor crane is used to lift a 600-kg concrete pipe. Determine the resultant internal loadings acting on the cross section at H.



Support Reactions: Referring to the free-body diagram of the hook, Fig. *a*.

$$\circlearrowleft + \Sigma M_F = 0; \quad D_x(0.3) - 600(9.81)(0.5) = 0 \qquad D_x = 9810\text{ N}$$

$$+\uparrow \Sigma F_y = 0; \quad D_y - 600(9.81) = 0 \qquad D_y = 5886\text{ N}$$

Subsequently, referring to the free-body diagram of member BCD, Fig. b,

$$\circlearrowleft + \Sigma M_B = 0; \quad F_{AC} \sin 75°(0.4) - 5886(1.8) = 0 \qquad F_{AC} = 27\,421.36\text{ N}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad B_x + 27\,421.36 \cos 75° - 9810 = 0 \qquad B_x = 2712.83\text{ N}$$

$$+\uparrow \Sigma F_y = 0; \quad 27\,421.36 \sin 75° - 5886 - B_y = 0 \qquad B_y = 20\,601\text{ N}$$

Internal Loadings: Using the results of $\mathbf{B}_x$ and $\mathbf{B}_y$, section BH of member BCD will be considered. Referring to the free-body diagram of this part shown in Fig. c,

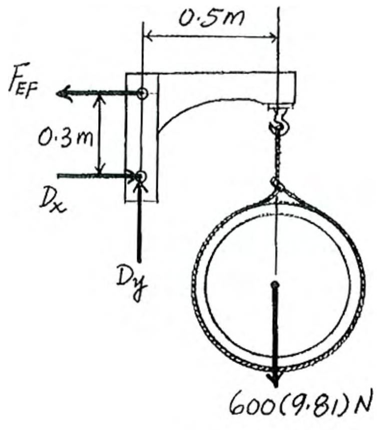
$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad N_H + 2712.83 = 0 \qquad N_H = -2712.83\text{ N} = -2.71\text{ kN} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \quad -V_H - 2060 = 0 \qquad V_H = -20601\text{ N} = -20.6\text{ kN} \qquad \textbf{Ans.}$$

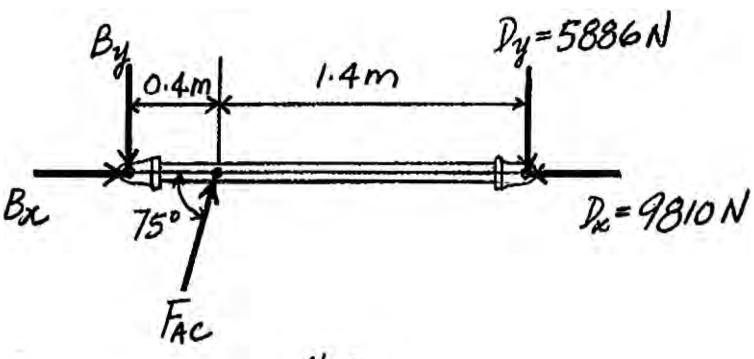
$$\circlearrowleft + \Sigma M_D = 0; \quad M_H + 20601(0.2) = 0 \qquad M_H = -4120.2\text{ N}\cdot\text{m}$$

$$= -4.12\text{ kN}\cdot\text{m} \qquad \textbf{Ans.}$$

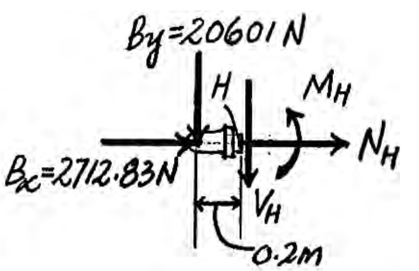
The negative signs indicates that N_H, V_H, and M_H act in the opposite sense to that shown on the free-body diagram.



(a)

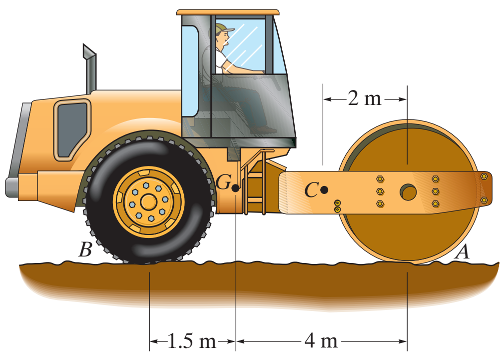


(b)



(c)

***1–24.** The machine is moving with a constant velocity. It has a total mass of 20 Mg, and its center of mass is located at G, excluding the front roller. If the front roller has a mass of 5 Mg, determine the resultant internal loadings acting on point C of each of the two side members that support the roller. Neglect the mass of the side members. The front roller is free to roll.



Support Reactions: We will only need to compute $\mathbf{N}_A$ by writing the moment equation of equilibrium about B with reference to the free-body diagram of the steamroller, Fig. *a*.

$$\circlearrowleft + \Sigma M_B = 0; \quad N_A(5.5) - 20(10^3)(9.81)(1.5) = 0 \qquad N_A = 53.51(10^3)\text{ N}$$

Internal Loadings: Using the result for N_A, the free-body diagram of the front roller shown in Fig. *b* will be considered.

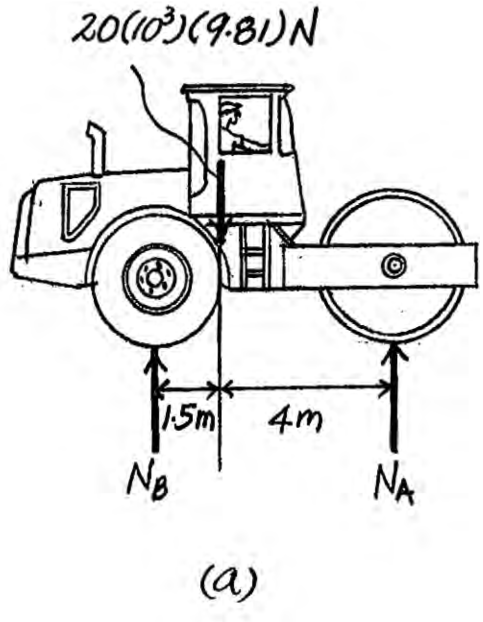
$$\overset{+}{\leftarrow} \Sigma F_x = 0; \quad 2N_C = 0 \qquad N_C = 0 \qquad \textbf{Ans.}$$

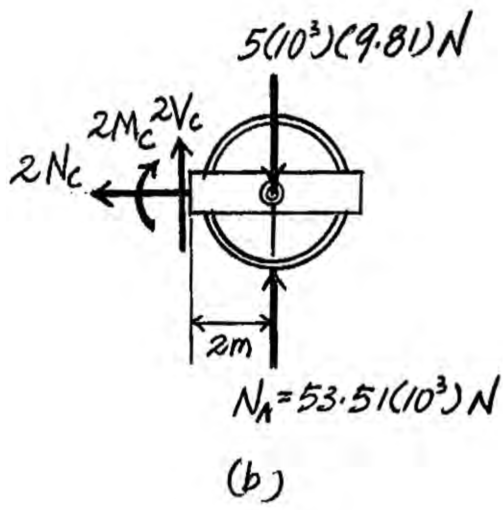
$$+\uparrow \Sigma F_y = 0; \quad 2V_C + 53.51(10^3) - 5(10^3)(9.81) = 0 \qquad V_C = -2229.55\text{ N}$$

$$= -2.23\text{ kN} \qquad \textbf{Ans.}$$

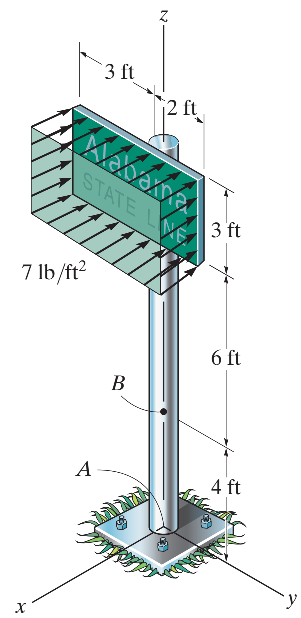
$$\circlearrowleft + \Sigma M_C = 0; \quad 53.51(10^3)(2) - 5(10^3)(9.81)(2) - 2M_C = 0 \quad M_C = 4459.10\text{ N}\cdot\text{m}$$

$$= 4.46\text{ kN}\cdot\text{m} \quad \textbf{Ans.}$$





•1–25. Determine the resultant internal loadings acting on the cross section through point B of the signpost. The post is fixed to the ground and a uniform pressure of 7 lb/ft^2 acts perpendicular to the face of the sign.



$\Sigma F_x = 0; \quad (V_B)_x - 105 = 0; \quad (V_B)_x = 105 \text{ lb}$ **Ans.**

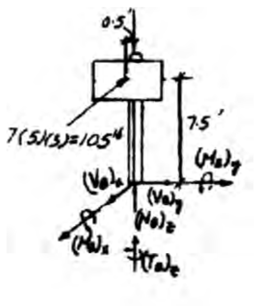
$\Sigma F_y = 0; \quad (V_B)_y = 0$ **Ans.**

$\Sigma F_z = 0; \quad (N_B)_z = 0$ **Ans.**

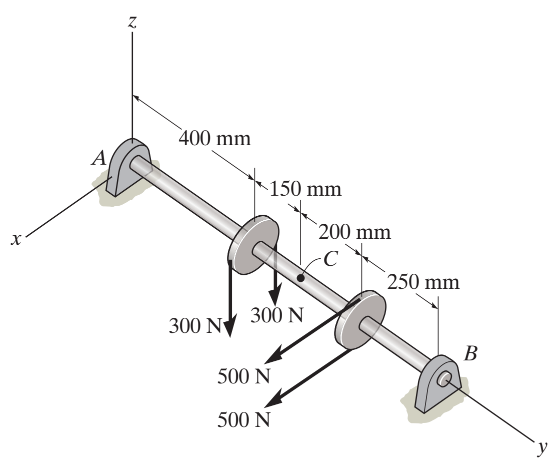
$\Sigma M_x = 0; \quad (M_B)_x = 0$ **Ans.**

$\Sigma M_y = 0; \quad (M_B)_y - 105(7.5) = 0; \quad (M_B)_y = 788 \text{ lb} \cdot \text{ft}$ **Ans.**

$\Sigma M_z = 0; \quad (T_B)_z - 105(0.5) = 0; \quad (T_B)_z = 52.5 \text{ lb} \cdot \text{ft}$ **Ans.**



1–26. The shaft is supported at its ends by two bearings A and B and is subjected to the forces applied to the pulleys fixed to the shaft. Determine the resultant internal loadings acting on the cross section located at point C. The 300-N forces act in the $-z$ direction and the 500-N forces act in the $+x$ direction. The journal bearings at A and B exert only x and z components of force on the shaft.



$\Sigma F_x = 0; \quad (V_C)_x + 1000 - 750 = 0; \quad (V_C)_x = -250 \text{ N}$ **Ans.**

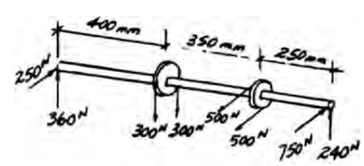
$\Sigma F_y = 0; \quad (N_C)_y = 0$ **Ans.**

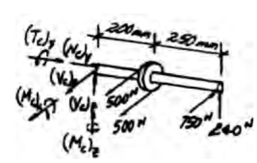
$\Sigma F_z = 0; \quad (V_C)_z + 240 = 0; \quad (V_C)_z = -240 \text{ N}$ **Ans.**

$\Sigma M_x = 0; \quad (M_C)_x + 240(0.45) = 0; \quad (M_C)_x = -108 \text{ N} \cdot \text{m}$ **Ans.**

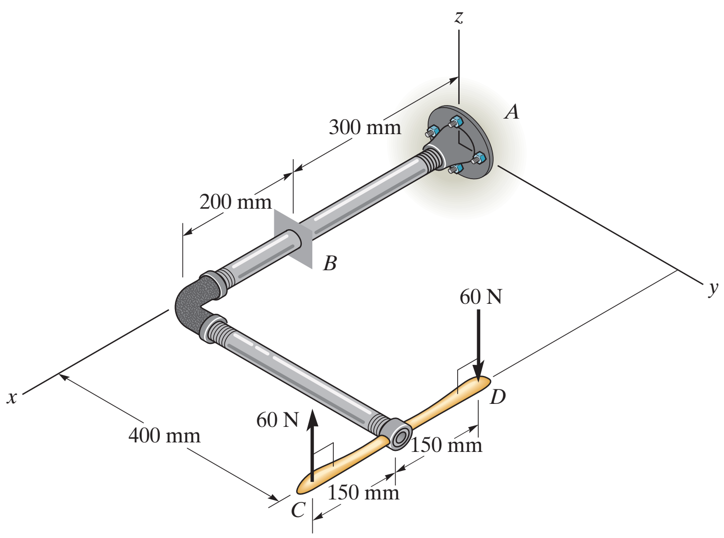
$\Sigma M_y = 0; \quad (T_C)_y = 0$ **Ans.**

$\Sigma M_z = 0; \quad (M_C)_z - 1000(0.2) + 750(0.45) = 0; \quad (M_C)_z = -138 \text{ N} \cdot \text{m}$ **Ans.**





1–27. The pipe has a mass of 12 kg/m. If it is fixed to the wall at A, determine the resultant internal loadings acting on the cross section at B. Neglect the weight of the wrench CD.



$\Sigma F_x = 0; \qquad (N_B)_x = 0$ **Ans.**

$\Sigma F_y = 0; \qquad (V_B)_y = 0$ **Ans.**

$\Sigma F_z = 0; \qquad (V_B)_z - 60 + 60 - (0.2)(12)(9.81) - (0.4)(12)(9.81) = 0$

$(V_B)_z = 70.6 \text{ N}$ **Ans.**

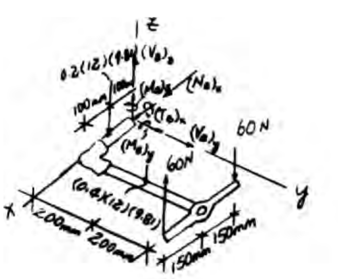
$\Sigma M_x = 0; \qquad (T_B)_x + 60(0.4) - 60(0.4) - (0.4)(12)(9.81)(0.2) = 0$

$(T_B)_x = 9.42 \text{ N} \cdot \text{m}$ **Ans.**

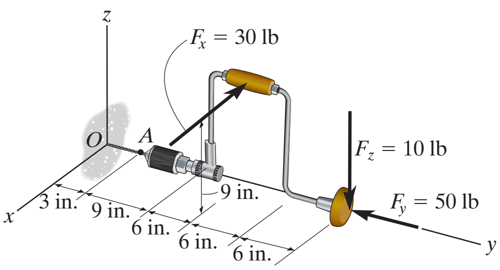
$\Sigma M_y = 0; \qquad (M_B)_y + (0.2)(12)(9.81)(0.1) + (0.4)(12)(9.81)(0.2) - 60(0.3) = 0$

$(M_B)_y = 6.23 \text{ N} \cdot \text{m}$ **Ans.**

$\Sigma M_z = 0; \qquad (M_B)_z = 0$ **Ans.**



***1–28.** The brace and drill bit is used to drill a hole at O. If the drill bit jams when the brace is subjected to the forces shown, determine the resultant internal loadings acting on the cross section of the drill bit at A.



Internal Loading: Referring to the free-body diagram of the section of the drill and brace shown in Fig. *a*,

$\Sigma F_x = 0;\quad (V_A)_x - 30 = 0 \qquad (V_A)_x = 30 \text{ lb}$ **Ans.**

$\Sigma F_y = 0;\quad (N_A)_y - 50 = 0 \qquad (N_A)_y = 50 \text{ lb}$ **Ans.**

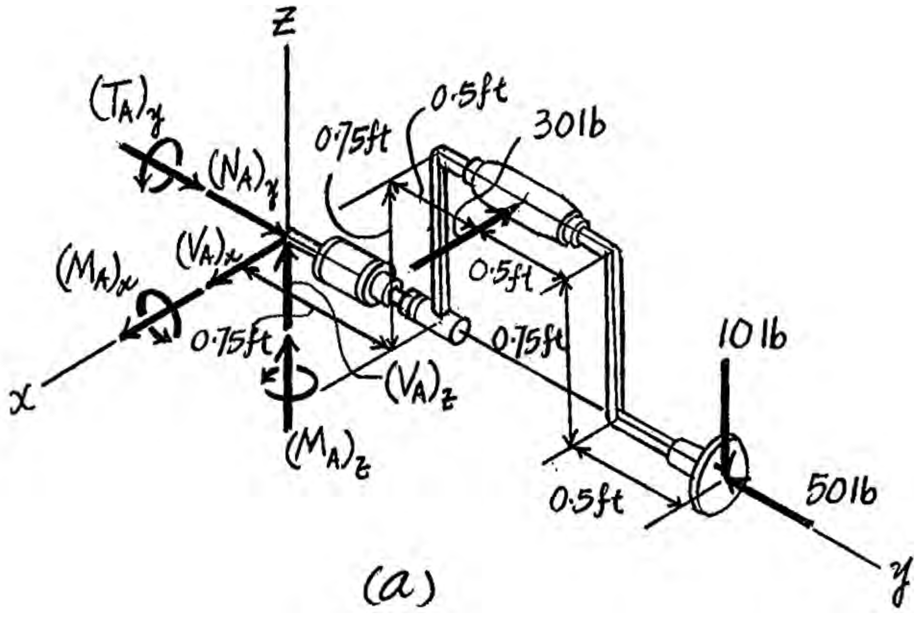
$\Sigma F_z = 0;\quad (V_A)_z - 10 = 0 \qquad (V_A)_z = 10 \text{ lb}$ **Ans.**

$\Sigma M_x = 0;\quad (M_A)_x - 10(2.25) = 0 \qquad (M_A)_x = 22.5 \text{ lb}\cdot\text{ft}$ **Ans.**

$\Sigma M_y = 0;\quad (T_A)_y - 30(0.75) = 0 \qquad (T_A)_y = 22.5 \text{ lb}\cdot\text{ft}$ **Ans.**

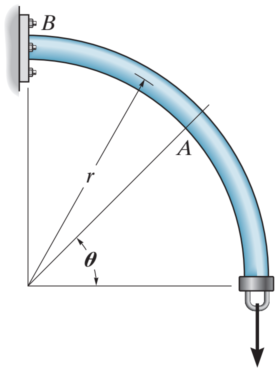
$\Sigma M_z = 0;\quad (M_A)_z + 30(1.25) = 0 \qquad (M_A)_z = -37.5 \text{ lb}\cdot\text{ft}$ **Ans.**

The negative sign indicates that $(M_A)_Z$ acts in the opposite sense to that shown on the free-body diagram.



(a)

•1–29. The curved rod has a radius r and is fixed to the wall at B. Determine the resultant internal loadings acting on the cross section through A which is located at an angle θ from the horizontal.



Equations of Equilibrium: For point A

$\searrow + \Sigma F_x = 0;\quad P\cos\theta - N_A = 0$

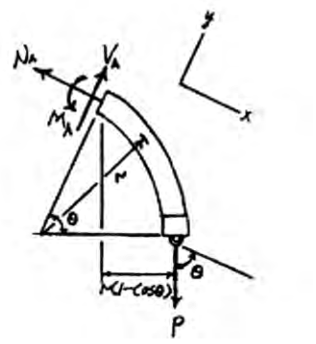
$$N_A = P\cos\theta \qquad \textbf{Ans.}$$

$\nearrow + \Sigma F_y = 0;\quad V_A - P\sin\theta = 0$

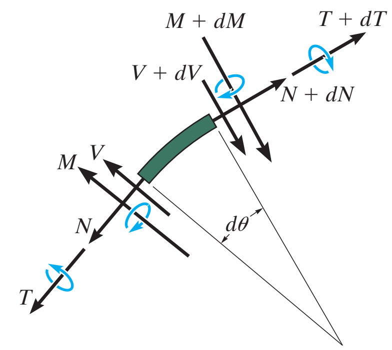
$$V_A = P\sin\theta \qquad \textbf{Ans.}$$

$\circlearrowleft + \Sigma M_A = 0;\quad M_A - P[r(1 - \cos\theta)] = 0$

$$M_A = Pr(1 - \cos\theta) \qquad \textbf{Ans.}$$



1–30. A differential element taken from a curved bar is shown in the figure. Show that $dN/d\theta = V$, $dV/d\theta = -N$, $dM/d\theta = -T$, and $dT/d\theta = M$.



$\Sigma F_x = 0;$

$$N \cos\frac{d\theta}{2} + V \sin\frac{d\theta}{2} - (N + dN)\cos\frac{d\theta}{2} + (V + dV)\sin\frac{d\theta}{2} = 0 \qquad \textbf{(1)}$$

$\Sigma F_y = 0;$

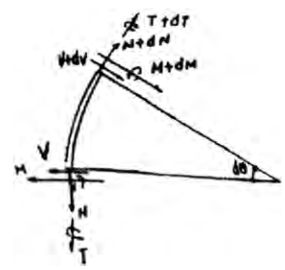
$$N \sin\frac{d\theta}{2} - V \cos\frac{d\theta}{2} + (N + dN)\sin\frac{d\theta}{2} + (V + dV)\cos\frac{d\theta}{2} = 0 \qquad \textbf{(2)}$$

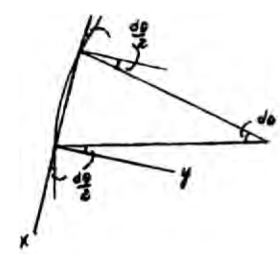
$\Sigma M_x = 0;$

$$T \cos\frac{d\theta}{2} + M \sin\frac{d\theta}{2} - (T + dT)\cos\frac{d\theta}{2} + (M + dM)\sin\frac{d\theta}{2} = 0 \qquad \textbf{(3)}$$

$\Sigma M_y = 0;$

$$T \sin\frac{d\theta}{2} - M \cos\frac{d\theta}{2} + (T + dT)\sin\frac{d\theta}{2} + (M + dM)\cos\frac{d\theta}{2} = 0 \qquad \textbf{(4)}$$





Since $\frac{d\theta}{2}$ is can add, then $\sin\frac{d\theta}{2} = \frac{d\theta}{2}, \cos\frac{d\theta}{2} = 1$

Eq. (1) becomes $V d\theta - dN + \frac{dV d\theta}{2} = 0$

Neglecting the second order term, $V d\theta - dN = 0$

$\frac{dN}{d\theta} = V$ QED

Eq. (2) becomes $N d\theta + dV + \frac{dN d\theta}{2} = 0$

Neglecting the second order term, $N d\theta + dV = 0$

$\frac{dV}{d\theta} = -N$ QED

Eq. (3) becomes $M d\theta - dT + \frac{dM d\theta}{2} = 0$

Neglecting the second order term, $M d\theta - dT = 0$

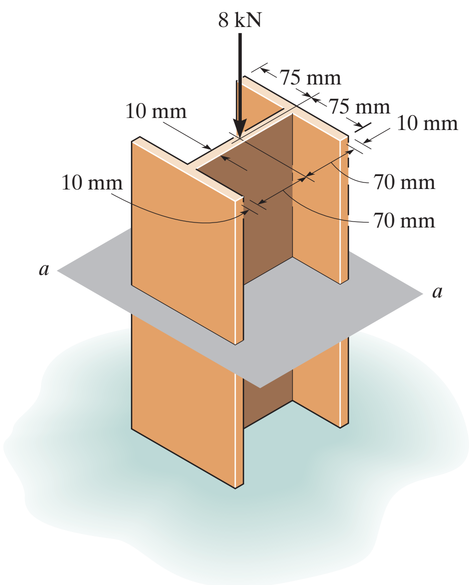
$\frac{dT}{d\theta} = M$ QED

Eq. (4) becomes $T d\theta + dM + \frac{dT d\theta}{2} = 0$

Neglecting the second order term, $T d\theta + dM = 0$

$\frac{dM}{d\theta} = -T$ QED

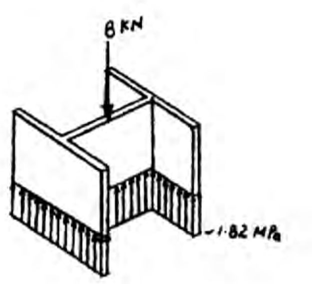
1–31. The column is subjected to an axial force of 8 kN, which is applied through the centroid of the cross-sectional area. Determine the average normal stress acting at section *a–a*. Show this distribution of stress acting over the area's cross section.



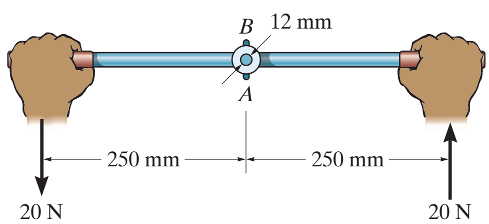
$$A = (2)(150)(10) + (140)(10)$$

$$= 4400 \text{ mm}^2 = 4.4\,(10^{-3})\text{ m}^2$$

$$\sigma = \frac{P}{A} = \frac{8\,(10^3)}{4.4\,(10^{-3})} = 1.82 \text{ MPa}$$ **Ans.**

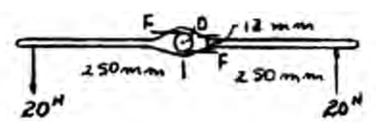


***1–32.** The lever is held to the fixed shaft using a tapered pin *AB*, which has a mean diameter of 6 mm. If a couple is applied to the lever, determine the average shear stress in the pin between the pin and lever.

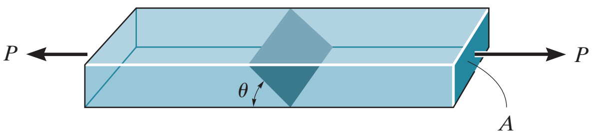


$$\circlearrowleft + \Sigma M_O = 0; \quad -F(12) + 20(500) = 0; \qquad F = 833.33 \text{ N}$$

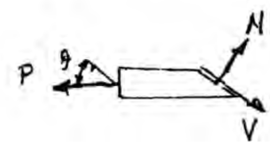
$$\tau_{\text{avg}} = \frac{V}{A} = \frac{833.33}{\frac{\pi}{4}\left(\frac{6}{1000}\right)^2} = 29.5 \text{ MPa}$$ **Ans.**



•1–33. The bar has a cross-sectional area A and is subjected to the axial load P. Determine the average normal and average shear stresses acting over the shaded section, which is oriented at θ from the horizontal. Plot the variation of these stresses as a function of θ $(0 \le \theta \le 90°)$.



Equations of Equilibrium:



$$\searrow + \Sigma F_x = 0; \qquad V - P\cos\theta = 0 \qquad V = P\cos\theta$$

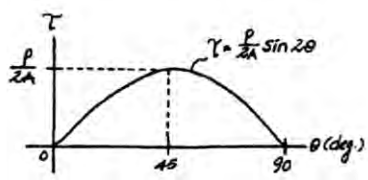
$$\nearrow + \Sigma F_y = 0; \qquad N - P\sin\theta = 0 \qquad N = P\sin\theta$$

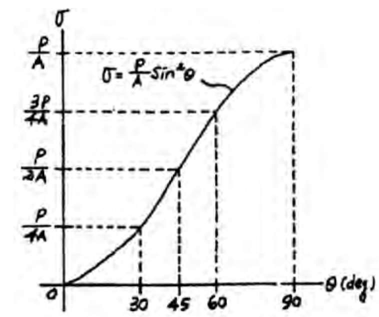
Average Normal Stress and Shear Stress: Area at θ plane, $A' = \dfrac{A}{\sin\theta}$.

$$\sigma = \frac{N}{A'} = \frac{P\sin\theta}{\frac{A}{\sin\theta}} = \frac{P}{A}\sin^2\theta \qquad \textbf{Ans.}$$

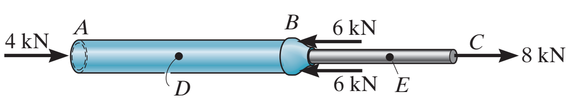
$$\tau_{\text{avg}} = \frac{V}{A'} = \frac{P\cos\theta}{\frac{A}{\sin\theta}}$$

$$= \frac{P}{A}\sin\theta\cos\theta = \frac{P}{2A}\sin 2\theta \qquad \textbf{Ans.}$$

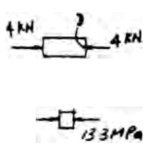




1–34. The built-up shaft consists of a pipe AB and solid rod BC. The pipe has an inner diameter of 20 mm and outer diameter of 28 mm. The rod has a diameter of 12 mm. Determine the average normal stress at points D and E and represent the stress on a volume element located at each of these points.

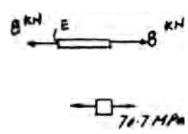


At D:



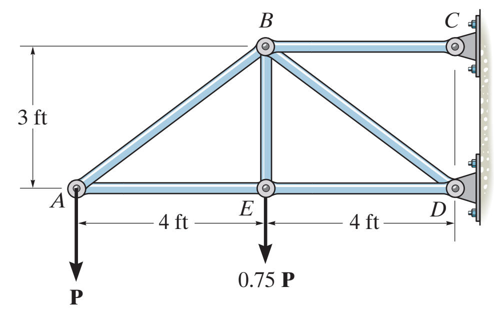
$$\sigma_D = \frac{P}{A} = \frac{4(10^3)}{\frac{\pi}{4}(0.028^2 - 0.02^2)} = 13.3 \text{ MPa} \quad \text{(C)} \qquad \textbf{Ans.}$$

At E:

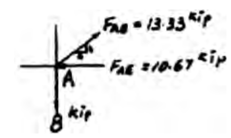


$$\sigma_E = \frac{P}{A} = \frac{8(10^3)}{\frac{\pi}{4}(0.012^2)} = 70.7 \text{ MPa} \quad \text{(T)} \qquad \textbf{Ans.}$$

1–35. The bars of the truss each have a cross-sectional area of 1.25 in^2. Determine the average normal stress in each member due to the loading $P = 8$ kip. State whether the stress is tensile or compressive.



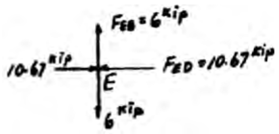
Joint A:



$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{13.33}{1.25} = 10.7 \text{ ksi} \qquad \text{(T)}$$ **Ans.**

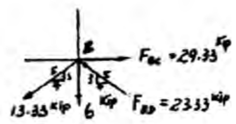
$$\sigma_{AE} = \frac{F_{AE}}{A_{AE}} = \frac{10.67}{1.25} = 8.53 \text{ ksi} \qquad \text{(C)}$$ **Ans.**

Joint E:



$$\sigma_{ED} = \frac{F_{ED}}{A_{ED}} = \frac{10.67}{1.25} = 8.53 \text{ ksi} \qquad \text{(C)}$$ **Ans.**

$$\sigma_{EB} = \frac{F_{EB}}{A_{EB}} = \frac{6.0}{1.25} = 4.80 \text{ ksi} \qquad \text{(T)}$$ **Ans.**

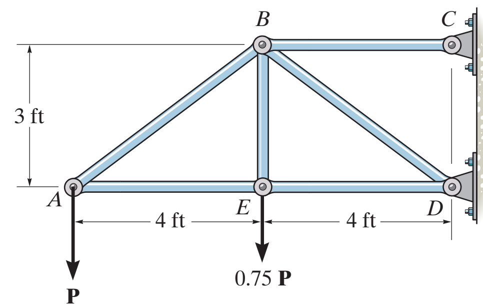


Joint B:

$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{29.33}{1.25} = 23.5 \text{ ksi} \qquad \text{(T)}$$ **Ans.**

$$\sigma_{BD} = \frac{F_{BD}}{A_{BD}} = \frac{23.33}{1.25} = 18.7 \text{ ksi} \qquad \text{(C)}$$ **Ans.**

***1–36.** The bars of the truss each have a cross-sectional area of 1.25 in^2. If the maximum average normal stress in any bar is not to exceed 20 ksi, determine the maximum magnitude P of the loads that can be applied to the truss.



Joint A:

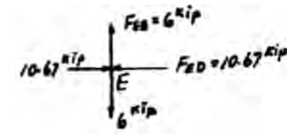
$$+\uparrow \Sigma F_y = 0; \qquad -P + \left(\frac{3}{5}\right)F_{AB} = 0$$

$$F_{AB} = (1.667)P$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad -F_{AE} + (1.667)P\left(\frac{4}{5}\right) = 0$$

$$F_{AE} = (1.333)P$$

Joint E:

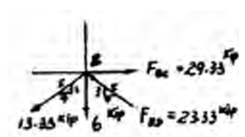


$$+\uparrow \Sigma F_y = 0; \qquad F_{EB} - (0.75)P = 0$$

$$F_{EB} = (0.75)P$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad (1.333)P - F_{ED} = 0$$

$$F_{ED} = (1.333)P$$



Joint B:

$$+\uparrow \Sigma F_y = 0; \qquad \left(\frac{3}{5}\right)F_{BD} - (0.75)P - (1.667)P\left(\frac{3}{5}\right) = 0$$

$$F_{BD} = (2.9167)P$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{BC} - (2.9167)P\left(\frac{4}{5}\right) - (1.667)P\left(\frac{4}{5}\right) = 0$$

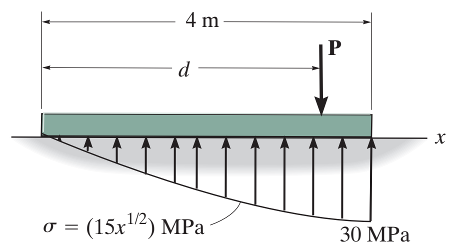
$$F_{BC} = (3.67)P$$

The highest stressed member is BC:

$$\sigma_{BC} = \frac{(3.67)P}{1.25} = 20$$

$P = 6.82$ kip **Ans.**

•**1–37.** The plate has a width of 0.5 m. If the stress distribution at the support varies as shown, determine the force **P** applied to the plate and the distance d to where it is applied.



The resultant force dF of the bearing pressure acting on the plate of area $dA = b\,dx = 0.5\,dx$, Fig. a,

$$dF = \sigma_b\, dA = (15x^{\frac{1}{2}})(10^6)(0.5dx) = 7.5(10^6)x^{\frac{1}{2}}\,dx$$

$$+\uparrow \Sigma F_y = 0; \qquad \int dF - P = 0$$

$$\int_0^{4m} 7.5(10^6)x^{\frac{1}{2}}\,dx - P = 0$$

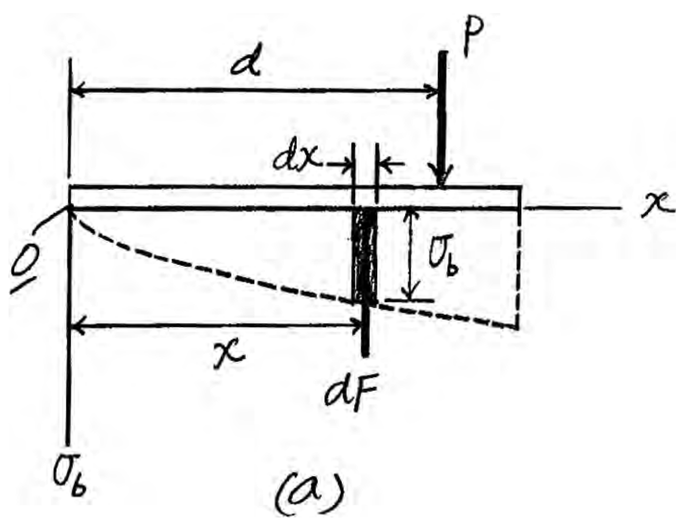
$$P = 40(10^6)\text{ N} = 40\text{ MN} \qquad \textbf{Ans.}$$

Equilibrium requires

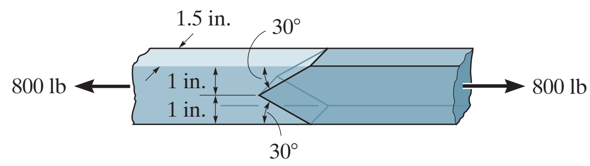
$$\circlearrowleft +\Sigma M_O = 0; \qquad \int x\,dF - Pd = 0$$

$$\int_0^{4m} x[7.5(10^6)x^{\frac{1}{2}}\,dx] - 40(10^6)\,d = 0$$

$$d = 2.40\text{ m} \qquad \textbf{Ans.}$$

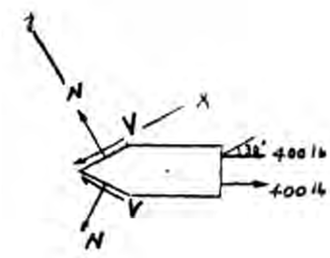


1–38. The two members used in the construction of an aircraft fuselage are joined together using a 30° fish-mouth weld. Determine the average normal and average shear stress on the plane of each weld. Assume each inclined plane supports a horizontal force of 400 lb.



$N - 400 \sin 30° = 0; \qquad N = 200 \text{ lb}$

$400 \cos 30° - V = 0; \qquad V = 346.41 \text{ lb}$

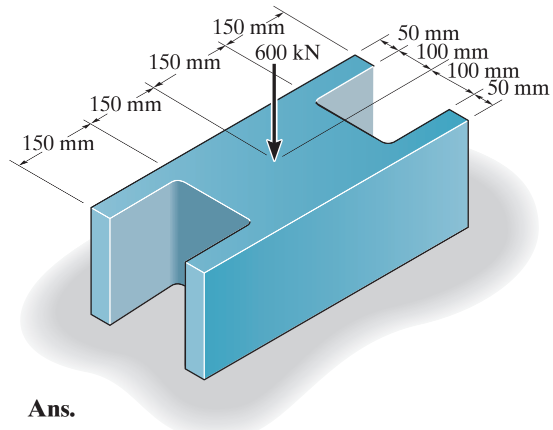


$$A' = \frac{1.5(1)}{\sin 30°} = 3 \text{ in}^2$$

$$\sigma = \frac{N}{A'} = \frac{200}{3} = 66.7 \text{ psi}$$ **Ans.**

$$\tau = \frac{V}{A'} = \frac{346.41}{3} = 115 \text{ psi}$$ **Ans.**

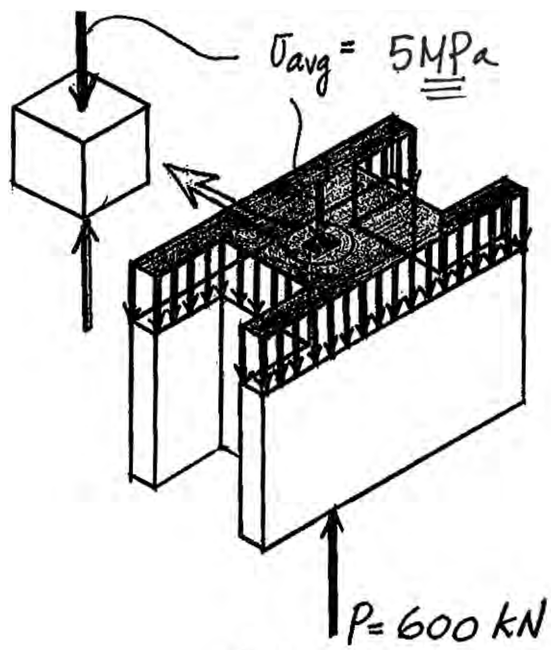
1–39. If the block is subjected to the centrally applied force of 600 kN, determine the average normal stress in the material. Show the stress acting on a differential volume element of the material.



The cross-sectional area of the block is $A = 0.6(0.3) - 0.3(0.2) = 0.12 \text{ m}^2$.

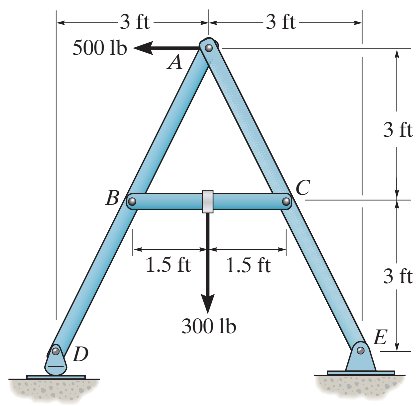
$$\sigma_{\text{avg}} = \frac{P}{A} = \frac{600(10^3)}{0.12} = 5(10^6) \text{ Pa} = 5 \text{ MPa}$$ **Ans.**

The average normal stress distribution over the cross-section of the block and the state of stress of a point in the block represented by a differential volume element are shown in Fig. *a*



(a)

***1–40.** The pins on the frame at B and C each have a diameter of 0.25 in. If these pins are subjected to *double shear*, determine the average shear stress in each pin.



Support Reactions: FBD(a)

$$\circlearrowleft +\Sigma M_g = 0; \qquad 500(6) + 300(3) - D_y(6) = 0$$

$$D_y = 650 \text{ lb}$$

$$\overset{+}{\leftarrow} \Sigma F_x = 0; \qquad 500 - E_x = 0 \qquad E_x = 500 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0; \qquad 650 - 300 - E_y = 0 \qquad E_y = 350 \text{ lb}$$

From FBD (c),

$$\circlearrowleft +\Sigma M_B = 0; \qquad C_y(3) - 300(1.5) = 0 \qquad C_y = 150 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0; \qquad B_y + 150 - 300 = 0 \qquad B_y = 150 \text{ lb}$$

From FBD (b)

$$\circlearrowleft +\Sigma M_A = 0; \qquad 150(1.5) + B_x(3) - 650(3) = 0$$

$$B_x = 575 \text{ lb}$$

From FBD (c),

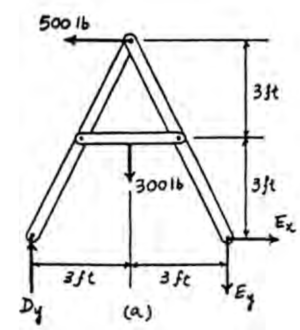
$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad C_x - 575 = 0 \qquad C_x = 575 \text{ lb}$$

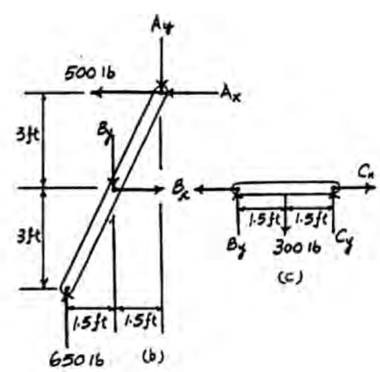
Hence, $F_B = F_C = \sqrt{575^2 + 150^2} = 594.24$ lb

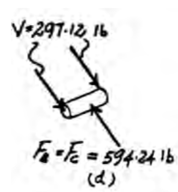
Average shear stress: Pins B and C are subjected to double shear as shown on FBD (d)

$$(\tau_B)_{avg} = (\tau_C)_{avg} = \frac{V}{A} = \frac{297.12}{\frac{\pi}{4}(0.25^2)}$$

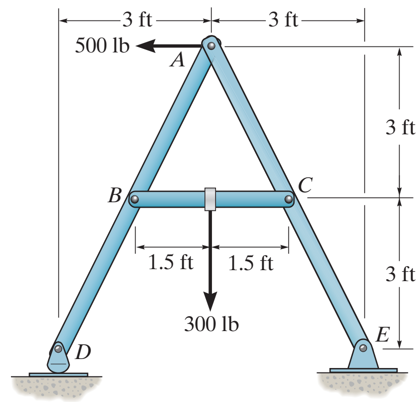
$$= 6053 \text{ psi} = 6.05 \text{ ksi} \qquad \textbf{Ans.}$$







•1–41. Solve Prob. 1–40 assuming that pins B and C are subjected to *single shear*.



Support Reactions: FBD(a)

$\circlearrowleft +\Sigma M_g = 0; \qquad 500(6) + 300(3) - D_y(6) = 0$

$$D_y = 650 \text{ lb}$$

$\overset{+}{\leftarrow} \Sigma F_x = 0; \qquad 500 - E_x = 0 \qquad E_x = 500 \text{ lb}$

$+\uparrow \Sigma F_y = 0; \qquad 650 - 300 - E_y = 0 \qquad E_y = 350 \text{ lb}$

From FBD (c),

$\circlearrowleft +\Sigma M_B = 0; \qquad C_y(3) - 300(1.5) = 0 \qquad C_y = 150 \text{ lb}$

$+\uparrow \Sigma F_y = 0; \qquad B_y + 150 - 300 = 0 \qquad B_y = 150 \text{ lb}$

From FBD (b)

$\circlearrowright +\Sigma M_A = 0; \qquad 150(1.5) + B_x(3) - 650(3) = 0$

$$B_x = 575 \text{ lb}$$

From FBD (c),

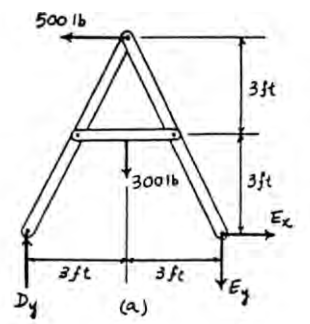
$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad C_x - 575 = 0 \qquad C_x = 575 \text{ lb}$

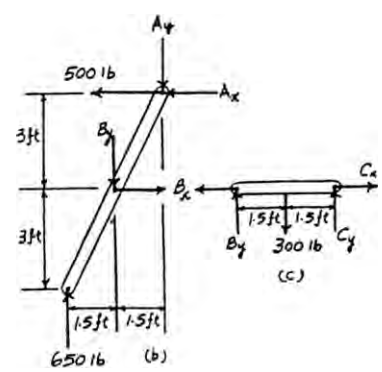
Hence, $F_B = F_C = \sqrt{575^2 + 150^2} = 594.24 \text{ lb}$

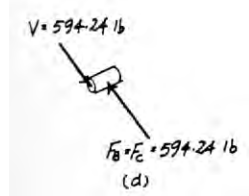
Average shear stress: Pins B and C are subjected to single shear as shown on FBD (d)

$$(\tau_B)_{avg} = (\tau_C)_{avg} = \frac{V}{A} = \frac{594.24}{\frac{\pi}{4}(0.25^2)}$$

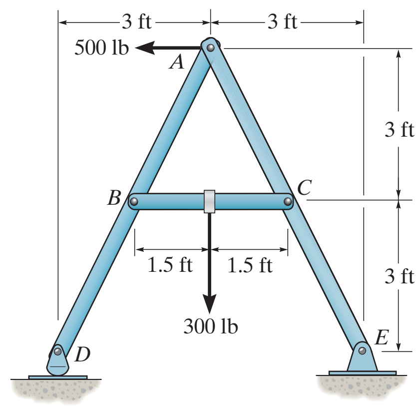
$$= 12106 \text{ psi} = 12.1 \text{ ksi} \qquad \textbf{Ans.}$$







1–42. The pins on the frame at D and E each have a diameter of 0.25 in. If these pins are subjected to *double shear,* determine the average shear stress in each pin.



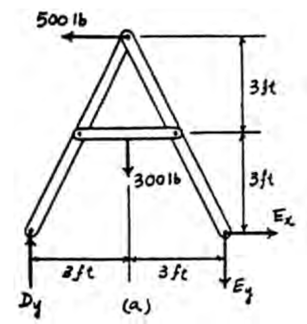
Support Reactions: FBD(a)

$$\circlearrowleft +\Sigma M_E = 0; \qquad 500(6) + 300(3) - D_y(6) = 0$$

$$D_y = 650 \text{ lb}$$

$$\overset{+}{\leftarrow} \Sigma F_x = 0; \qquad 500 - E_x = 0 \qquad E_x = 500 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0; \qquad 650 - 300 - E_y = 0 \qquad E_y = 350 \text{ lb}$$

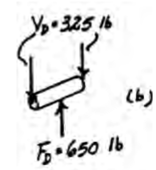


Average shear stress: Pins D and E are subjected to double shear as shown on FBD (b) and (c).

For Pin D, $F_D = D_y = 650$ lb then $V_D = \frac{F_D}{z} = 325$ lb

$$(\pi_D)_{\text{avg}} = \frac{V_D}{A_D} = \frac{325}{\frac{\pi}{4}(0.25)^2}$$

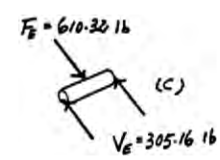
$$= 6621 \text{ psi} = 6.62 \text{ ksi} \qquad \textbf{Ans.}$$



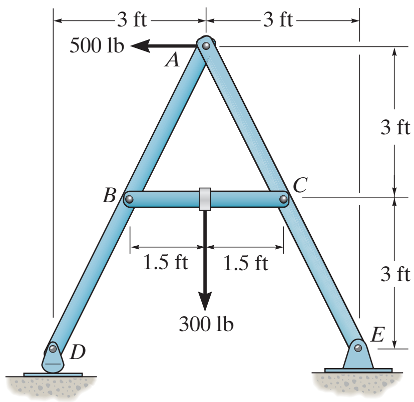
For Pin E, $F_E = \sqrt{500^2 + 350^2} = 610.32$ lb then $V_E = \frac{F_g}{z} = 305.16$ lb

$$(\tau_E)_{\text{avg}} = \frac{V_E}{A_E} = \frac{305.16}{\frac{\pi}{4}(0.25^2)}$$

$$= 6217 \text{ psi} = 6.22 \text{ ksi} \qquad \textbf{Ans.}$$



1–43. Solve Prob. 1–42 assuming that pins D and E are subjected to *single shear*.



Support Reactions: FBD(a)

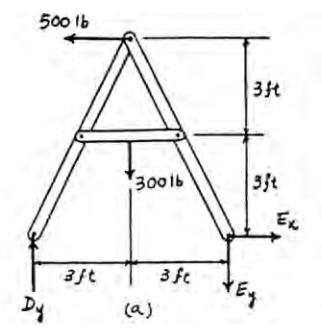
$$\zeta + \Sigma M_E = 0; \qquad 500(6) + 300(3) - D_y(6) = 0$$

$$D_y = 650 \text{ lb}$$

$$\overset{+}{\leftarrow} \Sigma F_x = 0; \qquad 500 - E_x = 0 \qquad E_x = 500 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0; \qquad 650 - 300 - E_y = 0 \qquad E_y = 350 \text{ lb}$$

Average shear stress: Pins D and E are subjected to single shear as shown on FBD (b) and (c).



For Pin D, $V_D = F_D = D_y = 650$ lb

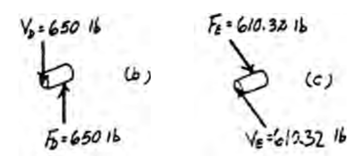
$$(\tau_D)_{avg} = \frac{V_D}{A_D} = \frac{650}{\frac{\pi}{4}(0.25^2)}$$

$$= 13242 \text{ psi} = 13.2 \text{ ksi} \qquad \textbf{Ans.}$$

For Pin E, $V_E = F_E = \sqrt{500^2 + 350^2} = 610.32$ lb

$$(\tau_E)_{avg} = \frac{V_E}{A_E} = \frac{610.32}{\frac{\pi}{4}(0.25^2)}$$

$$= 12433 \text{ psi} = 12.4 \text{ ksi} \qquad \textbf{Ans.}$$



***1–44.** A 175-lb woman stands on a vinyl floor wearing stiletto high-heel shoes. If the heel has the dimensions shown, determine the average normal stress she exerts on the floor and compare it with the average normal stress developed when a man having the same weight is wearing flat-heeled shoes. Assume the load is applied slowly, so that dynamic effects can be ignored. Also, assume the entire weight is supported only by the heel of one shoe.

Stiletto shoes:

$$A = \frac{1}{2}(\pi)(0.3)^2 + (0.6)(0.1) = 0.2014 \text{ in}^2$$

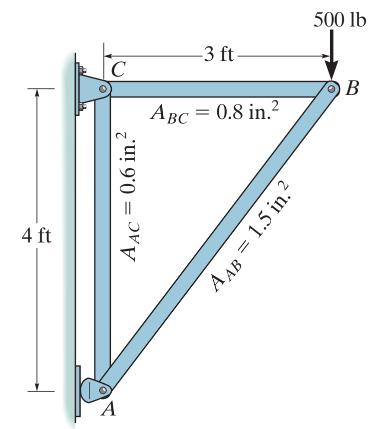
$$\sigma = \frac{P}{A} = \frac{175 \text{ lb}}{0.2014 \text{ in}^2} = 869 \text{ psi} \qquad \textbf{Ans.}$$

Flat-heeled shoes:

$$A = \frac{1}{2}(\pi)(1.2)^2 + 2.4(0.5) = 3.462 \text{ in}^2$$

$$\sigma = \frac{P}{A} = \frac{175 \text{ lb}}{3.462 \text{ in}^2} = 50.5 \text{ psi} \qquad \textbf{Ans.}$$

•1–45. The truss is made from three pin-connected members having the cross-sectional areas shown in the figure. Determine the average normal stress developed in each member when the truss is subjected to the load shown. State whether the stress is tensile or compressive.



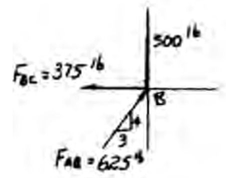
Joint *B*:

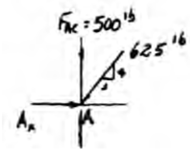
$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{625}{1.5} = 417 \text{ psi} \qquad \text{(C)} \qquad \textbf{Ans.}$$

$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{375}{0.8} = 469 \text{ psi} \qquad \text{(T)} \qquad \textbf{Ans.}$$

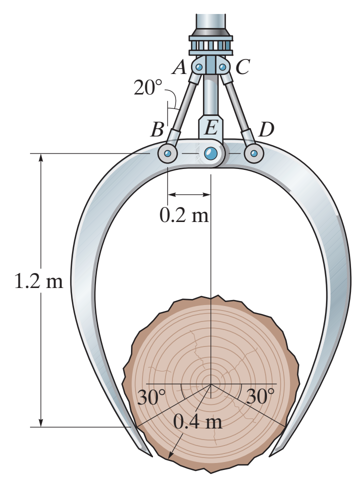
Joint *A*:

$$\sigma'_{AC} = \frac{F_{AC}}{A_{AC}} = \frac{500}{0.6} = 833 \text{ psi} \qquad \text{(T)} \qquad \textbf{Ans.}$$





1–46. Determine the average normal stress developed in links *AB* and *CD* of the smooth two-tine grapple that supports the log having a mass of 3 Mg. The cross-sectional area of each link is 400 mm^2.



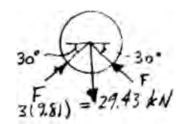
$$+\uparrow \Sigma F_y = 0; \qquad 2(F \sin 30°) - 29.43 = 0$$

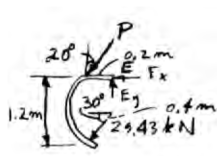
$$F = 29.43 \text{ kN}$$

$$\circlearrowleft + \Sigma M_E = 0; \quad P \cos 20°(0.2) - (29.43 \cos 30°)(1.2) + (29.43 \sin 30°)(0.4 \cos 30°) = 0$$

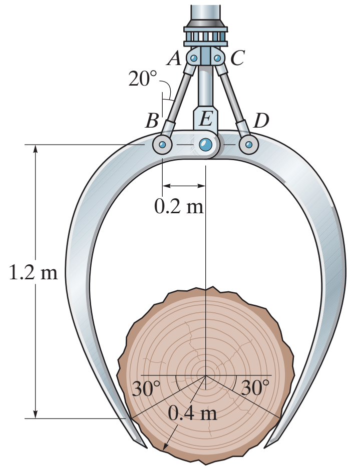
$$P = 135.61 \text{ kN}$$

$$\sigma = \frac{P}{A} = \frac{135.61(10^3)}{400(10^{-6})} = 339 \text{ MPa} \qquad \textbf{Ans.}$$





1–47. Determine the average shear stress developed in pins A and B of the smooth two-tine grapple that supports the log having a mass of 3 Mg. Each pin has a diameter of 25 mm and is subjected to double shear.



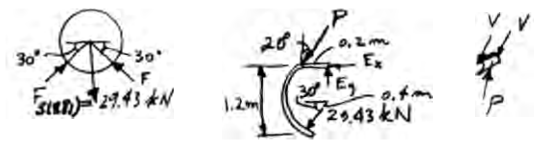
$$+\uparrow \Sigma F_y = 0; \qquad 2(F \sin 30^\circ) - 29.43 = 0$$

$$F = 29.43 \text{ kN}$$

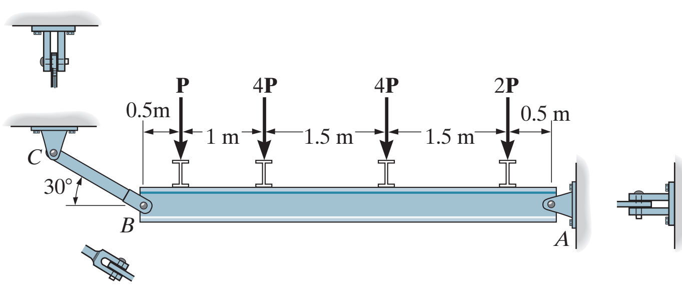
$$\circlearrowleft + \Sigma M_E = 0; \quad P \cos 20^\circ(0.2) - (29.43 \cos 30^\circ)(1.2) + (29.43 \sin 30^\circ)(0.4 \cos 30^\circ) = 0$$

$$P = 135.61 \text{ kN}$$

$$\tau_A = \tau_B = \frac{V}{A} = \frac{\frac{135.61(10^3)}{2}}{\frac{\pi}{4}(0.025)^2} = 138 \text{ MPa} \qquad \textbf{Ans.}$$



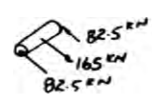
***1–48.** The beam is supported by a pin at A and a short link BC. If $P = 15$ kN, determine the average shear stress developed in the pins at A, B, and C. All pins are in double shear as shown, and each has a diameter of 18 mm.



For pins B and C:

$$\tau_B = \tau_C = \frac{V}{A} = \frac{82.5\,(10^3)}{\frac{\pi}{4}\left(\frac{18}{1000}\right)^2} = 324 \text{ MPa} \qquad \textbf{Ans.}$$

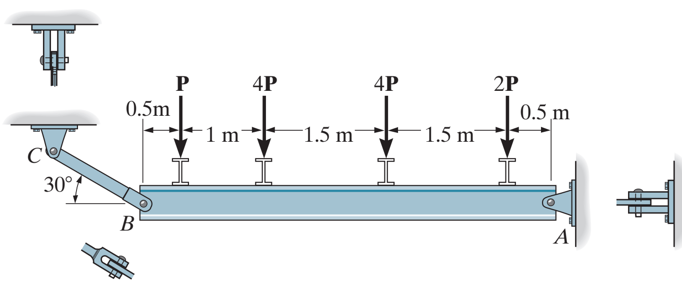
For pin A:

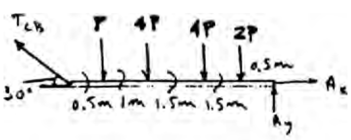


$$F_A = 2\sqrt{(82.5)^2 + (142.9)^2} = 165 \text{ kN}$$

$$\tau_A = \frac{V}{A} = \frac{82.5\,(10^3)}{\frac{\pi}{4}\left(\frac{18}{1000}\right)^2} = 324 \text{ MPa} \qquad \textbf{Ans.}$$

•1–49. The beam is supported by a pin at A and a short link BC. Determine the maximum magnitude P of the loads the beam will support if the average shear stress in each pin is not to exceed 80 MPa. All pins are in double shear as shown, and each has a diameter of 18 mm.





$\circlearrowleft +\Sigma M_A = 0; \qquad 2P(0.5) + 4P(2) + 4P(3.5) + P(4.5) - (T_{CB}\sin 30°)(5) = 0$

$T_{CB} = 11P$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad A_x - 11P\cos 30° = 0$

$A_x = 9.5263P$

$+\uparrow \Sigma F_y = 0; \qquad A_y - 11P + 11P\sin 30° = 0$

$A_y = 5.5P$

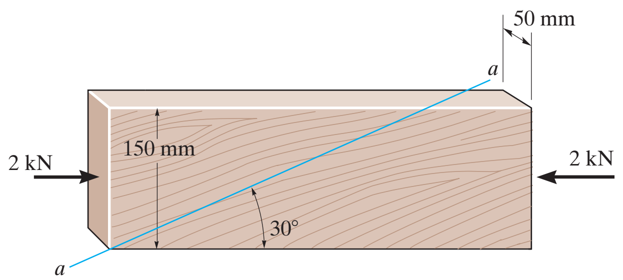
$F_A = \sqrt{(9.5263P)^2 + (5.5P)^2} = 11P$

Require;

$$\tau = \frac{V}{A}; \qquad 80(10^6) = \frac{11P/2}{\frac{\pi}{4}(0.018)^2}$$

$P = 3.70\text{ kN}$ **Ans.**

1–50. The block is subjected to a compressive force of 2 kN. Determine the average normal and average shear stress developed in the wood fibers that are oriented along section *a–a* at 30° with the axis of the block.



Force equilibrium equations written perpendicular and parallel to section *a–a* gives

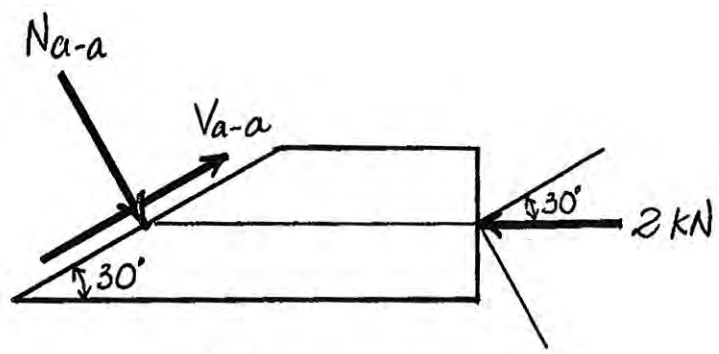
$+\nearrow\Sigma F_{x'} = 0; \qquad V_{a-a} - 2\cos 30° = 0 \qquad V_{a-a} = 1.732\text{ kN}$

$+\nwarrow\Sigma F_{y'} = 0; \qquad 2\sin 30° - N_{a-a} = 0 \qquad N_{a-a} = 1.00\text{ kN}$

The cross sectional area of section *a–a* is $A = \left(\dfrac{0.15}{\sin 30°}\right)(0.05) = 0.015\text{ m}^2$. Thus

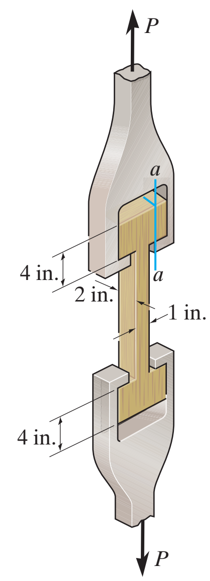
$$(\sigma_{a-a})_{\text{avg}} = \frac{N_{a-a}}{A} = \frac{1.00(10^3)}{0.015} = 66.67(10^3)\text{Pa} = 66.7\text{ kPa} \qquad \textbf{Ans.}$$

$$(\tau_{a-a})_{\text{avg}} = \frac{V_{a-a}}{A} = \frac{1.732(10^3)}{0.015} = 115.47(10^3)\text{Pa} = 115\text{ kPa} \qquad \textbf{Ans.}$$



(a)

1–51. During the tension test, the wooden specimen is subjected to an average normal stress of 2 ksi. Determine the axial force **P** applied to the specimen. Also, find the average shear stress developed along section *a–a* of the specimen.



Internal Loading: The normal force developed on the cross section of the middle portion of the specimen can be obtained by considering the free-body diagram shown in Fig. *a*.

$$+\uparrow \Sigma F_y = 0; \qquad \frac{P}{2} + \frac{P}{2} - N = 0 \qquad N = P$$

Referring to the free-body diagram shown in fig. *b*, the shear force developed in the shear plane *a–a* is

$$+\uparrow \Sigma F_y = 0; \qquad \frac{P}{2} - V_{a-a} = 0 \qquad V_{a-a} = \frac{P}{2}$$

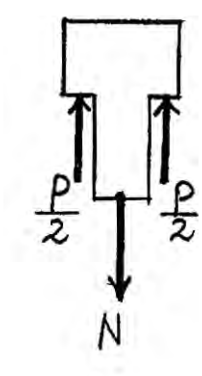
Average Normal Stress and Shear Stress: The cross-sectional area of the specimen is $A = 1(2) = 2\text{ in}^2$. We have

$$\sigma_{\text{avg}} = \frac{N}{A}; \qquad 2(10^3) = \frac{P}{2}$$

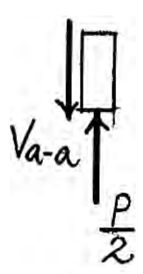
$$P = 4(10^3)\text{lb} = 4\text{ kip} \qquad \textbf{Ans.}$$

Using the result of **P**, $V_{a-a} = \frac{P}{2} = \frac{4(10^3)}{2} = 2(10^3)$ lb. The area of the shear plane is $A_{a-a} = 2(4) = 8\text{ in}^2$. We obtain

$$\left(\tau_{a-a}\right)_{\text{avg}} = \frac{V_{a-a}}{A_{a-a}} = \frac{2(10^3)}{8} = 250\text{ psi} \qquad \textbf{Ans.}$$

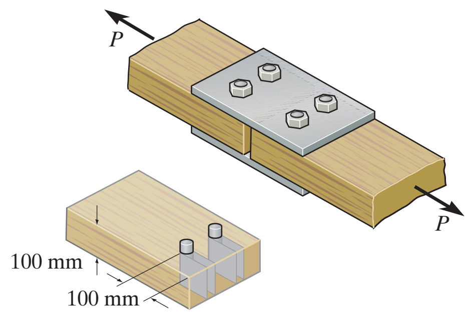


(a)



(b)

***1–52.** If the joint is subjected to an axial force of $P = 9$ kN, determine the average shear stress developed in each of the 6-mm diameter bolts between the plates and the members and along each of the four shaded shear planes.



Internal Loadings: The shear force developed on each shear plane of the bolt and the member can be determined by writing the force equation of equilibrium along the member's axis with reference to the free-body diagrams shown in Figs. *a*. and *b*, respectively.

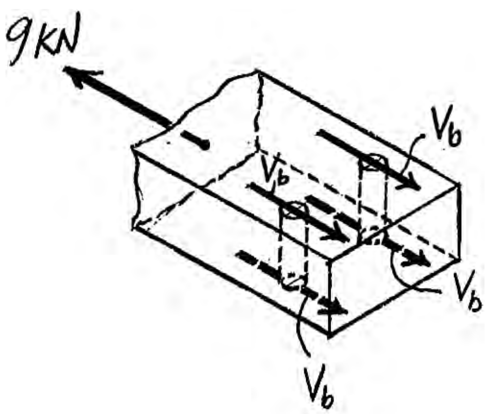
$\Sigma F_y = 0; \quad 4V_b - 9 = 0 \qquad V_b = 2.25 \text{ kN}$

$\Sigma F_y = 0; \quad 4V_p - 9 = 0 \qquad V_p = 2.25 \text{ kN}$

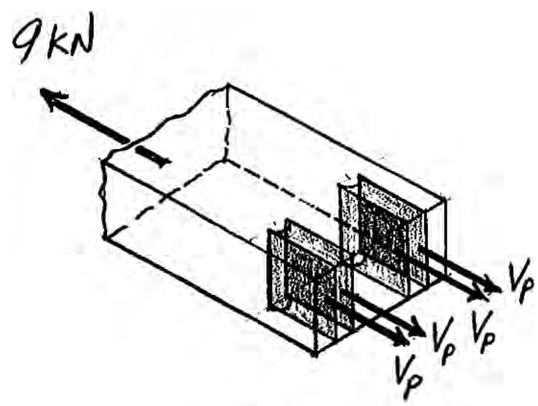
Average Shear Stress: The areas of each shear plane of the bolt and the member are $A_b = \frac{\pi}{4}(0.006^2) = 28.274(10^{-6})\text{m}^2$ and $A_p = 0.1(0.1) = 0.01 \text{ m}^2$, respectively. We obtain

$$\left(\tau_{\text{avg}}\right)_b = \frac{V_b}{A_b} = \frac{2.25(10^3)}{28.274(10^{-6})} = 79.6 \text{ MPa} \qquad \textbf{Ans.}$$

$$\left(\tau_{\text{avg}}\right)_p = \frac{V_p}{A_p} = \frac{2.25(10^3)}{0.01} = 225 \text{ kPa} \qquad \textbf{Ans.}$$

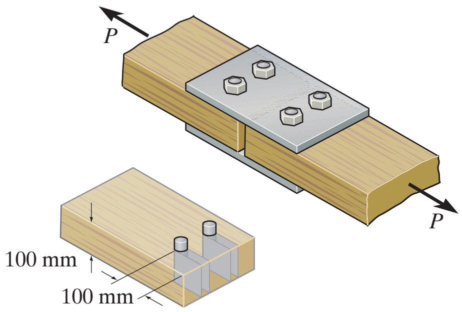


(a)



(b)

•1–53. The average shear stress in each of the 6-mm diameter bolts and along each of the four shaded shear planes is not allowed to exceed 80 MPa and 500 kPa, respectively. Determine the maximum axial force **P** that can be applied to the joint.



Internal Loadings: The shear force developed on each shear plane of the bolt and the member can be determined by writing the force equation of equilibrium along the member's axis with reference to the free-body diagrams shown in Figs. *a*. and *b*, respectively.

$\Sigma F_y = 0; \quad 4V_b - P = 0 \qquad V_b = P/4$

$\Sigma F_y = 0; \quad 4V_p - P = 0 \qquad V_p = P/4$

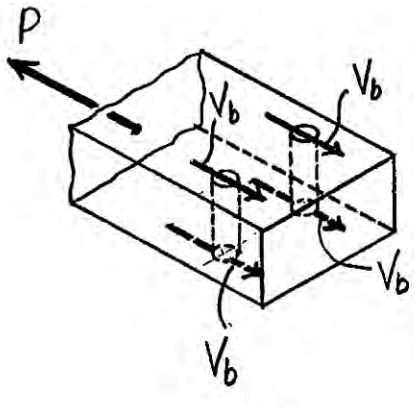
Average Shear Stress: The areas of each shear plane of the bolts and the members are $A_b = \frac{\pi}{4}(0.006^2) = 28.274(10^{-6})\text{m}^2$ and $A_p = 0.1(0.1) = 0.01\text{m}^2$, respectively. We obtain

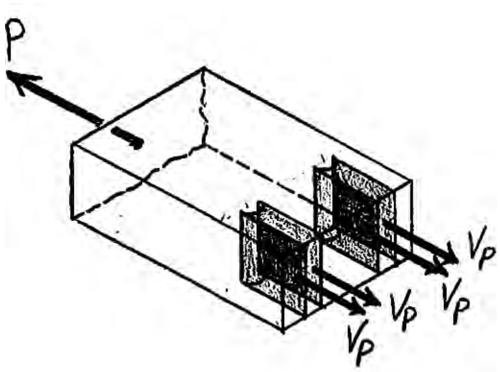
$$\left(\tau_{\text{allow}}\right)_b = \frac{V_b}{A_b}; \qquad 80(10^6) = \frac{P/4}{28.274(10^{-6})}$$

$$P = 9047 \text{ N} = 9.05 \text{ kN (controls)} \qquad \textbf{Ans.}$$

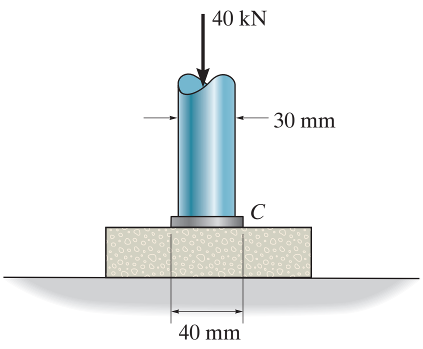
$$\left(\tau_{\text{allow}}\right)_p = \frac{V_p}{A_p}; \qquad 500(10^3) = \frac{P/4}{0.01}$$

$$P = 20\,000 \text{ N} = 20 \text{ kN}$$





1–54. The shaft is subjected to the axial force of 40 kN. Determine the average bearing stress acting on the collar C and the normal stress in the shaft.



Referring to the FBDs in Fig. *a*,

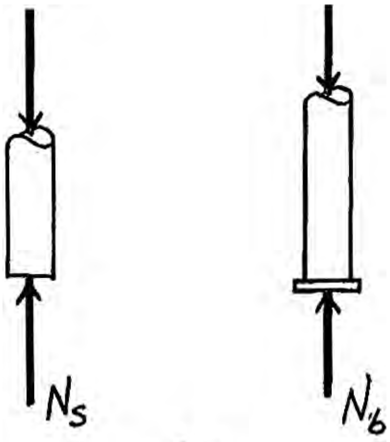
$+\uparrow \Sigma F_y = 0; \qquad N_s - 40 = 0 \qquad N_s = 40 \text{ kN}$

$+\uparrow \Sigma F_y = 0; \qquad N_b - 40 = 0 \qquad N_b = 40 \text{ kN}$

Here, the cross-sectional area of the shaft and the bearing area of the collar are $A_s = \frac{\pi}{4}(0.03^2) = 0.225(10^{-3})\pi \text{ m}^2$ and $A_b = \frac{\pi}{4}(0.04^2) = 0.4(10^{-3})\pi \text{ m}^2$. Thus,

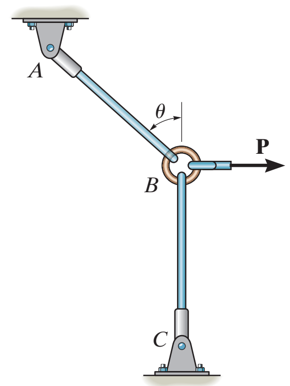
$$\left(\sigma_{\text{avg}}\right)_s = \frac{N_s}{A_s} = \frac{40(10^3)}{0.225(10^{-3})\pi} = 56.59(10^6) \text{ Pa} = 56.6 \text{ MPa}$$ **Ans.**

$$\left(\sigma_{\text{avg}}\right)_b = \frac{N_b}{A_b} = \frac{40(10^3)}{0.4(10^{-3})\pi} = 31.83(10^6)\text{Pa} = 31.8 \text{ MPa}$$ **Ans.**



(a)

1–55. Rods AB and BC each have a diameter of 5 mm. If the load of $P = 2$ kN is applied to the ring, determine the average normal stress in each rod if $\theta = 60°$.



Consider the equilibrium of joint B, Fig. a,

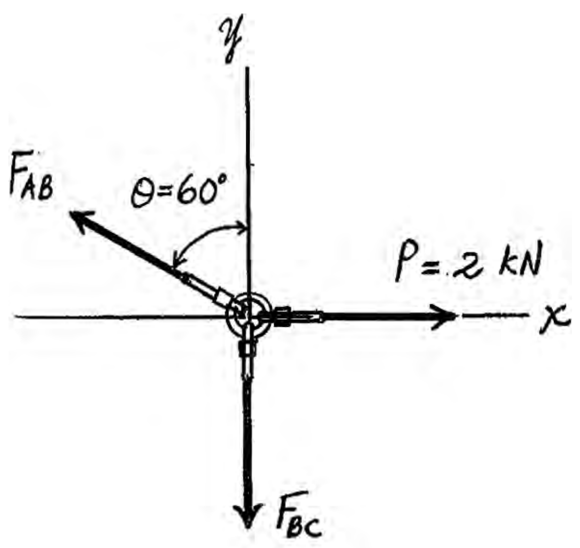
$\xrightarrow{+} \Sigma F_x = 0; \quad 2 - F_{AB} \sin 60° = 0 \quad F_{AB} = 2.309 \text{ kN}$

$+\uparrow \Sigma F_y = 0; \quad 2.309 \cos 60° - F_{BC} = 0 \quad F_{BC} = 1.155 \text{ kN}$

The cross-sectional area of wires AB and BC are $A_{AB} = A_{BC} = \frac{\pi}{4}(0.005^2) = 6.25(10^{-6})\pi \text{ m}^2$. Thus,

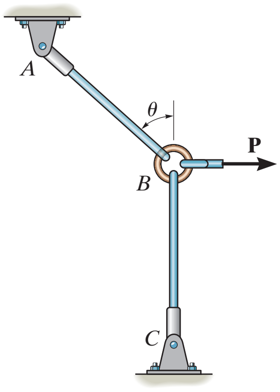
$$\left(\sigma_{avg}\right)_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{2.309(10^3)}{6.25(10^{-6})\pi} = 117.62(10^6) \text{ Pa} = 118 \text{ MPa} \qquad \textbf{Ans.}$$

$$\left(\sigma_{avg}\right)_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{1.155(10^3)}{6.25(10^{-6})\pi} = 58.81(10^6) \text{ Pa} = 58.8 \text{ MPa} \qquad \textbf{Ans.}$$



(a)

***1–56.** Rods AB and BC each have a diameter of 5 mm. Determine the angle θ of rod BC so that the average normal stress in rod AB is 1.5 times that in rod BC. What is the load **P** that will cause this to happen if the average normal stress in each rod is not allowed to exceed 100 MPa?



Consider the equilibrium of joint B, Fig. a,

$+\uparrow \Sigma F_y = 0; \qquad F_{AB}\cos\theta - F_{BC} = 0$ **(1)**

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad P - F_{AB}\sin\theta = 0$ **(2)**

The cross-sectional area of rods AB and BC are $A_{AB} = A_{BC} = \frac{\pi}{4}(0.005^2) = 6.25(10^{-6})\pi \text{ m}^2$. Since the average normal stress in rod AB is required to be 1.5 times to that of rod BC, then

$$(\sigma_{\text{avg}})_{AB} = 1.5\,(\sigma_{\text{avg}})_{BC}$$

$$\frac{F_{AB}}{A_{AB}} = 1.5\left(\frac{F_{BC}}{A_{BC}}\right)$$

$$\frac{F_{AB}}{6.25(10^{-6})\pi} = 1.5\left[\frac{F_{BC}}{6.25(10^{-6})\pi}\right]$$

$F_{AB} = 1.5\,F_{BC}$ **(3)**

Solving Eqs (1) and (3),

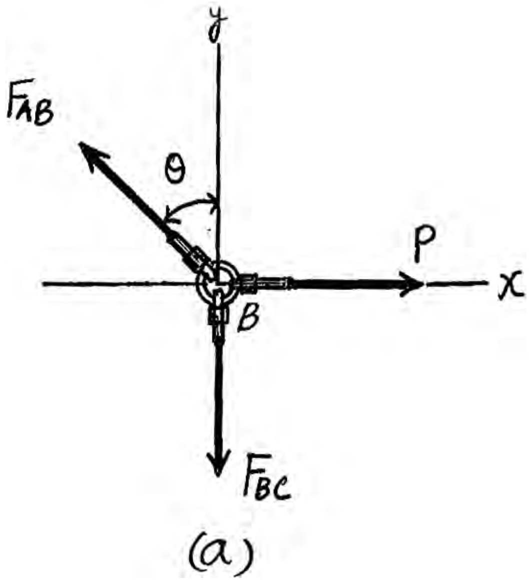
$\theta = 48.19° = 48.2°$ **Ans.**

Since wire AB will achieve the average normal stress of 100 MPa first when **P** increases, then

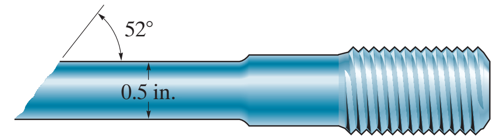
$$F_{AB} = \sigma_{\text{allow}}\,A_{AB} = \left[100(10^6)\right]\left[6.25(10^{-6})\pi\right] = 1963.50 \text{ N}$$

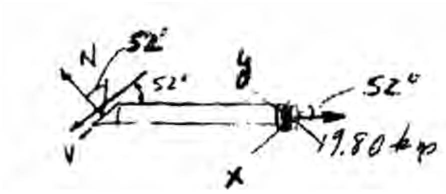
Substitute the result of F_{AB} and θ into Eq (2),

$P = 1.46$ kN **Ans.**



•1–57. The specimen failed in a tension test at an angle of 52° when the axial load was 19.80 kip. If the diameter of the specimen is 0.5 in., determine the average normal and average shear stress acting on the area of the inclined failure plane. Also, what is the average normal stress acting on the *cross section* when failure occurs?





$$\overset{+}{\swarrow}\ \Sigma F_x = 0; \qquad V - 19.80 \cos 52° = 0$$

$$V = 12.19 \text{ kip}$$

$$+\nwarrow \Sigma F_y = 0; \qquad N - 19.80 \sin 52° = 0$$

$$N = 15.603 \text{ kip}$$

Inclined plane:

$$\sigma' = \frac{P}{A}; \qquad \sigma' = \frac{15.603}{\frac{\pi(0.25)^2}{\sin 52°}} = 62.6 \text{ ksi} \qquad \textbf{Ans.}$$

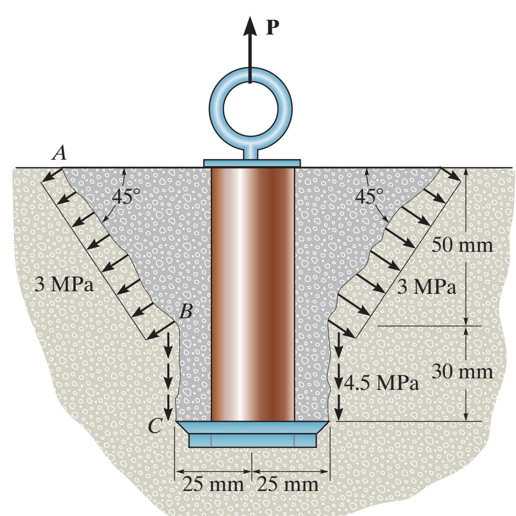
$$\tau'_{\text{avg}} = \frac{V}{A}; \qquad \tau'_{\text{avg}} = \frac{12.19}{\frac{\pi(0.25)^2}{\sin 52°}} = 48.9 \text{ ksi} \qquad \textbf{Ans.}$$

Cross section:

$$\sigma = \frac{P}{A}; \qquad \sigma = \frac{19.80}{\pi(0.25)^2} = 101 \text{ ksi} \qquad \textbf{Ans.}$$

$$\tau_{\text{avg}} = \frac{V}{A}; \qquad \tau_{\text{avg}} = 0 \qquad \textbf{Ans.}$$

1–58. The anchor bolt was pulled out of the concrete wall and the failure surface formed part of a frustum and cylinder. This indicates a shear failure occurred along the cylinder BC and tension failure along the frustum AB. If the shear and normal stresses along these surfaces have the magnitudes shown, determine the force **P** that must have been applied to the bolt.



Average Normal Stress:

For the frustum, $A = 2\pi\bar{x}L = 2\pi(0.025 + 0.025)\left(2\sqrt{0.05^2 + 0.05^2}\right)$

$$= 0.02221 \text{ m}^2$$

$$\sigma = \frac{P}{A}; \qquad 3\left(10^6\right) = \frac{F_1}{0.02221}$$

$$F_1 = 66.64 \text{ kN}$$

Average Shear Stress:

For the cylinder, $A = \pi(0.05)(0.03) = 0.004712 \text{ m}^2$

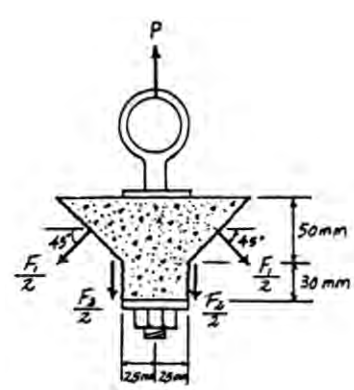
$$\tau_{\text{avg}} = \frac{V}{A}; \qquad 4.5\left(10^6\right) = \frac{F_2}{0.004712}$$

$$F_2 = 21.21 \text{ kN}$$

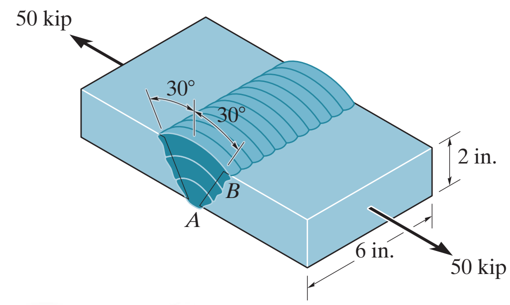
Equation of Equilibrium:

$$+\uparrow \Sigma F_y = 0; \qquad P - 21.21 - 66.64 \sin 45° = 0$$

$$P = 68.3 \text{ kN} \qquad \textbf{Ans.}$$



1–59. The open square butt joint is used to transmit a force of 50 kip from one plate to the other. Determine the average normal and average shear stress components that this loading creates on the face of the weld, section AB.



Equations of Equilibrium:

$\nwarrow^{+}\Sigma F_y = 0; \quad N - 50\cos 30° = 0 \quad N = 43.30 \text{ kip}$

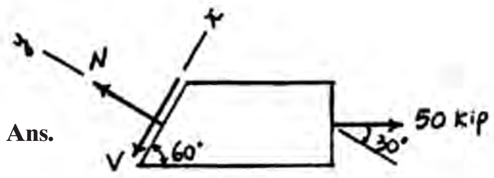
$^{+}\nearrow \Sigma F_x = 0; \quad -V + 50 \sin 30° = 0 \quad V = 25.0 \text{ kip}$

Average Normal and Shear Stress:

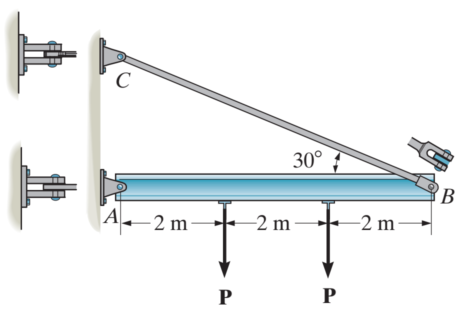
$$A' = \left(\frac{2}{\sin 60°}\right)(6) = 13.86 \text{ in}^2$$

$$\sigma = \frac{N}{A'} = \frac{43.30}{13.86} = 3.125 \text{ ksi}$$ **Ans.**

$$\tau_{avg} = \frac{V}{A'} = \frac{25.0}{13.86} = 1.80 \text{ ksi}$$ **Ans.**



***1–60.** If $P = 20$ kN, determine the average shear stress developed in the pins at A and C. The pins are subjected to double shear as shown, and each has a diameter of 18 mm.



Referring to the FBD of member AB, Fig. a

$\circlearrowleft + \Sigma M_A = 0; \quad F_{BC} \sin 30° (6) - 20(2) - 20(4) = 0 \quad F_{BC} = 40 \text{ kN}$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad A_x - 40 \cos 30° = 0 \quad A_x = 34.64 \text{ kN}$

$+\uparrow \Sigma F_y = 0; \quad A_y - 20 - 20 + 40 \sin 30° \quad A_y = 20 \text{ kN}$

Thus, the force acting on pin A is

$$F_A = \sqrt{A_x^{\,2} + A_y^{\,2}} = \sqrt{34.64^2 + 20^2} = 40 \text{ kN}$$

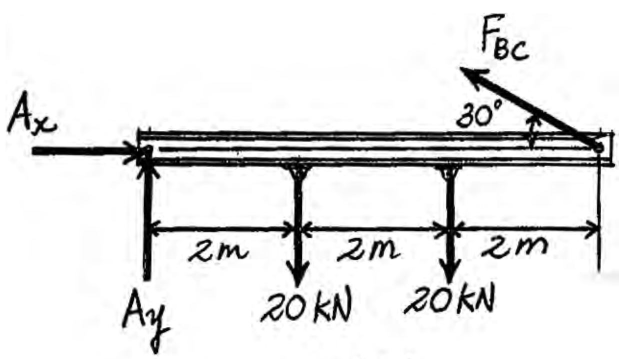
Pins A and C are subjected to double shear. Referring to their FBDs in Figs. b and c,

$$V_A = \frac{F_A}{2} = \frac{40}{2} = 20 \text{ kN} \qquad V_C = \frac{F_{BC}}{2} = \frac{40}{2} = 20 \text{ kN}$$

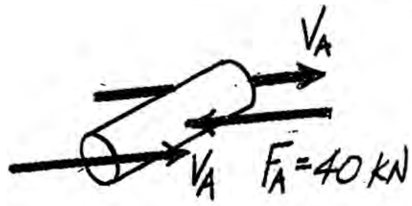
The cross-sectional area of Pins A and C are $A_A = A_C = \frac{\pi}{4}(0.018^2)$ $= 81(10^{-6})\pi \text{ m}^2$. Thus

$$\tau_A = \frac{V_A}{A_A} = \frac{20(10^3)}{81(10^{-6})\pi} = 78.59(10^6) \text{ Pa} = 78.6 \text{ MPa}$$ **Ans.**

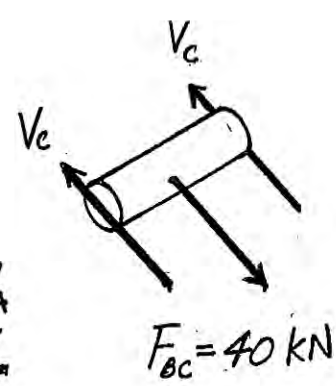
$$\tau_C = \frac{V_C}{A_C} = \frac{20(10^3)}{81(10^{-6})\pi} = 78.59(10^6) \text{ Pa} = 78.6 \text{ MPa}$$ **Ans.**



(a)

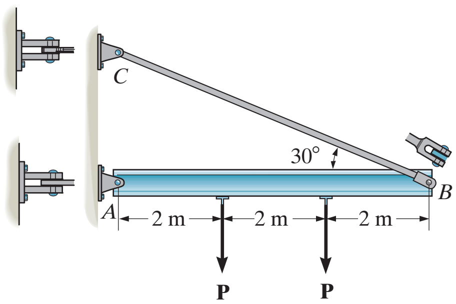


(b)



(c)

•1–61. Determine the maximum magnitude P of the load the beam will support if the average shear stress in each pin is not to allowed to exceed 60 MPa. All pins are subjected to double shear as shown, and each has a diameter of 18 mm.



Referring to the FBD of member AB, Fig. a,

$\circlearrowleft + \Sigma M_A = 0; \qquad F_{BC} \sin 30°(6) - P(2) - P(4) = 0 \qquad F_{BC} = 2P$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad A_x - 2P \cos 30° = 0 \qquad A_x = 1.732P$

$+\uparrow \Sigma F_y = 0; \qquad A_y - P - P + 2P \sin 30° = 0 \qquad A_y = P$

Thus, the force acting on pin A is

$$F_A = \sqrt{A_x^{\,2} + A_y^{\,2}} = \sqrt{(1.732P)^2 + P^2} = 2P$$

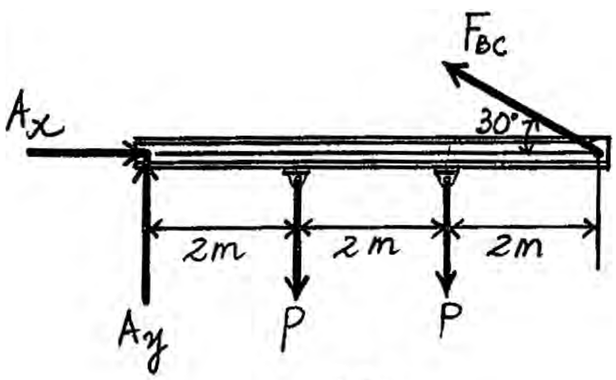
All pins are subjected to same force and double shear. Referring to the FBD of the pin, Fig. b,

$$V = \frac{F}{2} = \frac{2P}{2} = P$$

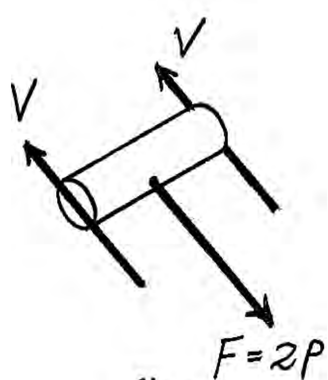
The cross-sectional area of the pin is $A = \frac{\pi}{4}(0.018^2) = 81.0(10^{-6})\pi\ \mathrm{m}^2$. Thus,

$$\tau_{\text{allow}} = \frac{V}{A}; \qquad 60(10^6) = \frac{P}{81.0(10^{-6})\pi}$$

$$P = 15\,268\ \mathrm{N} = 15.3\ \mathrm{kN} \qquad \textbf{Ans.}$$

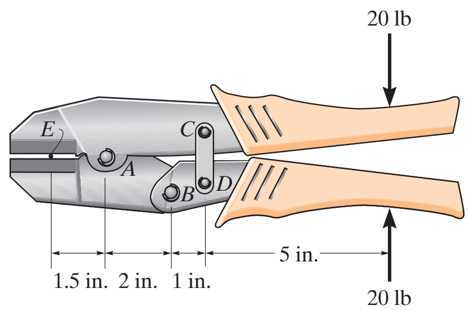


(a)



(b)

1–62. The crimping tool is used to crimp the end of the wire E. If a force of 20 lb is applied to the handles, determine the average shear stress in the pin at A. The pin is subjected to double shear and has a diameter of 0.2 in. Only a vertical force is exerted on the wire.

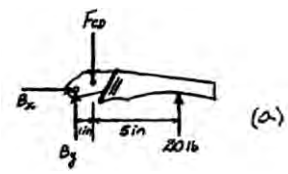


Support Reactions:

From FBD(a)

$\circlearrowleft + \Sigma M_D = 0; \qquad 20(5) - B_y(1) = 0 \qquad B_y = 100 \text{ lb}$

$\xrightarrow{+} \Sigma F_x = 0; \qquad B_x = 0$

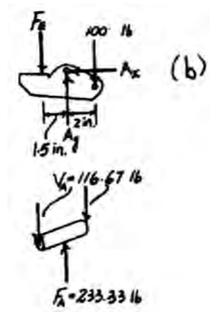


From FBD(b)

$\xrightarrow{+} \Sigma F_x = 0; \qquad A_x = 0$

$\circlearrowleft + \Sigma M_E = 0; \qquad A_y(1.5) - 100(3.5) = 0$

$$A_y = 233.33 \text{ lb}$$



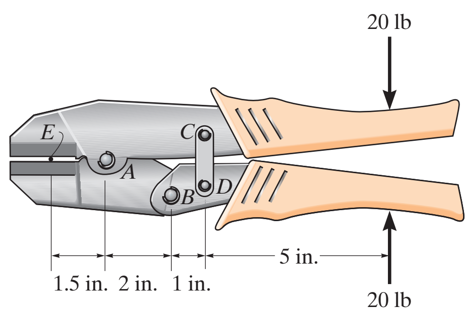
Average Shear Stress: Pin A is subjected to double shear. Hence,

$$V_A = \frac{F_A}{2} = \frac{A_y}{2} = 116.67 \text{ lb}$$

$$(\tau_A)_{\text{avg}} = \frac{V_A}{A_A} = \frac{116.67}{\frac{\pi}{4}(0.2^2)}$$

$$= 3714 \text{ psi} = 3.71 \text{ ksi} \qquad \textbf{Ans.}$$

1–63. Solve Prob. 1–62 for pin B. The pin is subjected to double shear and has a diameter of 0.2 in.



Support Reactions:

From FBD(a)

$\circlearrowleft + \Sigma M_D = 0; \qquad 20(5) - B_y(1) = 0 \qquad B_y = 100 \text{ lb}$

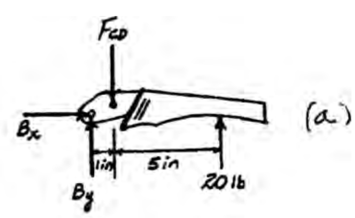
$\xrightarrow{+} \Sigma F_x = 0; \qquad B_x = 0$

Average Shear Stress: Pin B is subjected to double shear. Hence,

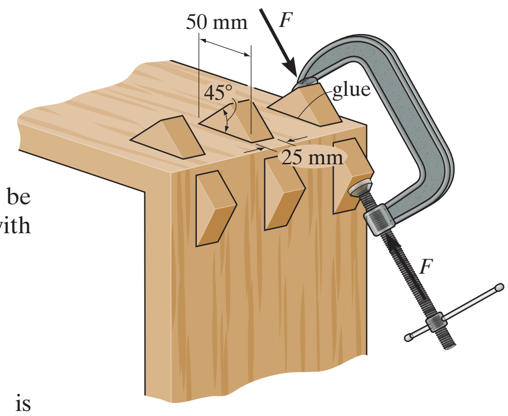
$$V_B = \frac{F_B}{2} = \frac{B_y}{2} = 50.0 \text{ lb}$$

$$(\tau_B)_{\text{avg}} = \frac{V_B}{A_B} = \frac{50.0}{\frac{\pi}{4}(0.2^2)}$$

$$= 1592 \text{ psi} = 1.59 \text{ ksi} \qquad \textbf{Ans.}$$



***1–64.** The triangular blocks are glued along each side of the joint. A C-clamp placed between two of the blocks is used to draw the joint tight. If the glue can withstand a maximum average shear stress of 800 kPa, determine the maximum allowable clamping force **F**.



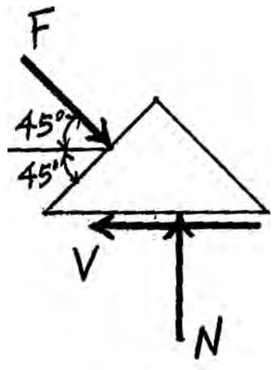
Internal Loadings: The shear force developed on the glued shear plane can be obtained by writing the force equation of equilibrium along the x axis with reference to the free-body diagram of the triangular block, Fig. *a*.

$$\xrightarrow{+}\Sigma F_x = 0; \qquad F\cos 45° - V = 0 \qquad V = \frac{\sqrt{2}}{2}F$$

Average Normal and Shear Stress: The area of the glued shear plane is $A = 0.05(0.025) = 1.25(10^{-3})\text{m}^2$. We obtain

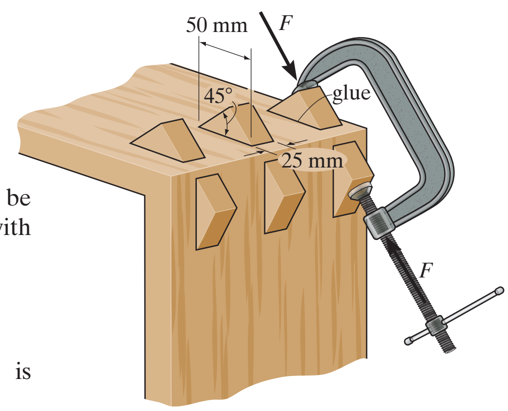
$$\tau_{avg} = \frac{V}{A}; \qquad 800(10^3) = \frac{\frac{\sqrt{2}}{2}F}{1.25(10^{-3})}$$

$$F = 1414 \text{ N} = 1.41 \text{ kN} \qquad \textbf{Ans.}$$



(a)

•1–65. The triangular blocks are glued along each side of the joint. A C-clamp placed between two of the blocks is used to draw the joint tight. If the clamping force is $F = 900$ N, determine the average shear stress developed in the glued shear plane.

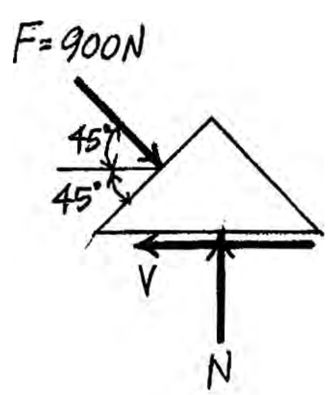


Internal Loadings: The shear force developed on the glued shear plane can be obtained by writing the force equation of equilibrium along the x axis with reference to the free-body diagram of the triangular block, Fig. *a*.

$$\xrightarrow{+}\Sigma F_x = 0; \qquad 900\cos 45° - V = 0 \qquad V = 636.40 \text{ N}$$

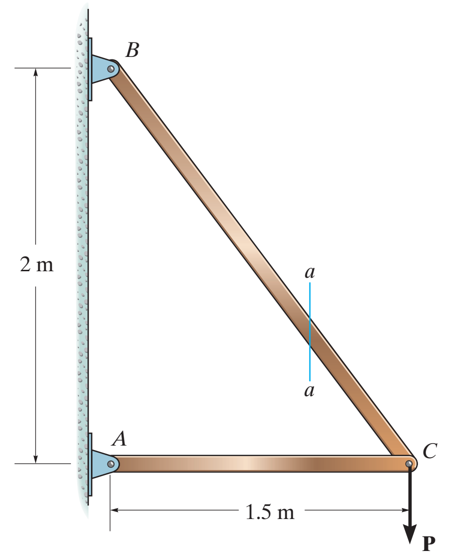
Average Normal and Shear Stress: The area of the glued shear plane is $A = 0.05(0.025) = 1.25(10^{-3})\text{m}^2$. We obtain

$$\tau_{avg} = \frac{V}{A} = \frac{636.40}{1.25(10^{-3})} = 509 \text{ kPa} \qquad \textbf{Ans.}$$



(a)

1–66. Determine the largest load **P** that can be a applied to the frame without causing either the average normal stress or the average shear stress at section *a–a* to exceed $\sigma = 150$ MPa and $\tau = 60$ MPa, respectively. Member *CB* has a square cross section of 25 mm on each side.



Analyse the equilibrium of joint *C* using the FBD Shown in Fig. *a*,

$$+\uparrow \Sigma F_y = 0; \qquad F_{BC}\left(\frac{4}{5}\right) - P = 0 \qquad F_{BC} = 1.25P$$

Referring to the FBD of the cut segment of member *BC* Fig. *b*.

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad N_{a-a} - 1.25P\left(\frac{3}{5}\right) = 0 \qquad N_{a-a} = 0.75P$$

$$+\uparrow \Sigma F_y = 0; \qquad 1.25P\left(\frac{4}{5}\right) - V_{a-a} = 0 \qquad V_{a-a} = P$$

The cross-sectional area of section *a–a* is $A_{a-a} = (0.025)\left(\frac{0.025}{3/5}\right) = 1.0417(10^{-3})\text{m}^2$. For Normal stress,

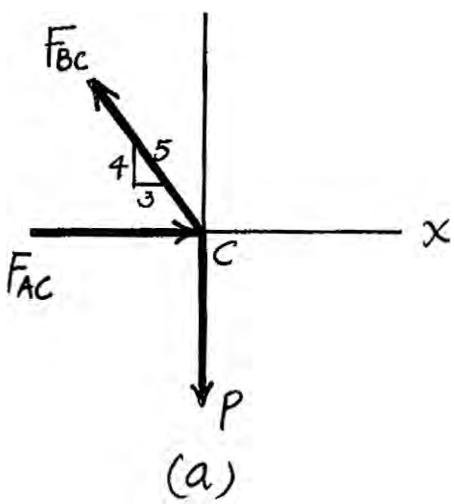
$$\sigma_{\text{allow}} = \frac{N_{a-a}}{A_{a-a}}; \qquad 150(10^6) = \frac{0.75P}{1.0417(10^{-3})}$$

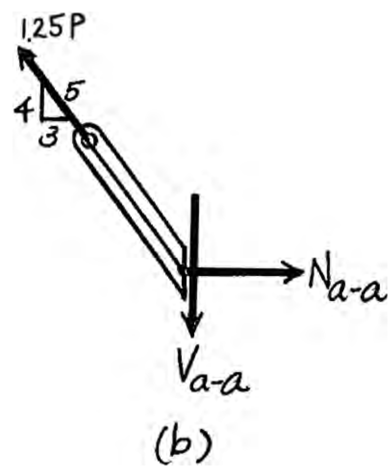
$$P = 208.33(10^3)\text{ N} = 208.33\text{ kN}$$

For Shear Stress

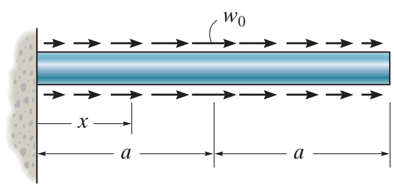
$$\tau_{\text{allow}} = \frac{V_{a-a}}{A_{a-a}}; \qquad 60(10^6) = \frac{P}{1.0417(10^{-3})}$$

$$P = 62.5(10^3)\text{ N} = 62.5\text{ kN (Controls!)}$$ **Ans.**





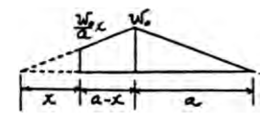
1–67. The prismatic bar has a cross-sectional area A. If it is subjected to a distributed axial loading that increases linearly from $w = 0$ at $x = 0$ to $w = w_0$ at $x = a$, and then decreases linearly to $w = 0$ at $x = 2a$, determine the average normal stress in the bar as a function of x for $0 \le x < a$.



Equation of Equilibrium:

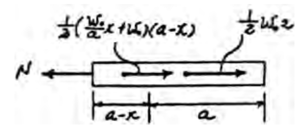
$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad -N + \frac{1}{2}\left(\frac{w_0}{a}x + w_0\right)(a - x) + \frac{1}{2} w_0 a = 0$$

$$N = \frac{w_0}{2a}\left(2a^2 - x^2\right)$$

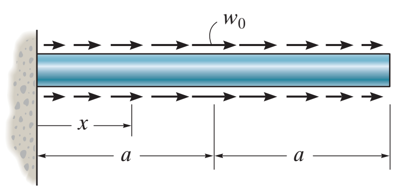


Average Normal Stress:

$$\sigma = \frac{N}{A} = \frac{\frac{w_0}{2a}\left(2a^2 - x^2\right)}{A} = \frac{w_0}{2aA}\left(2a^2 - x^2\right) \qquad \textbf{Ans.}$$



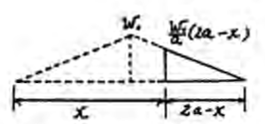
***1–68.** The prismatic bar has a cross-sectional area A. If it is subjected to a distributed axial loading that increases linearly from $w = 0$ at $x = 0$ to $w = w_0$ at $x = a$, and then decreases linearly to $w = 0$ at $x = 2a$, determine the average normal stress in the bar as a function of x for $a < x \le 2a$.



Equation of Equilibrium:

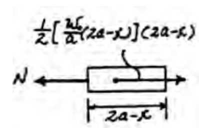
$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad -N + \frac{1}{2}\left[\frac{w_0}{a}(2a - x)\right](2a - x) = 0$$

$$N = \frac{w_0}{2a}(2a - x)^2$$

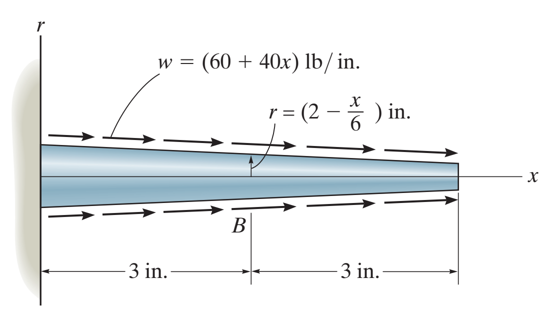


Average Normal Stress:

$$\sigma = \frac{N}{A} = \frac{\frac{w_0}{2a}(2a - x)^2}{A} = \frac{w_0}{2aA}(2a - x)^2 \qquad \textbf{Ans.}$$



•1–69. The tapered rod has a radius of $r = (2 - x/6)$ in. and is subjected to the distributed loading of $w = (60 + 40x)$ lb/in. Determine the average normal stress at the center of the rod, B.

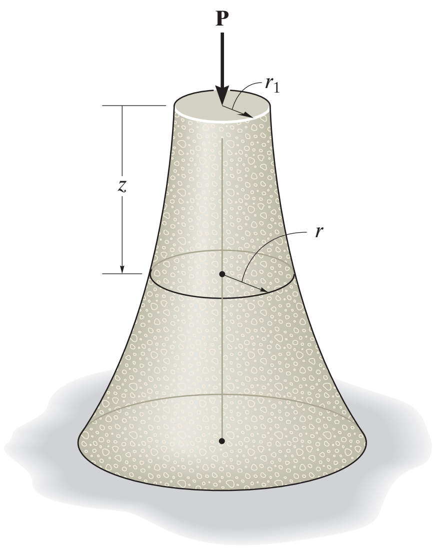


$$A = \pi\left(2 - \frac{3}{6}\right)^2 = 7.069 \text{ in}^2$$

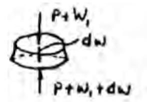
$$\Sigma F_x = 0; \qquad N - \int_3^6 (60 + 40x)\, dx = 0; \qquad N = 720 \text{ lb}$$

$$\sigma = \frac{720}{7.069} = 102 \text{ psi} \qquad \textbf{Ans.}$$

1–70. The pedestal supports a load **P** at its center. If the material has a mass density ρ, determine the radial dimension r as a function of z so that the average normal stress in the pedestal remains constant. The cross section is circular.



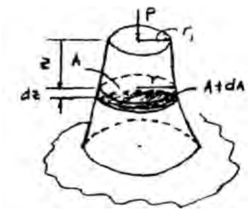
Require:



$$\sigma = \frac{P + W_1}{A} = \frac{P + W_1 + dW}{A + dA}$$

$$P\,dA + W_1\,dA = A\,dW$$

$$\frac{dW}{dA} = \frac{P + W_1}{A} = \sigma \qquad \textbf{(1)}$$



$$dA = \pi(r + dr)^2 - \pi r^2 = 2\pi\,r\,dr$$

$$dW = \pi r^2(\rho g)\,dt$$

From Eq. (1)

$$\frac{\pi r^2(\rho g)\,dz}{2\pi\,r\,dr} = \sigma$$

$$\frac{r\,\rho g\,dz}{2\,dr} = \sigma$$

$$\frac{\rho g}{2\sigma}\int_0^z dz = \int_{r_1}^{r}\frac{dr}{r}$$

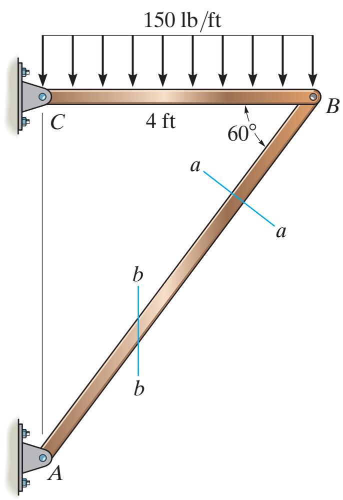
$$\frac{\rho g\,z}{2\sigma} = \ln\frac{r}{r_1}; \qquad r = r_1\,e^{(\frac{\pi}{2a})z}$$

However,

$$\sigma = \frac{P}{\pi\,r_1^2}$$

$$r = r_1\,e^{(\frac{\pi r_1^2 \rho g}{2P})z} \qquad \textbf{Ans.}$$

1–71. Determine the average normal stress at section *a–a* and the average shear stress at section *b–b* in member *AB*. The cross section is square, 0.5 in. on each side.



Consider the FBD of member *BC*, Fig. *a*,

$\circlearrowleft + \Sigma M_C = 0; \qquad F_{AB} \sin 60°(4) - 150(4)(2) = 0 \qquad F_{AB} = 346.41 \text{ lb}$

Referring to the FBD in Fig. *b*,

$\overset{+}{\swarrow} \Sigma F_{x'} = 0; \qquad N_{a-a} + 346.41 = 0 \qquad N_{a-a} = -346.41 \text{ lb}$

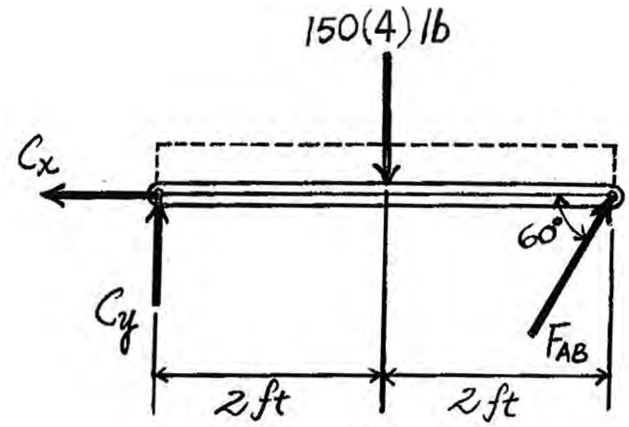
Referring to the FBD in Fig. *c*.

$+\uparrow \Sigma F_y = 0; \qquad V_{b-b} - 346.41 \sin 60° = 0 \qquad V_{b-b} = 300 \text{ lb}$

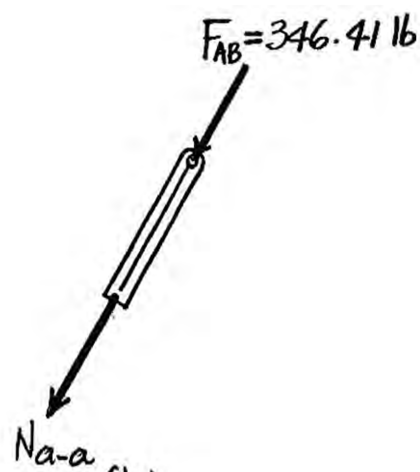
The cross-sectional areas of section *a–a* and *b–b* are $A_{a-a} = 0.5(0.5) = 0.25 \text{ in}^2$ and $A_{b-b} = 0.5\left(\dfrac{0.5}{\cos 60°}\right) = 0.5 \text{ in}^2$. Thus

$$\sigma_{a-a} = \frac{N_{a-a}}{A_{a-a}} = \frac{346.41}{0.25} = 1385.64 \text{ psi} = 1.39 \text{ ksi} \qquad \textbf{Ans.}$$

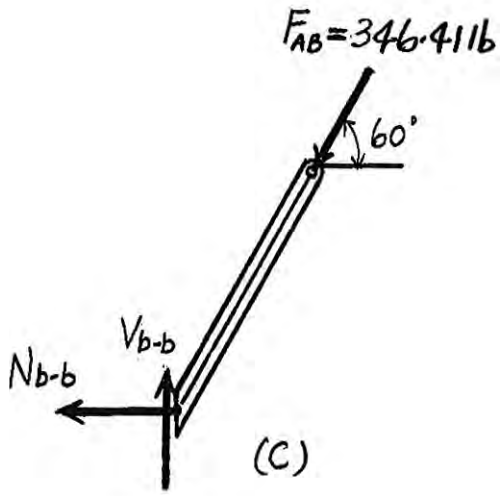
$$\tau_{b-b} = \frac{V_{b-b}}{A_{b-b}} = \frac{300}{0.5} = 600 \text{ psi} \qquad \textbf{Ans.}$$



(a)

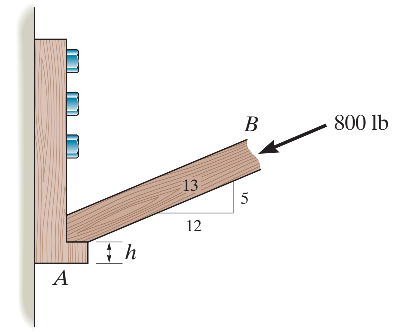


(b)



(c)

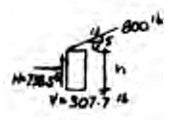
•1–73. Member B is subjected to a compressive force of 800 lb. If A and B are both made of wood and are $\frac{3}{8}$ in. thick, determine to the nearest $\frac{1}{4}$ in. the smallest dimension h of the horizontal segment so that it does not fail in shear. The average shear stress for the segment is $\tau_{allow} = 300$ psi.



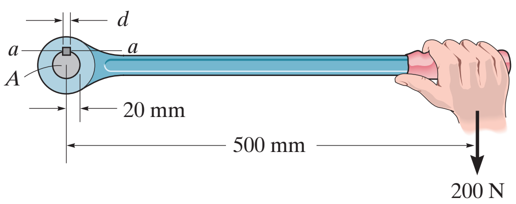
$$\tau_{allow} = 300 = \frac{307.7}{(\frac{3}{2})\,h}$$

$$h = 2.74 \text{ in.}$$

Use $h = 2\frac{3}{4}$ in. **Ans.**



1–74. The lever is attached to the shaft A using a key that has a width d and length of 25 mm. If the shaft is fixed and a vertical force of 200 N is applied perpendicular to the handle, determine the dimension d if the allowable shear stress for the key is $\tau_{allow} = 35$ MPa.

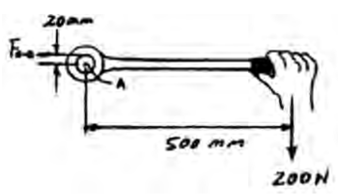


$$\circlearrowleft + \Sigma M_A = 0; \qquad F_{a-a}(20) - 200(500) = 0$$

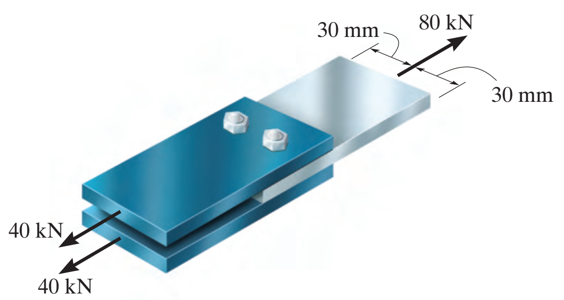
$$F_{a-a} = 5000 \text{ N}$$

$$\tau_{allow} = \frac{F_{a-a}}{A_{a-a}}; \qquad 35(10^6) = \frac{5000}{d(0.025)}$$

$$d = 0.00571 \text{ m} = 5.71 \text{ mm}$$ **Ans.**



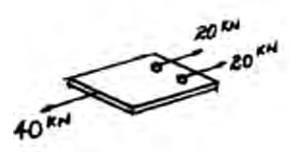
1–75. The joint is fastened together using two bolts. Determine the required diameter of the bolts if the failure shear stress for the bolts is $\tau_{fail} = 350$ MPa. Use a factor of safety for shear of F.S. = 2.5.



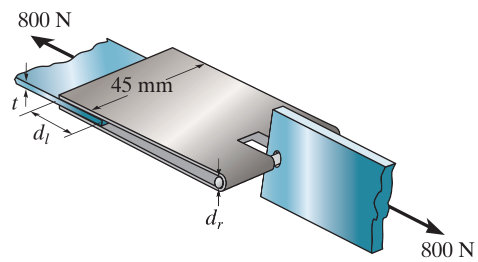
$$\frac{350(10^6)}{2.5} = 140(10^5)$$

$$\tau_{allow} = 140(10^6) = \frac{20(10^3)}{\frac{\pi}{4}\,d^2}$$

$$d = 0.0135 \text{ m} = 13.5 \text{ mm}$$ **Ans.**



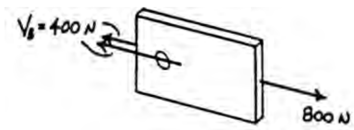
***1–76.** The lapbelt assembly is to be subjected to a force of 800 N. Determine (a) the required thickness t of the belt if the allowable tensile stress for the material is $(\sigma_t)_{\text{allow}} = 10\text{ MPa}$, (b) the required lap length d_l if the glue can sustain an allowable shear stress of $(\tau_{\text{allow}})_g = 0.75\text{ MPa}$, and (c) the required diameter d_r of the pin if the allowable shear stress for the pin is $(\tau_{\text{allow}})_p = 30\text{ MPa}$.



Allowable Normal Stress: Design of belt thickness.

$$(\sigma_t)_{\text{allow}} = \frac{P}{A}; \qquad 10\left(10^6\right) = \frac{800}{(0.045)t}$$

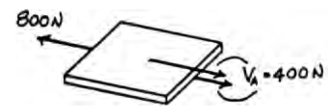
$$t = 0.001778\text{ m} = 1.78\text{ mm} \qquad \textbf{Ans.}$$



Allowable Shear Stress: Design of lap length.

$$(\tau_{\text{allow}})_g = \frac{V_A}{A}; \qquad 0.750\left(10^6\right) = \frac{400}{(0.045)\, d_t}$$

$$d_t = 0.01185\text{ m} = 11.9\text{ mm} \qquad \textbf{Ans.}$$

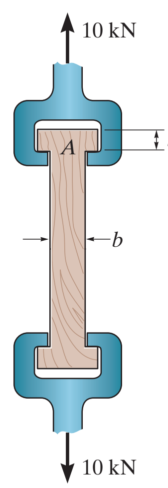


Allowable Shear Stress: Design of pin size.

$$(\tau_{\text{allow}})_P = \frac{V_B}{A}; \qquad 30\left(10^6\right) = \frac{400}{\frac{\pi}{4}\, d_r^2}$$

$$d_r = 0.004120\text{ m} = 4.12\text{ mm} \qquad \textbf{Ans.}$$

•1–77. The wood specimen is subjected to the pull of 10 kN in a tension testing machine. If the allowable normal stress for the wood is $(\sigma_t)_{\text{allow}} = 12\text{ MPa}$ and the allowable shear stress is $\tau_{\text{allow}} = 1.2\text{ MPa}$, determine the required dimensions b and t so that the specimen reaches these stresses simultaneously. The specimen has a width of 25 mm.



Allowable Shear Stress: Shear limitation

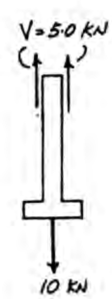
$$\tau_{\text{allow}} = \frac{V}{A}; \qquad 1.2\left(10^6\right) = \frac{5.00(10^3)}{(0.025)\, t}$$

$$t = 0.1667\text{ m} = 167\text{ mm} \qquad \textbf{Ans.}$$

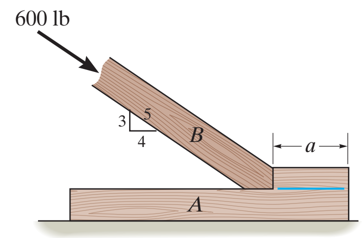
Allowable Normal Stress: Tension limitation

$$\sigma_{\text{allow}} = \frac{P}{A}; \qquad 12.0\left(10^6\right) = \frac{10(10^3)}{(0.025)\, b}$$

$$b = 0.03333\text{ m} = 33.3\text{ mm} \qquad \textbf{Ans.}$$



1–78. Member B is subjected to a compressive force of 600 lb. If A and B are both made of wood and are 1.5 in. thick, determine to the nearest 1/8 in. the smallest dimension a of the support so that the average shear stress along the blue line does not exceed $\tau_{\text{allow}} = 50$ psi. Neglect friction.



Consider the equilibrium of the FBD of member B, Fig. a,

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 600\left(\frac{4}{5}\right) - F_h = 0 \qquad F_h = 480 \text{ lb}$$

Referring to the FBD of the wood segment sectioned through glue line, Fig. b

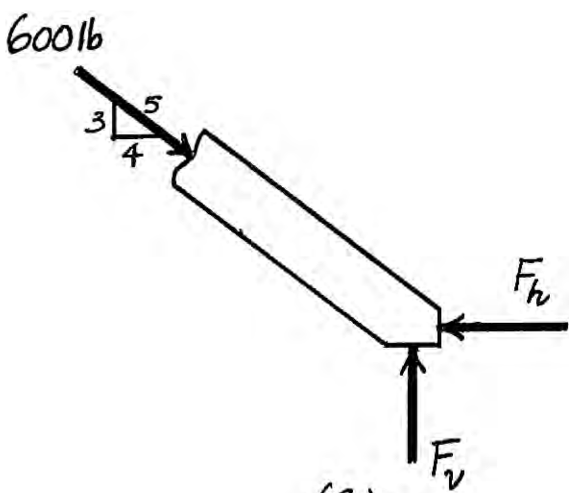
$$\xrightarrow{+} \Sigma F_x = 0; \qquad 480 - V = 0 \qquad V = 480 \text{ lb}$$

The area of shear plane is $A = 1.5(a)$. Thus,

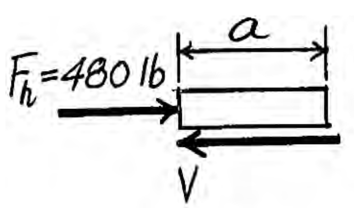
$$\tau_{\text{allow}} = \frac{V}{A}; \qquad 50 = \frac{480}{1.5a}$$

$$a = 6.40 \text{ in}$$

Use $a = 6\frac{1}{2}$ in. **Ans.**

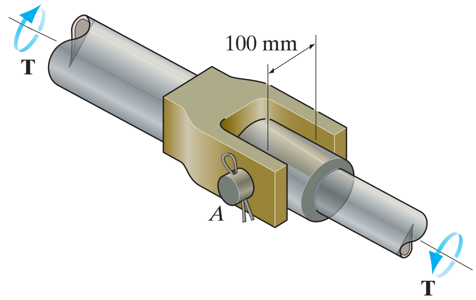


(a)



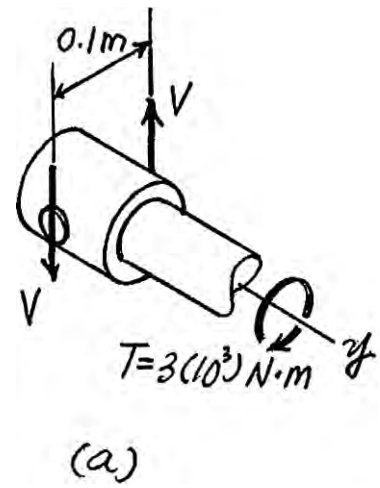
(b)

1–79. The joint is used to transmit a torque of $T = 3\ \text{kN} \cdot \text{m}$. Determine the required minimum diameter of the shear pin A if it is made from a material having a shear failure stress of $\tau_{\text{fail}} = 150$ MPa. Apply a factor of safety of 3 against failure.



Internal Loadings: The shear force developed on the shear plane of pin A can be determined by writing the moment equation of equilibrium along the y axis with reference to the free-body diagram of the shaft, Fig. *a*.

$$\Sigma M_y = 0; \quad V(0.1) - 3(10^3) = 0 \qquad V = 30(10^3)\text{N}$$



Allowable Shear Stress:

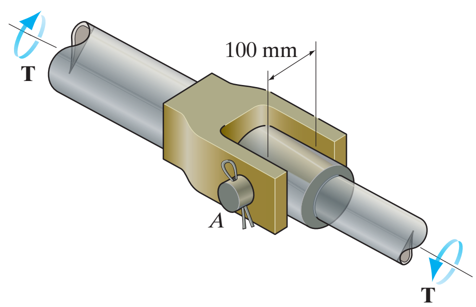
$$\tau_{\text{allow}} = \frac{\tau_{\text{fail}}}{\text{F.S.}} = \frac{150}{3} = 50\ \text{MPa}$$

Using this result,

$$\tau_{\text{allow}} = \frac{V}{A}; \qquad 50(10^6) = \frac{30(10^3)}{\frac{\pi}{4} d_A{}^2}$$

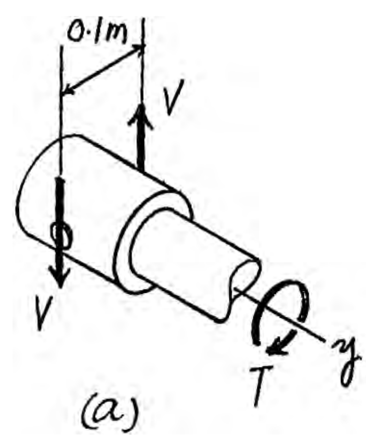
$$d_A = 0.02764\ \text{m} = 27.6\ \text{mm} \qquad \textbf{Ans.}$$

***1–80.** Determine the maximum allowable torque **T** that can be transmitted by the joint. The shear pin A has a diameter of 25 mm, and it is made from a material having a failure shear stress of $\tau_{\text{fail}} = 150$ MPa. Apply a factor of safety of 3 against failure.



Internal Loadings: The shear force developed on the shear plane of pin A can be determined by writing the moment equation of equilibrium along the y axis with reference to the free-body diagram of the shaft, Fig. *a*.

$$\Sigma M_y = 0; \quad V(0.1) - T = 0 \qquad V = 10T$$



Allowable Shear Stress:

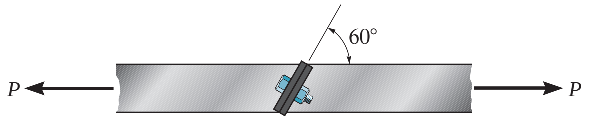
$$\tau_{\text{allow}} = \frac{\tau_{\text{fail}}}{\text{F.S.}} = \frac{150}{3} = 50\ \text{MPa}$$

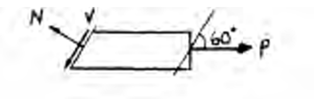
The area of the shear plane for pin A is $A_A = \frac{\pi}{4}(0.025^2) = 0.4909(10^{-3})\text{m}^2$. Using these results,

$$\tau_{\text{allow}} = \frac{V}{A_A}; \qquad 50(10^6) = \frac{10T}{0.4909(10^{-3})}$$

$$T = 2454.37\ \text{N} \cdot \text{m} = 2.45\ \text{kN} \cdot \text{m} \qquad \textbf{Ans.}$$

•1–81. The tension member is fastened together using *two* bolts, one on each side of the member as shown. Each bolt has a diameter of 0.3 in. Determine the maximum load P that can be applied to the member if the allowable shear stress for the bolts is $\tau_{\text{allow}} = 12$ ksi and the allowable average normal stress is $\sigma_{\text{allow}} = 20$ ksi.





$\nwarrow + \Sigma F_y = 0; \qquad N - P \sin 60° = 0$

$P = 1.1547\, N \qquad$ **(1)**

$\swarrow + \Sigma F_x = 0; \qquad V - P \cos 60° = 0$

$P = 2\, V \qquad$ **(2)**

Assume failure due to shear:

$$\tau_{\text{allow}} = 12 = \frac{V}{(2)\,\frac{\pi}{4}\,(0.3)^2}$$

$V = 1.696$ kip

From Eq. (2),

$P = 3.39$ kip

Assume failure due to normal force:

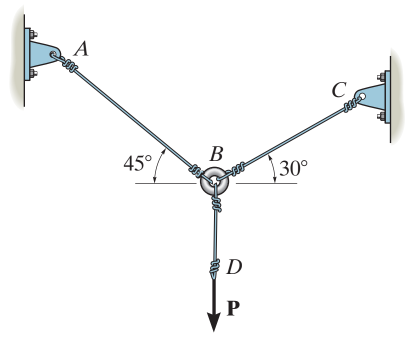
$$\sigma_{\text{allow}} = 20 = \frac{N}{(2)\,\frac{\pi}{4}\,(0.3)^2}$$

$N = 2.827$ kip

From Eq. (1),

$P = 3.26$ kip (controls) **Ans.**

1–82. The three steel wires are used to support the load. If the wires have an allowable tensile stress of $\sigma_{\text{allow}} = 165$ MPa, determine the required diameter of each wire if the applied load is $P = 6$ kN.



The force in wire BD is equal to the applied load; ie, $F_{BD} = P = 6$ kN. Analysing the equilibrium of joint B by referring to its FBD, Fig. a,

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_{BC}\cos 30° - F_{AB}\cos 45° = 0 \qquad \textbf{(1)}$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{BC}\sin 30° + F_{AB}\sin 45° - 6 = 0 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2),

$F_{AB} = 5.379$ kN $\qquad F_{BC} = 4.392$ kN

For wire BD,

$$\sigma_{\text{allow}} = \frac{F_{BD}}{A_{BD}}; \qquad 165(10^6) = \frac{6(10^3)}{\frac{\pi}{4}d_{BD}^{\,2}}$$

$$d_{BD} = 0.006804 \text{ m} = 6.804 \text{ mm}$$

Use $d_{BD} = 7.00$ mm **Ans.**

For wire AB,

$$\sigma_{\text{allow}} = \frac{F_{AB}}{A_{AB}}; \qquad 165(10^6) = \frac{5.379(10^3)}{\frac{\pi}{4}d_{AB}^{\,2}}$$

$$d_{AB} = 0.006443 \text{ m} = 6.443 \text{ mm}$$

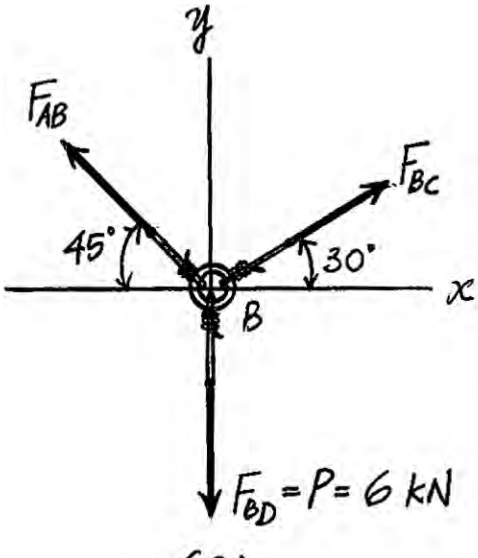
Use $d_{AB} = 6.50$ mm **Ans.**

For wire BC,

$$\sigma_{\text{allow}} = \frac{F_{BC}}{A_{BC}}; \qquad 165(10^6) = \frac{4.392(10^3)}{\frac{\pi}{4}d_{BC}^{\,2}}$$

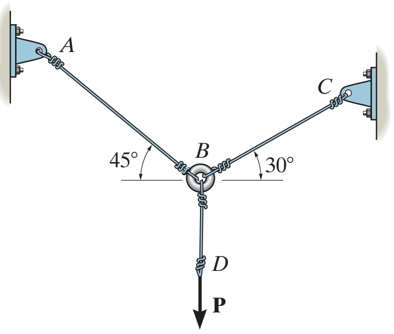
$$d_{BC} = 0.005822 \text{ m} = 5.822 \text{ mm}$$

$d_{BC} = 6.00$ mm **Ans.**



(a)

1–83. The three steel wires are used to support the load. If the wires have an allowable tensile stress of $\sigma_{\text{allow}} = 165$ MPa, and wire AB has a diameter of 6 mm, BC has a diameter of 5 mm, and BD has a diameter of 7 mm, determine the greatest force P that can be applied before one of the wires fails.



The force in wire BD is equal to the applied load; ie, $F_{BD} = P$. Analysing the equilibrium of joint B by referring to its FBD, Fig. a,

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_{BC} \cos 30° - F_{AB} \cos 45° = 0 \qquad \textbf{(1)}$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{BC} \sin 30° + F_{AB} \sin 45° - P = 0 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2),

$$F_{AB} = 0.8966\,P \qquad F_{BC} = 0.7321\,P$$

For wire BD,

$$\sigma_{\text{allow}} = \frac{F_{BD}}{A_{BD}}; \qquad 165(10^6) = \frac{P}{\frac{\pi}{4}(0.007^2)}$$

$$P = 6349.94 \text{ N} = 6.350 \text{ kN}$$

For wire AB,

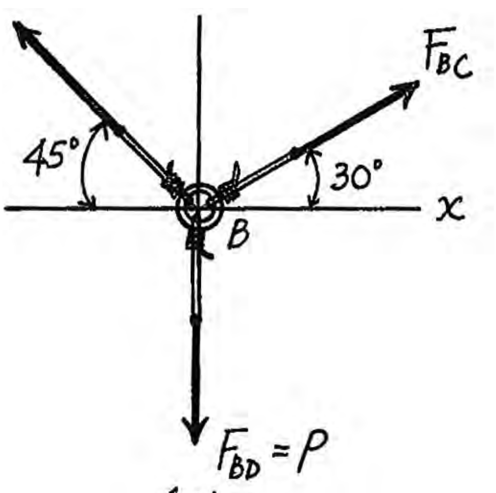
$$\sigma_{\text{allow}} = \frac{F_{AB}}{A_{AB}}; \qquad 165(10^6) = \frac{0.8966\,P}{\frac{\pi}{4}(0.006^2)}$$

$$P = 5203.42 \text{ N} = 5.203 \text{ kN}$$

For wire BC,

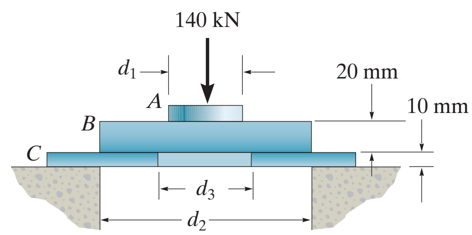
$$\sigma_{\text{allow}} = \frac{F_{BC}}{A_{BC}}; \qquad 165(10^6) = \frac{0.7321\,P}{\frac{\pi}{4}(0.005^2)}$$

$$P = 4425.60 \text{ N} = 4.43 \text{ kN (Controls!)} \qquad \textbf{Ans.}$$



(a)

***1–84.** The assembly consists of three disks A, B, and C that are used to support the load of 140 kN. Determine the smallest diameter d_1 of the top disk, the diameter d_2 within the support space, and the diameter d_3 of the hole in the bottom disk. The allowable bearing stress for the material is $(\sigma_{\text{allow}})_b = 350$ MPa and allowable shear stress is $\tau_{\text{allow}} = 125$ MPa.



Solution

Allowable Bearing Stress: Assume bearing failure for disk B.

$$(\sigma_b)_{\text{allow}} = \frac{P}{A}; \qquad 350\left(10^6\right) = \frac{140(10^3)}{\frac{\pi}{4}\, d_1^2}$$

$$d_1 = 0.02257 \text{ m} = 22.6 \text{ mm}$$

Allowable Shear Stress: Assume shear failure for disk C.

$$\tau_{\text{allow}} = \frac{V}{A}; \qquad 125\left(10^6\right) = \frac{140(10^3)}{\pi d_2\,(0.01)}$$

$$d_2 = 0.03565 \text{ m} = 35.7 \text{ mm} \qquad \textbf{Ans.}$$

Allowable Bearing Stress: Assume bearing failure for disk C.

$$(\sigma_b)_{\text{allow}} = \frac{P}{A}; \qquad 350\left(10^6\right) = \frac{140(10^3)}{\frac{\pi}{4}\left(0.03565^2 - d_3^2\right)}$$

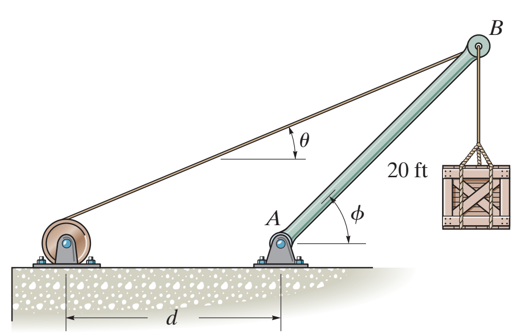
$$d_3 = 0.02760 \text{ m} = 27.6 \text{ mm} \qquad \textbf{Ans.}$$

Since $d_3 = 27.6 \text{ mm} > d_1 = 22.6$ mm, disk B might fail due to shear.

$$\tau = \frac{V}{A} = \frac{140(10^3)}{\pi(0.02257)(0.02)} = 98.7 \text{ MPa} < \tau_{\text{allow}} = 125 \text{ MPa } (\boldsymbol{O.K}\,!)$$

Therefore, $\qquad d_1 = 22.6$ mm $\qquad$ **Ans.**

•1–85. The boom is supported by the winch cable that has a diameter of 0.25 in. and an allowable normal stress of $\sigma_{\text{allow}} = 24$ ksi. Determine the greatest load that can be supported without causing the cable to fail when $\theta = 30°$ and $\phi = 45°$. Neglect the size of the winch.



$$\sigma = \frac{P}{A}; \qquad 24(10^3) = \frac{T}{\frac{\pi}{4}\,(0.25)^2};$$

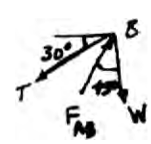
$$T = 1178.10 \text{ lb}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad -1178.10 \cos 30° + F_{AB} \sin 45° = 0$$

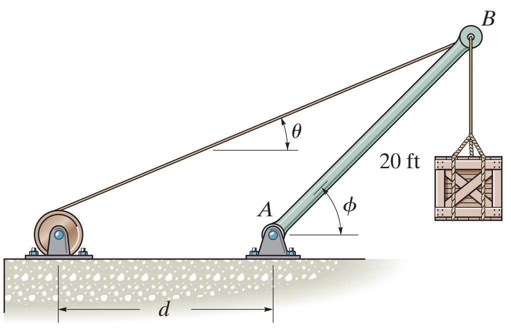
$$+\uparrow \Sigma F_y = 0; \qquad -W + F_{AB} \cos 45° - 1178.10 \sin 30° = 0$$

$$W = 431 \text{ lb} \qquad \textbf{Ans.}$$

$$F_{AB} = 1442.9 \text{ lb}$$



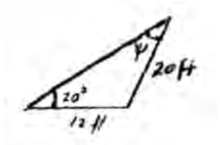
1–86. The boom is supported by the winch cable that has an allowable normal stress of $\sigma_{allow} = 24$ ksi. If it is required that it be able to slowly lift 5000 lb, from $\theta = 20°$ to $\theta = 50°$, determine the smallest diameter of the cable to the nearest $\frac{1}{16}$ in. The boom AB has a length of 20 ft. Neglect the size of the winch. Set $d = 12$ ft.



Maximum tension in cable occurs when $\theta = 20°$.

$$\frac{\sin 20°}{20} = \frac{\sin \psi}{12}$$

$$\psi = 11.842°$$

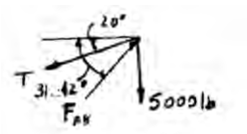


$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad -T \cos 20° + F_{AB} \cos 31.842° = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{AB} \sin 31.842° - T \sin 20° - 5000 = 0$$

$$T = 20\,698.3 \text{ lb}$$

$$F_{AB} = 22\,896 \text{ lb}$$

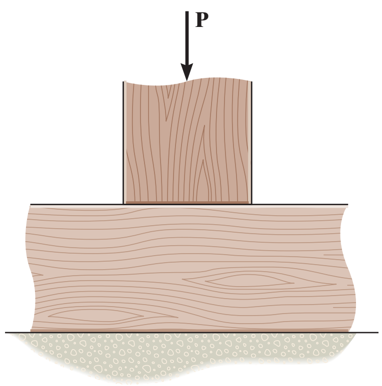


$$\sigma = \frac{P}{A}; \qquad 24(10^3) = \frac{20\,698.3}{\frac{\pi}{4}(d)^2}$$

$$d = 1.048 \text{ in.}$$

Use $\quad d = 1\frac{1}{16}$ in. **Ans.**

1–87. The 60 mm × 60 mm oak post is supported on the pine block. If the allowable bearing stresses for these materials are $\sigma_{oak} = 43$ MPa and $\sigma_{pine} = 25$ MPa, determine the greatest load P that can be supported. If a rigid bearing plate is used between these materials, determine its required area so that the maximum load P can be supported. What is this load?



For failure of pine block:

$$\sigma = \frac{P}{A}; \qquad 25(10^6) = \frac{P}{(0.06)(0.06)}$$

$$P = 90 \text{ kN}$$ **Ans.**

For failure of oak post:

$$\sigma = \frac{P}{A}; \qquad 43(10^6) = \frac{P}{(0.06)(0.06)}$$

$$P = 154.8 \text{ kN}$$

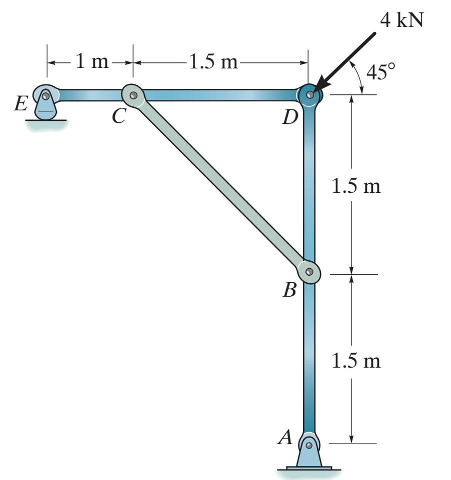
Area of plate based on strength of pine block:

$$\sigma = \frac{P}{A}; \qquad 25(10^6) = \frac{154.8(10)^3}{A}$$

$$A = 6.19(10^{-3})\text{m}^2$$ **Ans.**

$$P_{max} = 155 \text{ kN}$$ **Ans.**

***1–88.** The frame is subjected to the load of 4 kN which acts on member ABD at D. Determine the required diameter of the pins at D and C if the allowable shear stress for the material is $\tau_{allow} = 40$ MPa. Pin C is subjected to double shear, whereas pin D is subjected to single shear.



Referring to the FBD of member DCE, Fig. a,

$$\circlearrowleft + \Sigma M_E = 0; \qquad D_y(2.5) - F_{BC}\sin 45°\,(1) = 0 \qquad \textbf{(1)}$$

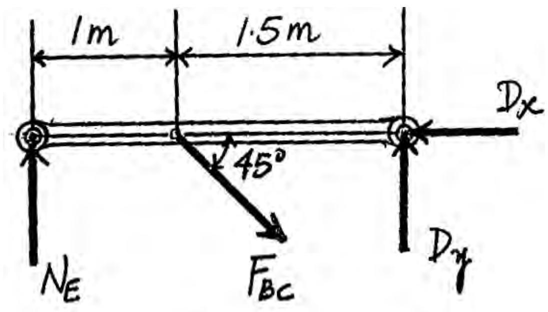
$$\xrightarrow{+} \Sigma F_x = 0 \qquad F_{BC}\cos 45° - D_x = 0 \qquad \textbf{(2)}$$

Referring to the FBD of member ABD, Fig. b,

$$\circlearrowleft + \Sigma M_A = 0; \qquad 4\cos 45°\,(3) + F_{BC}\sin 45°\,(1.5) - D_x\,(3) = 0 \qquad \textbf{(3)}$$

Solving Eqs (2) and (3),

$$F_{BC} = 8.00 \text{ kN} \qquad D_x = 5.657 \text{ kN}$$



(a)

Substitute the result of F_{BC} into (1)

$$D_y = 2.263 \text{ kN}$$

Thus, the force acting on pin D is

$$F_D = \sqrt{D_x^{\,2} + D_y^{\,2}} = \sqrt{5.657^2 + 2.263^2} = 6.093 \text{ kN}$$

Pin C is subjected to double shear white pin D is subjected to single shear. Referring to the FBDs of pins C, and D in Fig c and d, respectively,

$$V_C = \frac{F_{BC}}{2} = \frac{8.00}{2} = 4.00 \text{ kN} \qquad V_D = F_D = 6.093 \text{ kN}$$

For pin C,

$$\tau_{allow} = \frac{V_C}{A_C}; \qquad 40(10^6) = \frac{4.00(10^3)}{\frac{\pi}{4}\,d_C^{\,2}}$$

$$d_C = 0.01128 \text{ m} = 11.28 \text{ mm}$$

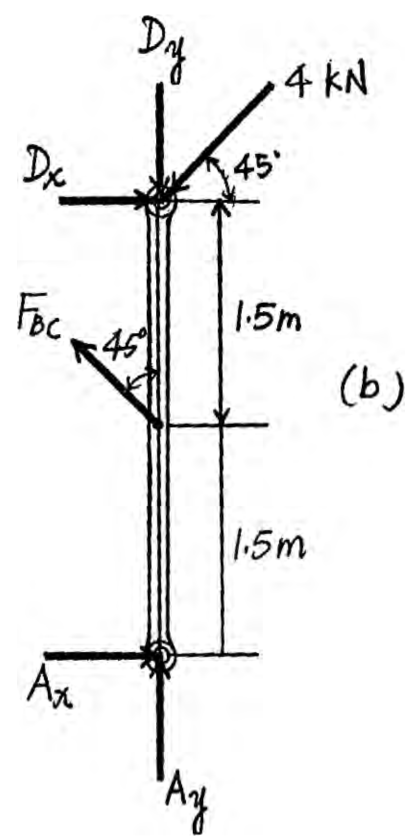
Use $d_C = 12$ mm **Ans.**

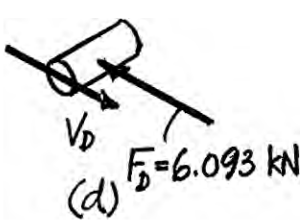
For pin D,

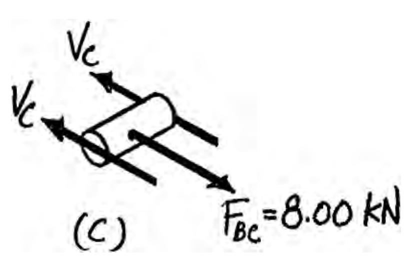
$$\tau_{allow} = \frac{V_D}{A_D}; \qquad 40(10^6) = \frac{6.093(10^3)}{\frac{\pi}{4}\,d_D^{\,2}}$$

$$d_D = 0.01393 \text{ m} = 13.93 \text{ mm}$$

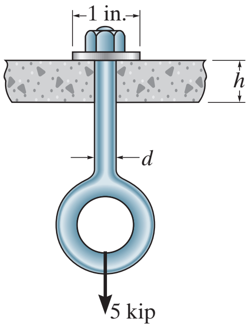
Use $d_D = 14$ mm **Ans.**







•1–89. The eye bolt is used to support the load of 5 kip. Determine its diameter d to the nearest $\frac{1}{8}$ in. and the required thickness h to the nearest $\frac{1}{8}$ in. of the support so that the washer will not penetrate or shear through it. The allowable normal stress for the bolt is $\sigma_{\text{allow}} = 21$ ksi and the allowable shear stress for the supporting material is $\tau_{\text{allow}} = 5$ ksi.



Allowable Normal Stress: Design of bolt size

$$\sigma_{\text{allow}} = \frac{P}{A_b}; \qquad 21.0(10^3) = \frac{5(10^3)}{\frac{\pi}{4}d^2}$$

$$d = 0.5506 \text{ in.}$$

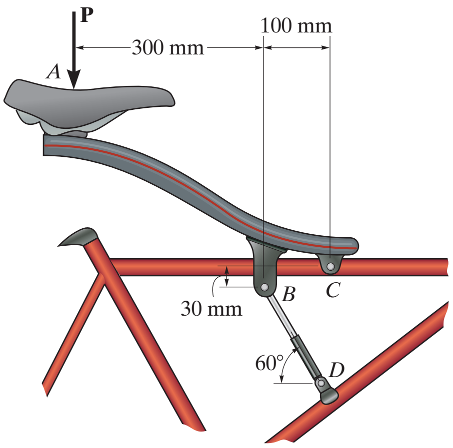
$$\text{Use } d = \frac{5}{8} \text{ in.} \qquad \textbf{Ans.}$$

Allowable Shear Stress: Design of support thickness

$$\tau_{\text{allow}} = \frac{V}{A}; \qquad 5(10^3) = \frac{5(10^3)}{\pi(1)(h)}$$

$$\text{Use } h = \frac{3}{8} \text{ in.} \qquad \textbf{Ans.}$$

1–90. The soft-ride suspension system of the mountain bike is pinned at C and supported by the shock absorber BD. If it is designed to support a load $P = 1500$ N, determine the required minimum diameter of pins B and C. Use a factor of safety of 2 against failure. The pins are made of material having a failure shear stress of $\tau_{\text{fail}} = 150$ MPa, and each pin is subjected to double shear.



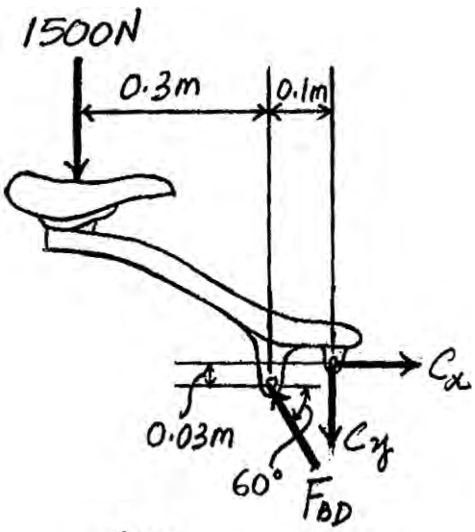
Internal Loadings: The forces acting on pins B and C can be determined by considering the equilibrium of the free-body diagram of the soft-ride suspension system shown in Fig. *a*.

$\circlearrowleft + \Sigma M_C = 0;\quad 1500(0.4) - F_{BD} \sin 60°(0.1) - F_{BD} \cos 60°(0.03) = 0$

$F_{BD} = 5905.36 \text{ N}$

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad C_x - 5905.36 \cos 60° = 0 \qquad C_x = 2952.68 \text{ N}$

$+\uparrow \Sigma F_y = 0;\quad 5905.36 \sin 60° - 1500 - C_y = 0 \qquad C_y = 3614.20 \text{ N}$



(a)

Thus,

$F_B = F_{BD} = 5905.36 \text{ N} \qquad F_C = \sqrt{{C_x}^2 + {C_y}^2} = \sqrt{2952.68^2 + 3614.20^2}$

$= 4666.98 \text{ N}$

Since **both** pins are in double shear,

$$V_B = \frac{F_B}{2} = \frac{5905.36}{2} = 2952.68 \text{ N} \qquad V_C = \frac{F_C}{2} = \frac{4666.98}{2} = 2333.49 \text{ N}$$

Allowable Shear Stress:

$$\tau_{\text{allow}} = \frac{\tau_{\text{fail}}}{\text{F.S.}} = \frac{150}{2} = 75 \text{ MPa}$$

Using this result,

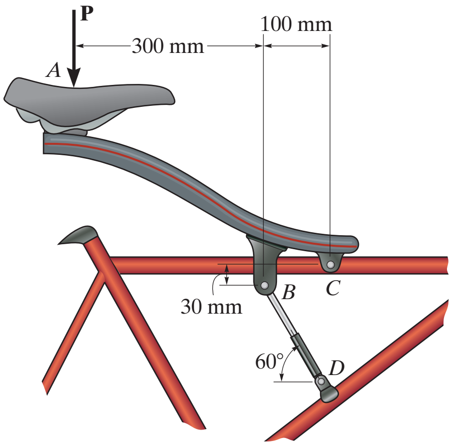
$$\tau_{\text{allow}} = \frac{V_B}{A_B};\qquad 75(10^6) = \frac{2952.68}{\frac{\pi}{4} {d_B}^2}$$

$$d_B = 0.007080 \text{ m} = 7.08 \text{ mm} \qquad \textbf{Ans.}$$

$$\tau_{\text{allow}} = \frac{V_C}{A_C};\qquad 75(10^6) = \frac{2333.49}{\frac{\pi}{4} {d_C}^2}$$

$$d_C = 0.006294 \text{ m} = 6.29 \text{ mm} \qquad \textbf{Ans.}$$

1–91. The soft-ride suspension system of the mountain bike is pinned at C and supported by the shock absorber BD. If it is designed to support a load of $P = 1500$ N, determine the factor of safety of pins B and C against failure if they are made of a material having a shear failure stress of $\tau_{\text{fail}} = 150$ MPa. Pin B has a diameter of 7.5 mm, and pin C has a diameter of 6.5 mm. Both pins are subjected to double shear.



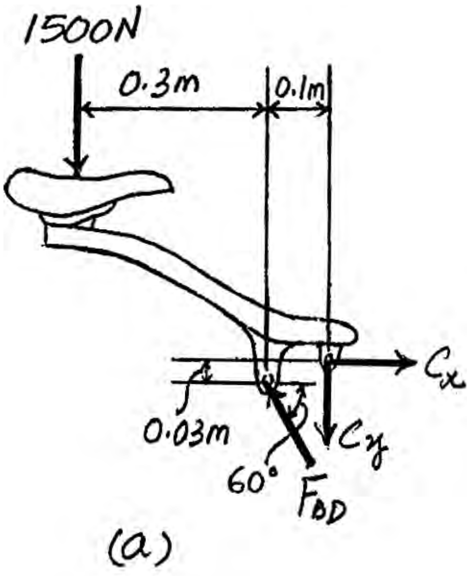
Internal Loadings: The forces acting on pins B and C can be determined by considering the equilibrium of the free-body diagram of the soft-ride suspension system shown in Fig. *a*.

$$+\Sigma M_C = 0; \qquad 1500(0.4) - F_{BD}\sin 60°(0.1) - F_{BD}\cos 60°(0.03) = 0$$

$$F_{BD} = 5905.36 \text{ N}$$

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad C_x - 5905.36\cos 60° = 0 \qquad C_x = 2952.68 \text{ N}$$

$$+\uparrow\Sigma F_y = 0; \qquad 5905.36\sin 60° - 1500 - C_y = 0 \quad C_y = 3614.20 \text{ N}$$



Thus,

$$F_B = F_{BD} = 5905.36 \text{ N} \qquad F_C = \sqrt{C_x^2 + C_y^2} = \sqrt{2952.68^2 + 3614.20^2}$$

$$= 4666.98 \text{ N}$$

Since both pins are in double shear,

$$V_B = \frac{F_B}{2} = \frac{5905.36}{2} = 2952.68\text{N} \qquad V_C = \frac{F_C}{2} = \frac{4666.98}{2} = 2333.49 \text{ N}$$

Allowable Shear Stress: The areas of the shear plane for pins B and C are $A_B = \frac{\pi}{4}(0.0075^2) = 44.179(10^{-6})\text{m}^2$ and $A_C = \frac{\pi}{4}(0.0065^2) = 33.183(10^{-6})\text{m}^2$. We obtain

$$\left(\tau_{\text{avg}}\right)_B = \frac{V_B}{A_B} = \frac{2952.68}{44.179(10^{-6})} = 66.84 \text{ MPa}$$

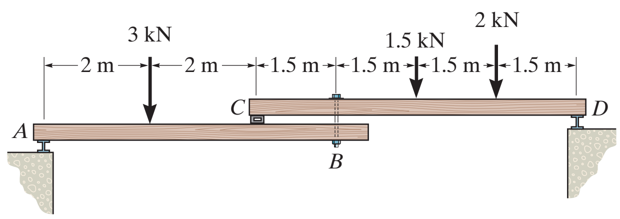
$$\left(\tau_{\text{avg}}\right)_C = \frac{V_C}{A_C} = \frac{2333.49}{33.183(10^{-6})} = 70.32 \text{ MPa}$$

Using these results,

$$(\text{F.S.})_B = \frac{\tau_{\text{fail}}}{\left(\tau_{\text{avg}}\right)_B} = \frac{150}{66.84} = 2.24 \qquad \textbf{Ans.}$$

$$(\text{F.S.})_C = \frac{\tau_{\text{fail}}}{\left(\tau_{\text{avg}}\right)_C} = \frac{150}{70.32} = 2.13 \qquad \textbf{Ans.}$$

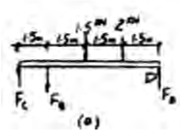
***1–92.** The compound wooden beam is connected together by a bolt at B. Assuming that the connections at A, B, C, and D exert only vertical forces on the beam, determine the required diameter of the bolt at B and the required outer diameter of its washers if the allowable tensile stress for the bolt is $(\sigma_t)_{\text{allow}} = 150$ MPa and the allowable bearing stress for the wood is $(\sigma_b)_{\text{allow}} = 28$ MPa. Assume that the hole in the washers has the same diameter as the bolt.



From FBD (a):

$$\circlearrowleft +\Sigma M_D = 0; \qquad F_B(4.5) + 1.5(3) + 2(1.5) - F_C(6) = 0$$

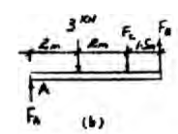
$$4.5\,F_B - 6\,F_C = -7.5 \qquad \textbf{(1)}$$



From FBD (b):

$$\circlearrowleft +\Sigma M_D = 0; \qquad F_B(5.5) - F_C(4) - 3(2) = 0$$

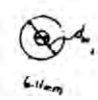
$$5.5\,F_B - 4\,F_C = 6 \qquad \textbf{(2)}$$



Solving Eqs. (1) and (2) yields

$F_B = 4.40$ kN; $\quad F_C = 4.55$ kN

For bolt:



$$\sigma_{\text{allow}} = 150(10^6) = \frac{4.40(10^3)}{\frac{\pi}{4}(d_B)^2}$$

$$d_B = 0.00611 \text{ m}$$

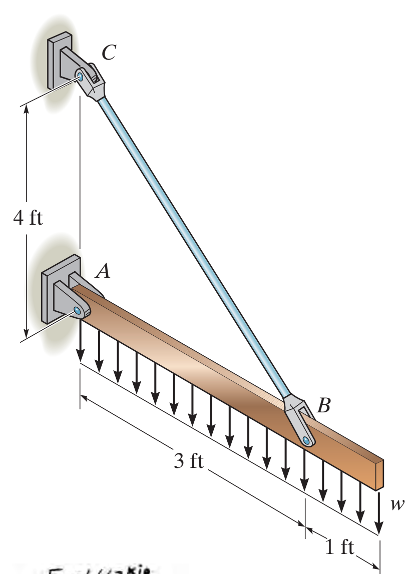
$$= 6.11 \text{ mm} \qquad \textbf{Ans.}$$

For washer:

$$\sigma_{\text{allow}} = 28(10^4) = \frac{4.40(10^3)}{\frac{\pi}{4}(d_w^2 - 0.00611^2)}$$

$$d_w = 0.0154 \text{ m} = 15.4 \text{ mm} \qquad \textbf{Ans.}$$

•1–93. The assembly is used to support the distributed loading of $w = 500$ lb/ft. Determine the factor of safety with respect to yielding for the steel rod BC and the pins at B and C if the yield stress for the steel in tension is $\sigma_y = 36$ ksi and in shear $\tau_y = 18$ ksi. The rod has a diameter of 0.40 in., and the pins each have a diameter of 0.30 in.



For rod BC:

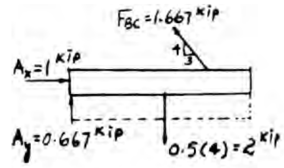
$$\sigma = \frac{P}{A} = \frac{1.667}{\frac{\pi}{4}(0.4^2)} = 13.26 \text{ ksi}$$

$$\text{F. S.} = \frac{\sigma_y}{\sigma} = \frac{36}{13.26} = 2.71 \qquad \textbf{Ans.}$$

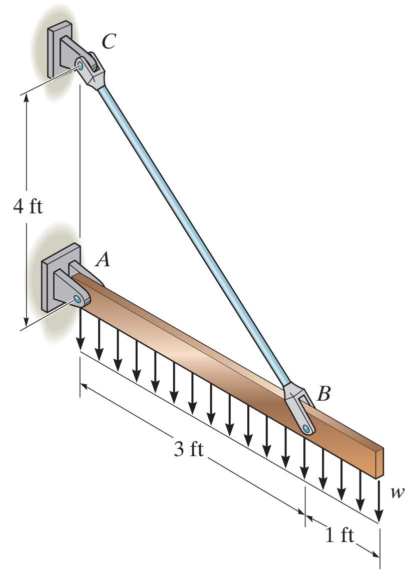
For pins B and C:

$$\tau = \frac{V}{A} = \frac{0.8333}{\frac{\pi}{4}(0.3^2)} = 11.79 \text{ ksi}$$

$$\text{F. S.} = \frac{\tau_y}{\tau} = \frac{18}{11.79} = 1.53 \qquad \textbf{Ans.}$$



1–94. If the allowable shear stress for each of the 0.30-in.-diameter steel pins at A, B, and C is $\tau_{\text{allow}} = 12.5$ ksi, and the allowable normal stress for the 0.40-in.-diameter rod is $\sigma_{\text{allow}} = 22$ ksi, determine the largest intensity w of the uniform distributed load that can be suspended from the beam.



Assume failure of pins B and C:

$$\tau_{\text{allow}} = 12.5 = \frac{1.667w}{\frac{\pi}{4}(0.3^2)}$$

$$w = 0.530 \text{ kip/ft} \qquad \text{(controls)}$$ **Ans.**

Assume failure of pins A:

$$F_A = \sqrt{(2w)^2 + (1.333w)^2} = 2.404\,w$$

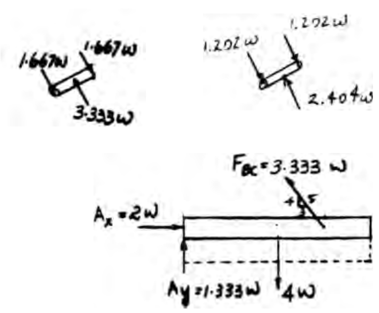
$$\tau_{\text{allow}} = 12.5 = \frac{1.202w}{\frac{\pi}{4}(0.3^2)}$$

$$w = 0.735 \text{ kip/ft}$$

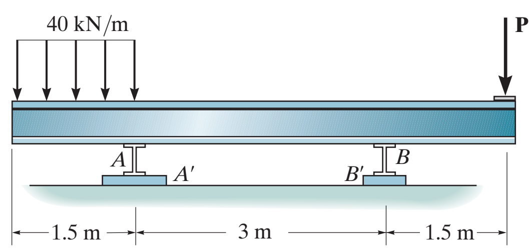
Assume failure of rod BC:

$$\sigma_{\text{allow}} = 22 = \frac{3.333w}{\frac{\pi}{4}(0.4^2)}$$

$$w = 0.829 \text{ kip/ft}$$



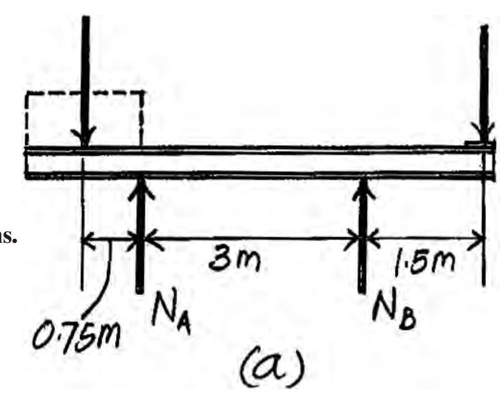
1–95. If the allowable bearing stress for the material under the supports at A and B is $(\sigma_b)_{\text{allow}} = 1.5$ MPa, determine the size of *square* bearing plates A' and B' required to support the load. Dimension the plates to the nearest mm. The reactions at the supports are vertical. Take $P = 100$ kN.



Referring to the FBD of the bean, Fig. a

$\zeta + \Sigma M_A = 0;\quad N_B(3) + 40(1.5)(0.75) - 100(4.5) = 0 \qquad N_B = 135$ kN

$\zeta + \Sigma M_B = 0;\quad 40(1.5)(3.75) - 100(1.5) - N_A(3) = 0 \qquad N_A = 25.0$ kN



For plate A',

$$(\sigma_b)_{\text{allow}} = \frac{N_A}{A_{A'}}; \qquad 1.5(10^6) = \frac{25.0(10^3)}{a_{A'}^2}$$

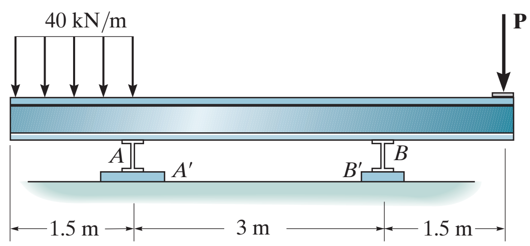
$$a_{A'} = 0.1291 \text{ m} = 130 \text{ mm}$$ **Ans.**

For plate B',

$$\sigma_{\text{allow}} = \frac{N_B}{A_{B'}}; \qquad 1.5(10^6) = \frac{135(10^3)}{a_{B'}^2}$$

$$a_{B'} = 0.300 \text{ m} = 300 \text{ mm}$$ **Ans.**

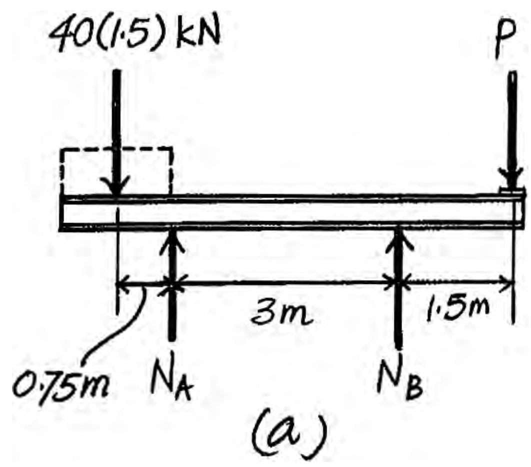
***1–96.** If the allowable bearing stress for the material under the supports at A and B is $(\sigma_b)_{\text{allow}} = 1.5$ MPa, determine the maximum load P that can be applied to the beam. The bearing plates A' and B' have square cross sections of 150 mm × 150 mm and 250 mm × 250 mm, respectively.



Referring to the FBD of the beam, Fig. a,

$\zeta + \Sigma M_A = 0; \quad N_B(3) + 40(1.5)(0.75) - P(4.5) = 0 \quad N_B = 1.5P - 15$

$\zeta + \Sigma M_B = 0; \quad 40(1.5)(3.75) - P(1.5) - N_A(3) = 0 \quad N_A = 75 - 0.5P$



For plate A',

$$(\sigma_b)_{\text{allow}} = \frac{N_A}{A_{A'}}; \qquad 1.5(10^6) = \frac{(75 - 0.5P)(10^3)}{0.15(0.15)}$$

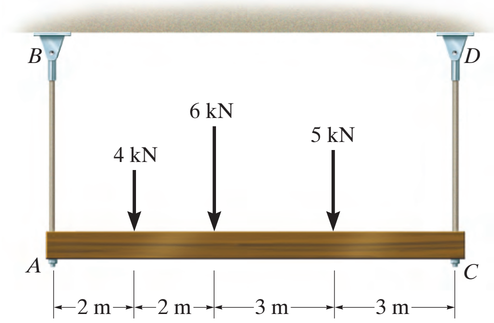
$$P = 82.5 \text{ kN}$$

For plate B',

$$(\sigma_b)_{\text{allow}} = \frac{N_B}{A_{B'}}; \qquad 1.5(10^6) = \frac{(1.5P - 15)(10^3)}{0.25(0.25)}$$

$$P = 72.5 \text{ kN} \quad \text{(Controls!)}$$ **Ans.**

•1–97. The rods AB and CD are made of steel having a failure tensile stress of $\sigma_{\text{fail}} = 510$ MPa. Using a factor of safety of F.S. = 1.75 for tension, determine their smallest diameter so that they can support the load shown. The beam is assumed to be pin connected at A and C.



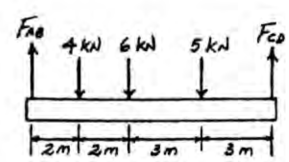
Support Reactions:

$\zeta + \Sigma M_A = 0; \quad F_{CD}(10) - 5(7) - 6(4) - 4(2) = 0$

$$F_{CD} = 6.70 \text{ kN}$$

$\zeta + \Sigma M_C = 0; \quad 4(8) + 6(6) + 5(3) - F_{AB}(10) = 0$

$$F_{AB} = 8.30 \text{ kN}$$



Allowable Normal Stress: Design of rod sizes

For rod AB

$$\sigma_{\text{allow}} = \frac{\sigma_{\text{fail}}}{\text{F.S}} = \frac{F_{AB}}{A_{AB}}; \qquad \frac{510(10^6)}{1.75} = \frac{8.30(10^3)}{\frac{\pi}{4} d_{AB}^2}$$

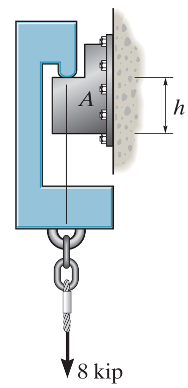
$$d_{AB} = 0.006022 \text{ m} = 6.02 \text{ mm}$$ **Ans.**

For rod CD

$$\sigma_{\text{allow}} = \frac{\sigma_{\text{fail}}}{\text{F.S}} = \frac{F_{CD}}{A_{CD}}; \qquad \frac{510(10^6)}{1.75} = \frac{6.70(10^3)}{\frac{\pi}{4} d_{CD}^2}$$

$$d_{CD} = 0.005410 \text{ m} = 5.41 \text{ mm}$$ **Ans.**

1–98. The aluminum bracket A is used to support the centrally applied load of 8 kip. If it has a constant thickness of 0.5 in., determine the smallest height h in order to prevent a shear failure. The failure shear stress is $\tau_{\text{fail}} = 23$ ksi. Use a factor of safety for shear of F.S. = 2.5.



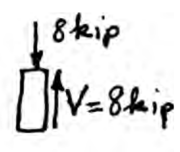
Equation of Equilibrium:

$$+\uparrow \Sigma F_y = 0; \qquad V - 8 = 0 \qquad V = 8.00 \text{ kip}$$

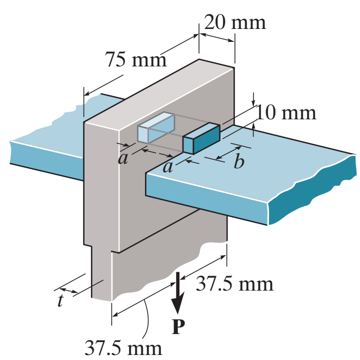
Allowable Shear Stress: Design of the support size

$$\tau_{\text{allow}} = \frac{\tau_{\text{fail}}}{\text{F.S}} = \frac{V}{A}; \qquad \frac{23(10^3)}{2.5} = \frac{8.00(10^3)}{h(0.5)}$$

$$h = 1.74 \text{ in.} \qquad \textbf{Ans.}$$



1–99. The hanger is supported using the rectangular pin. Determine the magnitude of the allowable suspended load **P** if the allowable bearing stress is $(\sigma_b)_{\text{allow}} = 220$ MPa, the allowable tensile stress is $(\sigma_t)_{\text{allow}} = 150$ MPa, and the allowable shear stress is $\tau_{\text{allow}} = 130$ MPa. Take $t = 6$ mm, $a = 5$ mm, and $b = 25$ mm.



Allowable Normal Stress: For the hanger

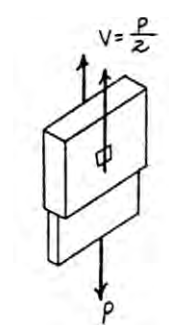
$$(\sigma_t)_{\text{allow}} = \frac{P}{A}; \qquad 150\left(10^6\right) = \frac{P}{(0.075)(0.006)}$$

$$P = 67.5 \text{ kN}$$

Allowable Shear Stress: The pin is subjected to double shear. Therefore, $V = \dfrac{P}{2}$

$$\tau_{\text{allow}} = \frac{V}{A}; \qquad 130\left(10^6\right) = \frac{P/2}{(0.01)(0.025)}$$

$$P = 65.0 \text{ kN}$$

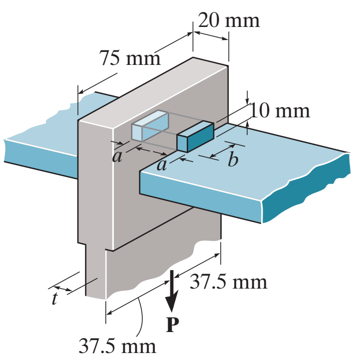


Allowable Bearing Stress: For the bearing area

$$(\sigma_b)_{\text{allow}} = \frac{P}{A}; \qquad 220\left(10^6\right) = \frac{P/2}{(0.005)(0.025)}$$

$$P = 55.0 \text{ kN } (\textit{Controls!}) \qquad \textbf{Ans.}$$

***1–100.** The hanger is supported using the rectangular pin. Determine the required thickness t of the hanger, and dimensions a and b if the suspended load is $P = 60$ kN. The allowable tensile stress is $(\sigma_t)_{\text{allow}} = 150$ MPa, the allowable bearing stress is $(\sigma_b)_{\text{allow}} = 290$ MPa, and the allowable shear stress is $\tau_{\text{allow}} = 125$ MPa.

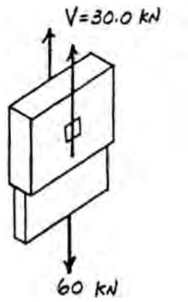


Allowable Normal Stress: For the hanger

$$(\sigma_t)_{\text{allow}} = \frac{P}{A}; \qquad 150\left(10^6\right) = \frac{60(10^3)}{(0.075)t}$$

$$t = 0.005333 \text{ m} = 5.33 \text{ mm} \qquad \textbf{Ans.}$$

Allowable Shear Stress: For the pin



$$\tau_{\text{allow}} = \frac{V}{A}; \qquad 125\left(10^6\right) = \frac{30(10^3)}{(0.01)b}$$

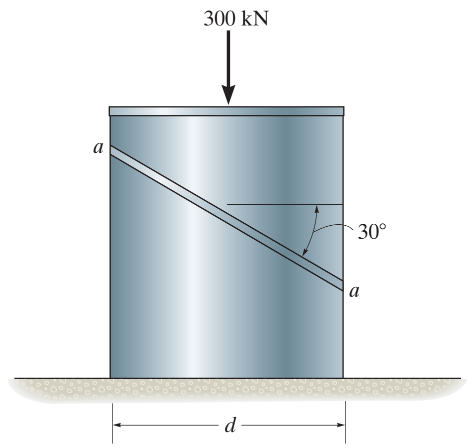
$$b = 0.0240 \text{ m} = 24.0 \text{ mm} \qquad \textbf{Ans.}$$

Allowable Bearing Stress: For the bearing area

$$(\sigma_b)_{\text{allow}} = \frac{P}{A}; \qquad 290\left(10^6\right) = \frac{30(10^3)}{(0.0240)\ a}$$

$$a = 0.00431 \text{ m} = 4.31 \text{ mm} \qquad \textbf{Ans.}$$

•1–101. The 200-mm-diameter aluminum cylinder supports a compressive load of 300 kN. Determine the average normal and shear stress acting on section *a–a*. Show the results on a differential element located on the section.



Referring to the FBD of the upper segment of the cylinder sectional through *a–a* shown in Fig. *a*,

$+\nearrow\Sigma F_{x'} = 0; \qquad N_{a-a} - 300\cos 30° = 0 \qquad N_{a-a} = 259.81 \text{ kN}$

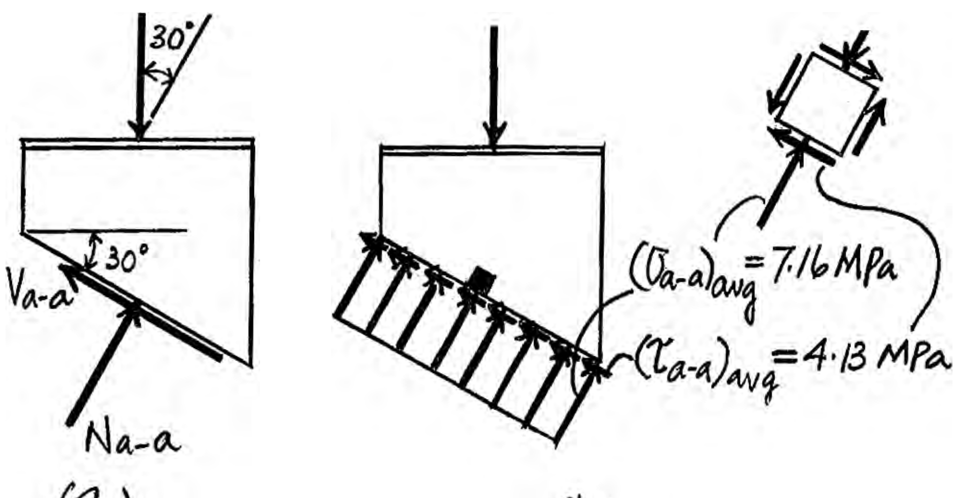
$+\nwarrow\Sigma F_{y'} = 0; \qquad V_{a-a} - 300\sin 30° = 0 \qquad V_{a-a} = 150 \text{ kN}$

Section *a–a* of the cylinder is an ellipse with $a = 0.1$ m and $b = \dfrac{0.1}{\cos 30°}$ m. Thus, $A_{a-a} = \pi ab = \pi(0.1)\left(\dfrac{0.1}{\cos 30°}\right) = 0.03628 \text{ m}^2$.

$$\left(\sigma_{a-a}\right)_{\text{avg}} = \frac{N_{a-a}}{A_{a-a}} = \frac{259.81(10^3)}{0.03628} = 7.162(10^6) \text{ Pa} = 7.16 \text{ MPa} \qquad \textbf{Ans.}$$

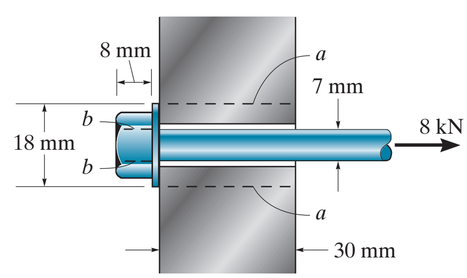
$$\left(\tau_{a-a}\right)_{\text{avg}} = \frac{V_{a-a}}{A_{a-a}} = \frac{150(10^3)}{0.03628} = 4.135(10^6) \text{ Pa} = 4.13 \text{ MPa} \qquad \textbf{Ans.}$$

The differential element representing the state of stress of a point on section *a–a* is shown in Fig. *b*



(a) (b)

1–102. The long bolt passes through the 30-mm-thick plate. If the force in the bolt shank is 8 kN, determine the average normal stress in the shank, the average shear stress along the cylindrical area of the plate defined by the section lines *a–a*, and the average shear stress in the bolt head along the cylindrical area defined by the section lines *b–b*.

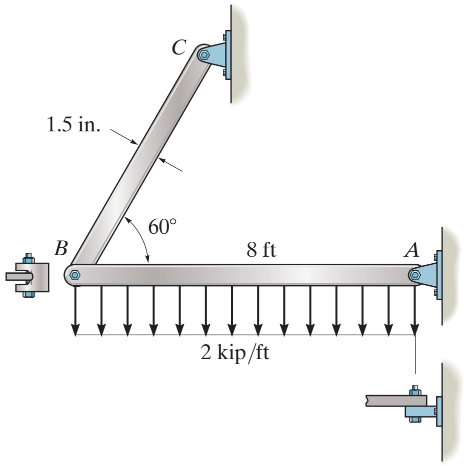


$$\sigma_s = \frac{P}{A} = \frac{8\,(10^3)}{\frac{\pi}{4}\,(0.007)^2} = 208 \text{ MPa} \qquad \textbf{Ans.}$$

$$(\tau_{\text{avg}})_a = \frac{V}{A} = \frac{8\,(10^3)}{\pi\,(0.018)(0.030)} = 4.72 \text{ MPa} \qquad \textbf{Ans.}$$

$$(\tau_{\text{avg}})_b = \frac{V}{A} = \frac{8\,(10^3)}{\pi\,(0.007)(0.008)} = 45.5 \text{ MPa} \qquad \textbf{Ans.}$$

1–103. Determine the required thickness of member BC and the diameter of the pins at A and B if the allowable normal stress for member BC is $\sigma_{\text{allow}} = 29$ ksi and the allowable shear stress for the pins is $\tau_{\text{allow}} = 10$ ksi.



Referring to the FBD of member AB, Fig. a,

$$\circlearrowleft + \Sigma M_A = 0; \qquad 2(8)(4) - F_{BC} \sin 60° \,(8) = 0 \qquad F_{BC} = 9.238 \text{ kip}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 9.238 \cos 60° - A_x = 0 \qquad A_x = 4.619 \text{ kip}$$

$$+\uparrow \Sigma F_y = 0; \qquad 9.238 \sin 60° - 2(8) + A_y = 0 \qquad A_y = 8.00 \text{ kip}$$

Thus, the force acting on pin A is

$$F_A = \sqrt{A_x^2 + A_y^2} = \sqrt{4.619^2 + 8.00^2} = 9.238 \text{ kip}$$

Pin A is subjected to single shear, Fig. c, while pin B is subjected to double shear, Fig. b.

$$V_A = F_A = 9.238 \text{ kip} \qquad V_B = \frac{F_{BC}}{2} = \frac{9.238}{2} = 4.619 \text{ kip}$$

For member BC

$$\sigma_{\text{allow}} = \frac{F_{BC}}{A_{BC}}; \qquad 29 = \frac{9.238}{1.5(t)} \qquad t = 0.2124 \text{ in.}$$

$$\text{Use } t = \frac{1}{4} \text{ in.}$$ **Ans.**

For pin A,

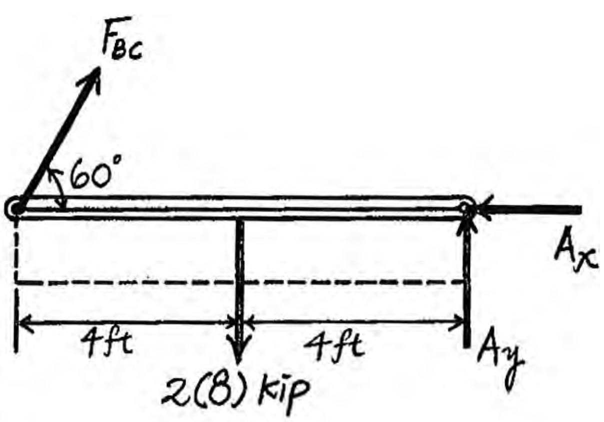
$$\tau_{\text{allow}} = \frac{V_A}{A_A}; \qquad 10 = \frac{9.238}{\frac{\pi}{4} d_A^2} \qquad d_A = 1.085 \text{ in.}$$

$$\text{Use } d_A = 1\frac{1}{8} \text{ in}$$ **Ans.**

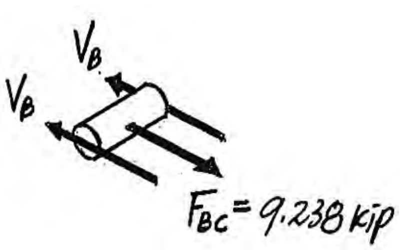
For pin B,

$$\tau_{\text{allow}} = \frac{V_B}{A_B}; \qquad 10 = \frac{4.619}{\frac{\pi}{4} d_B^2} \qquad d_B = 0.7669 \text{ in}$$

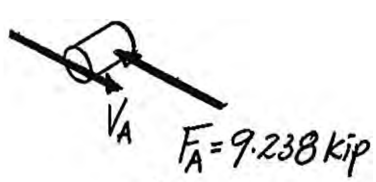
$$\text{Use } d_B = \frac{13}{16} \text{ in}$$ **Ans.**



(a)

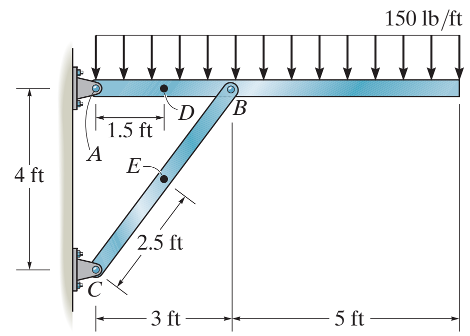


(b)



(c)

***1–104.** Determine the resultant internal loadings acting on the cross sections located through points *D* and *E* of the frame.



Segment *AD*:

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad N_D - 1.2 = 0; \quad N_D = 1.20 \text{ kip}$ **Ans.**

$+\downarrow \Sigma F_y = 0; \quad V_D + 0.225 + 0.4 = 0; \quad V_D = -0.625 \text{ kip}$ **Ans.**

$\circlearrowleft + \Sigma M_D = 0; \quad M_D + 0.225(0.75) + 0.4(1.5) = 0$

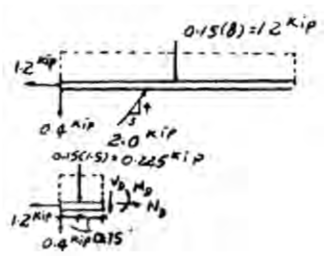
$M_D = -0.769 \text{ kip} \cdot \text{ft}$ **Ans.**

Segment *CE*:

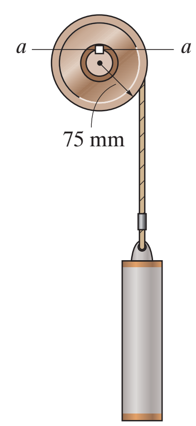
$\nearrow + \Sigma F_x = 0; \quad N_E + 2.0 = 0; \quad N_E = -2.00 \text{ kip}$ **Ans.**

$\searrow + \Sigma F_y = 0; \quad V_E = 0$ **Ans.**

$\circlearrowleft + \Sigma M_E = 0; \quad M_E = 0$ **Ans.**



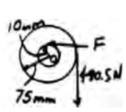
•1–105. The pulley is held fixed to the 20-mm-diameter shaft using a key that fits within a groove cut into the pulley and shaft. If the suspended load has a mass of 50 kg, determine the average shear stress in the key along section *a–a*. The key is 5 mm by 5 mm square and 12 mm long.

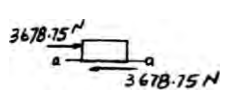


$\circlearrowleft + \Sigma M_O = 0; \quad F\,(10) - 490.5\,(75) = 0$

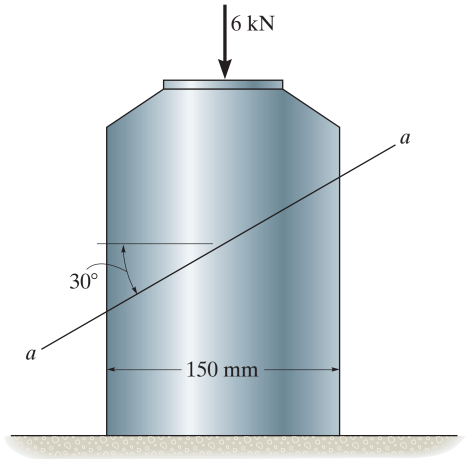
$F = 3678.75 \text{ N}$

$$\tau_{\text{avg}} = \frac{V}{A} = \frac{3678.75}{(0.005)(0.012)} = 61.3 \text{ MPa}$$ **Ans.**





1–106. The bearing pad consists of a 150 mm by 150 mm block of aluminum that supports a compressive load of 6 kN. Determine the average normal and shear stress acting on the plane through section *a–a*. Show the results on a differential volume element located on the plane.



Equation of Equilibrium:

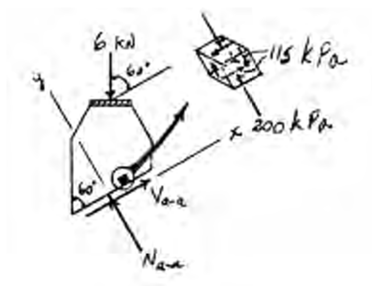
$+\nearrow \Sigma F_x = 0; \quad V_{a-a} - 6\cos 60° = 0 \quad V_{a-a} = 3.00 \text{ kN}$

$\nwarrow + \Sigma F_y = 0; \quad N_{a-a} - 6\sin 60° = 0 \quad N_{a-a} = 5.196 \text{ kN}$

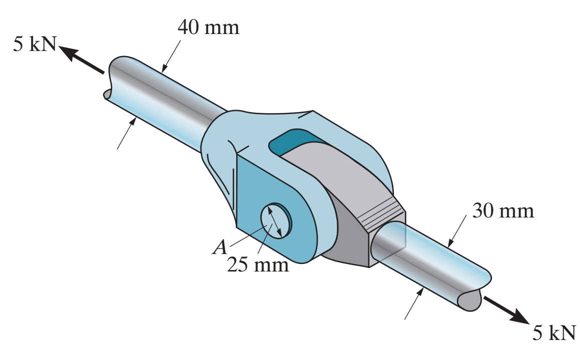
Averge Normal Stress And Shear Stress: The cross sectional Area at section *a–a* is $A = \left(\frac{0.15}{\sin 60°}\right)(0.15) = 0.02598 \text{ m}^2$.

$$\sigma_{a-a} = \frac{N_{a-a}}{A} = \frac{5.196(10^3)}{0.02598} = 200 \text{ kPa} \qquad \textbf{Ans.}$$

$$\tau_{a-a} = \frac{V_{a-a}}{A} = \frac{3.00(10^3)}{0.02598} = 115 \text{ kPa} \qquad \textbf{Ans.}$$



1–107. The yoke-and-rod connection is subjected to a tensile force of 5 kN. Determine the average normal stress in each rod and the average shear stress in the pin *A* between the members.

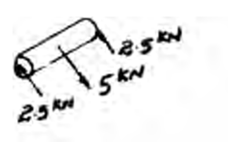


For the 40 – mm – dia rod:

$$\sigma_{40} = \frac{P}{A} = \frac{5\,(10^3)}{\frac{\pi}{4}\,(0.04)^2} = 3.98 \text{ MPa}$$

For the 30 – mm – dia rod:

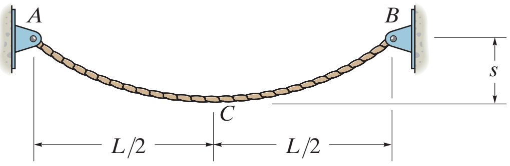
$$\sigma_{30} = \frac{V}{A} = \frac{5\,(10^3)}{\frac{\pi}{4}\,(0.03)^2} = 7.07 \text{ MPa} \qquad \textbf{Ans.}$$



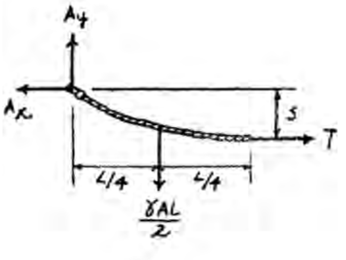
Average shear stress for pin *A*:

$$\tau_{\text{avg}} = \frac{P}{A} = \frac{2.5\,(10^3)}{\frac{\pi}{4}\,(0.025)^2} = 5.09 \text{ MPa} \qquad \textbf{Ans.}$$

***1–108.** The cable has a specific weight γ (weight/volume) and cross-sectional area A. If the sag s is small, so that its length is approximately L and its weight can be distributed uniformly along the horizontal axis, determine the average normal stress in the cable at its lowest point C.



Equation of Equilibrium:



$$\circlearrowleft +\Sigma M_A = 0; \qquad Ts - \frac{\gamma AL}{2}\left(\frac{L}{4}\right) = 0$$

$$T = \frac{\gamma AL^2}{8\,s}$$

Average Normal Stress:

$$\sigma = \frac{T}{A} = \frac{\frac{\gamma AL^2}{8\,s}}{A} = \frac{\gamma L^2}{8\,s}$$ **Ans.**

2–1. An air-filled rubber ball has a diameter of 6 in. If the air pressure within it is increased until the ball's diameter becomes 7 in., determine the average normal strain in the rubber.

$d_0 = 6$ in.

$d = 7$ in.

$$\varepsilon = \frac{\pi d - \pi d_0}{\pi d_0} = \frac{7 - 6}{6} = 0.167 \text{ in./in.}$$ **Ans.**

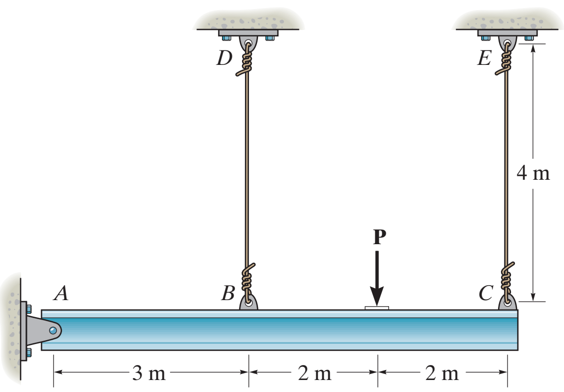
2–2. A thin strip of rubber has an unstretched length of 15 in. If it is stretched around a pipe having an outer diameter of 5 in., determine the average normal strain in the strip.

$L_0 = 15$ in.

$L = \pi(5 \text{ in.})$

$$\varepsilon = \frac{L - L_0}{L_0} = \frac{5\pi - 15}{15} = 0.0472 \text{ in./in.}$$ **Ans.**

2–3. The rigid beam is supported by a pin at A and wires BD and CE. If the load **P** on the beam causes the end C to be displaced 10 mm downward, determine the normal strain developed in wires CE and BD.

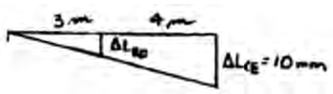


$$\frac{\Delta L_{BD}}{3} = \frac{\Delta L_{CE}}{7}$$

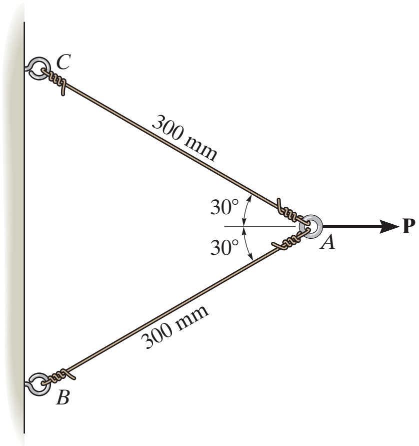
$$\Delta L_{BD} = \frac{3\,(10)}{7} = 4.286 \text{ mm}$$

$$\varepsilon_{CE} = \frac{\Delta L_{CE}}{L} = \frac{10}{4000} = 0.00250 \text{ mm/mm}$$ **Ans.**

$$\varepsilon_{BD} = \frac{\Delta L_{BD}}{L} = \frac{4.286}{4000} = 0.00107 \text{ mm/mm}$$ **Ans.**

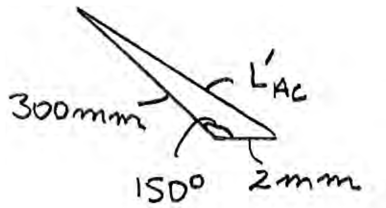


***2–4.** The two wires are connected together at A. If the force **P** causes point A to be displaced horizontally 2 mm, determine the normal strain developed in each wire.

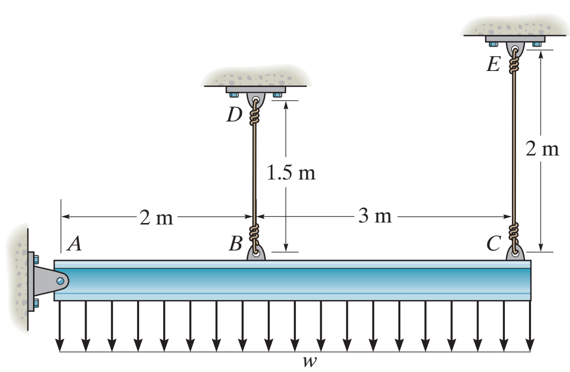


$$L'_{AC} = \sqrt{300^2 + 2^2 - 2(300)(2)\cos 150^\circ} = 301.734 \text{ mm}$$

$$\varepsilon_{AC} = \varepsilon_{AB} = \frac{L'_{AC} - L_{AC}}{L_{AC}} = \frac{301.734 - 300}{300} = 0.00578 \text{ mm/mm} \qquad \textbf{Ans.}$$



•2–5. The rigid beam is supported by a pin at A and wires BD and CE. If the distributed load causes the end C to be displaced 10 mm downward, determine the normal strain developed in wires CE and BD.



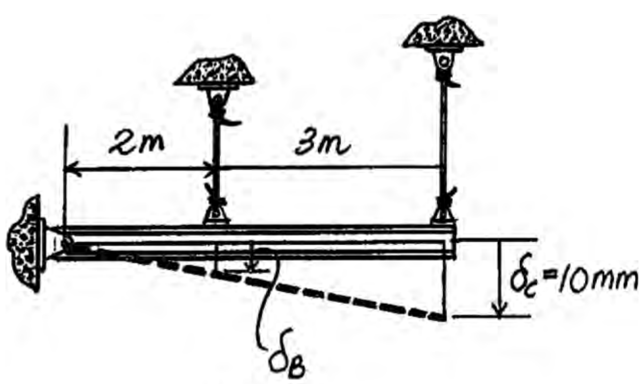
Since the vertical displacement of end C is small compared to the length of member AC, the vertical displacement δ_B of point B, can be approximated by referring to the similar triangle shown in Fig. a

$$\frac{\delta_B}{2} = \frac{10}{5}; \qquad \delta_B = 4 \text{ mm}$$

The unstretched lengths of wires BD and CE are $L_{BD} = 1500$ mm and $L_{CE} = 2000$ mm.

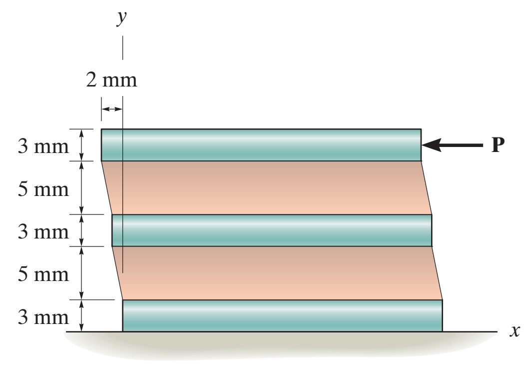
$$\left(\varepsilon_{avg}\right)_{BD} = \frac{\delta_B}{L_{BD}} = \frac{4}{1500} = 0.00267 \text{ mm/mm} \qquad \textbf{Ans.}$$

$$\left(\varepsilon_{avg}\right)_{CE} = \frac{\delta_C}{L_{CE}} = \frac{10}{2000} = 0.005 \text{ mm/mm} \qquad \textbf{Ans.}$$



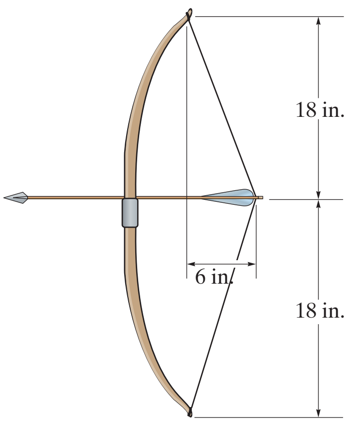
(a)

2–6. Nylon strips are fused to glass plates. When moderately heated the nylon will become soft while the glass stays approximately rigid. Determine the average shear strain in the nylon due to the load **P** when the assembly deforms as indicated.



$$\gamma = \tan^{-1}\left(\frac{2}{10}\right) = 11.31° = 0.197 \text{ rad}$$ **Ans.**

2–7. If the unstretched length of the bowstring is 35.5 in., determine the average normal strain in the string when it is stretched to the position shown.

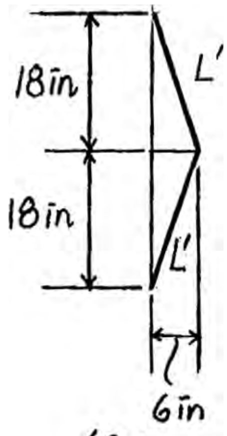


Geometry: Referring to Fig. *a*, the stretched length of the string is

$$L = 2L' = 2\sqrt{18^2 + 6^2} = 37.947 \text{ in.}$$

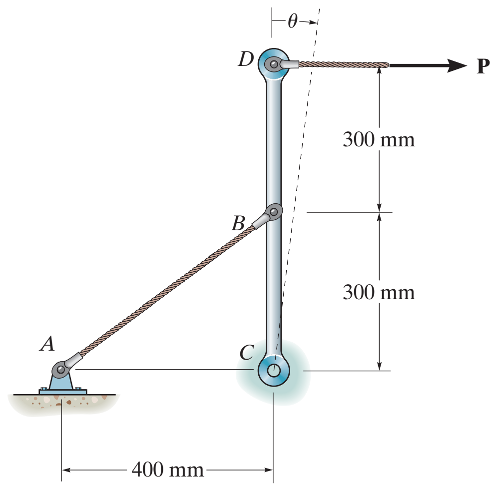
Average Normal Strain:

$$\varepsilon_{avg} = \frac{L - L_0}{L_0} = \frac{37.947 - 35.5}{35.5} = 0.0689 \text{ in./in.}$$ **Ans.**



(a)

***2–8.** Part of a control linkage for an airplane consists of a rigid member CBD and a flexible cable AB. If a force is applied to the end D of the member and causes it to rotate by $\theta = 0.3°$, determine the normal strain in the cable. Originally the cable is unstretched.



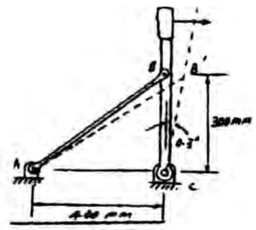
$$AB = \sqrt{400^2 + 300^2} = 500 \text{ mm}$$

$$AB' = \sqrt{400^2 + 300^2 - 2(400)(300)\cos 90.3°}$$

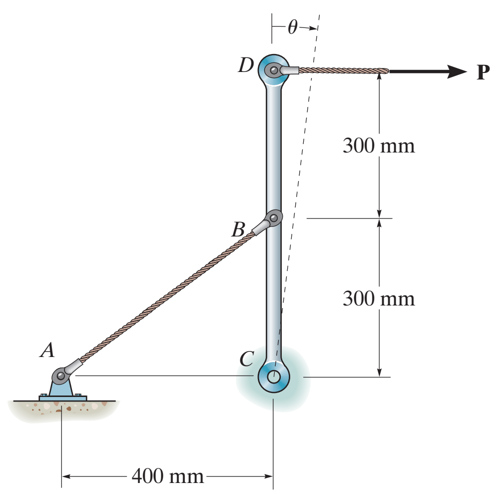
$$= 501.255 \text{ mm}$$

$$\varepsilon_{AB} = \frac{AB' - AB}{AB} = \frac{501.255 - 500}{500}$$

$$= 0.00251 \text{ mm/mm}$$ **Ans.**



•2–9. Part of a control linkage for an airplane consists of a rigid member CBD and a flexible cable AB. If a force is applied to the end D of the member and causes a normal strain in the cable of 0.0035 mm/mm, determine the displacement of point D. Originally the cable is unstretched.



$$AB = \sqrt{300^2 + 400^2} = 500 \text{ mm}$$

$$AB' = AB + \varepsilon_{AB}AB$$

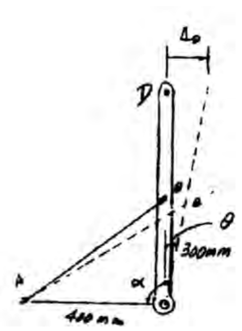
$$= 500 + 0.0035(500) = 501.75 \text{ mm}$$

$$501.75^2 = 300^2 + 400^2 - 2(300)(400)\cos\alpha$$

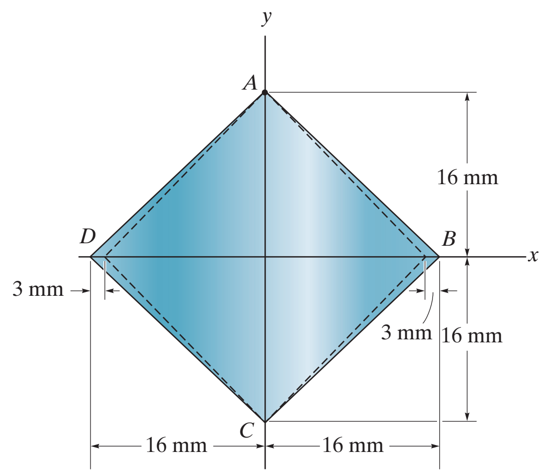
$$\alpha = 90.4185°$$

$$\theta = 90.4185° - 90° = 0.4185° = \frac{\pi}{180°}(0.4185) \text{ rad}$$

$$\Delta_D = 600(\theta) = 600\left(\frac{\pi}{180°}\right)(0.4185) = 4.38 \text{ mm}$$ **Ans.**



2–10. The corners *B* and *D* of the square plate are given the displacements indicated. Determine the shear strains at *A* and *B*.



Applying trigonometry to Fig. a

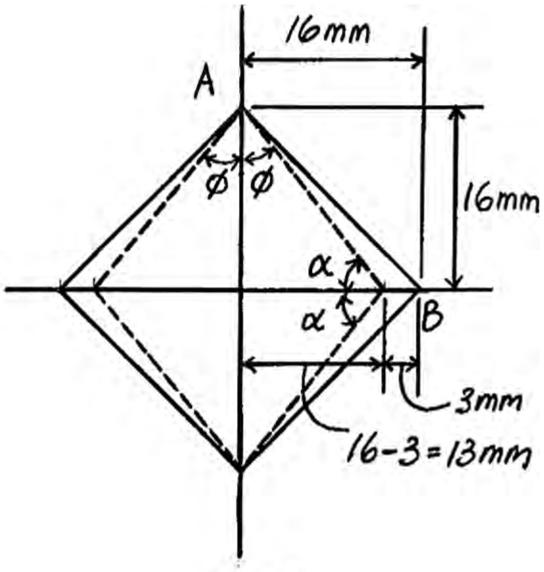
$$\phi = \tan^{-1}\left(\frac{13}{16}\right) = 39.09^\circ\left(\frac{\pi\ \text{rad}}{180^\circ}\right) = 0.6823\ \text{rad}$$

$$\alpha = \tan^{-1}\left(\frac{16}{13}\right) = 50.91^\circ\left(\frac{\pi\ \text{rad}}{180^\circ}\right) = 0.8885\ \text{rad}$$

By the definition of shear strain,

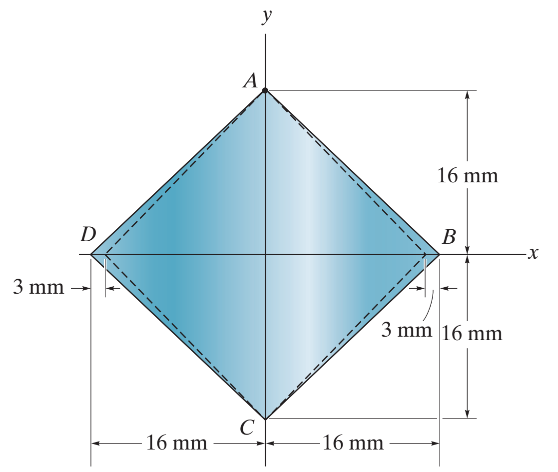
$$\left(\gamma_{xy}\right)_A = \frac{\pi}{2} - 2\phi = \frac{\pi}{2} - 2(0.6823) = 0.206\ \text{rad} \qquad \textbf{Ans.}$$

$$\left(\gamma_{xy}\right)_B = \frac{\pi}{2} - 2\alpha = \frac{\pi}{2} - 2(0.8885) = -0.206\ \text{rad} \qquad \textbf{Ans.}$$



(a)

2–11. The corners B and D of the square plate are given the displacements indicated. Determine the average normal strains along side AB and diagonal DB.



Referring to Fig. a,

$$L_{AB} = \sqrt{16^2 + 16^2} = \sqrt{512}\text{ mm}$$

$$L_{AB'} = \sqrt{16^2 + 13^2} = \sqrt{425}\text{ mm}$$

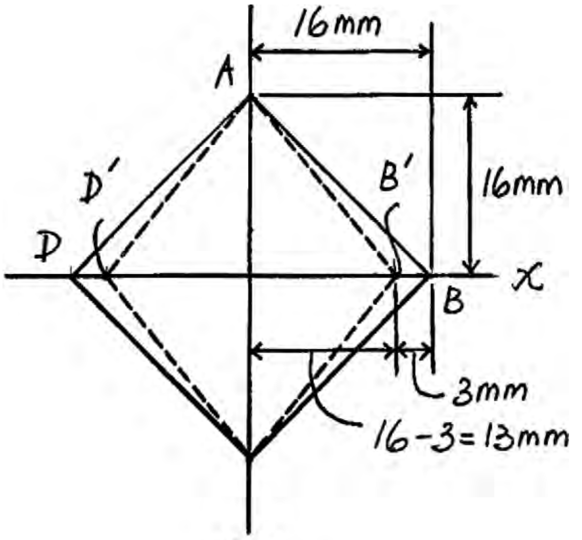
$$L_{BD} = 16 + 16 = 32\text{ mm}$$

$$L_{B'D'} = 13 + 13 = 26\text{ mm}$$

Thus,

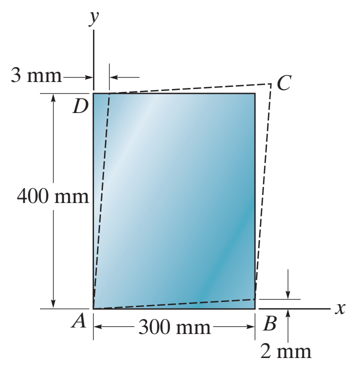
$$\left(\varepsilon_{\text{avg}}\right)_{AB} = \frac{L_{AB'} - L_{AB}}{L_{AB}} = \frac{\sqrt{425} - \sqrt{512}}{\sqrt{512}} = -0.0889\text{ mm/mm}$$ **Ans.**

$$\left(\varepsilon_{\text{avg}}\right)_{BD} = \frac{L_{B'D'} - L_{BD}}{L_{BD}} = \frac{26 - 32}{32} = -0.1875\text{ mm/mm}$$ **Ans.**



(a)

***2–12.** The piece of rubber is originally rectangular. Determine the average shear strain γ_{xy} at A if the corners B and D are subjected to the displacements that cause the rubber to distort as shown by the dashed lines.

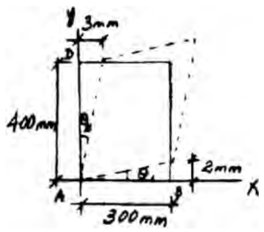


$$\theta_1 = \tan\theta_1 = \frac{2}{300} = 0.006667 \text{ rad}$$

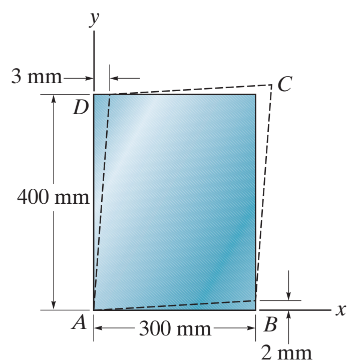
$$\theta_2 = \tan\theta_2 = \frac{3}{400} = 0.0075 \text{ rad}$$

$$\gamma_{xy} = \theta_1 + \theta_2$$

$$= 0.006667 + 0.0075 = 0.0142 \text{ rad} \qquad \textbf{Ans.}$$



•2–13. The piece of rubber is originally rectangular and subjected to the deformation shown by the dashed lines. Determine the average normal strain along the diagonal DB and side AD.



$$AD' = \sqrt{(400)^2 + (3)^2} = 400.01125 \text{ mm}$$

$$\phi = \tan^{-1}\left(\frac{3}{400}\right) = 0.42971^\circ$$

$$AB' = \sqrt{(300)^2 + (2)^2} = 300.00667$$

$$\varphi = \tan^{-1}\left(\frac{2}{300}\right) = 0.381966^\circ$$

$$\alpha = 90^\circ - 0.42971^\circ - 0.381966^\circ = 89.18832^\circ$$

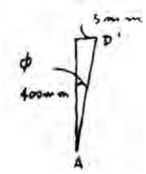
$$D'B' = \sqrt{(400.01125)^2 + (300.00667)^2 - 2(400.01125)(300.00667)\cos(89.18832^\circ)}$$

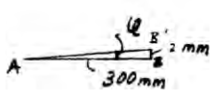
$$D'B' = 496.6014 \text{ mm}$$

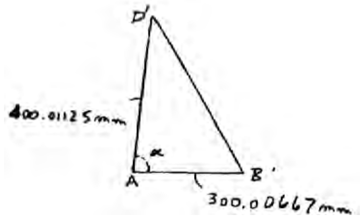
$$DB = \sqrt{(300)^2 + (400)^2} = 500 \text{ mm}$$

$$\varepsilon_{DB} = \frac{496.6014 - 500}{500} = -0.00680 \text{ mm/mm} \qquad \textbf{Ans.}$$

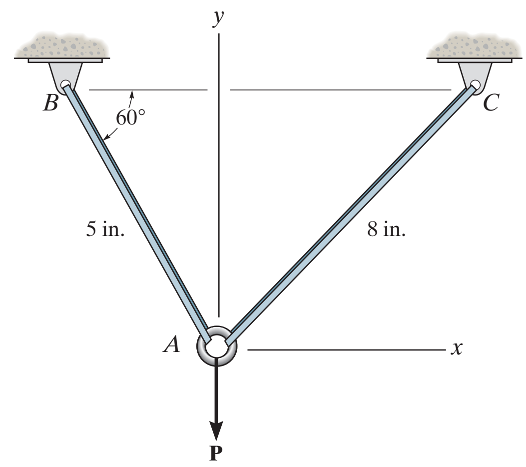
$$\varepsilon_{AD} = \frac{400.01125 - 400}{400} = 0.0281(10^{-3}) \text{ mm/mm} \qquad \textbf{Ans.}$$







2–14. Two bars are used to support a load. When unloaded, AB is 5 in. long, AC is 8 in. long, and the ring at A has coordinates $(0, 0)$. If a load $\mathbf{P}$ acts on the ring at A, the normal strain in AB becomes $\epsilon_{AB} = 0.02$ in./in., and the normal strain in AC becomes $\epsilon_{AC} = 0.035$ in./in. Determine the coordinate position of the ring due to the load.



Average Normal Strain:

$$L'_{AB} = L_{AB} + \varepsilon_{AB} L_{AB} = 5 + (0.02)(5) = 5.10 \text{ in.}$$

$$L'_{AC} = L_{AC} + \varepsilon_{AC} L_{AC} = 8 + (0.035)(8) = 8.28 \text{ in.}$$

Geometry:

$$a = \sqrt{8^2 - 4.3301^2} = 6.7268 \text{ in.}$$

$$5.10^2 = 9.2268^2 + 8.28^2 - 2(9.2268)(8.28)\cos\theta$$

$$\theta = 33.317^\circ$$

$$x' = 8.28 \cos 33.317^\circ = 6.9191 \text{ in.}$$

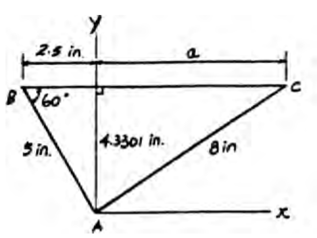
$$y' = 8.28 \sin 33.317^\circ = 4.5480 \text{ in.}$$

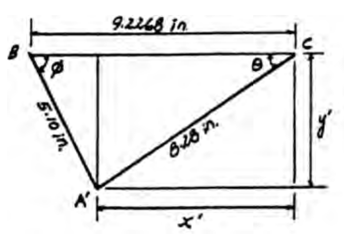
$$x = -(x' - a)$$

$$= -(6.9191 - 6.7268) = -0.192 \text{ in.}$$ **Ans.**

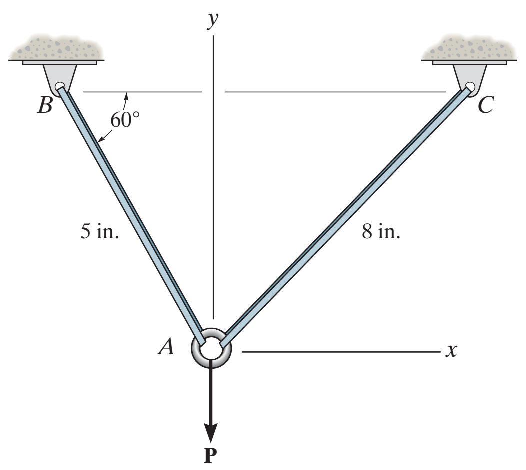
$$y = -(y' - 4.3301)$$

$$= -(4.5480 - 4.3301) = -0.218 \text{ in.}$$ **Ans.**





2–15. Two bars are used to support a load **P**. When unloaded, AB is 5 in. long, AC is 8 in. long, and the ring at A has coordinates (0, 0). If a load is applied to the ring at A, so that it moves it to the coordinate position (0.25 in., −0.73 in.), determine the normal strain in each bar.



Geometry:

$$a = \sqrt{8^2 - 4.3301^2} = 6.7268 \text{ in.}$$

$$L_{A'B} = \sqrt{(2.5 + 0.25)^2 + (4.3301 + 0.73)^2}$$

$$= 5.7591 \text{ in.}$$

$$L_{A'C} = \sqrt{(6.7268 - 0.25)^2 + (4.3301 + 0.73)^2}$$

$$= 8.2191 \text{ in.}$$

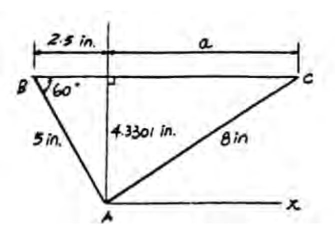
Average Normal Strain:

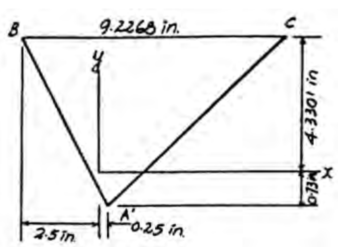
$$\varepsilon_{AB} = \frac{L_{A'B} - L_{AB}}{L_{AB}}$$

$$= \frac{5.7591 - 5}{5} = 0.152 \text{ in./in.}$$ **Ans.**

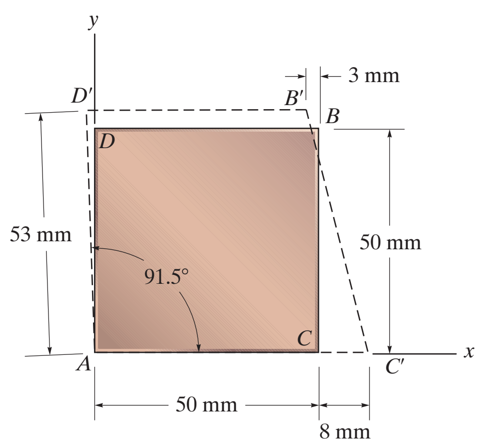
$$\varepsilon_{AC} = \frac{L_{A'C} - L_{AC}}{L_{AC}}$$

$$= \frac{8.2191 - 8}{8} = 0.0274 \text{ in./in.}$$ **Ans.**





***2–16.** The square deforms into the position shown by the dashed lines. Determine the average normal strain along each diagonal, AB and CD. Side $D'B'$ remains horizontal.



Geometry:

$$AB = CD = \sqrt{50^2 + 50^2} = 70.7107 \text{ mm}$$

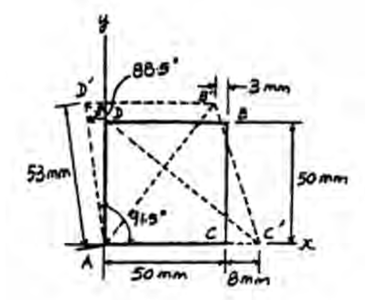
$$C'D' = \sqrt{53^2 + 58^2 - 2(53)(58)\cos 91.5°}$$

$$= 79.5860 \text{ mm}$$

$$B'D' = 50 + 53 \sin 1.5° - 3 = 48.3874 \text{ mm}$$

$$AB' = \sqrt{53^2 + 48.3874^2 - 2(53)(48.3874)\cos 88.5°}$$

$$= 70.8243 \text{ mm}$$



Average Normal Strain:

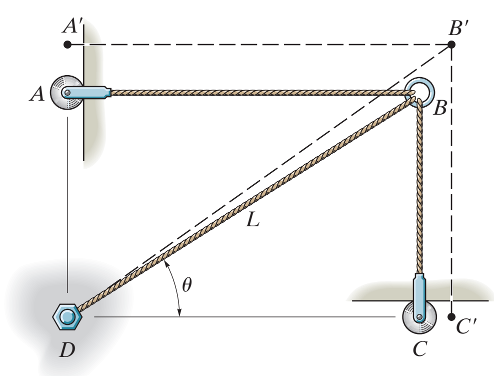
$$\varepsilon_{AB} = \frac{AB' - AB}{AB}$$

$$= \frac{70.8243 - 70.7107}{70.7107} = 1.61\left(10^{-3}\right) \text{ mm/mm}$$ **Ans.**

$$\varepsilon_{CD} = \frac{C'D' - CD}{CD}$$

$$= \frac{79.5860 - 70.7107}{70.7107} = 126\left(10^{-3}\right) \text{ mm/mm}$$ **Ans.**

•2–17. The three cords are attached to the ring at B. When a force is applied to the ring it moves it to point B', such that the normal strain in AB is ϵ_{AB} and the normal strain in CB is ϵ_{CB}. Provided these strains are small, determine the normal strain in DB. Note that AB and CB remain horizontal and vertical, respectively, due to the roller guides at A and C.



Coordinates of B $(L\cos\theta, L\sin\theta)$

Coordinates of B' $(L\cos\theta + \varepsilon_{AB} L\cos\theta, L\sin\theta + \varepsilon_{CB} L\sin\theta)$

$$L_{DB'} = \sqrt{(L\cos\theta + \varepsilon_{AB} L\cos\theta)^2 + (L\sin\theta + \varepsilon_{CB} L\sin\theta)^2}$$

$$L_{DB'} = L\sqrt{\cos^2\theta(1 + 2\varepsilon_{AB} + \varepsilon_{AB}^2) + \sin^2\theta(1 + 2\varepsilon_{CB} + \varepsilon_{CB}^2)}$$

Since ε_{AB} and ε_{CB} are small,

$$L_{DB'} = L\sqrt{1 + (2\,\varepsilon_{AB}\cos^2\theta + 2\varepsilon_{CB}\sin^2\theta)}$$

Use the binomial theorem,

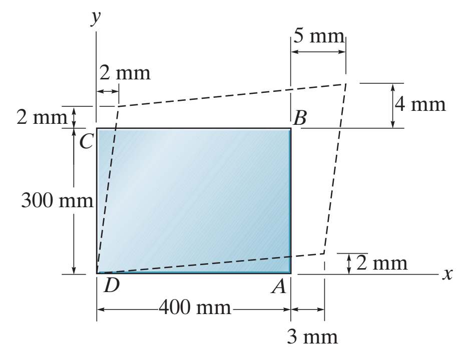
$$L_{DB'} = L\left(1 + \frac{1}{2}(2\,\varepsilon_{AB}\cos^2\theta + 2\varepsilon_{CB}\sin^2\theta)\right)$$

$$= L\,(1 + \varepsilon_{AB}\cos^2\theta + \varepsilon_{CB}\sin^2\theta)$$

Thus, $\varepsilon_{DB} = \dfrac{L(1 + \varepsilon_{AB}\cos^2\theta + \varepsilon_{CB}\sin^2\theta) - L}{L}$

$$\varepsilon_{DB} = \varepsilon_{AB}\cos^2\theta + \varepsilon_{CB}\sin^2\theta$$ **Ans.**

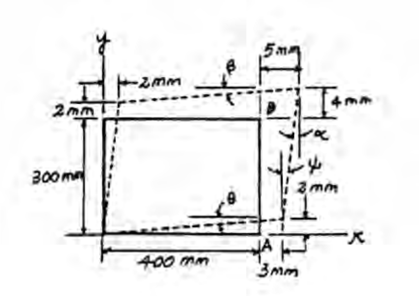
2–18. The piece of plastic is originally rectangular. Determine the shear strain γ_{xy} at corners A and B if the plastic distorts as shown by the dashed lines.



Geometry: For small angles,

$$\alpha = \psi = \frac{2}{302} = 0.00662252 \text{ rad}$$

$$\beta = \theta = \frac{2}{403} = 0.00496278 \text{ rad}$$



Shear Strain:

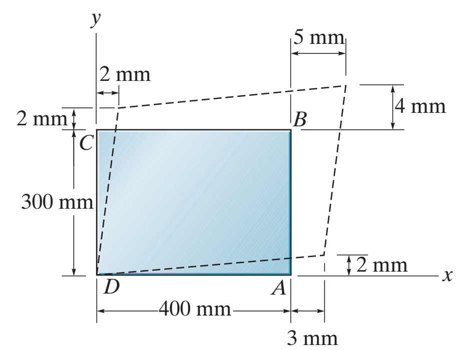
$$(\gamma_B)_{xy} = \alpha + \beta$$

$$= 0.0116 \text{ rad} = 11.6\left(10^{-3}\right) \text{ rad} \qquad \textbf{Ans.}$$

$$(\gamma_A)_{xy} = -(\theta + \psi)$$

$$= -0.0116 \text{ rad} = -11.6\left(10^{-3}\right) \text{ rad} \qquad \textbf{Ans.}$$

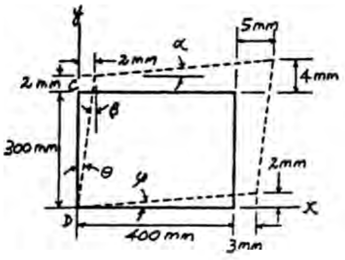
2–19. The piece of plastic is originally rectangular. Determine the shear strain γ_{xy} at corners D and C if the plastic distorts as shown by the dashed lines.



Geometry: For small angles,

$$\alpha = \psi = \frac{2}{403} = 0.00496278 \text{ rad}$$

$$\beta = \theta = \frac{2}{302} = 0.00662252 \text{ rad}$$



Shear Strain:

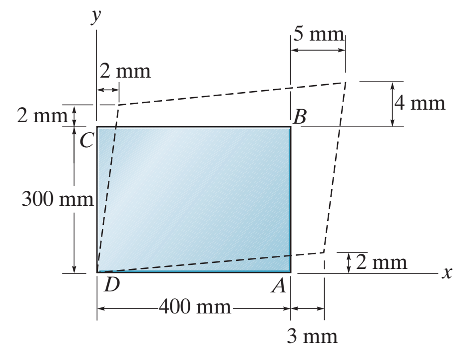
$$(\gamma_C)_{xy} = -(\alpha + \beta)$$

$$= -0.0116 \text{ rad} = -11.6\left(10^{-3}\right) \text{ rad} \qquad \textbf{Ans.}$$

$$(\gamma_D)_{xy} = \theta + \psi$$

$$= 0.0116 \text{ rad} = 11.6\left(10^{-3}\right) \text{ rad} \qquad \textbf{Ans.}$$

***2–20.** The piece of plastic is originally rectangular. Determine the average normal strain that occurs along the diagonals *AC* and *DB*.



Geometry:

$$AC = DB = \sqrt{400^2 + 300^2} = 500 \text{ mm}$$

$$DB' = \sqrt{405^2 + 304^2} = 506.4 \text{ mm}$$

$$A'C' = \sqrt{401^2 + 300^2} = 500.8 \text{ mm}$$

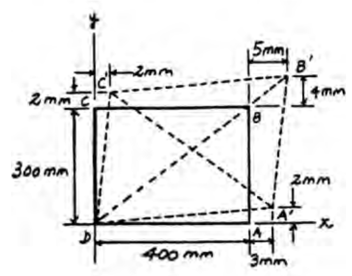
Average Normal Strain:

$$\varepsilon_{AC} = \frac{A'C' - AC}{AC} = \frac{500.8 - 500}{500}$$

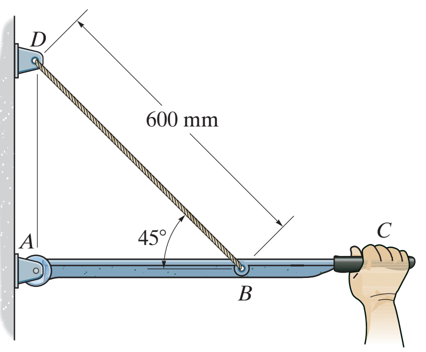
$$= 0.00160 \text{ mm/mm} = 1.60\left(10^{-3}\right) \text{ mm/mm} \qquad \textbf{Ans.}$$

$$\varepsilon_{DB} = \frac{DB' - DB}{DB} = \frac{506.4 - 500}{500}$$

$$= 0.0128 \text{ mm/mm} = 12.8\left(10^{-3}\right) \text{ mm/mm} \qquad \textbf{Ans.}$$



•2–21. The force applied to the handle of the rigid lever arm causes the arm to rotate clockwise through an angle of 3° about pin *A*. Determine the average normal strain developed in the wire. Originally, the wire is unstretched.



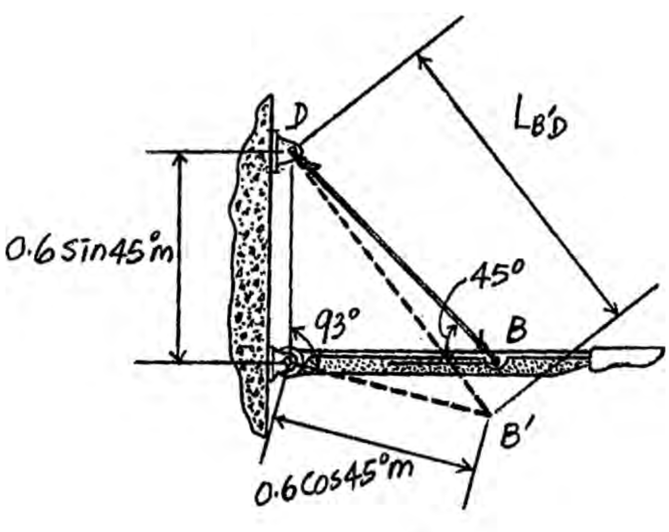
Geometry: Referring to Fig. *a*, the stretched length of $L_{B'D}$ can be determined using the consine law,

$$L_{B'D} = \sqrt{(0.6\cos 45°)^2 + (0.6\sin 45°)^2 - 2(0.6\cos 45°)(0.6\sin 45°)\cos 93°}$$

$$= 0.6155 \text{ m}$$

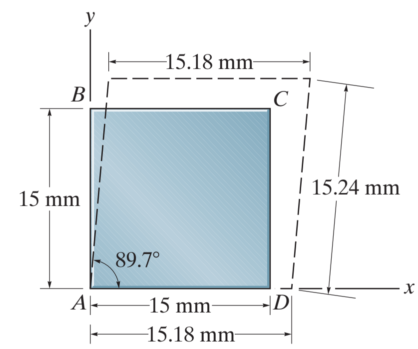
Average Normal Strain: The unstretched length of wire *BD* is $L_{BD} = 0.6$ m. We obtain

$$\varepsilon_{\text{avg}} = \frac{L_{B'D} - L_{BD}}{L_{BD}} = \frac{0.6155 - 0.6}{0.6} = 0.0258 \text{ m/m} \qquad \textbf{Ans.}$$



(a)

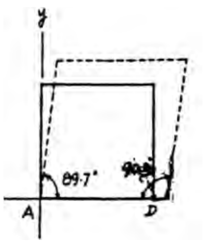
2–22. A square piece of material is deformed into the dashed position. Determine the shear strain γ_{xy} at A.



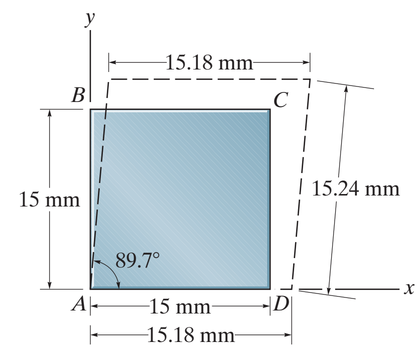
Shear Strain:

$$(\gamma_A)_{xy} = \frac{\pi}{2} - \left(\frac{89.7°}{180°}\right)\pi$$

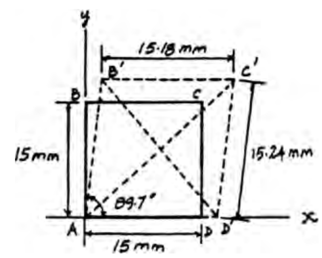
$$= 5.24\left(10^{-3}\right) \text{ rad} \qquad \textbf{Ans.}$$



2–23. A square piece of material is deformed into the dashed parallelogram. Determine the average normal strain that occurs along the diagonals AC and BD.



Geometry:



$$AC = BD = \sqrt{15^2 + 15^2} = 21.2132 \text{ mm}$$

$$AC' = \sqrt{15.18^2 + 15.24^2 - 2(15.18)(15.24)\cos 90.3°}$$

$$= 21.5665 \text{ mm}$$

$$B'D' = \sqrt{15.18^2 + 15.24^2 - 2(15.18)(15.24)\cos 89.7°}$$

$$= 21.4538 \text{ mm}$$

Average Normal Strain:

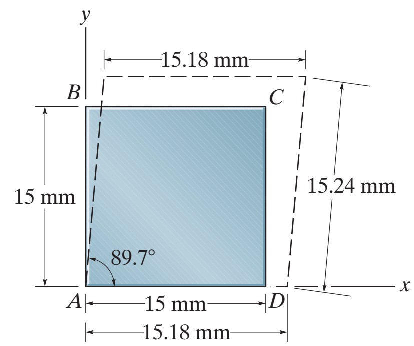
$$\varepsilon_{AC} = \frac{AC' - AC}{AC} = \frac{21.5665 - 21.2132}{21.2132}$$

$$= 0.01665 \text{ mm/mm} = 16.7\left(10^{-3}\right) \text{ mm/mm} \qquad \textbf{Ans.}$$

$$\varepsilon_{BD} = \frac{B'D' - BD}{BD} = \frac{21.4538 - 21.2132}{21.2132}$$

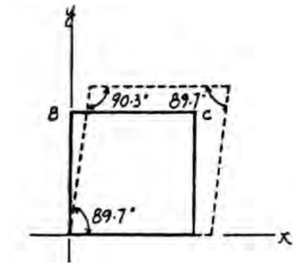
$$= 0.01134 \text{ mm/mm} = 11.3\left(10^{-3}\right) \text{ mm/mm} \qquad \textbf{Ans.}$$

***2–24.** A square piece of material is deformed into the dashed position. Determine the shear strain γ_{xy} at C.

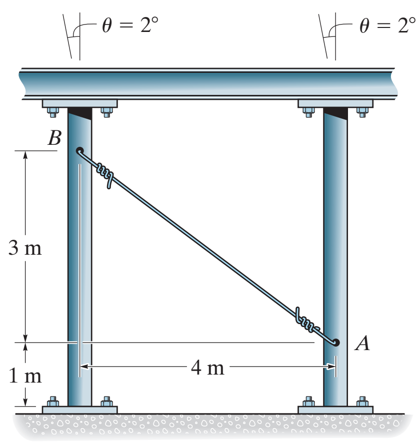


$$(\gamma_C)_{xy} = \frac{\pi}{2} - \left(\frac{89.7^\circ}{180^\circ}\right)\pi$$

$$= 5.24\left(10^{-3}\right) \text{ rad} \qquad \textbf{Ans.}$$



•2–25. The guy wire AB of a building frame is originally unstretched. Due to an earthquake, the two columns of the frame tilt $\theta = 2^\circ$. Determine the approximate normal strain in the wire when the frame is in this position. Assume the columns are rigid and rotate about their lower supports.



Geometry: The vertical displacement is negligible

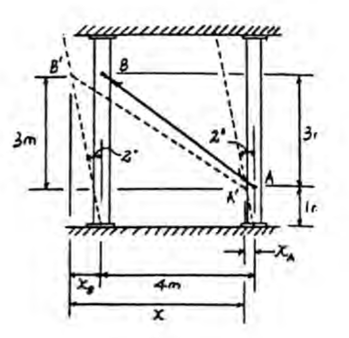
$$x_A = (1)\left(\frac{2^\circ}{180^\circ}\right)\pi = 0.03491 \text{ m}$$

$$x_B = (4)\left(\frac{2^\circ}{180^\circ}\right)\pi = 0.13963 \text{ m}$$

$$x = 4 + x_B - x_A = 4.10472 \text{ m}$$

$$A'B' = \sqrt{3^2 + 4.10472^2} = 5.08416 \text{ m}$$

$$AB = \sqrt{3^2 + 4^2} = 5.00 \text{ m}$$

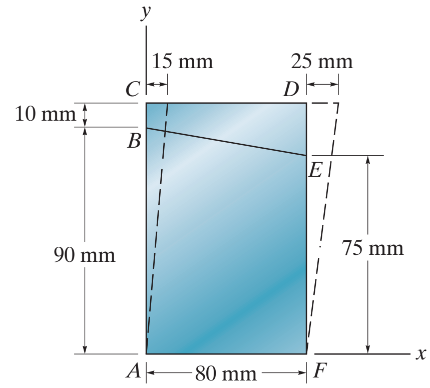


Average Normal Strain:

$$\varepsilon_{AB} = \frac{A'B' - AB}{AB}$$

$$= \frac{5.08416 - 5}{5} = 16.8\left(10^{-3}\right) \text{ m/m} \qquad \textbf{Ans.}$$

2–26. The material distorts into the dashed position shown. Determine (a) the average normal strains along sides *AC* and *CD* and the shear strain γ_{xy} at *F*, and (b) the average normal strain along line *BE*.



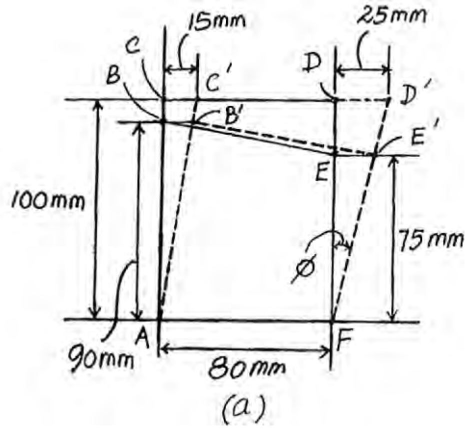
Referring to Fig. a,

$$L_{BE} = \sqrt{(90 - 75)^2 + 80^2} = \sqrt{6625}\text{ mm}$$

$$L_{AC'} = \sqrt{100^2 + 15^2} = \sqrt{10225}\text{ mm}$$

$$L_{C'D'} = 80 - 15 + 25 = 90\text{ mm}$$

$$\phi = \tan^{-1}\left(\frac{25}{100}\right) = 14.04°\left(\frac{\pi\text{ rad}}{180°}\right) = 0.2450\text{ rad.}$$



When the plate deforms, the vertical position of point *B* and E do not change.

$$\frac{L_{BB'}}{90} = \frac{15}{100}; \qquad L_{BB'} = 13.5\text{ mm}$$

$$\frac{L_{EE'}}{75} = \frac{25}{100}; \qquad L_{EE'} = 18.75\text{ mm}$$

$$L_{B'E'} = \sqrt{(90 - 75)^2 + (80 - 13.5 + 18.75)^2} = \sqrt{7492.5625}\text{ mm}$$

Thus,

$$\left(\varepsilon_{\text{avg}}\right)_{AC} = \frac{L_{AC'} - L_{AC}}{L_{AC}} = \frac{\sqrt{10225} - 100}{100} = 0.0112\text{ mm/mm} \qquad \textbf{Ans.}$$

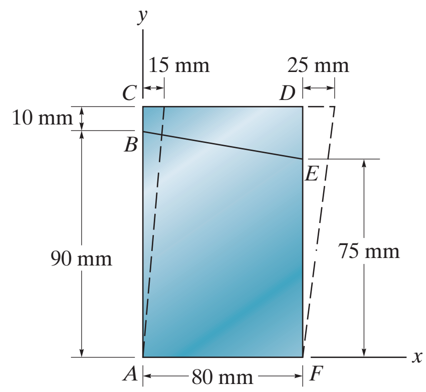
$$\left(\varepsilon_{\text{avg}}\right)_{CD} = \frac{L_{C'D'} - L_{CD}}{L_{CD}} = \frac{90-80}{80} = 0.125\text{ mm/mm} \qquad \textbf{Ans.}$$

$$\left(\varepsilon_{\text{avg}}\right)_{BE} = \frac{L_{B'E'} - L_{BE}}{L_{BE}} = \frac{\sqrt{7492.5625} - \sqrt{6625}}{\sqrt{6625}} = 0.0635\text{ mm/mm} \qquad \textbf{Ans.}$$

Referring to Fig. a, the angle at corner F becomes larger than 90° after the plate deforms. Thus, the shear strain is negative.

0.245 rad **Ans.**

2–27. The material distorts into the dashed position shown. Determine the average normal strain that occurs along the diagonals *AD* and *CF*.



The undeformed length of diagonals AD and CF are

$$L_{AD} = L_{CF} = \sqrt{80^2 + 100^2} = \sqrt{16400}\ \text{mm}$$

The deformed length of diagonals *AD* and *CF* are

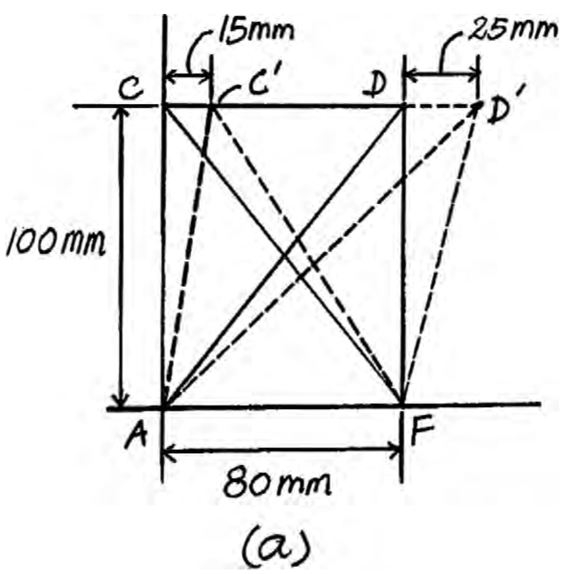
$$L_{AD'} = \sqrt{(80 + 25)^2 + 100^2} = \sqrt{21025}\ \text{mm}$$

$$L_{C'F} = \sqrt{(80 - 15)^2 + 100^2} = \sqrt{14225}\ \text{mm}$$

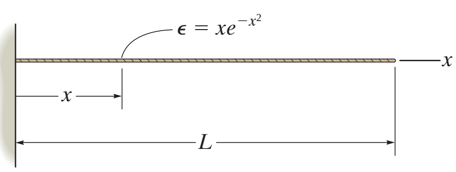
Thus,

$$\left(\varepsilon_{\text{avg}}\right)_{AD} = \frac{L_{AD'} - L_{AD}}{L_{AD}} = \frac{\sqrt{21025} - \sqrt{16400}}{\sqrt{16400}} = 0.132\ \text{mm/mm}$$ **Ans.**

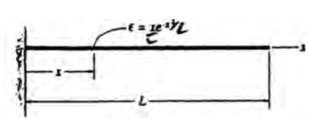
$$\left(\varepsilon_{\text{avg}}\right)_{CF} = \frac{L_{C'F} - L_{CF}}{L_{CF}} = \frac{\sqrt{14225} - \sqrt{16400}}{\sqrt{16400}} = -0.0687\ \text{mm/mm}$$ **Ans.**



***2–28.** The wire is subjected to a normal strain that is defined by $\epsilon = xe^{-x^2}$, where x is in millimeters. If the wire has an initial length L, determine the increase in its length.



$$\delta L = \varepsilon\, dx = x\, e^{-x^2}\, dx$$

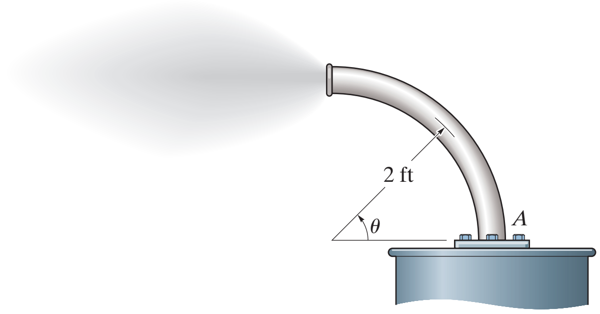


$$\Delta L = \int_0^L x\, e^{-x^2}\, dx$$

$$= -\left[\frac{1}{2} e^{-x^2}\right]\Big|_0^L = -\left[\frac{1}{2} e^{-L^2} - \frac{1}{2}\right]$$

$$= \frac{1}{2}\left[1 - e^{-L^2}\right]$$ **Ans.**

•2–29. The curved pipe has an original radius of 2 ft. If it is heated nonuniformly, so that the normal strain along its length is $\epsilon = 0.05 \cos \theta$, determine the increase in length of the pipe.



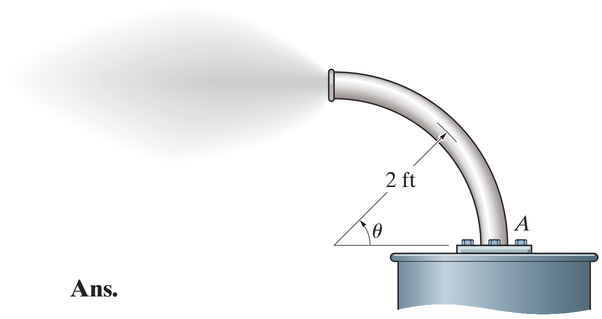
$$\varepsilon = 0.05 \cos\theta$$

$$\Delta L = \int \varepsilon \, dL$$

$$= \int_0^{90^\circ} (0.05 \cos \theta)(2\, d\theta)$$

$$= 0.1 \int_0^{90^\circ} \cos \theta \, d\theta = [0.1[\sin \theta]^0 \Big|^{90^\circ}_0] = 0.100 \text{ ft}$$ **Ans.**

2–30. Solve Prob. 2–29 if $\epsilon = 0.08 \sin \theta$.



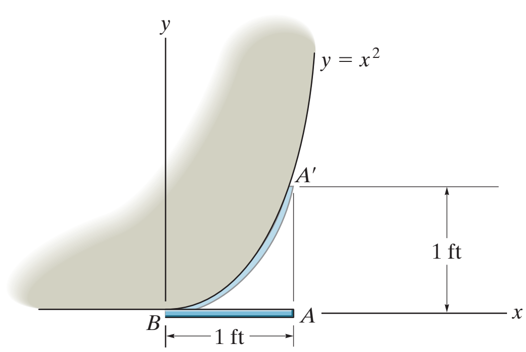
$$dL = 2\, d\theta \varepsilon = 0.08 \sin \theta$$

$$\Delta L = \int \varepsilon \, dL$$

$$= \int_0^{90^\circ} (0.08 \sin \theta)(2\, d\theta)$$

$$= 0.16 \int_0^{90^\circ} \sin \theta \, d\theta = 0.16[-\cos \theta]^0 \Big|^{90^\circ}_0 = 0.16 \text{ ft}$$ **Ans.**

2–31. The rubber band AB has an unstretched length of 1 ft. If it is fixed at B and attached to the surface at point A', determine the average normal strain in the band. The surface is defined by the function $y = (x^2)$ ft, where x is in feet.



Geometry:

$$L = \int_0^{1\text{ ft}} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

However $y = x^2$ then $\dfrac{dy}{dx} = 2x$

$$L = \int_0^{1\text{ ft}} \sqrt{1 + 4x^2}\, dx$$

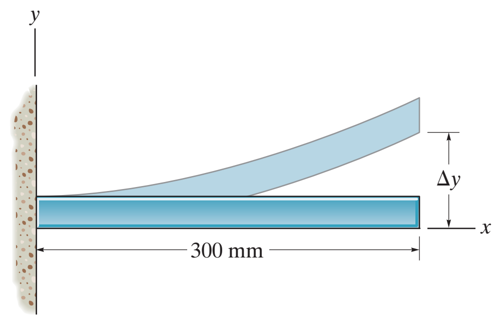
$$= \frac{1}{4}\left[2x\sqrt{1 + 4x^2} + \ln\left(2x + \sqrt{1 + 4x^2}\right)\right]\Big|_0^{1\text{ ft}}$$

$$= 1.47894 \text{ ft}$$

Average Normal Strain:

$$\varepsilon_{\text{avg}} = \frac{L - L_0}{L_0} = \frac{1.47894 - 1}{1} = 0.479 \text{ ft/ft}$$ **Ans.**

***2–32.** The bar is originally 300 mm long when it is flat. If it is subjected to a shear strain defined by $\gamma_{xy} = 0.02x$, where x is in meters, determine the displacement Δy at the end of its bottom edge. It is distorted into the shape shown, where no elongation of the bar occurs in the x direction.



Shear Strain:

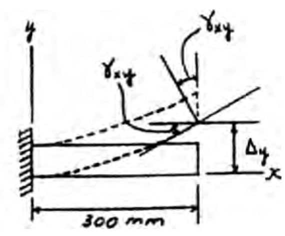
$$\frac{dy}{dx} = \tan \gamma_{xy}\,; \qquad \frac{dy}{dx} = \tan (0.02\,x)$$

$$\int_0^{\Delta y} dy = \int_0^{300\text{ mm}} \tan (0.02\,x)dx$$

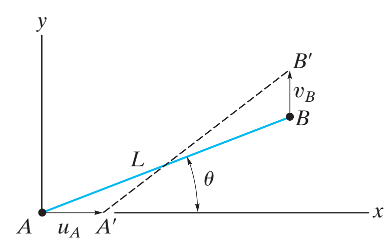
$$\Delta y = -50[\ln \cos (0.02x)]\Big|_0^{300\text{ mm}}$$

$$= 2.03 \text{ mm}$$

Ans.



•2–33. The fiber AB has a length L and orientation θ. If its ends A and B undergo very small displacements u_A and v_B, respectively, determine the normal strain in the fiber when it is in position $A'B'$.



Geometry:

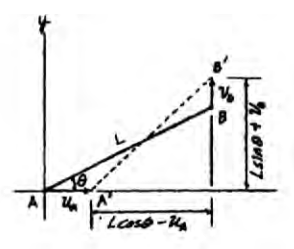
$$L_{A'B'} = \sqrt{(L\cos\theta - u_A)^2 + (L\sin\theta + v_B)^2}$$

$$= \sqrt{L^3 + u_A^2 + v_B^2 + 2L(v_B \sin\theta - u_A\cos\theta)}$$

Average Normal Strain:

$$\varepsilon_{AB} = \frac{L_{A'B'} - L}{L}$$

$$= \sqrt{1 + \frac{u_A^2 + v_B^2}{L^2} + \frac{2(v_B \sin\theta - u_A \cos\theta)}{L}} - 1$$



Neglecting higher terms u_A^2 and v_B^2

$$\varepsilon_{AB} = \left[1 + \frac{2(v_B \sin\theta - u_A\cos\theta)}{L}\right]^{\frac{1}{2}} - 1$$

Using the binomial theorem:

$$\varepsilon_{AB} = 1 + \frac{1}{2}\left(\frac{2v_B \sin\theta}{L} - \frac{2u_A\cos\theta}{L}\right) + \ldots - 1$$

$$= \frac{v_B \sin\theta}{L} - \frac{u_A \cos\theta}{L}$$

Ans.

2–34. If the normal strain is defined in reference to the final length, that is,

$$\epsilon'_n = \lim_{p \to p'} \left(\frac{\Delta s' - \Delta s}{\Delta s'} \right)$$

instead of in reference to the original length, Eq. 2–2, show that the difference in these strains is represented as a second-order term, namely, $\epsilon_n - \epsilon'_n = \epsilon_n \epsilon'_n$.

$$\varepsilon_B = \frac{\Delta S' - \Delta S}{\Delta S}$$

$$\begin{aligned}
\varepsilon_B - \varepsilon'_A &= \frac{\Delta S' - \Delta S}{\Delta S} - \frac{\Delta S' - \Delta S}{\Delta S'} \\
&= \frac{\Delta S'^2 - \Delta S \Delta S' - \Delta S' \Delta S + \Delta S^2}{\Delta S \Delta S'} \\
&= \frac{\Delta S'^2 + \Delta S^2 - 2\Delta S' \Delta S}{\Delta S \Delta S'} \\
&= \frac{(\Delta S' - \Delta S)^2}{\Delta S \Delta S'} = \left(\frac{\Delta S' - \Delta S}{\Delta S} \right) \left(\frac{\Delta S' - \Delta S}{\Delta S'} \right) \\
&= \varepsilon_A \varepsilon'_B \text{ (Q.E.D)}
\end{aligned}$$

•3–1. A concrete cylinder having a diameter of 6.00 in. and gauge length of 12 in. is tested in compression. The results of the test are reported in the table as load versus contraction. Draw the stress–strain diagram using scales of 1 in. = 0.5 ksi and 1 in. = $0.2(10^{-3})$ in./in. From the diagram, determine approximately the modulus of elasticity.

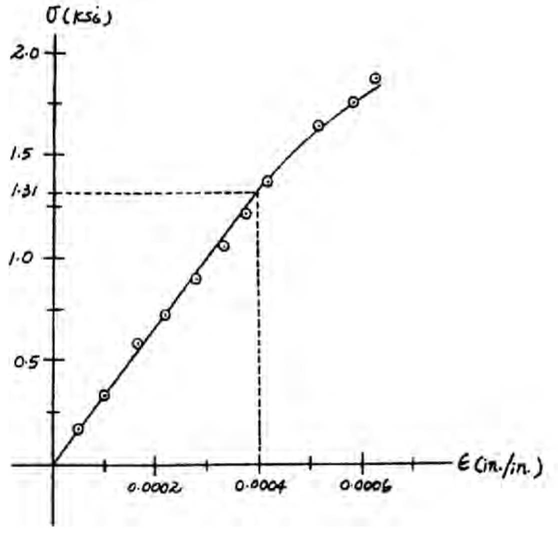
Load (kip)	Contraction (in.)
0	0
5.0	0.0006
9.5	0.0012
16.5	0.0020
20.5	0.0026
25.5	0.0034
30.0	0.0040
34.5	0.0045
38.5	0.0050
46.5	0.0062
50.0	0.0070
53.0	0.0075

Stress and Strain:

$\sigma = \dfrac{P}{A}$(ksi)	$\varepsilon = \dfrac{\delta L}{L}$(in./in.)
0	0
0.177	0.00005
0.336	0.00010
0.584	0.000167
0.725	0.000217
0.902	0.000283
1.061	0.000333
1.220	0.000375
1.362	0.000417
1.645	0.000517
1.768	0.000583
1.874	0.000625

Modulus of Elasticity: From the stress–strain diagram

$$E_{\text{approx}} = \frac{1.31 - 0}{0.0004 - 0} = 3.275\left(10^3\right) \text{ ksi} \qquad \textbf{Ans.}$$



3–2. Data taken from a stress–strain test for a ceramic are given in the table. The curve is linear between the origin and the first point. Plot the diagram, and determine the modulus of elasticity and the modulus of resilience.

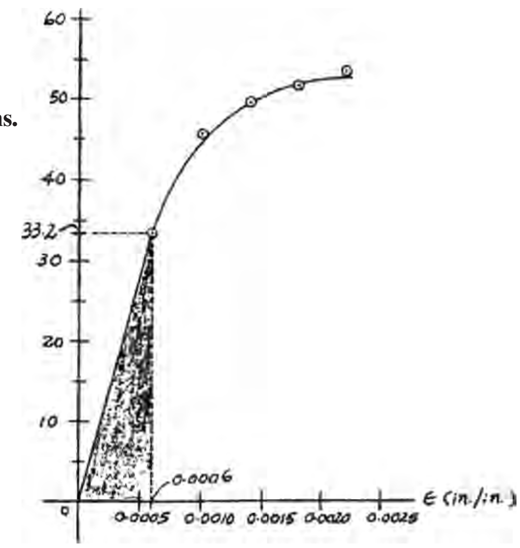
σ (ksi)	ϵ (in./in.)
0	0
33.2	0.0006
45.5	0.0010
49.4	0.0014
51.5	0.0018
53.4	0.0022

Modulus of Elasticity: From the stress–strain diagram

$$E = \frac{33.2 - 0}{0.0006 - 0} = 55.3\left(10^3\right)\ \text{ksi} \qquad \textbf{Ans.}$$

Modulus of Resilience: The modulus of resilience is equal to the area under the *linear portion* of the stress–strain diagram (shown shaded).

$$u_t = \frac{1}{2}(33.2)\left(10^3\right)\left(\frac{\text{lb}}{\text{in}^2}\right)\left(0.0006\,\frac{\text{in.}}{\text{in.}}\right) = 9.96\,\frac{\text{in}\cdot\text{lb}}{\text{in}^3} \qquad \textbf{Ans.}$$



3–3. Data taken from a stress–strain test for a ceramic are given in the table. The curve is linear between the origin and the first point. Plot the diagram, and determine approximately the modulus of toughness. The rupture stress is $\sigma_r = 53.4$ ksi.

σ (ksi)	ϵ (in./in.)
0	0
33.2	0.0006
45.5	0.0010
49.4	0.0014
51.5	0.0018
53.4	0.0022

Modulus of Toughness: The modulus of toughness is equal to the area under the stress–strain diagram (shown shaded).

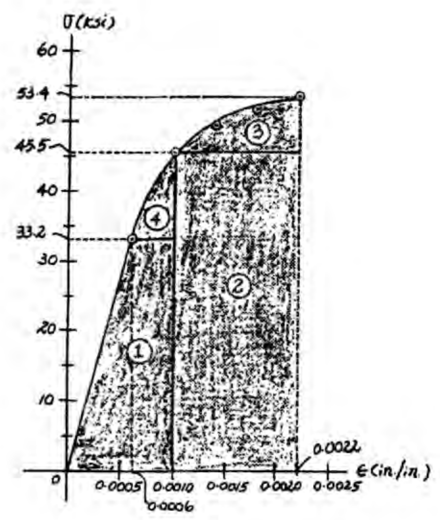
$$(u_t)_{\text{approx}} = \frac{1}{2}(33.2)\left(10^3\right)\left(\frac{\text{lb}}{\text{in}^2}\right)(0.0004 + 0.0010)\left(\frac{\text{in.}}{\text{in.}}\right)$$

$$+\ 45.5\left(10^3\right)\left(\frac{\text{lb}}{\text{in}^2}\right)(0.0012)\left(\frac{\text{in.}}{\text{in.}}\right)$$

$$+\ \frac{1}{2}(7.90)\left(10^3\right)\left(\frac{\text{lb}}{\text{in}^2}\right)(0.0012)\left(\frac{\text{in.}}{\text{in.}}\right)$$

$$+\ \frac{1}{2}(12.3)\left(10^3\right)\left(\frac{\text{lb}}{\text{in}^2}\right)(0.0004)\left(\frac{\text{in.}}{\text{in.}}\right)$$

$$= 85.0\,\frac{\text{in}\cdot\text{lb}}{\text{in}^3} \qquad \textbf{Ans.}$$



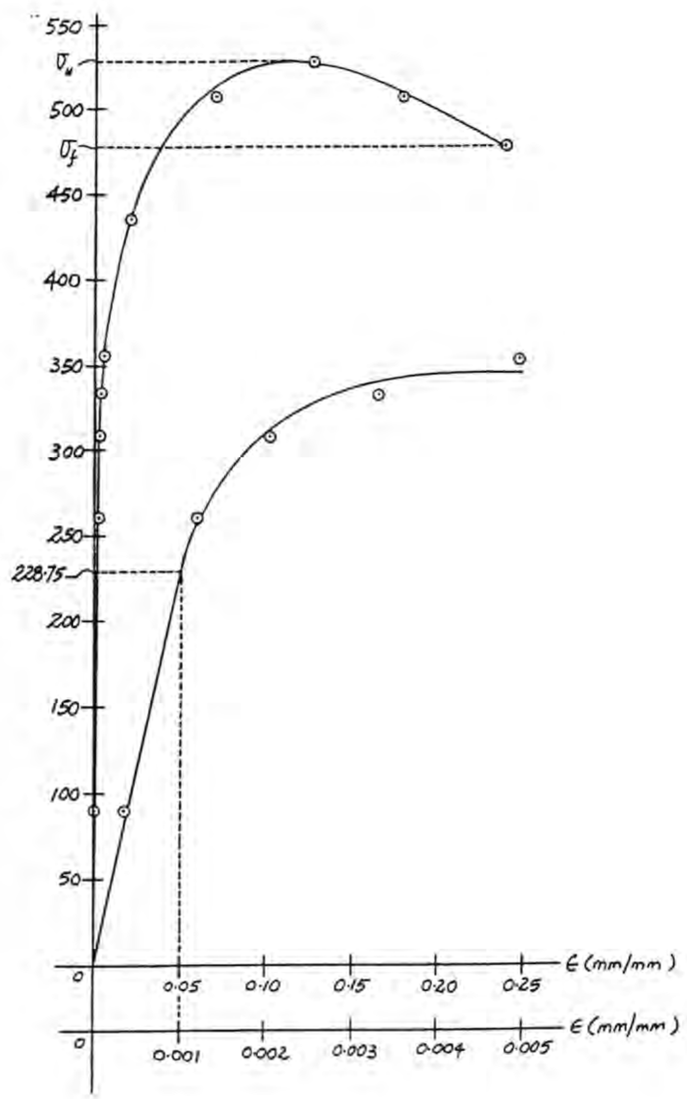
***3–4.** A tension test was performed on a specimen having an original diameter of 12.5 mm and a gauge length of 50 mm. The data are listed in the table. Plot the stress–strain diagram, and determine approximately the modulus of elasticity, the ultimate stress, and the fracture stress. Use a scale of 20 mm = 50 MPa and 20 mm = 0.05 mm/mm. Redraw the linear-elastic region, using the same stress scale but a strain scale of 20 mm = 0.001 mm/mm.

Load (kN)	Elongation (mm)
0	0
11.1	0.0175
31.9	0.0600
37.8	0.1020
40.9	0.1650
43.6	0.2490
53.4	1.0160
62.3	3.0480
64.5	6.3500
62.3	8.8900
58.8	11.9380

Stress and Strain:

$\sigma = \dfrac{P}{A}$(MPa) $\varepsilon = \dfrac{\delta L}{L}$(mm/mm)

0	0
90.45	0.00035
259.9	0.00120
308.0	0.00204
333.3	0.00330
355.3	0.00498
435.1	0.02032
507.7	0.06096
525.6	0.12700
507.7	0.17780
479.1	0.23876



Modulus of Elasticity: From the stress–strain diagram

$$(E)_{\text{approx}} = \frac{228.75(10^6) - 0}{0.001 - 0} = 229 \text{ GPa} \qquad \textbf{Ans.}$$

Ultimate and Fracture Stress: From the stress–strain diagram

$$(\sigma_\mu)_{\text{approx}} = 528 \text{ MPa} \qquad \textbf{Ans.}$$

$$(\sigma_f)_{\text{approx}} = 479 \text{ MPa} \qquad \textbf{Ans.}$$

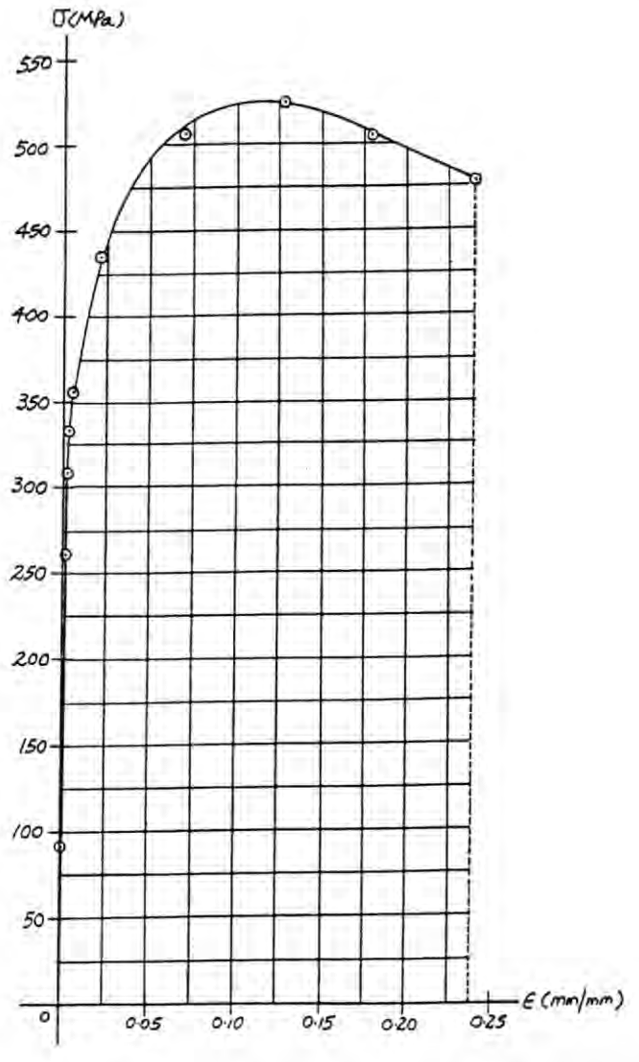
3–5. A tension test was performed on a steel specimen having an original diameter of 12.5 mm and gauge length of 50 mm. Using the data listed in the table, plot the stress–strain diagram, and determine approximately the modulus of toughness. Use a scale of 20 mm = 50 MPa and 20 mm = 0.05 mm/mm.

Load (kN)	Elongation (mm)
0	0
11.1	0.0175
31.9	0.0600
37.8	0.1020
40.9	0.1650
43.6	0.2490
53.4	1.0160
62.3	3.0480
64.5	6.3500
62.3	8.8900
58.8	11.9380

Stress and Strain:

$$\sigma = \frac{P}{A}(\text{MPa}) \quad \varepsilon = \frac{\delta L}{L}(\text{mm/mm})$$

0	0
90.45	0.00035
259.9	0.00120
308.0	0.00204
333.3	0.00330
355.3	0.00498
435.1	0.02032
507.7	0.06096
525.6	0.12700
507.7	0.17780
479.1	0.23876



Modulus of Toughness: The modulus of toughness is equal to the total area under the stress–strain diagram and can be approximated by counting the number of squares. The total number of squares is 187.

$$(u_t)_{\text{approx}} = 187(25)\left(10^6\right)\left(\frac{\text{N}}{\text{m}^2}\right)\left(0.025\,\frac{\text{m}}{\text{m}}\right) = 117 \ \text{MJ/m}^3 \qquad \textbf{Ans.}$$

3–6. A specimen is originally 1 ft long, has a diameter of 0.5 in., and is subjected to a force of 500 lb. When the force is increased from 500 lb to 1800 lb, the specimen elongates 0.009 in. Determine the modulus of elasticity for the material if it remains linear elastic.

Normal Stress and Strain: Applying $\sigma = \frac{P}{A}$ and $\varepsilon = \frac{\delta L}{L}$.

$$\sigma_1 = \frac{0.500}{\frac{\pi}{4}(0.5^2)} = 2.546 \text{ ksi}$$

$$\sigma_2 = \frac{1.80}{\frac{\pi}{4}(0.5^2)} = 9.167 \text{ ksi}$$

$$\Delta\varepsilon = \frac{0.009}{12} = 0.000750 \text{ in./in.}$$

Modulus of Elasticity:

$$E = \frac{\Delta\sigma}{\Delta\varepsilon} = \frac{9.167 - 2.546}{0.000750} = 8.83\left(10^3\right) \text{ ksi} \qquad \textbf{Ans.}$$

3–7. A structural member in a nuclear reactor is made of a zirconium alloy. If an axial load of 4 kip is to be supported by the member, determine its required cross-sectional area. Use a factor of safety of 3 relative to yielding. What is the load on the member if it is 3 ft long and its elongation is 0.02 in.? $E_{zr} = 14(10^3)$ ksi, $\sigma_Y = 57.5$ ksi. The material has elastic behavior.

Allowable Normal Stress:

$$\text{F.S.} = \frac{\sigma_y}{\sigma_{\text{allow}}}$$

$$3 = \frac{57.5}{\sigma_{\text{allow}}}$$

$$\sigma_{\text{allow}} = 19.17 \text{ ksi}$$

$$\sigma_{\text{allow}} = \frac{P}{A}$$

$$19.17 = \frac{4}{A}$$

$$A = 0.2087 \text{ in}^2 = 0.209 \text{ in}^2 \qquad \textbf{Ans.}$$

Stress–Strain Relationship: Applying Hooke's law with

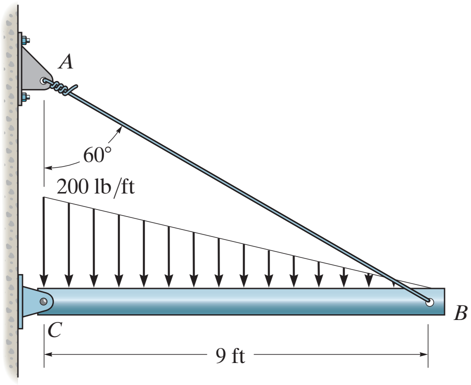
$$\varepsilon = \frac{\delta}{L} = \frac{0.02}{3\,(12)} = 0.000555 \text{ in./in.}$$

$$\sigma = E\varepsilon = 14\left(10^3\right)(0.000555) = 7.778 \text{ ksi}$$

Normal Force: Applying equation $\sigma = \frac{P}{A}$.

$$P = \sigma A = 7.778\,(0.2087) = 1.62 \text{ kip} \qquad \textbf{Ans.}$$

***3–8.** The strut is supported by a pin at C and an A-36 steel guy wire AB. If the wire has a diameter of 0.2 in., determine how much it stretches when the distributed load acts on the strut.



Here, we are only interested in determining the force in wire AB.

$$\circlearrowleft + \Sigma M_C = 0; \qquad F_{AB}\cos 60°(9) - \frac{1}{2}(200)(9)(3) = 0 \qquad F_{AB} = 600 \text{ lb}$$

The normal stress the wire is

$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{600}{\frac{\pi}{4}(0.2^2)} = 19.10(10^3) \text{ psi} = 19.10 \text{ ksi}$$

Since $\sigma_{AB} < \sigma_y = 36$ ksi, Hooke's Law can be applied to determine the strain in wire.

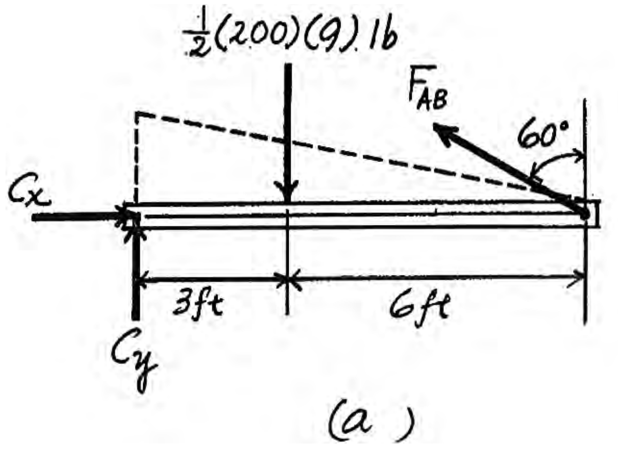
$$\sigma_{AB} = E\epsilon_{AB}; \qquad 19.10 = 29.0(10^3)\epsilon_{AB}$$

$$\epsilon_{AB} = 0.6586(10^{-3}) \text{ in/in}$$

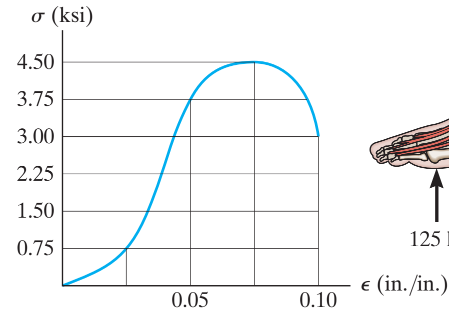
The unstretched length of the wire is $L_{AB} = \dfrac{9(12)}{\sin 60°} = 124.71$ in. Thus, the wire stretches

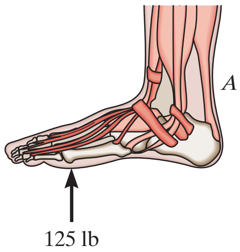
$$\delta_{AB} = \epsilon_{AB} L_{AB} = 0.6586(10^{-3})(124.71)$$

$$= 0.0821 \text{ in.}$$ **Ans.**



•3–9. The $\sigma-\epsilon$ diagram for a collagen fiber bundle from which a human tendon is composed is shown. If a segment of the Achilles tendon at A has a length of 6.5 in. and an approximate cross-sectional area of 0.229 in^2, determine its elongation if the foot supports a load of 125 lb, which causes a tension in the tendon of 343.75 lb.



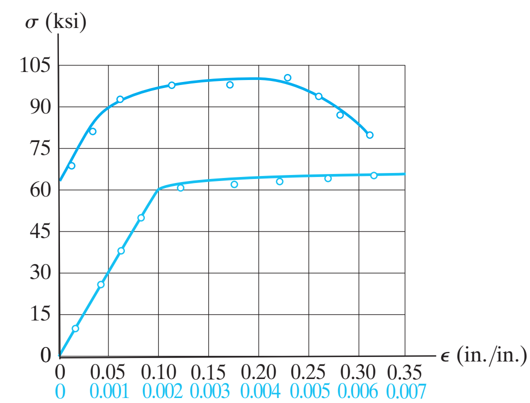


$$\sigma = \frac{P}{A} = \frac{343.75}{0.229} = 1.50 \text{ ksi}$$

From the graph $\varepsilon = 0.035$ in./in.

$$\delta = \varepsilon L = 0.035(6.5) = 0.228 \text{ in.} \qquad \textbf{Ans.}$$

3–10. The stress–strain diagram for a metal alloy having an original diameter of 0.5 in. and a gauge length of 2 in. is given in the figure. Determine approximately the modulus of elasticity for the material, the load on the specimen that causes yielding, and the ultimate load the specimen will support.



From the stress–strain diagram, Fig. *a*,

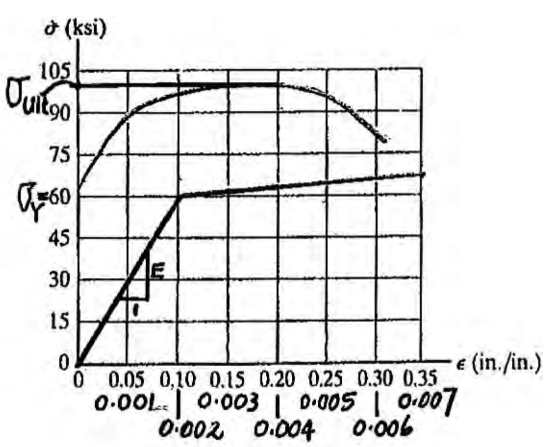
$$\frac{E}{1} = \frac{60 \text{ ksi} - 0}{0.002 - 0}; \qquad E = 30.0(10^3) \text{ ksi} \qquad \textbf{Ans.}$$

$$\sigma_y = 60 \text{ ksi} \qquad \sigma_{u/t} = 100 \text{ ksi}$$

Thus,

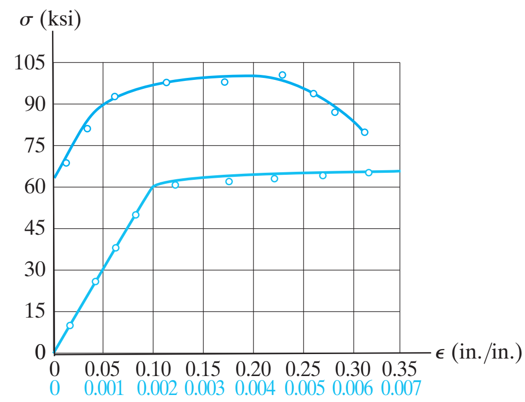
$$P_Y = \sigma_Y A = 60\left[\frac{\pi}{4}(0.5^2)\right] = 11.78 \text{ kip} = 11.8 \text{ kip} \qquad \textbf{Ans.}$$

$$P_{u/t} = \sigma_{u/t}\, A = 100\left[\frac{\pi}{4}(0.5^2)\right] = 19.63 \text{ kip} = 19.6 \text{ kip} \qquad \textbf{Ans.}$$



(a)

3–11. The stress–strain diagram for a steel alloy having an original diameter of 0.5 in. and a gauge length of 2 in. is given in the figure. If the specimen is loaded until it is stressed to 90 ksi, determine the approximate amount of elastic recovery and the increase in the gauge length after it is unloaded.



From the stress–strain diagram Fig. *a*, the modulus of elasticity for the steel alloy is

$$\frac{E}{1} = \frac{60 \text{ ksi} - 0}{0.002 - 0}; \qquad E = 30.0(10^3) \text{ ksi}$$

when the specimen is unloaded, its normal strain recovered along line AB, Fig. *a*, which has a gradient of E. Thus

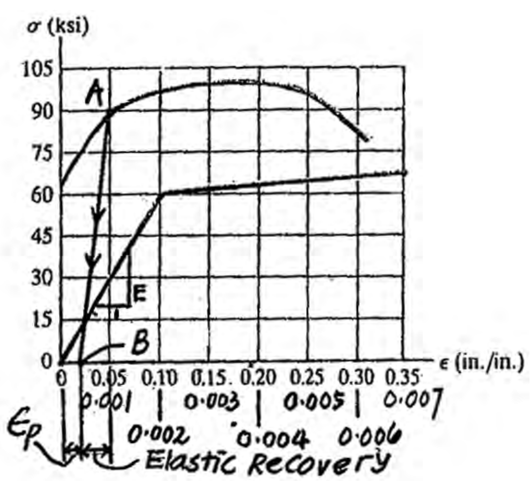
$$\textit{Elastic Recovery} = \frac{90}{E} = \frac{90 \text{ ksi}}{30.0(10^3) \text{ ksi}} = 0.003 \text{ in/in} \qquad \textbf{Ans.}$$

Thus, the permanent set is

$$\epsilon_P = 0.05 - 0.003 = 0.047 \text{ in/in}$$

Then, the increase in gauge length is

$$\Delta L = \epsilon_P L = 0.047(2) = 0.094 \text{ in} \qquad \textbf{Ans.}$$



(a)

***3–12.** The stress–strain diagram for a steel alloy having an original diameter of 0.5 in. and a gauge length of 2 in. is given in the figure. Determine approximately the modulus of resilience and the modulus of toughness for the material.

The Modulus of resilience is equal to the area under the stress–strain diagram up to the proportional limit.

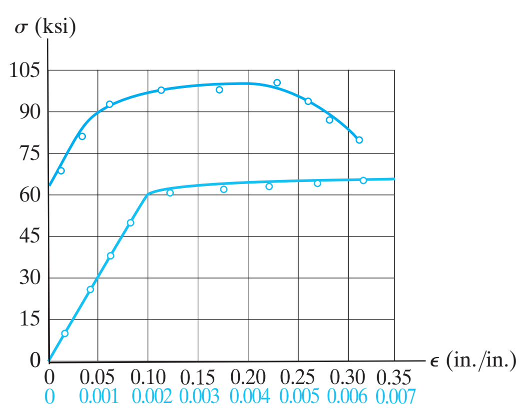
$$\sigma_{PL} = 60 \text{ ksi} \qquad \epsilon_{PL} = 0.002 \text{ in/in.}$$

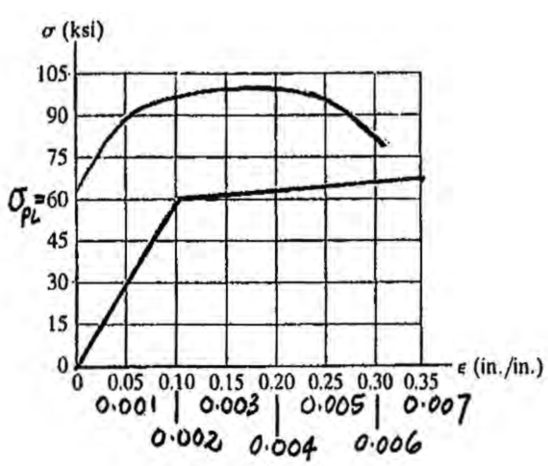
Thus,

$$(u_i)_r = \frac{1}{2}\sigma_{PL}\epsilon_{PL} = \frac{1}{2}\left[60(10^3)\right](0.002) = 60.0\ \frac{\text{in} \cdot \text{lb}}{\text{in}^3} \qquad \textbf{Ans.}$$

The modulus of toughness is equal to the area under the entire stress–strain diagram. This area can be approximated by counting the number of squares. The total number is 38. Thus,

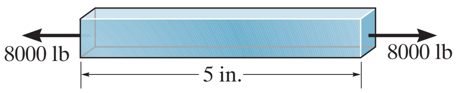
$$\left[(u_i)_t\right]_{\text{approx}} = 38\left[15(10^3)\ \frac{\text{lb}}{\text{in}^2}\right]\left(0.05\ \frac{\text{in}}{\text{in}}\right) = 28.5(10^3)\ \frac{\text{in} \cdot \text{lb}}{\text{in}^3} \qquad \textbf{Ans.}$$





(a)

•3–13. A bar having a length of 5 in. and cross-sectional area of 0.7 in^2 is subjected to an axial force of 8000 lb. If the bar stretches 0.002 in., determine the modulus of elasticity of the material. The material has linear-elastic behavior.



Normal Stress and Strain:

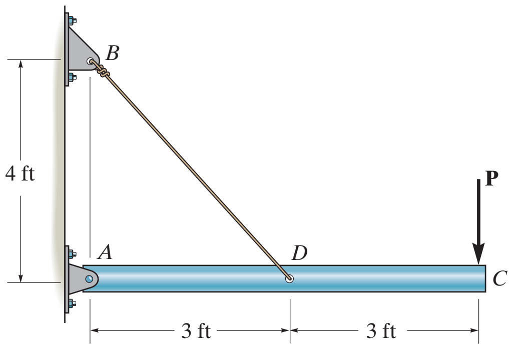
$$\sigma = \frac{P}{A} = \frac{8.00}{0.7} = 11.43 \text{ ksi}$$

$$\varepsilon = \frac{\delta L}{L} = \frac{0.002}{5} = 0.000400 \text{ in./in.}$$

Modulus of Elasticity:

$$E = \frac{\sigma}{\varepsilon} = \frac{11.43}{0.000400} = 28.6(10^3) \text{ ksi} \qquad \textbf{Ans.}$$

3–14. The rigid pipe is supported by a pin at A and an A-36 steel guy wire BD. If the wire has a diameter of 0.25 in., determine how much it stretches when a load of $P = 600$ lb acts on the pipe.



Here, we are only interested in determining the force in wire BD. Referring to the FBD in Fig. *a*

$$\circlearrowleft + \Sigma M_A = 0; \qquad F_{BD}\left(\tfrac{4}{5}\right)(3) - 600(6) = 0 \qquad F_{BD} = 1500 \text{ lb}$$

The normal stress developed in the wire is

$$\sigma_{BD} = \frac{F_{BD}}{A_{BD}} = \frac{1500}{\frac{\pi}{4}(0.25^2)} = 30.56(10^3) \text{ psi} = 30.56 \text{ ksi}$$

Since $\sigma_{BD} < \sigma_y = 36$ ksi, Hooke's Law can be applied to determine the strain in the wire.

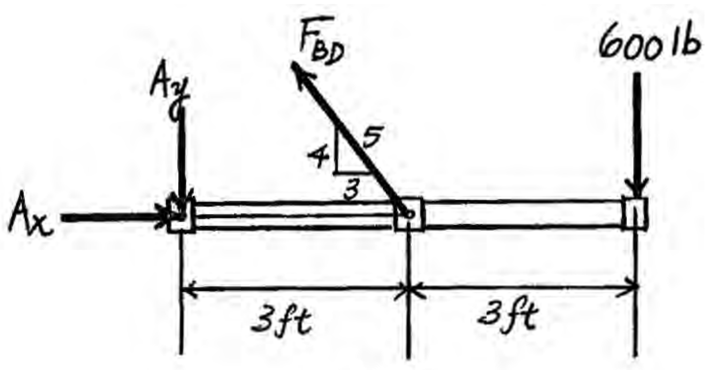
$$\sigma_{BD} = E\epsilon_{BD}; \qquad 30.56 = 29.0(10^3)\epsilon_{BD}$$

$$\epsilon_{BD} = 1.054(10^{-3}) \text{ in./in.}$$

The unstretched length of the wire is $L_{BD} = \sqrt{3^2 + 4^2} = 5\text{ft} = 60$ in. Thus, the wire stretches

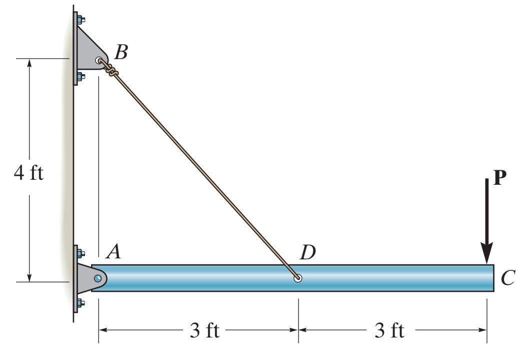
$$\delta_{BD} = \epsilon_{BD} L_{BD} = 1.054(10^{-3})(60)$$

$$= 0.0632 \text{ in} \qquad \textbf{Ans.}$$



(a)

3–15. The rigid pipe is supported by a pin at A and an A-36 guy wire BD. If the wire has a diameter of 0.25 in., determine the load P if the end C is displaced 0.075 in. downward.



Here, we are only interested in determining the force in wire BD. Referring to the FBD in Fig. a

$$\circlearrowleft + \Sigma M_A = 0; \qquad F_{BD}\left(\tfrac{4}{5}\right)(3) - P(6) = 0 \qquad F_{BD} = 2.50\,P$$

The unstretched length for wire BD is $L_{BD} = \sqrt{3^2 + 4^2} = 5\text{ ft} = 60\text{ in.}$ From the geometry shown in Fig. b, the stretched length of wire BD is

$$L_{BD'} = \sqrt{60^2 + 0.075^2 - 2(60)(0.075)\cos 143.13^\circ} = 60.060017$$

Thus, the normal strain is

$$\epsilon_{BD} = \frac{L_{BD'} - L_{BD}}{L_{BD}} = \frac{60.060017 - 60}{60} = 1.0003(10^{-3})\text{ in./in.}$$

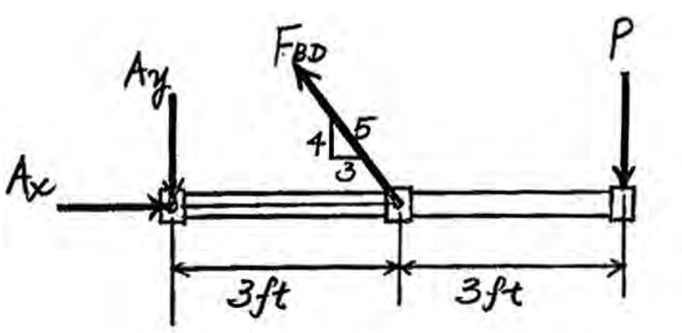
Then, the normal stress can be obtain by applying Hooke's Law.

$$\sigma_{BD} = E\epsilon_{BD} = 29(10^3)\left[1.0003(10^{-3})\right] = 29.01\text{ ksi}$$

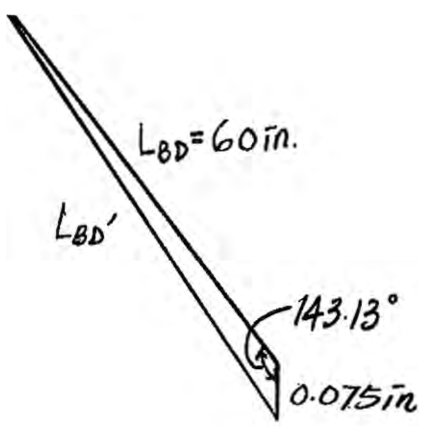
Since $\sigma_{BD} < \sigma_y = 36$ ksi, the result is valid.

$$\sigma_{BD} = \frac{F_{BD}}{A_{BD}}; \qquad 29.01(10^3) = \frac{2.50\,P}{\frac{\pi}{4}(0.25^2)}$$

$$P = 569.57\text{ lb} = 570\text{ lb}$$ **Ans.**

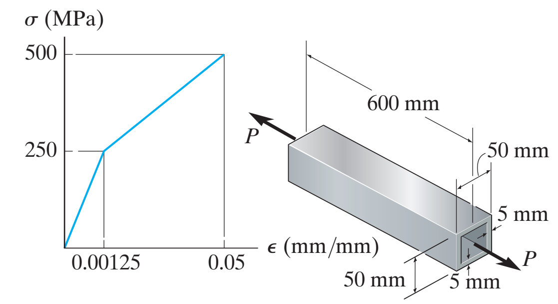


(a)



(b)

***3–16.** Determine the elongation of the square hollow bar when it is subjected to the axial force $P = 100$ kN. If this axial force is increased to $P = 360$ kN and released, find the permanent elongation of the bar. The bar is made of a metal alloy having a stress–strain diagram which can be approximated as shown.



Normal Stress and Strain: The cross-sectional area of the hollow bar is $A = 0.05^2 - 0.04^2 = 0.9(10^{-3})\text{m}^2$. When $P = 100\,\text{kN}$,

$$\sigma_1 = \frac{P}{A} = \frac{100(10^3)}{0.9(10^{-3})} = 111.11\text{ MPa}$$

From the stress–strain diagram shown in Fig. *a*, the slope of the straight line *OA* which represents the modulus of elasticity of the metal alloy is

$$E = \frac{250(10^6) - 0}{0.00125 - 0} = 200\text{ GPa}$$

Since $\sigma_1 < 250$ MPa, Hooke's Law can be applied. Thus

$$\sigma_1 = E\varepsilon_1; \quad 111.11(10^6) = 200(10^9)\varepsilon_1$$

$$\varepsilon_1 = 0.5556(10^{-3})\text{ mm/mm}$$

Thus, the elongation of the bar is

$$\delta_1 = \varepsilon_1 L = 0.5556(10^{-3})(600) = 0.333\text{ mm} \qquad \textbf{Ans.}$$

When $P = 360$ kN,

$$\sigma_2 = \frac{P}{A} = \frac{360(10^3)}{0.9(10^{-3})} = 400\text{ MPa}$$

From the geometry of the stress–strain diagram, Fig. *a*,

$$\frac{\varepsilon_2 - 0.00125}{400 - 250} = \frac{0.05 - 0.00125}{500 - 250} \qquad \varepsilon_2 = 0.0305\text{ mm/mm}$$

When $P = 360\,\text{kN}$ is removed, the strain recovers linearly along line *BC*, Fig. *a*, parallel to *OA*. Thus, the elastic recovery of strain is given by

$$\sigma_2 = E\varepsilon_r; \qquad 400(10^6) = 200(10^9)\varepsilon_r$$

$$\varepsilon_r = 0.002\text{ mm/mm}$$

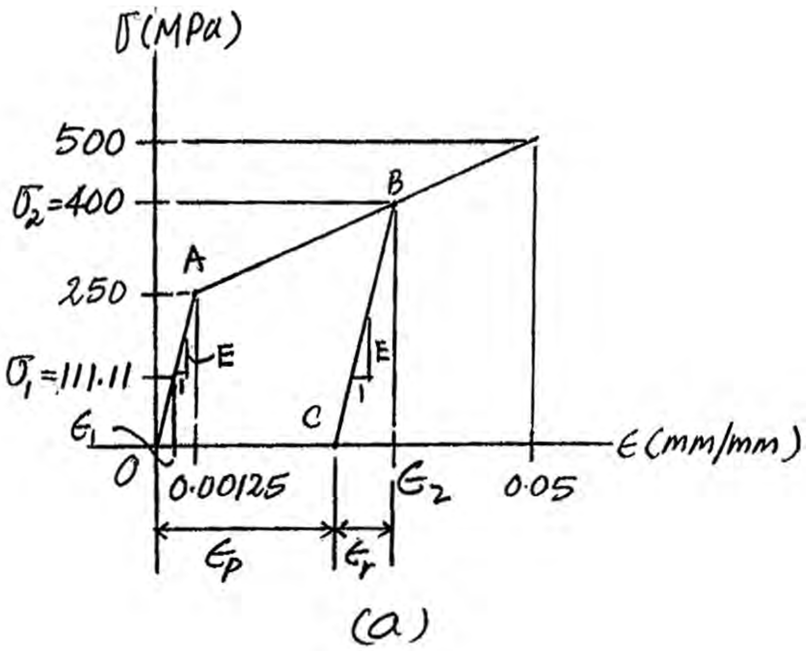
The permanent set is

$$\varepsilon_P = \varepsilon_2 - \varepsilon_r = 0.0305 - 0.002 = 0.0285\text{ mm/mm}$$

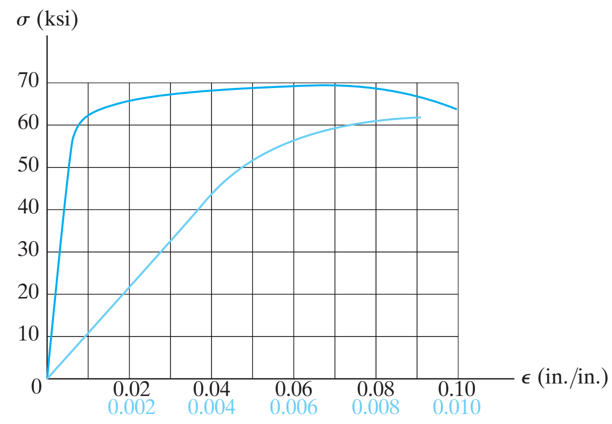
Thus, the permanent elongation of the bar is

$$\delta_P = \varepsilon_P L = 0.0285(600) = 17.1\text{ mm} \qquad \textbf{Ans.}$$

3–16. Continued



3–17. A tension test was performed on an aluminum 2014-T6 alloy specimen. The resulting stress–strain diagram is shown in the figure. Estimate (a) the proportional limit, (b) the modulus of elasticity, and (c) the yield strength based on a 0.2% strain offset method.



Proportional Limit and Yield Strength: From the stress–strain diagram, Fig. *a*,

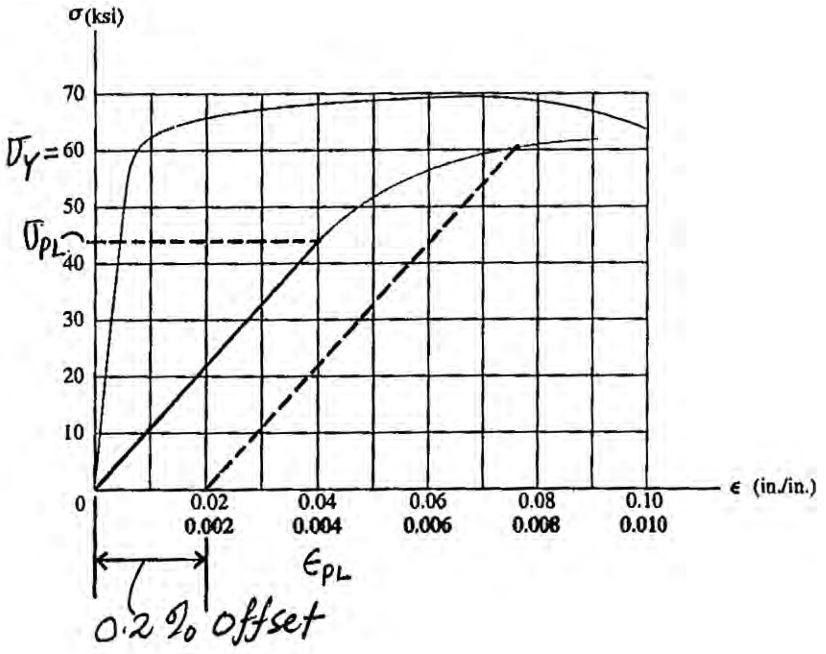
$$\sigma_{pl} = 44 \text{ ksi} \qquad \textbf{Ans.}$$

$$\sigma_Y = 60 \text{ ksi} \qquad \textbf{Ans.}$$

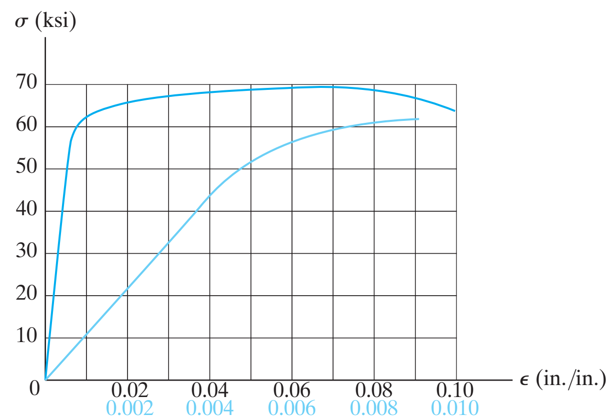
Modulus of Elasticity: From the stress–strain diagram, the corresponding strain for $\sigma_{PL} = 44$ ksi is $\varepsilon_{pl} = 0.004$ in./in. Thus,

$$E = \frac{44 - 0}{0.004 - 0} = 11.0(10^3) \text{ ksi} \qquad \textbf{Ans.}$$

Modulus of Resilience: The modulus of resilience is equal to the area under the



3–18. A tension test was performed on an aluminum 2014-T6 alloy specimen. The resulting stress–strain diagram is shown in the figure. Estimate (a) the modulus of resilience; and (b) modulus of toughness.



stress–strain diagram up to the proportional limit. From the stress–strain diagram,

$$\sigma_{pl} = 44 \text{ ksi} \qquad \varepsilon_{pl} = 0.004 \text{ in./in.}$$

Thus,

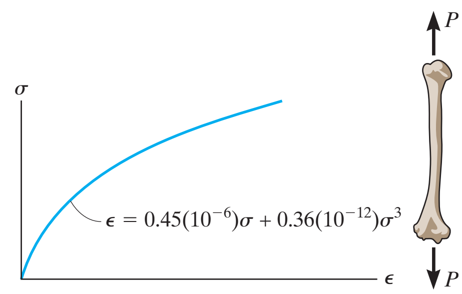
$$\left(U_i\right)_r = \frac{1}{2}\sigma_{pl}\varepsilon_{pl} = \frac{1}{2}(44)(10^3)(0.004) = 88\ \frac{\text{in} \cdot \text{lb}}{\text{in}^3} \qquad \textbf{Ans.}$$

Modulus of Toughness: The modulus of toughness is equal to the area under the entire stress–strain diagram. This area can be approximated by counting the number of squares. The total number of squares is 65. Thus,

$$\left[\left(U_i\right)_t\right]_{\text{approx}} = 65\left[10(10^3)\ \frac{\text{lb}}{\text{in}^2}\right]\left[0.01\frac{\text{in.}}{\text{in.}}\right] = 6.50(10^3)\ \frac{\text{in} \cdot \text{lb}}{\text{in}^3} \qquad \textbf{Ans.}$$

The stress–strain diagram for a bone is shown, and can be described by the equation

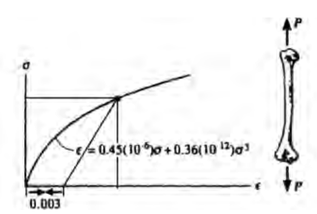
3–19. The stress–strain diagram for a bone is shown, and can be described by the equation $\epsilon = 0.45(10^{-6})\,\sigma + 0.36(10^{-12})\,\sigma^3$, where σ is in kPa. Determine the yield strength assuming a 0.3% offset.



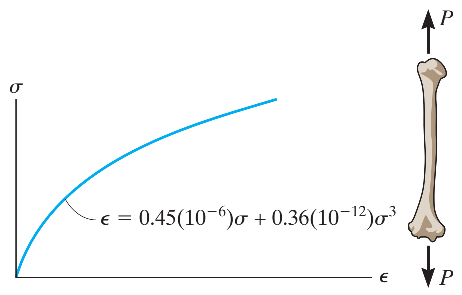
$$\varepsilon = 0.45(10^{-6})\sigma + 0.36(10^{-12})\sigma^3,$$

$$d\epsilon = \left(0.45(10^{-6}) + 1.08(10^{-12})\,\sigma^2\right)d\sigma$$

$$E = \left.\frac{d\sigma}{d\epsilon}\right|_{\sigma=0} = \frac{1}{0.45(10^{-6})} = 2.22 \text{ MPa} \qquad \textbf{Ans.}$$



***3–20.** The stress–strain diagram for a bone is shown and can be described by the equation $\epsilon = 0.45(10^{-6})\,\sigma + 0.36(10^{-12})\,\sigma^3$, where σ is in kPa. Determine the modulus of toughness and the amount of elongation of a 200-mm-long region just before it fractures if failure occurs at $\epsilon = 0.12$ mm/mm.



When $\varepsilon = 0.12$

$$120(10^3) = 0.45\,\sigma + 0.36(10^{-6})\sigma^3$$

Solving for the real root:

$$\sigma = 6873.52 \text{ kPa}$$

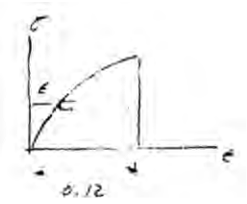
$$u_t = \int_A dA = \int_0^{6873.52} (0.12 - \varepsilon)d\sigma$$

$$u_t = \int_0^{6873.52} (0.12 - 0.45(10^{-6})\sigma - 0.36(10^{-12})\sigma^3)d\sigma$$

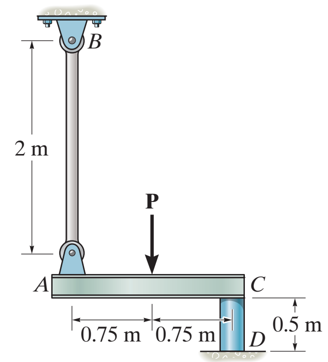
$$= 0.12\,\sigma - 0.225(10^{-6})\sigma^2 - 0.09(10^{-12})\sigma^4\Big|_0^{6873.52}$$

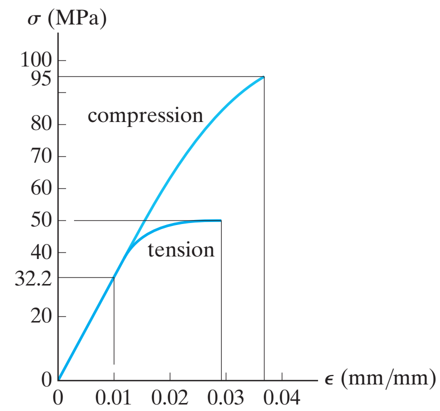
$$= 613 \text{ kJ/m}^3 \qquad \textbf{Ans.}$$

$$\delta = \varepsilon L = 0.12(200) = 24 \text{ mm} \qquad \textbf{Ans.}$$



•3–21. The stress–strain diagram for a polyester resin is given in the figure. If the rigid beam is supported by a strut AB and post CD, both made from this material, and subjected to a load of $P = 80$ kN, determine the angle of tilt of the beam when the load is applied. The diameter of the strut is 40 mm and the diameter of the post is 80 mm.





From the stress–strain diagram,

$$E = \frac{32.2(10)^6}{0.01} = 3.22(10^9)\ \text{Pa}$$

Thus,

$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{40(10^3)}{\frac{\pi}{4}(0.04)^2} = 31.83\ \text{MPa}$$

$$\varepsilon_{AB} = \frac{\sigma_{AB}}{E} = \frac{31.83(10^6)}{3.22(10^9)} = 0.009885\ \text{mm/mm}$$

$$\sigma_{CD} = \frac{F_{CD}}{A_{CD}} = \frac{40(10^3)}{\frac{\pi}{4}(0.08)^2} = 7.958\ \text{MPa}$$

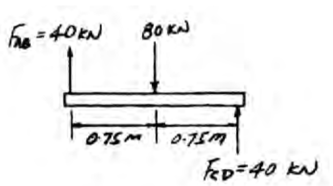
$$\varepsilon_{CD} = \frac{\sigma_{CD}}{E} = \frac{7.958(10^6)}{3.22(10^9)} = 0.002471\ \text{mm/mm}$$

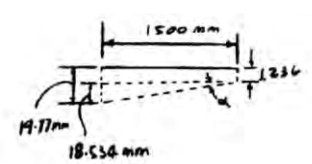
$$\delta_{AB} = \varepsilon_{AB}L_{AB} = 0.009885(2000) = 19.771\ \text{mm}$$

$$\delta_{CD} = \varepsilon_{CD}L_{CD} = 0.002471(500) = 1.236\ \text{mm}$$

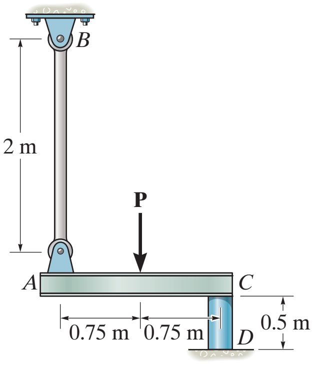
Angle of tilt α:

$$\tan\alpha = \frac{18.535}{1500}; \qquad \alpha = 0.708^\circ$$ **Ans.**





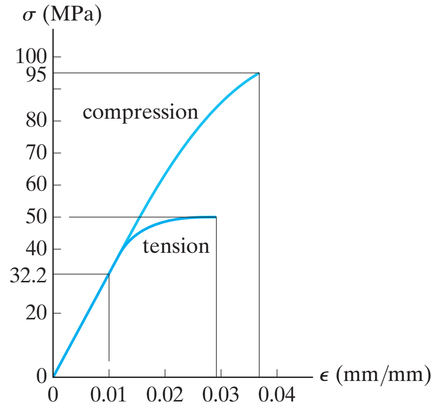
3–22. The stress–strain diagram for a polyester resin is given in the figure. If the rigid beam is supported by a strut *AB* and post *CD* made from this material, determine the largest load *P* that can be applied to the beam before it ruptures. The diameter of the strut is 12 mm and the diameter of the post is 40 mm.



Rupture of strut *AB*:

$$\sigma_R = \frac{F_{AB}}{A_{AB}}; \qquad 50(10^6) = \frac{P/2}{\frac{\pi}{4}(0.012)^2};$$

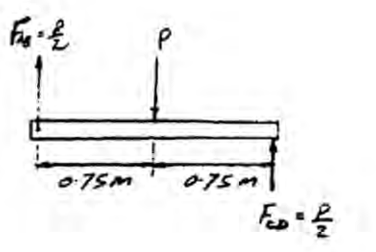
$$P = 11.3 \text{ kN (controls)}$$ **Ans.**



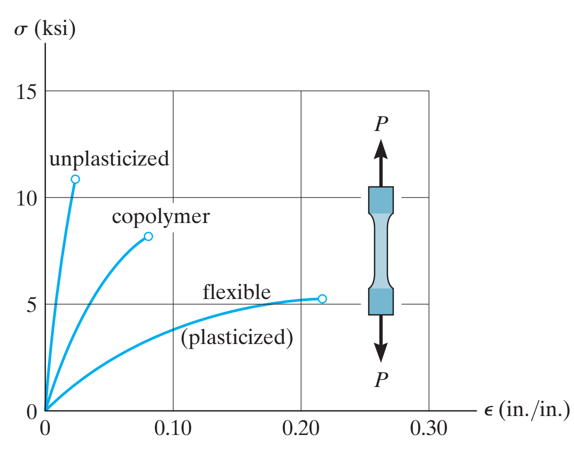
Rupture of post *CD*:

$$\sigma_R = \frac{F_{CD}}{A_{CD}}; \qquad 95(10^6) = \frac{P/2}{\frac{\pi}{4}(0.04)^2}$$

$$P = 239 \text{ kN}$$



3–23. By adding plasticizers to polyvinyl chloride, it is possible to reduce its stiffness. The stress–strain diagrams for three types of this material showing this effect are given below. Specify the type that should be used in the manufacture of a rod having a length of 5 in. and a diameter of 2 in., that is required to support at least an axial load of 20 kip and also be able to stretch at most $\frac{1}{4}$ in.



Normal Stress:

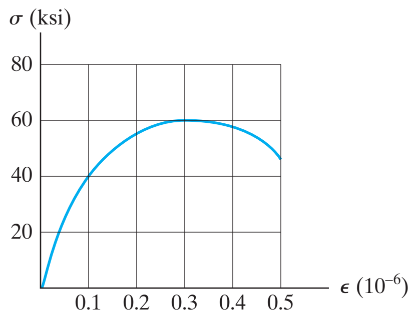
$$\sigma = \frac{P}{A} = \frac{20}{\frac{\pi}{4}(2^2)} = 6.366 \text{ ksi}$$

Normal Strain:

$$\varepsilon = \frac{0.25}{5} = 0.0500 \text{ in./in.}$$

From the stress–strain diagram, the *copolymer* will satisfy both stress and strain requirements. **Ans.**

***3–24.** The stress–strain diagram for many metal alloys can be described analytically using the Ramberg-Osgood three parameter equation $\epsilon = \sigma/E + k\sigma^n$, where E, k, and n are determined from measurements taken from the diagram. Using the stress–strain diagram shown in the figure, take $E = 30(10^3)$ ksi and determine the other two parameters k and n and thereby obtain an analytical expression for the curve.



Choose,

$$\sigma = 40 \text{ ksi}, \quad \varepsilon = 0.1$$

$$\sigma = 60 \text{ ksi}, \quad \varepsilon = 0.3$$

$$0.1 = \frac{40}{30(10^3)} + k(40)^n$$

$$0.3 = \frac{60}{30(10^3)} + k(60)^n$$

$$0.098667 = k(40)^n$$

$$0.29800 = k(60)^n$$

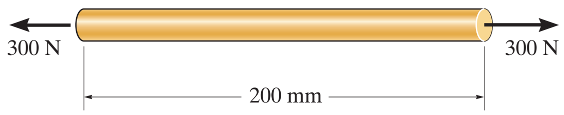
$$0.3310962 = (0.6667)^n$$

$$\ln(0.3310962) = n \ln(0.6667)$$

$$n = 2.73 \qquad \textbf{Ans.}$$

$$k = 4.23(10^{-6}) \qquad \textbf{Ans.}$$

•3–25. The acrylic plastic rod is 200 mm long and 15 mm in diameter. If an axial load of 300 N is applied to it, determine the change in its length and the change in its diameter. $E_p = 2.70$ GPa, $\nu_p = 0.4$.



$$\sigma = \frac{P}{A} = \frac{300}{\frac{\pi}{4}(0.015)^2} = 1.697 \text{ MPa}$$

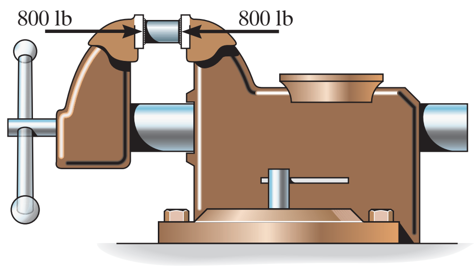
$$\varepsilon_{long} = \frac{\sigma}{E} = \frac{1.697(10^6)}{2.70(10^9)} = 0.0006288$$

$$\delta = \varepsilon_{long} L = 0.0006288(200) = 0.126 \text{ mm} \qquad \textbf{Ans.}$$

$$\varepsilon_{lat} = -V\varepsilon_{long} = -0.4(0.0006288) = -0.0002515$$

$$\Delta d = \varepsilon_{lat} d = -0.0002515(15) = -0.00377 \text{ mm} \qquad \textbf{Ans.}$$

3–26. The short cylindrical block of 2014-T6 aluminum, having an original diameter of 0.5 in. and a length of 1.5 in., is placed in the smooth jaws of a vise and squeezed until the axial load applied is 800 lb. Determine (a) the decrease in its length and (b) its new diameter.



a)

$$\sigma = \frac{P}{A} = \frac{800}{\frac{\pi}{4}(0.5)^2} = 4074.37 \text{ psi}$$

$$\varepsilon_{\text{long}} = \frac{\sigma}{E} = \frac{-4074.37}{10.6(10^6)} = -0.0003844$$

$$\delta = \varepsilon_{\text{long}}\, L = -0.0003844\,(1.5) = -0.577\,(10^{-3}) \text{ in.} \qquad \textbf{Ans.}$$

b)

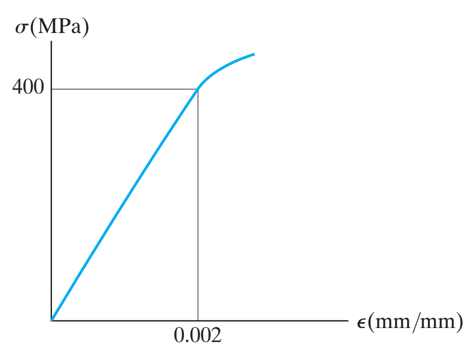
$$V = \frac{-\varepsilon_{\text{lat}}}{\varepsilon_{\text{long}}} = 0.35$$

$$\varepsilon_{\text{lat}} = -0.35\,(-0.0003844) = 0.00013453$$

$$\Delta d = \varepsilon_{\text{lat}}\, d = 0.00013453\,(0.5) = 0.00006727$$

$$d' = d + \Delta d = 0.5000673 \text{ in.} \qquad \textbf{Ans.}$$

3–27. The elastic portion of the stress–strain diagram for a steel alloy is shown in the figure. The specimen from which it was obtained had an original diameter of 13 mm and a gauge length of 50 mm. When the applied load on the specimen is 50 kN, the diameter is 12.99265 mm. Determine Poisson's ratio for the material.



Normal Stress:

$$\sigma = \frac{P}{A} = \frac{50(10^3)}{\frac{\pi}{4}(0.013^2)} = 376.70 \text{ Mpa}$$

Normal Strain: From the stress–strain diagram, the modulus of elasticity $E = \dfrac{400(10^6)}{0.002} = 200$ GPa. Applying Hooke's law

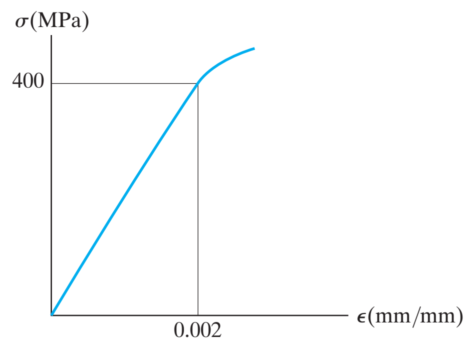
$$\varepsilon_{\text{long}} = \frac{\sigma}{E} = \frac{376.70(10^6)}{200(10^4)} = 1.8835\left(10^{-3}\right) \text{ mm/mm}$$

$$\varepsilon_{\text{lat}} = \frac{d - d_0}{d_0} = \frac{12.99265 - 13}{13} = -0.56538\left(10^{-3}\right) \text{ mm/mm}$$

Poisson's Ratio: The lateral and longitudinal strain can be related using Poisson's ratio.

$$V = -\frac{\varepsilon_{\text{lat}}}{\varepsilon_{\text{long}}} = -\frac{-0.56538(10^{-3})}{1.8835(10^{-3})} = 0.300 \qquad \textbf{Ans.}$$

***3–28.** The elastic portion of the stress–strain diagram for a steel alloy is shown in the figure. The specimen from which it was obtained had an original diameter of 13 mm and a gauge length of 50 mm. If a load of $P = 20$ kN is applied to the specimen, determine its diameter and gauge length. Take $\nu = 0.4$.



Normal Stress:

$$\sigma = \frac{P}{A} = \frac{20(10^3)}{\frac{\pi}{4}(0.013^2)} = 150.68\text{Mpa}$$

Normal Strain: From the Stress–Strain diagram, the modulus of elasticity $E = \frac{400(10^6)}{0.002} = 200$ GPa. Applying Hooke's Law

$$\varepsilon_{\text{long}} = \frac{\sigma}{E} = \frac{150.68(10^6)}{200(10^9)} = 0.7534\left(10^{-3}\right) \text{ mm/mm}$$

Thus,

$$\delta L = \varepsilon_{\text{long}} L_0 = 0.7534\left(10^{-3}\right)(50) = 0.03767 \text{ mm}$$

$$L = L_0 + \delta L = 50 + 0.03767 = 50.0377 \text{ mm} \qquad \textbf{Ans.}$$

Poisson's Ratio: The lateral and longitudinal can be related using poisson's ratio.

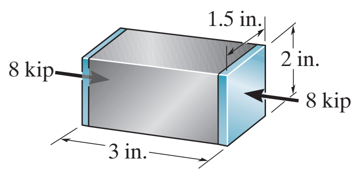
$$\varepsilon_{\text{lat}} = -\nu\varepsilon_{\text{long}} = -0.4(0.7534)\left(10^{-3}\right)$$

$$= -0.3014\left(10^{-3}\right) \text{ mm/mm}$$

$$\delta d = \varepsilon_{\text{lat}}\, d = -0.3014\left(10^{-3}\right)(13) = -0.003918 \text{ mm}$$

$$d = d_0 + \delta d = 13 + (-0.003918) = 12.99608 \text{ mm} \qquad \textbf{Ans.}$$

•3–29. The aluminum block has a rectangular cross section and is subjected to an axial compressive force of 8 kip. If the 1.5-in. side changed its length to 1.500132 in., determine Poisson's ratio and the new length of the 2-in. side. $E_{al} = 10(10^3)$ ksi.



$$\sigma = \frac{P}{A} = \frac{8}{(2)(1.5)} = 2.667 \text{ ksi}$$

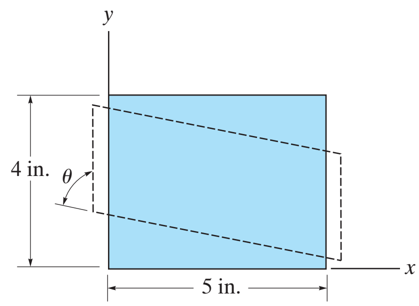
$$\varepsilon_{\text{long}} = \frac{\sigma}{E} = \frac{-2.667}{10(10^3)} = -0.0002667$$

$$\varepsilon_{\text{lat}} = \frac{1.500132 - 1.5}{1.5} = 0.0000880$$

$$\nu = \frac{-0.0000880}{-0.0002667} = 0.330 \qquad \textbf{Ans.}$$

$$h' = 2 + 0.0000880(2) = 2.000176 \text{ in.} \qquad \textbf{Ans.}$$

3–30. The block is made of titanium Ti-6A1-4V and is subjected to a compression of 0.06 in. along the y axis, and its shape is given a tilt of $\theta = 89.7°$. Determine ϵ_x, ϵ_y, and γ_{xy}.



Normal Strain:

$$\varepsilon_y = \frac{\delta L_y}{L_y} = \frac{-0.06}{4} = -0.0150 \text{ in./in.} \qquad \textbf{Ans.}$$

Poisson's Ratio: The lateral and longitudinal strain can be related using Poisson's ratio.

$$\varepsilon_x = -\nu\varepsilon_y = -0.36(-0.0150)$$

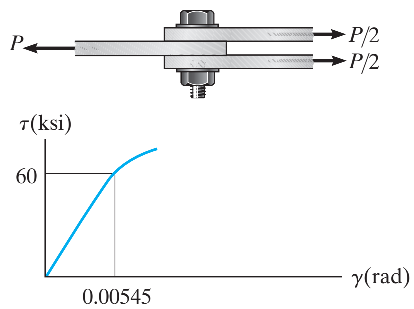
$$= 0.00540 \text{ in. /in.} \qquad \textbf{Ans.}$$

Shear Strain:

$$\beta = 180° - 89.7° = 90.3° = 1.576032 \text{ rad}$$

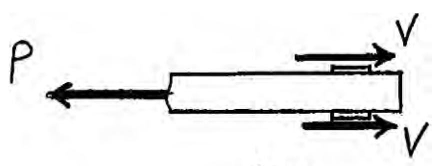
$$\gamma_{xy} = \frac{\pi}{2} - \beta = \frac{\pi}{2} - 1.576032 = -0.00524 \text{ rad} \qquad \textbf{Ans.}$$

3–31. The shear stress–strain diagram for a steel alloy is shown in the figure. If a bolt having a diameter of 0.75 in. is made of this material and used in the double lap joint, determine the modulus of elasticity E and the force P required to cause the material to yield. Take $\nu = 0.3$.



The shear force developed on the shear planes of the bolt can be determined by considering the equilibrium of the FBD shown in Fig. *a*

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad V + V - P = 0 \qquad V = \ = \frac{P}{2}$$



(a)

From the shear stress–strain diagram, the yield stress is $\tau_y = 60$ ksi. Thus,

$$\tau_y = \frac{V_y}{A}; \qquad 60 = \frac{P/2}{\frac{\pi}{4}\left(0.75^2\right)}$$

$$P = 53.01 \text{ kip} = 53.0 \text{ kip} \qquad \textbf{Ans.}$$

From the shear stress–strain diagram, the shear modulus is

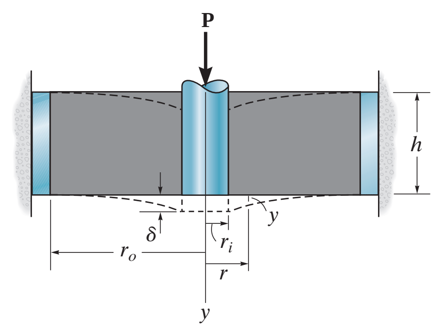
$$G = \frac{60 \text{ ksi}}{0.00545} = 11.01(10^3) \text{ ksi}$$

Thus, the modulus of elasticity is

$$G = \frac{E}{2(1 + \upsilon)}; \qquad 11.01(10^3) = \frac{E}{2(1 + 0.3)}$$

$$E = 28.6(10^3) \text{ ksi} \qquad \textbf{Ans.}$$

***3–32.** A shear spring is made by bonding the rubber annulus to a rigid fixed ring and a plug. When an axial load **P** is placed on the plug, show that the slope at point y in the rubber is $dy/dr = -\tan\gamma = -\tan(P/(2\pi hGr))$. For small angles we can write $dy/dr = -P/(2\pi hGr)$. Integrate this expression and evaluate the constant of integration using the condition that $y = 0$ at $r = r_o$. From the result compute the deflection $y = \delta$ of the plug.



Shear Stress–Strain Relationship: Applying Hooke's law with $\tau_A = \dfrac{P}{2\pi\, r\, h}$.

$$\gamma = \frac{\tau_A}{G} = \frac{P}{2\pi\, h\, G\, r}$$

$$\frac{dy}{dr} = -\tan\gamma = -\tan\left(\frac{P}{2\pi\, h\, G\, r}\right) \qquad \text{(Q.E.D)}$$

If γ is small, then $\tan\gamma = \gamma$. Therefore,

$$\frac{dy}{dr} = -\frac{P}{2\pi\, h\, G\, r}$$

$$y = -\frac{P}{2\pi\, h\, G}\int\frac{dr}{r}$$

$$y = -\frac{P}{2\pi\, h\, G}\ln r + C$$

At $r = r_o$, $\quad y = 0$

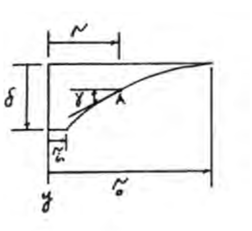
$$0 = -\frac{P}{2\pi\, h\, G}\ln r_o + C$$

$$C = \frac{P}{2\pi\, h\, G}\ln r_o$$

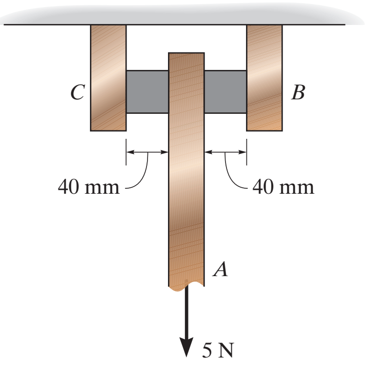
Then, $y = \dfrac{P}{2\pi\, h\, G}\ln\dfrac{r_o}{r}$

At $r = r_i$, $\quad y = \delta$

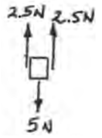
$$\delta = \frac{P}{2\pi\, h\, G}\ln\frac{r_o}{r_i} \qquad \textbf{Ans.}$$



•3–33. The support consists of three rigid plates, which are connected together using two symmetrically placed rubber pads. If a vertical force of 5 N is applied to plate A, determine the approximate vertical displacement of this plate due to shear strains in the rubber. Each pad has cross-sectional dimensions of 30 mm and 20 mm. $G_r = 0.20$ MPa.



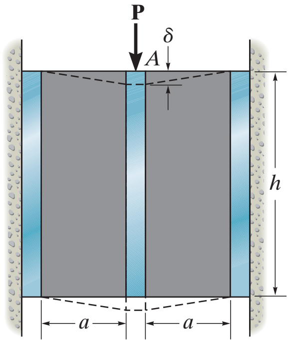
$$\tau_{avg} = \frac{V}{A} = \frac{2.5}{(0.03)(0.02)} = 4166.7 \text{ Pa}$$



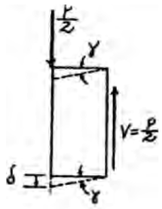
$$\gamma = \frac{\tau}{G} = \frac{4166.7}{0.2(10^6)} = 0.02083 \text{ rad}$$

$$\delta = 40(0.02083) = 0.833 \text{ mm}$$ **Ans.**

3–34. A shear spring is made from two blocks of rubber, each having a height h, width b, and thickness a. The blocks are bonded to three plates as shown. If the plates are rigid and the shear modulus of the rubber is G, determine the displacement of plate A if a vertical load **P** is applied to this plate. Assume that the displacement is small so that $\delta = a \tan \gamma \approx a\gamma$.



Average Shear Stress: The rubber block is subjected to a shear force of $V = \frac{P}{2}$.



$$\tau = \frac{V}{A} = \frac{\frac{P}{2}}{b\,h} = \frac{P}{2\,b\,h}$$

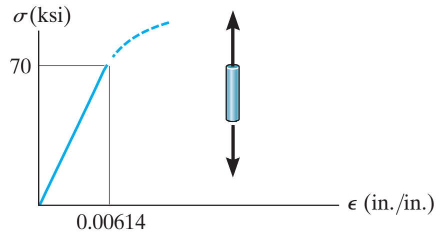
Shear Strain: Applying Hooke's law for shear

$$\gamma = \frac{\tau}{G} = \frac{\frac{P}{2\,b\,h}}{G} = \frac{P}{2\,b\,h\,G}$$

Thus,

$$\delta = a\,\gamma = \;= \frac{P\,a}{2\,b\,h\,G}$$ **Ans.**

3–35. The elastic portion of the tension stress–strain diagram for an aluminum alloy is shown in the figure. The specimen used for the test has a gauge length of 2 in. and a diameter of 0.5 in. When the applied load is 9 kip, the new diameter of the specimen is 0.49935 in. Compute the shear modulus G_{al} for the aluminum.



From the stress–strain diagram,

$$E_{al} = \frac{\sigma}{\varepsilon} = \frac{70}{0.00614} = 11400.65 \text{ ksi}$$

When specimen is loaded with a 9 - kip load,

$$\sigma = \frac{P}{A} = \frac{9}{\frac{\pi}{4}(0.5)^2} = 45.84 \text{ ksi}$$

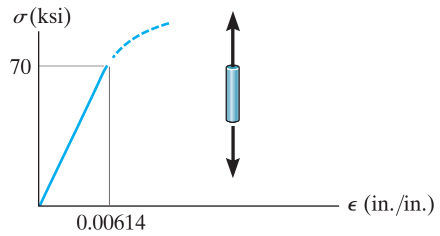
$$\varepsilon_{\text{long}} = \frac{\sigma}{E} = \frac{45.84}{11400.65} = 0.0040208 \text{ in./in.}$$

$$\varepsilon_{\text{lat}} = \frac{d' - d}{d} = \frac{0.49935 - 0.5}{0.5} = -0.0013 \text{ in./in.}$$

$$V = -\frac{\varepsilon_{\text{lat}}}{\varepsilon_{\text{long}}} = -\frac{-0.0013}{0.0040208} = 0.32332$$

$$G_{al} = \frac{E_{at}}{2(1 + v)} = \frac{11.4(10^3)}{2(1 + 0.32332)} = 4.31(10^3) \text{ ksi}$$ **Ans.**

***3–36.** The elastic portion of the tension stress–strain diagram for an aluminum alloy is shown in the figure. The specimen used for the test has a gauge length of 2 in. and a diameter of 0.5 in. If the applied load is 10 kip, determine the new diameter of the specimen. The shear modulus is $G_{al} = 3.8(10^3)$ ksi.



$$\sigma = \frac{P}{A} = \frac{10}{\frac{\pi}{4}(0.5)^2} = 50.9296 \text{ ksi}$$

From the stress–strain diagram

$$E = \frac{70}{0.00614} = 11400.65 \text{ ksi}$$

$$\varepsilon_{\text{long}} = \frac{\sigma}{E} = \frac{50.9296}{11400.65} = 0.0044673 \text{ in./in.}$$

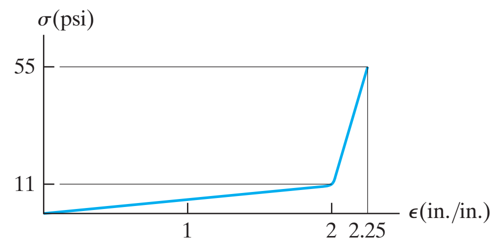
$$G = \frac{E}{2(1 + v)}; \qquad 3.8(10^3) = \frac{11400.65}{2(1 + v)}; \qquad v = 0.500$$

$$\varepsilon_{\text{lat}} = -v\varepsilon_{\text{long}} = -0.500(0.0044673) = -0.002234 \text{ in./in.}$$

$$\Delta d = \varepsilon_{\text{lat}}\, d = -0.002234(0.5) = -0.001117 \text{ in.}$$

$$d' = d + \Delta d = 0.5 - 0.001117 = 0.4989 \text{ in.}$$ **Ans.**

3–37. The $\sigma-\epsilon$ diagram for elastic fibers that make up human skin and muscle is shown. Determine the modulus of elasticity of the fibers and estimate their modulus of toughness and modulus of resilience.



$E = \frac{11}{2} = 5.5 \text{ psi}$ **Ans.**

$u_t = \frac{1}{2}(2)(11) + \frac{1}{2}(55 + 11)(2.25 - 2) = 19.25 \text{ psi}$ **Ans.**

$u_t = \frac{1}{2}(2)(11) = 11 \text{ psi}$ **Ans.**

3–38. A short cylindrical block of 6061-T6 aluminum, having an original diameter of 20 mm and a length of 75 mm, is placed in a compression machine and squeezed until the axial load applied is 5 kN. Determine (a) the decrease in its length and (b) its new diameter.

a) $\sigma = \frac{P}{A} = \frac{-5(10^3)}{\frac{\pi}{4}(0.02)^2} = -15.915 \text{ MPa}$

$\sigma = E\,\varepsilon_{long}; \qquad -15.915(10^6) = 68.9(10^9)\,\varepsilon_{long}$

$$\varepsilon_{long} = -0.0002310 \text{ mm/mm}$$

$\delta = \varepsilon_{long} L = -0.0002310(75) = -0.0173 \text{ mm}$ **Ans.**

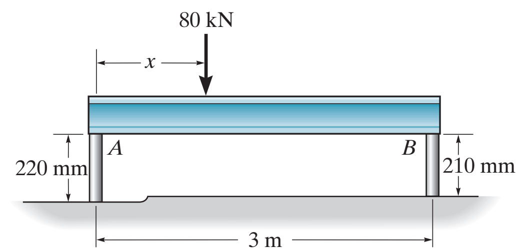
b) $v = -\frac{\varepsilon_{lat}}{\varepsilon_{long}}; \qquad 0.35 = -\frac{\varepsilon_{lat}}{-0.0002310}$

$$\varepsilon_{lat} = 0.00008085 \text{ mm/mm}$$

$\Delta d = \varepsilon_{lat}\, d = 0.00008085(20) = 0.0016 \text{ mm}$

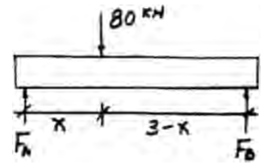
$d' = d + \Delta d = 20 + 0.0016 = 20.0016 \text{ mm}$ **Ans.**

3–39. The rigid beam rests in the horizontal position on two 2014-T6 aluminum cylinders having the *unloaded* lengths shown. If each cylinder has a diameter of 30 mm, determine the placement x of the applied 80-kN load so that the beam remains horizontal. What is the new diameter of cylinder A after the load is applied? $\nu_{al} = 0.35$.



$$\circlearrowleft + \Sigma M_A = 0; \qquad F_B(3) - 80(x) = 0; \qquad F_B = \frac{80x}{3} \qquad \textbf{(1)}$$

$$\circlearrowleft + \Sigma M_B = 0; \qquad -F_A(3) + 80(3 - x) = 0; \qquad F_A = \frac{80(3 - x)}{3} \qquad \textbf{(2)}$$



Since the beam is held horizontally, $\delta_A = \delta_B$

$$\sigma = \frac{P}{A}; \qquad \varepsilon = \frac{\sigma}{E} = \frac{\frac{P}{A}}{E}$$

$$\delta = \varepsilon L = \left(\frac{\frac{P}{A}}{E}\right) L = \frac{PL}{AE}$$

$$\delta_A = \delta_B; \qquad \frac{\frac{80(3 - x)}{3}(220)}{AE} = \frac{\frac{80x}{3}(210)}{AE}$$

$$80(3 - x)(220) = 80x(210)$$

$$x = 1.53 \text{ m} \qquad \textbf{Ans.}$$

From Eq. (2),

$$F_A = 39.07 \text{ kN}$$

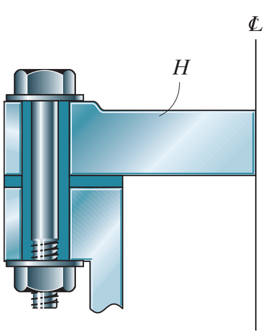
$$\sigma_A = \frac{F_A}{A} = \frac{39.07(10^3)}{\frac{\pi}{4}(0.03^2)} = 55.27 \text{ MPa}$$

$$\varepsilon_{long} = \frac{\sigma_A}{E} = -\frac{55.27(10^6)}{73.1(10^9)} = -0.000756$$

$$\varepsilon_{lat} = -\nu\varepsilon_{long} = -0.35(-0.000756) = 0.0002646$$

$$d'_A = d_A + d\,\varepsilon_{lat} = 30 + 30(0.0002646) = 30.008 \text{ mm} \qquad \textbf{Ans.}$$

*__3–40.__ The head H is connected to the cylinder of a compressor using six steel bolts. If the clamping force in each bolt is 800 lb, determine the normal strain in the bolts. Each bolt has a diameter of $\frac{3}{16}$ in. If $\sigma_Y = 40$ ksi and $E_{st} = 29(10^3)$ ksi, what is the strain in each bolt when the nut is unscrewed so that the clamping force is released?



Normal Stress:

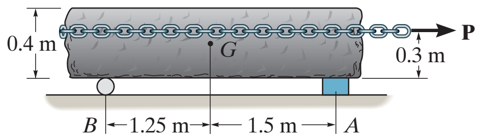
$$\sigma = \frac{P}{A} = \frac{800}{\frac{\pi}{4}\left(\frac{3}{16}\right)^2} = 28.97 \text{ ksi} < \sigma_\gamma = 40 \text{ ksi}$$

Normal Strain: Since $\sigma < \sigma_\gamma$, Hooke's law is still valid.

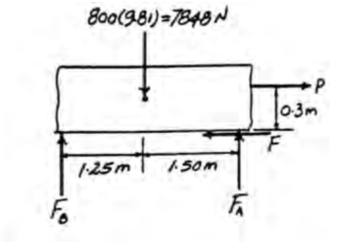
$$\varepsilon = \frac{\sigma}{E} = \frac{28.97}{29(10^3)} = 0.000999 \text{ in./in.} \qquad \textbf{Ans.}$$

If the nut is unscrewed, the load is zero. Therefore, the strain $\varepsilon = 0$ **Ans.**

•**3–41.** The stone has a mass of 800 kg and center of gravity at G. It rests on a pad at A and a roller at B. The pad is fixed to the ground and has a compressed height of 30 mm, a width of 140 mm, and a length of 150 mm. If the coefficient of static friction between the pad and the stone is $\mu_s = 0.8$, determine the approximate horizontal displacement of the stone, caused by the shear strains in the pad, before the stone begins to slip. Assume the normal force at A acts 1.5 m from G as shown. The pad is made from a material having $E = 4$ MPa and $\nu = 0.35$.



Equations of Equilibrium:



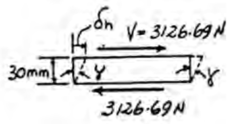
$$\circlearrowleft +\Sigma M_B = 0; \qquad F_A(2.75) - 7848(1.25) - P(0.3) = 0 \qquad \textbf{[1]}$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad P - F = 0 \qquad \textbf{[2]}$$

Note: The normal force at A does not act exactly at A. It has to shift due to friction.

Friction Equation:

$$F = \mu_s F_A = 0.8\, F_A \qquad \textbf{[3]}$$



Solving Eqs. [1], [2] and [3] yields:

$$F_A = 3908.37 \text{ N} \qquad F = P = 3126.69 \text{ N}$$

Average Shear Stress: The pad is subjected to a shear force of $V = F = 3126.69$ N.

$$\tau = \frac{V}{A} = \frac{3126.69}{(0.14)(0.15)} = 148.89 \text{ kPa}$$

Modulus of Rigidity:

$$G = \frac{E}{2(1 + v)} = \frac{4}{2(1 + 0.35)} = 1.481 \text{ MPa}$$

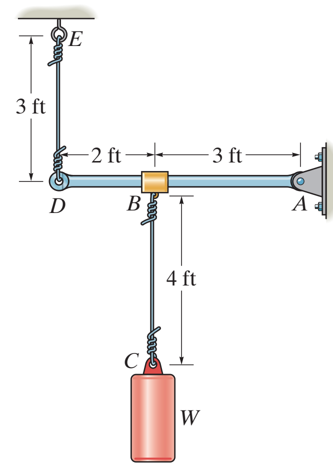
Shear Strain: Applying Hooke's law for shear

$$\gamma = \frac{\tau}{G} = \frac{148.89(10^3)}{1.481(10^6)} = 0.1005 \text{ rad}$$

Thus,

$$\delta_h = h\gamma = 30(0.1005) = 3.02 \text{ mm} \qquad \textbf{Ans.}$$

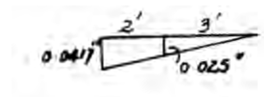
3–42. The bar DA is rigid and is originally held in the horizontal position when the weight W is supported from C. If the weight causes B to be displaced downward 0.025 in., determine the strain in wires DE and BC. Also, if the wires are made of A-36 steel and have a cross-sectional area of 0.002 in^2, determine the weight W.



$$\frac{3}{0.025} = \frac{5}{\delta}$$

$$\delta = 0.0417 \text{ in}$$

$$\varepsilon_{DE} = \frac{\delta}{L} = \frac{0.0417}{3(12)} = 0.00116 \text{ in./in.} \qquad \textbf{Ans.}$$

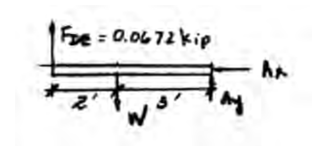


$$\sigma_{DE} = E\varepsilon_{DE} = 29(10^3)(0.00116) = 33.56 \text{ ksi}$$

$$F_{DE} = \sigma_{DE}A_{DE} = 33.56\,(0.002) = 0.0672 \text{ kip}$$

$$\circlearrowleft + \Sigma M_A = 0; \qquad -(0.0672)\,(5) + 3(W) = 0$$

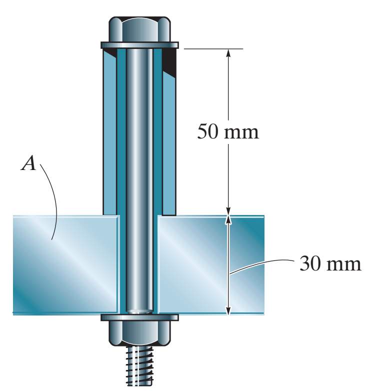
$$W = 0.112 \text{ kip} = 112 \text{ lb} \qquad \textbf{Ans.}$$



$$\sigma_{BC} = \frac{W}{A_{BC}} = \frac{0.112}{0.002} = 55.94 \text{ ksi}$$

$$\varepsilon_{BC} = \frac{\sigma_{BC}}{E} = \frac{55.94}{29\,(10^3)} = 0.00193 \text{ in./in.} \qquad \textbf{Ans.}$$

3–43. The 8-mm-diameter bolt is made of an aluminum alloy. It fits through a magnesium sleeve that has an inner diameter of 12 mm and an outer diameter of 20 mm. If the original lengths of the bolt and sleeve are 80 mm and 50 mm, respectively, determine the strains in the sleeve and the bolt if the nut on the bolt is tightened so that the tension in the bolt is 8 kN. Assume the material at A is rigid. $E_{al} = 70$ GPa, $E_{mg} = 45$ GPa.



Normal Stress:

$$\sigma_b = \frac{P}{A_b} = \frac{8(10^3)}{\frac{\pi}{4}(0.008^2)} = 159.15 \text{ MPa}$$

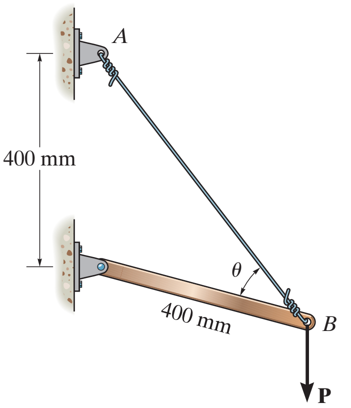
$$\sigma_s = \frac{P}{A_s} = \frac{8(10^3)}{\frac{\pi}{4}(0.02^2 - 0.012^2)} = 39.79 \text{ MPa}$$

Normal Strain: Applying Hooke's Law

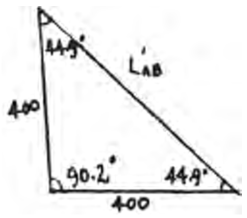
$$\varepsilon_b = \frac{\sigma_b}{E_{al}} = \frac{159.15(10^6)}{70(10^9)} = 0.00227 \text{ mm/mm} \qquad \textbf{Ans.}$$

$$\varepsilon_s = \frac{\sigma_s}{E_{mg}} = \frac{39.79(10^6)}{45(10^9)} = 0.000884 \text{ mm/mm} \qquad \textbf{Ans.}$$

***3–44.** The A-36 steel wire AB has a cross-sectional area of 10 mm^2 and is unstretched when $\theta = 45.0°$. Determine the applied load P needed to cause $\theta = 44.9°$.



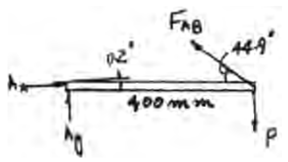
$$\frac{L'_{AB}}{\sin 90.2°} = \frac{400}{\sin 44.9°}$$



$$L'_{AB} = 566.67 \text{ mm}$$

$$L_{AB} = \frac{400}{\sin 45°} = 565.69$$

$$\varepsilon = \frac{L'_{AB} - L_{AB}}{L_{AB}} = \frac{566.67 - 565.69}{565.69} = 0.001744$$



$$\sigma = E\varepsilon = 200(10^9)\,(0.001744) = 348.76 \text{ MPa}$$

$$\circlearrowleft +\Sigma M_A = 0$$

$$P(400 \cos 0.2°) - F_{AB} \sin 44.9° \,(400) = 0 \qquad \textbf{(1)}$$

However,

$$F_{AB} = \sigma A = 348.76(10^6)(10)(10^{-6}) = 3.488 \text{ kN}$$

From Eq. (1),

$$P = 2.46 \text{ kN} \qquad \textbf{Ans.}$$