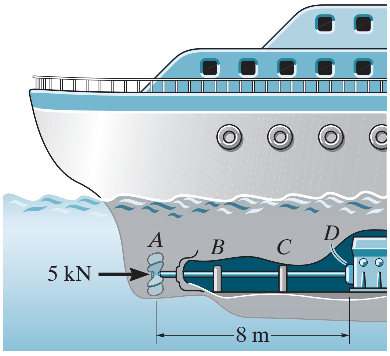
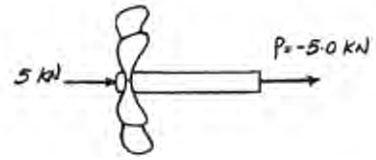
CONTENTS

•4–1. The ship is pushed through the water using an A-36 steel propeller shaft that is 8 m long, measured from the propeller to the thrust bearing D at the engine. If it has an outer diameter of 400 mm and a wall thickness of 50 mm, determine the amount of axial contraction of the shaft when the propeller exerts a force on the shaft of 5 kN. The bearings at B and C are journal bearings.



Internal Force: As shown on *FBD*.



Displacement:

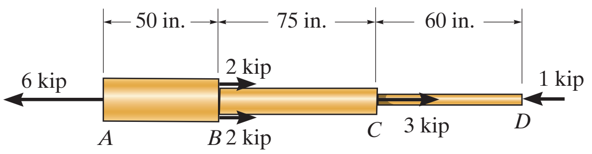
$$\delta_A = \frac{PL}{AE} = \frac{-5.00\,(10^3)(8)}{\frac{\pi}{4}\,(0.4^2 - 0.3^2)\,200(10^9)}$$

$$= -3.638(10^{-6})\text{ m}$$

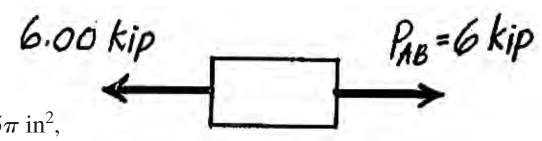
$$= -3.64\left(10^{-3}\right)\text{ mm} \qquad \textbf{Ans.}$$

Negative sign indicates that end A moves towards end D.

4–2. The copper shaft is subjected to the axial loads shown. Determine the displacement of end A with respect to end D. The diameters of each segment are $d_{AB} = 3$ in., $d_{BC} = 2$ in., and $d_{CD} = 1$ in. Take $E_{cu} = 18(10^3)$ ksi.



The normal forces developed in segment AB, BC and CD are shown in the *FBDS* of each segment in Fig. *a*, *b* and *c* respectively.



(a)

The cross-sectional area of segment AB, BC and CD are $A_{AB} = \frac{\pi}{4}(3^2) = 2.25\pi\text{ in}^2$, $A_{BC} = \frac{\pi}{4}(2^2) = \pi\text{ in}^2$ and $A_{CD} = \frac{\pi}{4}(1^2) = 0.25\pi\text{ in}^2$.

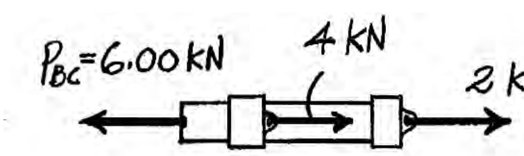
Thus,

$$\delta_{A/D} = \Sigma\frac{P_iL_i}{A_iE_i} = \frac{P_{AB}\,L_{AB}}{A_{AB}\,E_{Cu}} + \frac{P_{BC}\,L_{BC}}{A_{BC}\,E_{Cu}} + \frac{P_{CD}L_{CD}}{A_{CD}\,E_{Cu}}$$

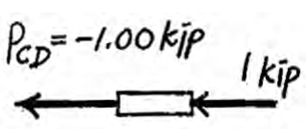
$$= \frac{6.00\,(50)}{(2.25\pi)\left[18(10^3)\right]} + \frac{2.00\,(75)}{\pi\left[18(10^3)\right]} + \frac{-1.00\,(60)}{(0.25\pi)\left[18(10^3)\right]}$$

$$= 0.766(10^{-3})\text{ in.} \qquad \textbf{Ans.}$$

The positive sign indicates that end A moves away from D.

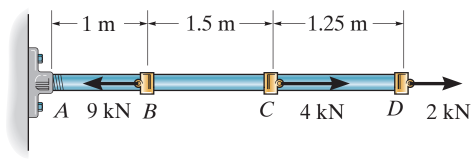


(b)



(c)

4–3. The A-36 steel rod is subjected to the loading shown. If the cross-sectional area of the rod is $50\ \text{mm}^2$, determine the displacement of its end D. Neglect the size of the couplings at B, C, and D.



The normal forces developed in segments AB, BC and CD are shown in the $FBDS$ of each segment in Fig. a, b and c, respectively.

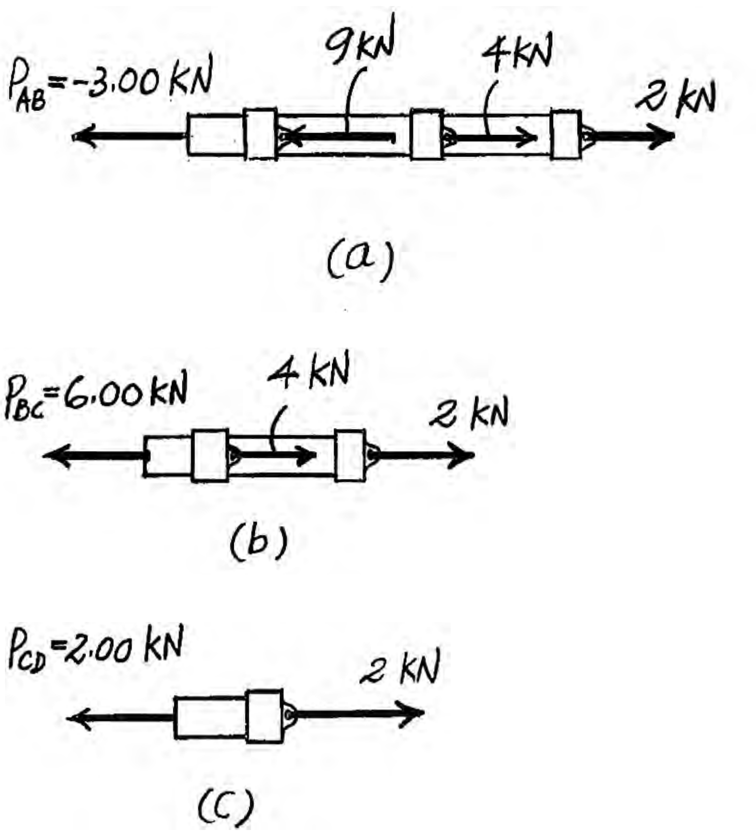
The cross-sectional areas of all the segments are $A = \left(50\ \text{mm}^2\right)\left(\dfrac{1\ \text{m}}{1000\ \text{mm}}\right)^2 = 50.0(10^{-6})\ \text{m}^2$.

$$\delta_D = \Sigma \frac{P_i L_i}{A_i E_i} = \frac{1}{A\,E_{SC}}\left(P_{AB}\,L_{AB} + P_{BC}\,L_{BC} + P_{CD}\,L_{CD}\right)$$

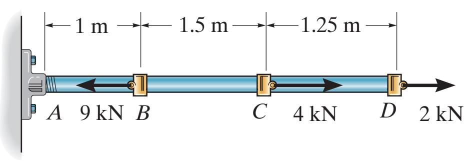
$$= \frac{1}{50.0(10^{-6})\left[200(10^9)\right]}\left[-3.00(10^3)(1) + 6.00(10^3)(1.5) + 2.00(10^3)(1.25)\right]$$

$$= 0.850(10^{-3})\ \text{m} = 0.850\ \text{mm} \qquad \textbf{Ans.}$$

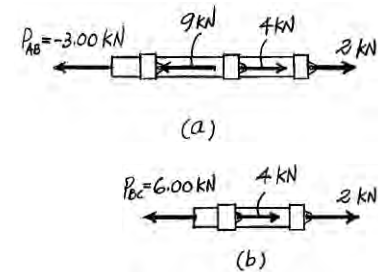
The positive sign indicates that end D moves away from the fixed support.



***4–4.** The A-36 steel rod is subjected to the loading shown. If the cross-sectional area of the rod is 50 mm^2, determine the displacement of C. Neglect the size of the couplings at B, C, and D.



The normal forces developed in segments AB and BC are shown the $FBDS$ of each segment in Fig. a and b, respectively. The cross-sectional area of these two segments are $A = \left(50 \text{ mm}^2\right)\left(\dfrac{1 \text{ m}}{10.00 \text{ mm}}\right)^2 = 50.0\,(10^{-6}) \text{ m}^2$. Thus,



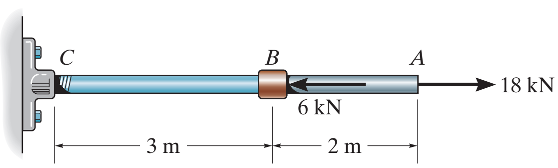
$$\delta_C = \Sigma \frac{P_i L_i}{A_i E_i} = \frac{1}{A\,E_{SC}}\left(P_{AB}\,L_{AB} + P_{BC}\,L_{BC}\right)$$

$$= \frac{1}{50.0(10^{-6})\left[200(10^9)\right]}\left[-3.00(10^3)(1) + 6.00(10^3)(1.5)\right]$$

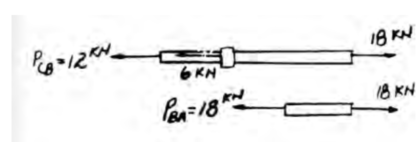
$$= 0.600\,(10^{-3}) \text{ m} = 0.600 \text{ mm}$$ **Ans.**

The positive sign indicates that coupling C moves away from the fixed support.

4–5. The assembly consists of a steel rod CB and an aluminum rod BA, each having a diameter of 12 mm. If the rod is subjected to the axial loadings at A and at the coupling B, determine the displacement of the coupling B and the end A. The unstretched length of each segment is shown in the figure. Neglect the size of the connections at B and C, and assume that they are rigid. $E_{st} = 200$ GPa, $E_{al} = 70$ GPa.



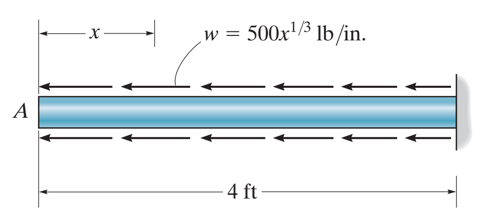
$$\delta_B = \frac{PL}{AE} = \frac{12(10^3)(3)}{\frac{\pi}{4}(0.012)^2(200)(10^9)} = 0.00159 \text{ m} = 1.59 \text{ mm}$$ **Ans.**

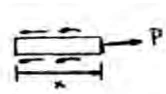


$$\delta_A = \Sigma\frac{PL}{AE} = \frac{12(10^3)(3)}{\frac{\pi}{4}(0.012)^2(200)(10^9)} + \frac{18(10^3)(2)}{\frac{\pi}{4}(0.012)^2(70)(10^9)}$$

$$= 0.00614 \text{ m} = 6.14 \text{ mm}$$ **Ans.**

4–6. The bar has a cross-sectional area of 3 in^2, and $E = 35(10^3)$ ksi. Determine the displacement of its end A when it is subjected to the distributed loading.



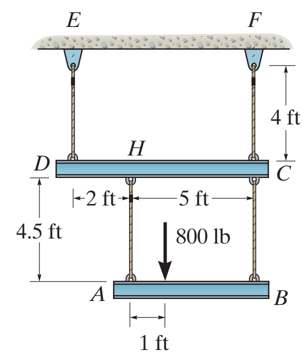


$$P(x) = \int_0^x w\,dx = 500\int_0^x x^{\frac{1}{3}}\,dx = \frac{1500}{4}\,x^{\frac{4}{3}}$$

$$\delta_A = \int_0^L \frac{P(x)\,dx}{AE} = \frac{1}{(3)(35)(10^6)}\int_0^{4(12)} \frac{1500}{4}\,x^{\frac{4}{3}}\,dx = \left(\frac{1500}{(3)(35)(10^8)(4)}\right)\left(\frac{3}{7}\right)(48)^{\frac{1}{3}}$$

$$\delta_A = 0.0128 \text{ in.}$$ **Ans.**

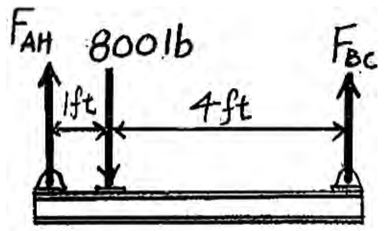
4–7. The load of 800 lb is supported by the four 304 stainless steel wires that are connected to the rigid members AB and DC. Determine the vertical displacement of the load if the members were horizontal before the load was applied. Each wire has a cross-sectional area of 0.05 in^2.



Referring to the *FBD* of member AB, Fig. a

$$\circlearrowleft +\Sigma M_A = 0; \qquad F_{BC}(5) - 800(1) = 0 \qquad F_{BC} = 160 \text{ lb}$$

$$\circlearrowleft +\Sigma M_B = 0; \qquad 800(4) - F_{AH}(5) = 0 \qquad F_{AH} = 640 \text{ lb}$$



(a)

Using the results of F_{BC} and F_{AH}, and referring to the *FBD* of member DC, Fig. b

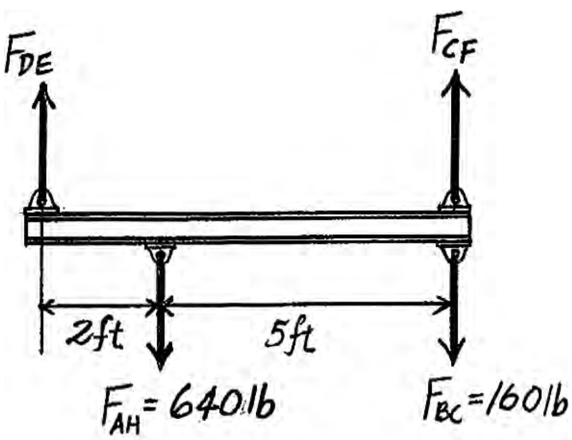
$$\circlearrowleft +\Sigma M_D = 0; \qquad F_{CF}(7) - 160(7) - 640(2) = 0 \qquad F_{CF} = 342.86 \text{ lb}$$

$$\circlearrowleft +\Sigma M_C = 0; \qquad 640(5) - F_{DE}(7) = 0 \qquad F_{DE} = 457.14 \text{ lb}$$

Since E and F are fixed,

$$\delta_D = \frac{F_{DE} L_{DE}}{A E_{st}} = \frac{457.14(4)(2)}{0.05\left[28.0\,(10^6)\right]} = 0.01567 \text{ in } \downarrow$$

$$\delta_C = \frac{F_{CF} L_{CF}}{A E_{st}} = \frac{342.86\,(4)(12)}{0.05\left[28.0\,(10^6)\right]} = 0.01176 \text{ in } \downarrow$$



(b)

From the geometry shown in Fig. c,

$$\delta_H = 0.01176 + \frac{5}{7}(0.01567 - 0.01176) = 0.01455 \text{ in } \downarrow$$

Subsequently,

$$\delta_{A/H} = \frac{F_{AH} L_{AH}}{A E_{st}} = \frac{640(4.5)(12)}{0.05\left[28.0(10^6)\right]} = 0.02469 \text{ in } \downarrow$$

$$\delta_{B/C} = \frac{F_{BC} L_{BC}}{A E_{st}} = \frac{160(4.5)(12)}{0.05\left[28.0(10^6)\right]} = 0.006171 \text{ in } \downarrow$$

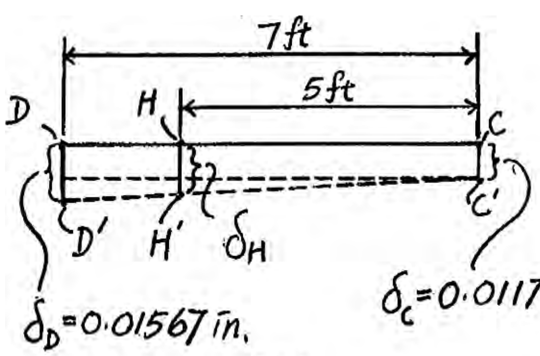
Thus,

$$(+\downarrow) \quad \delta_A = \delta_H + \delta_{A/H} = 0.01455 + 0.02469 = 0.03924 \text{ in } \downarrow$$

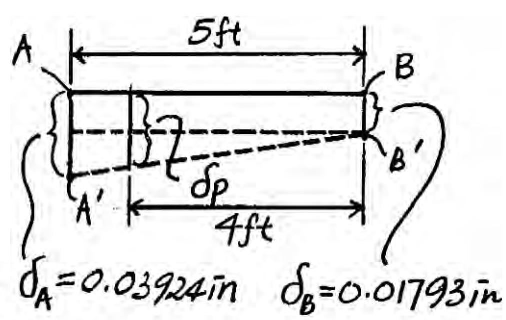
$$(+\downarrow) \quad \delta_B = \delta_C + \delta_{B/C} = 0.01176 + 0.006171 = 0.01793 \text{ in } \downarrow$$

From the geometry shown in Fig. d,

$$\delta_P = 0.01793 + \frac{4}{5}(0.03924 - 0.01793) = 0.0350 \text{ in } \downarrow \qquad \textbf{Ans.}$$

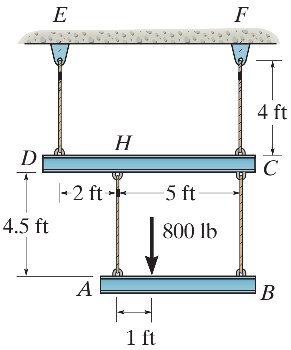


(c)



(d)

***4–8.** The load of 800 lb is supported by the four 304 stainless steel wires that are connected to the rigid members AB and DC. Determine the angle of tilt of each member after the load is applied. The members were originally horizontal, and each wire has a cross-sectional area of 0.05 in^2.



Referring to the *FBD* of member AB, Fig. *a*,

$\zeta + \Sigma M_A = 0; \quad F_{BC}(5) - 800(1) = 0 \quad F_{BC} = 160 \text{ lb}$

$\zeta + \Sigma M_B = 0; \quad 800(4) - F_{AH}(5) = 0 \quad F_{AH} = 640 \text{ lb}$

Using the results of F_{BC} and F_{AH} and referring to the *FBD* of member DC, Fig. *b*,

$\zeta + \Sigma M_D = 0; \quad F_{CF}(7) - 160(7) - 640(2) = 0 \quad F_{CF} = 342.86 \text{ lb}$

$\zeta + \Sigma M_C = 0; \quad 640(5) - F_{DE}(7) = 0 \quad F_{DE} = 457.14 \text{ lb}$

Since E and F are fixed,

$$\delta_D = \frac{F_{DE}\,L_{DE}}{A\,E_{st}} = \frac{457.14\,(4)(12)}{0.05\left[28.0(10^6)\right]} = 0.01567 \text{ in} \downarrow$$

$$\delta_C = \frac{F_{CF}\,L_{CF}}{A\,E_{st}} = \frac{342.86\,(4)(12)}{0.05\left[28.0(10^6)\right]} = 0.01176 \text{ in} \downarrow$$

From the geometry shown in Fig. *c*

$$\delta_H = 0.01176 + \frac{5}{7}(0.01567 - 0.01176) = 0.01455 \text{ in} \downarrow$$

$$\theta = \frac{0.01567 - 0.01176}{7(12)} = 46.6(10^{-6}) \text{ rad} \qquad \textbf{Ans.}$$

Subsequently,

$$\delta_{A/H} = \frac{F_{AH}\,L_{AH}}{A\,E_{st}} = \frac{640\,(4.5)(12)}{0.05\left[28.0(10^6)\right]} = 0.02469 \text{ in} \downarrow$$

$$\delta_{B/C} = \frac{F_{BC}\,L_{BC}}{A\,E_{st}} = \frac{160\,(4.5)(12)}{0.05\left[28.0(10^6)\right]} = 0.006171 \text{ in} \downarrow$$

Thus,

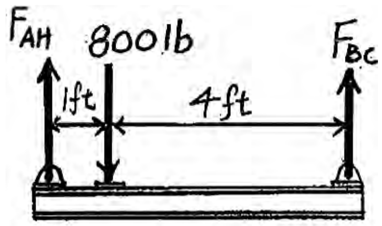
$$(+\downarrow) \quad \delta_A = \delta_H + \delta_{A/H} = 0.01455 + 0.02469 = 0.03924 \text{ in} \downarrow$$

$$(+\downarrow) \quad \delta_B = \delta_C + \delta_{B/C} = 0.01176 + 0.006171 = 0.01793 \text{ in} \downarrow$$

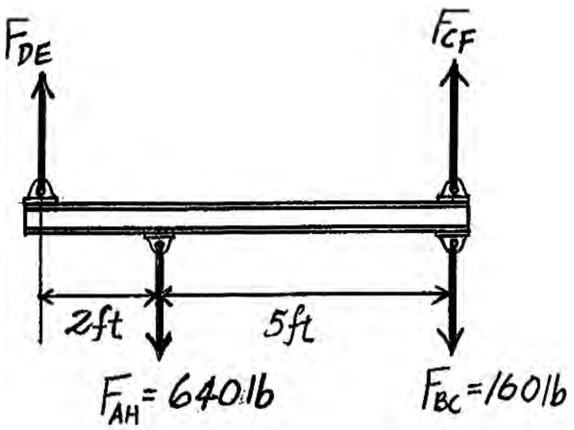
From the geometry shown in Fig. *d*

$$\phi = \frac{0.03924 - 0.01793}{5(12)} = 0.355(10^{-3}) \text{ rad} \qquad \textbf{Ans.}$$

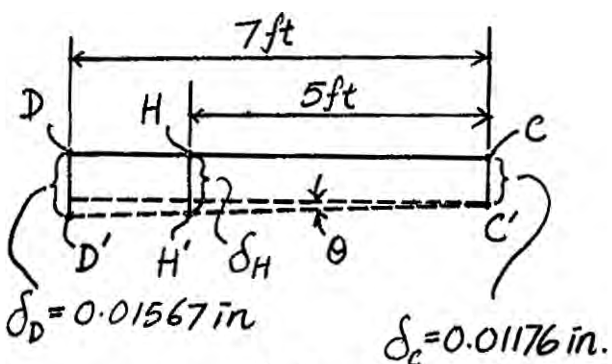
4–8. Continued



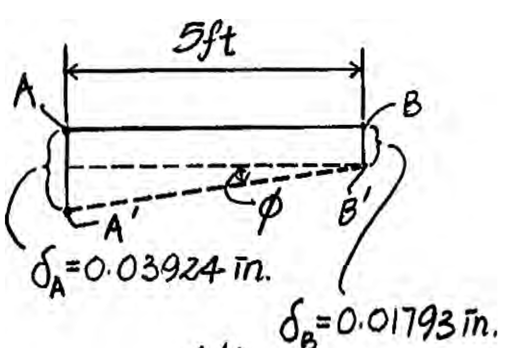
(a)



(b)

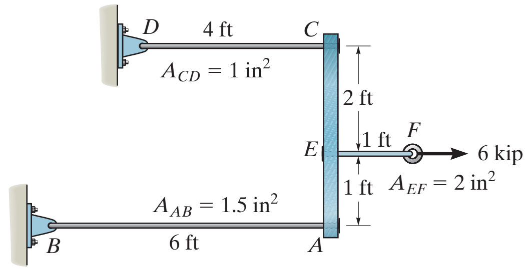


(c)



(d)

•4–9. The assembly consists of three titanium (Ti-6A1-4V) rods and a rigid bar *AC*. The cross-sectional area of each rod is given in the figure. If a force of 6 kip is applied to the ring *F*, determine the horizontal displacement of point *F*.

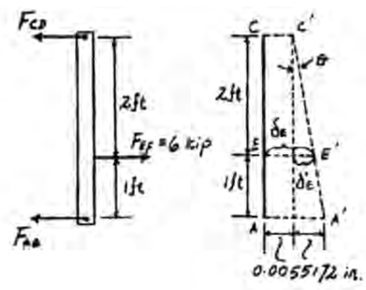


***Internal Force in the Rods*:**

$$\circlearrowleft +\Sigma M_A = 0; \qquad F_{CD}(3) - 6(1) = 0 \qquad F_{CD} = 2.00 \text{ kip}$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 6 - 2.00 - F_{AB} = 0 \qquad F_{AB} = 4.00 \text{ kip}$$

***Displacement*:**



$$\delta_C = \frac{F_{CD}\,L_{CD}}{A_{CD}\,E} = \frac{2.00(4)(12)}{(1)(17.4)(10^3)} = 0.0055172 \text{ in.}$$

$$\delta_A = \frac{F_{AB}\,L_{AB}}{A_{AB}\,E} = \frac{4.00(6)(12)}{(1.5)(17.4)(10^3)} = 0.0110344 \text{ in.}$$

$$\delta_{F/E} = \frac{F_{EF}\,L_{EF}}{A_{EF}\,E} = \frac{6.00(1)(12)}{(2)(17.4)(10^3)} = 0.0020690 \text{ in.}$$

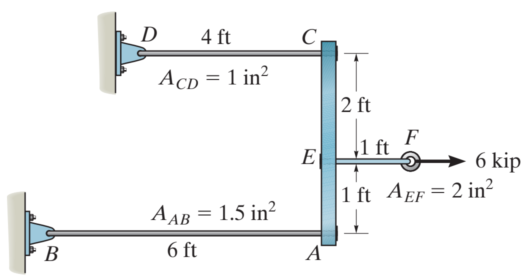
$$\frac{\delta'_E}{2} = \frac{0.0055172}{3}; \qquad \delta'_E = 0.0036782 \text{ in.}$$

$$\delta_E = \delta_C + \delta'_E = 0.0055172 + 0.0036782 = 0.0091954 \text{ in.}$$

$$\delta_F = \delta_E + \delta_{F/E}$$

$$= 0.0091954 + 0.0020690 = 0.0113 \text{ in.}$$ **Ans.**

4–10. The assembly consists of three titanium (Ti-6A1-4V) rods and a rigid bar *AC*. The cross-sectional area of each rod is given in the figure. If a force of 6 kip is applied to the ring *F*, determine the angle of tilt of bar *AC*.

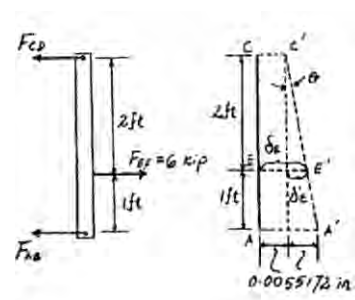


***Internal Force in the Rods*:**

$$\circlearrowleft +\Sigma M_A = 0; \qquad F_{CD}(3) - 6(1) = 0 \qquad F_{CD} = 2.00 \text{ kip}$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 6 - 2.00 - F_{AB} = 0 \qquad F_{AB} = 4.00 \text{ kip}$$

***Displacement*:**



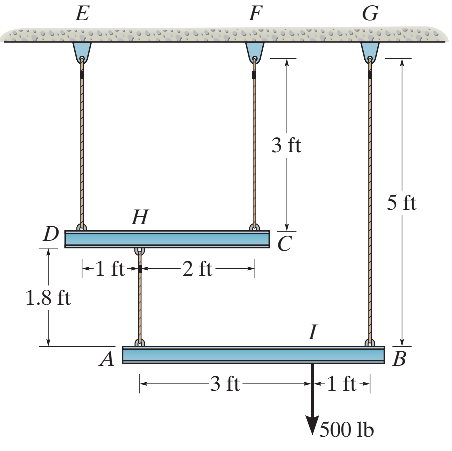
$$\delta_C = \frac{F_{CD}\,L_{CD}}{A_{CD}\,E} = \frac{2.00(4)(12)}{(1)(17.4)(10^3)} = 0.0055172 \text{ in.}$$

$$\delta_A = \frac{F_{AB}\,L_{AB}}{A_{AB}\,E} = \frac{4.00(6)(12)}{(1.5)(17.4)(10^3)} = 0.0110344 \text{ in.}$$

$$\theta = \tan^{-1}\frac{\delta_A - \delta_C}{3(12)} = \tan^{-1}\frac{0.0110344 - 0.0055172}{3(12)}$$

$$= 0.00878^\circ$$ **Ans.**

4–11. The load is supported by the four 304 stainless steel wires that are connected to the rigid members AB and DC. Determine the vertical displacement of the 500-lb load if the members were originally horizontal when the load was applied. Each wire has a cross-sectional area of 0.025 in^2.



Internal Forces in the wires:

FBD (b)

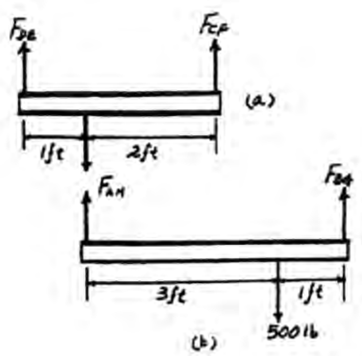
$$\circlearrowleft +\Sigma M_A = 0; \qquad F_{BC}(4) - 500(3) = 0 \qquad F_{BC} = 375.0\text{ lb}$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{AH} + 375.0 - 500 = 0 \qquad F_{AH} = 125.0\text{ lb}$$

FBD (a)

$$\circlearrowleft +\Sigma M_D = 0; \qquad F_{CF}(3) - 125.0(1) = 0 \qquad F_{CF} = 41.67\text{ lb}$$

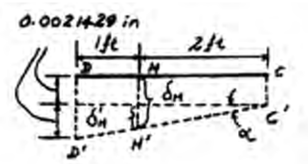
$$+\uparrow \Sigma F_y = 0; \qquad F_{DE} + 41.67 - 125.0 = 0 \qquad F_{DE} = 83.33\text{ lb}$$



Displacement:

$$\delta_D = \frac{F_{DE}L_{DE}}{A_{DE}E} = \frac{83.33(3)(12)}{0.025(28.0)(10^6)} = 0.0042857\text{ in.}$$

$$\delta_C = \frac{F_{CF}L_{CF}}{A_{CF}E} = \frac{41.67(3)(12)}{0.025(28.0)(10^6)} = 0.0021429\text{ in.}$$



$$\frac{\delta'_H}{2} = \frac{0.0021429}{3}; \qquad \delta'_H = 0.0014286\text{ in.}$$

$$\delta_H = 0.0014286 + 0.0021429 = 0.0035714\text{ in.}$$

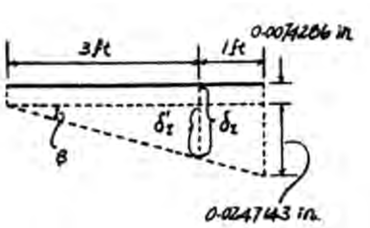
$$\delta_{A/H} = \frac{F_{AH}L_{AH}}{A_{AH}E} = \frac{125.0(1.8)(12)}{0.025(28.0)(10^6)} = 0.0038571\text{ in.}$$

$$\delta_A = \delta_H + \delta_{A/H} = 0.0035714 + 0.0038571 = 0.0074286\text{ in.}$$

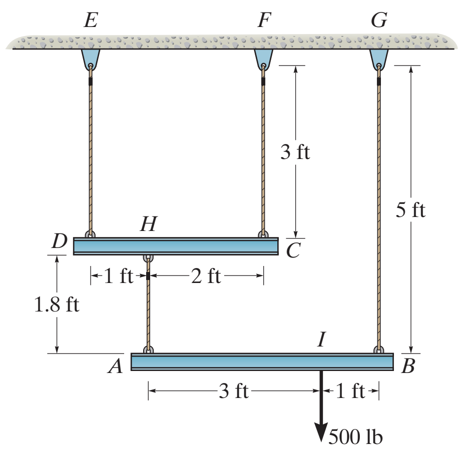
$$\delta_B = \frac{F_{BG}L_{BG}}{A_{BG}E} = \frac{375.0(5)(12)}{0.025(28.0)(10^6)} = 0.0321428\text{ in.}$$

$$\frac{\delta'_I}{3} = \frac{0.0247143}{4}; \qquad \delta'_I = 0.0185357\text{ in.}$$

$$\delta_I = 0.0074286 + 0.0185357 = 0.0260\text{ in.} \qquad \textbf{Ans.}$$



***4–12.** The load is supported by the four 304 stainless steel wires that are connected to the rigid members AB and DC. Determine the angle of tilt of each member after the 500-lb load is applied. The members were originally horizontal, and each wire has a cross-sectional area of 0.025 in^2.

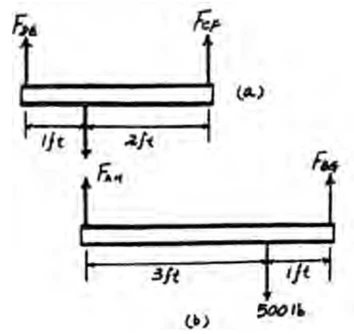


Internal Forces in the wires:

FBD (b)

$\circlearrowleft +\Sigma M_A = 0; \qquad F_{BG}(4) - 500(3) = 0 \qquad F_{BG} = 375.0 \text{ lb}$

$+\uparrow \Sigma F_y = 0; \qquad F_{AH} + 375.0 - 500 = 0 \qquad F_{AH} = 125.0 \text{ lb}$



FBD (a)

$\circlearrowleft +\Sigma M_D = 0; \qquad F_{CF}(3) - 125.0(1) = 0 \qquad F_{CF} = 41.67 \text{ lb}$

$+\uparrow \Sigma F_y = 0; \qquad F_{DE} + 41.67 - 125.0 = 0 \qquad F_{DE} = 83.33 \text{ lb}$

Displacement:

$$\delta_D = \frac{F_{DE}L_{DE}}{A_{DE}E} = \frac{83.33(3)(12)}{0.025(28.0)(10^6)} = 0.0042857 \text{ in.}$$

$$\delta_C = \frac{F_{CF}L_{CF}}{A_{CF}E} = \frac{41.67(3)(12)}{0.025(28.0)(10^6)} = 0.0021429 \text{ in.}$$

$$\frac{\delta'_H}{2} = \frac{0.0021429}{3}; \qquad \delta'_H = 0.0014286 \text{ in.}$$

$$\delta_H = \delta'_H + \delta_C = 0.0014286 + 0.0021429 = 0.0035714 \text{ in.}$$

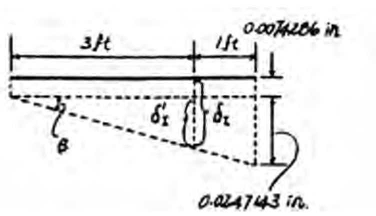
$$\tan\alpha = \frac{0.0021429}{36}; \qquad \alpha = 0.00341^\circ$$ **Ans.**

$$\delta_{A/H} = \frac{F_{AH}L_{AH}}{A_{AH}E} = \frac{125.0(1.8)(12)}{0.025(28.0)(10^6)} = 0.0038571 \text{ in.}$$

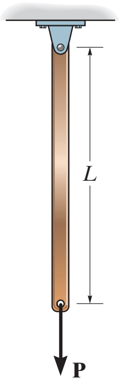
$$\delta_A = \delta_H + \delta_{A/H} = 0.0035714 + 0.0038571 = 0.0074286 \text{ in.}$$

$$\delta_B = \frac{F_{BG}L_{BG}}{A_{BG}E} = \frac{375.0(5)(12)}{0.025(28.0)(10^6)} = 0.0321428 \text{ in.}$$

$$\tan\beta = \frac{0.0247143}{48}; \qquad \beta = 0.0295^\circ$$ **Ans.**

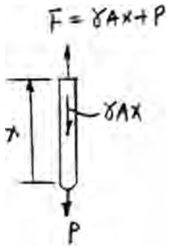


•4–13. The bar has a length L and cross-sectional area A. Determine its elongation due to the force **P** and its own weight. The material has a specific weight γ (weight/volume) and a modulus of elasticity E.

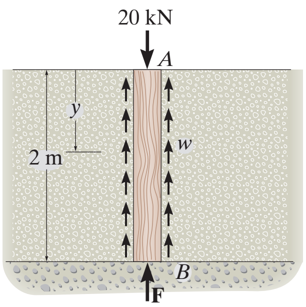


$$\delta = \int \frac{P(x)\,dx}{A(x)\,E} = \frac{1}{AE}\int_0^L (\gamma A x + P)\,dx$$

$$= \frac{1}{AE}\left(\frac{\gamma A L^2}{2} + PL\right) = \frac{\gamma L^2}{2E} + \frac{PL}{AE} \qquad \textbf{Ans.}$$

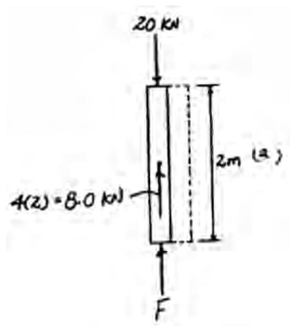


4–14. The post is made of Douglas fir and has a diameter of 60 mm. If it is subjected to the load of 20 kN and the soil provides a frictional resistance that is uniformly distributed along its sides of $w = 4$ kN/m, determine the force **F** at its bottom needed for equilibrium. Also, what is the displacement of the top of the post A with respect to its bottom B? Neglect the weight of the post.



Equation of Equilibrium: For entire post [FBD (a)]

$$+\uparrow \Sigma F_y = 0; \qquad F + 8.00 - 20 = 0 \qquad F = 12.0 \text{ kN} \qquad \textbf{Ans.}$$



Internal Force: FBD (b)

$$+\uparrow \Sigma F_y = 0; \qquad -F(y) + 4y - 20 = 0$$

$$F(y) = \{4y - 20\} \text{ kN}$$

Displacement:

$$\delta_{A/B} = \int_0^L \frac{F(y)dy}{A(y)E} = \frac{1}{AE}\int_0^{2\text{ m}} (4y - 20)dy$$

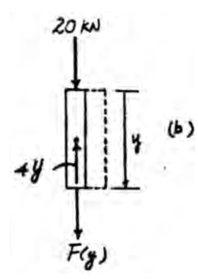
$$= \frac{1}{AE}\left(2y^2 - 20y\right)\Big|_0^{2\text{ m}}$$

$$= -\frac{32.0 \text{ kN}\cdot\text{m}}{AE}$$

$$= -\frac{32.0(10^3)}{\frac{\pi}{4}(0.06^2)\ 13.1\ (10^9)}$$

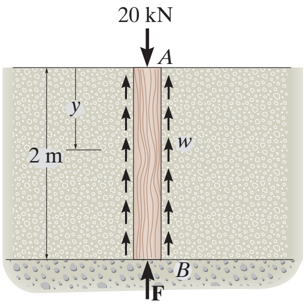
$$= -\,0.8639\left(10^{-3}\right)\text{ m}$$

$$= -\,0.864 \text{ mm} \qquad \textbf{Ans.}$$



Negative sign indicates that end A moves toward end B.

4–15. The post is made of Douglas fir and has a diameter of 60 mm. If it is subjected to the load of 20 kN and the soil provides a frictional resistance that is distributed along its length and varies linearly from $w = 0$ at $y = 0$ to $w = 3$ kN/m at $y = 2$ m, determine the force **F** at its bottom needed for equilibrium. Also, what is the displacement of the top of the post A with respect to its bottom B? Neglect the weight of the post.



Equation of Equilibrium: For entire post [FBD (a)]

$+\uparrow \Sigma F_y = 0; \qquad F + 3.00 - 20 = 0 \qquad F = 17.0 \text{ kN}$ **Ans.**

Internal Force: FBD (b)

$+\uparrow \Sigma F_y = 0; \qquad -F(y) + \frac{1}{2}\left(\frac{3y}{2}\right)y - 20 = 0$

$$F(y) = \left\{\frac{3}{4}y^2 - 20\right\} \text{kN}$$

Displacement:

$$\delta_{A/B} = \int_0^L \frac{F(y)\,dy}{A(y)E} = \frac{1}{AE}\int_0^{2\text{m}}\left(\frac{3}{4}y^2 - 20\right)dy$$

$$= \frac{1}{AE}\left(\frac{y^3}{4} - 20y\right)\Bigg|_0^{2\text{ m}}$$

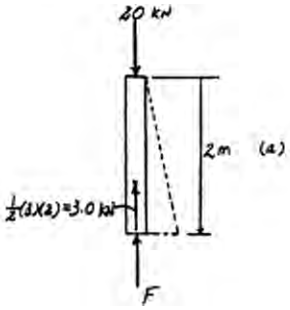
$$= -\frac{38.0 \text{ kN}\cdot\text{m}}{AE}$$

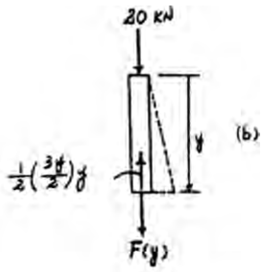
$$= -\frac{38.0(10^3)}{\frac{\pi}{4}(0.06^2)\,13.1\,(10^9)}$$

$$= -1.026\left(10^{-3}\right)\text{m}$$

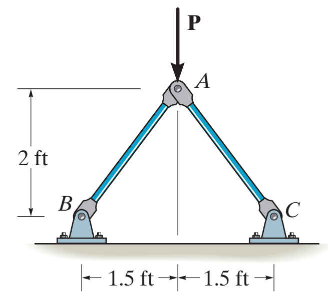
$$= -1.03 \text{ mm}$$ **Ans.**

Negative sign indicates that end A moves toward end B.





***4–16.** The linkage is made of two pin-connected A-36 steel members, each having a cross-sectional area of 1.5 in^2. If a vertical force of $P = 50$ kip is applied to point A, determine its vertical displacement at A.



Analysing the equilibrium of Joint A by referring to its FBD, Fig. a,

$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{AC}\left(\frac{3}{5}\right) - F_{AB}\left(\frac{3}{5}\right) = 0 \qquad F_{AC} = F_{AB} = F$$

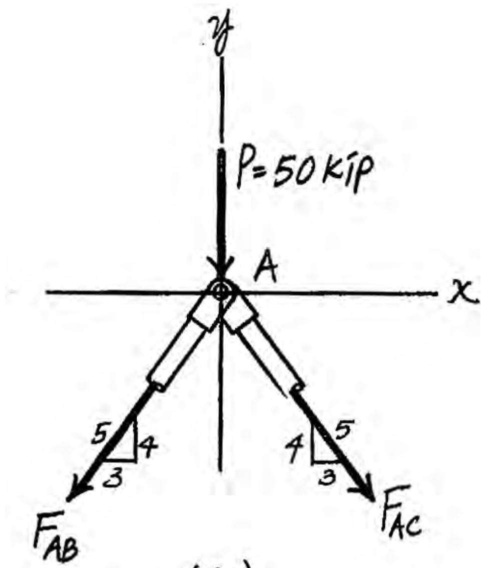
$$+\uparrow \Sigma F_y = 0 \qquad -2F\left(\frac{4}{5}\right) - 50 = 0 \qquad F = -31.25 \text{ kip}$$

The initial length of members AB and AC is $L = \sqrt{1.5^2 + 2^2} = (2.50 \text{ ft})\left(\frac{12 \text{ in}}{1 \text{ ft}}\right) = 30$ in. The axial deformation of members AB and AC is

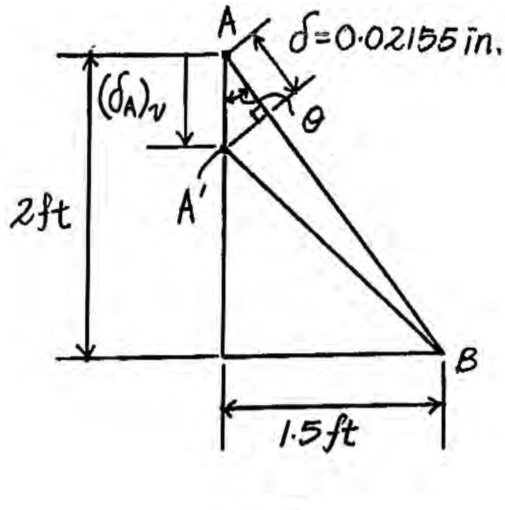
$$\delta = \frac{FL}{AE} = \frac{(-31.25)(30)}{(1.5)\left[29.0(10^3)\right]} = -0.02155 \text{ in.}$$

The negative sign indicates that end A moves toward B and C. From the geometry shown in Fig. b, $\theta = \tan^{-1}\left(\frac{1.5}{2}\right) = 36.87°$. Thus,

$$\left(\delta_A\right)_\gamma = \frac{\delta}{\cos\theta} = \frac{0.02155}{\cos 36.87°} = 0.0269 \text{ in.} \downarrow$$ **Ans.**

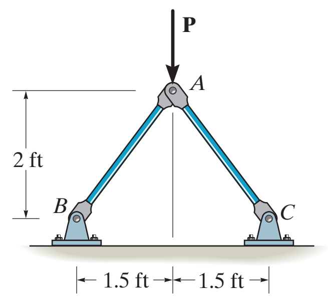


(a)



(b)

•4–17. The linkage is made of two pin-connected A-36 steel members, each having a cross-sectional area of 1.5 in^2. Determine the magnitude of the force **P** needed to displace point A 0.025 in. downward.



Analysing the equilibrium of joint A by referring to its FBD, Fig. a

$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{AC}\left(\frac{3}{5}\right) - F_{AB}\left(\frac{3}{5}\right) = 0 \qquad F_{AC} = F_{AB} = F$$

$$+\uparrow \Sigma F_y = 0; \qquad -2F\left(\frac{4}{5}\right) - P = 0 \qquad F = -0.625\,P$$

The initial length of members AB and AC are $L = \sqrt{1.5^2 + 2^2} = (2.50\text{ ft})\left(\frac{12\text{ in}}{1\text{ ft}}\right) = 30$ in. The axial deformation of members AB and AC is

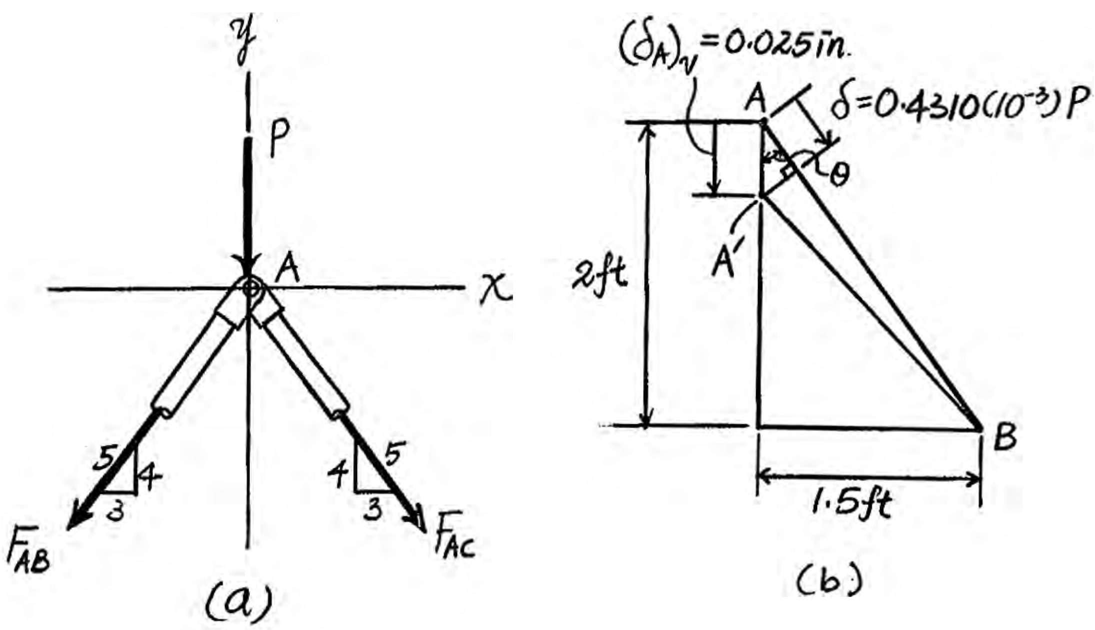
$$\delta = \frac{FL}{AE} = \frac{-0.625P(30)}{(1.5)\left[29.0(10^3)\right]} = -0.4310(10^{-3})\,P$$

The negative sign indicates that end A moves toward B and C. From the geometry shown in Fig. b, we obtain $\theta = \tan^{-1}\left(\frac{1.5}{2}\right) = 36.87°$. Thus

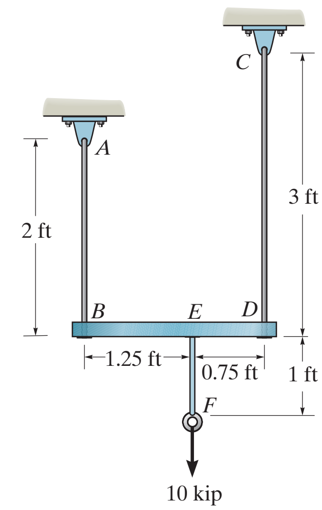
$$(\delta_A)_\gamma = \frac{\delta}{\cos\theta}$$

$$0.025 = \frac{0.4310(10^{-3})\,P}{\cos 36.87°}$$

$$P = 46.4\text{ kips} \qquad \textbf{Ans.}$$



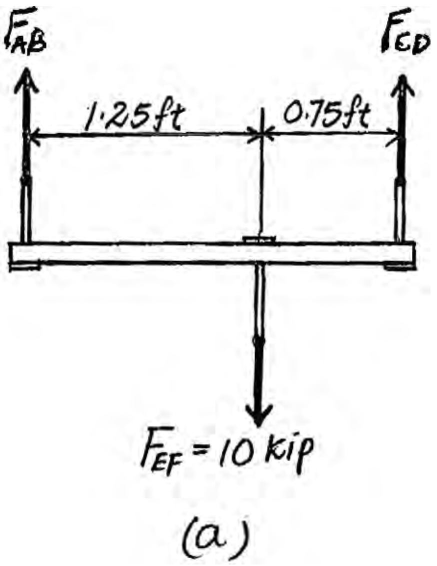
4–18. The assembly consists of two A-36 steel rods and a rigid bar BD. Each rod has a diameter of 0.75 in. If a force of 10 kip is applied to the bar as shown, determine the vertical displacement of the load.



Here, $F_{EF} = 10$ kip. Referring to the FBD shown in Fig. a,

$\zeta + \Sigma M_B = 0;\quad F_{CD}(2) - 10(1.25) = 0 \quad F_{CD} = 6.25$ kip

$\zeta + \Sigma M_D = 0;\quad 10(0.75) - F_{AB}(2) = 0 \quad F_{AB} = 3.75$ kip



The cross-sectional area of the rods is $A = \frac{\pi}{4}(0.75^2) = 0.140625\pi \text{ in}^2$. Since points A and C are fixed,

$$\delta_B = \frac{F_{AB} L_{AB}}{A\,E_{st}} = \frac{3.75\,(2)(12)}{0.140625\pi\left[29.0(10^3)\right]} = 0.007025 \text{ in.} \downarrow$$

$$\delta_D = \frac{F_{CD} L_{CD}}{A\,E_{st}} = \frac{6.25(3)(12)}{0.140625\pi\left[29.0(10^3)\right]} = 0.01756 \text{ in} \downarrow$$

From the geometry shown in Fig. b

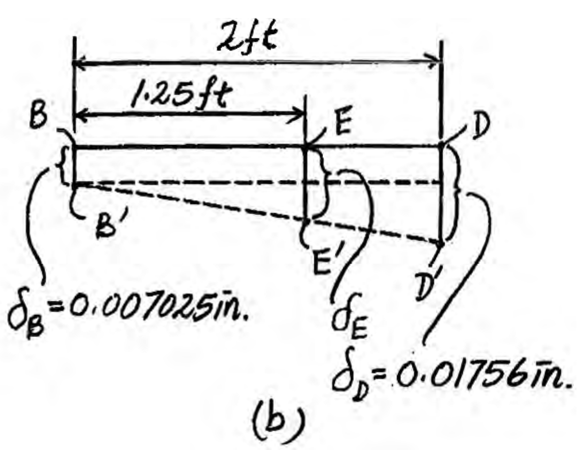
$$\delta_E = 0.007025 + \frac{1.25}{2}(0.01756 - 0.00725) = 0.01361 \text{ in.} \downarrow$$

Here,

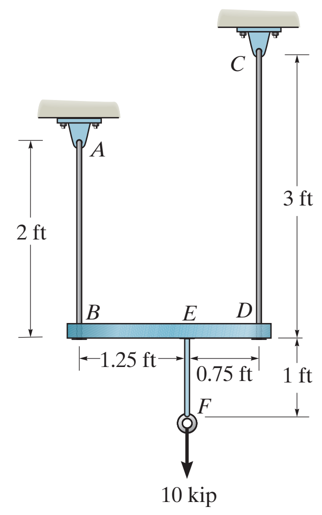
$$\delta_{F/E} = \frac{F_{EF} L_{EF}}{A\,E_{st}} = \frac{10\,(1)\,(12)}{0.140625\pi\left[29.0(10^3)\right]} = 0.009366 \text{ in} \downarrow$$

Thus,

$$(+\downarrow)\ \delta_F = \delta_E + \delta_{F/E} = 0.01361 + 0.009366 = 0.0230 \text{ in} \downarrow \qquad \textbf{Ans.}$$



4–19. The assembly consists of two A-36 steel rods and a rigid bar BD. Each rod has a diameter of 0.75 in. If a force of 10 kip is applied to the bar, determine the angle of tilt of the bar.



Here, $F_{EF} = 10$ kip. Referring to the FBD shown in Fig. a,

$\circlearrowleft + \Sigma M_B = 0; \qquad F_{CD}(2) - 10(1.25) = 0 \qquad F_{CD} = 6.25 \text{ kip}$

$\circlearrowleft + \Sigma M_D = 0; \qquad 10(0.75) - F_{AB}(2) = 0 \qquad F_{AB} = 3.75 \text{ kip}$

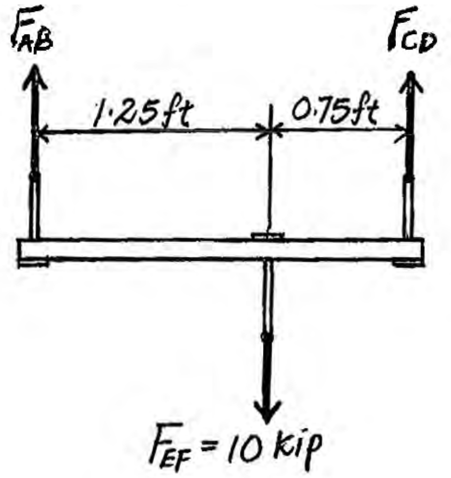
The cross-sectional area of the rods is $A = \frac{\pi}{4}(0.75^2) = 0.140625\pi \text{ in}^2$. Since points A and C are fixed then,

$$\delta_B = \frac{F_{AB}\,L_{AB}}{A\,E_{st}} = \frac{3.75\,(2)(12)}{0.140625\pi\left[29.0(10^3)\right]} = 0.007025 \text{ in} \downarrow$$

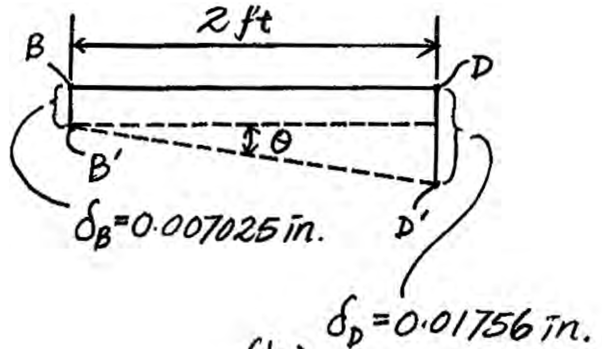
$$\delta_D = \frac{F_{CD}\,L_{CD}}{A\,E_{st}} = \frac{6.25\,(3)(12)}{0.140625\pi\left[29.0(10^3)\right]} = 0.01756 \text{ in} \downarrow$$

From the geometry shown in Fig. b,

$$\theta = \frac{0.01756 - 0.007025}{2(12)} = 0.439(10^{-3}) \text{ rad} \qquad \textbf{Ans.}$$

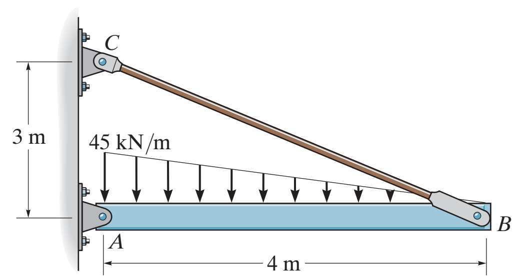


(a)



(b)

***4–20.** The rigid bar is supported by the pin-connected rod CB that has a cross-sectional area of 500 mm^2 and is made of A-36 steel. Determine the vertical displacement of the bar at B when the load is applied.



Force In The Rod. Referring to the *FBD* of member *AB*, Fig. *a*

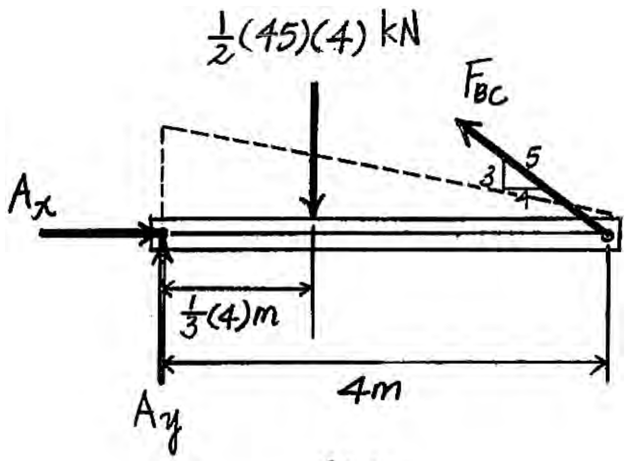
$$\zeta + \Sigma M_A = 0; \qquad F_{BC}\left(\frac{3}{5}\right)(4) - \frac{1}{2}(45)(4)\left[\frac{1}{3}(4)\right] = 0 \qquad F_{BC} = 50.0 \text{ kN}$$

Displacement. The initial length of rod BC is $L_{BC} = \sqrt{3^2 + 4^2} = 5$ m. The axial deformation of this rod is

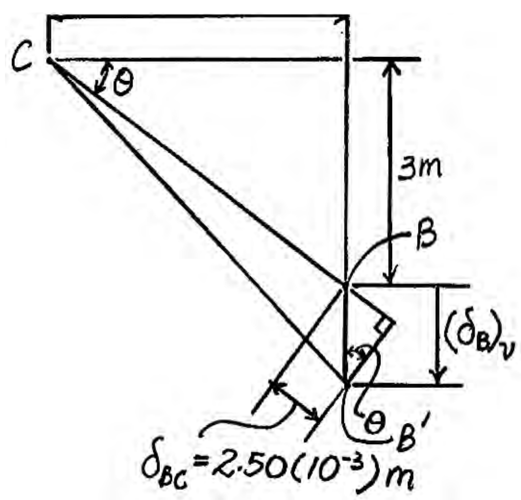
$$\delta_{BC} = \frac{F_{BC}\,L_{BC}}{A_{BC}\,E_{st}} = \frac{50.0(10^3)(5)}{0.5(10^{-3})\left[200(10^9)\right]} = 2.50\,(10^{-3})\text{ m}$$

From the geometry shown in Fig. b, $\theta = \tan^{-1}\left(\frac{3}{4}\right) = 36.87°$. Thus,

$$(\delta_B)_\gamma = \frac{\delta_{BC}}{\sin\theta} = \frac{2.50(10^{-3})}{\sin 36.87°} = 4.167\,(10^{-3})\text{ m} = 4.17\text{ mm} \qquad \textbf{Ans.}$$

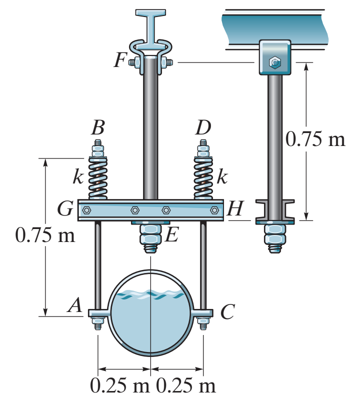


(a)



(b)

•4–21. A spring-supported pipe hanger consists of two springs which are originally unstretched and have a stiffness of $k = 60$ kN/m, three 304 stainless steel rods, AB and CD, which have a diameter of 5 mm, and EF, which has a diameter of 12 mm, and a rigid beam GH. If the pipe and the fluid it carries have a total weight of 4 kN, determine the displacement of the pipe when it is attached to the support.



Internal Force in the Rods:

FBD (a)

$$\circlearrowleft + \Sigma M_A = 0; \qquad F_{CD}(0.5) - 4(0.25) = 0 \qquad F_{CD} = 2.00 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{AB} + 2.00 - 4 = 0 \qquad F_{AB} = 2.00 \text{ kN}$$

FBD (b)

$$+\uparrow \Sigma F_y = 0; \qquad F_{EF} - 2.00 - 2.00 = 0 \qquad F_{EF} = 4.00 \text{ kN}$$

Displacement:

$$\delta_D = \delta_E = \frac{F_{EF}L_{EF}}{A_{EF}E} = \frac{4.00(10^3)(750)}{\frac{\pi}{4}(0.012)^2(193)(10^9)} = 0.1374 \text{ mm}$$

$$\delta_{A/B} = \delta_{C/D} = \frac{P_{CD}L_{CD}}{A_{CD}E} = \frac{2(10^3)(750)}{\frac{\pi}{4}(0.005)^2(193)(10^9)} = 0.3958 \text{ mm}$$

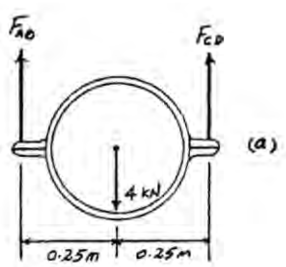
$$\delta_C = \delta_D + \delta_{C/D} = 0.1374 + 0.3958 = 0.5332 \text{ mm}$$

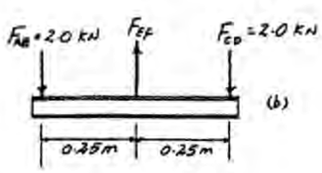
Displacement of the spring

$$\delta_{sp} = \frac{F_{sp}}{k} = \frac{2.00}{60} = 0.0333333 \text{ m} = 33.3333 \text{ mm}$$

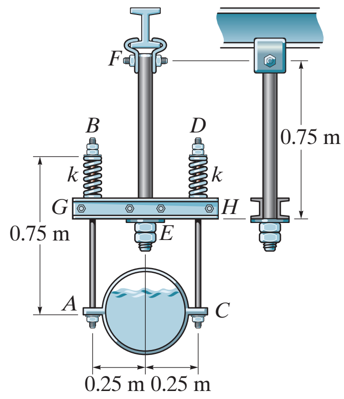
$$\delta_{\text{lat}} = \delta_C + \delta_{sp}$$

$$= 0.5332 + 33.3333 = 33.9 \text{ mm} \qquad \textbf{Ans.}$$





4–22. A spring-supported pipe hanger consists of two springs, which are originally unstretched and have a stiffness of $k = 60$ kN/m, three 304 stainless steel rods, AB and CD, which have a diameter of 5 mm, and EF, which has a diameter of 12 mm, and a rigid beam GH. If the pipe is displaced 82 mm when it is filled with fluid, determine the weight of the fluid.

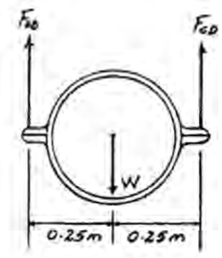


Internal Force in the Rods:

FBD (a)

$$\circlearrowleft +\Sigma M_A = 0; \qquad F_{CD}(0.5) - W(0.25) = 0 \qquad F_{CD} = \frac{W}{2}$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{AB} + \frac{W}{2} - W = 0 \qquad F_{AB} = \frac{W}{2}$$



FBD (b)

$$+\uparrow \Sigma F_y = 0; \qquad F_{EF} - \frac{W}{2} - \frac{W}{2} = 0 \qquad F_{EF} = W$$

Displacement:

$$\delta_D = \delta_E = \frac{F_{EF}L_{EF}}{A_{EF}E} = \frac{W(750)}{\frac{\pi}{4}(0.012)^2(193)(10^9)}$$

$$= 34.35988(10^{-6})\,W$$

$$\delta_{A/B} = \delta_{C/D} = \frac{F_{CD}L_{CD}}{A_{CD}E} = \frac{\frac{W}{2}(750)}{\frac{\pi}{4}(0.005)^2(193)(10^9)}$$

$$= 98.95644(10^{-6})\,W$$

$$\delta_C = \delta_D + \delta_{C/D}$$

$$= 34.35988(10^{-6})\,W + 98.95644(10^{-6})\,W$$

$$= 0.133316(10^{-3})\,W$$

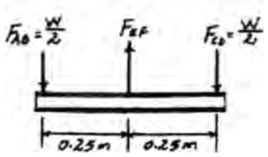
Displacement of the spring

$$\delta_{sp} = \frac{F_{sp}}{k} = \frac{\frac{W}{2}}{60(10^3)}(1000) = 0.008333\,W$$

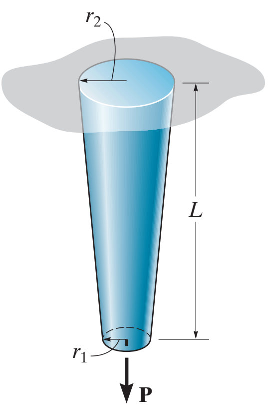
$$\delta_{\text{lat}} = \delta_C + \delta_{sp}$$

$$82 = 0.133316(10^{-3})\,W + 0.008333W$$

$$W = 9685 \text{ N} = 9.69 \text{ kN}$$ **Ans.**



4–23. The rod has a slight taper and length L. It is suspended from the ceiling and supports a load **P** at its end. Show that the displacement of its end due to this load is $\delta = PL/(\pi E r_2 r_1)$. Neglect the weight of the material. The modulus of elasticity is E.



$$r(x) = r_1 + \frac{r_2 - r_1}{L}x = \frac{r_1 L + (r_2 - r_1)x}{L}$$

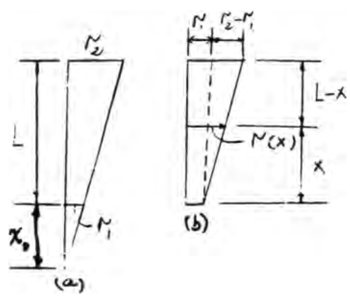
$$A(x) = \frac{\pi}{L^2}(r_1 L + (r_2 - r_1)x)^2$$

$$\delta = \int \frac{P\,dx}{A(x)E} = \frac{PL^2}{\pi E}\int_0^L \frac{dx}{[r_1 L + (r_2 - r_1)x]^2}$$

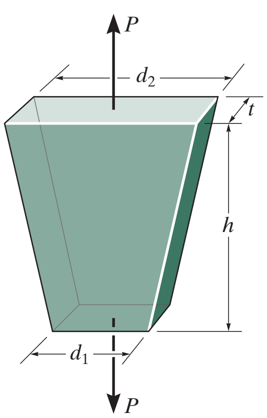
$$= -\frac{PL^2}{\pi E}\left[\frac{1}{(r_2 - r_2)(r_1 L + (r_2 - r_1)x)}\right]\Bigg|_0^L = -\frac{PL^2}{\pi E(r_2 - r_1)}\left[\frac{1}{r_1 L + (r_2 - r_1)L} - \frac{1}{r_1 L}\right]$$

$$= -\frac{PL^2}{\pi E(r_2 - r_1)}\left[\frac{1}{r_2 L} - \frac{1}{r_1 L}\right] = -\frac{PL^2}{\pi E(r_2 - r_1)}\left[\frac{r_1 - r_2}{r_2 r_1 L}\right]$$

$$= \frac{PL^2}{\pi E(r_2 - r_1)}\left[\frac{r_2 - r_1}{r_2 r_1 L}\right] = \frac{PL}{\pi E r_2 r_1}$$ **QED**



***4–24.** Determine the relative displacement of one end of the tapered plate with respect to the other end when it is subjected to an axial load P.



$$w = d_1 + \frac{d_2 - d_1}{h}x = \frac{d_1 h + (d_2 - d_1)x}{h}$$

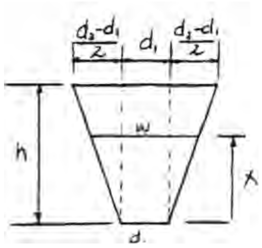
$$\delta = \int \frac{P(x)\,dx}{A(x)E} = \frac{P}{E}\int_0^h \frac{dx}{\frac{[d_1 h + (d_2 - d_1)x]t}{h}}$$

$$= \frac{Ph}{E\,t}\int_0^h \frac{dx}{d_1 h + (d_2 - d_1)x}$$

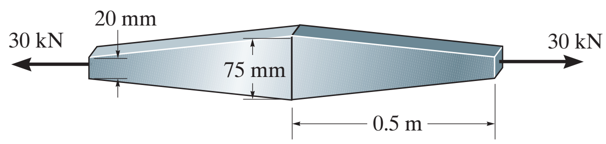
$$= \frac{Ph}{E\,t\,d_1 h}\int_0^h \frac{dx}{1 + \frac{d_2 - d_1}{d_1 h}x} = \frac{Ph}{E\,t\,d_1 h}\left(\frac{d_1 h}{d_2 - d_1}\right)\left[\ln\left(1 + \frac{d_2 - d_1}{d_1 h}x\right)\right]\Bigg|_0^h$$

$$= \frac{Ph}{E\,t(d_2 - d_1)}\left[\ln\left(1 + \frac{d_2 - d_1}{d_1}\right)\right] = \frac{Ph}{E\,t(d_2 - d_1)}\left[\ln\left(\frac{d_1 + d_2 - d_1}{d_1}\right)\right]$$

$$= \frac{Ph}{E\,t(d_2 - d_1)}\left[\ln\frac{d_2}{d_1}\right]$$ **Ans.**



4–25. Determine the elongation of the A-36 steel member when it is subjected to an axial force of 30 kN. The member is 10 mm thick. Use the result of Prob. 4–24.



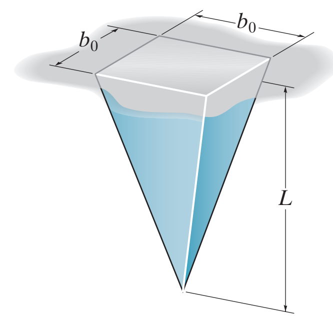
Using the result of prob. 4-24 by substituting $d_1 = 0.02$ m, $d_2 = 0.075$ m $t = 0.01$ m and $L = 0.5$ m.

$$\delta = 2\left[\frac{PL}{E_{st}\,t(d_2 - d_1)}\ln\frac{d_2}{d_1}\right]$$

$$= 2\left[\frac{30(10^3)\,(0.5)}{200(10^9)(0.01)(0.075 - 0.02)}\ln\left(\frac{0.075}{0.02}\right)\right]$$

$$= 0.360(10^{-3})\text{ m} = 0.360\text{ mm} \qquad \textbf{Ans.}$$

4–26. The casting is made of a material that has a specific weight γ and modulus of elasticity E. If it is formed into a pyramid having the dimensions shown, determine how far its end is displaced due to gravity when it is suspended in the vertical position.



Internal Forces:

$$+\uparrow \Sigma F_z = 0; \qquad P(z) - \frac{1}{3}\gamma A z = 0 \qquad P(z) = \frac{1}{3}\gamma A z$$

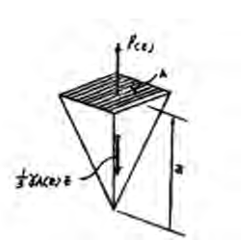
Displacement:

$$\delta = \int_0^L \frac{P(z)\,dz}{A(z)\,E}$$

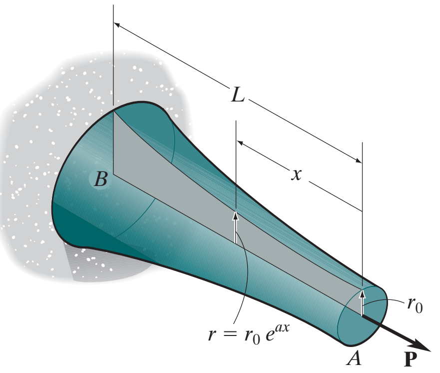
$$= \int_0^L \frac{\frac{1}{3}\gamma A z}{A\,E}\,dz$$

$$= \frac{\gamma}{3E}\int_0^L z\,dz$$

$$= \frac{\gamma L^2}{6E} \qquad \textbf{Ans.}$$



4–27. The circular bar has a variable radius of $r = r_0 e^{ax}$ and is made of a material having a modulus of elasticity of E. Determine the displacement of end A when it is subjected to the axial force **P**.



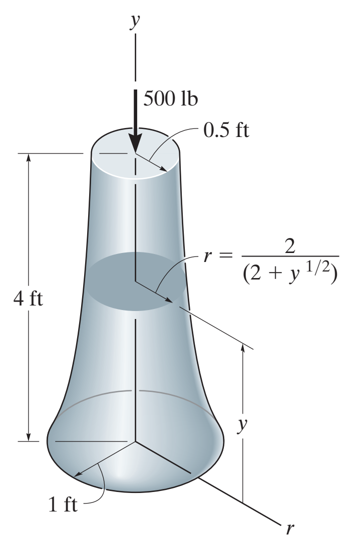
Displacements: The cross-sectional area of the bar as a function of x is $A(x) = \pi r^2 = \pi {r_0}^2 e^{2ax}$. We have

$$\delta = \int_0^L \frac{P(x)dx}{A(x)E} = \frac{P}{\pi {r_0}^2 E}\int_0^L \frac{dx}{e^{2ax}}$$

$$= \frac{P}{\pi {r_0}^2 E}\left[-\frac{1}{2ae^{2ax}}\right]\Bigg|_0^L$$

$$= -\frac{P}{2a\pi {r_0}^2 E}\left(1 - e^{-2aL}\right) \qquad \textbf{Ans.}$$

***4–28.** The pedestal is made in a shape that has a radius defined by the function $r = 2/(2 + y^{1/2})$ ft, where y is in feet. If the modulus of elasticity for the material is $E = 14(10^3)$ psi, determine the displacement of its top when it supports the 500-lb load.



$$\delta = \int \frac{P(y)\,dy}{A(y)\,E}$$

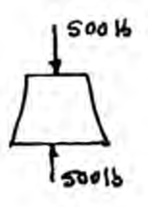
$$= \frac{500}{14(10^3)(144)}\int_0^4 \frac{dy}{\pi\left(\frac{2}{2 + y^{\frac{1}{2}}}\right)^2}$$

$$= 0.01974(10^{-3})\int_0^4 (4 + 4y^{\frac{1}{2}} + y)\,dy$$

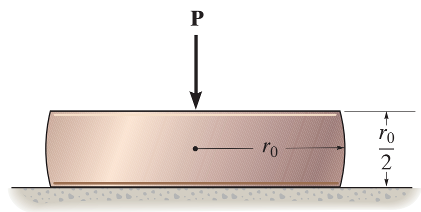
$$= 0.01974(10^{-3})\left[4y + 4\left(\frac{2}{3}\,y^{\frac{3}{2}}\right) + \frac{1}{2}\,y^2\right]_0^4$$

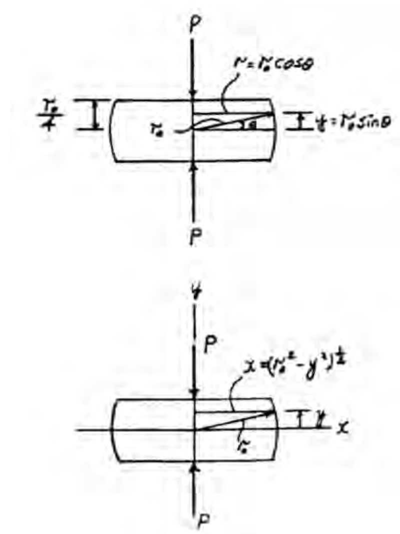
$$= 0.01974(10^{-3})(45.33)$$

$$= 0.8947(10^{-3})\text{ ft} = 0.0107\text{ in.} \qquad \textbf{Ans.}$$



•4–29. The support is made by cutting off the two opposite sides of a sphere that has a radius r_0. If the original height of the support is $r_0/2$, determine how far it shortens when it supports a load **P**. The modulus of elasticity is E.





Geometry:

$$A = \pi\, r^2 = \pi (r_0 \cos\theta)^2 = \pi\, r_0^2 \cos^2\theta$$

$$y = r_0 \sin\theta; \qquad dy = r_0 \cos\theta\, d\theta$$

Displacement:

$$\delta = \int_0^L \frac{P(y)\,dy}{A(y)\,E}$$

$$= 2\left[\frac{P}{E}\int_0^{\theta}\frac{r_0\cos\theta\,d\theta}{\pi\, r_0^2\cos^2\theta}\right] = 2\left[\frac{P}{\pi\, r_0\, E}\int_0^{\theta}\frac{d\theta}{\cos\theta}\right]$$

$$= \frac{2P}{\pi\, r_0\, E}\left[\ln(\sec\theta + \tan\theta)\right]\Big|_0^{\theta}$$

$$= \frac{2P}{\pi\, r_0\, E}\left[\ln(\sec\theta + \tan\theta)\right]$$

When $y = \frac{r_0}{4}$; $\qquad \theta = 14.48°$

$$\delta = \frac{2P}{\pi\, r_0\, E}\left[\ln(\sec 14.48° + \tan 14.48°)\right]$$

$$= \frac{0.511P}{\pi\, r_0\, E} \qquad \textbf{Ans.}$$

Also,

Geometry:

$$A(y) = \pi\, x^2 = \pi\,(r_0^2 - y^2)$$

Displacement:

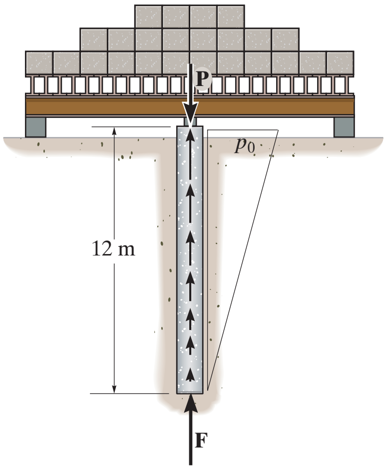
$$\delta = \int_0^L \frac{P(y)\,dy}{A(y)\,E}$$

$$= \frac{2P}{\pi E}\int_0^{\frac{0}{\pi}}\frac{dy}{r_0^2 - y^2} = \frac{2P}{\pi\, E}\left[\frac{1}{2r_0}\ln\frac{r_0 + y}{r_0 - y}\right]\Big|_0^{\frac{0}{\pi}}$$

$$= \frac{P}{\pi\, r_0\, E}\left[\ln 1.667 - \ln 1\right]$$

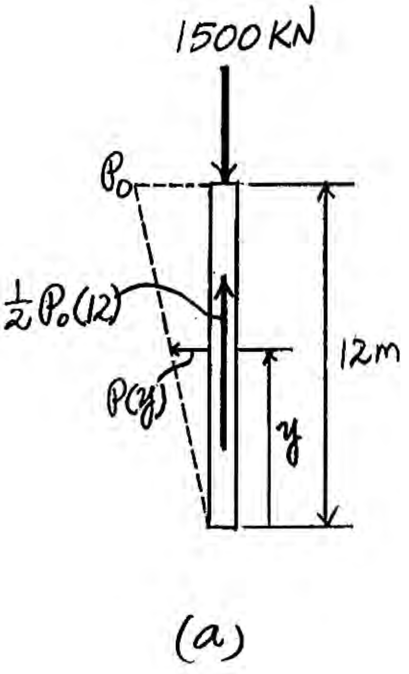
$$= \frac{0.511\,P}{\pi\, r_0\, E} \qquad \textbf{Ans.}$$

4–30. The weight of the kentledge exerts an axial force of $P = 1500$ kN on the 300-mm diameter high strength concrete bore pile. If the distribution of the resisting skin friction developed from the interaction between the soil and the surface of the pile is approximated as shown, and the resisting bearing force **F** is required to be zero, determine the maximum intensity p_0 kN/m for equilibrium. Also, find the corresponding elastic shortening of the pile. Neglect the weight of the pile.



Internal Loading: By considering the equilibrium of the pile with reference to its entire free-body diagram shown in Fig. *a*. We have

$+\uparrow \Sigma F_y = 0;\qquad \frac{1}{2}p_0(12) - 1500 = 0\qquad p_0 = 250\text{ kN/m}$ **Ans.**



Thus,

$$p(y) = \frac{250}{12}y = 20.83y\text{ kN/m}$$

The normal force developed in the pile as a function of y can be determined by considering the equilibrium of a section of the pile shown in Fig. *b*.

$+\uparrow \Sigma F_y = 0;\qquad \frac{1}{2}(20.83y)y - P(y) = 0\qquad P(y) = 10.42y^2\text{ kN}$

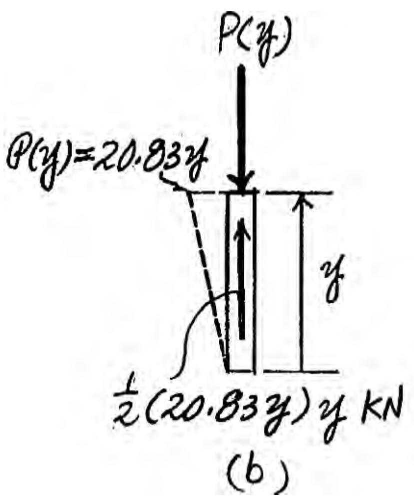
Displacement: The cross-sectional area of the pile is $A = \frac{\pi}{4}(0.3^2) = 0.0225\pi\text{ m}^2$. We have

$$\delta = \int_0^L \frac{P(y)dy}{A(y)E} = \int_0^{12\text{ m}} \frac{10.42(10^3)y^2dy}{0.0225\pi(29.0)(10^9)}$$

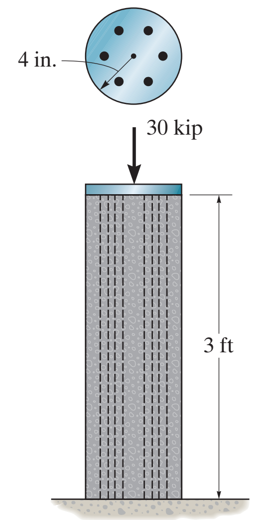
$$= \int_0^{12\text{ m}} 5.0816(10^{-6})y^2dy$$

$$= 1.6939(10^{-6})y^3\Big|_0^{12\text{ m}}$$

$$= 2.9270(10^{-3})\text{m} = 2.93\text{ mm}$$ **Ans.**



4–31. The column is constructed from high-strength concrete and six A-36 steel reinforcing rods. If it is subjected to an axial force of 30 kip, determine the average normal stress in the concrete and in each rod. Each rod has a diameter of 0.75 in.



Equations of Equilibrium:

$$+\uparrow \Sigma F_y = 0; \qquad 6P_{st} + P_{con} - 30 = 0 \qquad [1]$$

Compatibility:

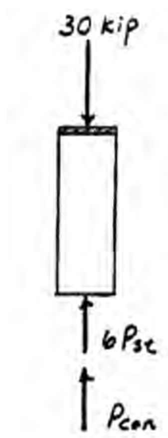
$$\delta_{st} = \delta_{con}$$

$$\frac{P_{st}(3)(12)}{\frac{\pi}{4}(0.75^2)(29.0)(10^3)} = \frac{P_{con}(3)(12)}{[\frac{\pi}{4}(8^2) - 6(\frac{\pi}{4})(0.75)^2](4.20)(10^3)}$$

$$P_{st} = 0.064065\, P_{con} \qquad [2]$$

Solving Eqs. [1] and [2] yields:

$$P_{st} = 1.388 \text{ kip} \qquad P_{con} = 21.670 \text{ kip}$$

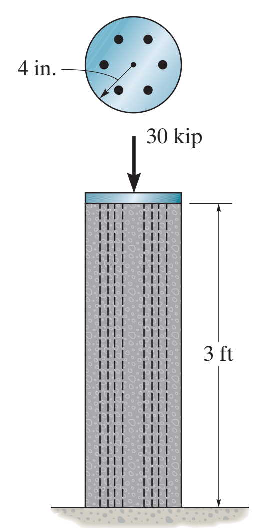


Average Normal Stress:

$$\sigma_{st} = \frac{P_{st}}{A_{st}} = \frac{1.388}{\frac{\pi}{4}(0.75^2)} = 3.14 \text{ ksi} \qquad \textbf{Ans.}$$

$$\sigma_{con} = \frac{P_{con}}{A_{con}} = \frac{21.670}{\frac{\pi}{4}(8^2) - 6\left(\frac{\pi}{4}\right)(0.75^2)} = 0.455 \text{ ksi} \qquad \textbf{Ans.}$$

***4–32.** The column is constructed from high-strength concrete and six A-36 steel reinforcing rods. If it is subjected to an axial force of 30 kip, determine the required diameter of each rod so that one-fourth of the load is carried by the concrete and three-fourths by the steel.



Equilibrium: The force of 30 kip is required to distribute in such a manner that 3/4 of the force is carried by steel and 1/4 of the force is carried by concrete. Hence

$$P_{st} = \frac{3}{4}(30) = 22.5 \text{ kip} \qquad P_{con} = \frac{1}{4}(30) = 7.50 \text{ kip}$$

Compatibility:

$$\delta_{st} = \delta_{con}$$

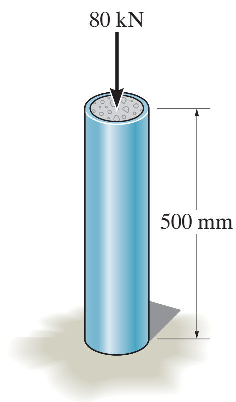
$$\frac{P_{st}L}{A_{st}E_{st}} = \frac{P_{con}\,L}{A_{con}\,E_{con}}$$

$$A_{st} = \frac{22.5A_{con}\,E_{con}}{7.50\,E_{st}}$$

$$6\left(\frac{\pi}{4}\right)d^2 = \frac{3\left[\frac{\pi}{4}(8^2) - 6\left(\frac{\pi}{4}\right)d^2\right](4.20)(10^3)}{29.0(10^3)}$$

$$d = 1.80 \text{ in.} \qquad \textbf{Ans.}$$

•4–33. The steel pipe is filled with concrete and subjected to a compressive force of 80 kN. Determine the average normal stress in the concrete and the steel due to this loading. The pipe has an outer diameter of 80 mm and an inner diameter of 70 mm. $E_{st} = 200$ GPa, $E_c = 24$ GPa.



$$+\uparrow \Sigma F_y = 0; \qquad P_{st} + P_{con} - 80 = 0 \qquad \textbf{(1)}$$

$$\delta_{st} = \delta_{con}$$

$$\frac{P_{st}\,L}{\frac{\pi}{4}(0.08^2 - 0.07^2)\,(200)\,(10^9)} = \frac{P_{con}\,L}{\frac{\pi}{4}(0.07^2)\,(24)\,(10^9)}$$

$$P_{st} = 2.5510\,P_{con} \qquad \textbf{(2)}$$

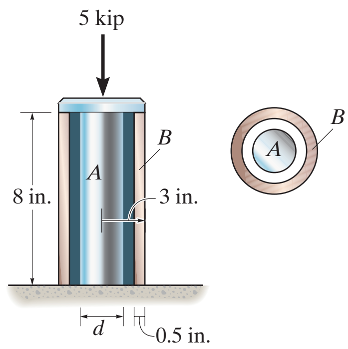
Solving Eqs. (1) and (2) yields

$$P_{st} = 57.47\text{ kN} \qquad P_{con} = 22.53\text{ kN}$$

$$\sigma_{st} = \frac{P_{st}}{A_{st}} = \frac{57.47\,(10^3)}{\frac{\pi}{4}\,(0.08^2 - 0.07^2)} = 48.8\text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_{con} = \frac{P_{con}}{A_{con}} = \frac{22.53\,(10^3)}{\frac{\pi}{4}\,(0.07^2)} = 5.85\text{ MPa} \qquad \textbf{Ans.}$$

4–34. The 304 stainless steel post A has a diameter of $d = 2$ in. and is surrounded by a red brass C83400 tube B. Both rest on the rigid surface. If a force of 5 kip is applied to the rigid cap, determine the average normal stress developed in the post and the tube.



Equations of Equilibrium:

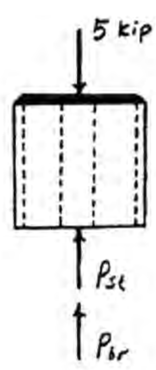
$$+\uparrow \Sigma F_y = 0; \qquad P_{st} + P_{br} - 5 = 0\,\textbf{[1]}$$

Compatibility:

$$\delta_{st} = \delta_{br}$$

$$\frac{P_{st}(8)}{\frac{\pi}{4}(2^2)(28.0)(10^3)} = \frac{P_{br}(8)}{\frac{\pi}{4}(6^2 - 5^2)(14.6)(10^3)}$$

$$P_{st} = 0.69738\,P_{br} \qquad \textbf{[2]}$$



Solving Eqs. [1] and [2] yields:

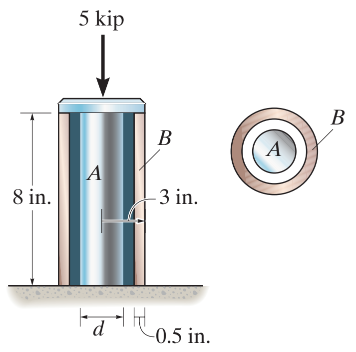
$$P_{br} = 2.9457\text{ kip} \qquad P_{st} = 2.0543\text{ kip}$$

Average Normal Stress:

$$\sigma_{br} = \frac{P_{br}}{A_{br}} = \frac{2.9457}{\frac{\pi}{4}(6^2 - 5^2)} = 0.341\text{ ksi} \qquad \textbf{Ans.}$$

$$\sigma_{st} = \frac{P_{st}}{A_{st}} = \frac{2.0543}{\frac{\pi}{4}(2^2)} = 0.654\text{ ksi} \qquad \textbf{Ans.}$$

4–35. The 304 stainless steel post A is surrounded by a red brass C83400 tube B. Both rest on the rigid surface. If a force of 5 kip is applied to the rigid cap, determine the required diameter d of the steel post so that the load is shared equally between the post and tube.



Equilibrium: The force of 5 kip is shared equally by the brass and steel. Hence

$$P_{st} = P_{br} = P = 2.50 \text{ kip}$$

Compatibility:

$$\delta_{st} = \delta_{br}$$

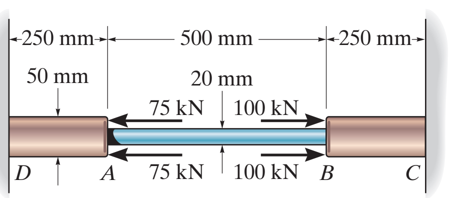
$$\frac{PL}{A_{st}E_{st}} = \frac{PL}{A_{br}E_{br}}$$

$$A_{st} = \frac{A_{br}E_{br}}{E_{st}}$$

$$\left(\frac{\pi}{4}\right)d^2 = \frac{\frac{\pi}{4}(6^2 - 5^2)(14.6)(10^3)}{28.0(10^3)}$$

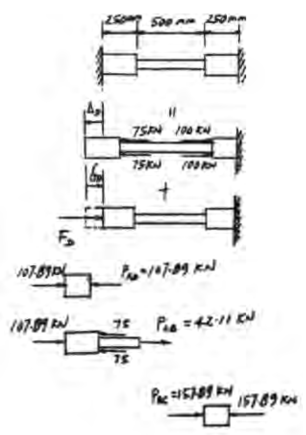
$$d = 2.39 \text{ in.}$$ **Ans.**

***4–36.** The composite bar consists of a 20-mm-diameter A-36 steel segment AB and 50-mm-diameter red brass C83400 end segments DA and CB. Determine the average normal stress in each segment due to the applied load.



$\overset{+}{\leftarrow} \Sigma F_x = 0; \quad F_C - F_D + 75 + 75 - 100 - 100 = 0$

$$F_C - F_D - 50 = 0 \qquad \textbf{(1)}$$



$\overset{+}{\leftarrow} \quad 0 = \Delta_D - \delta_D$

$$0 = \frac{150(0.5)}{\frac{\pi}{4}(0.02)^2(200)(10^9)} - \frac{50(0.25)}{\frac{\pi}{4}(0.05^2)(101)(10^9)}$$

$$- \frac{F_D(0.5)}{\frac{\pi}{4}(0.05^2)(101)(10^9)} - \frac{F_D(0.5)}{\frac{\pi}{4}(0.02^2)(200)(10^9)}$$

$$F_D = 107.89 \text{ kN}$$

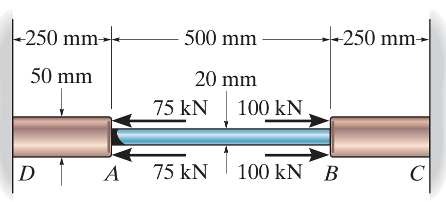
From Eq. (1), $F_C = 157.89$ kN

$$\sigma_{AD} = \frac{P_{AD}}{A_{AD}} = \frac{107.89(10^3)}{\frac{\pi}{4}(0.05^2)} = 55.0 \text{ MPa}$$ **Ans.**

$$\sigma_{AB} = \frac{P_{AB}}{A_{AB}} = \frac{42.11(10^3)}{\frac{\pi}{4}(0.02^2)} = 134 \text{ MPa}$$ **Ans.**

$$\sigma_{BC} = \frac{P_{BC}}{A_{BC}} = \frac{157.89(10^3)}{\frac{\pi}{4}(0.05^2)} = 80.4 \text{ MPa}$$ **Ans.**

•4–37. The composite bar consists of a 20-mm-diameter A-36 steel segment AB and 50-mm-diameter red brass C83400 end segments DA and CB. Determine the displacement of A with respect to B due to the applied load.

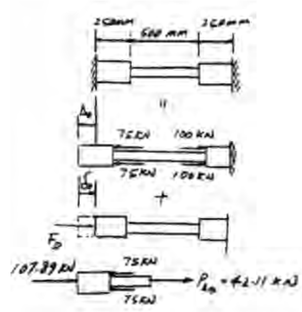


$$\overset{+}{\leftarrow} \qquad 0 = \Delta_D - \delta_D$$

$$0 = \frac{150(10^3)(500)}{\frac{\pi}{4}(0.02^2)(200)(10^9)} - \frac{50(10^3)(250)}{\frac{\pi}{4}(0.05^2)(101)(10^9)}$$

$$- \frac{F_D(500)}{\frac{\pi}{4}(0.05^2)(101)(10^9)} - \frac{F_D(500)}{\frac{\pi}{4}(0.02)^2(200)(10^9)}$$

$$F_D = 107.89 \text{ kN}$$

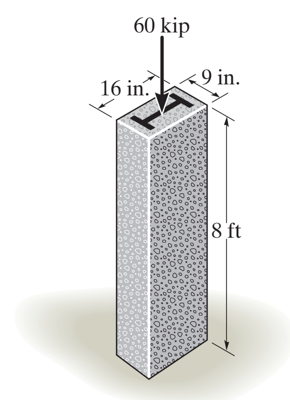


Displacement:

$$\delta_{A/B} = \frac{P_{AB}L_{AB}}{A_{AB}E_{st}} = \frac{42.11(10^3)(500)}{\frac{\pi}{4}(0.02^2)200(10^9)}$$

$$= 0.335 \text{ mm} \qquad \textbf{Ans.}$$

4–38. The A-36 steel column, having a cross-sectional area of 18 in^2, is encased in high-strength concrete as shown. If an axial force of 60 kip is applied to the column, determine the average compressive stress in the concrete and in the steel. How far does the column shorten? It has an original length of 8 ft.



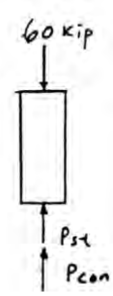
$$+\uparrow \Sigma F_y = 0; \qquad P_{st} + P_{con} - 60 = 0 \qquad \textbf{(1)}$$

$$\delta_{st} = \delta_{con}; \qquad \frac{P_{st}(8)(12)}{18(29)(10^3)} = \frac{P_{con}(8)(12)}{[(9)(16) - 18](4.20)(10^3)}$$

$$P_{st} = 0.98639\, P_{con} \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$P_{st} = 29.795 \text{ kip}; \qquad P_{con} = 30.205 \text{ kip}$$



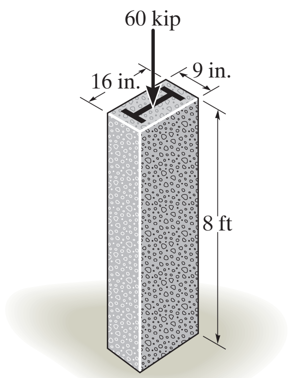
$$\sigma_{st} = \frac{P_{st}}{A_{st}} = \frac{29.795}{18} = 1.66 \text{ ksi} \qquad \textbf{Ans.}$$

$$\sigma_{con} = \frac{P_{con}}{A_{con}} = \frac{30.205}{9(16) - 18} = 0.240 \text{ ksi} \qquad \textbf{Ans.}$$

Either the concrete or steel can be used for the deflection calculation.

$$\delta = \frac{P_{st}L}{A_{st}E} = \frac{29.795(8)(12)}{18(29)(10^3)} = 0.0055 \text{ in.} \qquad \textbf{Ans.}$$

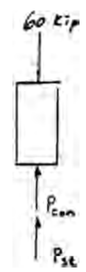
4–39. The A-36 steel column is encased in high-strength concrete as shown. If an axial force of 60 kip is applied to the column, determine the required area of the steel so that the force is shared equally between the steel and concrete. How far does the column shorten? It has an original length of 8 ft.



The force of 60 kip is shared equally by the concrete and steel. Hence

$$P_{st} = P_{con} = P = 30 \text{ kip}$$

$$\delta_{con} = \delta_{st}; \qquad \frac{PL}{A_{con}E_{con}} = \frac{PL}{A_{st}E_{st}}$$

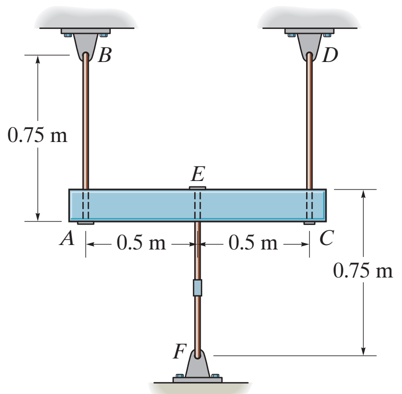


$$A_{st} = \frac{A_{con}E_{con}}{E_{st}} = \frac{[9(16) - A_{st}]\,4.20(10^3)}{29(10^3)}$$

$$= 18.2 \text{ in}^2 \qquad \textbf{Ans.}$$

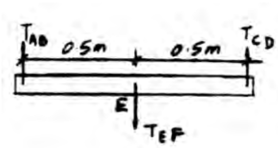
$$\delta = \frac{P_{st}L}{A_{st}E_{st}} = \frac{30(8)(12)}{18.2(29)(10^3)} = 0.00545 \text{ in.} \qquad \textbf{Ans.}$$

*__4–40.__ The rigid member is held in the position shown by three A-36 steel tie rods. Each rod has an unstretched length of 0.75 m and a cross-sectional area of 125 mm^2. Determine the forces in the rods if a turnbuckle on rod EF undergoes one full turn. The lead of the screw is 1.5 mm. Neglect the size of the turnbuckle and assume that it is rigid. *Note:* The lead would cause the rod, when *unloaded*, to shorten 1.5 mm when the turnbuckle is rotated one revolution.



$$\circlearrowleft + \Sigma M_E = 0; \qquad -T_{AB}(0.5) + T_{CD}(0.5) = 0$$

$$T_{AB} = T_{CD} = T \qquad \textbf{(1)}$$



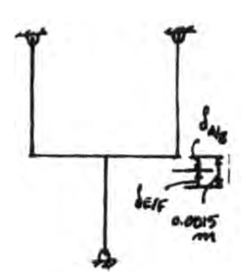
$$+\downarrow \Sigma F_y = 0; \qquad T_{EF} - 2T = 0$$

$$T_{EF} = 2T \qquad \textbf{(2)}$$

Rod EF shortens 1.5mm causing AB (and DC) to elongate. Thus:

$$0.0015 = \delta_{A/B} + \delta_{E/F}$$

$$0.0015 = \frac{T(0.75)}{(125)(10^{-6})(200)(10^9)} + \frac{2T(0.75)}{(125)(10^{-6})(200)(10^9)}$$



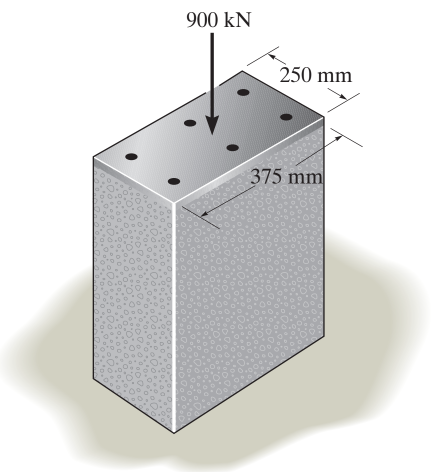
$$2.25T = 37500$$

$$T = 16666.67 \text{ N}$$

$$T_{AB} = T_{CD} = 16.7 \text{ kN} \qquad \textbf{Ans.}$$

$$T_{EF} = 33.3 \text{ kN} \qquad \textbf{Ans.}$$

•**4–41.** The concrete post is reinforced using six steel reinforcing rods, each having a diameter of 20 mm. Determine the stress in the concrete and the steel if the post is subjected to an axial load of 900 kN. $E_{st} = 200$ GPa, $E_c = 25$ GPa.



Referring to the *FBD* of the upper portion of the cut concrete post shown in Fig. *a*

$+\uparrow \Sigma F_y = 0; \qquad P_{con} + 6P_{st} - 900 = 0$ **(1)**

Since the steel rods and the concrete are firmly bonded, their deformation must be the same. Thus

$$\partial_{con} = \delta_{st}$$

$$\frac{P_{con}\,L}{A_{con}\,E_{con}} = \frac{P_{st}\,L}{A_{st}\,E_{st}}$$

$$\frac{P_{con}\,\cancel{L}}{\left[0.25(0.375) - 6(\frac{\pi}{4})(0.02^2)\right]\left[25(10^9)\right]} = \frac{P_{st}\,\cancel{L}}{(\frac{\pi}{4})(0.02^2)\left[200(10^9)\right]}$$

$$P_{con} = 36.552\,P_{st} \qquad \textbf{(2)}$$

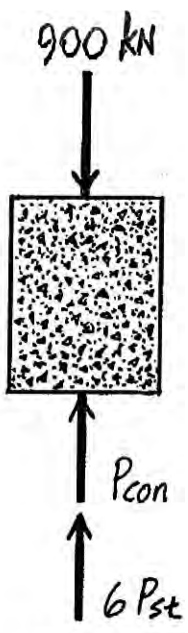
Solving Eqs (1) and (2) yields

$$P_{st} = 21.15\text{ kN} \qquad P_{con} = 773.10\text{ kN}$$

Thus,

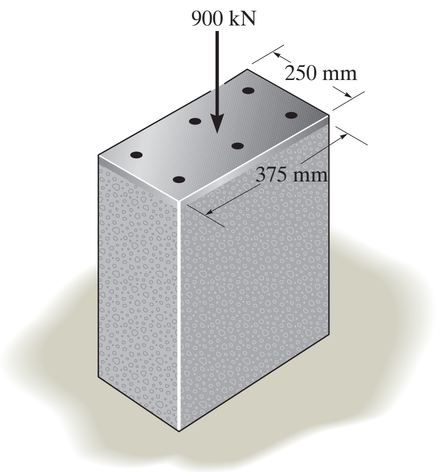
$$\sigma_{con} = \frac{P_{con}}{A_{con}} = \frac{773.10(10^3)}{0.15(0.375) - 6(\frac{\pi}{4})(0.02^2)} = 8.42\text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_{st} = \frac{P_{st}}{A_{st}} = \frac{21.15(10^3)}{\frac{\pi}{4}(0.02^2)} = 67.3\text{ MPa} \qquad \textbf{Ans.}$$



(a)

4–42. The post is constructed from concrete and six A-36 steel reinforcing rods. If it is subjected to an axial force of 900 kN, determine the required diameter of each rod so that one-fifth of the load is carried by the steel and four-fifths by the concrete. $E_{st} = 200$ GPa, $E_c = 25$ GPa.



The normal force in each steel rod is

$$P_{st} = \frac{\frac{1}{5}(900)}{6} = 30 \text{ kN}$$

The normal force in concrete is

$$P_{con} = \frac{4}{5}(900) = 720 \text{ kN}$$

Since the steel rods and the concrete are firmly bonded, their deformation must be the same. Thus

$$\delta_{con} = \delta_{st}$$

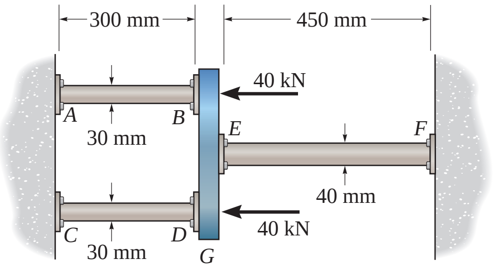
$$\frac{P_{con}\,L}{A_{con}\,E_{con}} = \frac{P_{st}\,L}{A_{st}\,E_{st}}$$

$$\frac{720\cancel{(10^3)}\,\cancel{L}}{\left[0.25(0.375) - 6(\frac{\pi}{4}d^2)\right]\left[25\cancel{(10^9)}\right]} = \frac{30\cancel{(10^3)}\cancel{L}}{\frac{\pi}{4}d^2\left[200\cancel{(10^9)}\right]}$$

$$49.5\pi\, d^2 = 0.09375$$

$$d = 0.02455 \text{ m} = 24.6 \text{ mm} \qquad \textbf{Ans.}$$

4–43. The assembly consists of two red brass C83400 copper alloy rods AB and CD of diameter 30 mm, a stainless 304 steel alloy rod EF of diameter 40 mm, and a rigid cap G. If the supports at A, C and F are rigid, determine the average normal stress developed in rods AB, CD and EF.



Equation of Equilibrium: Due to symmetry, $F_{AB} = F_{CD} = F$. Referring to the free-body diagram of the assembly shown in Fig. a,

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 2F + F_{EF} - 2\left[40(10^3)\right] = 0 \qquad \textbf{(1)}$$

Compatibility Equation: Using the method of superposition, Fig. b,

$$(\xrightarrow{+}) \quad 0 = -\delta_P + \delta_{EF}$$

$$0 = -\frac{40(10^3)(300)}{\frac{\pi}{4}(0.03^2)(101)(10^9)} + \left[\frac{F_{EF}(450)}{\frac{\pi}{4}(0.04^2)(193)(10^9)} + \frac{\left(F_{EF}/2\right)(300)}{\frac{\pi}{4}(0.03^2)(101)(10^9)}\right]$$

$$F_{EF} = 42\,483.23 \text{ N}$$

Substituting this result into Eq. (1),

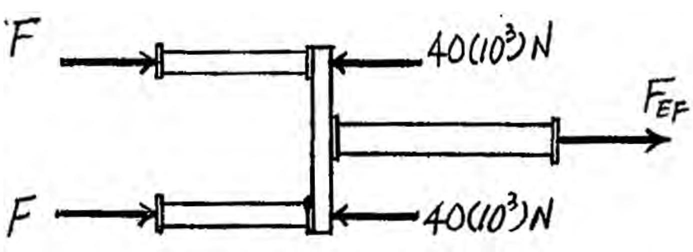
$$F = 18\,758.38 \text{ N}$$

4–43. Continued

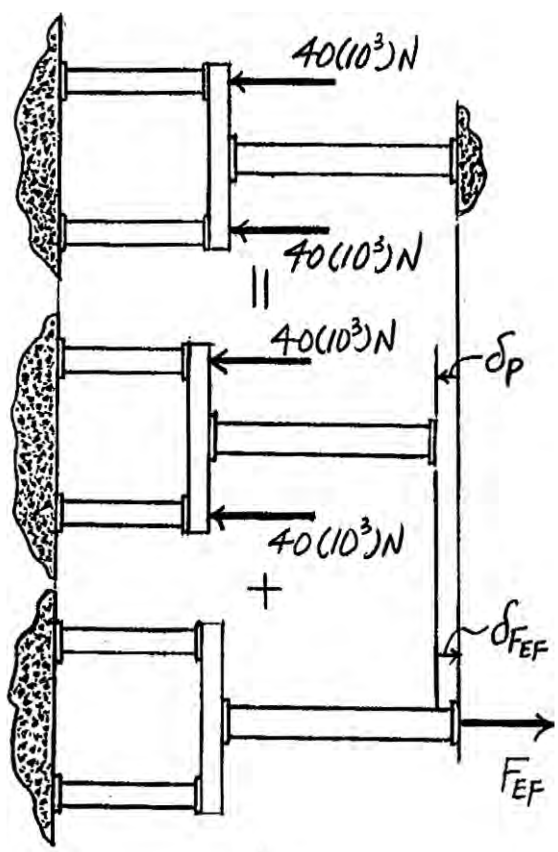
Normal Stress: We have,

$$\sigma_{AB} = \sigma_{CD} = \frac{F}{A_{CD}} = \frac{18\,758.38}{\frac{\pi}{4}(0.03^2)} = 26.5 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_{EF} = \frac{F_{EF}}{A_{EF}} = \frac{42\,483.23}{\frac{\pi}{4}(0.04^2)} = 33.8 \text{ MPa} \qquad \textbf{Ans.}$$

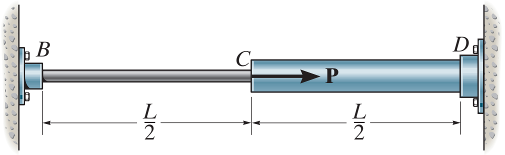


(a)



(b)

*__4–44.__ The two pipes are made of the same material and are connected as shown. If the cross-sectional area of BC is A and that of CD is $2A$, determine the reactions at B and D when a force $\mathbf{P}$ is applied at the junction C.



Equations of Equilibrium:

$$\xleftarrow{+} \Sigma F_x = 0; \qquad F_B + F_D - P = 0 \qquad \textbf{[1]}$$

Compatibility:

$$(\xrightarrow{+}) \qquad 0 = \delta_P - \delta_B$$

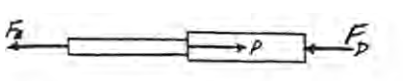
$$0 = \frac{P\left(\frac{L}{2}\right)}{2AE} - \left[\frac{F_B\left(\frac{L}{2}\right)}{AE} + \frac{F_B\left(\frac{L}{2}\right)}{2AE}\right]$$

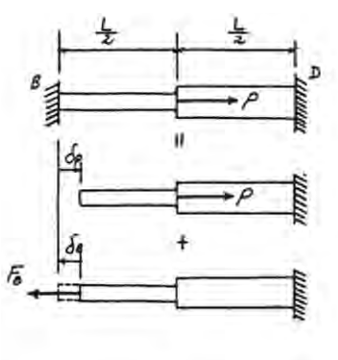
$$0 = \frac{PL}{4AE} - \frac{3F_B L}{4AE}$$

$$F_B = \frac{P}{3} \qquad \textbf{Ans.}$$

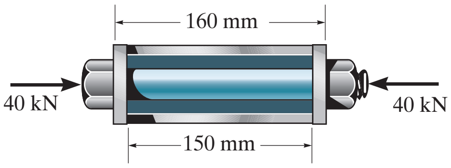
From Eq. [1]

$$F_D = \frac{2}{3}P \qquad \textbf{Ans.}$$





•**4–45.** The bolt has a diameter of 20 mm and passes through a tube that has an inner diameter of 50 mm and an outer diameter of 60 mm. If the bolt and tube are made of A-36 steel, determine the normal stress in the tube and bolt when a force of 40 kN is applied to the bolt. Assume the end caps are rigid.



Referring to the *FBD* of left portion of the cut assembly, Fig. *a*

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 40(10^3) - F_b - F_t = 0 \qquad \textbf{(1)}$$

Here, it is required that the bolt and the tube have the same deformation. Thus

$$\delta_t = \delta_b$$

$$\frac{F_t(150)}{\frac{\pi}{4}(0.06^2 - 0.05^2)\cancel{\left[200(10^9)\right]}} = \frac{F_b(160)}{\frac{\pi}{4}(0.02^2)\cancel{\left[200(10^9)\right]}}$$

$$F_t = 2.9333\, F_b \qquad \textbf{(2)}$$

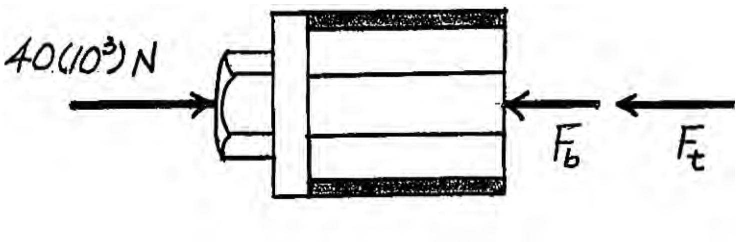
Solving Eqs (1) and (2) yields

$$F_b = 10.17\,(10^3)\text{ N} \qquad F_t = 29.83\,(10^3)\text{ N}$$

Thus,

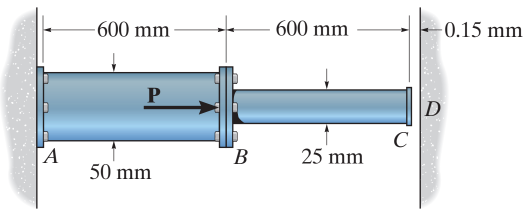
$$\sigma_b = \frac{F_b}{A_b} = \frac{10.17(10^3)}{\frac{\pi}{4}(0.02^2)} = 32.4\text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_t = \frac{F_t}{A_t} = \frac{29.83\,(10^3)}{\frac{\pi}{4}(0.06^2 - 0.05^2)} = 34.5\text{ MPa} \qquad \textbf{Ans.}$$



(a)

4–46. If the gap between C and the rigid wall at D is initially 0.15 mm, determine the support reactions at A and D when the force $\mathbf{P} = 200$ kN is applied. The assembly is made of A36 steel.



Equation of Equilibrium: Referring to the free-body diagram of the assembly shown in Fig. *a*,

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 200(10^3) - F_D - F_A = 0 \qquad \textbf{(1)}$$

Compatibility Equation: Using the method of superposition, Fig. *b*,

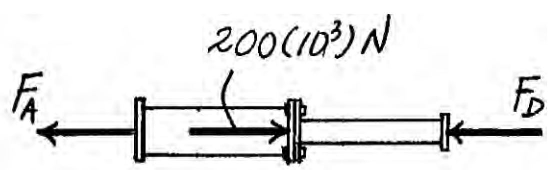
$$(\xrightarrow{+}) \qquad \delta = \delta_P - \delta_{F_D}$$

$$0.15 = \frac{200(10^3)(600)}{\frac{\pi}{4}(0.05^2)(200)(10^9)} - \left[\frac{F_D(600)}{\frac{\pi}{4}(0.05^2)(200)(10^9)} + \frac{F_D(600)}{\frac{\pi}{4}(0.025^2)(200)(10^9)}\right]$$

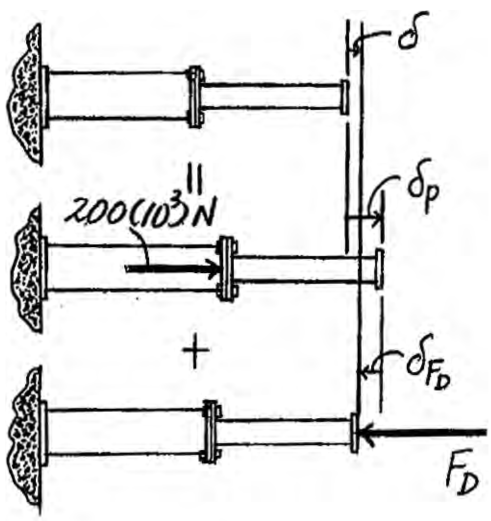
$$F_D = 20\,365.05 \text{ N} = 20.4 \text{ kN} \qquad \textbf{Ans.}$$

Substituting this result into Eq. (1),

$$F_A = 179\,634.95 \text{ N} = 180 \text{ kN} \qquad \textbf{Ans.}$$

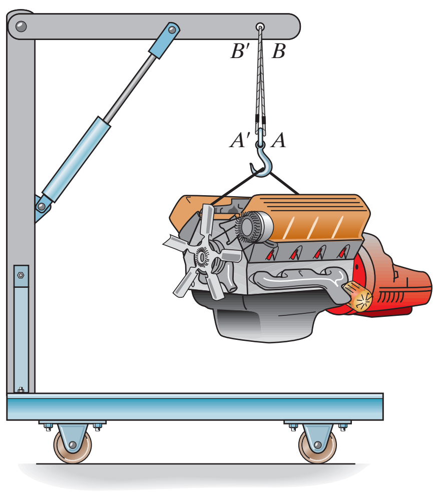


(a)



(b)

4–47. Two A-36 steel wires are used to support the 650-lb engine. Originally, AB is 32 in. long and $A'B'$ is 32.008 in. long. Determine the force supported by each wire when the engine is suspended from them. Each wire has a cross-sectional area of 0.01 in^2.



$$+\uparrow \Sigma F_y = 0; \qquad T_{A'B'} + T_{AB} - 650 = 0 \qquad \textbf{(1)}$$

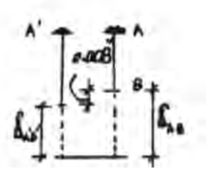
$$\delta_{AB} = \delta_{A'B'} + 0.008$$

$$\frac{T_{AB}\,(32)}{(0.01)(29)(10^6)} = \frac{T_{A'B'}\,(32.008)}{(0.01)(29)(10^6)} + 0.008$$

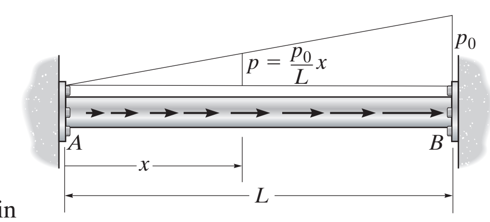
$$32T_{AB} - 32.008T_{A'B'} = 2320$$

$$T_{AB} = 361 \text{ lb} \qquad \textbf{Ans.}$$

$$T_{A'B'} = 289 \text{ lb} \qquad \textbf{Ans.}$$



***4–48.** Rod AB has a diameter d and fits snugly between the rigid supports at A and B when it is unloaded. The modulus of elasticity is E. Determine the support reactions at A and B if the rod is subjected to the linearly distributed axial load.



Equation of Equilibrium: Referring to the free-body diagram of rod AB shown in Fig. a,

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad \frac{1}{2}p_0 L - F_A - F_B = 0 \qquad \textbf{(1)}$$

Compatibility Equation: Using the method of superposition, Fig. b,

$$(\overset{+}{\rightarrow}) \qquad 0 = \delta_P - \delta_{F_A}$$

$$0 = \int_0^L \frac{P(x)dx}{AE} - \frac{F_A\,(L)}{AE}$$

$$0 = \int_0^L P(x)dx - F_A L$$

4–48. Continued

Here, $P(x) = \frac{1}{2}\left(\frac{p_0}{L}x\right)x = \frac{p_0}{2L}x^2$. Thus,

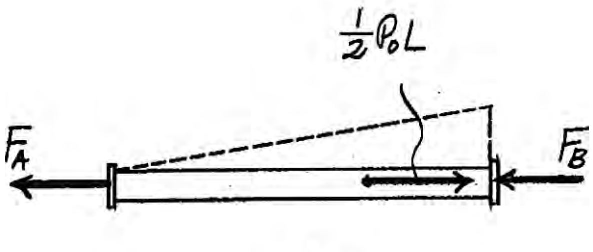
$$0 = \frac{p_0}{2L}\int_0^L x^2\,dx - F_A L$$

$$0 = \frac{p_0}{2L}\left(\frac{x^3}{3}\right)\Big|_0^L - F_{AL}$$

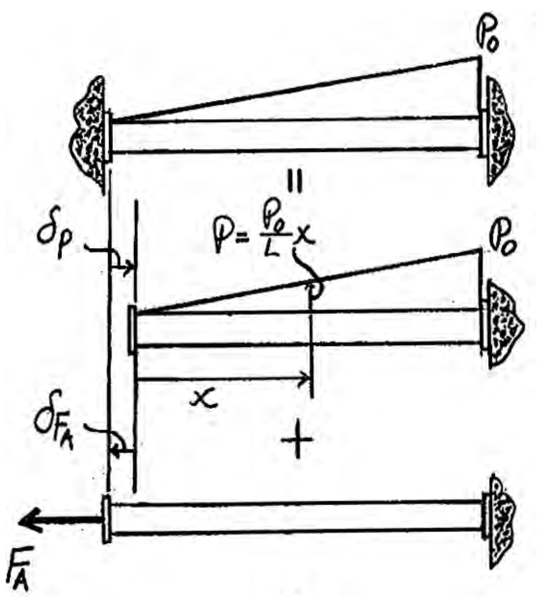
$$F_A = \frac{p_0 L}{6} \qquad \textbf{Ans.}$$

Substituting this result into Eq. (1),

$$F_B = \frac{p_0 L}{3} \qquad \textbf{Ans.}$$

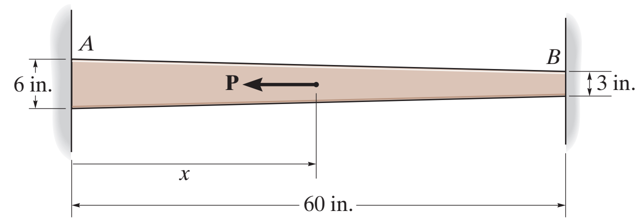


(a)



(b)

•4–49. The tapered member is fixed connected at its ends A and B and is subjected to a load $P = 7$ kip at $x = 30$ in. Determine the reactions at the supports. The material is 2 in. thick and is made from 2014-T6 aluminum.



$$\frac{y}{120 - x} = \frac{1.5}{60}$$

$$y = 3 - 0.025\,x$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_A + F_B - 7 = 0 \qquad \textbf{(1)}$$

$$\delta_{A/B} = 0$$

$$-\int_0^{30} \frac{F_A\,dx}{2(3 - 0.025\,x)(2)(E)} + \int_{30}^{60} \frac{F_B dx}{2(3 - 0.025\,x)(2)(E)} = 0$$

$$-F_A \int_0^{30} \frac{dx}{(3 - 0.025\,x)} + F_B \int_{30}^{60} \frac{dx}{(3 - 0.025x)} = 0$$

$$40\,F_A \ln(3 - 0.025\,x)|_0^{30} - 40\,F_B \ln(3 - 0.025x)|_{30}^{60} = 0$$

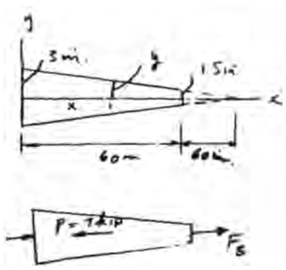
$$-F_A(0.2876) + 0.40547\,F_B = 0$$

$$F_A = 1.40942\,F_B$$

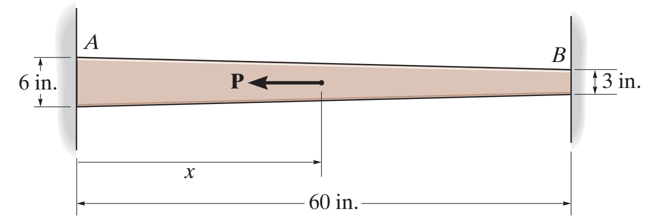
Thus, from Eq. (1).

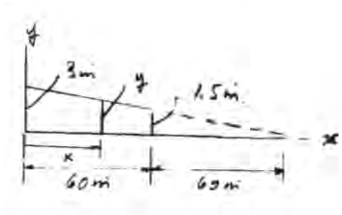
$$F_A = 4.09 \text{ kip} \qquad \textbf{Ans.}$$

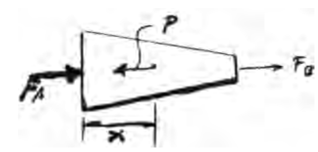
$$F_B = 2.91 \text{ kip} \qquad \textbf{Ans.}$$



4–50. The tapered member is fixed connected at its ends A and B and is subjected to a load **P**. Determine the location x of the load and its greatest magnitude so that the average normal stress in the bar does not exceed $\sigma_{\text{allow}} = 4$ ksi. The member is 2 in. thick.







$$\frac{y}{120 - x} = \frac{1.5}{60}$$

$$y = 3 - 0.025\,x$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_A + F_B - P = 0$$

$$\delta_{A/B} = 0$$

$$-\int_0^x \frac{F_A\,dx}{2(3 - 0.025\,x)(2)(E)} + \int_x^{60} \frac{F_B dx}{2(3 - 0.025\,x)(2)(E)} = 0$$

$$-F_A \int_0^x \frac{dx}{(3 - 0.025\,x)} + F_B \int_x^{60} \frac{dx}{(3 - 0.025\,x)} = 0$$

$$F_A(40)\ln(3 - 0.025\,x)\Big|_0^x - F_B(40)\ln(3 - 0.025x)\Big|_x^{60} = 0$$

$$F_A \ln\left(1 - \frac{0.025\,x}{3}\right) = -F_B \ln\left(2 - \frac{0.025x}{1.5}\right)$$

For greatest magnitude of P require,

$$4 = \frac{F_A}{2(3 - 0.025\,x)(2)}; \qquad F_A = 48 - 0.4\,x$$

$$4 = \frac{F_B}{2(3)}; \qquad F_B = 24 \text{ kip}$$

Thus,

$$(48 - 0.4\,x)\ln\left(1 - \frac{0.025\,x}{3}\right) = -24\ln\left(2 - \frac{0.025\,x}{1.5}\right)$$

Solving by trial and error,

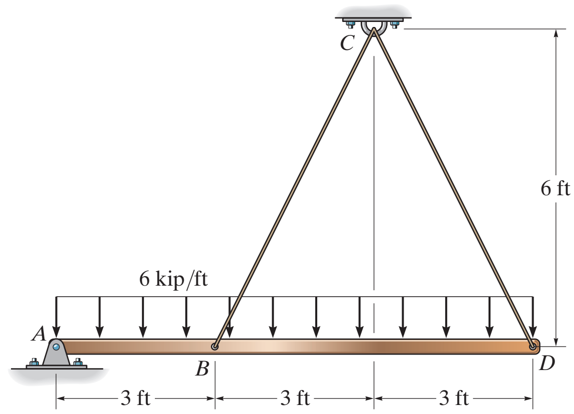
$$x = 28.9 \text{ in.} \qquad \textbf{Ans.}$$

Therefore,

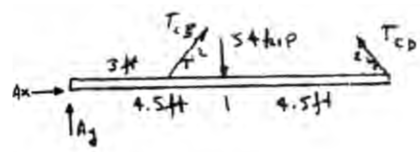
$$F_A = 36.4 \text{ kip}$$

$$P = 60.4 \text{ kip} \qquad \textbf{Ans.}$$

4–51. The rigid bar supports the uniform distributed load of 6 kip/ft. Determine the force in each cable if each cable has a cross-sectional area of 0.05 in^2, and $E = 31(10^3)$ ksi.



$$\zeta + \Sigma M_A = 0; \qquad T_{CB}\left(\frac{2}{\sqrt{5}}\right)(3) - 54(4.5) + T_{CD}\left(\frac{2}{\sqrt{5}}\right)9 = 0 \qquad \textbf{(1)}$$

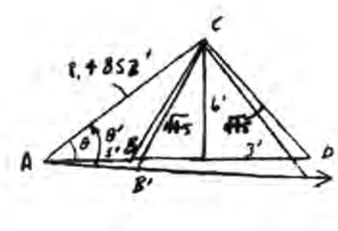


$$\theta = \tan^{-1}\frac{6}{6} = 45°$$

$$L_{BC'}^2 = (3)^2 + (8.4853)^2 - 2(3)(8.4853)\cos\theta'$$

Also,

$$L_{D'C'}^2 = (9)^2 + (8.4853)^2 - 2(9)(8.4853)\cos\theta' \qquad \textbf{(2)}$$



Thus, eliminating $\cos\theta'$.

$$-L_{B'C'}^2(0.019642) + 1.5910 = -L_{D'C'}^2(0.0065473) + 1.001735$$

$$L_{B'C'}^2(0.019642) = 0.0065473\, L_{D'C'}^2 + 0.589256$$

$$L_{B'C'}^2 = 0.333\, L_{D'C'}^2 + 30$$

But,

$$L_{B'C} = \sqrt{45} + \delta_{BC'}\,, \qquad L_{D'C} = \sqrt{45} + \delta_{DC'}$$

Neglect squares or δ'_B since small strain occurs.

$$L_{D'C}^2 = (\sqrt{45} + \delta_{BC})^2 = 45 + 2\sqrt{45}\,\delta_{BC}$$

$$L_{D'C}^2 = (\sqrt{45} + \delta_{DC})^2 = 45 + 2\sqrt{45}\,\delta_{DC}$$

$$45 + 2\sqrt{45}\,\delta_{BC} = 0.333(45 + 2\sqrt{45}\,\delta_{DC}) + 30$$

$$2\sqrt{45}\,\delta_{BC} = 0.333(2\sqrt{45}\,\delta_{DC})$$

$$\delta_{DC} = 3\delta_{BC}$$

Thus,

$$\frac{T_{CD}\sqrt{45}}{AE} = 3\,\frac{T_{CB}\sqrt{45}}{AE}$$

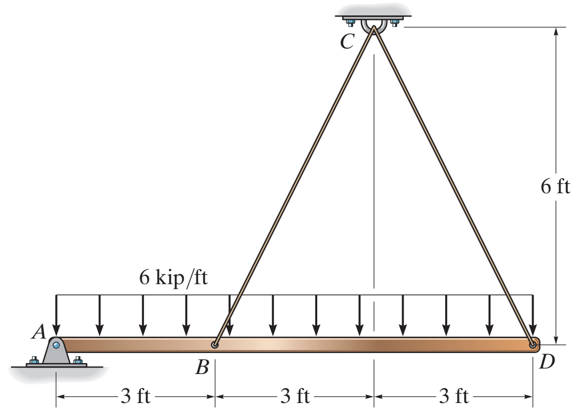
$$T_{CD} = 3\,T_{CB}$$

From Eq. (1).

$$T_{CD} = 27.1682\text{ kip} = 27.2\text{ kip} \qquad \textbf{Ans.}$$

$$T_{CB} = 9.06\text{ kip} \qquad \textbf{Ans.}$$

***4–52.** The rigid bar is originally horizontal and is supported by two cables each having a cross-sectional area of $0.05\ \text{in}^2$, and $E = 31(10^3)$ ksi. Determine the slight rotation of the bar when the uniform load is applied.



See solution of Prob. 4-51.

$$T_{CD} = 27.1682 \text{ kip}$$

$$\delta_{DC} = \frac{T_{CD}\sqrt{45}}{0.05(31)(10^3)} = \frac{27.1682\sqrt{45}}{0.05(31)(10^3)} = 0.1175806 \text{ ft}$$

Using Eq. (2) of Prob. 4-51,

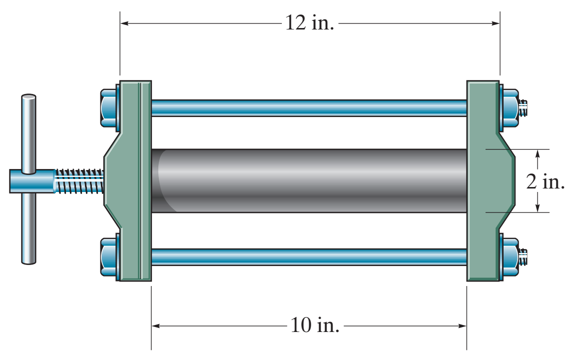
$$(\sqrt{45} + 0.1175806)^2 = (9)^2 + (8.4852)^2 - 2(9)(8.4852)\cos\theta'$$

$$\theta' = 45.838°$$

Thus,

$$\Delta\theta = 45.838° - 45° = 0.838°$$ **Ans.**

•4–53. The press consists of two rigid heads that are held together by the two A-36 steel $\frac{1}{2}$-in.-diameter rods. A 6061-T6-solid-aluminum cylinder is placed in the press and the screw is adjusted so that it just presses up against the cylinder. If it is then tightened one-half turn, determine the average normal stress in the rods and in the cylinder. The single-threaded screw on the bolt has a lead of 0.01 in. *Note*: The lead represents the distance the screw advances along its axis for one complete turn of the screw.



$$\xrightarrow{+} \Sigma F_x = 0; \qquad 2F_{st} - F_{al} = 0$$

$$\delta_{st} = 0.005 - \delta_{al}$$

$$\frac{F_{st}(12)}{(\frac{\pi}{4})(0.5)^2(29)(10^3)} = 0.005 - \frac{F_{al}(10)}{\pi(1)^2(10)(10^3)}$$

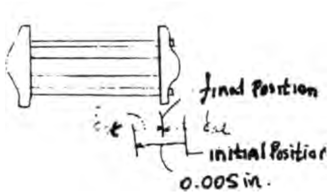
Solving,

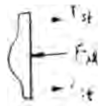
$$F_{st} = 1.822 \text{ kip}$$

$$F_{al} = 3.644 \text{ kip}$$

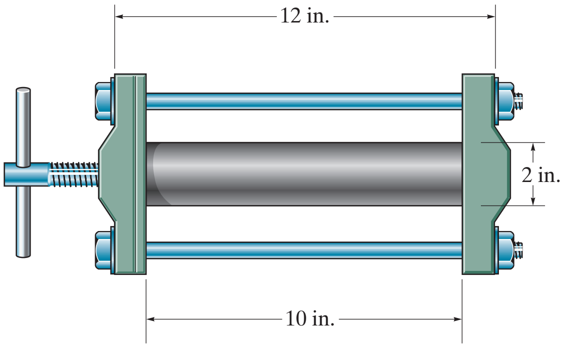
$$\sigma_{rod} = \frac{F_{st}}{A_{st}} = \frac{1.822}{(\frac{\pi}{4})(0.5)^2} = 9.28 \text{ ksi}$$ **Ans.**

$$\sigma_{cyl} = \frac{F_{al}}{A_{al}} = \frac{3.644}{\pi(1)^2} = 1.16 \text{ ksi}$$ **Ans.**





4–54. The press consists of two rigid heads that are held together by the two A-36 steel $\frac{1}{2}$-in.-diameter rods. A 6061-T6-solid-aluminum cylinder is placed in the press and the screw is adjusted so that it just presses up against the cylinder. Determine the angle through which the screw can be turned before the rods or the specimen begin to yield. The single-threaded screw on the bolt has a lead of 0.01 in. *Note*: The lead represents the distance the screw advances along its axis for one complete turn of the screw.



$$\xrightarrow{+} \Sigma F_x = 0; \qquad 2F_{st} - F_{al} = 0$$

$$\delta_{st} = d - \delta_{al}$$

$$\frac{F_{st}(12)}{(\frac{\pi}{4})(0.5)^2(29)(10^3)} = d - \frac{F_{al}(10)}{\pi(1)^2(10)(10^3)} \qquad \textbf{(1)}$$

Assume steel yields first,

$$\sigma_Y = 36 = \frac{F_{st}}{(\frac{\pi}{4})(0.5)^2}; \qquad F_{st} = 7.068 \text{ kip}$$

Then $F_{al} = 14.137$ kip;

$$\sigma_{al} = \frac{14.137}{\pi(1)^2} = 4.50 \text{ ksi}$$

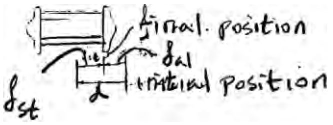
4.50 ksi $<$ 37 ksi steel yields first as assumed. From Eq. (1),

$$d = 0.01940 \text{ in.}$$

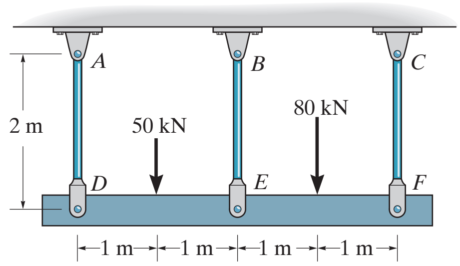
Thus,

$$\frac{\theta}{360°} = \frac{0.01940}{0.01}$$

$$\theta = 698° \qquad \textbf{Ans.}$$



4–55. The three suspender bars are made of A-36 steel and have equal cross-sectional areas of 450 mm^2. Determine the average normal stress in each bar if the rigid beam is subjected to the loading shown.



Referring to the *FBD* of the rigid beam, Fig. *a*,

$$+\uparrow\Sigma F_y = 0; \qquad F_{AD} + F_{BE} + F_{CF} - 50(10^3) - 80(10^3) = 0 \qquad \textbf{(1)}$$

$$\circlearrowleft + \Sigma M_D = 0; \qquad F_{BE}(2) + F_{CF}(4) - 50(10^3)(1) - 80(10^3)(3) = 0 \qquad \textbf{(2)}$$

Referring to the geometry shown in Fig. *b*,

$$\delta_{BE} = \delta_{AD} + \left(\frac{\delta_{CF} - \delta_{AD}}{4}\right)(2)$$

$$\delta_{BE} = \frac{1}{2}\left(\delta_{AD} + \delta_{CF}\right)$$

$$\frac{F_{BE}\cancel{L}}{\cancel{AE}} = \frac{1}{2}\left(\frac{F_{AD}\cancel{L}}{\cancel{AE}} + \frac{F_{CF}\cancel{L}}{\cancel{AE}}\right)$$

$$F_{AD} + F_{CF} = 2F_{BE} \qquad \textbf{(3)}$$

Solving Eqs. (1), (2) and (3) yields

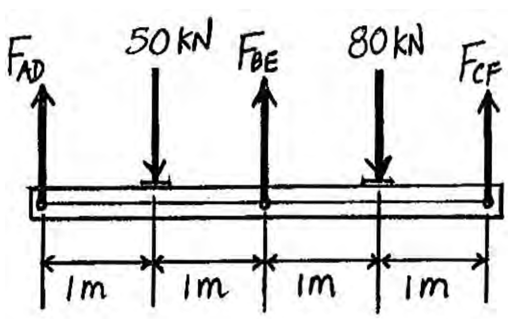
$$F_{BE} = 43.33(10^3)\text{ N} \qquad F_{AD} = 35.83(10^3)\text{ N} \qquad F_{CF} = 50.83(10^3)\text{ N}$$

Thus,

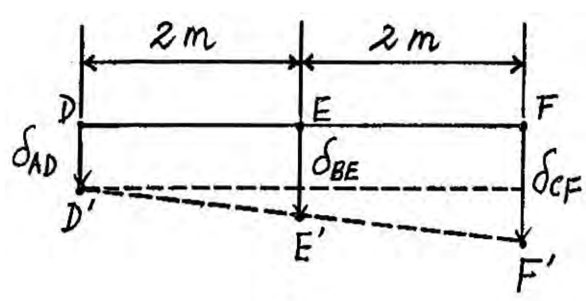
$$\sigma_{BE} = \frac{F_{BE}}{A} = \frac{43.33(10^3)}{0.45(10^{-3})} = 96.3\text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_{AD} = \frac{F_{AD}}{A} = \frac{35.83(10^3)}{0.45(10^{-3})} = 79.6\text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_{CF} = 113\text{ MPa} \qquad \textbf{Ans.}$$

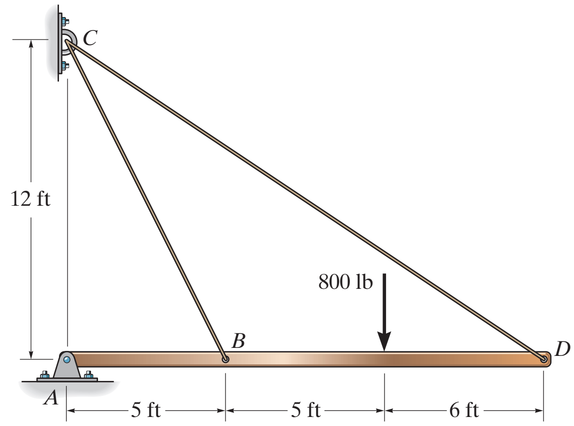


(a)



(b)

***4–56.** The rigid bar supports the 800-lb load. Determine the normal stress in each A-36 steel cable if each cable has a cross-sectional area of 0.04 in^2.



Referring to the *FBD* of the rigid bar, Fig. *a*,

$$\circlearrowleft + \Sigma M_A = 0; \qquad F_{BC}\left(\frac{12}{13}\right)(5) + F_{CD}\left(\frac{3}{5}\right)(16) - 800(10) = 0 \qquad \textbf{(1)}$$

The unstretched length of wires BC and CD are $L_{BC} = \sqrt{12^2 + 5^2} = 13$ ft and $L_{CD} = \sqrt{12^2 + 16^2} = 20$ ft. The stretches of wires BC and CD are

$$\delta_{BC} = \frac{F_{BC}\,L_{BC}}{AE} = \frac{F_{BC}\,(13)}{AE} \qquad \delta_{CD} = \frac{F_{CD}\,L_{CD}}{AE} = \frac{F_{CD}(20)}{AE}$$

Referring to the geometry shown in Fig. b, the vertical displacement of the points on the rigid bar is $\delta\gamma = \dfrac{\delta}{\cos\theta}$. For points B and D, $\cos\theta_B = \dfrac{12}{13}$ and $\cos\theta_D = \dfrac{3}{5}$. Thus, the vertical displacement of points B and D are

$$\left(\delta_B\right)_\gamma = \frac{\delta_{BC}}{\cos\theta_B} = \frac{F_{BC}\,(13)/AE}{12/13} = \frac{169\,F_{BC}}{12AE}$$

$$\left(\delta_D\right)_\gamma = \frac{\delta_{CD}}{\cos\theta_D} = \frac{F_{CD}\,(20)/AE}{3/5} = \frac{100\,F_{CD}}{3\,AE}$$

The similar triangles shown in Fig. c give

$$\frac{\left(\delta_B\right)_\gamma}{5} = \frac{\left(\delta_D\right)_\gamma}{16}$$

$$\frac{1}{5}\left(\frac{169\,F_{BC}}{12\,\cancel{AE}}\right) = \frac{1}{16}\left(\frac{100\,F_{CD}}{3\cancel{AE}}\right)$$

$$F_{BC} = \frac{125}{169}F_{CD} \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2), yields

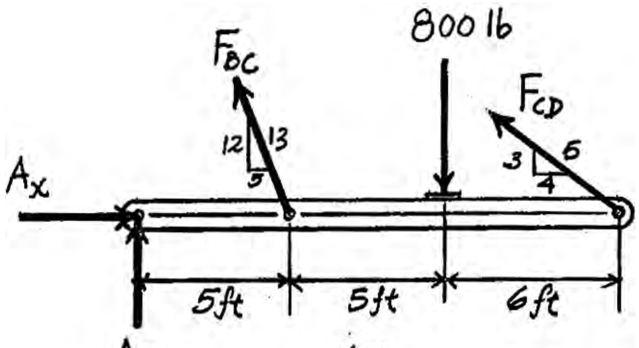
$$F_{CD} = 614.73\text{ lb} \qquad F_{BC} = 454.69\text{ lb}$$

Thus,

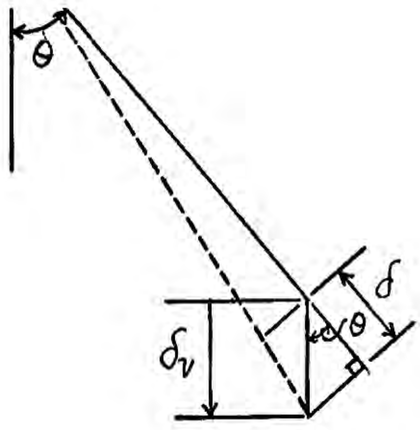
$$\sigma_{CD} = \frac{F_{CD}}{A_{CD}} = \frac{614.73}{0.04} = 15.37(10^3)\text{ psi} = 15.4\text{ ksi} \qquad \textbf{Ans.}$$

$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{454.69}{0.04} = 11.37(10^3)\text{ psi} = 11.4\text{ ksi} \qquad \textbf{Ans.}$$

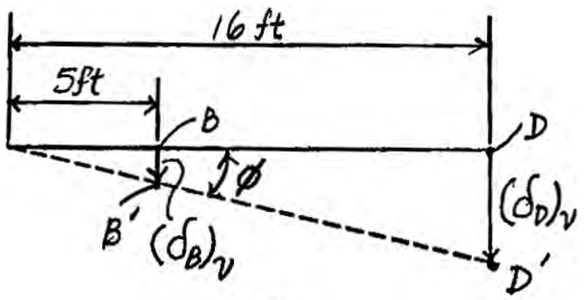
4–56. Continued



(a)

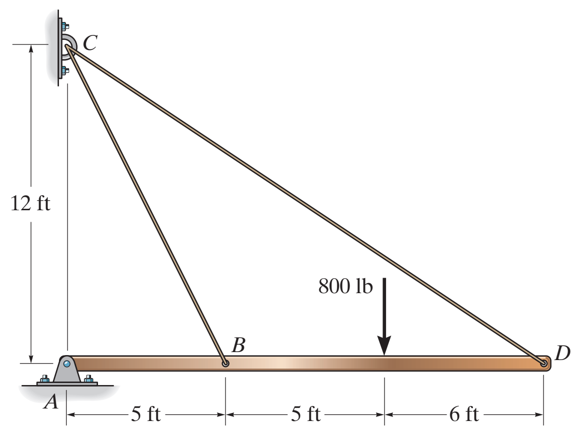


(b)



(c)

•4–57. The rigid bar is originally horizontal and is supported by two A-36 steel cables each having a cross-sectional area of 0.04 in^2. Determine the rotation of the bar when the 800-lb load is applied.



Referring to the *FBD* of the rigid bar Fig. *a*,

$$\zeta + \Sigma M_A = 0; \qquad F_{BC}\left(\frac{12}{13}\right)(5) + F_{CD}\left(\frac{3}{5}\right)(16) - 800(10) = 0 \qquad \textbf{(1)}$$

The unstretched length of wires BC and CD are $L_{BC} = \sqrt{12^2 + 5^2} = 13$ ft and $L_{CD} = \sqrt{12^2 + 16^2} = 20$ ft. The stretch of wires BC and CD are

$$\delta_{BC} = \frac{F_{BC}\,L_{BC}}{A\,E} = \frac{F_{BC}\,(13)}{A\,E} \qquad \delta_{CD} = \frac{F_{CD}\,L_{CD}}{A\,E} = \frac{F_{CD}(20)}{A\,E}$$

Referring to the geometry shown in Fig. *b*, the vertical displacement of the points on the rigid bar is $\delta_\gamma = \dfrac{\delta}{\cos\theta}$. For points B and D, $\cos\theta_B = \dfrac{12}{13}$ and $\cos\theta_D = \dfrac{3}{5}$. Thus, the vertical displacement of points B and D are

$$(\delta_B)_\gamma = \frac{\delta_{BC}}{\cos\theta_B} = \frac{F_{BC}\,(13)/AE}{12/13} = \frac{169\,F_{BC}}{12AE}$$

$$(\delta_D)_\gamma = \frac{\delta_{CD}}{\cos\theta_D} = \frac{F_{CD}\,(20)/AE}{3/5} = \frac{100\,F_{CD}}{3\,AE}$$

The similar triangles shown in Fig. *c* gives

$$\frac{(\delta_B)_\gamma}{5} = \frac{(\delta_D)_\gamma}{16}$$

$$\frac{1}{5}\left(\frac{169\,F_{BC}}{12\,AE}\right) = \frac{1}{16}\left(\frac{100\,F_{CD}}{3\,AE}\right)$$

$$F_{BC} = \frac{125}{169}F_{CD} \qquad \textbf{(2)}$$

Solving Eqs (1) and (2), yields

$$F_{CD} = 614.73\text{ lb} \qquad F_{BC} = 454.69\text{ lb}$$

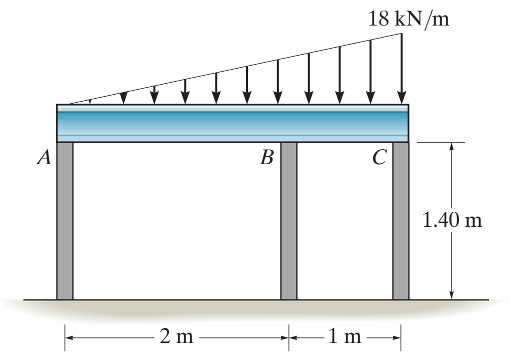
Thus,

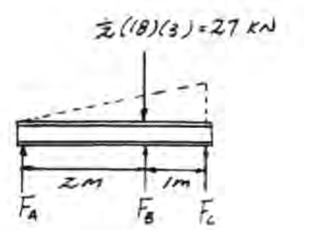
$$(\delta_D)_\gamma = \frac{100(614.73)}{3(0.04)\left[29.0\,(10^6)\right]} = 0.01766\text{ ft}$$

Then

$$\theta = \left(\frac{0.01766\text{ ft}}{16\text{ ft}}\right)\left(\frac{180^\circ}{\pi}\right) = 0.0633^\circ \qquad \textbf{Ans.}$$

4–58. The horizontal beam is assumed to be rigid and supports the distributed load shown. Determine the vertical reactions at the supports. Each support consists of a wooden post having a diameter of 120 mm and an unloaded (original) length of 1.40 m. Take $E_w = 12$ GPa.





$\circlearrowleft + \Sigma M_B = 0; \qquad F_C(1) - F_A(2) = 0 \qquad$ **(1)**

$+\uparrow \Sigma F_y = 0; \qquad F_A + F_B + F_C - 27 = 0 \qquad$ **(2)**

$$\frac{\delta_B - \delta_A}{2} = \frac{\delta_C - \delta_A}{3}; \qquad 3\delta_B - \delta_A = 2\delta_C$$

$$\frac{3F_BL}{AE} - \frac{F_AL}{AE} = \frac{2F_CL}{AE}; \qquad 3F_B - F_A = 2F_C \qquad \textbf{(3)}$$

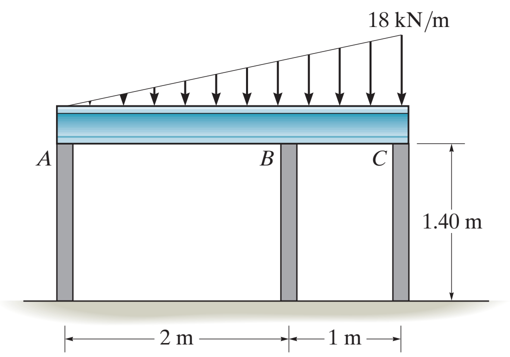
Solving Eqs. (1)–(3) yields :

$F_A = 5.79$ kN **Ans.**

$F_B = 9.64$ kN **Ans.**

$F_C = 11.6$ kN **Ans.**

4–59. The horizontal beam is assumed to be rigid and supports the distributed load shown. Determine the angle of tilt of the beam after the load is applied. Each support consists of a wooden post having a diameter of 120 mm and an unloaded (original) length of 1.40 m. Take $E_w = 12$ GPa.

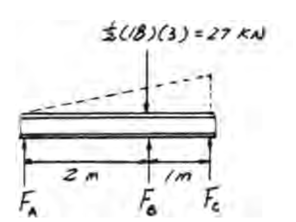


$\circlearrowleft + \Sigma M_B = 0; \qquad F_C(1) - F_A(2) = 0 \qquad$ **(1)**

$\uparrow + \Sigma F_y = 0; \qquad F_A + F_B + F_C - 27 = 0 \qquad$ **(2)**

$$\frac{\delta_B - \delta_A}{2} = \frac{\delta_C - \delta_A}{3}; \qquad 3\delta_B - \delta_A = 2\delta_C$$

$$\frac{3F_BL}{AE} - \frac{F_AL}{AE} = \frac{2F_CL}{AE}; \qquad 3F_B - F_A = 2F_C \qquad \textbf{(3)}$$



Solving Eqs. (1)–(3) yields :

$F_A = 5.7857$ kN; $\quad F_B = 9.6428$ kN; $\quad F_C = 11.5714$ kN

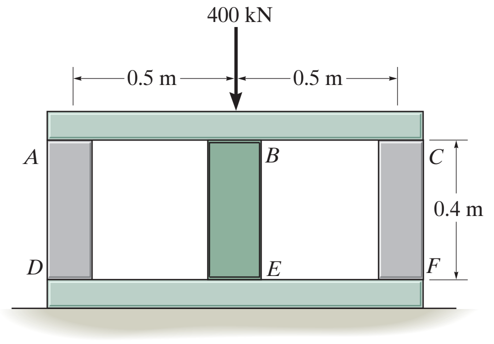
$$\delta_A = \frac{F_AL}{AE} = \frac{5.7857(10^3)(1.40)}{\frac{\pi}{4}(0.12^2)12(10^9)} = 0.0597(10^{-3})\text{ m}$$

$$\delta_C = \frac{F_CL}{AE} = \frac{11.5714(10^3)(1.40)}{\frac{\pi}{4}(0.12^2)12(10^9)} = 0.1194(10^{-3})\text{ m}$$

$$\tan\theta = \frac{0.1194 - 0.0597}{3}(10^{-3})$$

$\theta = 0.0199(10^{-3})$ rad $= 1.14(10^{-3})°$ **Ans.**

*4–60. The assembly consists of two posts AD and CF made of A-36 steel and having a cross-sectional area of 1000 mm^2, and a 2014-T6 aluminum post BE having a cross-sectional area of 1500 mm^2. If a central load of 400 kN is applied to the rigid cap, determine the normal stress in each post. There is a small gap of 0.1 mm between the post BE and the rigid member ABC.



Equation of Equilibrium. Due to symmetry, $\mathbf{F}_{AD} = \mathbf{F}_{CF} = \mathbf{F}$. Referring to the FBD of the rigid cap, Fig. a,

$$+\uparrow \Sigma F_y = 0; \qquad F_{BE} + 2F - 400(10^3) = 0 \qquad \textbf{(1)}$$

Compatibility Equation. Referring to the initial and final position of rods AD (CF) and BE, Fig. b,

$$\delta = 0.1 + \delta_{BE}$$

$$\frac{F(400)}{1(10^{-3})\left[200(10^9)\right]} = 0.1 + \frac{F_{BE}(399.9)}{1.5(10^{-3})\left[73.1(10^9)\right]}$$

$$F = 1.8235\, F_{BE} + 50(10^3) \qquad \textbf{(2)}$$

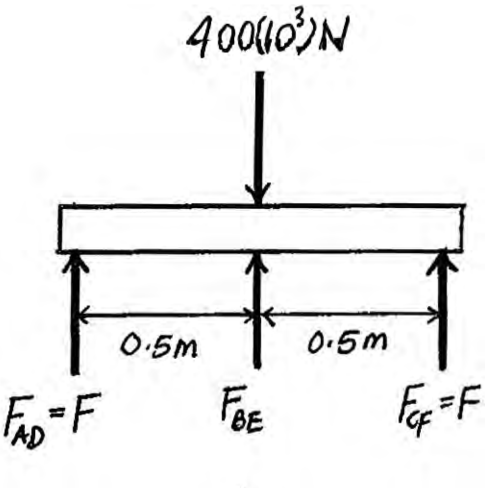
Solving Eqs (1) and (2) yield

$$F_{BE} = 64.56(10^3)\text{ N} \qquad F = 167.72(10^3)\text{ N}$$

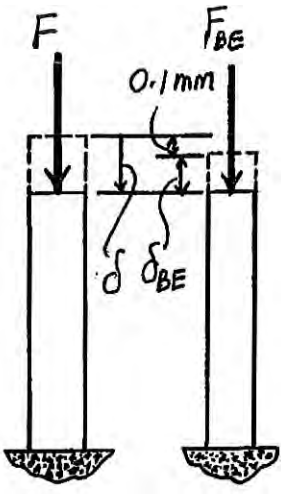
Normal Stress.

$$\sigma_{AD} = \sigma_{CF} = \frac{F}{A_{st}} = \frac{167.72(10^3)}{1(10^{-3})} = 168\text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_{BE} = \frac{F_{BE}}{A_{al}} = \frac{64.56(10^3)}{1.5(10^{-3})} = 43.0\text{ MPa} \qquad \textbf{Ans.}$$

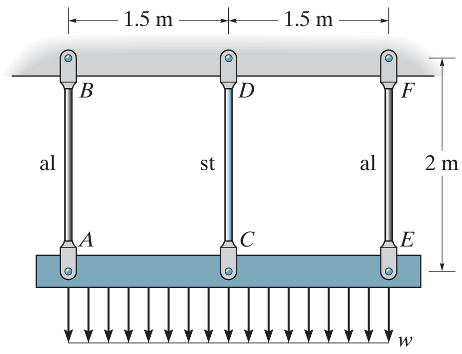


(a)



(b)

•4–61. The distributed loading is supported by the three suspender bars. AB and EF are made of aluminum and CD is made of steel. If each bar has a cross-sectional area of $450\ \text{mm}^2$, determine the maximum intensity w of the distributed loading so that an allowable stress of $(\sigma_{\text{allow}})_{\text{st}} = 180\ \text{MPa}$ in the steel and $(\sigma_{\text{allow}})_{\text{al}} = 94\ \text{MPa}$ in the aluminum is not exceeded. $E_{\text{st}} = 200\ \text{GPa}$, $E_{\text{al}} = 70\ \text{GPa}$. Assume ACE is rigid.



$\circlearrowleft + \Sigma M_C = 0; \qquad F_{EF}(1.5) - F_{AB}(1.5) = 0$

$$F_{EF} = F_{AB} = F$$

$+\uparrow \Sigma F_y = 0; \qquad 2F + F_{CD} - 3w = 0 \qquad \textbf{(1)}$

Compatibility condition :

$$\delta_A = \delta_C$$

$$\frac{FL}{A(70)(10^9)} = \frac{F_{CD}L}{A(200)(10^9)}; \qquad F = 0.35\,F_{CD} \qquad \textbf{(2)}$$

Assume failure of AB and EF:

$$F = (\sigma_{\text{allow}})_{\text{al}}\,A$$

$$= 94(10^6)(450)(10^{-6})$$

$$= 42300\ \text{N}$$

From Eq. (2) $F_{CD} = 120857.14\ \text{N}$

From Eq. (1) $w = 68.5\ \text{kN/m}$

Assume failure of CD:

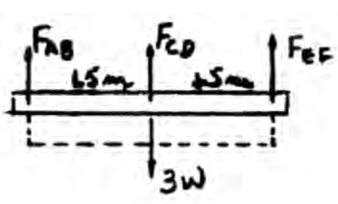
$$F_{CD} = (\sigma_{\text{allow}})_{\text{st}}\,A$$

$$= 180(10^6)(450)(10^{-6})$$

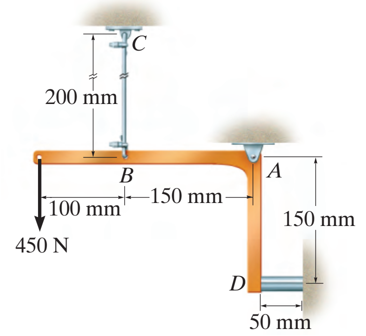
$$= 81000\ \text{N}$$

From Eq. (2) $F = 28350\ \text{N}$

From Eq. (1) $w = 45.9\ \text{kN/m}$ (controls) **Ans.**



4–62. The rigid link is supported by a pin at A, a steel wire BC having an unstretched length of 200 mm and cross-sectional area of 22.5 mm^2, and a short aluminum block having an unloaded length of 50 mm and cross-sectional area of 40 mm^2. If the link is subjected to the vertical load shown, determine the average normal stress in the wire and the block. $E_{st} = 200$ GPa, $E_{al} = 70$ GPa.



Equations of Equilibrium:

$$\zeta + \Sigma M_A = 0; \qquad 450(250) - F_{BC}(150) - F_D(150) = 0$$

$$750 - F_{BC} - F_D = 0 \qquad \textbf{[1]}$$

Compatibility:

$$\delta_{BC} = \delta_D$$

$$\frac{F_{BC}(200)}{22.5(10^{-6})200(10^9)} = \frac{F_D(50)}{40(10^{-6})70(10^9)}$$

$$F_{BC} = 0.40179\, F_D \qquad \textbf{[2]}$$

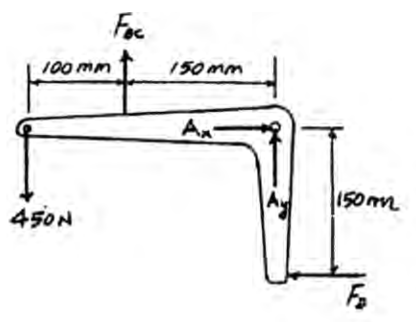
Solving Eqs. [1] and [2] yields:

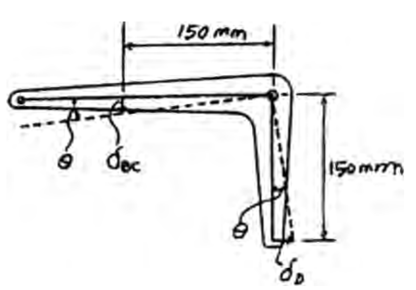
$$F_D = 535.03 \text{ N} \qquad F_{BC} = 214.97 \text{ N}$$

Average Normal Stress:

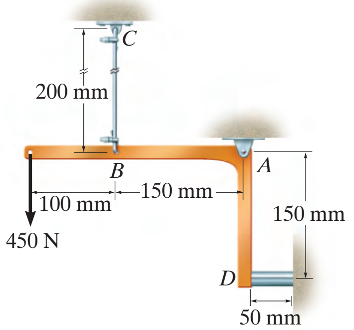
$$\sigma_D = \frac{F_D}{A_D} = \frac{535.03}{40(10^{-6})} = 13.4 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{214.97}{22.5(10^{-6})} = 9.55 \text{ MPa} \qquad \textbf{Ans.}$$





4–63. The rigid link is supported by a pin at A, a steel wire BC having an unstretched length of 200 mm and cross-sectional area of 22.5 mm^2, and a short aluminum block having an unloaded length of 50 mm and cross-sectional area of 40 mm^2. If the link is subjected to the vertical load shown, determine the rotation of the link about the pin A. Report the answer in radians. $E_{st} = 200$ GPa, $E_{al} = 70$ GPa.



Equations of Equilibrium:

$$\circlearrowleft + \Sigma M_A = 0; \qquad 450(250) - F_{BC}(150) - F_D(150) = 0$$

$$750 - F_{BC} - F_D = 0 \qquad \textbf{[1]}$$

Compatibility:

$$\delta_{BC} = \delta_D$$

$$\frac{F_{BC}(200)}{22.5(10^{-6})200(10^9)} = \frac{F_D(50)}{40(10^{-6})70(10^9)}$$

$$F_{BC} = 0.40179\, F_D \qquad \textbf{[2]}$$

Solving Eqs. [1] and [2] yields :

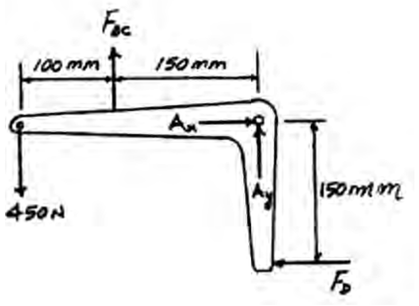
$$F_D = 535.03 \text{ N} \qquad F_{BC} = 214.97 \text{ N}$$

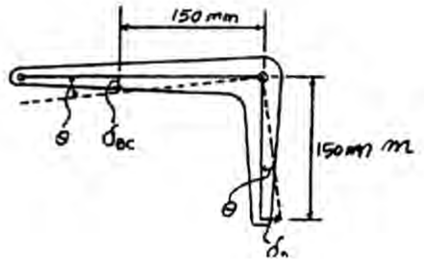
Displacement:

$$\delta_D = \frac{F_D L_D}{A_D E_{al}} = \frac{535.03(50)}{40(10^{-6})(70)(10^9)} = 0.009554 \text{ mm}$$

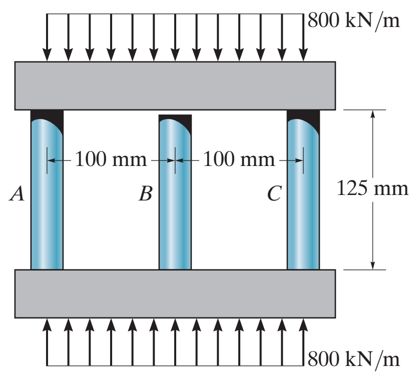
$$\tan\theta = \frac{\delta_D}{150} = \frac{0.009554}{150}$$

$$\theta = 63.7(10^{-6}) \text{ rad} = 0.00365° \qquad \textbf{Ans.}$$





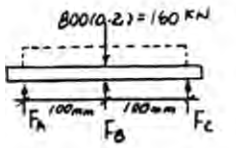
***4–64.** The center post B of the assembly has an original length of 124.7 mm, whereas posts A and C have a length of 125 mm. If the caps on the top and bottom be considered rigid, determine the average normal stress in each post. The posts are made of aluminum and have a cross-sectional area of 400 mm^2. $E_{al} = 70$ GPa.



$\circlearrowleft + \Sigma M_B = 0; \qquad -F_A(100) + F_C(100) = 0$

$F_A = F_C = F$ **(1)**

$+\uparrow \Sigma F_y = 0; \qquad 2F + F_B - 160 = 0$ **(2)**



$\delta_A = \delta_B + 0.0003$

$$\frac{F\,(0.125)}{400\,(10^{-6})(70)(10^6)} = \frac{F_B\,(0.1247)}{400\,(10^{-6})(70)(10^6)} + 0.0003$$

$0.125\,F - 0.1247 F_B = 8.4$ **(3)**

Solving Eqs. (2) and (3)

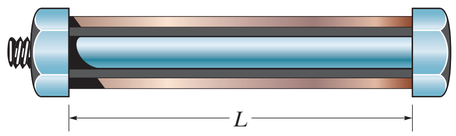
$F = 75.762$ kN

$F_B = 8.547$ kN

$$\sigma_A = \sigma_C = \frac{75.726\,(10^3)}{400(10^{-6})} = 189 \text{ MPa}$$ **Ans.**

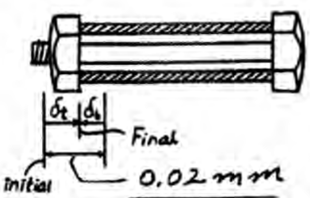
$$\sigma_B = \frac{8.547\,(10^3)}{400\,(10^{-6})} = 21.4 \text{ MPa}$$ **Ans.**

•4–65. The assembly consists of an A-36 steel bolt and a C83400 red brass tube. If the nut is drawn up snug against the tube so that $L = 75$ mm, then turned an additional amount so that it advances 0.02 mm on the bolt, determine the force in the bolt and the tube. The bolt has a diameter of 7 mm and the tube has a cross-sectional area of 100 mm^2.



Equilibrium: Since no external load is applied, the force acting on the tube and the bolt is the same.

Compatibility:

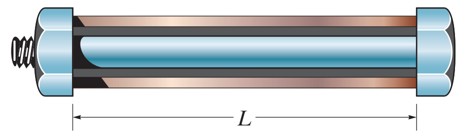


$$0.02 = \delta_t + \delta_b$$

$$0.02 = \frac{P(75)}{100(10^{-6})(101)(10^9)} + \frac{P(75)}{\frac{\pi}{4}(0.007^2)(200)(10^9)}$$

$P = 1164.83 \text{ N} = 1.16 \text{ kN}$ **Ans.**

4–66. The assembly consists of an A-36 steel bolt and a C83400 red brass tube. The nut is drawn up snug against the tube so that $L = 75$ mm. Determine the maximum additional amount of advance of the nut on the bolt so that none of the material will yield. The bolt has a diameter of 7 mm and the tube has a cross-sectional area of 100 mm^2.



Allowable Normal Stress:

$$(\sigma_Y)_{st} = 250\left(10^6\right) = \frac{P_{st}}{\frac{\pi}{4}(0.007)^2}$$

$$P_{st} = 9.621 \text{ kN}$$

$$(\sigma_Y)_{br} = 70.0\left(10^6\right) = \frac{P_{br}}{100(10^{-6})}$$

$$P_{br} = 7.00 \text{ kN}$$

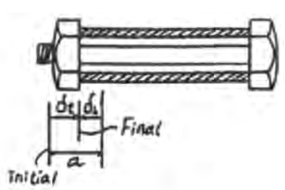
Since $P_{st} > P_{br}$, by comparison he brass will yield first.

Compatibility:

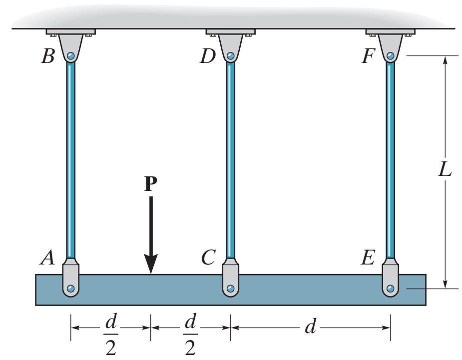
$$a = \delta_t + \delta_b$$

$$= \frac{7.00(10^3)(75)}{100(10^{-6})(101)(10^9)} + \frac{7.00(10^3)(75)}{\frac{\pi}{4}(0.007)^2(200)(10^9)}$$

$$= 0.120 \text{ mm} \qquad \textbf{Ans.}$$



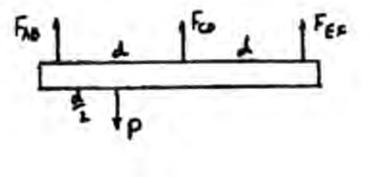
4–67. The three suspender bars are made of the same material and have equal cross-sectional areas A. Determine the average normal stress in each bar if the rigid beam ACE is subjected to the force **P**.



$$\zeta + \Sigma M_A = 0; \qquad F_{CD}(d) + F_{EF}(2d) - P\left(\frac{d}{2}\right) = 0$$

$$F_{CD} + 2F_{EF} = \frac{P}{2} \qquad \textbf{(1)}$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{AB} + F_{CD} + F_{EF} - P = 0 \qquad \textbf{(2)}$$

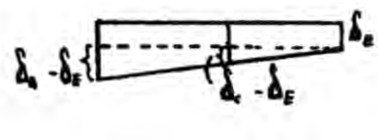


$$\frac{\delta_C - \delta_E}{d} = \frac{\delta_A - \delta_E}{2d}$$

$$2\delta_C = \delta_A + \delta_E$$

$$\frac{2F_{CD}L}{AE} = \frac{F_{AB}L}{AE} + \frac{F_{EF}L}{AE}$$

$$2F_{CD} - F_{AB} - F_{EF} = 0 \qquad \textbf{(3)}$$



Solving Eqs. (1), (2) and (3) yields

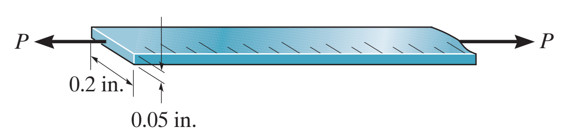
$$F_{AB} = \frac{7P}{12} \qquad F_{CD} = \frac{P}{3} \qquad F_{EF} = \frac{P}{12}$$

$$\sigma_{AB} = \frac{7P}{12A} \qquad \textbf{Ans.}$$

$$\sigma_{CD} = \frac{P}{3A} \qquad \textbf{Ans.}$$

$$\sigma_{EF} = \frac{P}{12A} \qquad \textbf{Ans.}$$

***4–68.** A steel surveyor's tape is to be used to measure the length of a line. The tape has a rectangular cross section of 0.05 in. by 0.2 in. and a length of 100 ft when $T_1 = 60°\text{F}$ and the tension or pull on the tape is 20 lb. Determine the true length of the line if the tape shows the reading to be 463.25 ft when used with a pull of 35 lb at $T_2 = 90°\text{F}$. The ground on which it is placed is flat. $\alpha_{st} = 9.60(10^{-6})/°\text{F}$, $E_{st} = 29(10^3)$ ksi.

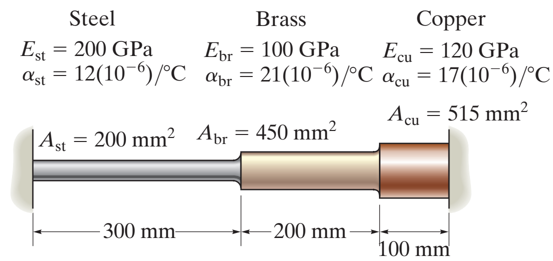


$$\delta_T = \alpha \Delta T L = 9.6(10^{-6})(90 - 60)(463.25) = 0.133416 \text{ ft}$$

$$\delta = \frac{PL}{AE} = \frac{(35 - 20)(463.25)}{(0.2)(0.05)(29)(10^6)} = 0.023961 \text{ ft}$$

$$L = 463.25 + 0.133416 + 0.023961 = 463.41 \text{ ft} \qquad \textbf{Ans.}$$

•4–69. Three bars each made of different materials are connected together and placed between two walls when the temperature is $T_1 = 12°\text{C}$. Determine the force exerted on the (rigid) supports when the temperature becomes $T_2 = 18°\text{C}$. The material properties and cross-sectional area of each bar are given in the figure.

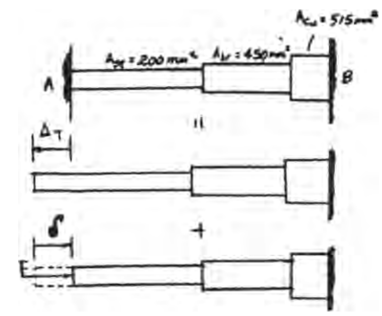


$(\overset{+}{\leftarrow})\qquad 0 = \Delta_T - \delta$

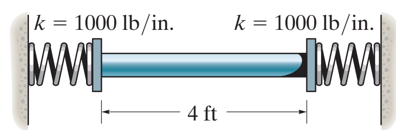
$$0 = 12(10^{-6})(6)(0.3) + 21\,(10^{-6})(6)(0.2) + 17\,(10^{-6})(6)(0.1)$$

$$- \frac{F(0.3)}{200(10^{-6})(200)(10^9)} - \frac{F(0.2)}{450(10^{-6})(100)(10^9)} - \frac{F(0.1)}{515(10^{-6})(120)(10^9)}$$

$F = 4203\ \text{N} = 4.20\ \text{kN}$ **Ans.**



4–70. The rod is made of A-36 steel and has a diameter of 0.25 in. If the rod is 4 ft long when the springs are compressed 0.5 in. and the temperature of the rod is $T = 40°\text{F}$, determine the force in the rod when its temperature is $T = 160°\text{F}$.



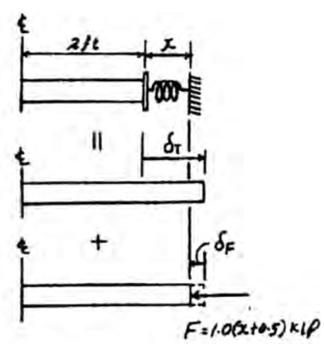
Compatibility:

$(\overset{+}{\rightarrow})\qquad x = \delta_T - \delta_F$

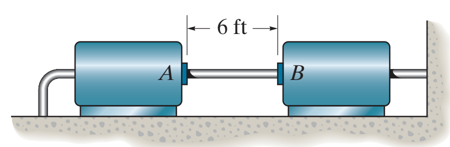
$$x = 6.60(10^{-6})(160 - 40)(2)(12) - \frac{1.00(0.5)(2)(12)}{\frac{\pi}{4}(0.25^2)(29.0)(10^3)}$$

$x = 0.01869$ in.

$F = 1.00(0.01869 + 0.5) = 0.519$ kip **Ans.**



4–71. A 6-ft-long steam pipe is made of A-36 steel with $\sigma_Y = 40$ ksi. It is connected directly to two turbines A and B as shown. The pipe has an outer diameter of 4 in. and a wall thickness of 0.25 in. The connection was made at $T_1 = 70°\text{F}$. If the turbines' points of attachment are assumed rigid, determine the force the pipe exerts on the turbines when the steam and thus the pipe reach a temperature of $T_2 = 275°\text{F}$.

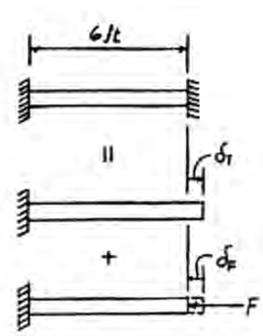


Compatibility:

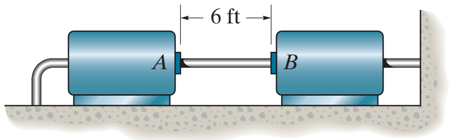
$(\overset{+}{\rightarrow})\qquad 0 = \delta_T - \delta_F$

$$0 = 6.60(10^{-6})(275 - 70)(6)(12) - \frac{F(6)(12)}{\frac{\pi}{4}(4^2 - 3.5^2)(29.0)(10^3)}$$

$F = 116$ kip **Ans.**



***4–72.** A 6-ft-long steam pipe is made of A-36 steel with $\sigma_Y = 40$ ksi. It is connected directly to two turbines A and B as shown. The pipe has an outer diameter of 4 in. and a wall thickness of 0.25 in. The connection was made at $T_1 = 70°\text{F}$. If the turbines' points of attachment are assumed to have a stiffness of $k = 80(10^3)$ kip/in., determine the force the pipe exerts on the turbines when the steam and thus the pipe reach a temperature of $T_2 = 275°\text{F}$.



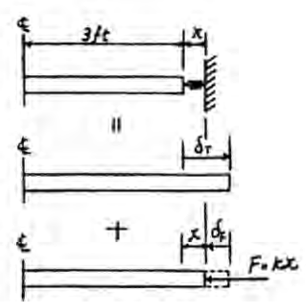
Compatibility:

$$x = \delta_T - \delta_F$$

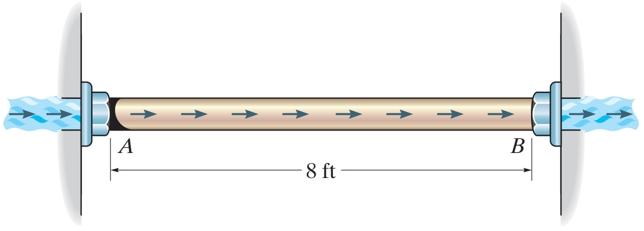
$$x = 6.60(10^{-6})(275 - 70)(3)(12) - \frac{80(10^3)(x)(3)(12)}{\frac{\pi}{4}(4^2 - 3.5^2)(29.0)(10^3)}$$

$$x = 0.001403 \text{ in.}$$

$$F = k\,x = 80(10^3)(0.001403) = 112 \text{ kip}$$ **Ans.**



•4–73. The pipe is made of A-36 steel and is connected to the collars at A and B. When the temperature is 60° F, there is no axial load in the pipe. If hot gas traveling through the pipe causes its temperature to rise by $\Delta T = (40 + 15x)°\text{F}$, where x is in feet, determine the average normal stress in the pipe. The inner diameter is 2 in., the wall thickness is 0.15 in.



Compatibility:

$$0 = \delta_T - \delta_F \qquad \text{Where} \qquad \delta_T = \int_0^L \alpha\,\Delta T\,dx$$

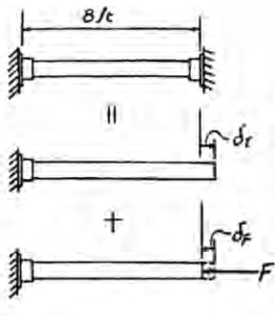
$$0 = 6.60\left(10^{-6}\right)\int_0^{8\text{ft}} (40 + 15\,x)\,dx - \frac{F(8)}{A(29.0)(10^3)}$$

$$0 = 6.60\left(10^{-6}\right)\left[40(8) + \frac{15(8)^2}{2}\right] - \frac{F(8)}{A(29.0)(10^3)}$$

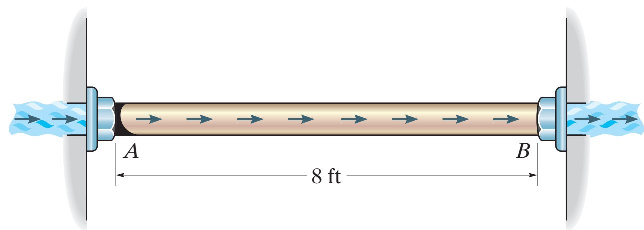
$$F = 19.14\,A$$

Average Normal Stress:

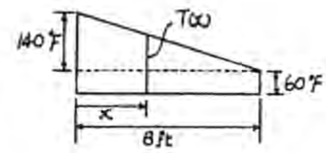
$$\sigma = \frac{19.14\,A}{A} = 19.1 \text{ ksi}$$ **Ans.**



4–74. The bronze C86100 pipe has an inner radius of 0.5 in. and a wall thickness of 0.2 in. If the gas flowing through it changes the temperature of the pipe uniformly from $T_A = 200°\text{F}$ at A to $T_B = 60°\text{F}$ at B, determine the axial force it exerts on the walls. The pipe was fitted between the walls when $T = 60°\text{F}$.

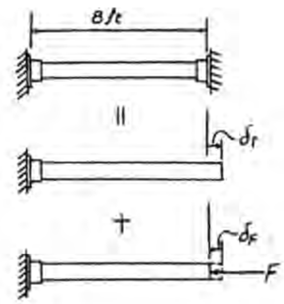


Temperature Gradient:



$$T(x) = 60 + \left(\frac{8 - x}{8}\right)140 = 200 - 17.5x$$

Compatibility:



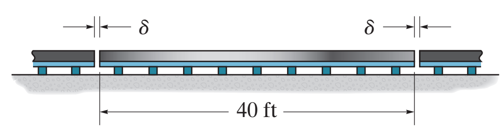
$$0 = \delta_T - \delta_F \qquad \text{Where} \qquad \delta_T = \int \alpha \Delta T\, dx$$

$$0 = 9.60\left(10^{-6}\right)\int_0^{2\text{ft}} [(200 - 17.5x) - 60]\, dx - \frac{F(8)}{\frac{\pi}{4}(1.4^2 - 1^2)15.0(10^3)}$$

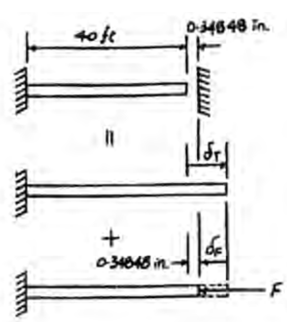
$$0 = 9.60\left(10^{-6}\right)\int_0^{2\text{ft}} (140 - 17.5x)\, dx - \frac{F(8)}{\frac{\pi}{4}(1.4^2 - 1^2)\,15.0(10^3)}$$

$F = 7.60$ kip **Ans.**

4–75. The 40-ft-long A-36 steel rails on a train track are laid with a small gap between them to allow for thermal expansion. Determine the required gap δ so that the rails just touch one another when the temperature is increased from $T_1 = -20°\text{F}$ to $T_2 = 90°\text{F}$. Using this gap, what would be the axial force in the rails if the temperature were to rise to $T_3 = 110°\text{F}$? The cross-sectional area of each rail is 5.10 in^2.



Thermal Expansion: Note that since adjacent rails expand, each rail will be required to expand $\frac{\delta}{2}$ on each end, or δ for the entine rail.



$$\delta = \alpha \Delta T L = 6.60(10^{-6})[90 - (-20)](40)(12)$$

$$= 0.34848 \text{ in.} = 0.348 \text{ in.}$$ **Ans.**

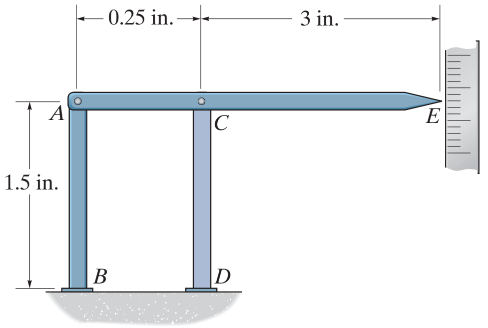
Compatibility:

$$(\overset{+}{\rightarrow}) \qquad 0.34848 = \delta_T - \delta_F$$

$$0.34848 = 6.60(10^{-6})[110 - (-20)](40)(12) - \frac{F(40)(12)}{5.10(29.0)(10^3)}$$

$F = 19.5$ kip **Ans.**

***4–76.** The device is used to measure a change in temperature. Bars AB and CD are made of A-36 steel and 2014-T6 aluminum alloy respectively. When the temperature is at 75°F, ACE is in the horizontal position. Determine the vertical displacement of the pointer at E when the temperature rises to 150°F.



Thermal Expansion:

$$\left(\delta_T\right)_{CD} = \alpha_{al}\Delta T L_{CD} = 12.8(10^{-6})(150 - 75)(1.5) = 1.44(10^{-3})\text{ in.}$$

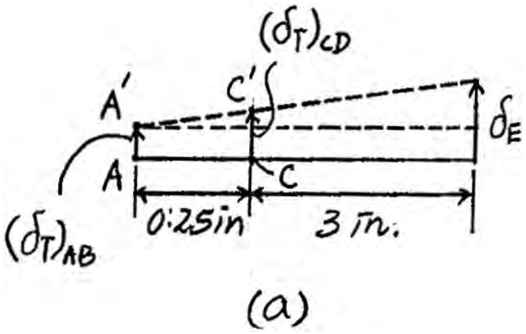
$$\left(\delta_T\right)_{AB} = \alpha_{st}\Delta T L_{AB} = 6.60(10^{-6})(150 - 75)(1.5) = 0.7425(10^{-3})\text{ in.}$$

From the geometry of the deflected bar AE shown Fig. b,

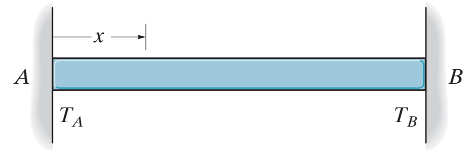
$$\delta_E = \left(\delta_T\right)_{AB} + \left[\frac{\left(\delta_T\right)_{CD} - \left(\delta_T\right)_{AB}}{0.25}\right](3.25)$$

$$= 0.7425(10^{-3}) + \left[\frac{1.44(10^{-3}) - 0.7425(10^{-3})}{0.25}\right](3.25)$$

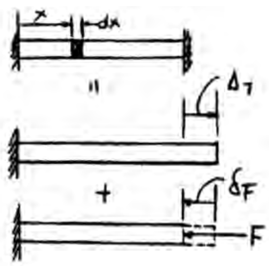
$$= 0.00981\text{ in.}$$ **Ans.**



•4–77. The bar has a cross-sectional area A, length L, modulus of elasticity E, and coefficient of thermal expansion α. The temperature of the bar changes uniformly along its length from T_A at A to T_B at B so that at any point x along the bar $T = T_A + x(T_B - T_A)/L$. Determine the force the bar exerts on the rigid walls. Initially no axial force is in the bar and the bar has a temperature of T_A.



$$\xrightarrow{+} \quad 0 = \Delta_T - \delta_F \qquad \textbf{(1)}$$



However,

$$d\Delta_T = \alpha\Delta_T\,dx = \alpha\left(T_A + \frac{T_B - T_A}{L}x - T_A\right)dx$$

$$\Delta_T = \alpha\int_0^L \frac{T_B - T_A}{L}x\,dx = \alpha\left[\frac{T_B - T_A}{2L}x^2\right]\Bigg|_0^L$$

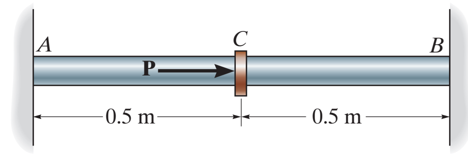
$$= \alpha\left[\frac{T_B - T_A}{2}L\right] = \frac{\alpha L}{2}(T_B - T_A)$$

From Eq.(1).

$$0 = \frac{\alpha L}{2}(T_B - T_A) - \frac{FL}{AE}$$

$$F = \frac{\alpha AE}{2}(T_B - T_A)$$ **Ans.**

4–78. The A-36 steel rod has a diameter of 50 mm and is lightly attached to the rigid supports at A and B when $T_1 = 80°\text{C}$. If the temperature becomes $T_2 = 20°\text{C}$ and an axial force of $P = 200$ kN is applied to its center, determine the reactions at A and B.



Referring to the *FBD* of the rod, Fig. *a*

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad F_B - F_A + 200(10^3) = 0 \qquad \textbf{(1)}$$

When the rod is unconstrained at B, it has a free contraction of $\delta_T = \alpha_{st}\,\Delta T L = 12(10^{-6})(80 - 20)(1000) = 0.72$ mm. Also, under force $\mathbf{P}$ and F_B with unconstrained at B, the deformation of the rod are

$$\delta_P = \frac{PL_{AC}}{AE} = \frac{200(10^3)(500)}{\frac{\pi}{4}(0.05^2)\left[200(10^9)\right]} = 0.2546 \text{ mm}$$

$$\delta_{F_B} = \frac{F_B\,L_{AB}}{AE} = \frac{F_B\,(1000)}{\frac{\pi}{4}(0.05^2)\left[200(10^9)\right]} = 2.5465(10^{-6})\,F_B$$

Using the method of super position, Fig. *b*,

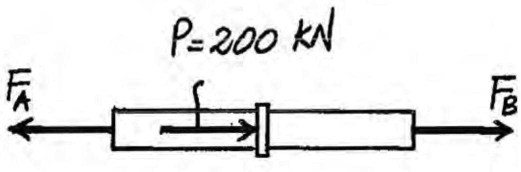
$$\left(\overset{+}{\rightarrow}\right) \qquad 0 = -\delta_T + \delta_P + \delta_{F_B}$$

$$0 = -0.72 + 0.2546 + 2.5465(10^{-6})\,F_B$$

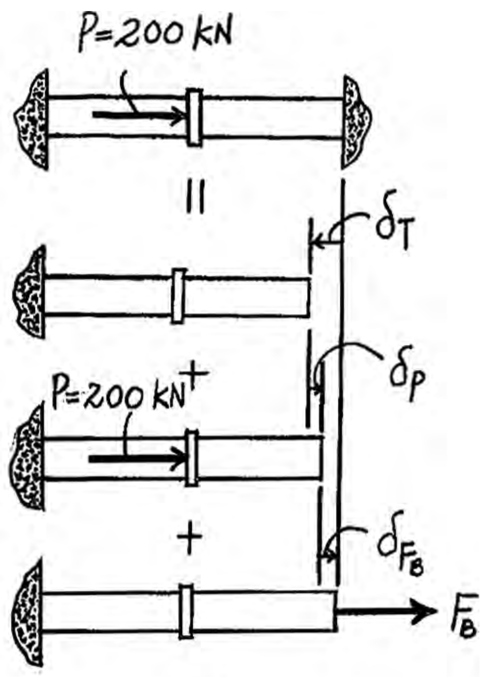
$$F_B = 182.74(10^3)\text{ N} = 183 \text{ kN} \qquad \textbf{Ans.}$$

Substitute the result of $\mathbf{F}_B$ into Eq (1),

$$F_A = 382.74(10^3)\text{ N} = 383 \text{ kN} \qquad \textbf{Ans.}$$

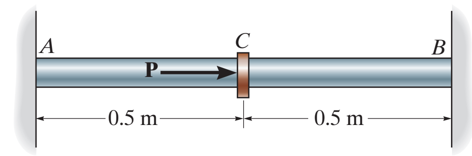


(a)



(b)

4–79. The A-36 steel rod has a diameter of 50 mm and is lightly attached to the rigid supports at A and B when $T_1 = 50°\text{C}$. Determine the force P that must be applied to the collar at its midpoint so that, when $T_2 = 30°\text{C}$, the reaction at B is zero.



When the rod is unconstrained at B, it has a free contraction of $\delta_T = \alpha_{st}\,\Delta T L = 12(10^{-6})(50 - 30)(1000) = 0.24\text{ mm}$. Also, under force **P** with unconstrained at B, the deformation of the rod is

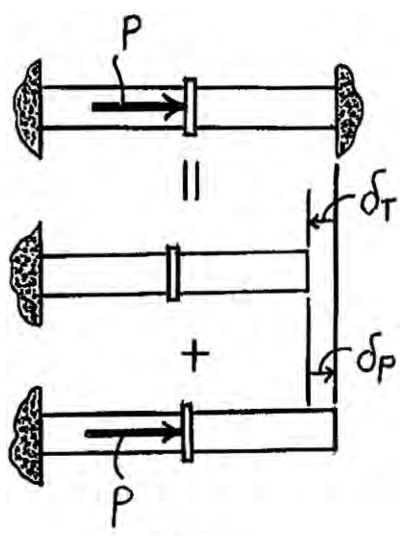
$$\delta_P = \frac{PL_{AC}}{AE} = \frac{P(500)}{\frac{\pi}{4}(0.05^2)\left[200(10^9)\right]} = 1.2732(10^{-6})\,P$$

Since $\mathbf{F}_B$ is required to be zero, the method of superposition, Fig. b, gives

$$(\overset{+}{\rightarrow}) \qquad 0 = -\delta_T + \delta_P$$

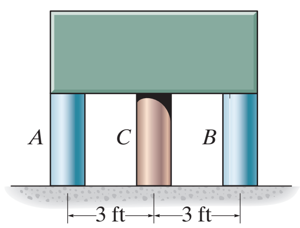
$$0 = -0.24 + 1.2732(10^{-6})P$$

$$P = 188.50(10^3)\text{ N} = 188\text{ kN}$$ **Ans.**



(a)

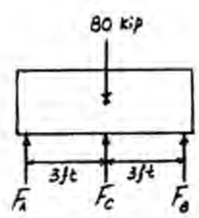
*__4–80.__ The rigid block has a weight of 80 kip and is to be supported by posts A and B, which are made of A-36 steel, and the post C, which is made of C83400 red brass. If all the posts have the same original length before they are loaded, determine the average normal stress developed in each post when post C is heated so that its temperature is increased by 20°F. Each post has a cross-sectional area of 8 in^2.



Equations of Equilibrium:

$$\circlearrowleft + \Sigma M_C = 0; \qquad F_B(3) - F_A(3) = 0 \qquad F_A = F_B = F$$

$$+\uparrow \Sigma F_y = 0; \qquad 2F + F_C - 80 = 0 \qquad \textbf{[1]}$$

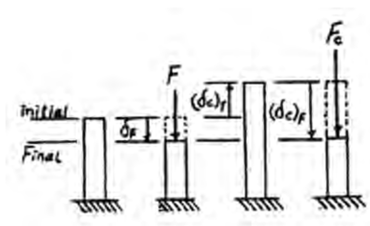


Compatibility:

$$(+\downarrow) \qquad (\delta_C)_F - (\delta_C)_T = \delta_F$$

$$\frac{F_C L}{8(14.6)(10^3)} - 9.80\left(10^{-5}\right)(20)L = \frac{FL}{8(29.0)(10^3)}$$

$$8.5616\,F_C - 4.3103\,F = 196 \qquad \textbf{[2]}$$



Solving Eqs. [1] and [2] yields:

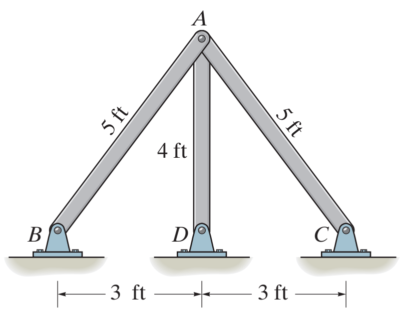
$$F = 22.81\text{ kip} \qquad F_C = 34.38\text{ kip}$$

average Normal Sress:

$$\sigma_A = \sigma_B = \frac{F}{A} = \frac{22.81}{8} = 2.85\text{ ksi}$$ **Ans.**

$$\sigma_C = \frac{F_C}{A} = \frac{34.38}{8} = 4.30\text{ ksi}$$ **Ans.**

•4–81. The three bars are made of A-36 steel and form a pin-connected truss. If the truss is constructed when $T_1 = 50°\text{F}$, determine the force in each bar when $T_2 = 110°\text{F}$. Each bar has a cross-sectional area of 2 in^2.



$$(\delta_T{}')_{AB} - (\delta_F{}')_{AB} = (\delta_T)_{AD} + (\delta_F)_{AD} \qquad \textbf{(1)}$$

However, $\delta_{AB} = \delta'_{AB}\cos\theta$;

$$\delta'_{AB} = \frac{\delta_{AB}}{\cos\theta} = \frac{5}{4}\delta_{AB}$$

Substitute into Eq. (1)

$$\frac{5}{4}(\delta_T)_{AB} - \frac{5}{4}(\delta_F)_{AB} = (\delta_T)_{AD} + (\delta_F)_{AD}$$

$$\frac{5}{4}\left[6.60(10^{-6})(110° - 50°)(5)(12) - \frac{F_{AB}(5)(12)}{2(29)(10^3)}\right]$$

$$= 6.60(10^{-6})(110° - 50°)(4)(12) + \frac{F_{AD}(4)(12)}{2(29)(10^3)}$$

$$620.136 = 75F_{AB} + 48F_{AD} \qquad \textbf{(2)}$$

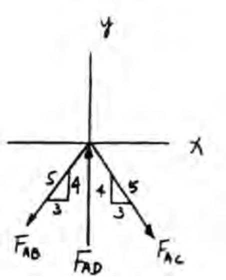
$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad \frac{3}{5}F_{AC} - \frac{3}{5}F_{AB} = 0; \qquad F_{AC} = F_{AB}$$

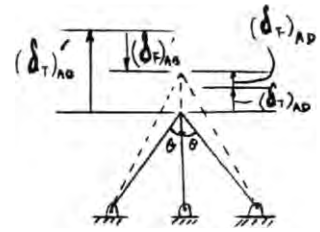
$$+\uparrow\Sigma F_y = 0; \qquad F_{AD} - 2\left(\frac{4}{5}F_{AB}\right) = 0 \qquad \textbf{(3)}$$

Solving Eqs. (2) and (3) yields :

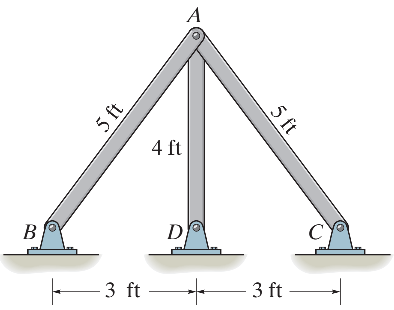
$F_{AD} = 6.54$ kip **Ans.**

$F_{AC} = F_{AB} = 4.09$ kip **Ans.**





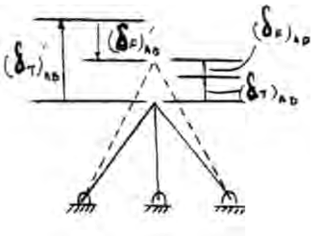
4–82. The three bars are made of A-36 steel and form a pin-connected truss. If the truss is constructed when $T_1 = 50°\text{F}$, determine the vertical displacement of joint A when $T_2 = 150°\text{F}$. Each bar has a cross-sectional area of 2 in^2.



$$(\delta_T{}')_{AB} - (\delta_F{}')_{AB} = (\delta_T)_{AD} + (\delta_F)_{AD} \quad \textbf{(1)}$$

However, $\delta_{AB} = \delta'_{AB}\cos\theta$;

$$\delta'_{AB} = \frac{\delta_{AB}}{\cos\theta} = \frac{5}{4}\delta_{AB}$$



Substitute into Eq. (1)

$$\frac{5}{4}(\delta_T)_{AB} - \frac{5}{4}(\delta_T)_{AB} = (\delta_T)_{AD} + (\delta_F)_{AD}$$

$$\frac{5}{4}\left[6.60(10^{-6})(150° - 50°)(5)(12) - \frac{F_{AB}(5)(12)}{2(29)(10^3)}\right]$$

$$= 6.60(10^{-6})(150° - 50°)(4)(12) + \frac{F_{AD}(4)(12)}{2(29)(10^3)}$$

$$239.25 - 6.25F_{AB} = 153.12 + 4\,F_{AD}$$

$$4\,F_{AD} + 6.25F_{AB} = 86.13 \quad \textbf{(2)}$$

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad \frac{3}{5}F_{AC} - \frac{3}{5}F_{AB} = 0; \qquad F_{AC} = F_{AB}$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{AD} - 2\left(\frac{4}{5}F_{AB}\right) = 0;$$

$$F_{AD} = 1.6F_{AB} \quad \textbf{(3)}$$

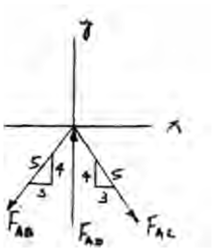
Solving Eqs. (2) and (3) yields:

$F_{AB} = 6.8086$ kip: $\qquad F_{AD} = 10.8939$ kip

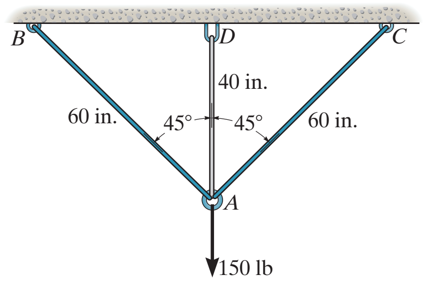
$$(\delta_A)_r = (\delta_T)_{AD} + (\delta_T)_{AD}$$

$$= 6.60(10^{-6})(150° - 50°)(4)(12) + \frac{10.8939(4)(12)}{2(29)(10^3)}$$

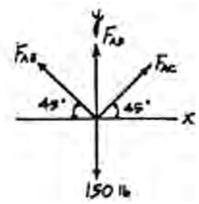
$$= 0.0407 \text{ in.} \uparrow \qquad \textbf{Ans.}$$



4–83. The wires AB and AC are made of steel, and wire AD is made of copper. Before the 150-lb force is applied, AB and AC are each 60 in. long and AD is 40 in. long. If the temperature is increased by 80°F, determine the force in each wire needed to support the load. Take $E_{st} = 29(10^3)$ ksi, $E_{cu} = 17(10^3)$ ksi, $\alpha_{st} = 8(10^{-6})/°\text{F}$, $\alpha_{cu} = 9.60(10^{-6})/°\text{F}$. Each wire has a cross-sectional area of $0.0123\ \text{in}^2$.



Equations of Equilibrium:



$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{AC}\cos 45° - F_{AB}\cos 45° = 0$$

$$F_{AC} = F_{AB} = F$$

$$+\uparrow \Sigma F_y = 0; \qquad 2F\sin 45° + F_{AD} - 150 = 0 \qquad \textbf{[1]}$$

Compatibility:

$$(\delta_{AC})_T = 8.0\left(10^{-6}\right)(80)(60) = 0.03840\ \text{in.}$$

$$(\delta_{AC})_{T_r} = \frac{(\delta_{AC})_T}{\cos 45°} = \frac{0.03840}{\cos 45°} = 0.05431\ \text{in.}$$

$$(\delta_{AD})_T = 9.60\left(10^{-6}\right)(80)(40) = 0.03072\ \text{in.}$$

$$\delta_0 = (\delta_{AC})_{T_r} - (\delta_{AD})_T = 0.05431 - 0.03072 = 0.02359\ \text{in.}$$

$$(\delta_{AD})_F = (\delta_{AC})_{F_r} + \delta_0$$

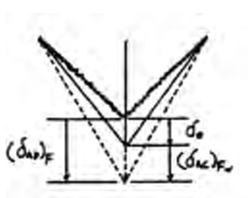
$$\frac{F_{AD}(40)}{0.0123(17.0)(10^6)} = \frac{F(60)}{0.0123(29.0)(10^6)\cos 45°} + 0.02359$$

$$0.1913F_{AD} - 0.2379F = 23.5858 \qquad \textbf{[2]}$$

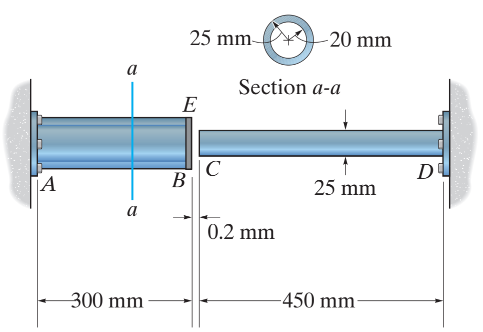
Solving Eq. [1] and [2] yields:

$F_{AC} = F_{AB} = F = 10.0\ \text{lb}$ **Ans.**

$F_{AD} = 136\ \text{lb}$ **Ans.**



***4–84.** The AM1004-T61 magnesium alloy tube AB is capped with a rigid plate E. The gap between E and end C of the 6061-T6 aluminum alloy solid circular rod CD is 0.2 mm when the temperature is at 30° C. Determine the normal stress developed in the tube and the rod if the temperature rises to 80° C. Neglect the thickness of the rigid cap.



Compatibility Equation: If tube AB and rod CD are unconstrained, they will have a free expansion of $\left(\delta_T\right)_{AB} = \alpha_{mg}\,\Delta T L_{AB} = 26(10^{-6})(80 - 30)(300) = 0.39$ mm and $\left(\delta_T\right)_{CD} = \alpha_{al}\Delta T L_{CD} = 24(10^{-6})(80 - 30)(450) = 0.54$ mm. Referring to the deformation diagram of the tube and the rod shown in Fig. a,

$$\delta = \left[\left(\delta_T\right)_{AB} - \left(\delta_F\right)_{AB}\right] + \left[\left(\delta_T\right)_{CD} - \left(\delta_F\right)_{CD}\right]$$

$$0.2 = \left[0.39 - \frac{F(300)}{\pi\left(0.025^2 - 0.02^2\right)(44.7)(10^9)}\right] + \left[0.54 - \frac{F(450)}{\frac{\pi}{4}\left(0.025^2\right)(68.9)(10^9)}\right]$$

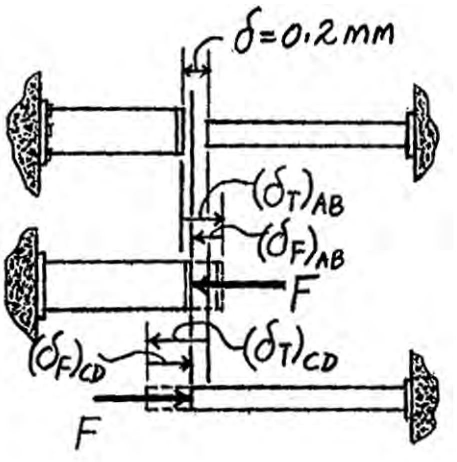
$$F = 32\,017.60 \text{ N}$$

Normal Stress:

$$\sigma_{AB} = \frac{F}{A_{AB}} = \frac{32\,017.60}{\pi\left(0.025^2 - 0.02^2\right)} = 45.3 \text{ MPa} \qquad \textbf{Ans.}$$

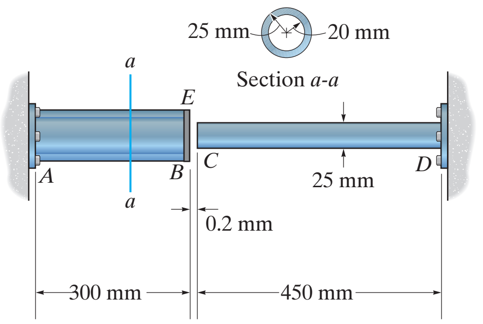
$$\sigma_{CD} = \frac{F}{A_{CD}} = \frac{32\,017.60}{\frac{\pi}{4}\left(0.025^2\right)} = 65.2 \text{ MPa} \qquad \textbf{Ans.}$$

$$F = 107\,442.47 \text{ N}$$



(a)

•4–85. The AM1004-T61 magnesium alloy tube AB is capped with a rigid plate. The gap between E and end C of the 6061-T6 aluminum alloy solid circular rod CD is 0.2 mm when the temperature is at 30° C. Determine the highest temperature to which it can be raised without causing yielding either in the tube or the rod. Neglect the thickness of the rigid cap.



Then

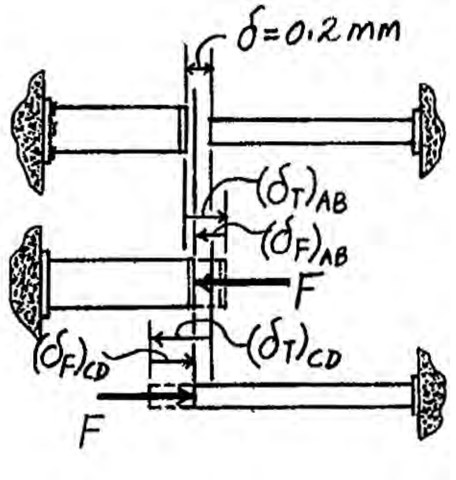
$$\sigma_{CD} = \frac{F}{A_{CD}} = \frac{107\,442.47}{\frac{\pi}{4}\left(0.025^2\right)} = 218.88\text{MPa} < (\sigma_Y)_{al} \qquad \textbf{(O.K.!)}$$

Compatibility Equation: If tube AB and rod CD are unconstrained, they will have a free expansion of $\left(\delta_T\right)AB = \alpha_{mg}\,\Delta T L_{AB} = 26(10^{-6})(T - 30)(300) = 7.8(10^{-6})(T - 30)$ and $\left(\delta_T\right)_{CD} = \alpha_{al}\,\Delta T L_{CD} = 24(10^{-6})(T - 30)(450) = 0.0108(T - 30)$. Referring to the deformation diagram of the tube and the rod shown in Fig. *a*,

$$\delta = \left[\left(\delta_T\right)_{AB} - \left(\delta_F\right)_{AB}\right] + \left[\left(\delta_T\right)_{CD} - \left(\delta_F\right)_{CD}\right]$$

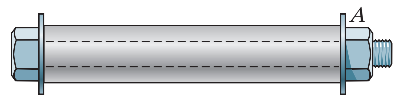
$$0.2 = \left[7.8(10^{-3})(T - 30) - \frac{107\,442.47(300)}{\pi\left(0.025^2 - 0.02^2\right)(44.7)(10^9)}\right] + \left[0.0108(T - 30) - \frac{107\,442.47(450)}{\frac{\pi}{4}\left(0.025^2\right)(68.9)(10^9)}\right]$$

$$T = 172°\text{ C} \qquad \textbf{Ans.}$$



(a)

4–86. The steel bolt has a diameter of 7 mm and fits through an aluminum sleeve as shown. The sleeve has an inner diameter of 8 mm and an outer diameter of 10 mm. The nut at A is adjusted so that it just presses up against the sleeve. If the assembly is originally at a temperature of $T_1 = 20°C$ and then is heated to a temperature of $T_2 = 100°C$, determine the average normal stress in the bolt and the sleeve. $E_{st} = 200$ GPa, $E_{al} = 70$ GPa, $\alpha_{st} = 14(10^{-6})/°C$, $\alpha_{al} = 23(10^{-6})/°C$.



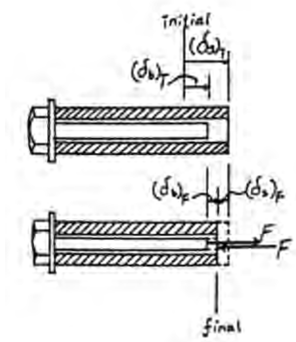
Compatibility:

$$(\delta_s)_T - (\delta_b)_T = (\delta_s)_F + (\delta_b)_F$$

$$23(10^{-6})(100 - 20)L - 14(10^{-6})(100 - 20)L$$

$$= \frac{FL}{\frac{\pi}{4}(0.01^2 - 0.008^2)70(10^9)} + \frac{FL}{\frac{\pi}{4}(0.007^2)200(10^9)}$$

$$F = 1133.54 \text{ N}$$

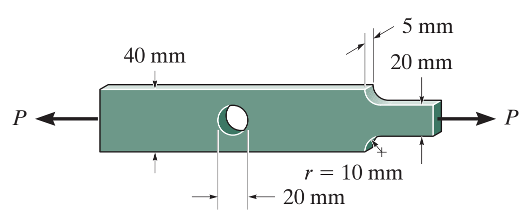


Average Normal Stress:

$$\sigma_s = \frac{F}{A_s} = \frac{1133.54}{\frac{\pi}{4}(0.01^2 - 0.008^2)} = 40.1 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_b = \frac{F}{A_b} = \frac{1133.54}{\frac{\pi}{4}(0.007^2)} = 29.5 \text{ MPa} \qquad \textbf{Ans.}$$

4–87. Determine the maximum normal stress developed in the bar when it is subjected to a tension of $P = 8$ kN.



For the fillet:

$$\frac{w}{h} = \frac{40}{20} = 2 \qquad \frac{r}{h} = \frac{10}{20} = 0.5$$

From Fig. 10-24. $K = 1.4$

$$\sigma_{max} = K\sigma_{avg}$$

$$= 1.4\left(\frac{8\,(10^3)}{0.02\,(0.005)}\right)$$

$$= 112 \text{ MPa}$$

For the hole:

$$\frac{r}{w} = \frac{10}{40} = 0.25$$

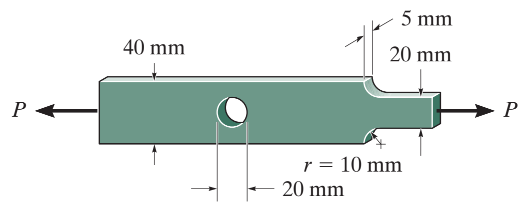
From Fig. 4-25. $K = 2.375$

$$\sigma_{max} = K\sigma_{avg}$$

$$= 2.375\left(\frac{8\,(10^3)}{(0.04 - 0.02)(0.005)}\right)$$

$$= 190 \text{ MPa} \qquad \textbf{Ans.}$$

***4–88.** If the allowable normal stress for the bar is $\sigma_{\text{allow}} = 120$ MPa, determine the maximum axial force P that can be applied to the bar.



Assume failure of the fillet.

$$\frac{w}{h} = \frac{40}{20} = 2; \qquad \frac{r}{h} = \frac{10}{20} = 0.5$$

From Fig. 4-24. $K = 1.4$

$$\sigma_{\text{allow}} = \sigma_{\text{max}} = K\sigma_{\text{avg}}$$

$$120(10^6) = 1.4\left(\frac{P}{0.02(0.005)}\right)$$

$$P = 8.57 \text{ kN}$$

Assume failure of the hole.

$$\frac{r}{w} = \frac{10}{20} = 0.25$$

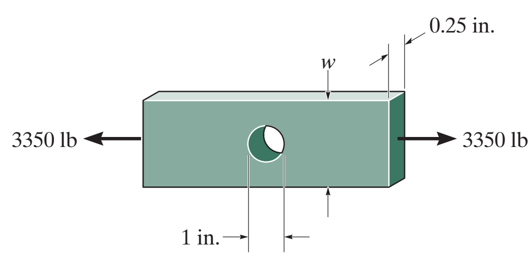
From Fig. 4-25. $K = 2.375$

$$\sigma_{\text{allow}} = \sigma_{\text{max}} = K\sigma_{\text{avg}}$$

$$120(10^4) = 2.375\left(\frac{P}{(0.04 - 0.02)(0.005)}\right)$$

$$P = 5.05 \text{ kN (controls)}$$ **Ans.**

•4–89. The member is to be made from a steel plate that is 0.25 in. thick. If a 1-in. hole is drilled through its center, determine the approximate width w of the plate so that it can support an axial force of 3350 lb. The allowable stress is $\sigma_{\text{allow}} = 22$ ksi.



$$\sigma_{\text{allow}} = \sigma_{\text{max}} = K\sigma_{\text{avg}}$$

$$22 = K\left[\frac{3.35}{(w - 1)(0.25)}\right]$$

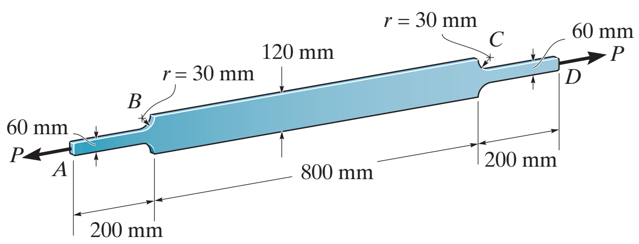
$$w = \frac{3.35K + 5.5}{5.5}$$

By trial and error, from Fig. 4-25, choose $\frac{r}{w} = 0.2; \qquad K = 2.45$

$$w = \frac{3.35(2.45) + 5.5}{5.5} = 2.49 \text{ in.}$$ **Ans.**

Since $\frac{r}{w} = \frac{0.5}{2.49} = 0.2$ OK

4–90. The A-36 steel plate has a thickness of 12 mm. If there are shoulder fillets at B and C, and $\sigma_{allow} = 150$ MPa, determine the maximum axial load P that it can support. Calculate its elongation, neglecting the effect of the fillets.



Maximum Normal Stress at fillet:

$$\frac{r}{h} = \frac{30}{60} = 0.5 \qquad \text{and} \qquad \frac{w}{h} = \frac{120}{60} = 2$$

From the text, $K = 1.4$

$$\sigma_{max} = \sigma_{allow} = K\sigma_{avg}$$

$$150(10^6) = 1.4\left[\frac{P}{0.06(0.012)}\right]$$

$$P = 77142.86 \text{ N} = 77.1 \text{ kN} \qquad \textbf{Ans.}$$

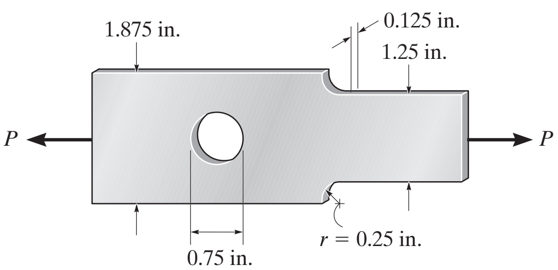
Displacement:

$$\delta = \Sigma\frac{PL}{AE}$$

$$= \frac{77142.86(400)}{(0.06)(0.012)(200)(10^9)} + \frac{77142.86(800)}{(0.12)(0.012)(200)(10^9)}$$

$$= 0.429 \text{ mm} \qquad \textbf{Ans.}$$

4–91. Determine the maximum axial force P that can be applied to the bar. The bar is made from steel and has an allowable stress of $\sigma_{allow} = 21$ ksi.



Assume failure of the fillet.

$$\frac{r}{h} = \frac{0.25}{1.25} = 0.2 \qquad \frac{w}{h} = \frac{1.875}{1.25} = 1.5$$

From Fig. 4-24, $K = 1.73$

$$\sigma_{allow} = \sigma_{max} = K\sigma_{avg}$$

$$21 = 1.73\left(\frac{P}{1.25\,(0.125)}\right)$$

$$P = 1.897 \text{ kip}$$

Assume failure of the hole.

$$\frac{r}{w} = \frac{0.375}{1.875} = 0.20$$

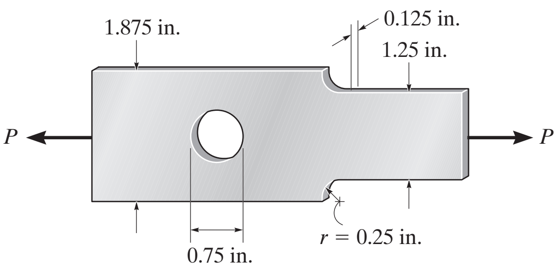
From Fig. 4-25, $K = 2.45$

$$\sigma_{allow} = \sigma_{max} = K\sigma_{avg}$$

$$21 = 2.45\left(\frac{P}{(1.875 - 0.75)(0.125)}\right)$$

$$P = 1.21 \text{ kip} \quad \text{(controls)} \qquad \textbf{Ans.}$$

***4–92.** Determine the maximum normal stress developed in the bar when it is subjected to a tension of $P = 2$ kip.



At fillet:

$$\frac{r}{h} = \frac{0.25}{1.25} = 0.2 \qquad \frac{w}{h} = \frac{1.875}{1.25} = 1.5$$

From Fig. 4-24, $K = 1.73$

$$\sigma_{\max} = K\left(\frac{P}{A}\right) = 1.73\left[\frac{2}{1.25(0.125)}\right] = 22.1 \text{ ksi}$$

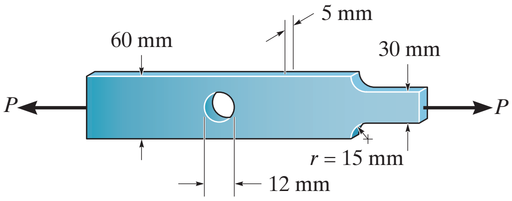
At hole:

$$\frac{r}{w} = \frac{0.375}{1.875} = 0.20$$

From Fig. 4-25, $K = 2.45$

$$\sigma_{\max} = 2.45\left[\frac{2}{(1.875 - 0.75)(0.125)}\right] = 34.8 \text{ ksi} \quad \text{(Controls)}$$ **Ans.**

•4–93. Determine the maximum normal stress developed in the bar when it is subjected to a tension of $P = 8$ kN.



Maximum Normal Stress at fillet:

$$\frac{r}{h} = \frac{15}{30} = 0.5 \quad \text{and} \quad \frac{w}{h} = \frac{60}{30} = 2$$

From the text, $K = 1.4$

$$\sigma_{\max} = K\sigma_{\text{avg}} = K\frac{P}{h\,t}$$

$$= 1.4\left[\frac{8(10^3)}{(0.03)(0.005)}\right] = 74.7 \text{ MPa}$$

Maximum Normal Stress at the hole:

$$\frac{r}{w} = \frac{6}{60} = 0.1$$

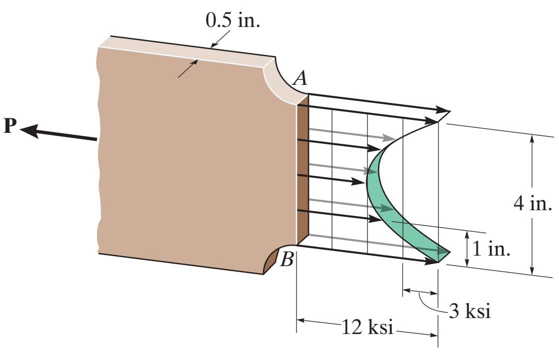
From the text, $K = 2.65$

$$\sigma_{\max} = K\,\sigma_{\text{avg}} = K\frac{P}{(w - 2r)\,t}$$

$$= 2.65\left[\frac{8(10^3)}{(0.06 - 0.012)(0.005)}\right]$$

$$= 88.3 \text{ MPa} \quad (\textit{Controls})$$ **Ans.**

4–94. The resulting stress distribution along section AB for the bar is shown. From this distribution, determine the approximate resultant axial force P applied to the bar. Also, what is the stress-concentration factor for this geometry?



$$P = \int \sigma dA = \text{Volume under curve}$$

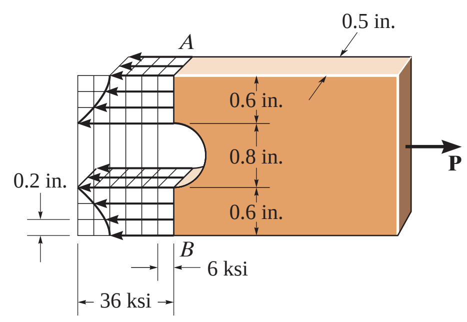
Number of squares = 10

$$P = 10(3)(1)(0.5) = 15 \text{ kip}$$ **Ans.**

$$\sigma_{avg} = \frac{P}{A} = \frac{15 \text{ kip}}{(4 \text{ in.})(0.5 \text{ in.})} = 7.5 \text{ ksi}$$

$$K = \frac{\sigma_{max}}{\sigma_{avg}} = \frac{12 \text{ ksi}}{7.5 \text{ ksi}} = 1.60$$ **Ans.**

4–95. The resulting stress distribution along section AB for the bar is shown. From this distribution, determine the approximate resultant axial force P applied to the bar. Also, what is the stress-concentration factor for this geometry?



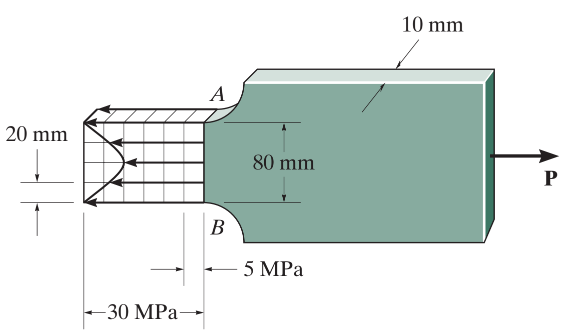
Number of squares = 28

$$P = 28(6)(0.2)(0.5) = 16.8 \text{ kip}$$ **Ans.**

$$\sigma_{avg} = \frac{P}{A} = \frac{16.8}{2(0.6)(0.5)} = 28 \text{ ksi}$$

$$K = \frac{\sigma_{max}}{\sigma_{avg}} = \frac{36}{28} = 1.29$$ **Ans.**

***4–96.** The resulting stress distribution along section AB for the bar is shown. From this distribution, determine the approximate resultant axial force P applied to the bar. Also, what is the stress-concentration factor for this geometry?



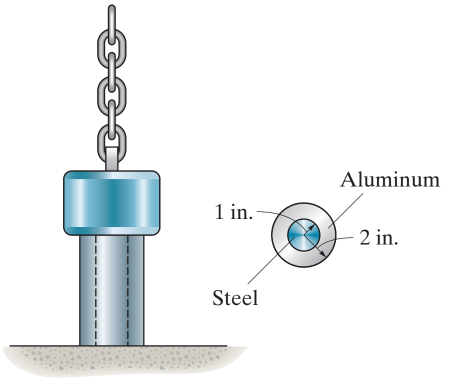
Number of squares = 19

$$P = 19(5)(10^6)(0.02)(0.01) = 19 \text{ kN}$$ **Ans.**

$$\sigma_{avg} = \frac{P}{A} = \frac{19(10^3)}{0.08(0.01)} = 23.75 \text{ MPa}$$

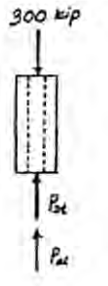
$$K = \frac{\sigma_{max}}{\sigma_{avg}} = \frac{30 \text{ MPa}}{23.75 \text{ MPa}} = 1.26$$ **Ans.**

•4–97. The 300-kip weight is slowly set on the top of a post made of 2014-T6 aluminum with an A-36 steel core. If both materials can be considered elastic perfectly plastic, determine the stress in each material.



Equations of Equilibrium:

$+\uparrow \Sigma F_y = 0; \qquad P_{st} + P_{al} - 300 = 0$ **[1]**



Elastic Analysis: Assume both materials still behave elastically under the load.

$$\delta_{st} = \delta_{al}$$

$$\frac{P_{st} L}{\frac{\pi}{4}(2)^2(29)(10^3)} = \frac{P_{al} L}{\frac{\pi}{4}(4^2 - 2^2)(10.6)(10^3)}$$

$$P_{st} = 0.9119\, P_{al}$$

Solving Eqs. [1] and [2] yields:

$$P_{al} = 156.91 \text{ kip} \qquad P_{st} = 143.09 \text{ kip}$$

Average Normal Stress:

$$\sigma_{al} = \frac{P_{al}}{A_{al}} = \frac{156.91}{\frac{\pi}{4}(4^2 - 2^2)}$$

$$= 16.65 \text{ ksi} < (\sigma_Y)_{al} = 60.0 \text{ ksi} \qquad \textbf{(OK!)}$$

$$\sigma_{st} = \frac{P_{st}}{A_{st}} = \frac{143.09}{\frac{\pi}{4}(2^2)}$$

$$= 45.55 \text{ ksi} > (\sigma_Y)_{st} = 36.0 \text{ ksi}$$

Therefore, the steel core yields and so the elastic analysis is invalid. The stress in the steel is

$$\sigma_{st} = (\sigma_y)_{st} = 36.0 \text{ ksi} \qquad \textbf{Ans.}$$

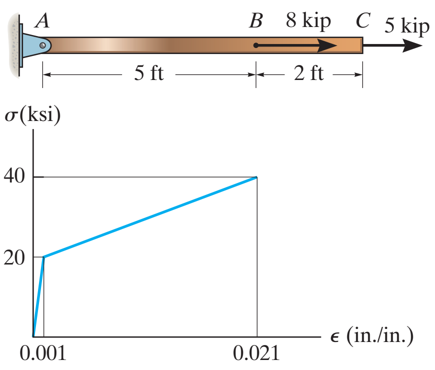
$$P_{st} = (\sigma_Y)_{st} A_{st} = 36.0\left(\frac{\pi}{4}\right)\left(2^2\right) = 113.10 \text{ kip}$$

From Eq. [1] $P_{al} = 186.90$ kip

$$\sigma_{al} = \frac{P_{al}}{A_{al}} = \frac{186.90}{\frac{\pi}{4}(4^2 - 2^2)} = 19.83 \text{ ksi} < (\sigma_Y)_{al} = 60.0 \text{ ksi}$$

Then $\sigma_{al} = 19.8$ ksi **Ans.**

4–98. The bar has a cross-sectional area of 0.5 in^2 and is made of a material that has a stress–strain diagram that can be approximated by the two line segments shown. Determine the elongation of the bar due to the applied loading.



Average Normal Stress and Strain: For segment BC

$$\sigma_{BC} = \frac{P_{BC}}{A_{BC}} = \frac{5}{0.5} = 10.0 \text{ ksi}$$

$$\frac{10.0}{\varepsilon_{BC}} = \frac{20}{0.001}; \qquad \varepsilon_{BC} = \frac{0.001}{20}(10.0) = 0.00050 \text{ in./in.}$$

Average Normal Stress and Strain: For segment AB

$$\sigma_{AB} = \frac{P_{AB}}{A_{AB}} = \frac{13}{0.5} = 26.0 \text{ ksi}$$

$$\frac{26.0 - 20}{\varepsilon_{AB} - 0.001} = \frac{40 - 20}{0.021 - 0.001}$$

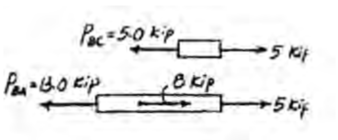
$$\varepsilon_{AB} = 0.0070 \text{ in./in.}$$

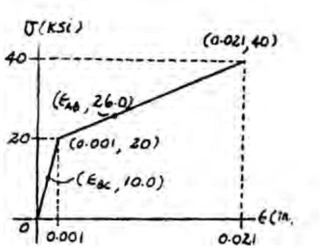
Elongation:

$$\delta_{BC} = \varepsilon_{BC} L_{BC} = 0.00050(2)(12) = 0.0120 \text{ in.}$$

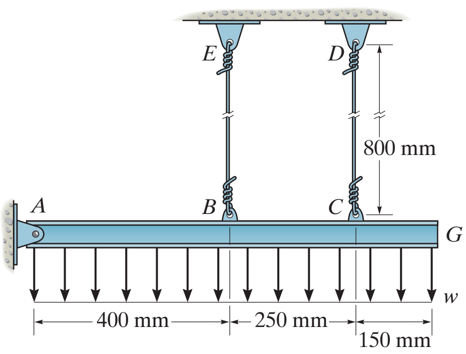
$$\delta_{AB} = \varepsilon_{AB} L_{AB} = 0.0070(5)(12) = 0.420 \text{ in.}$$

$$\delta_{\text{Tot}} = \delta_{BC} + \delta_{AB} = 0.0120 + 0.420 = 0.432 \text{ in.}$$ **Ans.**





4–99. The rigid bar is supported by a pin at A and two steel wires, each having a diameter of 4 mm. If the yield stress for the wires is $\sigma_Y = 530$ MPa, and $E_{st} = 200$ GPa, determine the intensity of the distributed load w that can be placed on the beam and will just cause wire EB to yield. What is the displacement of point G for this case? For the calculation, assume that the steel is elastic perfectly plastic.



Equations of Equilibrium:

$$\circlearrowleft + \Sigma M_A = 0; \qquad F_{BE}(0.4) + F_{CD}(0.65) - 0.8w\,(0.4) = 0$$

$$0.4\,F_{BE} + 0.65F_{CD} = 0.32w \qquad \textbf{[1]}$$

Plastic Analysis: Wire CD will yield first followed by wire BE. When both wires yield

$$F_{BE} = F_{CD} = (\sigma_\gamma)A$$

$$= 530\left(10^6\right)\left(\frac{\pi}{4}\right)\left(0.004^2\right) = 6.660 \text{ kN}$$

Substituting the results into Eq. [1] yields:

$$w = 21.9 \text{ kN/m} \qquad \textbf{Ans.}$$

Displacement: When wire BE achieves yield stress, the corresponding yield strain is

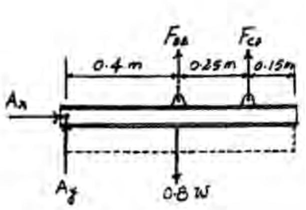
$$\varepsilon_\gamma = \frac{\sigma_\gamma}{E} = \frac{530(10^6)}{200(10^9)} = 0.002650 \text{ mm/mm}$$

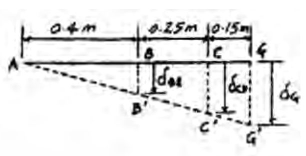
$$\delta_{BE} = \varepsilon_\gamma L_{BE} = 0.002650(800) = 2.120 \text{ mm}$$

From the geometry

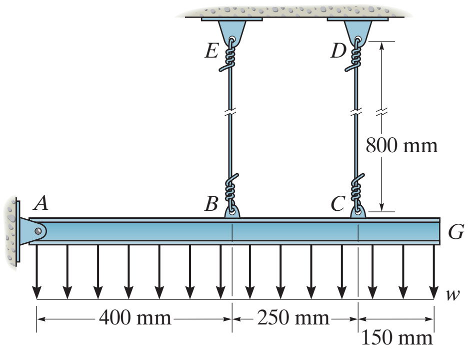
$$\frac{\delta_G}{0.8} = \frac{\delta_{BE}}{0.4}$$

$$\delta_G = 2\delta_{BE} = 2(2.120) = 4.24 \text{ mm} \qquad \textbf{Ans.}$$





*__4–100.__ The rigid bar is supported by a pin at A and two steel wires, each having a diameter of 4 mm. If the yield stress for the wires is $\sigma_Y = 530$ MPa, and $E_{st} = 200$ GPa, determine (a) the intensity of the distributed load w that can be placed on the beam that will cause only one of the wires to start to yield and (b) the smallest intensity of the distributed load that will cause both wires to yield. For the calculation, assume that the steel is elastic perfectly plastic.



Equations of Equilibrium:

$\zeta + \Sigma M_A = 0; \qquad F_{BE}(0.4) + F_{CD}(0.65) - 0.8w\,(0.4) = 0$

$$0.4\,F_{BE} + 0.65\,F_{CD} = 0.32w \qquad \textbf{[1]}$$

(a) By observation, wire CD will yield first.

Then $F_{CD} = \sigma_\gamma A = 530\left(10^6\right)\left(\frac{\pi}{4}\right)\left(0.004^2\right) = 6.660$ kN.

From the geometry

$$\frac{\delta_{BE}}{0.4} = \frac{\delta_{CD}}{0.65}; \qquad \delta_{CD} = 1.625\delta_{BE}$$

$$\frac{F_{CD}L}{AE} = 1.625\frac{F_{BE}L}{AE}$$

$$F_{CD} = 1.625\,F_{BE} \qquad \textbf{[2]}$$

Using $F_{CD} = 6.660$ kN and solving Eqs. [1] and [2] yields:

$$F_{BE} = 4.099 \text{ kN}$$

$$w = 18.7 \text{ kN/m} \qquad \textbf{Ans.}$$

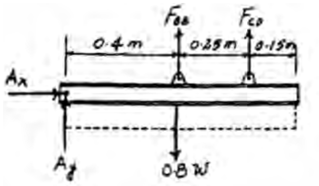
(b) When both wires yield

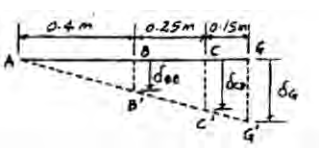
$$F_{BE} = F_{CD} = (\sigma_\gamma)A$$

$$= 530\left(10^6\right)\left(\frac{\pi}{4}\right)\left(0.004^2\right) = 6.660 \text{ kN}$$

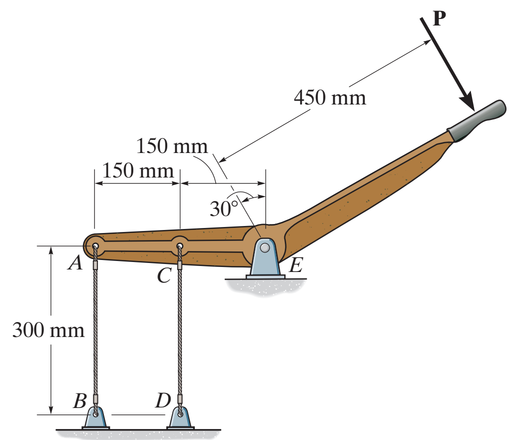
Substituting the results into Eq. [1] yields:

$$w = 21.9 \text{ kN/m} \qquad \textbf{Ans.}$$





•4–101. The rigid lever arm is supported by two A-36 steel wires having the same diameter of 4 mm. If a force of $P = 3$ kN is applied to the handle, determine the force developed in both wires and their corresponding elongations. Consider A-36 steel as an elastic-perfectly plastic material.



Equation of Equilibrium. Refering to the free-body diagram of the lever shown in Fig. *a*,

$$\circlearrowleft + \Sigma M_E = 0; \qquad F_{AB}(300) + F_{CD}(150) - 3\left(10^3\right)(450) = 0$$

$$2F_{AB} + F_{CD} = 9\left(10^3\right) \qquad \textbf{(1)}$$

Elastic Analysis. Assuming that both wires AB and CD behave as linearly elastic, the compatibility equation can be written by referring to the geometry of Fig. *b*.

$$\delta_{AB} = \left(\frac{300}{150}\right)\delta_{CD}$$

$$\delta_{AB} = 2\delta_{CD} \qquad \textbf{(2)}$$

$$\frac{F_{AB}\,L}{AE} = 2\left(\frac{F_{CD}\,L}{AE}\right)$$

$$F_{AB} = 2F_{CD} \qquad \textbf{(3)}$$

Solving Eqs. (1) and (3),

$$F_{CD} = 1800\,\text{N} \qquad F_{AB} = 3600\,\text{N}$$

Normal Stress.

$$\sigma_{CD} = \frac{F_{CD}}{A_{CD}} = \frac{1800}{\frac{\pi}{4}\left(0.004^2\right)} = 143.24\text{ MPa} < (\sigma_Y)_{st} \qquad \textbf{(O.K.)}$$

$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{3600}{\frac{\pi}{4}\left(0.004^2\right)} = 286.48\text{ MPa} > (\sigma_Y)_{st} \qquad \textbf{(N.G.)}$$

Since wire AB yields, the elastic analysis is not valid. The solution must be reworked using

$$F_{AB} = (\sigma_Y)_{st}\,A_{AB} = 250\left(10^6\right)\left[\frac{\pi}{4}\left(0.004^2\right)\right]$$

$$= 3141.59\,\text{N} = 3.14\,\text{kN} \qquad \textbf{Ans.}$$

Substituting this result into Eq. (1),

$$F_{CD} = 2716.81\,\text{N} = 2.72\text{ kN} \qquad \textbf{Ans.}$$

$$\sigma_{CD} = \frac{F_{CD}}{A_{CD}} = \frac{2716.81}{\frac{\pi}{4}\left(0.004^2\right)} = 216.20\text{ MPa} < (\sigma_Y)_{st} \qquad \textbf{(O.K.)}$$

4–101. Continued

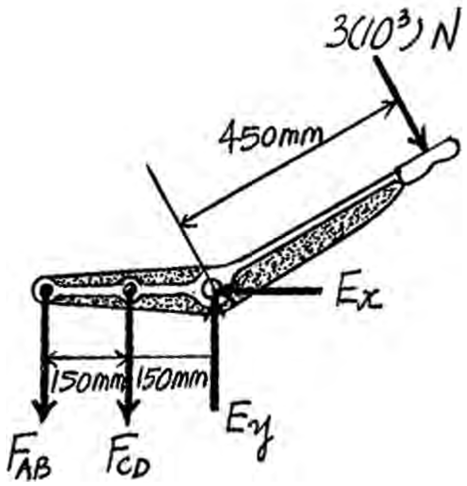
Since wire CD is linearly elastic, its elongation can be determined by

$$\delta_{CD} = \frac{F_{CD}L_{CD}}{A_{CD}E_{st}} = \frac{2716.81(300)}{\frac{\pi}{4}\left(0.004^2\right)(200)\left(10^9\right)}$$

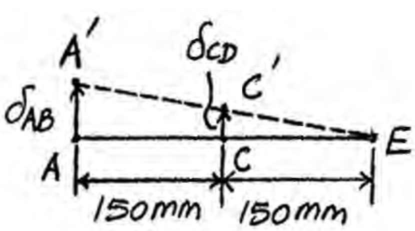
$$= 0.3243 \text{ mm} = 0.324 \text{ mm} \qquad \textbf{Ans.}$$

From Eq. (2),

$$\delta_{AB} = 2\delta_{CD} = 2(0.3243) = 0.649 \text{ mm} \qquad \textbf{Ans.}$$

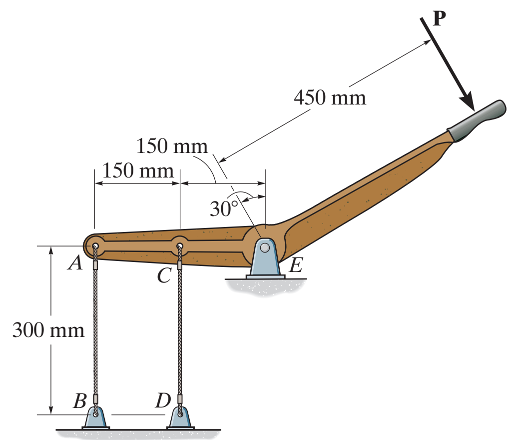


(a)



(b)

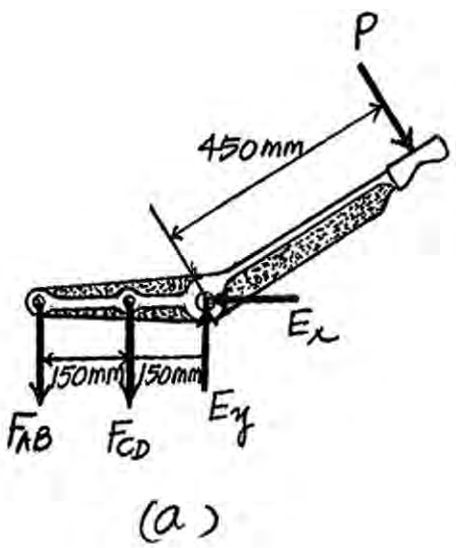
4–102. The rigid lever arm is supported by two A-36 steel wires having the same diameter of 4 mm. Determine the smallest force **P** that will cause (a) only one of the wires to yield; (b) both wires to yield. Consider A-36 steel as an elastic-perfectly plastic material.



Equation of Equilibrium. Refering to the free-body diagram of the lever arm shown in Fig. *a*,

$$\circlearrowleft + \Sigma M_E = 0; \qquad F_{AB}(300) + F_{CD}(150) - P(450) = 0$$

$$2F_{AB} + F_{CD} = 3P \qquad \textbf{(1)}$$



Elastic Analysis. The compatibility equation can be written by referring to the geometry of Fig. *b*.

$$\delta_{AB} = \left(\frac{300}{150}\right)\delta_{CD}$$

$$\delta_{AB} = 2\delta_{CD}$$

$$\frac{F_{AB}L}{AE} = 2\left(\frac{F_{CD}L}{AE}\right)$$

$$F_{CD} = \frac{1}{2}F_{AB} \qquad \textbf{(2)}$$

Assuming that wire AB is about to yield first,

$$F_{AB} = (\sigma_Y)_{st}\,A_{AB} = 250\left(10^6\right)\left[\frac{\pi}{4}\left(0.004^2\right)\right] = 3141.59\,\text{N}$$

From Eq. (2),

$$F_{CD} = \frac{1}{2}(3141.59) = 1570.80\,\text{N}$$

Substituting the result of F_{AB} and F_{CD} into Eq. (1),

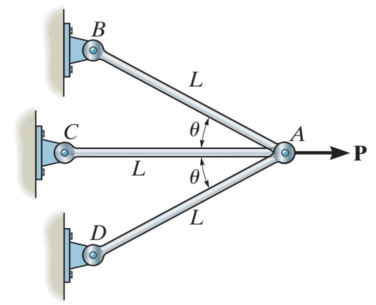
$$P = 2618.00\,\text{N} = 2.62\,\text{kN} \qquad \textbf{Ans.}$$

Plastic Analysis. Since both wires AB and CD are required to yield,

$$F_{AB} = F_{CD} = (\sigma_Y)_{st}\,A = 250\left(10^6\right)\left[\frac{\pi}{4}\left(0.004^2\right)\right] = 3141.59\,\text{N}$$

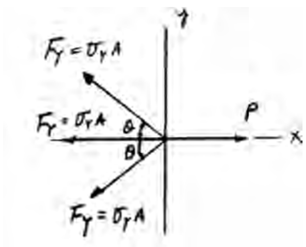
Substituting this result into Eq. (1),

4–103. The three bars are pinned together and subjected to the load **P**. If each bar has a cross-sectional area A, length L, and is made from an elastic perfectly plastic material, for which the yield stress is σ_Y, determine the largest load (ultimate load) that can be supported by the bars, i.e., the load P that causes all the bars to yield. Also, what is the horizontal displacement of point A when the load reaches its ultimate value? The modulus of elasticity is E.



$$P = 3141.59\,\text{N} = 3.14\,\text{kN} \qquad \textbf{Ans.}$$

When all bars yield, the force in each bar is, $F_Y = \sigma_Y A$



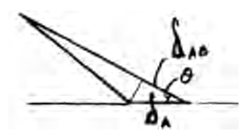
$$\xrightarrow{+} \Sigma F_x = 0; \qquad P - 2\sigma_Y A\cos\theta - \sigma_Y A = 0$$

$$P = \sigma_Y A(2\cos\theta + 1) \qquad \textbf{Ans.}$$

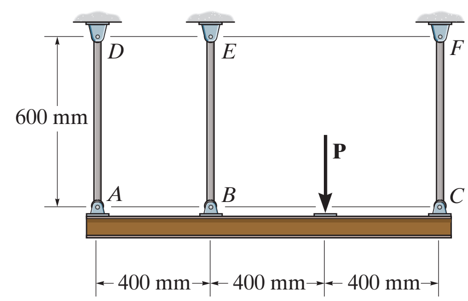
Bar AC will yield first followed by bars AB and AD.

$$\delta_{AB} = \delta_{AD} = \frac{F_Y(L)}{AE} = \frac{\sigma_Y AL}{AE} = \frac{\sigma_Y L}{E}$$

$$\delta_A = \frac{\delta_{AB}}{\cos\theta} = \frac{\sigma_Y L}{E\cos\theta} \qquad \textbf{Ans.}$$



***4–104.** The rigid beam is supported by three 25-mm diameter A-36 steel rods. If the beam supports the force of $P = 230$ kN, determine the force developed in each rod. Consider the steel to be an elastic perfectly-plastic material.



Equation of Equilibrium. Referring to the free-body diagram of the beam shown in Fig. *a*,

$$+\uparrow \Sigma F_y = 0; \qquad F_{AD} + F_{BE} + F_{CF} - 230\left(10^3\right) = 0 \qquad \textbf{(1)}$$

$$\circlearrowleft +\Sigma M_A = 0; \qquad F_{BE}(400) + F_{CF}(1200) - 230\left(10^3\right)(800) = 0$$

$$F_{BE} + 3F_{CF} = 460\left(10^3\right) \qquad \textbf{(2)}$$

Elastic Analysis. Referring to the deflection diagram of the beam shown in Fig. *b*, the compatibility equation can be written as

$$\delta_{BE} = \delta_{AD} + \left(\frac{\delta_{CF} - \delta_{AD}}{1200}\right)(400)$$

$$\delta_{BE} = \frac{2}{3}\delta_{AD} + \frac{1}{3}\delta_{CF}$$

$$\frac{F_{BE}L}{AE} = \frac{2}{3}\left(\frac{F_{CD}L}{AE}\right) + \frac{1}{3}\left(\frac{F_{CF}L}{AE}\right)$$

$$F_{BE} = \frac{2}{3}F_{AD} + \frac{1}{3}F_{CF} \qquad \textbf{(3)}$$

Solving Eqs. (1), (2), and (3)

$F_{CF} = 131\,428.57$ N $\qquad F_{BE} = 65\,714.29$ N $\qquad F_{AD} = 32\,857.14$ N

4–104. Continued

Normal Stress.

$$\sigma_{CF} = \frac{F_{CF}}{A_{CF}} = \frac{131428.57}{\frac{\pi}{4}\left(0.025^2\right)} = 267.74 \text{ MPa} > (\sigma_Y)_{st} \qquad \textbf{(N.G.)}$$

$$\sigma_{BE} = \frac{F_{BE}}{A_{BE}} = \frac{65714.29}{\frac{\pi}{4}\left(0.025^2\right)} = 133.87 \text{ MPa} < (\sigma_Y)_{st} \qquad \textbf{(O.K.)}$$

$$\sigma_{AD} = \frac{F_{AD}}{A_{AD}} = \frac{32857.14}{\frac{\pi}{4}\left(0.025^2\right)} = 66.94 \text{ MPa} < (\sigma_Y)_{st} \qquad \textbf{(O.K.)}$$

Since rod CF yields, the elastic analysis is not valid. The solution must be reworked using

$$F_{CF} = (\sigma_Y)_{st}\, A_{CF} = 250\left(10^6\right)\left[\frac{\pi}{4}\left(0.025^2\right)\right] = 122\,718.46\,\text{N} = 123\,\text{kN} \qquad \textbf{Ans.}$$

Substituting this result into Eq. (2),

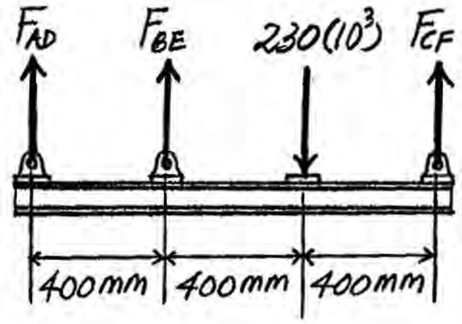
$$F_{BE} = 91844.61 \text{ N} = 91.8 \text{ kN} \qquad \textbf{Ans.}$$

Substituting the result for F_{CF} and F_{BE} into Eq. (1),

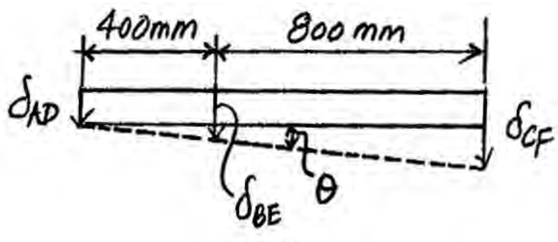
$$F_{AD} = 15436.93 \text{ N} = 15.4 \text{ kN} \qquad \textbf{Ans.}$$

$$\sigma_{BE} = \frac{F_{BE}}{A_{BE}} = \frac{91844.61}{\frac{\pi}{4}\left(0.025^2\right)} = 187.10 \text{ MPa} < (\sigma_Y)_{st} \qquad \textbf{(O.K.)}$$

$$\sigma_{AD} = \frac{F_{AD}}{A_{AD}} = \frac{15436.93}{\frac{\pi}{4}\left(0.025^2\right)} = 31.45 \text{ MPa} < (\sigma_Y)_{st} \qquad \textbf{(O.K.)}$$

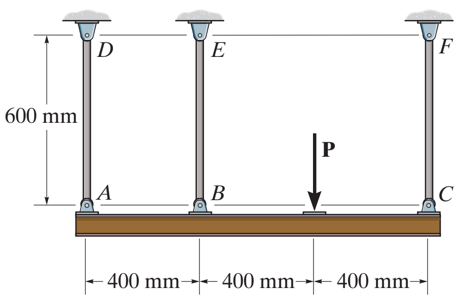


(a)



(b)

•**4–105.** The rigid beam is supported by three 25-mm diameter A-36 steel rods. If the force of $P = 230$ kN is applied on the beam and removed, determine the residual stresses in each rod. Consider the steel to be an elastic perfectly-plastic material.



Equation of Equilibrium. Referring to the free-body diagram of the beam shown in Fig. *a*,

$$+\uparrow \Sigma F_y = 0; \qquad F_{AD} + F_{BE} + F_{CF} - 230\left(10^3\right) = 0 \qquad \textbf{(1)}$$

$$\zeta + \Sigma M_A = 0; \qquad F_{BE}(400) + F_{CF}(1200) - 230\left(10^3\right)(800) = 0$$

$$F_{BE} + 3F_{CF} = 460\left(10^3\right) \qquad \textbf{(2)}$$

Elastic Analysis. Referring to the deflection diagram of the beam shown in Fig. *b*, the compatibility equation can be written as

$$\delta_{BE} = \delta_{AD} + \left(\frac{\delta_{CF} - \delta_{AD}}{1200}\right)(400)$$

$$\delta_{BE} = \frac{2}{3}\delta_{AD} + \frac{1}{3}\delta_{CF} \qquad \textbf{(3)}$$

$$\frac{F_{BE}L}{AE} = \frac{2}{3}\left(\frac{F_{CD}L}{AE}\right) + \frac{1}{3}\left(\frac{F_{CF}L}{AE}\right)$$

$$F_{BE} = \frac{2}{3}F_{AD} + \frac{1}{3}F_{CF} \qquad \textbf{(4)}$$

Solving Eqs. (1), (2), and (4)

$$F_{CF} = 131428.57\text{ N} \qquad F_{BE} = 65714.29\text{ N} \qquad F_{AD} = 32857.14\text{ N}$$

Normal Stress.

$$\sigma_{CF} = \frac{F_{CF}}{A_{CF}} = \frac{131428.57}{\frac{\pi}{4}\left(0.025^2\right)} = 267.74\text{ MPa (T)} > (\sigma_Y)_{st} \qquad \textbf{(N.G.)}$$

$$\sigma_{BE} = \frac{F_{BE}}{A_{BE}} = \frac{65714.29}{\frac{\pi}{4}\left(0.025^2\right)} = 133.87\text{ MPa (T)} < (\sigma_Y)_{st} \qquad \textbf{(O.K.)}$$

$$\sigma_{AD} = \frac{F_{AD}}{A_{AD}} = \frac{32857.14}{\frac{\pi}{4}\left(0.025^2\right)} = 66.94\text{ MPa (T)} < (\sigma_Y)_{st} \qquad \textbf{(O.K.)}$$

Since rod *CF* yields, the elastic analysis is not valid. The solution must be reworked using

$$\sigma_{CF} = (\sigma_Y)_{st} = 250\text{ MPa (T)}$$

$$F_{CF} = \sigma_{CF}A_{CF} = 250\left(10^6\right)\left[\frac{\pi}{4}\left(0.025^2\right)\right] = 122718.46\text{ N}$$

4–105. Continued

Substituting this result into Eq. (2),

$$F_{BE} = 91844.61 \text{ N}$$

Substituting the result for $\mathbf{F}_{CF}$ and $\mathbf{F}_{BE}$ into Eq. (1),

$$F_{AD} = 15436.93\text{N}$$

$$\sigma_{BE} = \frac{F_{BE}}{A_{BE}} = \frac{91844.61}{\frac{\pi}{4}\left(0.025^2\right)} = 187.10 \text{ MPa (T)} < (\sigma_Y)_{st} \qquad \textbf{(O.K.)}$$

$$\sigma_{AD} = \frac{F_{AD}}{A_{AD}} = \frac{15436.93}{\frac{\pi}{4}\left(0.025^2\right)} = 31.45 \text{ MPa (T)} < (\sigma_Y)_{st} \qquad \textbf{(O.K.)}$$

Residual Stresses. The process of removing **P** can be represented by applying the force **P**′, which has a magnitude equal to that of **P** but is opposite in sense, Fig. *c*. Since the process occurs in a linear manner, the corresponding normal stress must have the same magnitude but opposite sense to that obtained from the elastic analysis. Thus,

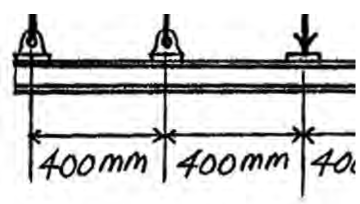
$\sigma'_{CF} = 267.74$ MPa (C) $\quad \sigma'_{BE} = 133.87$ MPa (C) $\quad \sigma'_{AD} = 66.94$ MPa (C)

Considering the tensile stress as positive and the compressive stress as negative,

$$(\sigma_{CF})_r = \sigma_{CF} + \sigma'_{CF} = 250 + (-267.74) = -17.7 \text{ MPa} = 17.7 \text{ MPa (C)} \qquad \textbf{Ans.}$$

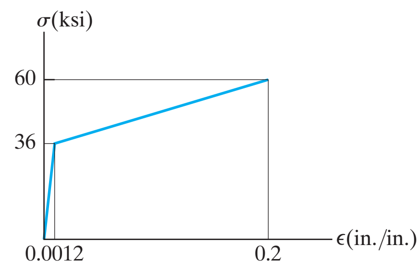
$$(\sigma_{BE})_r = \sigma_{BE} + \sigma'_{BE} = 187.10 + (-133.87) = 53.2 \text{ MPa (T)} \qquad \textbf{Ans.}$$

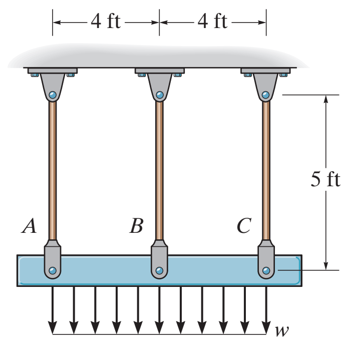
$$(\sigma_{AD})_r = \sigma_{AD} + \sigma'_{AD} = 31.45 + (-66.94) = -35.5 \text{ MPa} = 35.5 \text{ MPa (C)} \qquad \textbf{Ans.}$$



(a)

4–106. The distributed loading is applied to the rigid beam, which is supported by the three bars. Each bar has a cross-sectional area of 1.25 in^2 and is made from a material having a stress–strain diagram that can be approximated by the two line segments shown. If a load of $w = 25$ kip/ft is applied to the beam, determine the stress in each bar and the vertical displacement of the beam.





$\circlearrowleft + \Sigma M_B = 0; \qquad F_C(4) - F_A(4) = 0;$

$$F_A = F_C = F$$

$+\uparrow \Sigma F_y = 0; \qquad 2F + F_B - 200 = 0 \qquad$ **(1)**

Since the loading and geometry are symmetrical, the bar will remain horizontal. Therefore, the displacement of the bars is the same and hence, the force in each bar is the same. From Eq. (1).

$F = F_B = 66.67$ kip

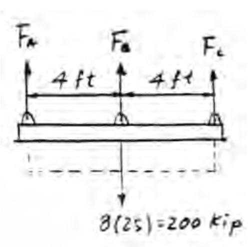
Thus,

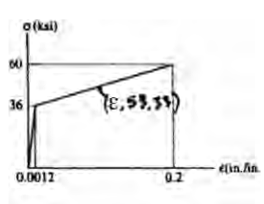
$$\sigma_A = \sigma_B = \sigma_C = \frac{66.67}{1.25} = 53.33 \text{ ksi}$$ **Ans.**

From the stress-strain diagram:

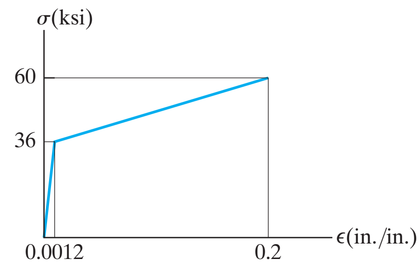
$$\frac{53.33 - 36}{\varepsilon - 0.0012} = \frac{60 - 36}{0.2 - 0.0012}; \qquad \varepsilon = 0.14477 \text{ in./in.}$$

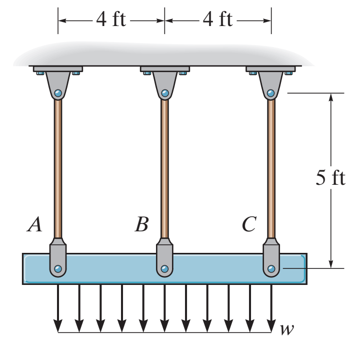
$$\delta = \varepsilon L = 0.14477(5)(12) = 8.69 \text{ in.}$$ **Ans.**





4–107. The distributed loading is applied to the rigid beam, which is supported by the three bars. Each bar has a cross-sectional area of 0.75 in^2 and is made from a material having a stress–strain diagram that can be approximated by the two line segments shown. Determine the intensity of the distributed loading w needed to cause the beam to be displaced downward 1.5 in.





$\circlearrowleft + \Sigma M_B = 0; \qquad F_C(4) - F_A(4) = 0; \qquad F_A = F_C = F$

$+\uparrow \Sigma F_y = 0; \qquad 2F + F_B - 8w = 0$ **(1)**

Since the system and the loading are symmetrical, the bar will remain horizontal. Hence the displacement of the bars is the same and the force supported by each bar is the same.

From Eq. (1),

$F_B = F = 2.6667\,w$ **(2)**

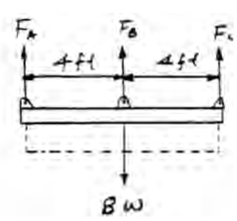
From the stress-strain diagram:

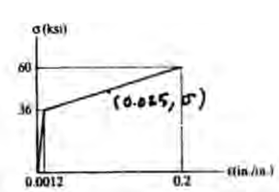
$$\varepsilon = \frac{1.5}{5\,(12)} = 0.025 \text{ in./in.}$$

$$\frac{\sigma - 36}{0.025 - 0.0012} = \frac{60 - 36}{0.2 - 0.0012}; \qquad \sigma = 38.87 \text{ ksi}$$

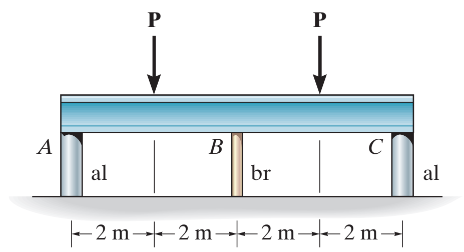
Hence $F = \sigma A = 38.87\,(0.75) = 29.15$ kip

From Eq. (2), $w = 10.9$ kip/ft **Ans.**



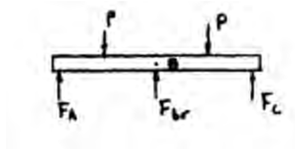


***4–108.** The rigid beam is supported by the three posts A, B, and C of equal length. Posts A and C have a diameter of 75 mm and are made of aluminum, for which $E_{al} = 70$ GPa and $(\sigma_Y)_{al} = 20$ MPa. Post B has a diameter of 20 mm and is made of brass, for which $E_{br} = 100$ GPa and $(\sigma_Y)_{br} = 590$ MPa. Determine the smallest magnitude of **P** so that (a) only rods A and C yield and (b) all the posts yield.



$\Sigma M_B = 0; \qquad F_A = F_C = F_{al}$

$+\uparrow \Sigma F_y = 0; \qquad F_{at} + 2F_{at} - 2P = 0$ **(1)**



(a) Post A and C will yield,

$$F_{al} = (\sigma_t)_{al} A$$

$$= 20(10^4)(\tfrac{\pi}{a})(0.075)^2$$

$$= 88.36 \text{ kN}$$

$$(E_{al})_r = \frac{(\sigma_r)_{al}}{E_{al}} = \frac{20(10^4)}{70(10^4)} = 0.0002857$$

Compatibility condition:

$$\delta_{br} = \delta_{al}$$

$$= 0.0002857(L)$$

$$\frac{F_{br}\,(L)}{\frac{\pi}{4}(0.02)^2(100)(10^4)} = 0.0002857\,L$$

$$F_{br} = 8.976 \text{ kN}$$

$$\sigma_{br} = \frac{8.976(10^3)}{\frac{\pi}{4}(0.02^3)} = 28.6 \text{ MPa} < \sigma_r \qquad \textbf{OK.}$$

From Eq. (1),

$$8.976 + 2(88.36) - 2P = 0$$

$$P = 92.8 \text{ kN} \qquad \textbf{Ans.}$$

(b) All the posts yield:

$$F_{br} = (\sigma_r)_{br} A$$

$$= (590)(10^4)(\tfrac{\pi}{4})(0.02^2)$$

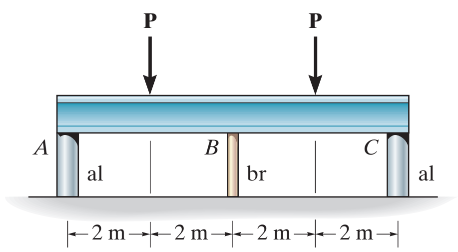
$$= 185.35 \text{ kN}$$

$$F_{al} = 88.36 \text{ kN}$$

From Eq. (1); $185.35 + 2(88.36) - 2P = 0$

$$P = 181 \text{ kN} \qquad \textbf{Ans.}$$

•4–109. The rigid beam is supported by the three posts A, B, and C. Posts A and C have a diameter of 60 mm and are made of aluminum, for which $E_{al} = 70$ GPa and $(\sigma_Y)_{al} = 20$ MPa. Post B is made of brass, for which $E_{br} = 100$ GPa and $(\sigma_Y)_{br} = 590$ MPa. If $P = 130$ kN, determine the largest diameter of post B so that all the posts yield at the same time.



$$+\uparrow \Sigma F_y = 0; \qquad 2(F_\gamma)_{al} + F_{br} - 260 = 0 \qquad \textbf{(1)}$$

$$(F_{al})_\gamma = (\sigma_\gamma)_{al} A$$

$$= 20(10^6)(\tfrac{\pi}{4})(0.06)^2 = 56.55 \text{ kN}$$

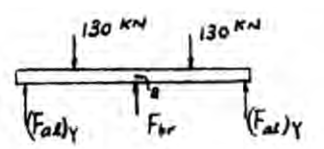
From Eq. (1),

$$2(56.55) + F_{br} - 260 = 0$$

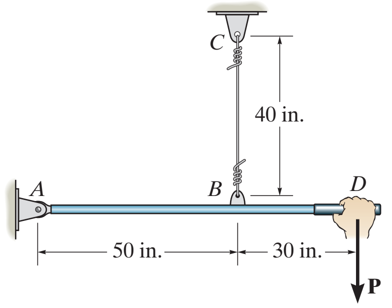
$$F_{br} = 146.9 \text{ kN}$$

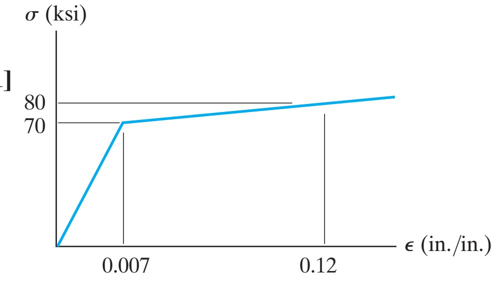
$$(\sigma_\gamma)_{br} = 590(10^6) = \frac{146.9(10^3)}{\frac{\pi}{4}(d_B)^3}$$

$$d_B = 0.01779 \text{ m} = 17.8 \text{ mm} \qquad \textbf{Ans.}$$



4–110. The wire BC has a diameter of 0.125 in. and the material has the stress–strain characteristics shown in the figure. Determine the vertical displacement of the handle at D if the pull at the grip is slowly increased and reaches a magnitude of (a) $P = 450$ lb, (b) $P = 600$ lb.





Equations of Equilibrium:

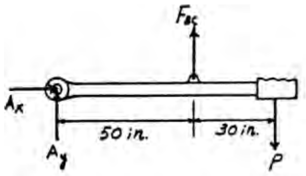
$\zeta + \Sigma M_A = 0; \qquad F_{BC}(50) - P(80) = 0$ **[1]**

(a) From Eq. [1] when $P = 450$ lb, $\qquad F_{BC} = 720$ lb

Average Normal Stress and Strain:

$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{720}{\frac{\pi}{4}(0.125^2)} = 58.67 \text{ ksi}$$

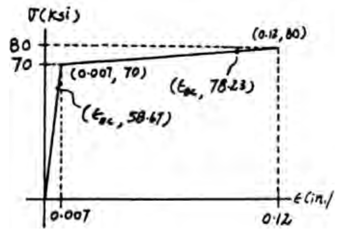
From the Stress–Strain diagram



$$\frac{58.67}{\varepsilon_{BC}} = \frac{70}{0.007}; \qquad \varepsilon_{BC} = 0.005867 \text{ in./in.}$$

Displacement:

$$\delta_{BC} = \varepsilon_{BC} L_{BC} = 0.005867(40) = 0.2347 \text{ in.}$$



$$\frac{\delta_D}{80} = \frac{\delta_{BC}}{50}; \qquad \delta_D = \frac{8}{5}(0.2347) = 0.375 \text{ in.}$$ **Ans.**

(b) From Eq. [1] when $P = 600$ lb, $\qquad F_{BC} = 960$ lb

Average Normal Stress and Strain:

$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{960}{\frac{\pi}{4}(0.125)^2} = 78.23 \text{ ksi}$$

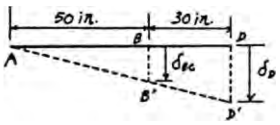
From Stress–Strain diagram

$$\frac{78.23 - 70}{\varepsilon_{BC} - 0.007} = \frac{80 - 70}{0.12 - 0.007} \qquad \varepsilon_{BC} = 0.09997 \text{ in./in.}$$

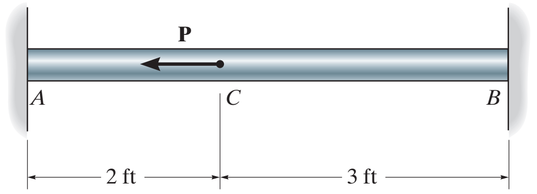
Displacement:

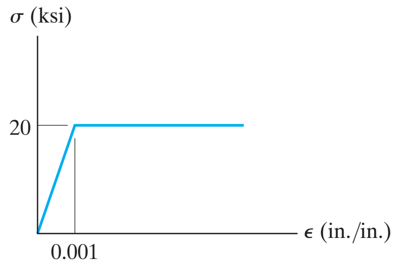
$$\delta_{BC} = \varepsilon_{BC} L_{BC} = 0.09997(40) = 3.9990 \text{ in.}$$

$$\frac{\delta_D}{80} = \frac{\delta_{BC}}{50}; \qquad \delta_D = \frac{8}{5}(3.9990) = 6.40 \text{ in.}$$ **Ans.**



4–111. The bar having a diameter of 2 in. is fixed connected at its ends and supports the axial load **P**. If the material is elastic perfectly plastic as shown by the stress–strain diagram, determine the smallest load P needed to cause segment CB to yield. If this load is released, determine the permanent displacement of point C.





When P is increased, region AC will become plastic first, then CB will become plastic. Thus,

$$F_A = F_B = \sigma A = 20(\pi)(1)^2 = 62.832 \text{ kip}$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_A + F_B - P = 0 \qquad \textbf{(1)}$$

$$P = 2(62.832) = 125.66 \text{ kip}$$

$$P = 126 \text{ kip} \qquad \textbf{Ans.}$$

The deflection of point C is,

$$\delta_C = \varepsilon L = (0.001)(3)(12) = 0.036 \text{ in.} \leftarrow$$

Consider the reverse of P on the bar.

$$\frac{F_A'(2)}{AE} = \frac{F_B'(3)}{AE}$$

$$F_A' = 1.5\, F_B'$$

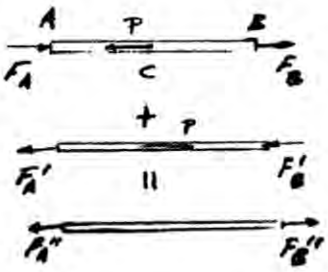
So that from Eq. (1)

$$F_B' = 0.4P$$

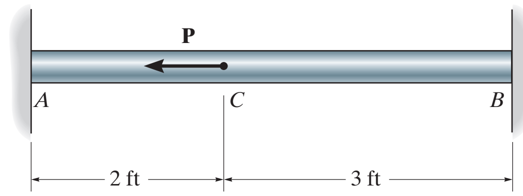
$$F_A' = 0.6P$$

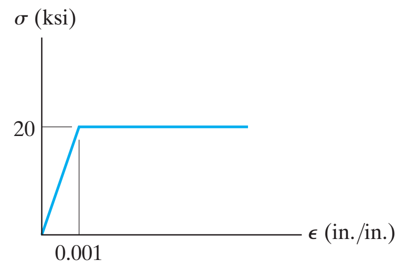
$$\delta_C' = \frac{F_B' L}{AE} = \frac{0.4(P)(3)(12)}{AE} = \frac{0.4(125.66)(3)(12)}{\pi(1)^2(20/0.001)} = 0.02880 \text{ in.} \rightarrow$$

$$\Delta\delta = 0.036 - 0.0288 = 0.00720 \text{ in.} \leftarrow \qquad \textbf{Ans.}$$



***4–112.** Determine the elongation of the bar in Prob. 4–111 when both the load **P** and the supports are removed.





When P is increased, region AC will become plastic first, then CB will become plastic. Thus,

$$F_A = F_B = \sigma A = 20(\pi)(1)^2 = 62.832 \text{ kip}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_A + F_B - P = 0 \qquad \textbf{(1)}$$

$$P = 2(62.832) = 125.66 \text{ kip}$$

$$P = 126 \text{ kip} \qquad \textbf{Ans.}$$

The deflection of point C is,

$$\delta_C = \varepsilon L = (0.001)(3)(12) = 0.036 \text{ in.} \leftarrow$$

Consider the reverse of P on the bar.

$$\frac{F_A'(2)}{AE} = \frac{F_B'(3)}{AE}$$

$$F_A' = 1.5\, F_B'$$

So that from Eq. (1)

$$F_B' = 0.4P$$

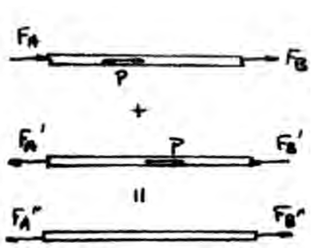
$$F_A' = 0.6P$$

The resultant reactions are

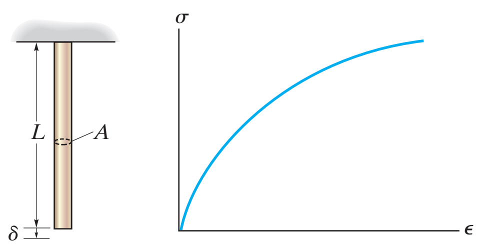
$$F_A'' = F_B'' = -62.832 + 0.6(125.66) = 62.832 - 0.4(125.66) = 12.568 \text{ kip}$$

When the supports are removed the elongation will be,

$$\delta = \frac{PL}{AE} = \frac{12.568(5)(12)}{\pi(1)^2(20/0.001)} = 0.0120 \text{ in.} \qquad \textbf{Ans.}$$



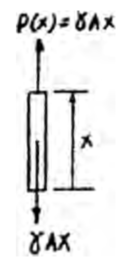
•4–113. A material has a stress–strain diagram that can be described by the curve $\sigma = c\epsilon^{1/2}$. Determine the deflection δ of the end of a rod made from this material if it has a length L, cross-sectional area A, and a specific weight γ.



$$\sigma = c\,\varepsilon^{\frac{1}{2}}; \qquad \sigma^2 = c^2\,\varepsilon$$

$$\sigma^2(x) = c^2\varepsilon(x) \tag{1}$$

$$\text{However } \sigma(x) = \frac{P(x)}{A}; \qquad \varepsilon(x) = \frac{d\delta}{dx}$$



From Eq. (1),

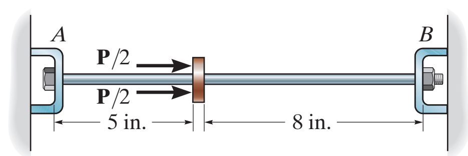
$$\frac{P^2(x)}{A^2} = c^2\frac{d\delta}{dx}; \qquad \frac{d\delta}{dx} = \frac{P^2(x)}{A^2c^2}$$

$$\delta = \frac{1}{A^2c^2}\int P^2(x)\,dx = \frac{1}{A^2c^2}\int_0^L (\gamma Ax)^2\,dx$$

$$= \frac{\gamma^2}{c^2}\int_0^L x^2\,dx = \frac{\gamma^2}{c^2}\frac{x^3}{3}\bigg|_0^L$$

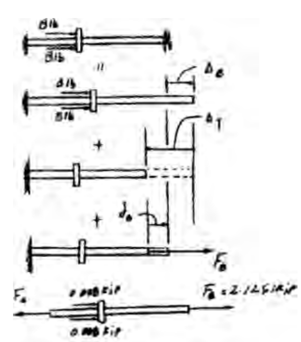
$$\delta = \frac{\gamma^3L^3}{3c^2}$$ **Ans.**

4–114. The 2014-T6 aluminum rod has a diameter of 0.5 in. and is lightly attached to the rigid supports at A and B when $T_1 = 70°\text{F}$. If the temperature becomes $T_2 = -10°\text{F}$, and an axial force of $P = 16$ lb is applied to the rigid collar as shown, determine the reactions at A and B.



$$\stackrel{+}{\rightarrow} \quad 0 = \Delta_B - \Delta_T + \delta_B$$

$$0 = \frac{0.016(5)}{\frac{\pi}{4}(0.5^2)(10.6)(10^3)} - 12.8(10^{-6})[70° - (-10°)](13) + \frac{F_B(13)}{\frac{\pi}{4}(0.5^2)(10.6)(10^3)}$$

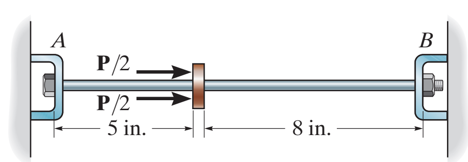


$$F_B = 2.1251 \text{ kip} = 2.13 \text{ kip}$$ **Ans.**

$$\stackrel{+}{\rightarrow} \Sigma F_x = 0; \qquad 2(0.008) + 2.1251 - F_A = 0$$

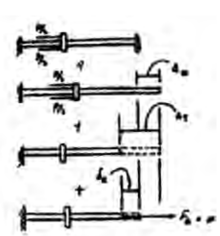
$$F_A = 2.14 \text{ kip}$$ **Ans.**

4–115. The 2014-T6 aluminum rod has a diameter of 0.5 in. and is lightly attached to the rigid supports at A and B when $T_1 = 70°\text{F}$. Determine the force P that must be applied to the collar so that, when $T = 0°\text{F}$, the reaction at B is zero.



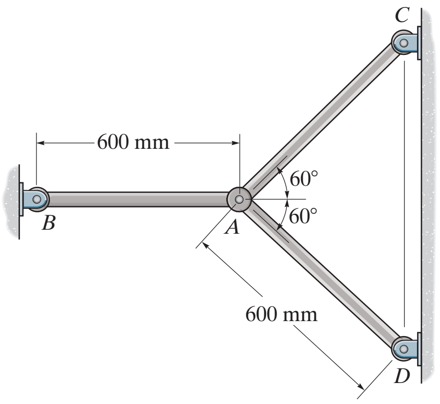
$$\stackrel{+}{\rightarrow} \quad 0 = \Delta_B - \Delta_T + \delta_B$$

$$0 = \frac{P(5)}{\frac{\pi}{4}(0.5^2)(10.6)(10^3)} - 12.8(10^{-6})[(70)(13)] + 0$$



$$P = 4.85 \text{ kip}$$ **Ans.**

***4–116.** The rods each have the same 25-mm diameter and 600-mm length. If they are made of A-36 steel, determine the forces developed in each rod when the temperature increases to 50° C.



Equation of Equilibrium: Referring to the free-body diagram of joint A shown in Fig. a,

$$+\uparrow \Sigma F_x = 0; \qquad F_{AD} \sin 60° - F_{AC} \sin 60° = 0 \qquad F_{AC} = F_{AD} = F$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_{AB} - 2F \cos 60° = 0$$

$$F_{AB} = F \qquad \textbf{(1)}$$

Compatibility Equation: If AB and AC are unconstrained, they will have a free expansion of $\left(\delta_T\right)_{AB} = \left(\delta_T\right)_{AC} = \alpha_{st} \Delta T L = 12(10^{-6})(50)(600) = 0.36$ mm. Referring to the initial and final position of joint A,

$$\delta_{F_{AB}} - \left(\delta_T\right)_{AB} = \left(\delta_T'\right)_{AC} - \delta_{F_{AC}}'$$

Due to symmetry, joint A **will** displace horizontally, and $\delta_{AC}' = \dfrac{\delta_{AC}}{\cos 60°} = 2\delta_{AC}$. Thus, $\left(\delta_T'\right)_{AC} = 2(\delta_T)_{AC}$ and $\delta_{F_{AC}}' = 2\delta_{F_{AC}}$. Thus, this equation becomes

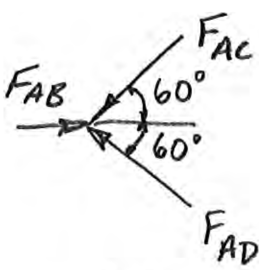
$$\delta_{F_{AB}} - \left(\delta_T\right)_{AB} = 2\left(\delta_T\right)_{AC} - 2\delta_{AC}$$

$$\frac{F_{AB}(600)}{\frac{\pi}{4}\left(0.025^2\right)(200)(10^9)} - 0.36 = 2(0.36) - 2\left[\frac{F(600)}{\frac{\pi}{4}\left(0.025^2\right)(200)(10^9)}\right]$$

$$F_{AB} + 2F = 176\,714.59 \qquad \textbf{(2)}$$

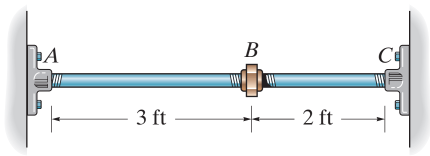
Solving Eqs. (1) and (2),

$$F_{AB} = F_{AC} = F_{AD} = 58\,904.86\,\text{N} = 58.9\,\text{kN} \qquad \textbf{Ans.}$$

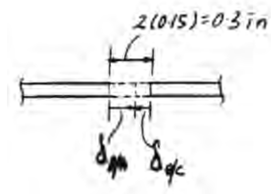


(a)

•4–117. Two A-36 steel pipes, each having a cross-sectional area of $0.32\ \text{in}^2$, are screwed together using a union at B as shown. Originally the assembly is adjusted so that no load is on the pipe. If the union is then tightened so that its screw, having a lead of 0.15 in., undergoes two full turns, determine the average normal stress developed in the pipe. Assume that the union at B and couplings at A and C are rigid. Neglect the size of the union. *Note:* The lead would cause the pipe, when *unloaded*, to shorten 0.15 in. when the union is rotated one revolution.



The loads acting on both segments AB and BC are the same since no external load acts on the system.



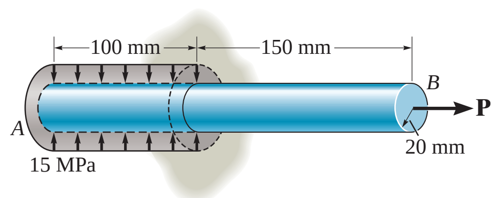
$$0.3 = \delta_{B/A} + \delta_{B/C}$$

$$0.3 = \frac{P(3)(12)}{0.32(29)(10^3)} + \frac{P(2)(12)}{0.32(29)(10^3)}$$

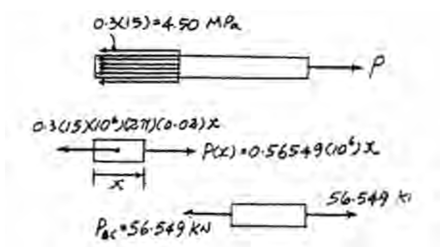
$$P = 46.4 \text{ kip}$$

$$\sigma_{AB} = \sigma_{BC} = \frac{P}{A} = \frac{46.4}{0.32} = 145 \text{ ksi} \qquad \textbf{Ans.}$$

4–118. The brass plug is force-fitted into the rigid casting. The uniform normal bearing pressure on the plug is estimated to be 15 MPa. If the coefficient of static friction between the plug and casting is $\mu_s = 0.3$, determine the axial force P needed to pull the plug out. Also, calculate the displacement of end B relative to end A just before the plug starts to slip out. $E_{br} = 98$ GPa.



Equations of Equilibrium:



$$\xrightarrow{+} \Sigma F_x = 0; \qquad P - 4.50(10^6)(2)(\pi)(0.02)(0.1) = 0$$

$$P = 56.549 \text{ kN} = 56.5 \text{ kN} \qquad \textbf{Ans.}$$

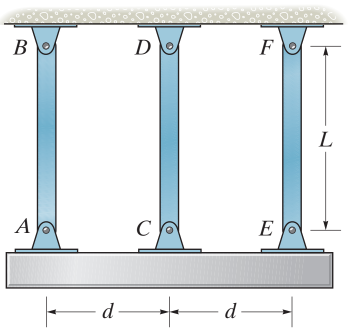
Displacement:

$$\delta_{B/A} = \Sigma \frac{PL}{AE}$$

$$= \frac{56.549(10^3)(0.15)}{\pi(0.02^2)(98)(10^9)} + \int_0^{0.1\text{ m}} \frac{0.56549(10^6)\, x\, dx}{\pi(0.02^2)(98)(10^9)}$$

$$= 0.00009184 \text{ m} = 0.0918 \text{ mm} \qquad \textbf{Ans.}$$

4–119. The assembly consists of two bars AB and CD of the same material having a modulus of elasticity E_1 and coefficient of thermal expansion α_1, and a bar EF having a modulus of elasticity E_2 and coefficient of thermal expansion α_2. All the bars have the same length L and cross-sectional area A. If the rigid beam is originally horizontal at temperature T_1, determine the angle it makes with the horizontal when the temperature is increased to T_2.



Equations of Equilibrium:

$$\zeta + \Sigma M_C = 0; \qquad F_{AB} = F_{EF} = F$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{CD} - 2F = 0 \qquad \textbf{[1]}$$

Compatibility:

$$\delta_{AB} = (\delta_{AB})_T - (\delta_{AB})_F \qquad \delta_{CD} = (\delta_{CD})_T + (\delta_{CD})_F$$

$$\delta_{EF} = (\delta_{EF})_T - (\delta_{EF})_F$$

From the geometry

$$\frac{\delta_{CD} - \delta_{AB}}{d} = \frac{\delta_{EF} - \delta_{AB}}{2d}$$

$$2\delta_{CD} = \delta_{EF} + \delta_{AB}$$

$$2\left[(\delta_{CD})_T + (\delta_{CD})_F\right] = (\delta_{EF})_T - (\delta_{EF})_F + (\delta_{AB})_T - (\delta_{AB})_F$$

$$2\left[\alpha_1 (T_2 - T_1) L + \frac{F_{CD}(L)}{AE_1}\right]$$

$$= \alpha_2 (T_2 - T_1) L - \frac{F(L)}{AE_2} + \alpha_1 (T_2 - T_1) L - \frac{F(L)}{AE_1} \qquad \textbf{[2]}$$

Substitute Eq. [1] into [2].

$$2\alpha_1 (T_2 - T_1) L + \frac{4FL}{AE_1} = \alpha_2 (T_2 - T_1)L - \frac{FL}{AE_2} + \alpha_1 (T_2 - T_1)L - \frac{FL}{AE_1}$$

$$\frac{5F}{AE_1} + \frac{F}{AE_2} = \alpha_2 (T_2 - T_1) - \alpha_1 (T_2 - T_1)$$

$$F\left(\frac{5E_2 + E_1}{AE_1E_2}\right) = (T_2 - T_1)(\alpha_2 - \alpha_1)\,; \qquad F = \frac{AE_1E_2(T_2 - T_1)(\alpha_2 - \alpha_1)}{5E_2 + E_1}$$

$$(\delta_{EF})_T = \alpha_2 (T_2 - T_1) L$$

$$(\delta_{EF})_F = \frac{AE_1E_2(T_2 - T_1)(\alpha_2 - \alpha_1)(L)}{AE_2(5E_2 + E_1)} = \frac{E_1 (T_2 - T_1)(\alpha_2 - \alpha_1)(L)}{5E_2 + E_1}$$

$$\delta_{EF} = (\delta_{EF})_T - (\delta_{EF})_F = \frac{\alpha_2 L(T_2 - T_1)(5E_2 - E_1) - E_1L(T_2 - T_1)(\alpha_2 - \alpha_1)}{5E_2 + E_1}$$

$$(\delta_{AB})_T = \alpha_1 (T_2 - T_1) L$$

$$(\delta_{AB})_F = \frac{AE_1E_2(T_2 - T_1)(\alpha_2 - \alpha_1)(L)}{AE_1(5E_2 + E_1)} = \frac{E_2(T_2 - T_1)(\alpha_2 - \alpha_1)(L)}{5E_2 + E_1}$$

$$\delta_{AB} = (\delta_{AB})_T - (\delta_{AB})_F = \frac{\alpha_1 L(5E_2 + E_1)(T_2 - T_1) - E_2 L(T_2 - T_1)(\alpha_2 - \alpha_1)}{5E_2 + E_1}$$

4–119. Continued

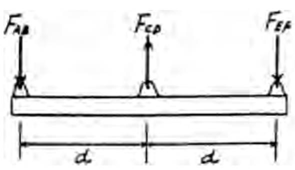
$$\delta_{EF} - \delta_{AB} = \frac{L(T_2 - T_1)}{5E_2 + E_1}[\alpha_2 (5E_2 + E_1) - E_1 (\alpha_2 - \alpha_1) - \alpha_1 (5E_2 + E_1) + E_2 (\alpha_2 - \alpha_1)]$$

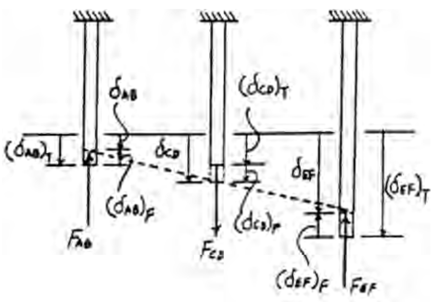
$$= \frac{L(T_2 - T_1)}{5E_2 + E_1}\left[(5E_2 + E_1)(\alpha_2 - \alpha_1) + (\alpha_2 - \alpha_1)(E_2 - E_1)\right]$$

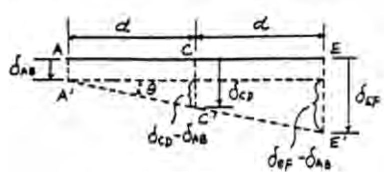
$$= \frac{L(T_2 - T_1)(\alpha_2 - \alpha_1)}{5E_2 + E_1}(5E_2 + E_1 + E_2 - E_1)$$

$$= \frac{L(T_2 - T_1)(\alpha_2 - \alpha_1)(6E_2)}{5E_2 + E_1}$$

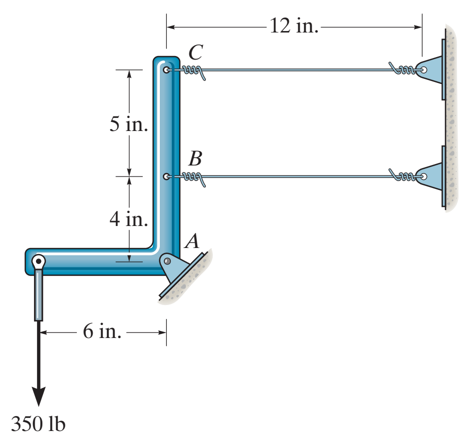
$$\theta = \frac{\delta_{EF} - \delta_{AB}}{2d} = \frac{3E_2 L(T_2 - T_1)(\alpha_2 - \alpha_1)}{d(5E_2 + E_1)} \qquad \textbf{Ans.}$$







***4–120.** The rigid link is supported by a pin at A and two A-36 steel wires, each having an unstretched length of 12 in. and cross-sectional area of 0.0125 in^2. Determine the force developed in the wires when the link supports the vertical load of 350 lb.



Equations of Equilibrium:

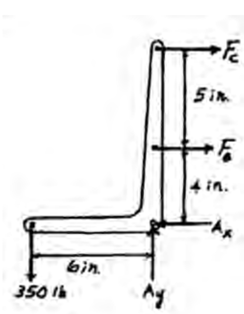
$$\curvearrowleft +\Sigma M_A = 0; \qquad -F_C(9) - F_B(4) + 350(6) = 0 \qquad \textbf{[1]}$$

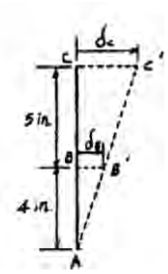
Compatibility:

$$\frac{\delta_B}{4} = \frac{\delta_C}{9}$$

$$\frac{F_B(L)}{4AE} = \frac{F_C(L)}{9AE}$$

$$9F_B - 4F_C = 0, \qquad \textbf{[2]}$$



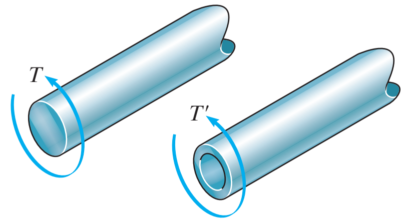


Solving Eqs. [1] and [2] yields:

$$F_B = 86.6 \text{ lb} \qquad \textbf{Ans.}$$

$$F_C = 195 \text{ lb} \qquad \textbf{Ans.}$$

•5–1. A shaft is made of a steel alloy having an allowable shear stress of $\tau_{allow} = 12$ ksi. If the diameter of the shaft is 1.5 in., determine the maximum torque **T** that can be transmitted. What would be the maximum torque **T′** if a 1-in.-diameter hole is bored through the shaft? Sketch the shear-stress distribution along a radial line in each case.

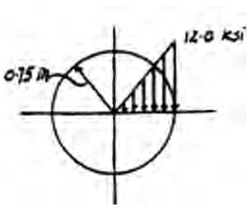


Allowable Shear Stress: Applying the torsion formula

$$\tau_{max} = \tau_{allow} = \frac{Tc}{J}$$

$$12 = \frac{T\,(0.75)}{\frac{\pi}{2}\,(0.75^4)}$$

$$T = 7.95 \text{ kip} \cdot \text{in.} \qquad \textbf{Ans.}$$



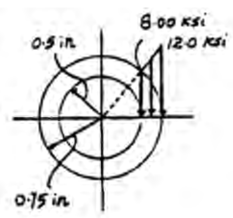
Allowable Shear Stress: Applying the torsion formula

$$\tau_{max} = \tau_{allow} = \frac{T'c}{J}$$

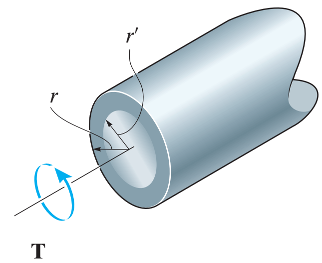
$$12 = \frac{T'\,(0.75)}{\frac{\pi}{2}\,(0.75^4 - 0.5^4)}$$

$$T' = 6.381 \text{ kip} \cdot \text{in.} = 6.38 \text{ kip} \cdot \text{in.} \qquad \textbf{Ans.}$$

$$\tau_{\rho = 0.5\text{ in}} = \frac{T'\rho}{J} = \frac{6.381(0.5)}{\frac{\pi}{2}\,(0.75^4 - 0.5^4)} = 8.00 \text{ ksi}$$



5–2. The solid shaft of radius r is subjected to a torque $\mathbf{T}$. Determine the radius r' of the inner core of the shaft that resists one-half of the applied torque $(T/2)$. Solve the problem two ways: (a) by using the torsion formula, (b) by finding the resultant of the shear-stress distribution.



a) $$\tau_{max} = \frac{Tc}{J} = \frac{Tr}{\frac{\pi}{2}r^4} = \frac{2T}{\pi r^3}$$

$$\tau = \frac{(\frac{T}{2})r'}{\frac{\pi}{2}(r')^4} = \frac{T}{\pi(r')^3}$$

Since $\tau = \dfrac{r'}{r}\tau_{max}$; $\quad \dfrac{T}{\pi(r')^3} = \dfrac{r'}{r}\left(\dfrac{2T}{\pi r^3}\right)$

$$r' = \frac{r}{2^{\frac{1}{4}}} = 0.841\,r$$ **Ans.**

b) $$\int_0^{\frac{T}{2}} dT = 2\pi \int_0^{r'} \tau\rho^2\,d\rho$$

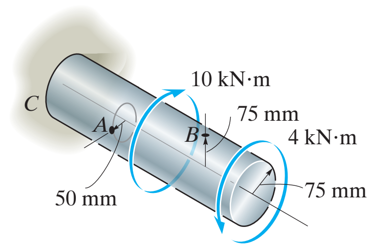
$$\int_0^{\frac{T}{2}} dT = 2\pi \int_0^{r'} \frac{\rho}{r}\tau_{max}\,\rho^2\,d\rho$$

$$\int_0^{\frac{T}{2}} dT = 2\pi \int_0^{r'} \frac{\rho}{r}\left(\frac{2T}{\pi r^3}\right)\rho^2\,d\rho$$

$$\frac{T}{2} = \frac{4T}{r^4}\int_0^{r'} \rho^3\,d\rho$$

$$r' = \frac{r}{2^{\frac{1}{4}}} = 0.841r$$ **Ans.**

5–3. The solid shaft is fixed to the support at C and subjected to the torsional loadings shown. Determine the shear stress at points A and B and sketch the shear stress on volume elements located at these points.



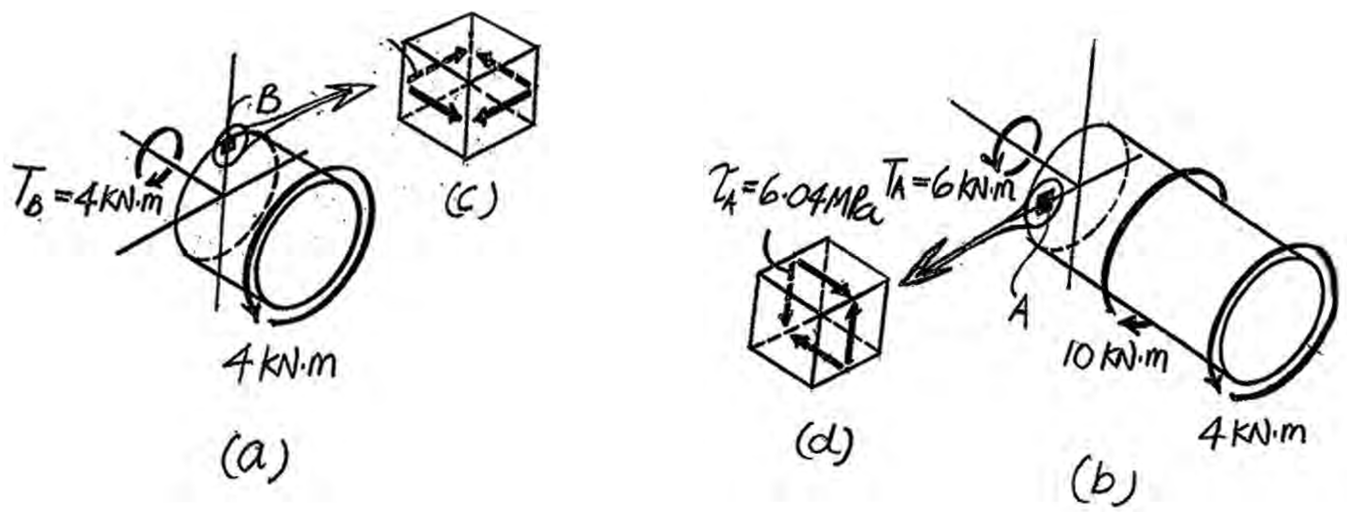
The internal torques developed at Cross-sections pass through point B and A are shown in Fig. *a* and *b*, respectively.

The polar moment of inertia of the shaft is $J = \frac{\pi}{2}(0.075^4) = 49.70(10^{-6})\ \text{m}^4$. For point B, $\rho_B = C = 0.075$ Thus,

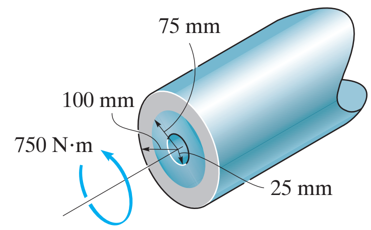
$$\tau_B = \frac{T_B c}{J} = \frac{4(10^3)(0.075)}{49.70(10^{-6})} = 6.036(10^6)\ \text{Pa} = 6.04\ \text{MPa} \qquad \textbf{Ans.}$$

From point A, $\rho_A = 0.05$ m.

$$\tau_A = \frac{T_A \rho_A}{J} = \frac{6(10^3)(0.05)}{49.70\ (10^{-6})} = 6.036(10^6)\ \text{Pa} = 6.04\ \text{MPa}. \qquad \textbf{Ans.}$$



***5–4.** The tube is subjected to a torque of 750 N·m. Determine the amount of this torque that is resisted by the gray shaded section. Solve the problem two ways: (a) by using the torsion formula, (b) by finding the resultant of the shear-stress distribution.

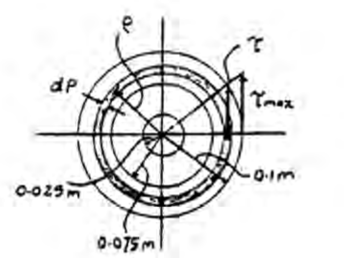


a) ***Applying Torsion Formula:***

$$\tau_{max} = \frac{Tc}{J} = \frac{750(0.1)}{\frac{\pi}{2}(0.1^4 - 0.025^4)} = 0.4793 \text{ MPa}$$

$$\tau_{max} = 0.4793\left(10^6\right) = \frac{T'(0.1)}{\frac{\pi}{2}(0.1^4 - 0.075^4)}$$

$$T' = 515 \text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$



b) ***Integration Method:***

$$\tau = \left(\frac{\rho}{c}\right)\tau_{max} \quad \text{and} \quad dA = 2\pi\rho\, d\rho$$

$$dT' = \rho\tau\, dA = \rho\tau(2\pi\rho\, d\rho) = 2\pi\tau\rho^2\, d\rho$$

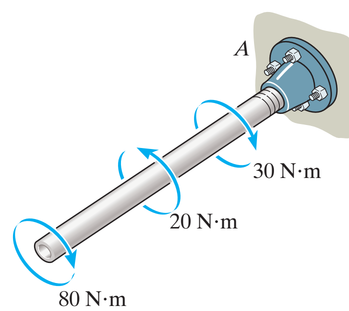
$$T' = \int 2\pi\tau\rho^2\, d\rho = 2\pi\int_{0.075\text{m}}^{0.1\text{m}} \tau_{max}\left(\frac{\rho}{c}\right)\rho^2\, d\rho$$

$$= \frac{2\pi\tau_{max}}{c}\int_{0.075\text{m}}^{0.1\text{m}} \rho^3\, d\rho$$

$$= \frac{2\pi(0.4793)(10^6)}{0.1}\left[\frac{\rho^4}{4}\right]\Bigg|_{0.075\text{ m}}^{0.1\text{ m}}$$

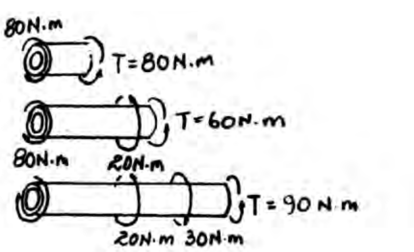
$$= 515 \text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$

5–5. The copper pipe has an outer diameter of 40 mm and an inner diameter of 37 mm. If it is tightly secured to the wall at A and three torques are applied to it as shown, determine the absolute maximum shear stress developed in the pipe.

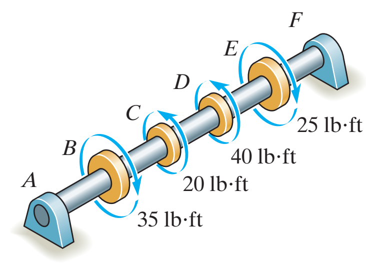


$$\tau_{max} = \frac{T_{max}\, c}{J} = \frac{90(0.02)}{\frac{\pi}{2}(0.02^4 - 0.0185^4)}$$

$$= 26.7 \text{ MPa} \qquad \textbf{Ans..}$$

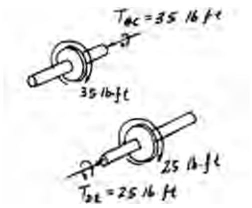


5–6. The solid shaft has a diameter of 0.75 in. If it is subjected to the torques shown, determine the maximum shear stress developed in regions BC and DE of the shaft. The bearings at A and F allow free rotation of the shaft.

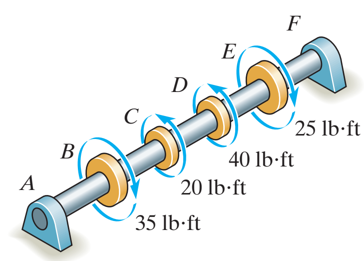


$$(\tau_{BC})_{\max} = \frac{T_{BC}\,c}{J} = \frac{35(12)(0.375)}{\frac{\pi}{2}(0.375)^4} = 5070 \text{ psi} = 5.07 \text{ ksi}$$ **Ans.**

$$(\tau_{DE})_{\max} = \frac{T_{DE}\,c}{J} = \frac{25(12)(0.375)}{\frac{\pi}{2}(0.375)^4} = 3621 \text{ psi} = 3.62 \text{ ksi}$$ **Ans.**



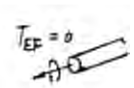
5–7. The solid shaft has a diameter of 0.75 in. If it is subjected to the torques shown, determine the maximum shear stress developed in regions CD and EF of the shaft. The bearings at A and F allow free rotation of the shaft.

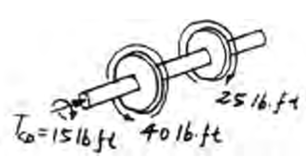


$$(\tau_{EF})_{\max} = \frac{T_{EF}\,c}{J} = 0$$ **Ans.**

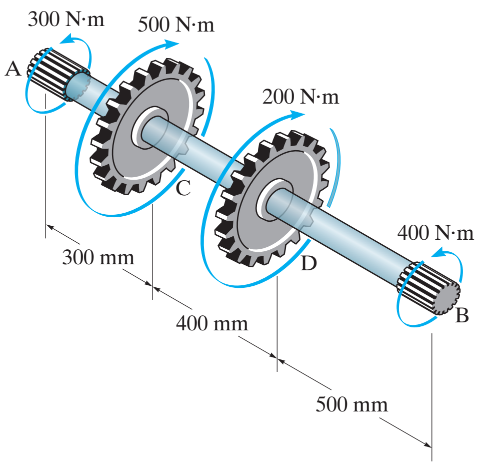
$$(\tau_{CD})_{\max} = \frac{T_{CD}\,c}{J} = \frac{15(12)(0.375)}{\frac{\pi}{2}(0.375)^4}$$

$$= 2173 \text{ psi} = 2.17 \text{ ksi}$$ **Ans.**





***5–8.** The solid 30-mm-diameter shaft is used to transmit the torques applied to the gears. Determine the absolute maximum shear stress on the shaft.

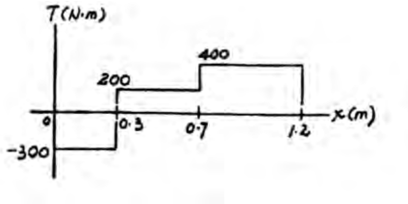


Internal Torque: As shown on torque diagram.

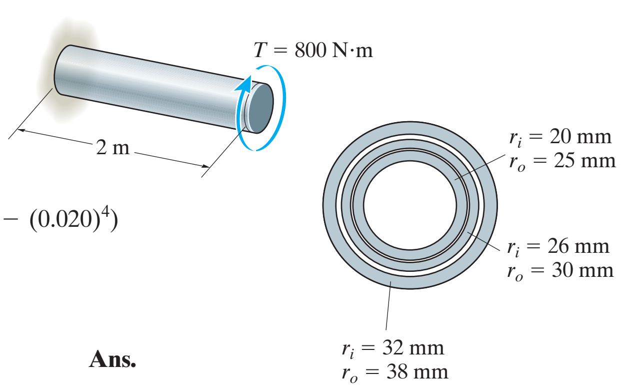
Maximum Shear Stress: From the torque diagram $T_{max} = 400 \text{ N} \cdot \text{m}$. Then, applying torsion Formula.

$$\tau_{\substack{abs\\max}} = \frac{T_{max}\, c}{J}$$

$$= \frac{400(0.015)}{\frac{\pi}{2}(0.015^4)} = 75.5 \text{ MPa} \qquad \textbf{Ans.}$$



•5–9. The shaft consists of three concentric tubes, each made from the same material and having the inner and outer radii shown. If a torque of $T = 800 \text{ N} \cdot \text{m}$ is applied to the rigid disk fixed to its end, determine the maximum shear stress in the shaft.

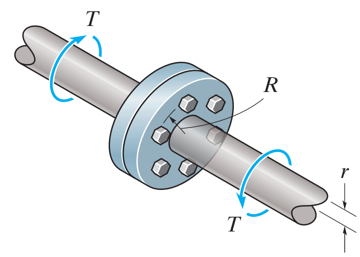


$$J = \frac{\pi}{2}((0.038)^4 - (0.032)^4) + \frac{\pi}{2}((0.030)^4 - (0.026)^4) + \frac{\pi}{2}((0.025)^4 - (0.020)^4)$$

$$J = 2.545(10^{-6}) \text{ m}^4$$

$$\tau_{max} = \frac{Tc}{J} = \frac{800(0.038)}{2.545(10^{-6})} = 11.9 \text{ MPa} \qquad \textbf{Ans.}$$

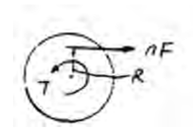
5–10. The coupling is used to connect the two shafts together. Assuming that the shear stress in the bolts is *uniform*, determine the number of bolts necessary to make the maximum shear stress in the shaft equal to the shear stress in the bolts. Each bolt has a diameter d.



n is the number of bolts and F is the shear force in each bolt.

$$T - nFR = 0; \qquad F = \frac{T}{nR}$$

$$\tau_{avg} = \frac{F}{A} = \frac{\frac{T}{nR}}{(\frac{\pi}{4})d^2} = \frac{4T}{nR\pi d^2}$$



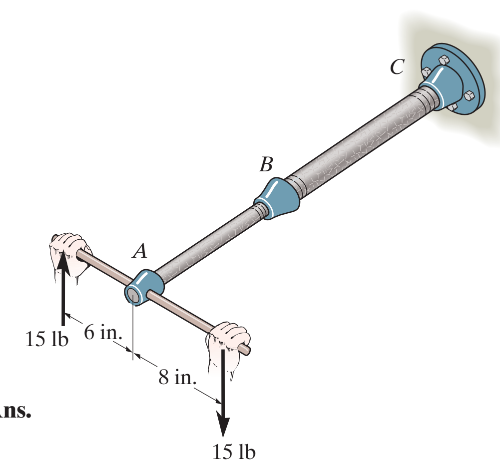
Maximum shear stress for the shaft:

$$\tau_{max} = \frac{Tc}{J} = \frac{Tr}{\frac{\pi}{2}r^4} = \frac{2T}{\pi r^3}$$

$$\tau_{avg} = \tau_{max}; \qquad \frac{4T}{nR\pi d^2} = \frac{2T}{\pi r^3}$$

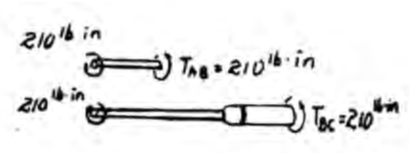
$$n = \frac{2r^3}{Rd^2}$$ **Ans.**

5–11. The assembly consists of two sections of galvanized steel pipe connected together using a reducing coupling at B. The smaller pipe has an outer diameter of 0.75 in. and an inner diameter of 0.68 in., whereas the larger pipe has an outer diameter of 1 in. and an inner diameter of 0.86 in. If the pipe is tightly secured to the wall at C, determine the maximum shear stress developed in each section of the pipe when the couple shown is applied to the handles of the wrench.

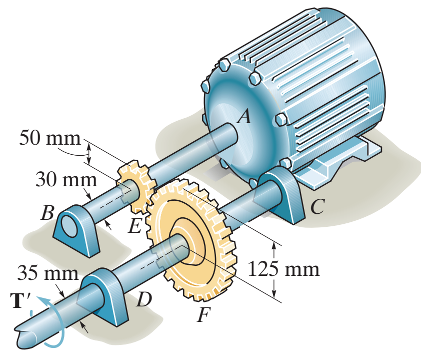


$$\tau_{AB} = \frac{Tc}{J} = \frac{210(0.375)}{\frac{\pi}{2}(0.375^4 - 0.34^4)} = 7.82 \text{ ksi}$$ **Ans.**

$$\tau_{BC} = \frac{Tc}{J} = \frac{210(0.5)}{\frac{\pi}{2}(0.5^4 - 0.43^4)} = 2.36 \text{ ksi}$$ **Ans.**



***5–12.** The motor delivers a torque of 50 N·m to the shaft AB. This torque is transmitted to shaft CD using the gears at E and F. Determine the equilibrium torque $\mathbf{T'}$ on shaft CD and the maximum shear stress in each shaft. The bearings B, C, and D allow free rotation of the shafts.



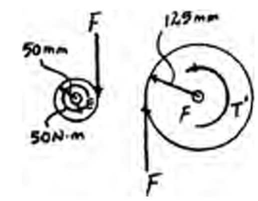
Equilibrium:

$\circlearrowleft +\Sigma M_E = 0; \qquad 50 - F(0.05) = 0 \qquad F = 1000\ \text{N}$

$\circlearrowleft +\Sigma M_F = 0; \qquad T' - 1000(0.125) = 0$

$$T' = 125\ \text{N}\cdot\text{m} \qquad \textbf{Ans.}$$

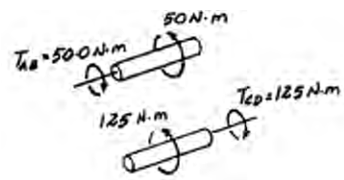
Internal Torque: As shown on FBD.



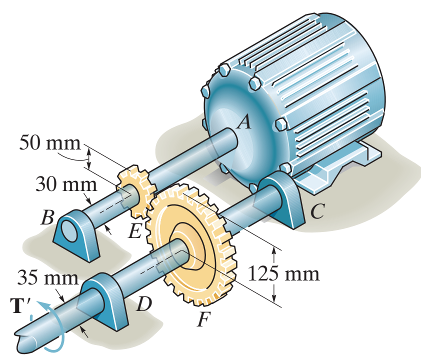
Maximum Shear Stress: Applying torsion Formula.

$$(\tau_{AB})_{\max} = \frac{T_{AB}\,c}{J} = \frac{50.0(0.015)}{\frac{\pi}{2}(0.015^4)} = 9.43\ \text{MPa} \qquad \textbf{Ans.}$$

$$(\tau_{CD})_{\max} = \frac{T_{CD}c}{J} = \frac{125(0.0175)}{\frac{\pi}{2}(0.0175^4)} = 14.8\ \text{MPa} \qquad \textbf{Ans.}$$



•5–13. If the applied torque on shaft CD is $T' = 75$ N·m, determine the absolute maximum shear stress in each shaft. The bearings B, C, and D allow free rotation of the shafts, and the motor holds the shafts fixed from rotating.



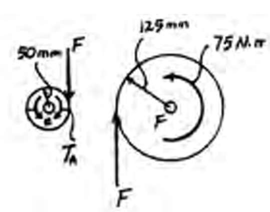
Equilibrium:

$\circlearrowleft +\Sigma M_F = 0; \qquad 75 - F(0.125) = 0; \qquad F = 600\ \text{N}$

$\circlearrowleft +\Sigma M_E = 0; \qquad 600(0.05) - T_A = 0$

$$T_A = 30.0\ \text{N}\cdot\text{m}$$

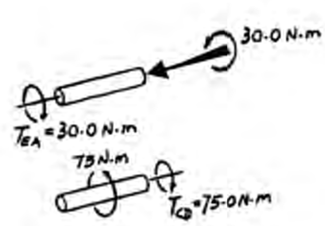
Internal Torque: As shown on FBD.



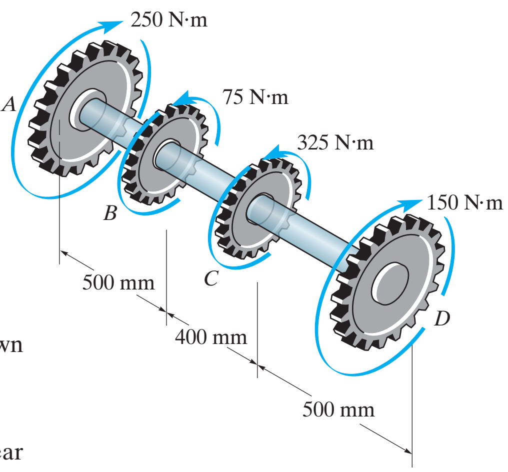
Maximum Shear Stress: Applying the torsion formula

$$(\tau_{EA})_{\max} = \frac{T_{EA}\,c}{J} = \frac{30.0(0.015)}{\frac{\pi}{2}(0.015^4)} = 5.66\ \text{MPa} \qquad \textbf{Ans.}$$

$$(\tau_{CD})_{\max} = \frac{T_{CD}c}{J} = \frac{75.0(0.0175)}{\frac{\pi}{2}(0.0175^4)} = 8.91\ \text{MPa} \qquad \textbf{Ans.}$$



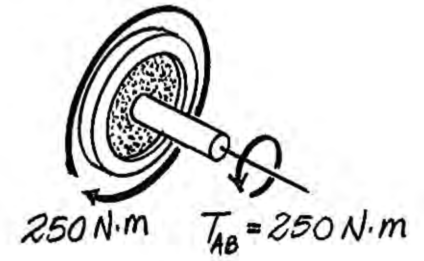
5–14. The solid 50-mm-diameter shaft is used to transmit the torques applied to the gears. Determine the absolute maximum shear stress in the shaft.



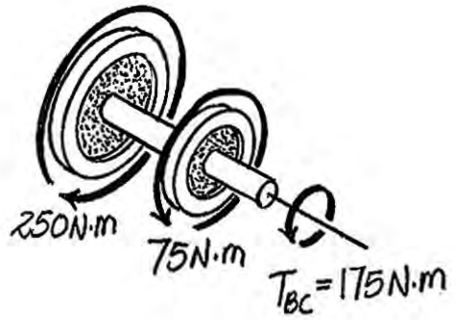
The internal torque developed in segments AB, BC and CD of the shaft are shown in Figs. a, b and c.

The maximum torque occurs in segment AB. Thus, the absolute maximum shear stress occurs in this segment. The polar moment of inertia of the shaft is $J = \frac{\pi}{2}(0.025^4) = 0.1953\pi(10^{-6})\text{m}^4$. Thus,

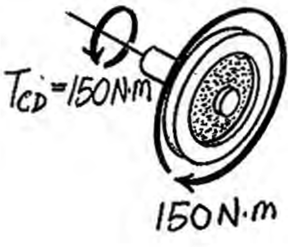
$$\left(\tau_{\max}\right)_{\text{abs}} = \frac{T_{AB}\,c}{J} = \frac{250(0.025)}{0.1953\pi(10^{-6})} = 10.19(10^6)\text{Pa} = 10.2\text{ MPa} \qquad \textbf{Ans.}$$



(a)

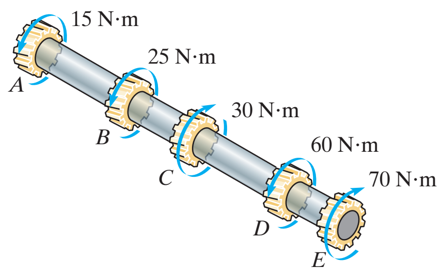


(b)



(c)

5–15. The solid shaft is made of material that has an allowable shear stress of $\tau_{allow} = 10$ MPa. Determine the required diameter of the shaft to the nearest mm.

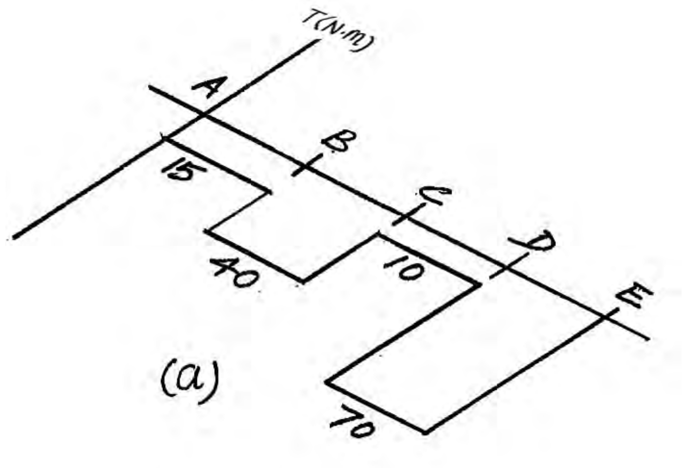


The internal torques developed in each segment of the shaft are shown in the torque diagram, Fig. a.

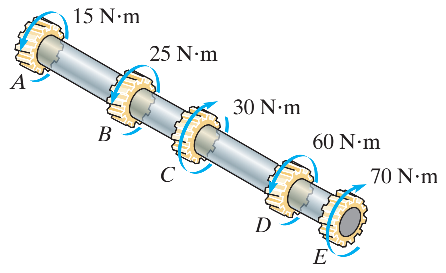
Segment DE is critical since it is subjected to the greatest internal torque. The polar moment of inertia of the shaft is $J = \frac{\pi}{2}\left(\frac{d}{2}\right)^4 = \frac{\pi}{32}d^4$. Thus,

$$\tau_{allow} = \frac{T_{DE}\,c}{J}; \qquad 10(10^6) = \frac{70\left(\frac{d}{2}\right)}{\frac{\pi}{32}d^4}$$

$$d = 0.03291 \text{ m} = 32.91 \text{ mm} = 33 \text{ mm} \qquad \textbf{Ans.}$$



***5–16.** The solid shaft has a diameter of 40 mm. Determine the absolute maximum shear stress in the shaft and sketch the shear-stress distribution along a radial line of the shaft where the shear stress is maximum.

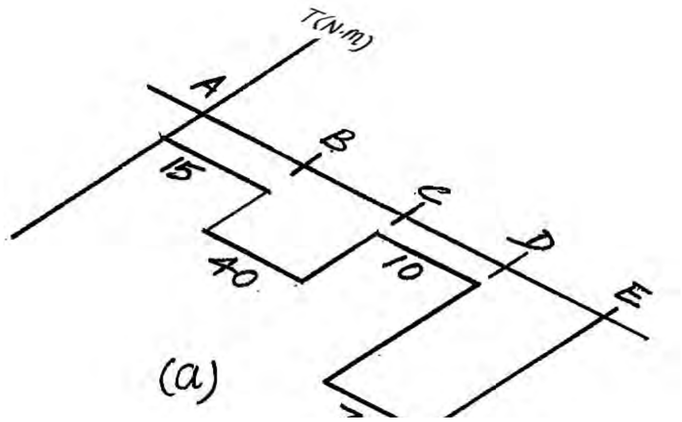


The internal torque developed in each segment of the shaft are shown in the torque diagram, Fig. a.

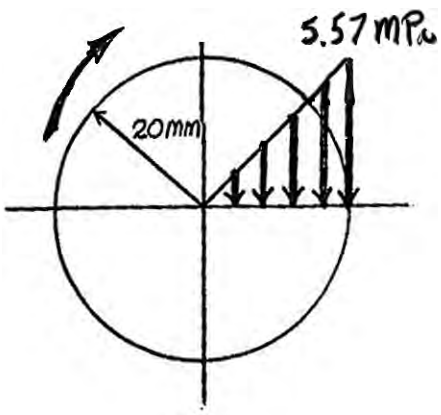
Since segment DE subjected to the greatest torque, the absolute maximum shear stress occurs here. The polar moment of inertia of the shaft is $J = \frac{\pi}{2}(0.02^4) = 80(10^{-9})\pi \text{ m}^4$. Thus,

$$\tau_{max} = \frac{T_{DE}\, c}{J} = \frac{70(0.02)}{80(10^{-9})\pi} = 5.57(10^6) \text{ Pa} = 5.57 \text{ MPa} \qquad \textbf{Ans.}$$

The shear stress distribution along the radial line is shown in Fig. b.

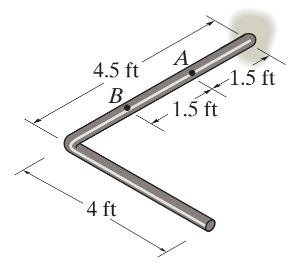


(a)



(b)

•5–17. The rod has a diameter of 1 in. and a weight of 10 lb/ft. Determine the maximum torsional stress in the rod at a section located at A due to the rod's weight.



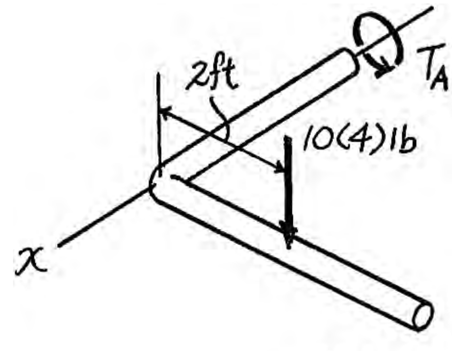
Here, we are only interested in the internal torque. Thus, other components of the internal loading are not indicated in the FBD of the cut segment of the rod, Fig. a.

$$\Sigma M_x = 0; \qquad T_A - 10(4)(2) = 0 \qquad T_A = 80 \text{ lb} \cdot \text{ft}\left(\frac{12\text{in}}{1\text{ft}}\right) = 960 \text{ lb} \cdot \text{in.}$$

The polar moment of inertia of the cross section at A is $J = \frac{\pi}{2}(0.5^4) = 0.03125\pi \text{ in}^4$.

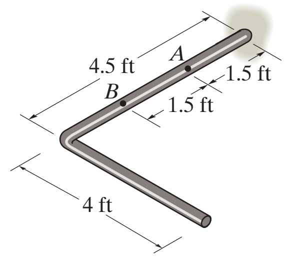
Thus

$$\tau_{max} = \frac{T_A\, c}{J} = \frac{960\ (0.5)}{0.03125\pi} = 4889.24 \text{ psi} = 4.89 \text{ ksi} \qquad \textbf{Ans.}$$



(a)

5–18. The rod has a diameter of 1 in. and a weight of 15 lb/ft. Determine the maximum torsional stress in the rod at a section located at B due to the rod's weight.

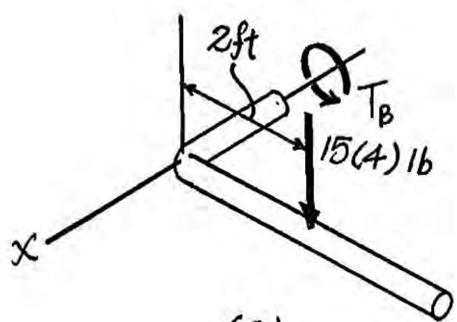


Here, we are only interested in the internal torque. Thus, other components of the internal loading are not indicated in the FBD of the cut segment of the rod, Fig. a.

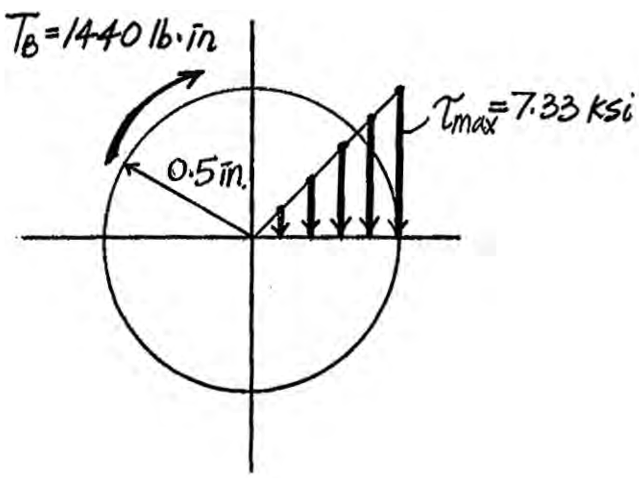
$$\Sigma M_x = 0; \qquad T_B - 15(4)(2) = 0 \qquad T_B = 120\ \text{lb}\cdot\text{ft}\left(\frac{12\ \text{in}}{1\text{ft}}\right) = 1440\ \text{lb}\cdot\text{in}.$$

The polar moment of inertia of the cross-section at B is $J = \frac{\pi}{2}(0.5^4) = 0.03125\pi\ \text{in}^4$. Thus,

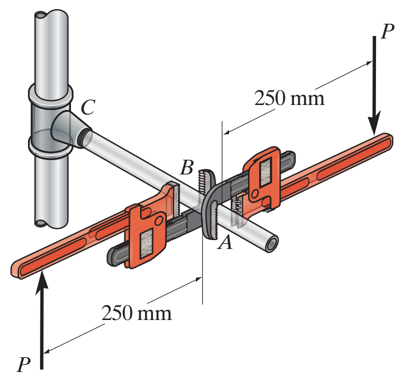
$$\tau_{\max} = \frac{T_B\, c}{J} = \frac{1440(0.5)}{0.03125\pi} = 7333.86\ \text{psi} = 7.33\ \text{ksi} \qquad \textbf{Ans.}$$



(a)



5–19. Two wrenches are used to tighten the pipe. If $P = 300$ N is applied to each wrench, determine the maximum torsional shear stress developed within regions AB and BC. The pipe has an outer diameter of 25 mm and inner diameter of 20 mm. Sketch the shear stress distribution for both cases.



Internal Loadings: The internal torque developed in segments AB and BC of the pipe can be determined by writing the moment equation of equilibrium about the x axis by referring to their respective free - body diagrams shown in Figs. *a* and *b*.

$$\Sigma M_x = 0;\ T_{AB} - 300(0.25) = 0 \qquad T_{AB} = 75\,\text{N}\cdot\text{m}$$

And

$$\Sigma M_x = 0;\ T_{BC} - 300(0.25) - 300(0.25) = 0 \qquad T_{BC} = 150\,\text{N}\cdot\text{m}$$

Allowable Shear Stress: The polar moment of inertia of the pipe is $J = \frac{\pi}{2}\left(0.0125^4 - 0.01^4\right) = 22.642(10^{-9})\text{m}^4$.

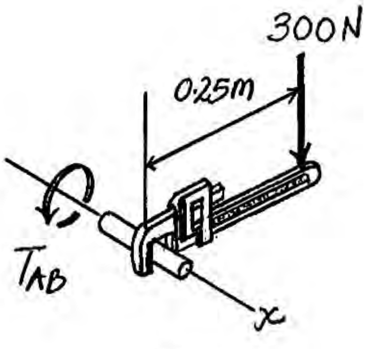
$$\left(\tau_{\max}\right)_{AB} = \frac{T_{AB}\,c}{J} = \frac{75(0.0125)}{22.642(10^{-9})} = 41.4\text{ MPa} \qquad \textbf{Ans.}$$

$$\left(\tau_{AB}\right)_{\rho=0.01\text{ m}} = \frac{T_{AB}\,\rho}{J} = \frac{75(0.01)}{22.642(10^{-9})} = 33.1\text{ MPa}$$

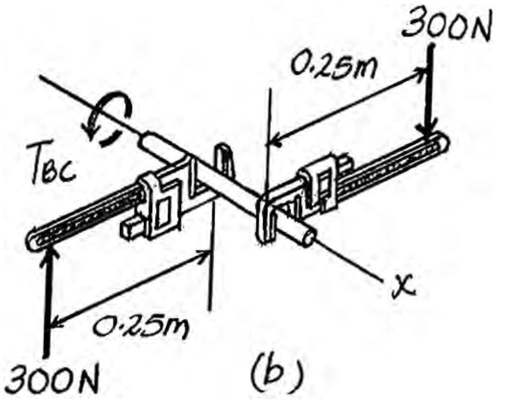
$$\left(\tau_{\max}\right)_{BC} = \frac{T_{BC}\,c}{J} = \frac{150(0.0125)}{22.642(10^{-9})} = 82.8\text{ MPa} \qquad \textbf{Ans.}$$

$$\left(\tau_{BC}\right)_{\rho=0.01\text{ m}} = \frac{T_{BC}\,\rho}{J} = \frac{150(0.01)}{22.642(10^{-9})} = 66.2\text{ MPa}$$

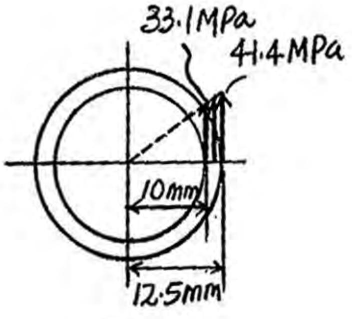
The shear stress distribution along the radial line of segments AB and BC of the pipe is shown in Figs. *c* and *d*, respectively.



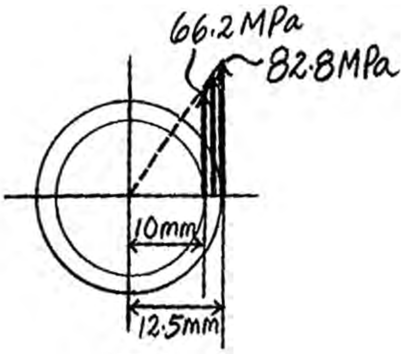
(a)



(b)

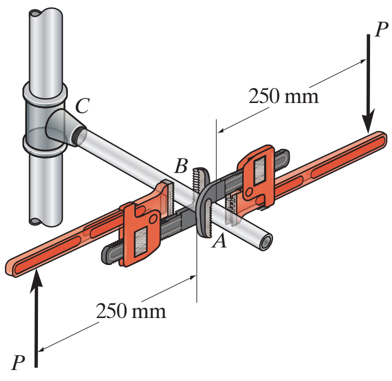


(c)



(d)

***5–20.** Two wrenches are used to tighten the pipe. If the pipe is made from a material having an allowable shear stress of $\tau_{allow} = 85$ MPa, determine the allowable maximum force **P** that can be applied to each wrench. The pipe has an outer diameter of 25 mm and inner diameter of 20 mm.



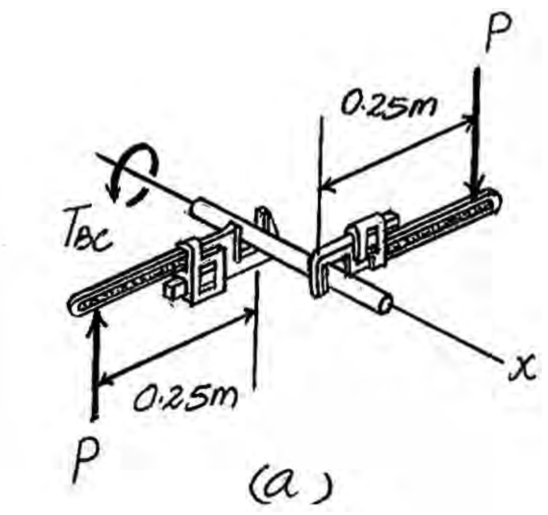
Internal Loading: By observation, segment BC of the pipe is critical since it is subjected to a greater internal torque than segment AB. Writing the moment equation of equilibrium about the x axis by referring to the free-body diagram shown in Fig. a, we have

$$\Sigma M_x = 0; \quad T_{BC} - P(0.25) - P(0.25) = 0 \qquad T_{BC} = 0.5P$$

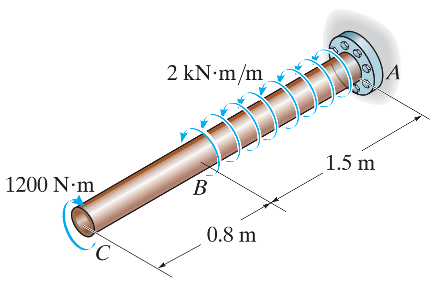
Allowable Shear Stress: The polar moment of inertia of the pipe is $J = \frac{\pi}{2}\left(0.0125^4 - 0.01^4\right) = 22.642(10^{-9})\text{m}^4$

$$\tau_{allow} = \frac{T_{BC}\,c}{J}; \qquad 85(10^6) = \frac{0.5P(0.0125)}{22.642(10^{-9})}$$

$$P = 307.93\text{N} = 308\text{N}$$ **Ans.**



•5–21. The 60-mm-diameter solid shaft is subjected to the distributed and concentrated torsional loadings shown. Determine the absolute maximum and minimum shear stresses on the outer surface of the shaft and specify their locations, measured from the fixed end A.



The internal torque for segment BC is Constant $T_{BC} = 1200\ \text{N}\cdot\text{m}$, Fig. a. However, the internal for segment AB varies with x, Fig. b.

$$T_{AB} - 2000x + 1200 = 0 \qquad T_{AB} = (2000x - 1200)\ \text{N}\cdot\text{m}$$

The minimum shear stress occurs when the internal torque is zero in segment AB. By setting $T_{AB} = 0$,

$$0 = 2000x - 1200 \qquad x = 0.6\ \text{m} \qquad \textbf{Ans.}$$

And

$$d = 1.5\ \text{m} - 0.6\ \text{m} = 0.9\ \text{m} \qquad \textbf{Ans.}$$

$$\tau_{\min} = 0 \qquad \textbf{Ans.}$$

The maximum shear stress occurs when the internal torque is the greatest. This occurs at fixed support A where

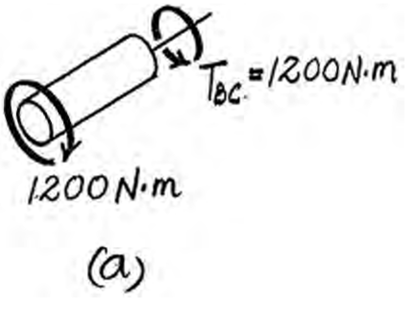
$$d = 0 \qquad \textbf{Ans.}$$

At this location,

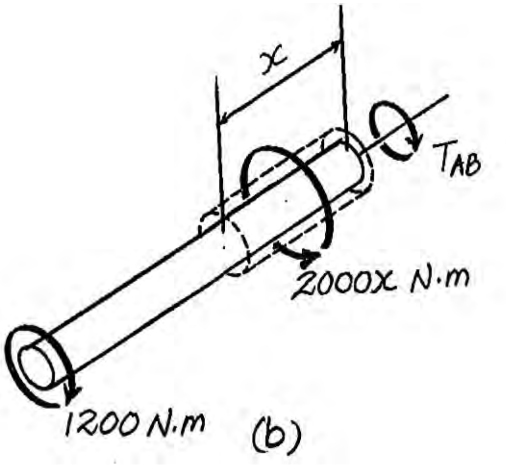
$$(T_{AB})_{\max} = 2000(1.5) - 1200 = 1800\ \text{N}\cdot\text{m}$$

The polar moment of inertia of the rod is $J = \frac{\pi}{2}(0.03^4) = 0.405(10^{-6})\pi$. Thus,

$$\tau_{\max} = \frac{(T_{AB})_{\max}\, c}{J} = \frac{1800(0.03)}{0.405(10^{-6})\pi} = 42.44(10^6)\text{Pa} = 42.4\ \text{MPa} \qquad \textbf{Ans.}$$

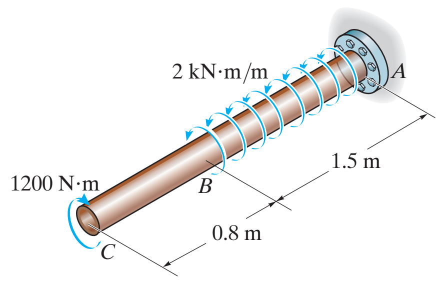


(a)



(b)

5–22. The solid shaft is subjected to the distributed and concentrated torsional loadings shown. Determine the required diameter d of the shaft to the nearest mm if the allowable shear stress for the material is $\tau_{\text{allow}} = 50$ MPa.



The internal torque for segment BC is constant $T_{BC} = 1200\ \text{N}\cdot\text{m}$, Fig. a. However, the internal torque for segment AB varies with x, Fig. b.

$$T_{AB} - 2000x + 1200 = 0 \quad T_{AB} = (2000x - 1200)\ \text{N}\cdot\text{m}$$

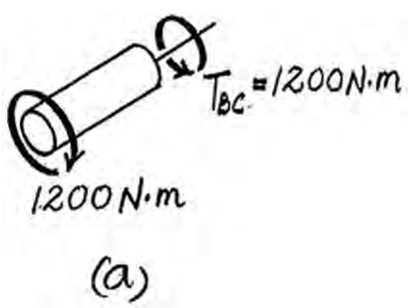
For segment AB, the maximum internal torque occurs at fixed support A where $x = 1.5$ m. Thus,

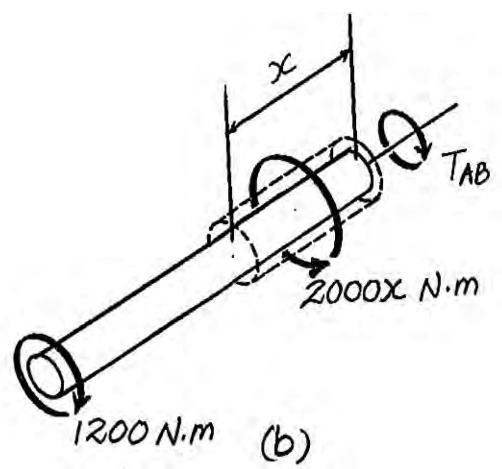
$$\left(T_{AB}\right)_{\max} = 2000(1.5) - 1200 = 1800\ \text{N}\cdot\text{m}$$

Since $\left(T_{AB}\right)_{\max} > T_{BC}$, the critical cross-section is at A. The polar moment of inertia of the rod is $J = \frac{\pi}{2}\left(\frac{d}{2}\right)^4 = \frac{\pi d^4}{32}$. Thus,

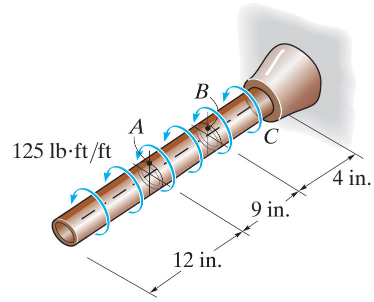
$$\tau_{\text{allow}} = \frac{Tc}{J}; \qquad 50(10^6) = \frac{1800(d/2)}{\pi d^4/32}$$

$$d = 0.05681\ \text{m} = 56.81\ \text{mm} = 57\ \text{mm} \qquad \textbf{Ans.}$$





***5–24.** The copper pipe has an outer diameter of 2.50 in. and an inner diameter of 2.30 in. If it is tightly secured to the wall at C and a uniformly distributed torque is applied to it as shown, determine the shear stress developed at points A and B. These points lie on the pipe's outer surface. Sketch the shear stress on volume elements located at A and B.



Internal Torque: As shown on FBD.

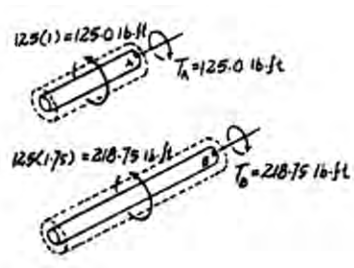
Maximum Shear Stress: Applying the torsion formula

$$\tau_A = \frac{T_A\, c}{J}$$

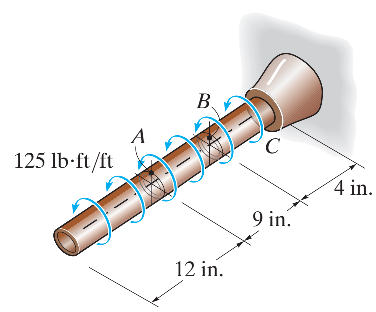
$$= \frac{125.0(12)(1.25)}{\frac{\pi}{2}(1.25^4 - 1.15^4)} = 1.72 \text{ ksi} \qquad \textbf{Ans.}$$

$$\tau_B = \frac{T_B\, c}{J}$$

$$= \frac{218.75(12)(1.25)}{\frac{\pi}{2}(1.25^4 - 1.15^4)} = 3.02 \text{ ksi} \qquad \textbf{Ans.}$$



•5–25. The copper pipe has an outer diameter of 2.50 in. and an inner diameter of 2.30 in. If it is tightly secured to the wall at C and it is subjected to the uniformly distributed torque along its entire length, determine the absolute maximum shear stress in the pipe. Discuss the validity of this result.



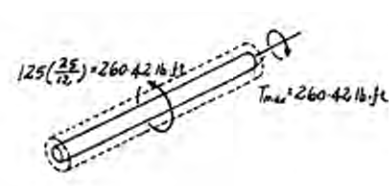
Internal Torque: The maximum torque occurs at the support C.

$$T_{\max} = (125 \text{ lb}\cdot\text{ft/ft})\left(\frac{25 \text{ in.}}{12 \text{ in./ft}}\right) = 260.42 \text{ lb}\cdot\text{ft}$$

Maximum Shear Stress: Applying the torsion formula

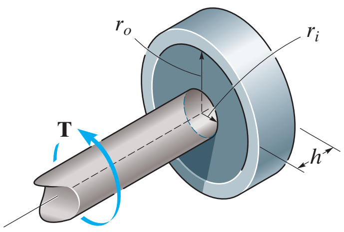
$$\tau_{\substack{\text{abs}\\ \text{max}}} = \frac{T_{\max}\, c}{J}$$

$$= \frac{260.42(12)(1.25)}{\frac{\pi}{2}(1.25^4 - 1.15^4)} = 3.59 \text{ ksi} \qquad \textbf{Ans.}$$



According to Saint-Venant's principle, application of the torsion formula should be as points sufficiently removed from the supports or points of concentrated loading.

5–26. A cylindrical spring consists of a rubber annulus bonded to a rigid ring and shaft. If the ring is held fixed and a torque **T** is applied to the shaft, determine the maximum shear stress in the rubber.

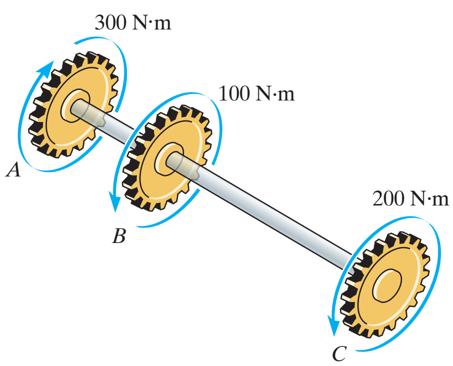


$$\tau = \frac{F}{A} = \frac{\frac{T}{r}}{2\,\pi\,r\,h} = \frac{T}{2\pi\,r^2\,h}$$

Shear stress is maximum when r is the smallest, i.e. $r = r_i$. Hence,

$$\tau_{\max} = \frac{T}{2\pi\,r_i^{\,2}\,h}$$ **Ans.**

5–27. The A-36 steel shaft is supported on smooth bearings that allow it to rotate freely. If the gears are subjected to the torques shown, determine the maximum shear stress developed in the segments AB and BC. The shaft has a diameter of 40 mm.

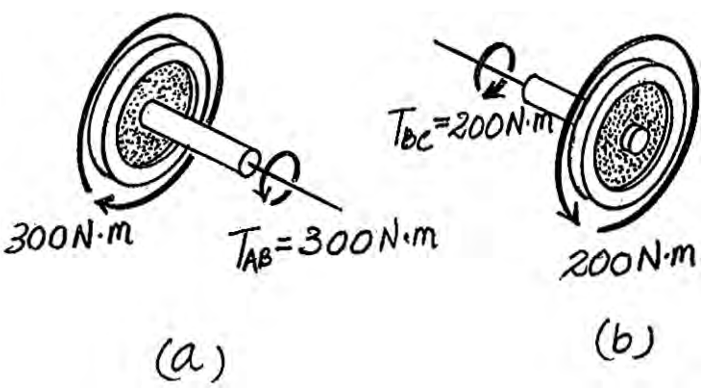


The internal torque developed in segments AB and BC are shown in their respective FBDs, Figs. a and b.

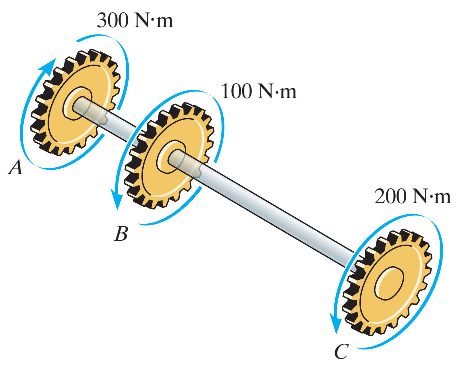
The polar moment of inertia of the shaft is $J = \frac{\pi}{2}(0.02^4) = 80(10^{-9})\pi$ m^4. Thus,

$$\left(\tau_{AB}\right)_{\max} = \frac{T_{AB}\,c}{J} = \frac{300(0.02)}{80(10^{-9})\pi} = 23.87(10^6)\text{Pa} = 23.9\text{ MPa}$$ **Ans.**

$$\left(\tau_{BC}\right)_{\max} = \frac{T_{BC}\,c}{J} = \frac{200(0.02)}{80(10^{-9})\pi} = 15.92(10^6)\text{ Pa} = 15.9\text{ MPa}$$ **Ans.**



***5–28.** The A-36 steel shaft is supported on smooth bearings that allow it to rotate freely. If the gears are subjected to the torques shown, determine the required diameter of the shaft to the nearest mm if $\tau_{\text{allover}} = 60$ MPa.

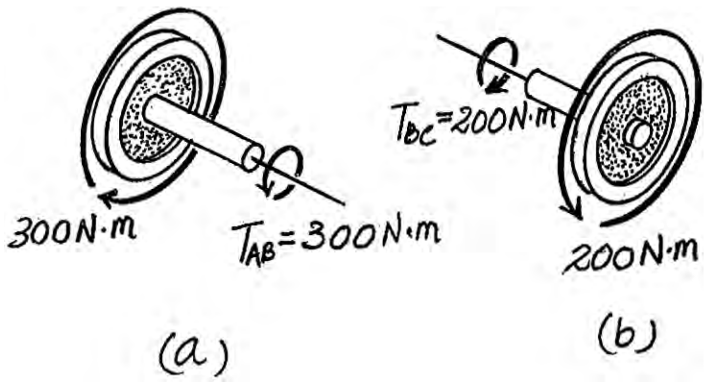


The internal torque developed in segments AB and BC are shown in their respective FBDs, Fig. *a* and *b*

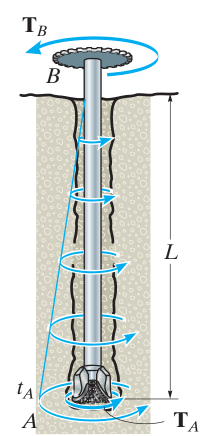
Here, segment AB is critical since its internal torque is the greatest. The polar moment of inertia of the shaft is $J = \frac{\pi}{2}\left(\frac{d}{2}\right)^4 = \frac{\pi d^4}{32}$. Thus,

$$\tau_{\text{allow}} = \frac{T_C}{J}; \qquad 60(10^6) = \frac{300(d/2)}{\pi d^4/32}$$

$$d = 0.02942 \text{ m} = 30 \text{ mm} \qquad \textbf{Ans.}$$



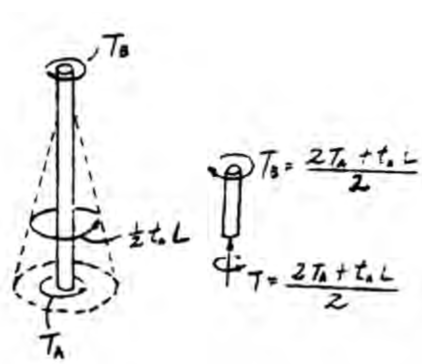
•5–29. When drilling a well at constant angular velocity, the bottom end of the drill pipe encounters a torsional resistance T_A. Also, soil along the sides of the pipe creates a distributed frictional torque along its length, varying uniformly from zero at the surface B to t_A at A. Determine the minimum torque T_B that must be supplied by the drive unit to overcome the resisting torques, and compute the maximum shear stress in the pipe. The pipe has an outer radius r_o and an inner radius r_i.



$$T_A + \frac{1}{2}t_A L - T_B = 0$$

$$T_B = \frac{2T_A + t_A L}{2} \qquad \textbf{Ans.}$$

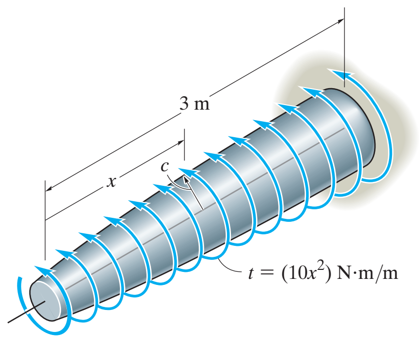
Maximum shear stress: The maximum torque is within the region above the distributed torque.



$$\tau_{\max} = \frac{Tc}{J}$$

$$\tau_{\max} = \frac{\left[\frac{(2T_A + t_A L)}{2}\right](r_0)}{\frac{\pi}{2}(r_0^4 - r_i^4)} = \frac{(2T_A + t_A L)r_0}{\pi(r_0^4 - r_i^4)} \qquad \textbf{Ans.}$$

5–30. The shaft is subjected to a distributed torque along its length of $t = (10x^2)$ N·m/m, where x is in meters. If the maximum stress in the shaft is to remain constant at 80 MPa, determine the required variation of the radius c of the shaft for $0 \leq x \leq 3$ m.

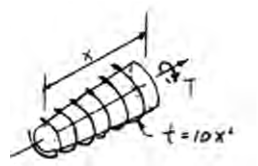


$$T = \int t\,dx = \int_0^x 10\,x^2 dx = \frac{10}{3}x^3$$

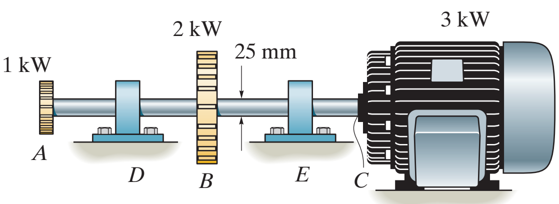
$$\tau = \frac{Tc}{J}; \qquad 80(10^6) = \frac{(\frac{10}{3})x^3\,c}{\frac{\pi}{2}\,c^4}$$

$$c^3 = 26.526(10^{-9})\,x^3$$

$$c = (2.98\,x)\text{ mm}$$ **Ans.**



5–31. The solid steel shaft AC has a diameter of 25 mm and is supported by smooth bearings at D and E. It is coupled to a motor at C, which delivers 3 kW of power to the shaft while it is turning at 50 rev/s. If gears A and B remove 1 kW and 2 kW, respectively, determine the maximum shear stress developed in the shaft within regions AB and BC. The shaft is free to turn in its support bearings D and E.



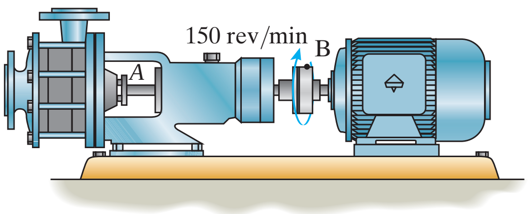
$$T_C = \frac{P}{\omega} = \frac{3(10^3)}{50(2\pi)} = 9.549\text{ N}\cdot\text{m}$$

$$T_A = \frac{1}{3}T_C = 3.183\text{ N}\cdot\text{m}$$

$$(\tau_{AB})_{max} = \frac{T_C}{J} = \frac{3.183\,(0.0125)}{\frac{\pi}{2}(0.0125^4)} = 1.04\text{ MPa}$$ **Ans.**

$$(\tau_{BC})_{max} = \frac{T_C}{J} = \frac{9.549\,(0.0125)}{\frac{\pi}{2}(0.0125^4)} = 3.11\text{ MPa}$$ **Ans.**

***5–32.** The pump operates using the motor that has a power of 85 W. If the impeller at B is turning at 150 rev/min, determine the maximum shear stress developed in the 20-mm-diameter transmission shaft at A.



Internal Torque:

$$\omega = 150 \frac{\text{rev}}{\text{min}}\left(\frac{2\pi \text{ rad}}{\text{rev}}\right)\frac{1 \text{ min}}{60 \text{ s}} = 5.00\pi \text{ rad/s}$$

$$P = 85 \text{ W} = 85 \text{ N}\cdot\text{m/s}$$

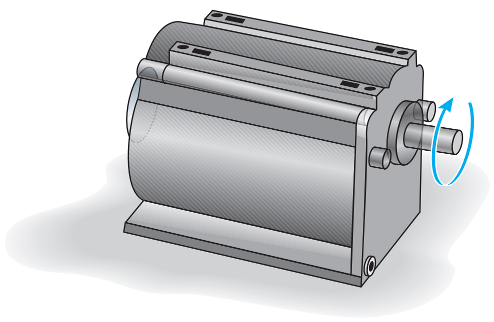
$$T = \frac{P}{\omega} = \frac{85}{5.00\pi} = 5.411 \text{ N}\cdot\text{m}$$

Maximum Shear Stress: Applying torsion formula

$$\tau_{\max} = \frac{T\,c}{J}$$

$$= \frac{5.411\,(0.01)}{\frac{\pi}{2}(0.01^4)} = 3.44 \text{ MPa}$$ **Ans.**

•5–33. The gear motor can develop 2 hp when it turns at 450 rev/min. If the shaft has a diameter of 1 in., determine the maximum shear stress developed in the shaft.



The angular velocity of the shaft is

$$\omega = \left(450 \frac{\cancel{\text{rev}}}{\cancel{\text{min}}}\right)\left(\frac{2\pi \text{ rad}}{1 \cancel{\text{rev}}}\right)\left(\frac{1 \cancel{\text{min}}}{60 \text{ s}}\right) = 15\pi \text{ rad/s}$$

and the power is

$$P = 2 \text{ hp}\left(\frac{550 \text{ ft}\cdot\text{lb/s}}{1 \text{ hp}}\right) = 1100 \text{ ft}\cdot\text{lb/s}$$

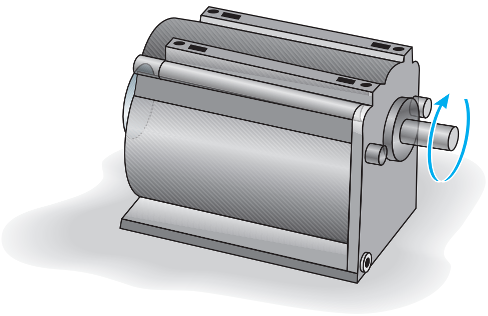
Then

$$T = \frac{P}{\omega} = \frac{1100}{15\pi} = 23.34 \text{ lb}\cdot\text{ft}\left(\frac{12 \text{ in}}{1\text{ft}}\right) = 280.11 \text{ lb}\cdot\text{in}$$

The polar moment of inertia of the shaft is $J = \frac{\pi}{2}(0.5^4) = 0.03125\pi \text{ in}^4$. Thus,

$$\tau_{\max} = \frac{T\,c}{J} = \frac{280.11\,(0.5)}{0.03125\pi} = 1426.60 \text{ psi} = 1.43 \text{ ksi}$$ **Ans.**

5–34. The gear motor can develop 3 hp when it turns at 150 rev/min. If the allowable shear stress for the shaft is $\tau_{\text{allow}} = 12$ ksi, determine the smallest diameter of the shaft to the nearest $\frac{1}{8}$ in. that can be used.



The angular velocity of the shaft is

$$\omega = \left(150\,\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 5\pi\text{ rad/s}$$

and the power is

$$P = (3\text{ hp})\left(\frac{550\text{ ft}\cdot\text{lb/s}}{1\text{ hp}}\right) = 1650\text{ ft}\cdot\text{lb/s}$$

Then

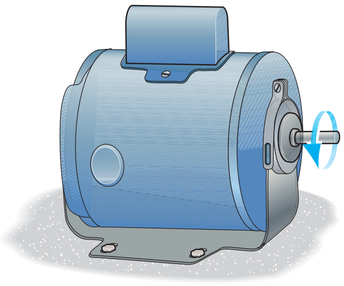
$$T = \frac{P}{\omega} = \frac{1650}{5\pi} = (105.04\text{ lb}\cdot\text{ft})\left(\frac{12\text{ in}}{1\text{ ft}}\right) = 1260.51\text{ lb}\cdot\text{in}$$

The polar moment of inertia of the shaft is $J = \frac{\pi}{2}\left(\frac{d}{2}\right)^4 = \frac{\pi d^4}{32}$. Thus,

$$\tau_{\text{allow}} = \frac{T\,c}{J}; \qquad 12(10^3) = \frac{1260.51\,(d/2)}{\pi d^4/32}$$

$$d = 0.8118\text{ in.} = \frac{7}{8}\text{ in.} \qquad \textbf{Ans.}$$

5–35. The 25-mm-diameter shaft on the motor is made of a material having an allowable shear stress of $\tau_{\text{allow}} = 75$ MPa. If the motor is operating at its maximum power of 5 kW, determine the minimum allowable rotation of the shaft.



Allowable Shear Stress: The polar moment of inertia of the shaft is $J = \frac{\pi}{2}\left(0.0125^4\right) = 38.3495(10^{-9})\text{ m}^4$.

$$\tau_{\text{allow}} = \frac{Tc}{J}; \qquad 75(10^6) = \frac{T(0.0125)}{38.3495(10^{-9})}$$

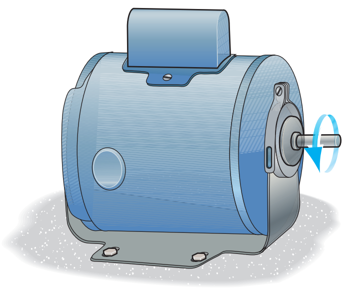
$$T = 230.10\text{ N}\cdot\text{m}$$

Internal Loading:

$$T = \frac{P}{\omega}; \qquad 230.10 = \frac{5(10^3)}{\omega}$$

$$\omega = 21.7\text{ rad/s} \qquad \textbf{Ans.}$$

***5–36.** The drive shaft of the motor is made of a material having an allowable shear stress of $\tau_{allow} = 75$ MPa. If the outer diameter of the tubular shaft is 20 mm and the wall thickness is 2.5 mm, determine the maximum allowable power that can be supplied to the motor when the shaft is operating at an angular velocity of 1500 rev/min.



Internal Loading: The angular velocity of the shaft is

$$\omega = \left(1500\,\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 50\pi\text{ rad/s}$$

We have

$$T = \frac{P}{\omega} = \frac{P}{50\pi}$$

Allowable Shear Stress: The polar moment of inertia of the shaft is $J = \frac{\pi}{2}\left(0.01^4 - 0.0075^4\right) = 10.7379(10^{-9})\text{ m}^4$.

$$\tau_{allow} = \frac{Tc}{J}; \qquad 75(10^6) = \frac{\left(\frac{P}{50\pi}\right)(0.01)}{10.7379(10^{-9})}$$

$$P = 12\,650.25\text{ W} = 12.7\text{ kW} \qquad \textbf{Ans.}$$

•5–37. A ship has a propeller drive shaft that is turning at 1500 rev/min while developing 1800 hp. If it is 8 ft long and has a diameter of 4 in., determine the maximum shear stress in the shaft caused by torsion.

Internal Torque:

$$\omega = 1500\,\frac{\text{rev}}{\text{min}}\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\frac{1\text{ min}}{60\text{ s}} = 50.0\,\pi\text{ rad/s}$$

$$P = 1800\text{ hp}\left(\frac{550\text{ ft}\cdot\text{lb/s}}{1\text{ hp}}\right) = 990\,000\text{ ft}\cdot\text{lb/s}$$

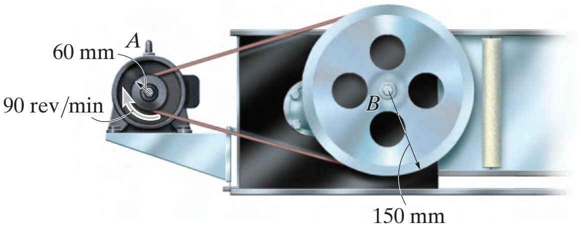
$$T = \frac{P}{\omega} = \frac{990\,000}{50.0\pi} = 6302.54\text{ lb}\cdot\text{ft}$$

Maximum Shear Stress: Applying torsion formula

$$\tau_{max} = \frac{Tc}{J} = \frac{6302.54(12)(2)}{\frac{\pi}{2}(2^4)}$$

$$= 6018\text{ psi} = 6.02\text{ ksi} \qquad \textbf{Ans.}$$

5–38. The motor A develops a power of 300 W and turns its connected pulley at 90 rev/min. Determine the required diameters of the steel shafts on the pulleys at A and B if the allowable shear stress is $\tau_{allow} = 85$ MPa.



Internal Torque: For shafts A and B

$$\omega_A = 90\,\frac{\text{rev}}{\text{min}}\left(\frac{2\pi\text{ rad}}{\text{rev}}\right)\frac{1\text{ min}}{60\text{ s}} = 3.00\pi\text{ rad/s}$$

$$P = 300\text{ W} = 300\text{ N}\cdot\text{m/s}$$

$$T_A = \frac{P}{\omega_A} = \frac{300}{3.00\pi} = 31.83\text{ N}\cdot\text{m}$$

$$\omega_B = \omega_A\left(\frac{r_A}{r_B}\right) = 3.00\pi\left(\frac{0.06}{0.15}\right) = 1.20\pi\text{ rad/s}$$

$$P = 300\text{ W} = 300\text{ N}\cdot\text{m/s}$$

$$T_B = \frac{P}{\omega_B} = \frac{300}{1.20\pi} = 79.58\text{ N}\cdot\text{m}$$

Allowable Shear Stress: For shaft A

$$\tau_{max} = \tau_{allow} = \frac{T_A c}{J}$$

$$85\left(10^6\right) = \frac{31.83\left(\frac{d_A}{2}\right)}{\frac{\pi}{2}\left(\frac{d_A}{2}\right)^4}$$

$$d_A = 0.01240\text{ m} = 12.4\text{ mm}$$ **Ans.**

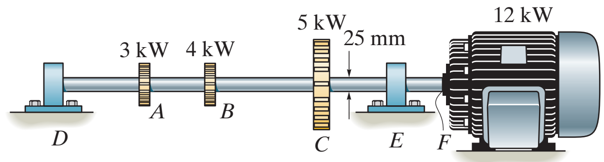
For shaft B

$$\tau_{max} = \tau_{allow} = \frac{T_B c}{J}$$

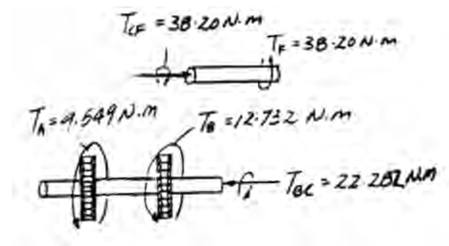
$$85\left(10^6\right) = \frac{79.58\left(\frac{d_B}{2}\right)}{\frac{\pi}{2}\left(\frac{d_B}{2}\right)^4}$$

$$d_B = 0.01683\text{ m} = 16.8\text{ mm}$$ **Ans.**

5–39. The solid steel shaft DF has a diameter of 25 mm and is supported by smooth bearings at D and E. It is coupled to a motor at F, which delivers 12 kW of power to the shaft while it is turning at 50 rev/s. If gears A, B, and C remove 3 kW, 4 kW, and 5 kW respectively, determine the maximum shear stress developed in the shaft within regions CF and BC. The shaft is free to turn in its support bearings D and E.



$$\omega = 50\,\frac{\text{rev}}{\text{s}}\left[\frac{2\pi\ \text{rad}}{\text{rev}}\right] = 100\,\pi\ \text{rad/s}$$



$$T_F = \frac{P}{\omega} = \frac{12(10^3)}{100\,\pi} = 38.20\ \text{N}\cdot\text{m}$$

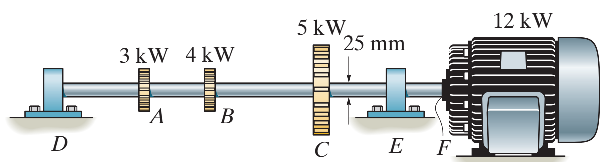
$$T_A = \frac{P}{\omega} = \frac{3(10^3)}{100\,\pi} = 9.549\ \text{N}\cdot\text{m}$$

$$T_B = \frac{P}{\omega} = \frac{4(10^3)}{100\,\pi} = 12.73\ \text{N}\cdot\text{m}$$

$$(\tau_{\max})_{CF} = \frac{T_{CF}\,c}{J} = \frac{38.20(0.0125)}{\frac{\pi}{2}(0.0125^4)} = 12.5\ \text{MPa}$$ **Ans.**

$$(\tau_{\max})_{BC} = \frac{T_{BC}\,c}{J} = \frac{22.282(0.0125)}{\frac{\pi}{2}(0.0125^4)} = 7.26\ \text{MPa}$$ **Ans.**

***5–40.** Determine the absolute maximum shear stress developed in the shaft in Prob. 5–39.

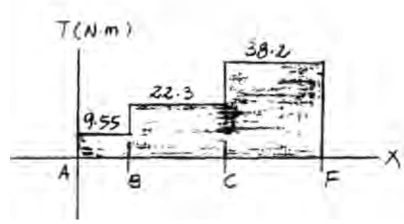


$$\omega = 50\,\frac{\text{rev}}{\text{s}}\left[\frac{2\pi\ \text{rad}}{\text{rev}}\right] = 100\,\pi\ \text{rad/s}$$

$$T_F = \frac{P}{\omega} = \frac{12(10^3)}{100\pi} = 38.20\ \text{N}\cdot\text{m}$$

$$T_A = \frac{P}{\omega} = \frac{3(10^3)}{100\pi} = 9.549\ \text{N}\cdot\text{m}$$

$$T_B = \frac{P}{\omega} = \frac{4(10^3)}{100\pi} = 12.73\ \text{N}\cdot\text{m}$$

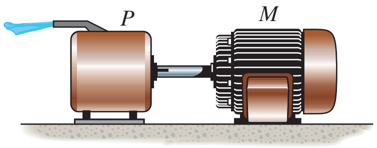


From the torque diagram,

$$T_{\max} = 38.2\ \text{N}\cdot\text{m}$$

$$\tau_{\substack{\text{abs}\\ \text{max}}} = \frac{Tc}{J} = \frac{38.2(0.0125)}{\frac{\pi}{2}(0.0125^4)} = 12.5\ \text{MPa}$$ **Ans.**

•5–41. The A-36 steel tubular shaft is 2 m long and has an outer diameter of 50 mm. When it is rotating at 40 rad/s, it transmits 25 kW of power from the motor M to the pump P. Determine the smallest thickness of the tube if the allowable shear stress is $\tau_{\text{allow}} = 80$ MPa.



The internal torque in the shaft is

$$T = \frac{P}{\omega} = \frac{25(10^3)}{40} = 625 \text{ N} \cdot \text{m}$$

The polar moment of inertia of the shaft is $J = \frac{\pi}{2}(0.025^4 - C_i{}^4)$. Thus,

$$\tau_{\text{allow}} = \frac{Tc}{J}; \qquad 80(10^6) = \frac{625(0.025)}{\frac{\pi}{2}(0.025^4 - C_i{}^4)}$$

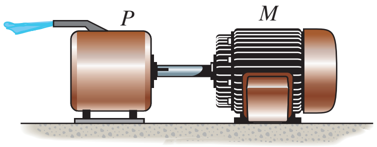
$$C_i = 0.02272 \text{ m}$$

So that

$$t = 0.025 - 0.02272$$

$$= 0.002284 \text{ m} = 2.284 \text{ mm} = 2.5 \text{ mm} \qquad \textbf{Ans.}$$

5–42. The A-36 solid tubular steel shaft is 2 m long and has an outer diameter of 60 mm. It is required to transmit 60 kW of power from the motor M to the pump P. Determine the smallest angular velocity the shaft can have if the allowable shear stress is $\tau_{\text{allow}} = 80$ MPa.



The polar moment of inertia of the shaft is $J = \frac{\pi}{2}(0.03^4) = 0.405(10^{-6})\pi \text{ m}^4$. Thus,

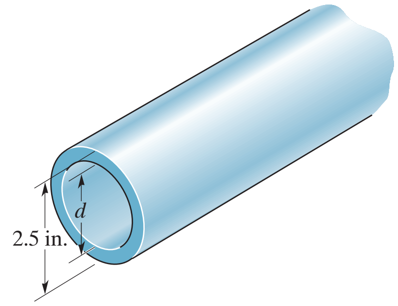
$$\tau_{\text{allow}} = \frac{Tc}{J}; \qquad 80(10^6) = \frac{T(0.03)}{0.405(10^{-6})\pi}$$

$$T = 3392.92 \text{ N} \cdot \text{m}$$

$$P = T\omega; \qquad 60(10^3) = 3392.92\,\omega$$

$$\omega = 17.68 \text{ rad/s} = 17.7 \text{ rad/s} \qquad \textbf{Ans.}$$

5–43. A steel tube having an outer diameter of 2.5 in. is used to transmit 35 hp when turning at 2700 rev/min. Determine the inner diameter d of the tube to the nearest $\frac{1}{8}$ in. if the allowable shear stress is $\tau_{\text{allow}} = 10$ ksi.



$$\omega = \frac{2700(2\pi)}{60} = 282.74 \text{ rad/s}$$

$$P = T\omega$$

$$35(550) = T(282.74)$$

$$T = 68.083 \text{ lb} \cdot \text{ft}$$

$$\tau_{\max} = \tau_{\text{allow}} = \frac{Tc}{J}$$

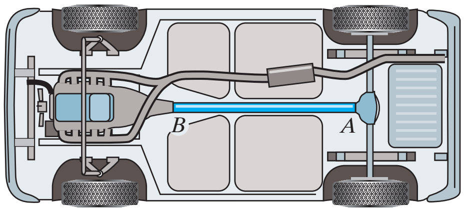
$$10(10^3) = \frac{68.083(12)(1.25)}{\frac{\pi}{2}(12.5^4 - {c_i}^4)}$$

$$c_i = 1.2416 \text{ in.}$$

$$d = 2.48 \text{ in.}$$

Use $d = 2\frac{1}{2}$ in. **Ans.**

***5–44.** The drive shaft AB of an automobile is made of a steel having an allowable shear stress of $\tau_{\text{allow}} = 8$ ksi. If the outer diameter of the shaft is 2.5 in. and the engine delivers 200 hp to the shaft when it is turning at 1140 rev/min, determine the minimum required thickness of the shaft's wall.



$$\omega = \frac{1140(2\pi)}{60} = 119.38 \text{ rad/s}$$

$$P = T\omega$$

$$200(550) = T(119.38)$$

$$T = 921.42 \text{ lb} \cdot \text{ft}$$

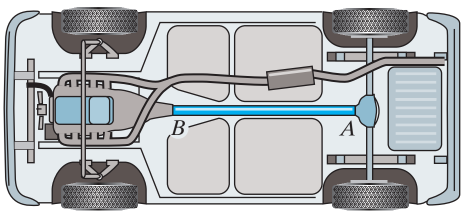
$$\tau_{\text{allow}} = \frac{Tc}{J}$$

$$8(10^3) = \frac{921.42(12)(1.25)}{\frac{\pi}{2}(1.25^4 - r_i^4)}, \qquad r_i = 1.0762 \text{ in.}$$

$$t = r_o - r_i = 1.25 - 1.0762$$

$$t = 0.174 \text{ in.}$$ **Ans.**

•5–45. The drive shaft AB of an automobile is to be designed as a thin-walled tube. The engine delivers 150 hp when the shaft is turning at 1500 rev/min. Determine the minimum thickness of the shaft's wall if the shaft's outer diameter is 2.5 in. The material has an allowable shear stress of $\tau_{allow} = 7$ ksi.



$$\omega = \frac{1500(2\pi)}{60} = 157.08 \text{ rad/s}$$

$$P = T\omega$$

$$150(550) = T(157.08)$$

$$T = 525.21 \text{ lb} \cdot \text{ft}$$

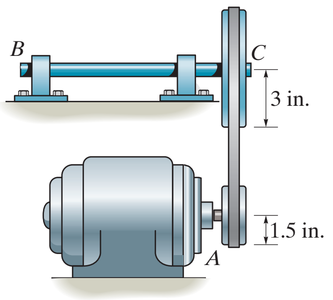
$$\tau_{allow} = \frac{Tc}{J}$$

$$7(10^3) = \frac{525.21(12)(1.25)}{\frac{\pi}{2}(1.25^4 - r_i^4)}, \qquad r_i = 1.1460 \text{ in.}$$

$$t = r_o - r_i = 1.25 - 1.1460$$

$$t = 0.104 \text{ in.}$$ **Ans.**

5–46. The motor delivers 15 hp to the pulley at A while turning at a constant rate of 1800 rpm. Determine to the nearest $\frac{1}{8}$ in. the smallest diameter of shaft BC if the allowable shear stress for steel is $\tau_{allow} = 12$ ksi. The belt does not slip on the pulley.



The angular velocity of shaft BC can be determined using the pulley ratio that is

$$\omega_{BC} = \left(\frac{r_A}{r_C}\right)\omega_A = \left(\frac{1.5}{3}\right)\left(1800\,\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 30\pi \text{ rad/s}$$

The power is

$$P = (15 \text{ hp})\left(\frac{550 \text{ ft} \cdot \text{n/s}}{1 \text{ hp}}\right) = 8250 \text{ ft} \cdot \text{lb/s}$$

Thus,

$$T = \frac{P}{\omega} = \frac{8250}{30\pi} = (87.54 \text{ lb} \cdot \text{ft})\left(\frac{12 \text{ in.}}{1 \text{ ft}}\right) = 1050.42 \text{ lb} \cdot \text{in}$$

The polar moment of inertia of the shaft is $J = \frac{\pi}{2}\left(\frac{d}{2}\right)^4 = \frac{\pi d^4}{32}$. Thus,

$$\tau_{allow} = \frac{Tc}{J}; \qquad 12(10^3) = \frac{1050.42(d/2)}{\pi d^4/32}$$

$$d = 0.7639 \text{ in} = \frac{7}{8} \text{ in.}$$ **Ans.**

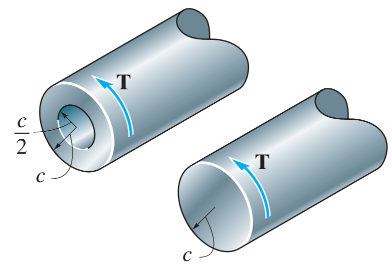
5–47. The propellers of a ship are connected to a A-36 steel shaft that is 60 m long and has an outer diameter of 340 mm and inner diameter of 260 mm. If the power output is 4.5 MW when the shaft rotates at 20 rad/s, determine the maximum torsional stress in the shaft and its angle of twist.

$$T = \frac{P}{\omega} = \frac{4.5(10^6)}{20} = 225(10^3)\ \text{N}\cdot\text{m}$$

$$\tau_{\max} = \frac{Tc}{J} = \frac{225(10^3)(0.170)}{\frac{\pi}{2}[(0.170)^4 - (0.130)^4]} = 44.3\ \text{MPa}$$ **Ans.**

$$\phi = \frac{TL}{JG} = \frac{225\left(10^3\right)(60)}{\frac{\pi}{2}[(0.170)^4 - (0.130)^4)75(10^9)} = 0.2085\ \text{rad} = 11.9°$$ **Ans.**

***5–48.** A shaft is subjected to a torque **T**. Compare the effectiveness of using the tube shown in the figure with that of a solid section of radius *c*. To do this, compute the percent increase in torsional stress and angle of twist per unit length for the tube versus the solid section.



Shear stress:

For the tube,

$$(\tau_t)_{\max} = \frac{T\,c}{J_t}$$

For the solid shaft,

$$(\tau_s)_{\max} = \frac{T\,c}{J_s}$$

$$\%\ \text{increase in shear stress} = \frac{(\tau_s)_{\max} - (\tau_t)_{\max}}{(\tau_t)_{\max}}(100) = \frac{\frac{Tc}{J_t} - \frac{Tc}{J_s}}{\frac{Tc}{J_s}}(100)$$

$$= \frac{J_s - J_t}{J_t}(100) = \frac{\frac{\pi}{2}c^4 - [\frac{\pi}{2}[c^4 - (\frac{\pi}{2})^4]]}{\frac{\pi}{2}[c^4 - (\frac{\pi}{2})^4]}(100)$$

$$= 6.67\ \%$$ **Ans.**

Angle of twist:

For the tube,

$$\phi_t = \frac{TL}{J_t(G)}$$

For the shaft,

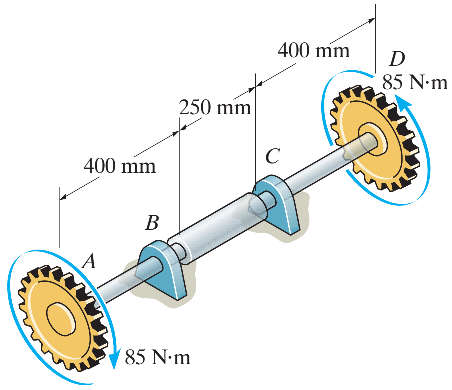
$$\phi_s = \frac{TL}{J_s(G)}$$

$$\%\ \text{increase in}\ \phi = \frac{\phi_t - \phi_s}{\phi_s}(100\%) = \frac{\frac{TL}{J_t(G)} - \frac{TL}{J_s(G)}}{\frac{TL}{J_s(G)}}(100\%)$$

$$= \frac{J_s - J_t}{J_t}(100\%) = \frac{\frac{\pi}{2}c^4 - [\frac{\pi}{2}[c^4 - (\frac{\pi}{2})^4]]}{\frac{\pi}{2}[c^4 - (\frac{\pi}{2})^4]}(100\%)$$

$$= 6.67\ \%$$ **Ans.**

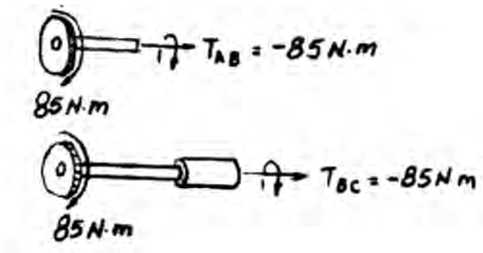
•5–49. The A-36 steel axle is made from tubes *AB* and *CD* and a solid section *BC*. It is supported on smooth bearings that allow it to rotate freely. If the gears, fixed to its ends, are subjected to 85-N·m torques, determine the angle of twist of gear *A* relative to gear *D*. The tubes have an outer diameter of 30 mm and an inner diameter of 20 mm. The solid section has a diameter of 40 mm.



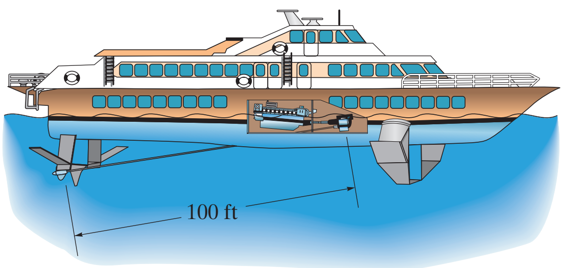
$$\phi_{ND} = \Sigma \frac{TL}{JG}$$

$$= \frac{2(85)(0.4)}{\frac{\pi}{2}(0.015^4 - 0.01^4)(75)(10^9)} + \frac{(85)(0.25)}{\frac{\pi}{2}(0.02^4)(75)(10^9)}$$

$$= 0.01534 \text{ rad} = 0.879° \qquad \textbf{Ans.}$$



5–50. The hydrofoil boat has an A-36 steel propeller shaft that is 100 ft long. It is connected to an in-line diesel engine that delivers a maximum power of 2500 hp and causes the shaft to rotate at 1700 rpm. If the outer diameter of the shaft is 8 in. and the wall thickness is $\frac{3}{8}$ in., determine the maximum shear stress developed in the shaft. Also, what is the "wind up," or angle of twist in the shaft at full power?



Internal Torque:

$$\omega = 1700 \frac{\text{rev}}{\text{min}}\left(\frac{2\pi \text{ rad}}{\text{rev}}\right)\frac{1 \text{ min}}{60 \text{ s}} = 56.67\pi \text{ rad/s}$$

$$P = 2500 \text{ hp}\left(\frac{550 \text{ ft} \cdot \text{lb/s}}{1 \text{ hp}}\right) = 1\,375\,000 \text{ ft} \cdot \text{lb/s}$$

$$T = \frac{P}{\omega} = \frac{1\,375\,000}{56.67\pi} = 7723.7 \text{ lb} \cdot \text{ft}$$

Maximum Shear Stress: Applying torsion Formula.

$$\tau_{\max} = \frac{Tc}{J}$$

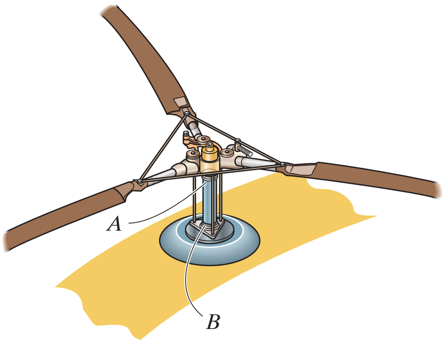
$$= \frac{7723.7(12)(4)}{\frac{\pi}{2}(4^4 - 3.625^4)} = 2.83 \text{ ksi} \qquad \textbf{Ans.}$$

Angle of Twist:

$$\phi = \frac{TL}{JG} = \frac{7723.7(12)(100)(12)}{\frac{\pi}{2}(4^4 - 3.625^4)11.0(10^6)}$$

$$= 0.07725 \text{ rad} = 4.43° \qquad \textbf{Ans.}$$

5–51. The engine of the helicopter is delivering 600 hp to the rotor shaft AB when the blade is rotating at 1200 rev/min. Determine to the nearest $\frac{1}{8}$ in. the diameter of the shaft AB if the allowable shear stress is $\tau_{\text{allow}} = 8$ ksi and the vibrations limit the angle of twist of the shaft to 0.05 rad. The shaft is 2 ft long and made from L2 steel.



$$\omega = \frac{1200(2)(\pi)}{60} = 125.66 \text{ rad/s}$$

$$P = T\omega$$

$$600(550) = T(125.66)$$

$$T = 2626.06 \text{ lb} \cdot \text{ft}$$

Shear - stress failure

$$\tau_{\text{allow}} = \frac{Tc}{J}$$

$$8(10^3) = \frac{2626.06(12)c}{\frac{\pi}{2}c^4}$$

$$c = 1.3586 \text{ in.}$$

Angle of twist limitation

$$\phi = \frac{TL}{JG}$$

$$0.05 = \frac{2626.06(12)(2)(12)}{\frac{\pi}{2}c^4(11.0)(10^6)}$$

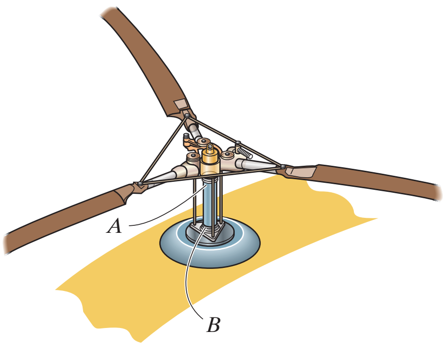
$$c = 0.967 \text{ in.}$$

Shear - stress failure controls the design.

$$d = 2c = 2\,(1.3586) = 2.72 \text{ in.}$$

Use $d = 2.75$ in. **Ans.**

***5–52.** The engine of the helicopter is delivering 600 hp to the rotor shaft AB when the blade is rotating at 1200 rev/min. Determine to the nearest $\frac{1}{8}$ in. the diameter of the shaft AB if the allowable shear stress is $\tau_{allow} = 10.5$ ksi and the vibrations limit the angle of twist of the shaft to 0.05 rad. The shaft is 2 ft long and made from L2 steel.



$$\omega = \frac{1200(2)(\pi)}{60} = 125.66 \text{ rad/s}$$

$$P = T\omega$$

$$600(550) = T(125.66)$$

$$T = 2626.06 \text{ lb} \cdot \text{ft}$$

Shear - stress failure

$$\tau_{allow} = 10.5(10)^3 = \frac{2626.06(12)c}{\frac{\pi}{2}c^4}$$

$$c = 1.2408 \text{ in.}$$

Angle of twist limitation

$$\phi = \frac{TL}{JG}$$

$$0.05 = \frac{2626.06(12)(2)(12)}{\frac{\pi}{2}c^4\,(11.0)(10^6)}$$

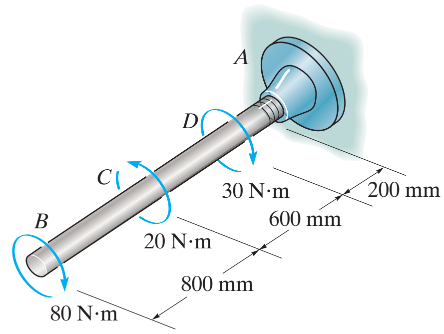
$$c = 0.967 \text{ in.}$$

Shear stress failure controls the design

$$d = 2c = 2\,(1.2408) = 2.48 \text{ in.}$$

Use $d = 2.50$ in. **Ans.**

•5–53. The 20-mm-diameter A-36 steel shaft is subjected to the torques shown. Determine the angle of twist of the end B.



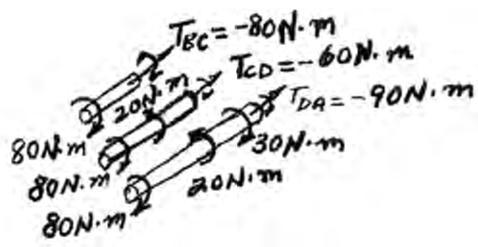
Internal Torque: As shown on FBD.

Angle of Twist:

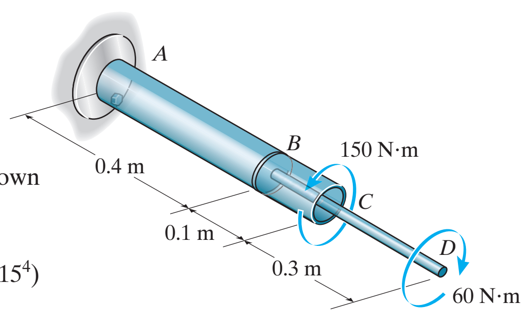
$$\phi_B = \sum \frac{TL}{JG}$$

$$= \frac{1}{\frac{\pi}{2}(0.01^4)(75.0)(10^9)}\left[-80.0(0.8) + (-60.0)(0.6) + (-90.0)(0.2)\right]$$

$$= -0.1002 \text{ rad} = |5.74^\circ|$$ **Ans.**



5–54. The assembly is made of A-36 steel and consists of a solid rod 20 mm in diameter fixed to the inside of a tube using a rigid disk at B. Determine the angle of twist at D. The tube has an outer diameter of 40 mm and wall thickness of 5 mm.



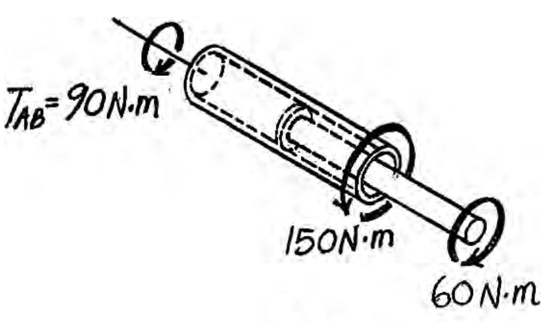
The internal torques developed in segments AB and BD of the assembly are shown in Fig. *a* and *b*

The polar moment of inertia of solid rod and tube are $J_{AB} = \frac{\pi}{2}(0.02^4 - 0.015^4) = 54.6875(10^{-9})\pi \text{ m}^4$ and $J_{BD} = \frac{\pi}{2}(0.01^4) = 5(10^{-9})\pi \text{ m}^4$. Thus,

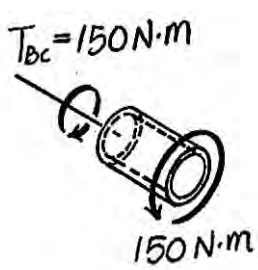
$$\phi_D = \Sigma \frac{T_i L_i}{J_i G_i} = \frac{T_{AB} L_{AB}}{J_{AB} G_{st}} + \frac{T_{BD} L_{BD}}{J_{BD} G_{st}}$$

$$= \frac{90(0.4)}{54.6875(10^{-9})\pi\,[75(10^9)]} + \frac{-60\,(0.4)}{5(10^{-9})\pi\,[75(10^9)]}$$

$$= -0.01758 \text{ rad} = 1.01^\circ$$ **Ans.**

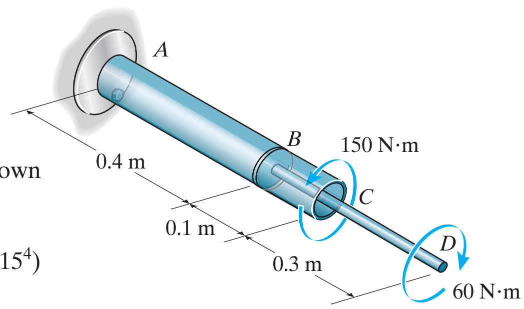


(a)



(b)

5–55. The assembly is made of A-36 steel and consists of a solid rod 20 mm in diameter fixed to the inside of a tube using a rigid disk at B. Determine the angle of twist at C. The tube has an outer diameter of 40 mm and wall thickness of 5 mm.



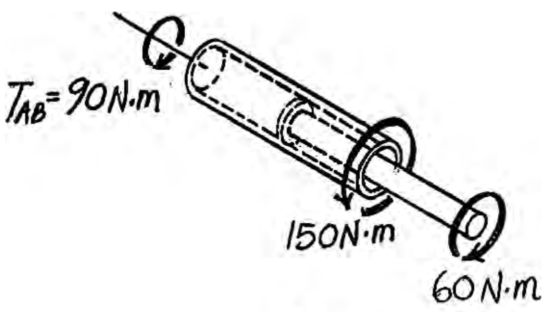
The internal torques developed in segments AB and BC of the assembly are shown in Figs. *a* and *b*.

The polar moment of inertia of the tube is $J = \dfrac{\pi}{2}(0.02^4 - 0.015^4) = 54.6875\,(10^{-9})\pi\ \text{m}^4$. Thus,

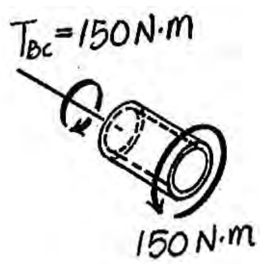
$$\phi_C = \Sigma \frac{T_i L_i}{J_i G_i} = \frac{T_{AB} L_{AB}}{JG_{st}} + \frac{T_{BC} L_{BC}}{J\,G_{st}}$$

$$= \frac{1}{54.6875(10^{-9})\pi\,[75(10^9)]}\Big[90(0.4) + 150(0.1)\Big]$$

$$= 0.003958 \text{ rad} = 0.227° \qquad \textbf{Ans.}$$

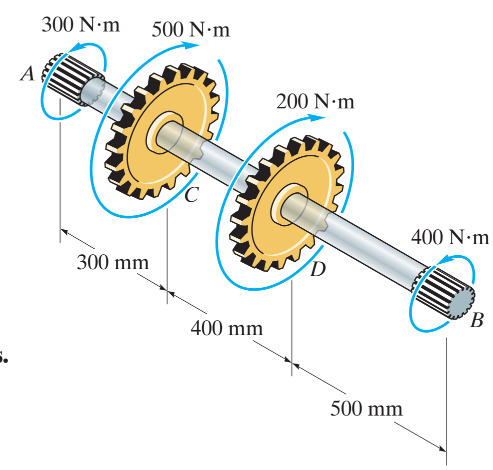


(a)



(b)

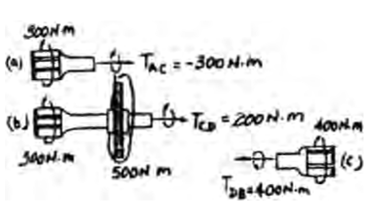
***5–56.** The splined ends and gears attached to the A-36 steel shaft are subjected to the torques shown. Determine the angle of twist of end B with respect to end A. The shaft has a diameter of 40 mm.



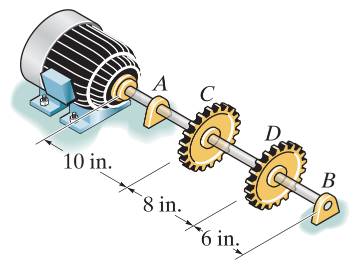
$$\phi_{B/A} = \Sigma \frac{TL}{JG} = \frac{-300(0.3)}{JG} + \frac{200(0.4)}{JG} + \frac{400(0.5)}{JG}$$

$$= \frac{190}{JG} = \frac{190}{\frac{\pi}{2}(0.02^4)(75)(10^9)}$$

$$= 0.01008 \text{ rad} = 0.578° \qquad \textbf{Ans.}$$



•5–57. The motor delivers 40 hp to the 304 stainless steel shaft while it rotates at 20 Hz. The shaft is supported on smooth bearings at A and B, which allow free rotation of the shaft. The gears C and D fixed to the shaft remove 25 hp and 15 hp, respectively. Determine the diameter of the shaft to the nearest $\frac{1}{8}$ in. if the allowable shear stress is $\tau_{\text{allow}} = 8$ ksi and the allowable angle of twist of C with respect to D is 0.20°.



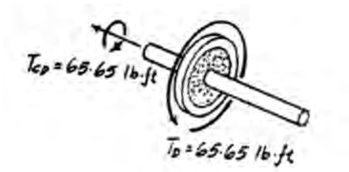
External Applied Torque: Applying $T = \dfrac{P}{2\pi f}$, we have

$$T_M = \frac{40(550)}{2\pi(20)} = 175.07 \text{ lb} \cdot \text{ft} \qquad T_C = \frac{25(550)}{2\pi(20)} = 109.42 \text{ lb} \cdot \text{ft}$$

$$T_D = \frac{15(550)}{2\pi(20)} = 65.65 \text{ lb} \cdot \text{ft}$$

Internal Torque: As shown on FBD.

Allowable Shear Stress: Assume failure due to shear stress. By observation, section AC is the critical region.



$$\tau_{\max} = \tau_{\text{allow}} = \frac{Tc}{J}$$

$$8(10^3) = \frac{175.07(12)\left(\frac{d}{2}\right)}{\frac{\pi}{2}\left(\frac{d}{2}\right)^4}$$

$$d = 1.102 \text{ in.}$$

Angle of Twist: Assume failure due to angle of twist limitation.

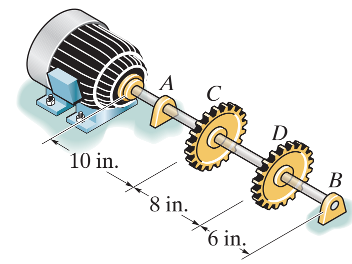
$$\phi_{C/D} = \frac{T_{CD}L_{CD}}{JG}$$

$$\frac{0.2(\pi)}{180} = \frac{65.65(12)(8)}{\frac{\pi}{2}\left(\frac{d}{2}\right)^4 (11.0)(10^6)}$$

$$d = 1.137 \text{ in. } (\textit{controls}\,!)$$

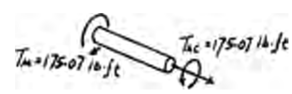
Use $d = 1\frac{1}{4}$ in. **Ans.**

5–58. The motor delivers 40 hp to the 304 stainless steel solid shaft while it rotates at 20 Hz. The shaft has a diameter of 1.5 in. and is supported on smooth bearings at A and B, which allow free rotation of the shaft. The gears C and D fixed to the shaft remove 25 hp and 15 hp, respectively. Determine the absolute maximum stress in the shaft and the angle of twist of gear C with respect to gear D.



External Applied Torque: Applying $T = \dfrac{P}{2\pi f}$, we have

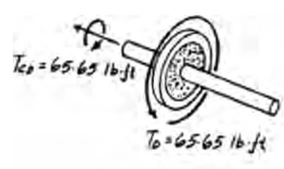
$$T_M = \frac{40(550)}{2\pi(20)} = 175.07\ \text{lb}\cdot\text{ft} \qquad T_C = \frac{25(550)}{2\pi(20)} = 109.42\ \text{lb}\cdot\text{ft}$$



$$T_D = \frac{15(550)}{2\pi(20)} = 65.65\ \text{lb}\cdot\text{ft}$$

Internal Torque: As shown on FBD.

Allowable Shear Stress: The maximum torque occurs within region AC of the shaft where $T_{\max} = T_{AC} = 175.07\ \text{lb}\cdot\text{ft}$.



$$\tau_{\substack{\text{abs}\\ \text{max}}} = \frac{T_{\max}\, c}{J} = \frac{175.07(12)(0.75)}{\frac{\pi}{2}(0.75^4)} = 3.17\ \text{ksi} \qquad \textbf{Ans.}$$

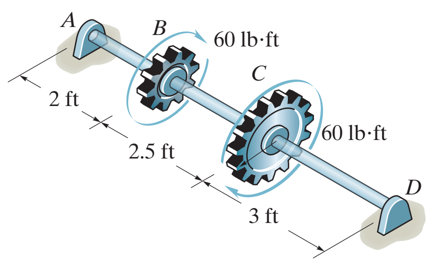
Angle of Twist:

$$\phi_{C/D} = \frac{T_{CD}\, L_{CD}}{JG}$$

$$= \frac{65.65(12)(8)}{\frac{\pi}{2}(0.75^4)(11.0)(10^6)}$$

$$= 0.001153\ \text{rad} = 0.0661^\circ \qquad \textbf{Ans.}$$

5–59. The shaft is made of A-36 steel. It has a diameter of 1 in. and is supported by bearings at A and D, which allow free rotation. Determine the angle of twist of B with respect to D.



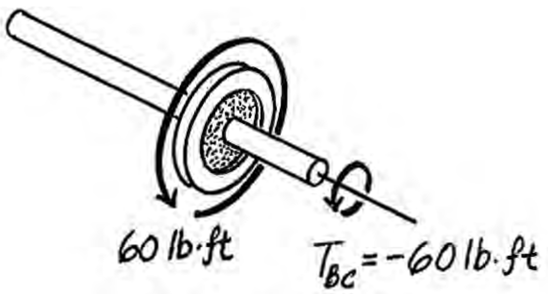
The internal torques developed in segments BC and CD are shown in Figs. *a* and *b*.

The polar moment of inertia of the shaft is $J = \frac{\pi}{2}(0.5^4) = 0.03125\pi \text{ in}^4$. Thus,

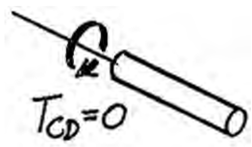
$$\phi_{B/D} = \sum \frac{T_i L_i}{J_i G_i} = \frac{T_{BC}\,L_{BC}}{J\,G_{st}} + \frac{T_{CD}\,L_{CD}}{J\,G_{st}}$$

$$= \frac{-60(12)(2.5)(12)}{(0.03125\pi)[11.0(10^6)]} + 0$$

$$= -0.02000 \text{ rad} = 1.15°$$ **Ans.**

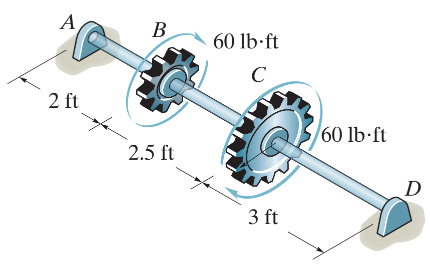


(a)



(b)

***5–60.** The shaft is made of A-36 steel. It has a diameter of 1 in. and is supported by bearings at A and D, which allow free rotation. Determine the angle of twist of gear C with respect to B.



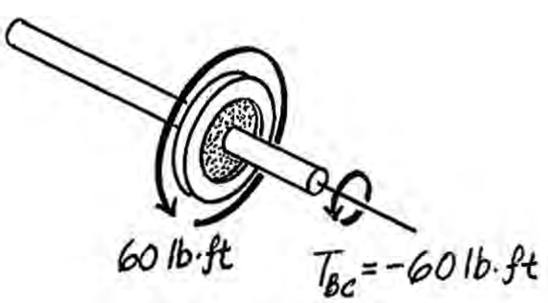
The internal torque developed in segment BC is shown in Fig. *a*

The polar moment of inertia of the shaft is $J = \frac{\pi}{2}(0.5^4) = 0.03125\pi \text{ in}^4$. Thus,

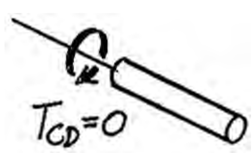
$$\phi_{C/B} = \frac{T_{BC}\,L_{BC}}{J\,G_{st}} = \frac{-60(12)(2.5)(12)}{(0.03125\pi)[11.0(10^6)]}$$

$$= -0.02000 \text{ rad}$$

$$= 1.15°$$ **Ans.**

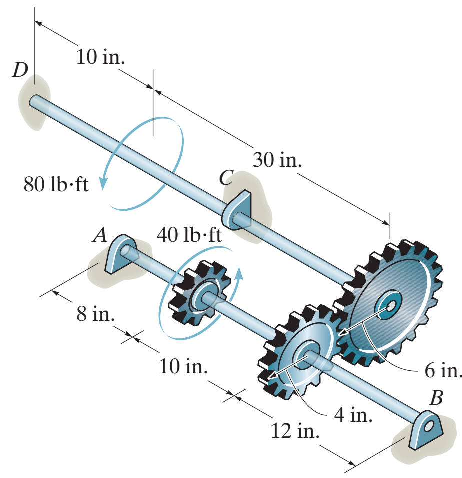


(a)

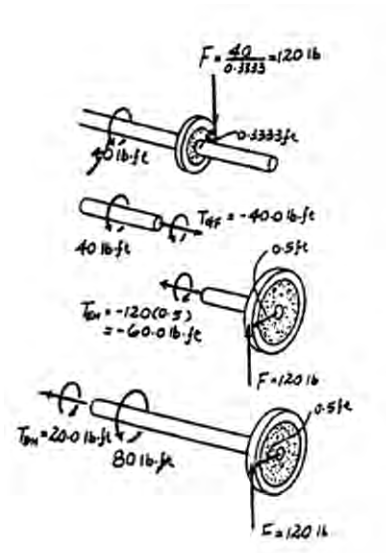


(b)

•5–61. The two shafts are made of A-36 steel. Each has a diameter of 1 in., and they are supported by bearings at A, B, and C, which allow free rotation. If the support at D is fixed, determine the angle of twist of end B when the torques are applied to the assembly as shown.



Internal Torque: As shown on FBD.



Angle of Twist:

$$\phi_E = \sum \frac{TL}{JG}$$

$$= \frac{1}{\frac{\pi}{2}(0.5^4)(11.0)(10^5)}[-60.0(12)(30) + 20.0(12)(10)]$$

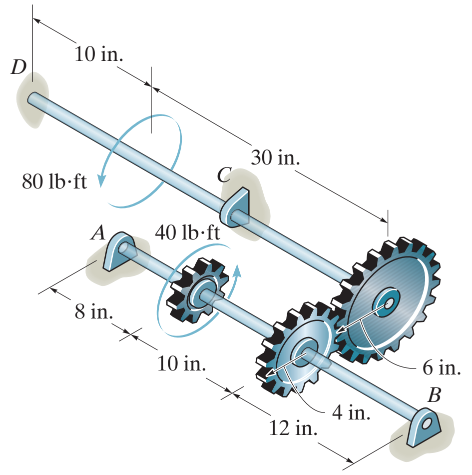
$$= -0.01778 \text{ rad} = 0.01778 \text{ rad}$$

$$\phi_F = \frac{6}{4}\phi_E = \frac{6}{4}(0.01778) = 0.02667 \text{ rad}$$

Since there is no torque applied between F and B then

$$\phi_B = \phi_F = 0.02667 \text{ rad} = 1.53^\circ$$ **Ans.**

5–62. The two shafts are made of A-36 steel. Each has a diameter of 1 in., and they are supported by bearings at A, B, and C, which allow free rotation. If the support at D is fixed, determine the angle of twist of end A when the torques are applied to the assembly as shown.



Internal Torque: As shown on FBD.

Angle of Twist:

$$\phi_E = \sum \frac{TL}{JG}$$

$$= \frac{1}{\frac{\pi}{2}(0.5^4)(11.0)(10^6)}[-60.0(12)(30) + 20.0(12)(10)]$$

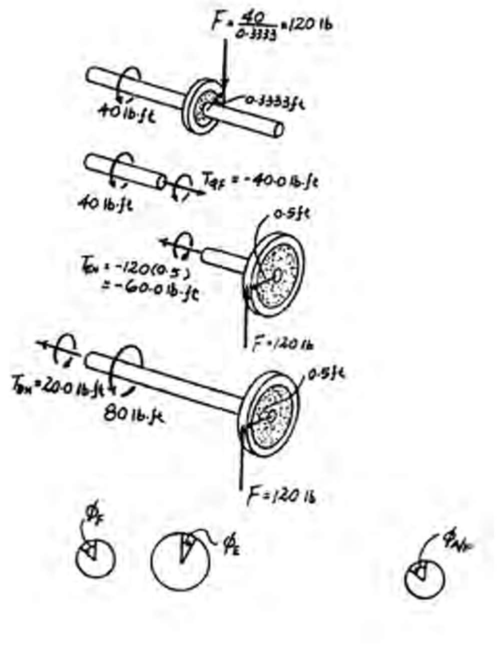
$$= -0.01778 \text{ rad} = 0.01778 \text{ rad}$$

$$\phi_F = \frac{6}{4}\phi_E = \frac{6}{4}(0.01778) = 0.02667 \text{ rad}$$

$$\phi_{A/F} = \frac{T_{GF}\, L_{GF}}{JG}$$

$$= \frac{-40(12)(10)}{\frac{\pi}{2}(0.5^4)(11.0)(10^6)}$$

$$= -0.004445 \text{ rad} = 0.004445 \text{ rad}$$

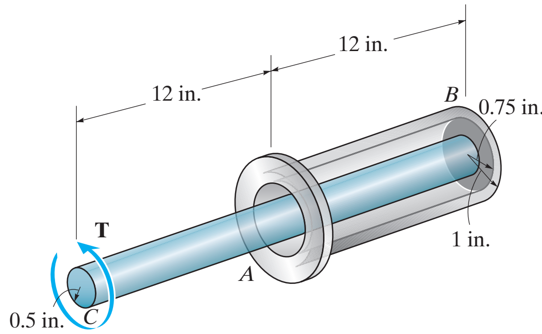


$$\phi_A = \phi_F + \phi_{A/F}$$

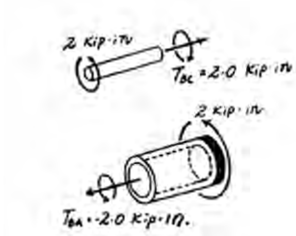
$$= 0.02667 + 0.004445$$

$$= 0.03111 \text{ rad} = 1.78° \qquad \textbf{Ans.}$$

5–63. The device serves as a compact torsional spring. It is made of A-36 steel and consists of a solid inner shaft CB which is surrounded by and attached to a tube AB using a rigid ring at B. The ring at A can also be assumed rigid and is fixed from rotating. If a torque of $T = 2$ kip·in. is applied to the shaft, determine the angle of twist at the end C and the maximum shear stress in the tube and shaft.



Internal Torque: As shown on FBD.



Maximum Shear Stress:

$$(\tau_{BC})_{\max} = \frac{T_{BC}\,c}{J} = \frac{2.00(0.5)}{\frac{\pi}{2}(0.5^4)} = 10.2 \text{ ksi} \qquad \textbf{Ans.}$$

$$(\tau_{BA})_{\max} = \frac{T_{BA}\,c}{J} = \frac{2.00(1)}{\frac{\pi}{2}(1^4 - 0.75^4)} = 1.86 \text{ ksi} \qquad \textbf{Ans.}$$

Angle of Twist:

$$\phi_B = \frac{T_{BA}\,L_{BA}}{JG}$$

$$= \frac{(2.00)(12)}{\frac{\pi}{2}(1^4 - 0.75^4)11.0(10^3)} = 0.002032 \text{ rad}$$

$$\phi_{C/B} = \frac{T_{BC}\,L_{BC}}{JG}$$

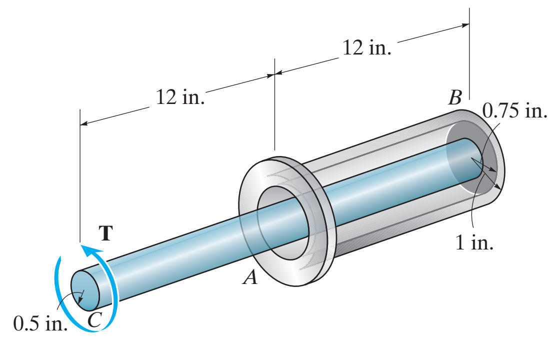
$$= \frac{2.00(24)}{\frac{\pi}{2}(0.5^4)11.0(10^3)} = 0.044448 \text{ rad}$$

$$\phi_C = \phi_B + \phi_{C/B}$$

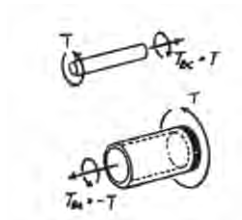
$$= 0.002032 + 0.044448$$

$$= 0.04648 \text{ rad} = 2.66° \qquad \textbf{Ans.}$$

***5–64.** The device serves as a compact torsion spring. It is made of A-36 steel and consists of a solid inner shaft CB which is surrounded by and attached to a tube AB using a rigid ring at B. The ring at A can also be assumed rigid and is fixed from rotating. If the allowable shear stress for the material is $\tau_{allow} = 12$ ksi and the angle of twist at C is limited to $\phi_{allow} = 3°$, determine the maximum torque T that can be applied at the end C.



Internal Torque: As shown on FBD.



Allowable Shear Stress: Assume failure due to shear stress.

$$\tau_{max} = \tau_{allow} = \frac{T_{BC}\, c}{J}$$

$$12.0 = \frac{T\,(0.5)}{\frac{\pi}{2}(0.5^4)}$$

$$T = 2.356 \text{ kip} \cdot \text{in}$$

$$\tau_{max} = \tau_{allow} = \frac{T_{BA}\, c}{J}$$

$$12.0 = \frac{T\,(1)}{\frac{\pi}{2}(1^4 - 0.75^4)}$$

$$T = 12.89 \text{ kip} \cdot \text{in}$$

Angle of Twist: Assume failure due to angle of twist limitation.

$$\phi_B = \frac{T_{BA} L_{BA}}{JG} = \frac{T(12)}{\frac{\pi}{2}(1^4 - 0.75^4)\, 11.0(10^3)}$$

$$= 0.001016T$$

$$\phi_{C/B} = \frac{T_{BC} L_{BC}}{JG} = \frac{T(24)}{\frac{\pi}{2}(0.5^4) 11.0(10^3)}$$

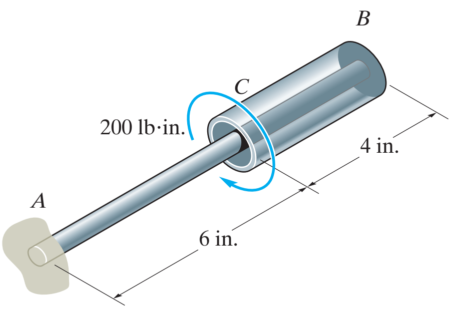
$$= 0.022224T$$

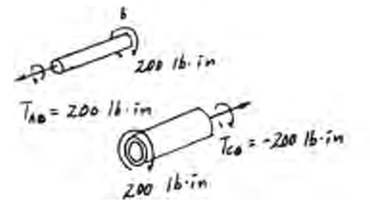
$$(\phi_C)_{allow} = \phi_B + \phi_{C/B}$$

$$\frac{3(\pi)}{180} = 0.001016T + 0.022224T$$

$$T = 2.25 \text{ kip} \cdot \text{in } (\textit{controls}\,!)$$ **Ans.**

•5–65. The A-36 steel assembly consists of a tube having an outer radius of 1 in. and a wall thickness of 0.125 in. Using a rigid plate at B, it is connected to the solid 1-in-diameter shaft AB. Determine the rotation of the tube's end C if a torque of 200 lb · in. is applied to the tube at this end. The end A of the shaft is fixed supported.





$$\phi_B = \frac{T_{AB}L}{JG} = \frac{200(10)}{\frac{\pi}{2}(0.5)^4(11.0)(10^6)} = 0.001852 \text{ rad}$$

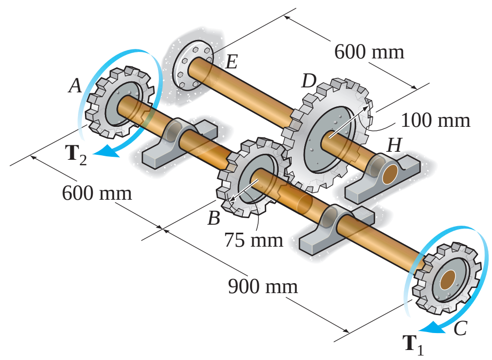
$$\phi_{C/B} = \frac{T_{CB}L}{JG} = \frac{-200(4)}{\frac{\pi}{2}(1^4 - 0.875^4)(11.0)(10^6)} = -0.0001119 \text{ rad}$$

$$\phi_C = \phi_B + \phi_{C/B}$$

$$= 0.001852 + 0.0001119$$

$$= 0.001964 \text{ rad} = 0.113° \qquad \textbf{Ans.}$$

5–66. The 60-mm diameter shaft ABC is supported by two journal bearings, while the 80-mm diameter shaft EH is fixed at E and supported by a journal bearing at H. If $T_1 = 2\ \text{kN}\cdot\text{m}$ and $T_2 = 4\ \text{kN}\cdot\text{m}$, determine the angle of twist of gears A and C. The shafts are made of A-36 steel.



Equilibrium: Referring to the free - body diagram of shaft ABC shown in Fig. a

$$\Sigma M_x = 0; \quad F(0.075) - 4(10^3) - 2(10^3) = 0 \qquad F = 80(10^3)\,\text{N}$$

Internal Loading: Referring to the free - body diagram of gear D in Fig. b,

$$\Sigma M_x = 0; \quad 80(10^3)(0.1) - T_{DH} = 0 \qquad T_{DH} = 8(10^3)\,\text{N}\cdot\text{m}$$

Also, from the free - body diagram of gear A, Fig. c,

$$\Sigma M_x = 0; \quad T_{AB} - 4(10^3) = 0 \qquad T_{AB} = 4\left(10^3\right)\text{N}\cdot\text{m}$$

And from the free - body diagram of gear C, Fig. d,

$$\Sigma M_x = 0; -T_{BC} - 2\left(10^3\right) = 0 \qquad T_{BC} = -2(10^3)\,\text{N}\cdot\text{m}$$

Angle of Twist: The polar moment of inertia of segments AB, BC and DH of the shaft are $J_{AB} = J_{BC} = \frac{\pi}{2}\left(0.03^4\right) = 0.405(10^{-6})\pi\,\text{m}^4$ and $J_{DH} = \frac{\pi}{2}\left(0.04^4\right) = 1.28(10^{-6})\pi\,\text{m}^4$. We have

$$\phi_D = \frac{T_{DH}\,L_{DH}}{J_{DH}G_{st}} = \frac{8(10^3)(0.6)}{1.28(10^{-6})\pi(75)(10^9)} = 0.01592\text{ rad}$$

Then, using the gear ratio,

$$\phi_B = \phi_D\left(\frac{r_D}{r_B}\right) = 0.01592\left(\frac{100}{75}\right) = 0.02122\text{ rad}$$

Also,

$$\phi_{C/B} = \frac{T_{BC}\,L_{BC}}{J_{BC}G_{st}} = \frac{-2(10^3)(0.9)}{0.405(10^{-6})\pi(75)(10^9)} = -0.01886\text{ rad} = 0.01886\text{ rad}$$

$$\phi_{A/B} = \frac{T_{AB}L_{AB}}{J_{AB}\,G_{st}} = \frac{4(10^3)(0.6)}{0.405(10^{-6})\pi(75)(10^9)} = 0.02515\text{ rad}$$

Thus,

$$\phi_A = \phi_B + \phi_{A/B}$$

$$\phi_A = 0.02122 + 0.02515$$

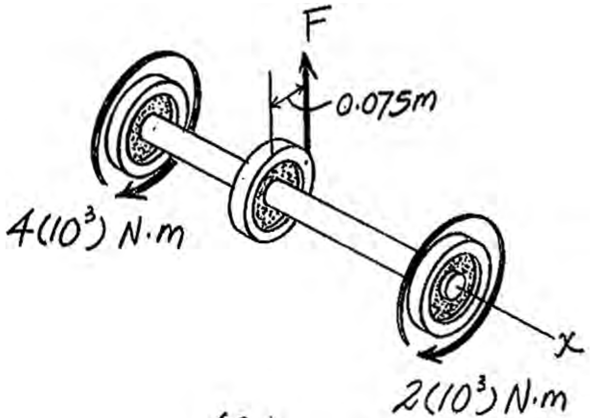
$$= 0.04637\text{ rad} = 2.66° \qquad \textbf{Ans.}$$

$$\phi_C = \phi_B + \phi_{C/B}$$

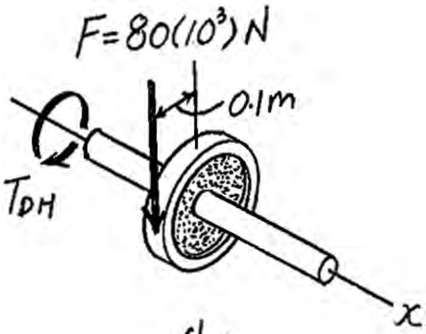
$$\phi_C = 0.02122 + 0.01886$$

$$= 0.04008\text{ rad} = 2.30° \qquad \textbf{Ans.}$$

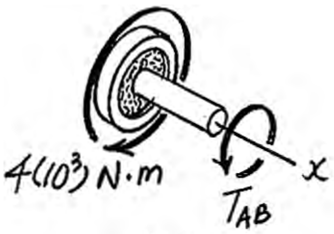
5–66. Continued



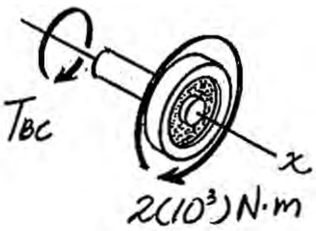
(a)



(b)

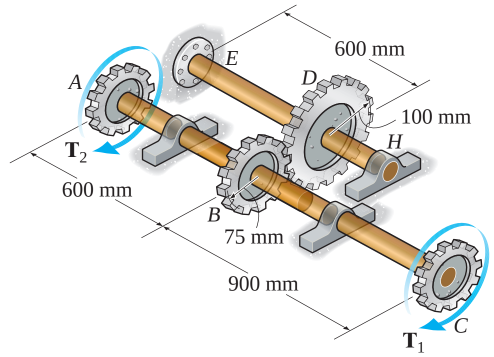


(c)



(d)

5–67. The 60-mm diameter shaft ABC is supported by two journal bearings, while the 80-mm diameter shaft EH is fixed at E and supported by a journal bearing at H. If the angle of twist at gears A and C is required to be 0.04 rad, determine the magnitudes of the torques T_1 and T_2. The shafts are made of A-36 steel.



Equilibrium: Referring to the free - body diagram of shaft ABC shown in Fig. a

$$\Sigma M_x = 0; \quad F(0.075) - T_1 - T_2 = 0 \qquad F = 13.333(T_1 + T_2)$$

Internal Loading: Referring to the free - body diagram of gear D in Fig. b,

$$\Sigma M_x = 0; \quad 13.333(T_1 + T_2)(0.1) - T_{DE} = 0 \qquad T_{DE} = 1.333(T_1 + T_2)$$

Also, from the free - body diagram of gear A, Fig. c,

$$\Sigma M_x = 0; \quad T_{AB} - T_2 = 0 \qquad T_{AB} = T_2$$

and from the free - body diagram of gear C, Fig. d

$$\Sigma M_x = 0; \quad T_{BC} - T_1 = 0 \qquad T_{BC} = T_1$$

Angle of Twist: The polar moments of inertia of segments AB, BC and DH of the shaft are $J_{AB} = J_{BC} = \frac{\pi}{2}\left(0.03^4\right) = 0.405(10^{-6})\pi \text{m}^4$ and $J_{DH} = \frac{\pi}{2}\left(0.04^4\right) = 1.28(10^{-6})\pi \text{m}^4$. We have

$$\phi_D = \frac{T_{DE}L_{DH}}{J_{DE}G_{st}} = \frac{1.333(T_1 + T_2)(0.6)}{1.28(10^{-6})\pi(75)(10^9)} = 2.6258(10^{-6})(T_1 + T_2)$$

Then, using the gear ratio,

$$\phi_B = \phi_D\left(\frac{r_D}{r_B}\right) = 2.6258(10^{-6})(T_1 + T_2)\left(\frac{100}{75}\right) = 3.5368(10^{-6})(T_1 + T_2)$$

Also,

$$\phi_{C/B} = \frac{T_{BC}L_{BC}}{J_{BC}G_{st}} = \frac{T_1(0.9)}{0.405(10^{-6})\pi(75)(10^9)} = 9.4314(10^{-6})T_1$$

$$\phi_{A/B} = \frac{T_{AB}L_{AB}}{J_{AB}G_{st}} = \frac{T_2(0.6)}{0.405(10^{-6})\pi(75)(10^9)} = 6.2876(10^{-6})T_2$$

Here, it is required that $\phi_A = \phi_C = 0.04$ rad. Thus,

$$\phi_A = \phi_B + \phi_{A/B}$$

$$0.04 = 3.5368(10^{-6})(T_1 + T_2) + 6.2876(10^{-6})T_2$$

$$T_1 + 2.7778T_2 = 11309.73 \qquad (1)$$

$$\phi_C = \phi_B + \phi_{C/B}$$

$$0.04 = 3.5368(10^{-6})(T_1 + T_2) + 9.4314(10^{-6})T_1$$

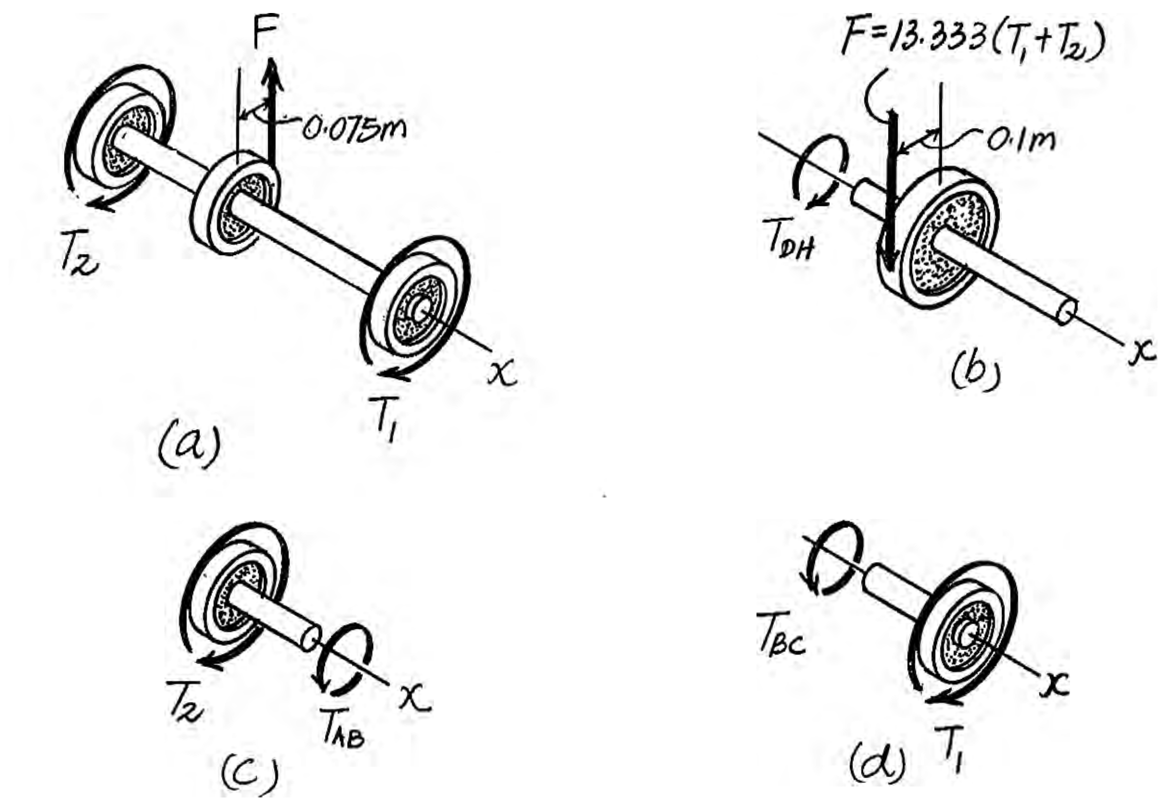
$$3.6667T_1 + T_2 = 11309.73 \qquad (2)$$

5–67. Continued

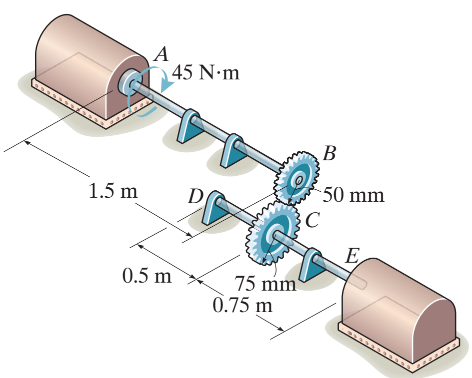
Solving Eqs. (1) and (2),

$$T_1 = 2188.98\ \text{N}\cdot\text{m} = 2.19\ \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$

$$T_2 = 3283.47\ \text{N}\cdot\text{m} = 3.28\ \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$



***5–68.** The 30-mm-diameter shafts are made of L2 tool steel and are supported on journal bearings that allow the shaft to rotate freely. If the motor at A develops a torque of $T = 45\ \text{N}\cdot\text{m}$ on the shaft AB, while the turbine at E is fixed from turning, determine the amount of rotation of gears B and C.



Internal Torque: As shown on FBD.

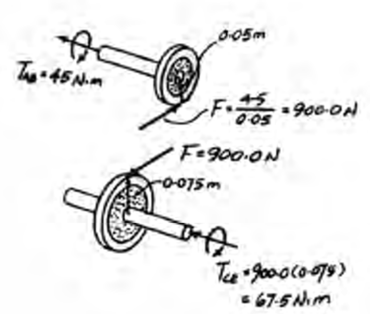
Angle of Twist:

$$\phi_C = \frac{T_{CE}\,L_{CE}}{JG}$$

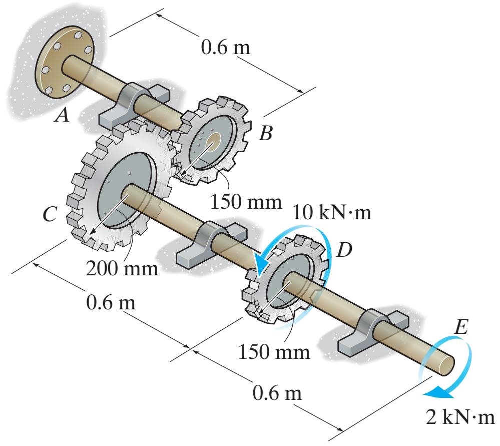
$$= \frac{67.5(0.75)}{\frac{\pi}{2}(0.015^4)75.0(10^3)}$$

$$= 0.008488\ \text{rad} = 0.486^\circ \qquad \textbf{Ans.}$$

$$\phi_B = \frac{75}{50}\quad \phi_C = 0.729^\circ \qquad \textbf{Ans.}$$



•5–69. The shafts are made of A-36 steel and each has a diameter of 80 mm. Determine the angle of twist at end E.



Equilibrium: Referring to the free - body diagram of shaft CDE shown in Fig. a,

$\Sigma M_x = 0;\quad 10(10^3) - 2(10^3) - F(0.2) = 0 \qquad\qquad F = 40(10^3)\text{ N}$

Internal Loading: Referring to the free - body diagram of gear B, Fig. b,

$\Sigma M_x = 0;\quad -T_{AB} - 40(10^3)(0.15) = 0 \qquad\qquad T_{AB} = -6(10^3)\text{ N}\cdot\text{m}$

Referring to the free - body diagram of gear D, Fig. c,

$\Sigma M_x = 0;\quad 10(10^3) - 2(10^3) - T_{CD} = 0 \qquad\qquad T_{CD} = 8(10^3)\text{ N}\cdot\text{m}$

Referring to the free - body diagram of shaft DE, Fig. d,

$\Sigma M_x = 0;\quad -T_{DE} - 2(10^3) = 0 \qquad\qquad T_{DE} = -2(10^3)\text{ N}\cdot\text{m}$

Angle of Twist: The polar moment of inertia of the shafts are $J = \frac{\pi}{2}\left(0.04^4\right) = 1.28(10^{-6})\pi\text{ m}^4$.

We have

$$\phi_B = \frac{T_{AB}L_{AB}}{JG_{st}} = \frac{-6(10^3)(0.6)}{1.28(10^{-6})\pi(75)(10^9)} = -0.01194\text{ rad} = 0.01194\text{ rad}$$

Using the gear ratio,

$$\phi_C = \phi_B\left(\frac{r_B}{r_C}\right) = 0.01194\left(\frac{150}{200}\right) = 0.008952\text{ rad}$$

Also,

$$\phi_{E/C} = \Sigma\frac{T_iL_i}{J_iG_i} = \frac{T_{CD}L_{CD}}{JG_{st}} + \frac{T_{DE}L_{DE}}{JG_{st}}$$

$$= \frac{0.6}{1.28(10^{-6})\pi(75)(10^9)}\left\{8(10^3) + \left[-2(10^3)\right]\right\}$$

$$= 0.01194\text{ rad}$$

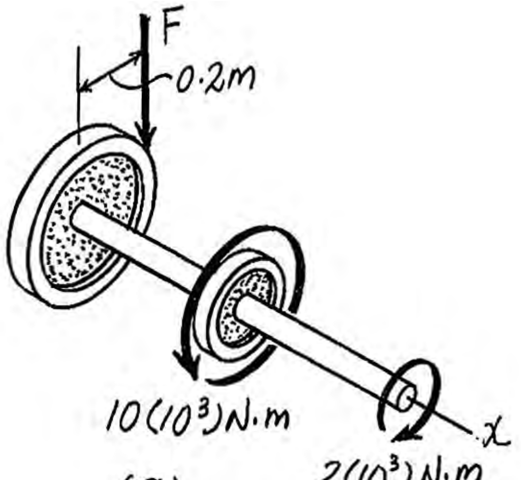
Thus,

$$\phi_E = \phi_C + \phi_{E/C}$$

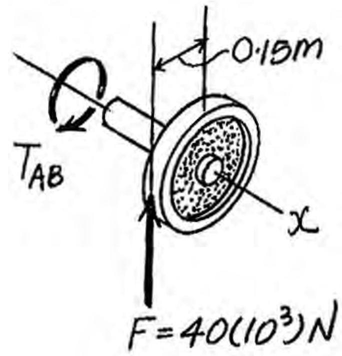
$$\phi_E = 0.008952 + 0.01194$$

$$= 0.02089\text{ rad} = 1.20^\circ \qquad\qquad \textbf{Ans.}$$

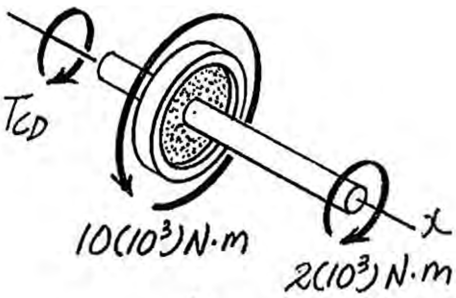
5–69. Continued



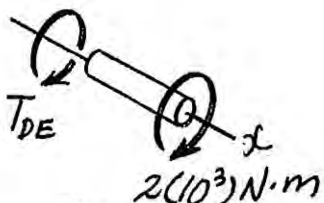
(a)



(b)

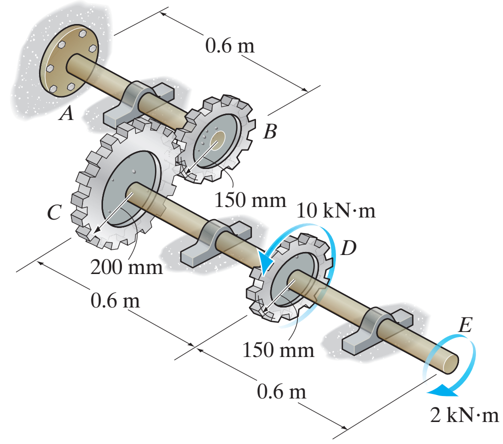


(c)



(d)

5–70. The shafts are made of A-36 steel and each has a diameter of 80 mm. Determine the angle of twist of gear D.



Equilibrium: Referring to the free-body diagram of shaft CDE shown in Fig. a,

$\Sigma M_x = 0; \quad 10(10^3) - 2(10^3) - F(0.2) = 0 \qquad F = 40(10^3)\text{ N}$

Internal Loading: Referring to the free - body diagram of gear B, Fig. b,

$\Sigma M_x = 0; \quad -T_{AB} - 40(10^3)(0.15) = 0 \qquad T_{AB} = -6(10^3)\text{ N}\cdot\text{m}$

Referring to the free - body diagram of gear D, Fig. c,

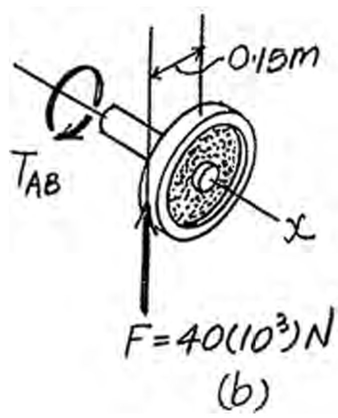
$\Sigma M_x = 0; \quad 10(10^3) - 2(10^3) - T_{CD} = 0 \qquad T_{CD} = 8(10^3)\text{ N}\cdot\text{m}$

Angle of Twist: The polar moment of inertia of the shafts are $J = \frac{\pi}{2}\left(0.04^4\right) = 1.28(10^{-6})\pi\text{ m}^4$. We have

$$\phi_B = \frac{T_{AB}L_{AB}}{JG_{st}} = \frac{-6(10^3)(0.6)}{1.28(10^{-6})\pi(75)(10^9)} = -0.01194\text{ rad} = 0.01194\text{ rad}$$

Using the gear ratio,

$$\phi_C = \phi_B\left(\frac{r_B}{r_C}\right) = 0.01194\left(\frac{150}{200}\right) = 0.008952\text{ rad}$$



Also,

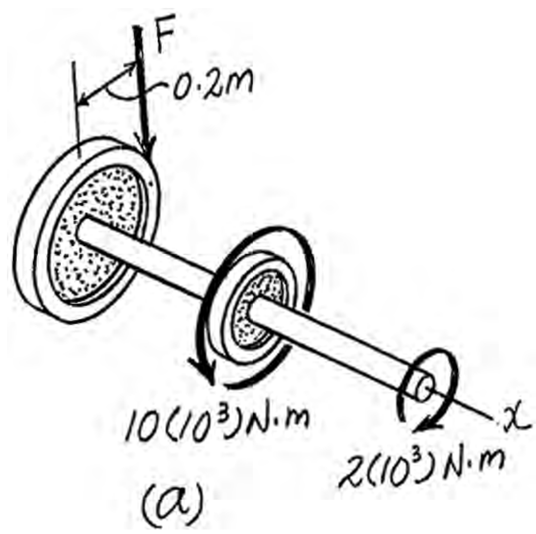
$$\phi_{D/C} = \frac{T_{CD}L_{CD}}{JG_{st}} = \frac{8(10^3)(0.6)}{1.28(10^{-6})\pi(75)(10^9)} = 0.01592\text{ rad}$$

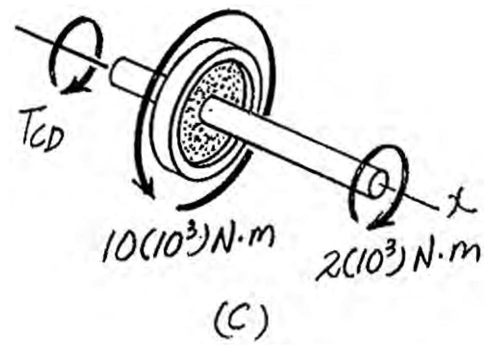
Thus,

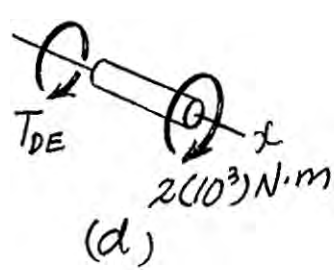
$$\phi_D = \phi_C + \phi_{D/C}$$

$$\phi_D = 0.008952 + 0.01592$$

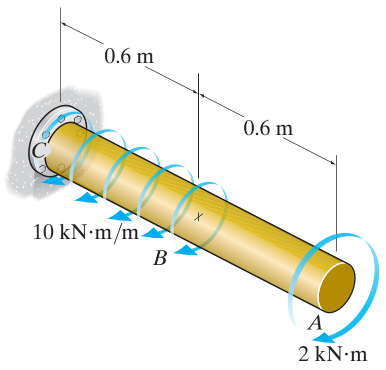
$$= 0.02487\text{ rad} = 1.42°$$ **Ans.**







***5–72.** The 80-mm diameter shaft is made of 6061-T6 aluminum alloy and subjected to the torsional loading shown. Determine the angle of twist at end A.



Equilibrium: Referring to the free - body diagram of segment AB shown in Fig. a,

$\Sigma M_x = 0;\quad -T_{AB} - 2(10^3) = 0 \qquad\qquad T_{AB} = -2(10^3)\,\text{N}\cdot\text{m}$

And the free - body diagram of segment BC, Fig. b,

$\Sigma M_x = 0;\quad -T_{BC} - 10(10^3)x - 2(10^3) = 0 \qquad\qquad T_{BC} = -\left[10(10^3)x + 2(10^3)\right]\text{N}\cdot\text{m}$

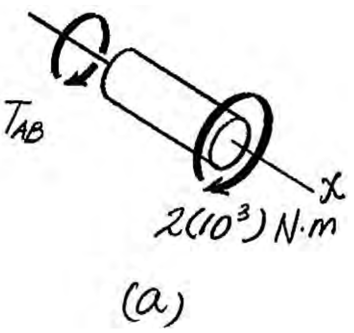
Angle of Twist: The polar moment of inertia of the shaft is $J = \frac{\pi}{2}\left(0.04^2\right) = 1.28(10^{-6})\pi\ \text{m}^4$. We have

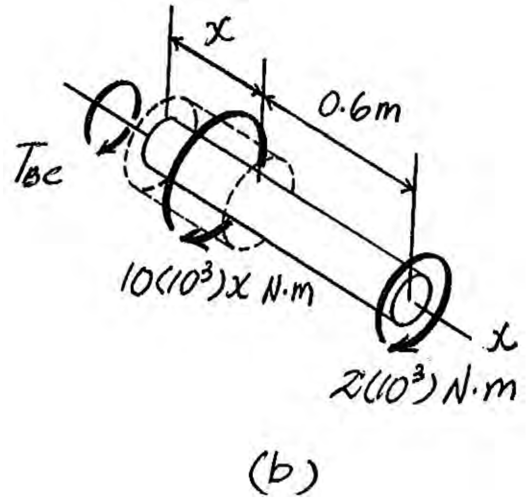
$$\phi_A = \Sigma\frac{T_iL_i}{J_iG_i} = \frac{T_{AB}L_{AB}}{JG_{al}} + \int_0^{L_{BC}}\frac{T_{BC}\,dx}{JG_{al}}$$

$$= \frac{-2(10^3)(0.6)}{1.28(10^{-6})\pi(26)(10^9)} + \int_0^{0.6\text{ m}}\frac{-\left[10(10^3)x + 2(10^3)\right]dx}{1.28(10^{-6})\pi(26)(10^9)}$$

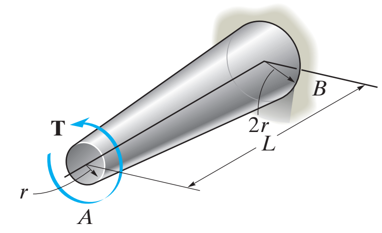
$$= -\frac{1}{1.28(10^{-6})\pi(26)(10^9)}\left\{1200 + \left[5(10^3)x^2 + 2(10^3)x\right]\Big|_0^{0.6\text{m}}\right\}$$

$$= -0.04017\text{ rad} = 2.30^\circ$$ **Ans.**





•5–73. The tapered shaft has a length L and a radius r at end A and $2r$ at end B. If it is fixed at end B and is subjected to a torque T, determine the angle of twist of end A. The shear modulus is G.

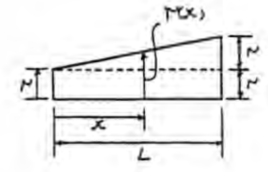


Geometry:

$$r(x) = r + \frac{r}{L}x = \frac{rL + rx}{L}$$

$$J(x) = \frac{\pi}{2}\left(\frac{rL + rx}{L}\right)^4 = \frac{\pi r^4}{2L^4}(L + x)^4$$

Angle of Twist:



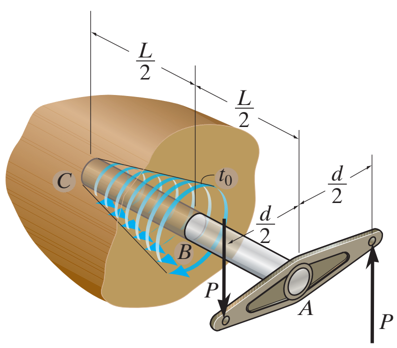
$$\phi = \int_0^L \frac{T\,dx}{J(x)G}$$

$$= \frac{2TL^4}{\pi r^4 G}\int_0^L \frac{dx}{(L + x)^4}$$

$$= \frac{2TL^4}{\pi r^4 G}\left[-\frac{1}{3(L + x)^3}\right]\Bigg|_0^L$$

$$= \frac{7TL}{12\pi r^4 G}$$ **Ans.**

5–74. The rod ABC of radius c is embedded into a medium where the distributed torque reaction varies linearly from zero at C to t_0 at B. If couple forces P are applied to the lever arm, determine the value of t_0 for equilibrium. Also, find the angle of twist of end A. The rod is made from material having a shear modulus of G.



Equilibrium: Referring to the free-body diagram of the entire rod shown in Fig. a,

$$\Sigma M_x = 0; \quad Pd - \frac{1}{2}(t_0)\left(\frac{L}{2}\right) = 0$$

$$t_o = \frac{4Pd}{L}$$ **Ans.**

Internal Loading: The distributed torque expressed as a function of x, measured from the left end, is $t = \left(\frac{t_o}{L/2}\right)x = \left(\frac{4Pd/L}{L/2}\right)x = \left(\frac{8Pd}{L^2}\right)x$. Thus, the resultant torque within region x of the shaft is

$$T_R = \frac{1}{2}tx = \frac{1}{2}\left[\left(\frac{8Pd}{L^2}\right)x\right]x = \frac{4Pd}{L^2}x^2$$

Referring to the free - body diagram shown in Fig. b,

$$\Sigma M_x = 0; \quad T_{BC} - \frac{4Pd}{L^2}x^2 = 0 \qquad T_{BC} = \frac{4Pd}{L^2}x^2$$

Referring to the free - body diagram shown in Fig. c,

$$\Sigma M_x = 0; \quad Pd - T_{AB} = 0 \qquad T_{AB} = Pd$$

5–74. Continued

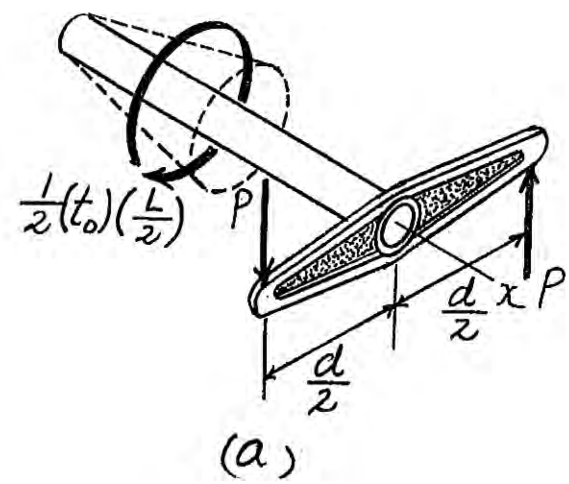
Angle of Twist:

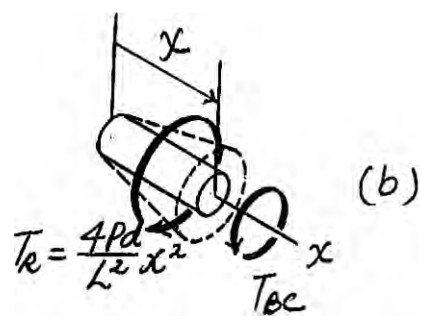
$$\phi = \Sigma \frac{T_i L_i}{J_i G_i} = \int_0^{L_{BC}} \frac{T_{BC}\,dx}{JG} + \frac{T_{AB}\,L_{AB}}{JG}$$

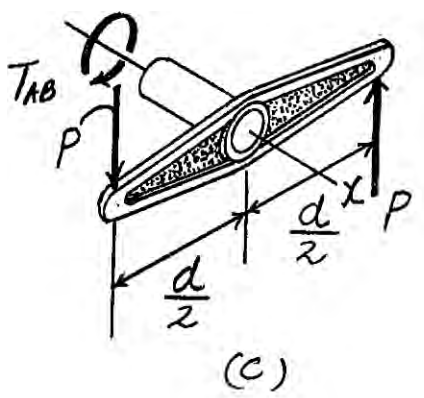
$$= \int_0^{L/2} \frac{\frac{4Pd}{L^2} x^2 dx}{\left(\frac{\pi}{2} c^4\right) G} + \frac{Pd(L/2)}{\left(\frac{\pi}{2} c^4\right) G}$$

$$= \frac{8Pd}{\pi c^4 L^2 G} \left(\frac{x^3}{3}\right)\Bigg|_0^{L/2} + \frac{PLd}{\pi c^4 G}$$

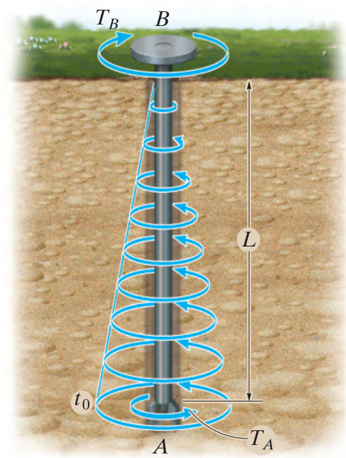
$$= \frac{4PLd}{3\pi c^4 G}$$ **Ans.**





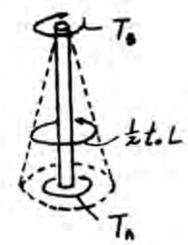


5–75. When drilling a well, the deep end of the drill pipe is assumed to encounter a torsional resistance T_A. Furthermore, soil friction along the sides of the pipe creates a linear distribution of torque per unit length, varying from zero at the surface B to t_0 at A. Determine the necessary torque T_B that must be supplied by the drive unit to turn the pipe. Also, what is the relative angle of twist of one end of the pipe with respect to the other end at the instant the pipe is about to turn? The pipe has an outer radius r_o and an inner radius r_i. The shear modulus is G.



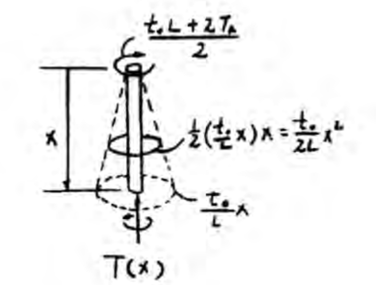
$$\frac{1}{2}t_0 L + T_A - T_B = 0$$

$$T_B = \frac{t_0 L + 2T_A}{2}$$ **Ans.**



$$T(x) + \frac{t_0}{2L}x^2 - \frac{t_0 L + 2T_A}{2} = 0$$

$$T(x) = \frac{t_0 L + 2T_A}{2} - \frac{t_0}{2L}x^2$$



$$\phi = \int \frac{T(x)\,dx}{J\,G}$$

$$= \frac{1}{J\,G}\int_0^L \left(\frac{t_0 L + 2T_A}{2} - \frac{t_0}{2L}x^2\right)dx$$

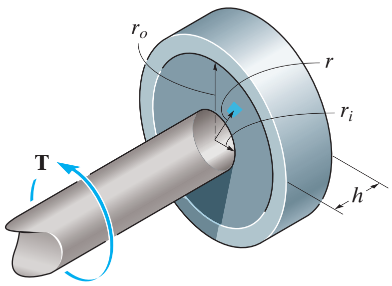
$$= \frac{1}{JG}\left[\frac{t_0 L + 2T_A}{2}x - \frac{t_0}{6L}x^3\right]\Bigg|_0^L$$

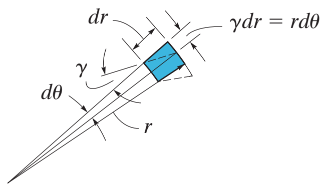
$$= \frac{t_0 L^2 + 3T_A L}{3JG}$$

However, $J = \frac{\pi}{2}(r_o{}^4 - r_i{}^4)$

$$\phi = \frac{2L(t_0 L + 3T_A)}{3\pi(r_o{}^4 - r_i{}^4)G}$$ **Ans.**

***5–76.** A cylindrical spring consists of a rubber annulus bonded to a rigid ring and shaft. If the ring is held fixed and a torque T is applied to the rigid shaft, determine the angle of twist of the shaft. The shear modulus of the rubber is G. *Hint:* As shown in the figure, the deformation of the element at radius r can be determined from $r d\theta = dr\gamma$. Use this expression along with $\tau = T/(2\pi r^2 h)$ from Prob. 5–26, to obtain the result.





$$r\,d\theta = \gamma\,dr$$

$$d\theta = \frac{\gamma dr}{r} \qquad (1)$$

From Prob. 5-26,

$$\tau = \frac{T}{2\pi\, r^2 h} \qquad \text{and} \qquad \gamma = \frac{\tau}{G}$$

$$\gamma = \frac{T}{2\pi\, r^2 hG}$$

From (1),

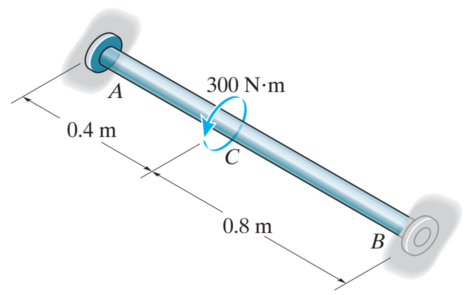
$$d\theta = \frac{T}{2\pi\, hG}\frac{dr}{r^3}$$

$$\theta = \frac{T}{2\pi\, hG}\int_{r_i}^{r_o}\frac{dr}{r^3} = \frac{T}{2\pi\, hG}\left[-\frac{1}{2\,r^2}\right]\Bigg|_{r_i}^{r_o}$$

$$= \frac{T}{2\pi\, hG}\left[-\frac{1}{2r_o^2} + \frac{1}{2r_i^2}\right]$$

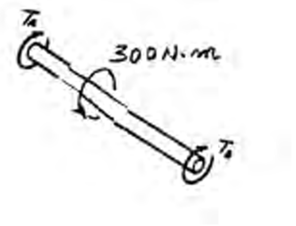
$$= \frac{T}{4\pi\, hG}\left[\frac{1}{r_i^2} - \frac{1}{r_o^2}\right] \qquad \textbf{Ans.}$$

•5–77. The A-36 steel shaft has a diameter of 50 mm and is fixed at its ends A and B. If it is subjected to the torque, determine the maximum shear stress in regions AC and CB of the shaft.



Equilibrium:

$$T_A + T_B - 300 = 0 \qquad \textbf{[1]}$$



Compatibility:

$$\phi_{C/A} = \phi_{C/B}$$

$$\frac{T_A(0.4)}{JG} = \frac{T_B(0.8)}{JG}$$

$$T_A = 2.00T_B \qquad \textbf{[2]}$$

Solving Eqs. [1] and [2] yields:

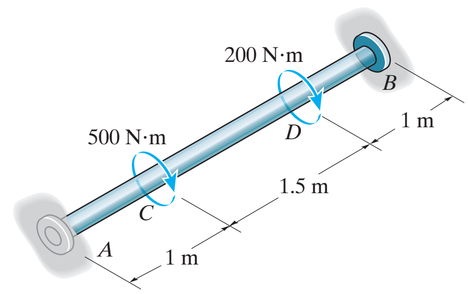
$$T_A = 200\ \text{N}\cdot\text{m} \qquad T_B = 100\ \text{N}\cdot\text{m}$$

Maximum Shear stress:

$$(\tau_{AC})_{\max} = \frac{T_A c}{J} = \frac{200(0.025)}{\frac{\pi}{2}(0.025^4)} = 8.15\ \text{MPa} \qquad \textbf{Ans.}$$

$$(\tau_{CB})_{\max} = \frac{T_B c}{J} = \frac{100(0.025)}{\frac{\pi}{2}(0.025^4)} = 4.07\ \text{MPa} \qquad \textbf{Ans.}$$

5–78. The A-36 steel shaft has a diameter of 60 mm and is fixed at its ends A and B. If it is subjected to the torques shown, determine the absolute maximum shear stress in the shaft.



Referring to the FBD of the shaft shown in Fig. *a*,

$$\Sigma M_x = 0; \qquad T_A + T_B - 500 - 200 = 0 \qquad \textbf{(1)}$$

Using the method of superposition, Fig. *b*

$$\phi_A = (\phi_A)_{T_A} - (\phi_A)_T$$

$$0 = \frac{T_A(3.5)}{JG} - \left[\frac{500\,(1.5)}{JG} + \frac{700\,(1)}{JG}\right]$$

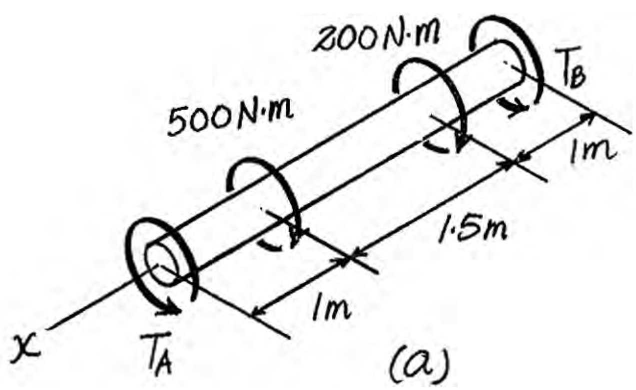
$$T_A = 414.29\ \mathrm{N \cdot m}$$

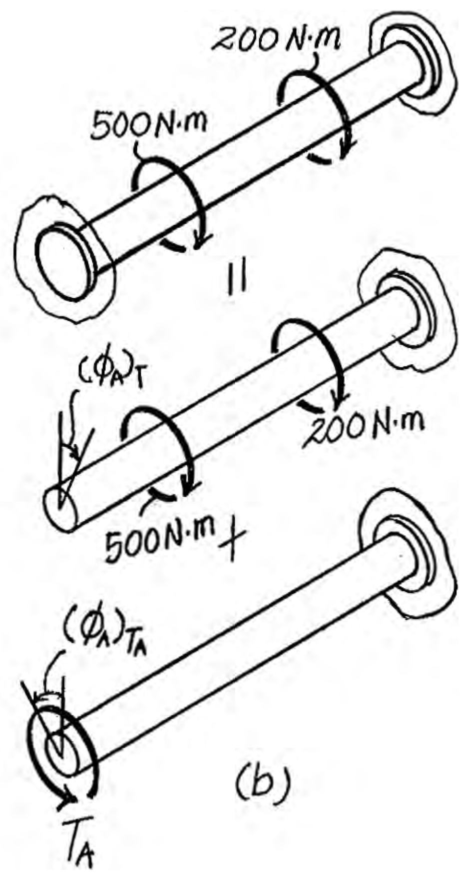
Substitute this result into Eq (1),

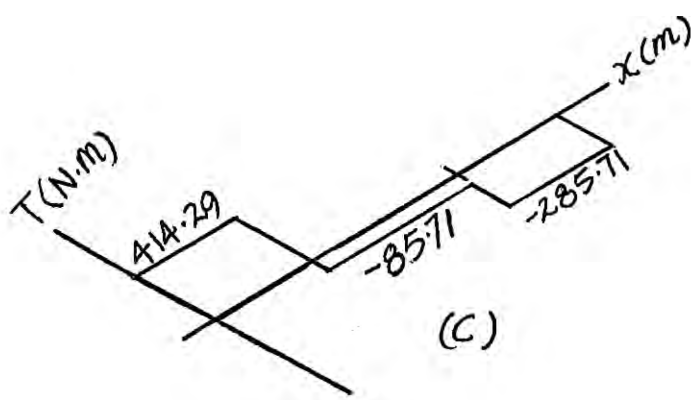
$$T_B = 285.71\ \mathrm{N \cdot m}$$

Referring to the torque diagram shown in Fig. *c*, segment *AC* is subjected to maximum internal torque. Thus, the absolute maximum shear stress occurs here.

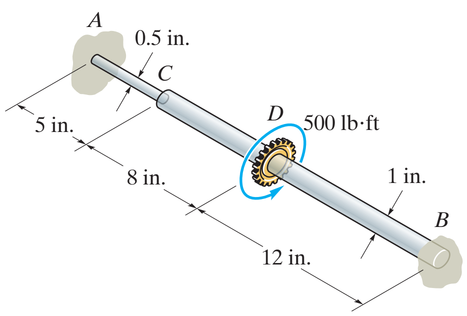
$$\tau Abs = \frac{T_{AC}\,c}{J} = \frac{414.29\,(0.03)}{\frac{\pi}{2}(0.03)^4} = 9.77\ \mathrm{MPa} \qquad \textbf{Ans.}$$







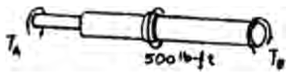
5–79. The steel shaft is made from two segments: AC has a diameter of 0.5 in, and CB has a diameter of 1 in. If it is fixed at its ends A and B and subjected to a torque of determine the maximum shear stress in the shaft. $G_{st} = 10.8(10^3)$ ksi.



Equilibrium:

$$T_A + T_B - 500 = 0 \qquad \textbf{(1)}$$

Compatibility condition:



$$\phi_{D/A} = \phi_{D/B}$$

$$\frac{T_A(5)}{\frac{\pi}{2}(0.25^4)G} + \frac{T_A(8)}{\frac{\pi}{2}(0.5^4)G} = \frac{T_B(12)}{\frac{\pi}{2}(0.5^4)G}$$

$$1408\, T_A = 192\, T_B \qquad \textbf{(2)}$$

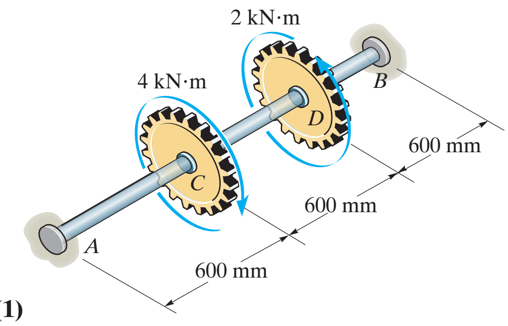
Solving Eqs. (1) and (2) yields

$$T_A = 60 \text{ lb} \cdot \text{ft} \qquad T_B = 440 \text{ lb} \cdot \text{ft}$$

$$\tau_{AC} = \frac{Tc}{J} = \frac{60(12)(0.25)}{\frac{\pi}{2}(0.25^4)} = 29.3 \text{ ksi} \qquad \text{(max)} \qquad \textbf{Ans.}$$

$$\tau_{DB} = \frac{Tc}{J} = \frac{440(12)(0.5)}{\frac{\pi}{2}(0.5^4)} = 26.9 \text{ ksi}$$

***5–80.** The shaft is made of A-36 steel, has a diameter of 80 mm, and is fixed at B while A is loose and can rotate 0.005 rad before becoming fixed. When the torques are applied to C and D, determine the maximum shear stress in regions AC and CD of the shaft.



Referring to the FBD of the shaft shown in Fig. *a*,

$\Sigma M_x = 0; \qquad T_A + T_B + 2 - 4 = 0$ **(1)**

Using the method of superposition, Fig. *b*,

$$\phi_A = (\phi_A)_T - (\theta_A)_{T_A}$$

$$0.005 = \left[\frac{4(10^3)(0.6)}{\frac{\pi}{2}(0.04^4)\left[75(10^9)\right]} + \frac{2(10^3)(0.6)}{\frac{\pi}{2}(0.04^4)\left[75(10^9)\right]}\right] - \frac{T_A(1.8)}{\frac{\pi}{2}(0.04^4)\left[75(10^9)\right]}$$

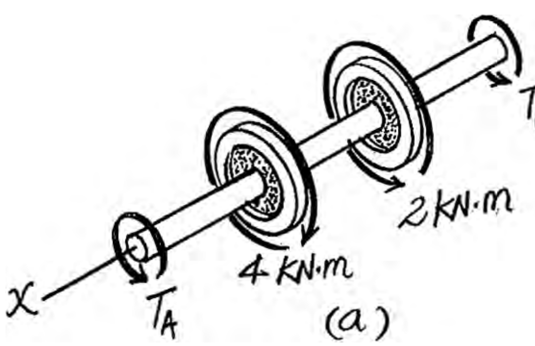
$T_A = 1162.24\ \text{N}\cdot\text{m} = 1.162\ \text{kN}\cdot\text{m}$

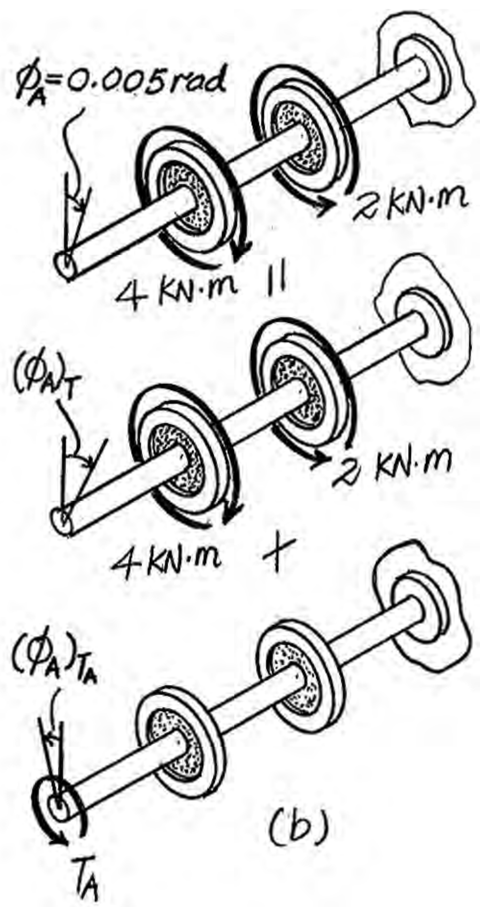
Substitute this result into Eq (1),

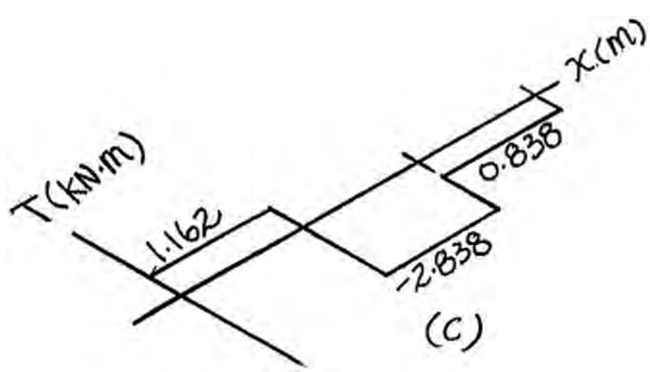
$T_B = 0.838\ \text{kN}\cdot\text{m}$

Referring to the torque diagram shown in Fig. *c*, segment CD is subjected to a maximum internal torque. Thus, the absolute maximum shear stress occurs here.

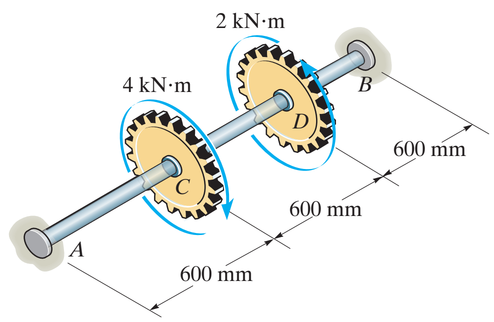
$$\tau_{\$\$\$} = \frac{T_{CD}\,c}{J} = \frac{2.838\,(10^3)(0.04)}{\frac{\pi}{2}\,(0.04)^4} = 28.23\,(10^6)\ \text{Pa} = 28.2\ \text{MPa}$$ **Ans.**





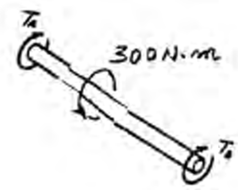


•5–81. The shaft is made of A-36 steel and has a diameter of 80 mm. It is fixed at B and the support at A has a torsional stiffness of $k = 0.5\ \text{MN}\cdot\text{m/rad}$. If it is subjected to the gear torques shown, determine the absolute maximum shear stress in the shaft.



Referring to the FBD of the shaft shown in Fig. a,

$$\Sigma M_x = 0; \qquad T_A + T_B + 2 - 4 = 0 \qquad \textbf{(1)}$$



Using the method of superposition, Fig. b,

$$\phi_A = (\phi_A)_T - (\phi_A)_{T_A}$$

$$\frac{T_A}{0.5(10^6)} = \left[\frac{4(10^3)(0.6)}{\frac{\pi}{2}(0.04^4)\left[75(10^9)\right]} + \frac{2(10^3)(0.6)}{\frac{\pi}{2}(0.04^4)\left[75(10^9)\right]}\right] - \frac{T_A(1.8)}{\frac{\pi}{2}(0.04^4)\left[75(10^9)\right]}$$

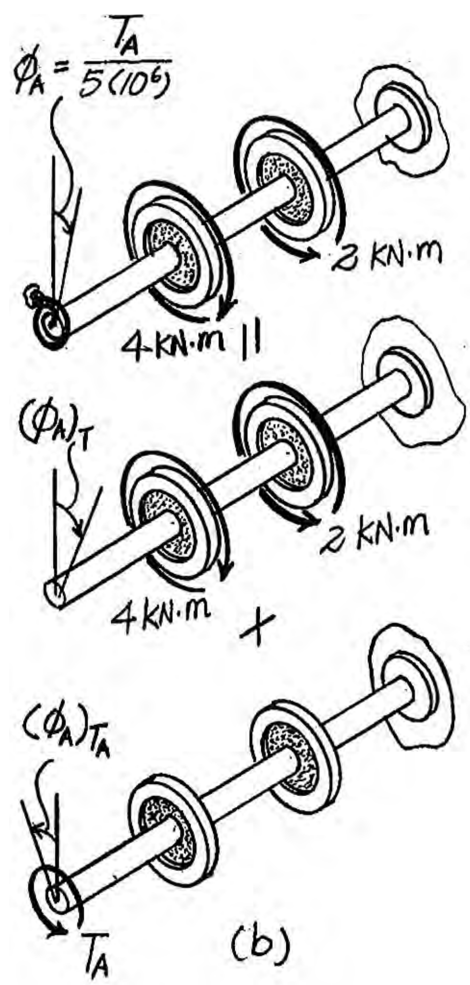
$$T_A = 1498.01\ \text{N}\cdot\text{m} = 1.498\ \text{kN}\cdot\text{m}$$

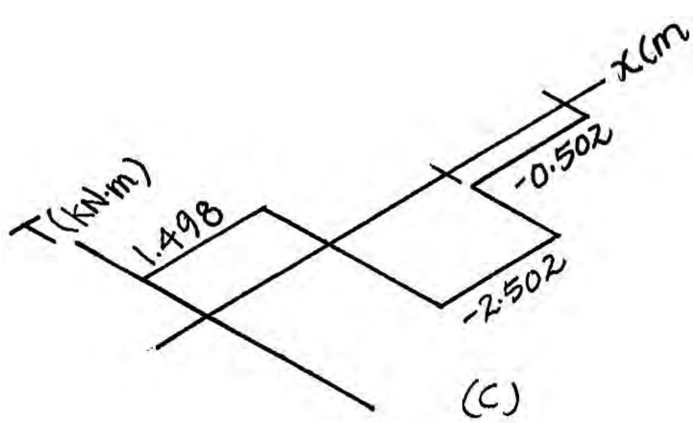
Substituting this result into Eq (1),

$$T_B = 0.502\ \text{kN}\cdot\text{m}$$

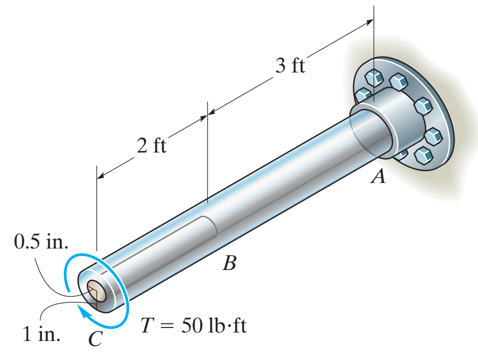
Referring to the torque diagram shown in Fig. c, segment CD subjected to maximum internal torque. Thus, the maximum shear stress occurs here.

$$\tau_{\$\$\$} = \frac{T_{CD}\,C}{J} = \frac{2.502(10^3)(0.04)}{\frac{\pi}{2}(0.04)^4} = \$\$\$ = 24.9\ \text{MPa} \qquad \textbf{Ans.}$$





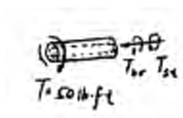
5–82. The shaft is made from a solid steel section AB and a tubular portion made of steel and having a brass core. If it is fixed to a rigid support at A, and a torque of $T = 50\ \text{lb}\cdot\text{ft}$ is applied to it at C, determine the angle of twist that occurs at C and compute the maximum shear stress and maximum shear strain in the brass and steel. Take $G_{st} = 11.5(10^3)$ ksi, $G_{br} = 5.6(10^3)$ ksi.



Equilibrium:

$$T_{br} + T_{st} - 50 = 0 \qquad \textbf{(1)}$$

Both the steel tube and brass core undergo the same angle of twist $\phi_{C/B}$



$$\phi_{C/B} = \frac{TL}{JG} = \frac{T_{st}\,(2)(12)}{\frac{\pi}{2}\,(0.5^4)(5.6)(10^4)} = \frac{T_{st}\,(2)(12)}{\frac{\pi}{2}\,(1^4 - 0.5^4)(11.5)(10^6)}$$

$$T_{br} = 0.032464\, T_{st} \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields:

$T_{st} = 48.428\ \text{lb}\cdot\text{ft}; \qquad T_{br} = 1.572\ \text{lb}\cdot\text{ft}$

$$\phi_C = \Sigma\frac{TL}{JG} = \frac{1.572(12)(2)(12)}{\frac{\pi}{2}\,(0.5^4)(5.6)(10^6)} + \frac{50(12)(3)(12)}{\frac{\pi}{2}(1^4)(11.5)(10^6)}$$

$$= 0.002019\ \text{rad} = 0.116^\circ \qquad \textbf{Ans.}$$

$$(\tau_{st})_{\max\,AB} = \frac{T_{AB}c}{J} = \frac{50(12)(1)}{\frac{\pi}{2}(1^4)} = 382\ \text{psi}$$

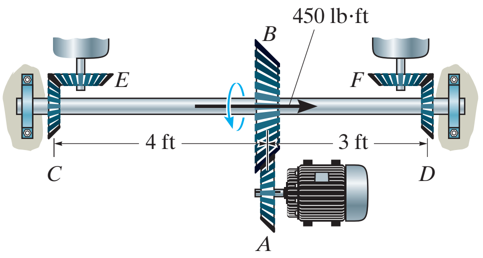
$$(\tau_{st})_{\max\,BC} = \frac{T_{st}\,c}{J} = \frac{48.428(12)(1)}{\frac{\pi}{2}\,(1^4 - 0.5^4)} = 394.63\ \text{psi} = 395\ \text{psi (Max)} \qquad \textbf{Ans.}$$

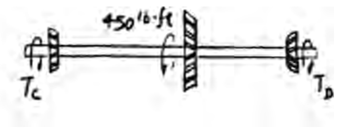
$$(\gamma_{st})_{\max} = \frac{(\tau_{st})_{\max}}{G} = \frac{394.63}{11.5(10^6)} = 343.(10^{-6})\ \text{rad} \qquad \textbf{Ans.}$$

$$(\tau_{br})_{\max} = \frac{T_{br}\,c}{J} = \frac{1.572(12)(0.5)}{\frac{\pi}{2}\,(0.5^4)} = 96.07\ \text{psi} = 96.1\ \text{psi (Max)} \qquad \textbf{Ans.}$$

$$(\gamma_{br})_{\max} = \frac{(\tau_{br})_{\max}}{G} = \frac{96.07}{5.6(10^6)} = 17.2(10^{-6})\ \text{rad} \qquad \textbf{Ans.}$$

5–83. The motor A develops a torque at gear B of 450 lb · ft, which is applied along the axis of the 2-in.-diameter steel shaft CD. This torque is to be transmitted to the pinion gears at E and F. If these gears are temporarily fixed, determine the maximum shear stress in segments CB and BD of the shaft. Also, what is the angle of twist of each of these segments? The bearings at C and D only exert force reactions on the shaft and do not resist torque. $G_{st} = 12(10^3)$ ksi.





Equilibrium:

$$T_C + T_D - 450 = 0 \qquad \textbf{(1)}$$

Compatibility condition:

$$\phi_{B/C} = \phi_{B/D}$$

$$\frac{T_C(4)}{JG} = \frac{T_D(3)}{JG}$$

$$T_C = 0.75\,T_D \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2), yields

$$T_D = 257.14 \text{ lb} \cdot \text{ft}$$

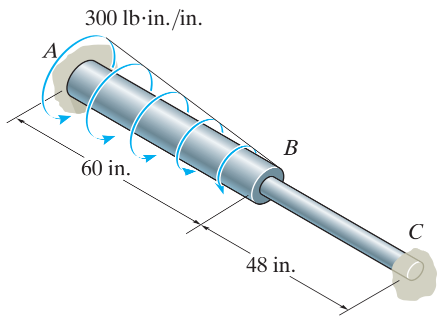
$$T_C = 192.86 \text{ lb} \cdot \text{ft}$$

$$(\tau_{BC})_{max} = \frac{192.86(12)(1)}{\frac{\pi}{2}(1^4)} = 1.47 \text{ ksi} \qquad \textbf{Ans.}$$

$$(\tau_{BD})_{max} = \frac{257.14(12)(1)}{\frac{\pi}{2}(1^4)} = 1.96 \text{ ksi} \qquad \textbf{Ans.}$$

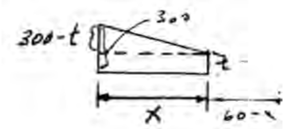
$$\phi = \frac{192.86(12)(4)(12)}{\frac{\pi}{2}(1^4)(12)(10^6)} = 0.00589 \text{ rad} = 0.338^\circ \qquad \textbf{Ans.}$$

***5–84.** A portion of the A-36 steel shaft is subjected to a linearly distributed torsional loading. If the shaft has the dimensions shown, determine the reactions at the fixed supports A and C. Segment AB has a diameter of 1.5 in. and segment BC has a diameter of 0.75 in.



Equilibrium:

$$T_A + T_C - 9000 = 0 \qquad \textbf{(1)}$$

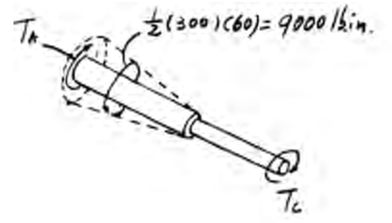


$$T_R = t\,x + \frac{1}{2}(300 - t)x = 150x + \frac{t\,x}{2}$$

But $\dfrac{t}{60 - x} = \dfrac{300}{60}$; $\qquad t = 5(60 - x)$

$$T_R = 150\,x + \frac{1}{2}[5(60 - x)]x$$

$$= (300x - 2.5x^2)\text{ lb}\cdot\text{in.}$$



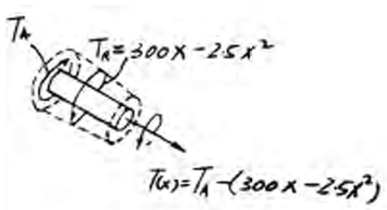
Compatibility condition:

$$\phi_{B/A} = \phi_{B/C}$$

$$\phi_{B/A} = \int \frac{T(x)\,dx}{JG} = \frac{1}{JG}\int_0^{60} [T_A - (300x - 2.5x^2)]\,dx$$

$$= \frac{1}{JG}[T_A x - 150x^2 + 0.8333x^3]\Big|_0^{60}$$

$$= \frac{60T_A - 360\,000}{JG}$$



$$\frac{60T_A - 360\,000}{\frac{\pi}{2}(0.75^4)G} = \frac{T_C(48)}{\frac{\pi}{2}(0.375^4)G}$$

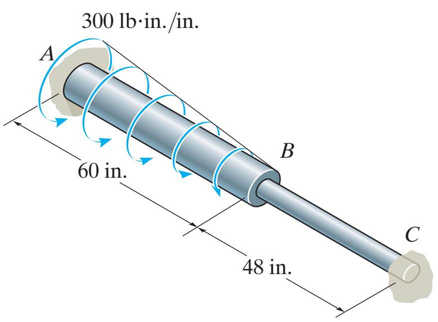
$$60T_A - 768T_C = 360\,000 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields:

$T_C = 217.4\text{ lb}\cdot\text{in.} = 18.1\text{ lb}\cdot\text{ft}$ **Ans.**

$T_A = 8782.6\text{ lb}\cdot\text{in.} = 732\text{ lb}\cdot\text{ft}$ **Ans.**

•5–85. Determine the rotation of joint B and the absolute maximum shear stress in the shaft in Prob. 5–84.

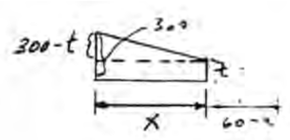


Equilibrium:

$$T_A + T_C - 9000 = 0 \qquad \textbf{(1)}$$

$$T_R = tx + \frac{1}{2}(300 - t)x = 150x + \frac{tx}{2}$$

But $\dfrac{t}{60 - x} = \dfrac{300}{60}$; $\qquad t = 5(60 - x)$



$$T_R = 150x + \frac{1}{2}[5(60 - x)]x$$

$$= (300x - 2.5x^2)\ \text{lb}\cdot\text{in.}$$

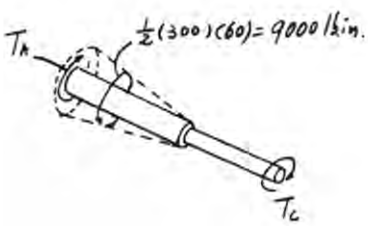
Compatibility condition:

$$\phi_{B/A} = \phi_{B/C}$$

$$\phi_{B/A} = \int \frac{T(x)\,dx}{JG} = \frac{1}{JG}\int_0^{60} [T_A - (300x - 2.5x^2)]\,dx$$

$$= \frac{1}{JG}[T_A x - 150x^2 + 0.8333x^3]\Big|_0^{60}$$

$$= \frac{60T_A - 360\,000}{JG}$$



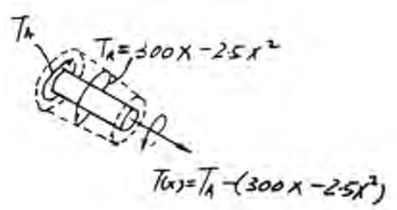
$$\frac{60T_A - 360\,000}{\frac{\pi}{2}(0.75^4)G} = \frac{T_C(48)}{\frac{\pi}{2}(0.375^4)G}$$

$$60T_A - 768T_C = 360\,000 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields:

$$T_C = 217.4\ \text{lb}\cdot\text{in.} = 18.1\ \text{lb}\cdot\text{ft}$$

$$T_A = 8782.6\ \text{lb}\cdot\text{in.} = 732\ \text{lb}\cdot\text{ft}$$



For segment BC:

$$\phi_B = \phi_{B/C} = \frac{T_C L}{JG} = \frac{217.4(48)}{\frac{\pi}{2}(0.375)^4(11.0)(10^6)} = 0.030540\ \text{rad}$$

$$\phi_B = 1.75^\circ \qquad \textbf{Ans.}$$

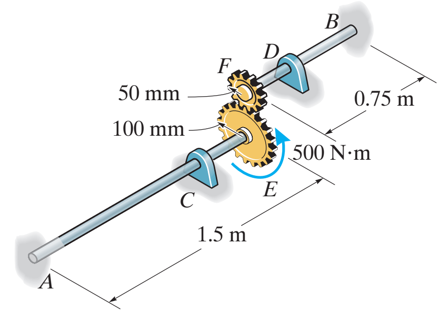
$$\tau_{max} = \frac{T_C}{J} = \frac{217.4(0.375)}{\frac{\pi}{2}(0.375)^4} = 2.62\ \text{ksi}$$

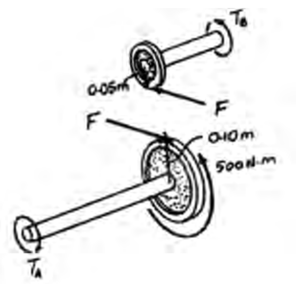
For segment AB,

$$\tau_{max} = \frac{T_C}{J} = \frac{8782.6(0.75)}{\frac{\pi}{2}(0.75)^4} = 13.3\ \text{ksi}$$

$$\tau^{abs}_{max} = 13.3\ \text{ksi} \qquad \textbf{Ans.}$$

5–86. The two shafts are made of A-36 steel. Each has a diameter of 25 mm and they are connected using the gears fixed to their ends. Their other ends are attached to fixed supports at A and B. They are also supported by journal bearings at C and D, which allow free rotation of the shafts along their axes. If a torque of 500 N · m is applied to the gear at E as shown, determine the reactions at A and B.





Equilibrium:

$$T_A + F(0.1) - 500 = 0 \qquad \textbf{[1]}$$

$$T_B - F(0.05) = 0 \qquad \textbf{[2]}$$

From Eqs. [1] and [2]

$$T_A + 2T_B - 500 = 0 \qquad \textbf{[3]}$$

Compatibility:

$$0.1\phi_E = 0.05\phi_F$$

$$\phi_E = 0.5\phi_F$$

$$\frac{T_A(1.5)}{JG} = 0.5\left[\frac{T_B(0.75)}{JG}\right]$$

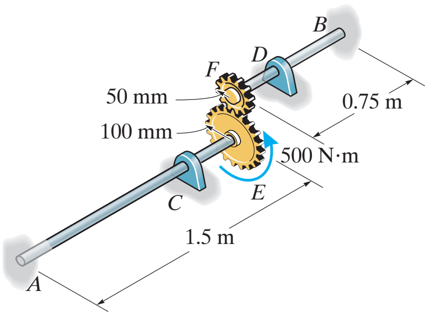
$$T_A = 0.250T_B \qquad \textbf{[4]}$$

Solving Eqs. [3] and [4] yields:

$$T_B = 222 \text{ N} \cdot \text{m} \qquad \textbf{Ans.}$$

$$T_A = 55.6 \text{ N} \cdot \text{m} \qquad \textbf{Ans.}$$

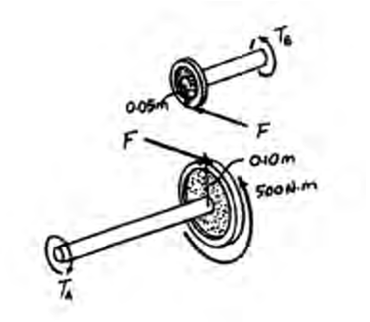
5–87. Determine the rotation of the gear at E in Prob. 5–86.



Equilibrium:

$$T_A + F(0.1) - 500 = 0 \qquad \textbf{[1]}$$

$$T_B - F(0.05) = 0 \qquad \textbf{[2]}$$



From Eqs. [1] and [2]

$$T_A + 2T_B - 500 = 0 \qquad \textbf{[3]}$$

Compatibility:

$$0.1\phi_E = 0.05\phi_F$$

$$\phi_E = 0.5\phi_F$$

$$\frac{T_A(1.5)}{JG} = 0.5\left[\frac{T_B(0.75)}{JG}\right]$$

$$T_A = 0.250T_B \qquad \textbf{[4]}$$

Solving Eqs. [3] and [4] yields:

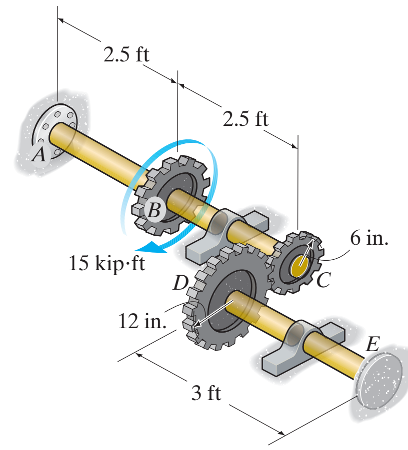
$$T_B = 222.22\ \text{N}\cdot\text{m} \qquad T_A = 55.56\ \text{N}\cdot\text{m}$$

Angle of Twist:

$$\phi_E = \frac{T_A L}{JG} = \frac{55.56(1.5)}{\frac{\pi}{2}(0.0125^4)(75.0)(10^9)}$$

$$= 0.02897\ \text{rad} = 1.66° \qquad \textbf{Ans.}$$

***5–88.** The shafts are made of A-36 steel and have the same diameter of 4 in. If a torque of 15 kip · ft is applied to gear B, determine the absolute maximum shear stress developed in the shaft.



Equilibrium: Referring to the free - body diagrams of shafts ABC and DE shown in Figs. a and b, respectively, we have

$$\Sigma M_x = 0; \quad T_A + F(0.5) - 15 = 0 \qquad \textbf{(1)}$$

and

$$\Sigma M_x = 0; \quad F(1) - T_E = 0 \qquad \textbf{(2)}$$

Internal Loadings: The internal torques developed in segments AB and BC of shaft ABC and shaft DE are shown in Figs. c, d, and e, respectively.

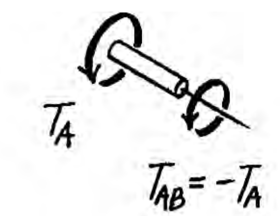
Compatibility Equation:

$$\phi_C r_C = \phi_D r_D$$

$$\left(\frac{T_{AB}L_{AB}}{JG_{st}} + \frac{T_{BC}L_{BC}}{JG_{st}}\right)r_C = \left(\frac{T_{DE}\,L_{DE}}{JG_{st}}\right)r_D$$

$$\left[-T_A(2.5) + F(0.5)(2.5)\right](0.5) = -T_E(3)(1)$$

$$T_A - 0.5F = 2.4T_E \qquad \textbf{(3)}$$



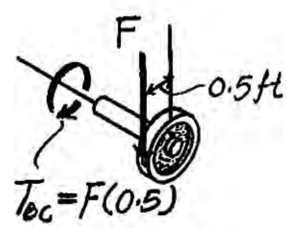
(c)

Solving Eqs. (1), (2), and (3), we have

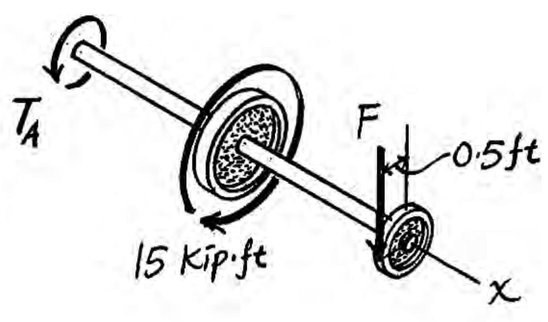
$F = 4.412$ kip $\qquad T_E = 4.412$ kip · ft $\qquad T_A = 12.79$ kip · ft

Maximum Shear Stress: By inspection, segment AB of shaft ABC is subjected to the greater torque.

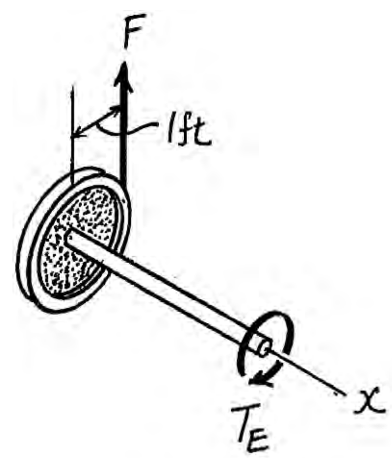
$$\left(\tau_{\max}\right)_{\text{abs}} = \frac{T_{AB}c}{J_{st}} = \frac{12.79(12)(2)}{\frac{\pi}{2}\left(2^4\right)} = 12.2 \text{ ksi} \qquad \textbf{Ans.}$$



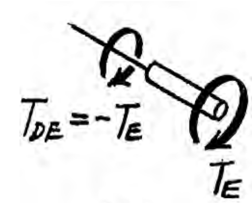
(d)



(a)

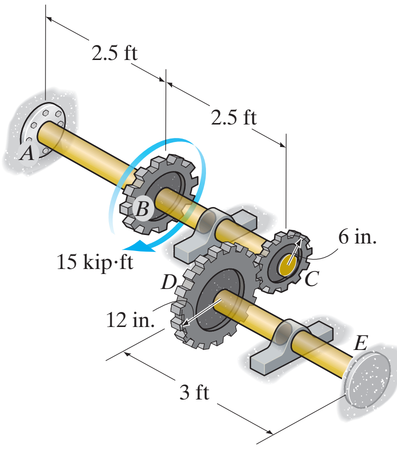


(b)



(e)

•5–89. The shafts are made of A-36 steel and have the same diameter of 4 in. If a torque of 15 kip · ft is applied to gear B, determine the angle of twist of gear B.



Equilibrium: Referring to the free - body diagrams of shafts ABC and DE shown in Figs. a and b, respectively,

$$\Sigma M_x = 0; \quad T_A + F(0.5) - 15 = 0 \qquad \textbf{(1)}$$

and

$$\Sigma M_x = 0; \quad F(1) - T_E = 0 \qquad \textbf{(2)}$$

Internal Loadings: The internal torques developed in segments AB and BC of shaft ABC and shaft DE are shown in Figs. c, d, and e, respectively.

Compatibility Equation: It is required that

$$\phi_C r_C = \phi_D r_D$$

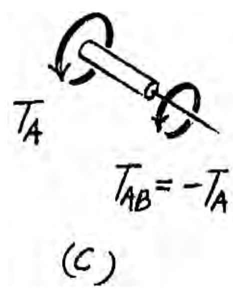
$$\left(\frac{T_{AB}\,L_{AB}}{JG_{st}} + \frac{T_{BC}\,L_{BC}}{JG_{st}}\right) r_C = \left(\frac{T_{DE}\,L_{DE}}{JG_{st}}\right) r_D$$

$$\left[-T_A(2.5) + F(0.5)(2.5)\right](0.5) = -T_E(3)(1)$$

$$T_A - 0.5F = 2.4T_E \qquad \textbf{(3)}$$

Solving Eqs. (1), (2), and (3),

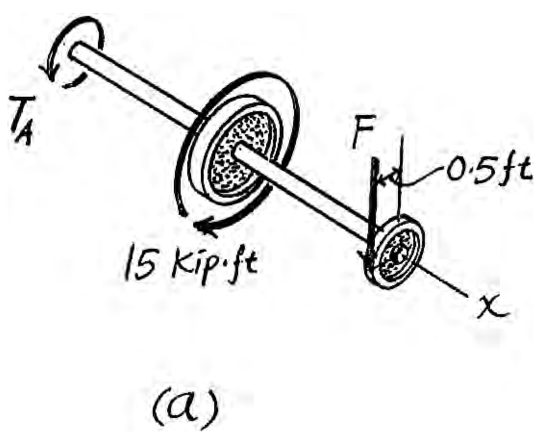
$F = 4.412$ kip $\qquad T_E = 4.412$ kip · ft $\qquad T_A = 12.79$ kip · ft

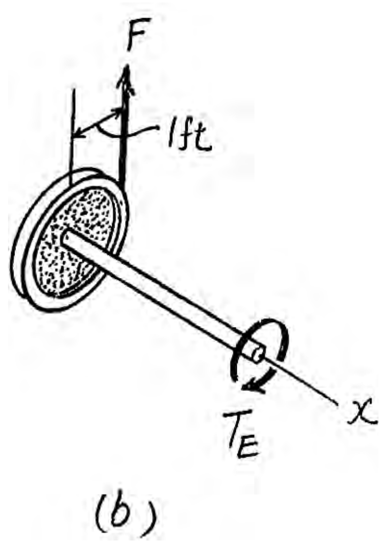


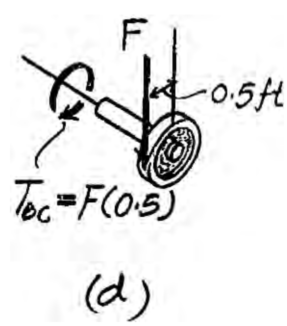
Angle of Twist: Here, $T_{AB} = -T_A = -12.79$ kip · ft

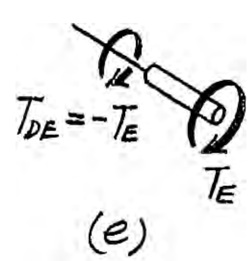
$$\phi_B = \frac{T_{AB}\,L_{AB}}{JG_{st}} = \frac{-12.79(12)(2.5)(12)}{\frac{\pi}{2}\left(2^4\right)(11.0)(10^3)}$$

$$= -0.01666 \text{ rad} = 0.955° \qquad \textbf{Ans.}$$

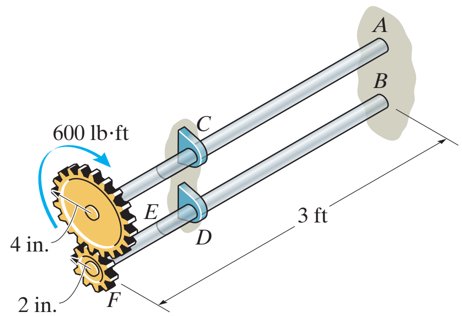


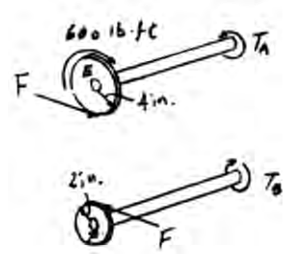






5–90. The two 3-ft-long shafts are made of 2014-T6 aluminum. Each has a diameter of 1.5 in. and they are connected using the gears fixed to their ends. Their other ends are attached to fixed supports at A and B. They are also supported by bearings at C and D, which allow free rotation of the shafts along their axes. If a torque of 600 lb · ft is applied to the top gear as shown, determine the maximum shear stress in each shaft.





$$T_A + F\left(\frac{4}{12}\right) - 600 = 0 \qquad (1)$$

$$T_B - F\left(\frac{2}{12}\right) = 0 \qquad (2)$$

From Eqs. (1) and (2)

$$T_A + 2T_B - 600 = 0 \qquad (3)$$

$$4(\phi_E) = 2(\phi_F); \qquad \phi_E = 0.5\phi_F$$

$$\frac{T_A L}{JG} = 0.5\left(\frac{T_B L}{JG}\right); \qquad T_A = 0.5T_B \qquad (4)$$

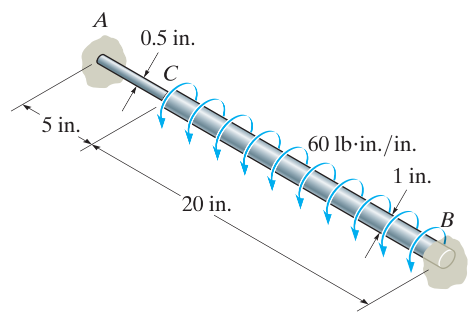
Solving Eqs. (3) and (4) yields:

$$T_B = 240 \text{ lb} \cdot \text{ft}; \qquad T_A = 120 \text{ lb} \cdot \text{ft}$$

$$(\tau_{BD})_{\max} = \frac{T_B\, c}{J} = \frac{240(12)(0.75)}{\frac{\pi}{2}(0.75^4)} = 4.35 \text{ ksi} \qquad \textbf{Ans.}$$

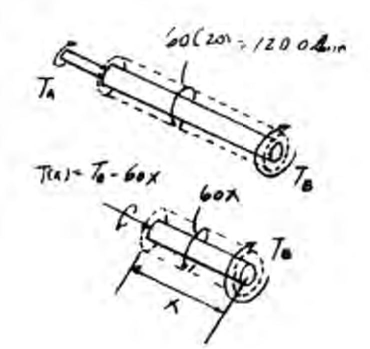
$$(\tau_{AC})_{\max} = \frac{T_A\, c}{J} = \frac{120(12)(0.75)}{\frac{\pi}{2}(0.75^4)} = 2.17 \text{ ksi} \qquad \textbf{Ans.}$$

5–91. The A-36 steel shaft is made from two segments: AC has a diameter of 0.5 in. and CB has a diameter of 1 in. If the shaft is fixed at its ends A and B and subjected to a uniform distributed torque of 60 lb·in./in. along segment CB, determine the absolute maximum shear stress in the shaft.



Equilibrium:

$$T_A + T_B - 60(20) = 0 \qquad \textbf{(1)}$$



Compatibility condition:

$$\phi_{C/B} = \phi_{C/A}$$

$$\phi_{C/B} = \int \frac{T(x)\,dx}{JG} = \int_0^{20} \frac{(T_B - 60x)\,dx}{\frac{\pi}{2}(0.5^4)(11.0)(10^6)}$$

$$= 18.52(10^{-6})T_B - 0.011112$$

$$18.52(10^{-6})T_B - 0.011112 = \frac{T_A(5)}{\frac{\pi}{2}(0.25^4)(11.0)(10^6)}$$

$$18.52(10^{-6})T_B - 74.08(10^{-6})T_A = 0.011112$$

$$18.52T_B - 74.08T_A = 11112 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields:

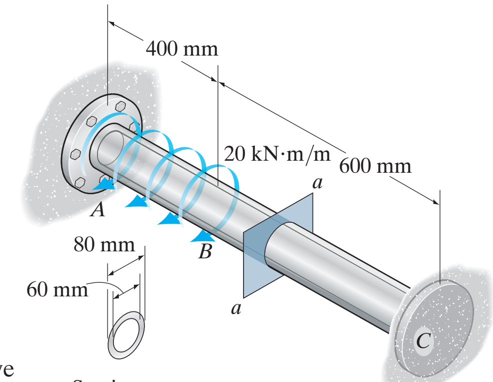
$$T_A = 120.0 \text{ lb}\cdot\text{in.}; \qquad T_B = 1080 \text{ lb}\cdot\text{in.}$$

$$(\tau_{max})_{BC} = \frac{T_B\,c}{J} = \frac{1080(0.5)}{\frac{\pi}{2}(0.5^4)} = 5.50 \text{ ksi}$$

$$(\tau_{max})_{AC} = \frac{T_A\,c}{J} = \frac{120.0(0.25)}{\frac{\pi}{2}(0.25^4)} = 4.89 \text{ ksi}$$

$$\tau^{abs}_{max} = 5.50 \text{ ksi} \qquad \textbf{Ans.}$$

***5–92.** If the shaft is subjected to a uniform distributed torque of $t = 20\ \mathrm{kN \cdot m/m}$, determine the maximum shear stress developed in the shaft. The shaft is made of 2014-T6 aluminum alloy and is fixed at A and C.



Section a–a

Equilibrium: Referring to the free - body diagram of the shaft shown in Fig. a, we have

$$\Sigma M_x = 0; \quad T_A + T_C - 20(10^3)(0.4) = 0 \qquad \textbf{(1)}$$

Compatibility Equation: The resultant torque of the distributed torque within the region x of the shaft is $T_R = 20(10^3)x\ \mathrm{N \cdot m}$. Thus, the internal torque developed in the shaft as a function of x when end C is free is $T(x) = 20(10^3)x\ \mathrm{N \cdot m}$, Fig. b. Using the method of superposition, Fig. c,

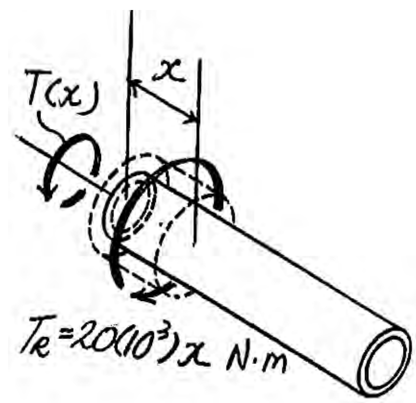
$$\phi_C = \left(\phi_C\right)_t - \left(\phi_C\right)_{T_C}$$

$$0 = \int_0^{0.4\ \mathrm{m}} \frac{T(x)dx}{JG} - \frac{T_C L}{JG}$$

$$0 = \int_0^{0.4\ \mathrm{m}} \frac{20(10^3)xdx}{JG} - \frac{T_C(1)}{JG}$$

$$0 = 20(10^3)\left(\frac{x^2}{2}\right)\Bigg|_0^{0.4\ \mathrm{m}} - T_C$$

$$T_C = 1600\ \mathrm{N \cdot m}$$



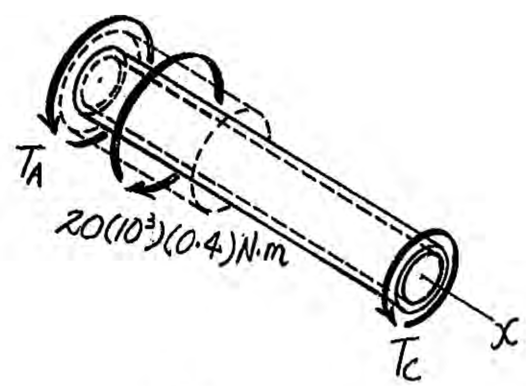
(b)

Substituting this result into Eq. (1),

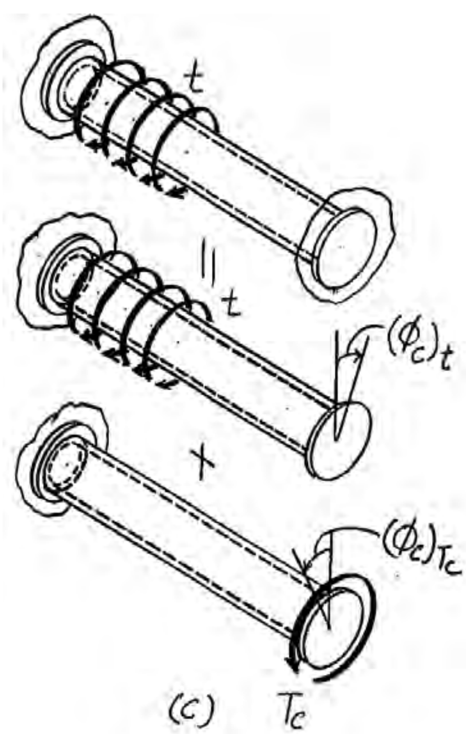
$$T_A = 6400\ \mathrm{N \cdot m}$$

Maximum Shear Stress: By inspection, the maximum internal torque occurs at support A. Thus,

$$\left(\tau_{\max}\right)_{\text{abs}} = \frac{T_A c}{J} = \frac{6400(0.04)}{\frac{\pi}{2}\left(0.04^4 - 0.03^4\right)} = 93.1\ \mathrm{MPa} \qquad \textbf{Ans.}$$

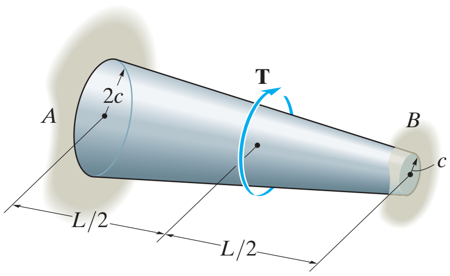


(a)



(c)

•5–93. The tapered shaft is confined by the fixed supports at A and B. If a torque **T** is applied at its mid-point, determine the reactions at the supports.



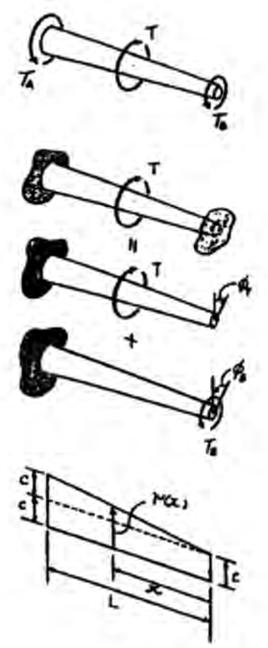
Equilibrium:

$$T_A + T_B - T = 0 \qquad \textbf{[1]}$$

Section Properties:

$$r(x) = c + \frac{c}{L}x = \frac{c}{L}(L + x)$$

$$J(x) = \frac{\pi}{2}\left[\frac{c}{L}(L + x)\right]^4 = \frac{\pi c^4}{2L^4}(L + x)^4$$



Angle of Twist:

$$\phi_T = \int \frac{T\,dx}{J(x)G} = \int_{\frac{\pi}{2}}^{L} \frac{T\,dx}{\frac{\pi c^4}{2L^4}(L + x)^4 G}$$

$$= \frac{2TL^4}{\pi c^4 G}\int_{\frac{\pi}{2}}^{L} \frac{dx}{(L + x)^4}$$

$$= -\frac{2TL^4}{3\pi c^4 G}\left[\frac{1}{(L + x)^3}\right]\Bigg|_{\frac{\pi}{2}}^{L}$$

$$= \frac{37TL}{324\,\pi c^4 G}$$

$$\phi_B = \int \frac{T\,dx}{J(x)G} = \int_0^L \frac{T_B\,dx}{\frac{\pi c^4}{2L^4}(L + x)^4 G}$$

$$= \frac{2T_B L^4}{\pi c^4 G}\int_0^L \frac{dx}{(L + x)^4}$$

$$= -\frac{2T_B L^4}{3\pi c^4 G}\left[\frac{1}{(L + x)^3}\right]\Bigg|_0^L$$

$$= \frac{7T_B L}{12\pi c^4 G}$$

Compatibility:

$$0 = \phi_T - \phi_B$$

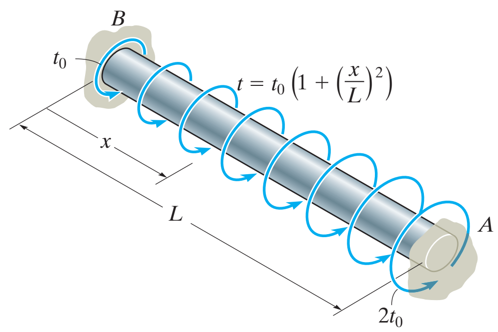
$$0 = \frac{37TL}{324\pi c^4 G} - \frac{7T_B L}{12\pi c^4 G}$$

$$T_B = \frac{37}{189}T \qquad \textbf{Ans.}$$

Substituting the result into Eq. [1] yields:

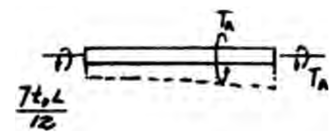
$$T_A = \frac{152}{189}T \qquad \textbf{Ans.}$$

5–94. The shaft of radius c is subjected to a distributed torque t, measured as torque/length of shaft. Determine the reactions at the fixed supports A and B.



$$T(x) = \int_0^x t_0\left(1 + \frac{x^2}{L^2}\right)dx = t_0\left(x + \frac{x^2}{3L^2}\right) \qquad \textbf{(1)}$$

By superposition:

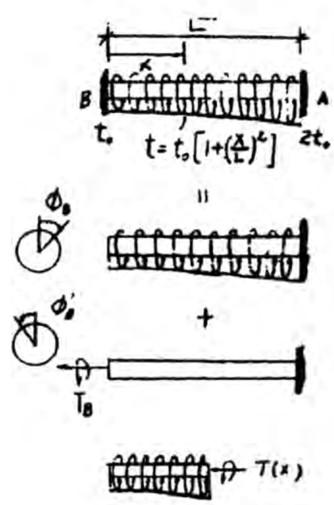


$$0 = \phi_B - \phi_B$$

$$0 = \int_0^L \frac{t_0\left(x + \frac{x^3}{3L^2}\right)dx}{JG} - \frac{T_B(L)}{JG} = \frac{7t_0L^2}{12} - T_B(L)$$

$$T_B = \frac{7t_0L}{12} \qquad \textbf{Ans.}$$

From Eq. (1),

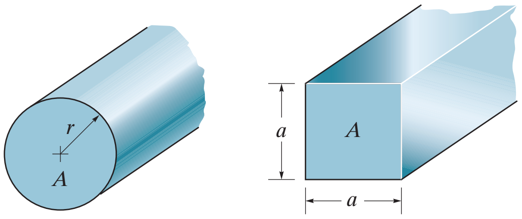


$$T_A = t_0\left(L + \frac{L^3}{3L^2}\right) = \frac{4t_0L}{3}$$

$$T_A + \frac{7t_0L}{12} - \frac{4t_0L}{3} = 0$$

$$T_A = \frac{3t_0L}{4} \qquad \textbf{Ans.}$$

5–95. Compare the values of the maximum elastic shear stress and the angle of twist developed in 304 stainless steel shafts having circular and square cross sections. Each shaft has the same cross-sectional area of 9 in^2, length of 36 in., and is subjected to a torque of 4000 lb · in.



Maximum Shear Stress:

For circular shaft

$$A = \pi c^2 = 9; \qquad c = \left(\frac{9}{\pi}\right)^{\frac{1}{2}}$$

$$(\tau_c)_{\max} = \frac{Tc}{J} = \frac{Tc}{\frac{\pi}{2}c^4} = \frac{2T}{\pi\, c^3} = \frac{2(4000)}{\pi\left(\frac{9}{x}\right)^{\frac{1}{2}}} = 525 \text{ psi} \qquad \textbf{Ans.}$$

For rectangular shaft

$$A = a^2 = 9\,; \qquad a = 3 \text{ in.}$$

$$(\tau_r)_{\max} = \frac{4.81T}{a^3} = \frac{4.81(4000)}{3^3} = 713 \text{ psi} \qquad \textbf{Ans.}$$

Angle of Twist:

For circular shaft

$$\phi_c = \frac{TL}{JG} = \frac{4000(36)}{\frac{\pi}{2}\left(\frac{9}{\pi}\right)^2 11.0(10^6)}$$

$$= 0.001015 \text{ rad} = 0.0582° \qquad \textbf{Ans.}$$

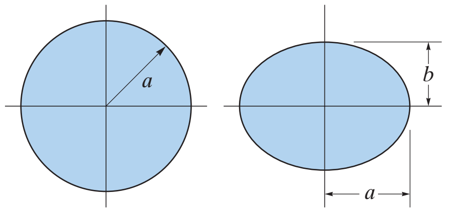
For rectangular shaft

$$\phi_r = \frac{7.10\,TL}{a^4 G} = \frac{7.10(4000)(36)}{3^4(11.0)(10^6)}$$

$$= 0.001147 \text{ rad} = 0.0657° \qquad \textbf{Ans.}$$

The rectangular shaft has a greater maximum shear stress and angle of twist.

***5–96.** If $a = 25$ mm and $b = 15$ mm, determine the maximum shear stress in the circular and elliptical shafts when the applied torque is $T = 80$ N · m. By what percentage is the shaft of circular cross section more efficient at withstanding the torque than the shaft of elliptical cross section?



For the circular shaft:

$$(\tau_{\max})_c = \frac{T\,c}{J} = \frac{80(0.025)}{\frac{\pi}{2}(0.025^4)} = 3.26 \text{ MPa}$$ **Ans.**

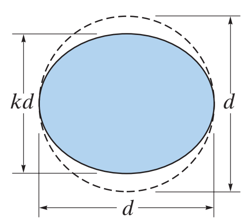
For the elliptical shaft:

$$(\tau_{\max})_c = \frac{2T}{\pi\, a\, b^2} = \frac{2(80)}{\pi(0.025)(0.015^2)} = 9.05 \text{ MPa}$$ **Ans.**

$$\% \text{ more efficient} = \frac{(\tau_{\max})_c - (\tau_{\max})_c}{(\tau_{\max})_c}(100\%)$$

$$= \frac{9.05 - 3.26}{3.26}(100\%) = 178\ \%$$ **Ans.**

•5–97. It is intended to manufacture a circular bar to resist torque; however, the bar is made elliptical in the process of manufacturing, with one dimension smaller than the other by a factor k as shown. Determine the factor by which the maximum shear stress is increased.



For the circular shaft:

$$(\tau_{\max})_c = \frac{Tc}{J} = \frac{T\left(\frac{d}{2}\right)}{\frac{\pi}{2}\left(\frac{d}{2}\right)^4} = \frac{16T}{\pi\, d^3}$$

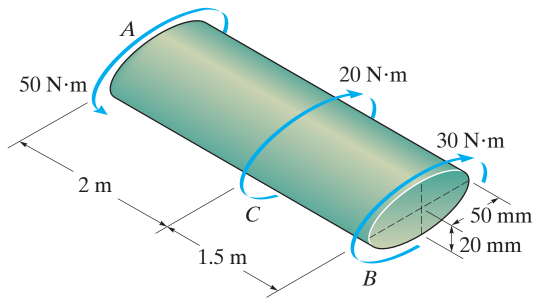
For the elliptical shaft:

$$(\tau_{\max})_c = \frac{2T}{\pi\, a\, b^2} = \frac{2T}{\pi\left(\frac{d}{2}\right)\left(\frac{kd}{2}\right)^2} = \frac{16T}{\pi\, k^2\, d^3}$$

$$\text{Factor of increase in shear stress} = \frac{(\tau_{\max})_c}{(\tau_{\max})_c} = \frac{\frac{16T}{\pi\, k^2\, d^3}}{\frac{16T}{\pi\, d^3}}$$

$$= \frac{1}{k^2}$$ **Ans.**

5–98. The shaft is made of red brass C83400 and has an elliptical cross section. If it is subjected to the torsional loading shown, determine the maximum shear stress within regions AC and BC, and the angle of twist ϕ of end B relative to end A.



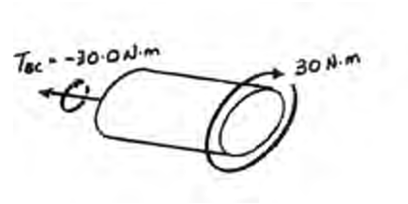
Maximum Shear Stress:

$$(\tau_{BC})_{max} = \frac{2T_{BC}}{\pi\, a\, b^2} = \frac{2(30.0)}{\pi(0.05)(0.02^2)}$$

$$= 0.955 \text{ MPa} \qquad \textbf{Ans.}$$

$$(\tau_{AC})_{max} = \frac{2T_{AC}}{\pi\, a\, b^2} = \frac{2(50.0)}{\pi(0.05)(0.02^2)}$$

$$= 1.59 \text{ MPa} \qquad \textbf{Ans.}$$



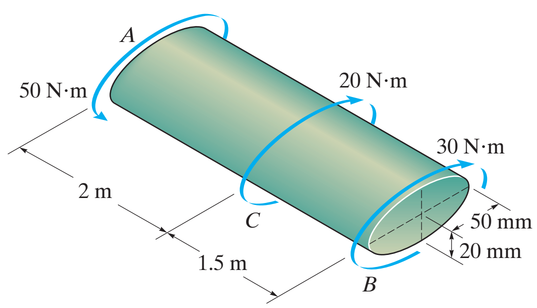
Angle of Twist:

$$\phi_{B/A} = \sum \frac{(a^2 + b^2)T\,L}{\pi\, a^3 b^3 G}$$

$$= \frac{(0.05^2 + 0.02^2)}{\pi(0.05^3)(0.02^3)(37.0)(10^9)}[(-30.0)(1.5) + (-50.0)(2)]$$

$$= -0.003618 \text{ rad} = 0.207° \qquad \textbf{Ans.}$$

5–99. Solve Prob. 5–98 for the maximum shear stress within regions AC and BC, and the angle of twist ϕ of end B relative to C.



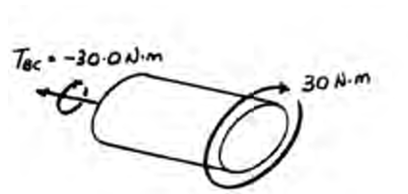
Maximum Shear Stress:

$$(\tau_{BC})_{max} = \frac{2T_{BC}}{\pi\, a\, b^2} = \frac{2(30.0)}{\pi(0.05)(0.02^2)}$$

$$= 0.955 \text{ MPa} \qquad \textbf{Ans.}$$

$$(\tau_{AC})_{max} = \frac{2T_{AC}}{\pi\, a\, b^2} = \frac{2(50.0)}{\pi(0.05)(0.02^2)}$$

$$= 1.59 \text{ MPa} \qquad \textbf{Ans.}$$



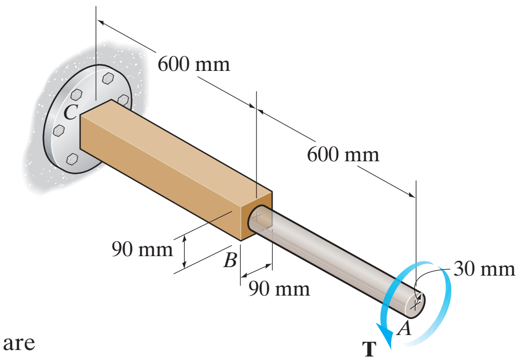
Angle of Twist:

$$\phi_{B/C} = \frac{(a^2 + b^2)\,T_{BC}\,L}{\pi\, a^3\, b^3\, G}$$

$$= \frac{(0.05^2 + 0.02^2)(-30.0)(1.5)}{\pi(0.05^3)(0.02^3)(37.0)(10^9)}$$

$$= -0.001123 \text{ rad} = |\,0.0643°| \qquad \textbf{Ans.}$$

***5–100.** Segments AB and BC of the shaft have circular and square cross sections, respectively. If end A is subjected to a torque of $T = 2\ \text{kN}\cdot\text{m}$, determine the absolute maximum shear stress developed in the shaft and the angle of twist of end A. The shaft is made from A-36 steel and is fixed at C.



Internal Loadings: The internal torques developed in segments AB and BC are shown in Figs. a, and b, respectively.

Maximum Shear Stress: For segment AB,

$$\left(\tau_{\max}\right)_{AB} = \frac{T_{AB}\,c}{J} - \frac{2(10^3)(0.03)}{\frac{\pi}{2}\left(0.03^4\right)} = 47.2\ \text{MPa (max)} \qquad \textbf{Ans.}$$

For segment BC,

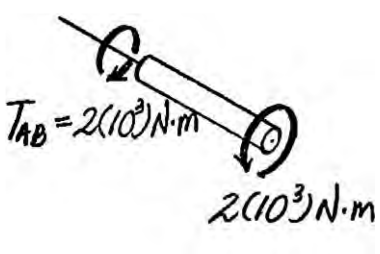
$$\left(\tau_{\max}\right)_{BC} = \frac{4.81T_{BC}}{a^3} = \frac{4.81\left[2(10^3)\right]}{(0.09)^3} = 13.20\ \text{MPa}$$

Angle of Twist:

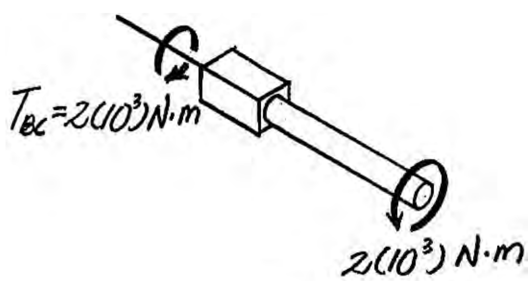
$$\phi_A = \frac{T_{AB}L_{AB}}{JG} + \frac{7.10T_{BC}L_{BC}}{a^4G}$$

$$= \frac{2(10^3)(0.6)}{\frac{\pi}{2}\left(0.03^4\right)(75)(10^9)} + \frac{7.10(2)(10^3)(0.6)}{(0.09)^4(75)(10^9)}$$

$$= 0.01431\ \text{rad} = 0.820° \qquad \textbf{Ans.}$$

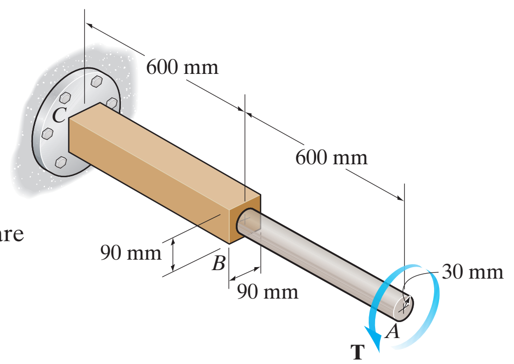


(a)



(b)

•5–101. Segments AB and BC of the shaft have circular and square cross sections, respectively. The shaft is made from A-36 steel with an allowable shear stress of $\tau_{allow} = 75$ MPa, and an angle of twist at end A which is not allowed to exceed 0.02 rad. Determine the maximum allowable torque $\mathbf{T}$ that can be applied at end A. The shaft is fixed at C.

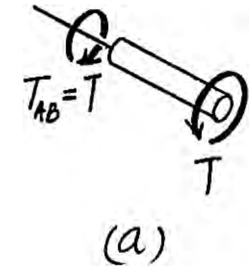


Internal Loadings: The internal torques developed in segments AB and BC are shown in Figs. a, and b, respectively.

Allowable Shear Stress: For segment AB,

$$\tau_{allow} = \frac{T_{AB}c}{J}; \qquad 75(10^6) = \frac{T(0.03)}{\frac{\pi}{2}\left(0.03^4\right)}$$

$$T = 3180.86 \text{ N} \cdot \text{m}$$

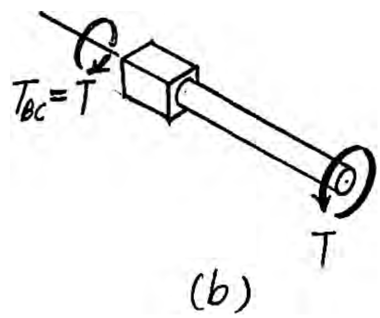


For segment BC,

$$\tau_{allow} = \frac{4.81T_{BC}}{a^3}; \qquad 75(10^6) = \frac{4.81T}{(0.09)^3}$$

$$T = 11\,366.94 \text{ N} \cdot \text{m}$$

Angle of Twist:

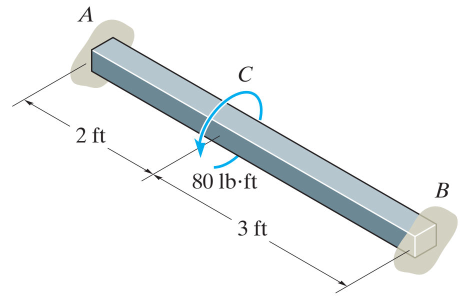


$$\phi_A = \frac{T_{AB}L_{AB}}{JG} + \frac{7.10T_{BC}L_{BC}}{a^4G}$$

$$0.02 = \frac{T(0.6)}{\frac{\pi}{2}\left(0.03^4\right)(75)(10^9)} + \frac{7.10T(0.6)}{(0.09)^4(75)(10^9)}$$

$$T = 2795.90 \text{ N} \cdot \text{m} = 2.80 \text{ kN} \cdot \text{m (controls)}$$ **Ans.**

5–102. The aluminum strut is fixed between the two walls at A and B. If it has a 2 in. by 2 in. square cross section, and it is subjected to the torque of 80 lb · ft at C, determine the reactions at the fixed supports. Also, what is the angle of twist at C? $G_{al} = 3.8(10^3)$ ksi.



By superposition:

$$0 = \phi - \phi_B$$

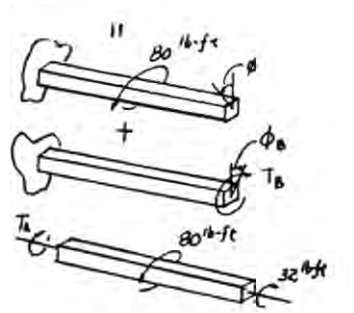
$$0 = \frac{7.10(80)(2)}{a^4G} - \frac{7.10(T_B)(5)}{a^4G}$$

$$T_B = 32 \text{ lb} \cdot \text{ft}$$ **Ans.**

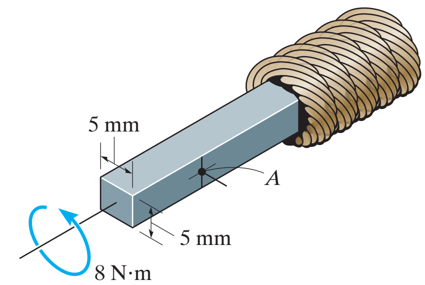
$$T_A + 32 - 80 = 0$$

$$T_A = 48 \text{ lb} \cdot \text{ft}$$ **Ans.**

$$\phi_C = \frac{7.10(32)(12)(3)(12)}{(2^4)(3.8)(10^6)} = 0.00161 \text{ rad} = 0.0925°$$ **Ans.**

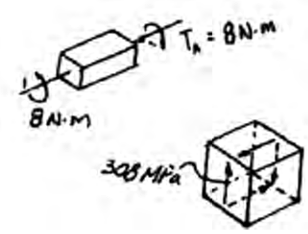


5–103. The square shaft is used at the end of a drive cable in order to register the rotation of the cable on a gauge. If it has the dimensions shown and is subjected to a torque of 8 N · m, determine the shear stress in the shaft at point A. Sketch the shear stress on a volume element located at this point.

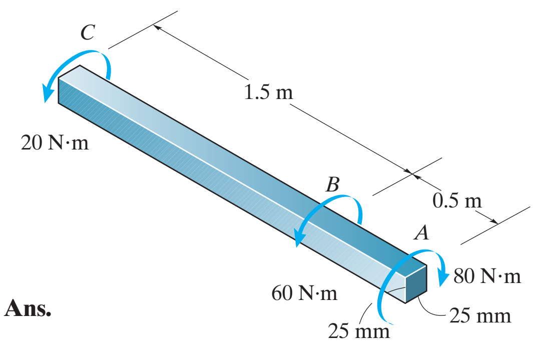


Maximum shear stress:

$$(\tau_{\max})_A = \frac{4.81T}{a^3} = \frac{4.81(8)}{(0.005)^3} = 308 \text{ MPa}$$ **Ans.**



***5–104.** The 6061-T6 aluminum bar has a square cross section of 25 mm by 25 mm. If it is 2 m long, determine the maximum shear stress in the bar and the rotation of one end relative to the other end.



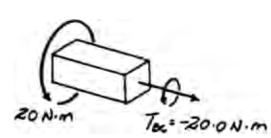
Maximum Shear Stress:

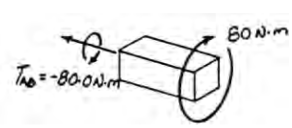
$$\tau_{\max} = \frac{4.81T_{\max}}{a^3} = \frac{4.81(80.0)}{(0.025^3)} = 24.6 \text{ MPa}$$ **Ans.**

Angle of Twist:

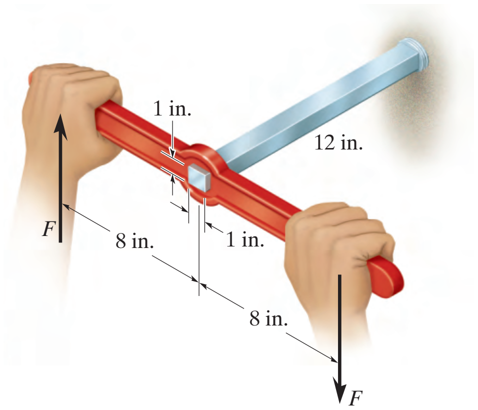
$$\phi_{A/C} = \sum \frac{7.10TL}{a^4G} = \frac{7.10(-20.0)(1.5)}{(0.025^4)(26.0)(10^9)} + \frac{7.10(-80.0)(0.5)}{(0.025^4)(26.0)(10^9)}$$

$$= -0.04894 \text{ rad} = |\,2.80°\,|$$ **Ans.**





•5–105. The steel shaft is 12 in. long and is screwed into the wall using a wrench. Determine the largest couple forces F that can be applied to the shaft without causing the steel to yield. $\tau_Y = 8$ ksi.



$$F(16) - T = 0 \qquad \textbf{(1)}$$

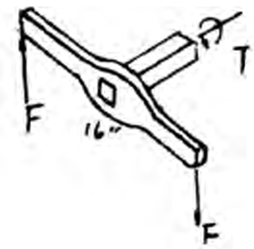
$$\tau_{\max} = \tau_Y = \frac{4.81T}{a^3}$$

$$8(10^3) = \frac{4.81T}{(1)^3}$$

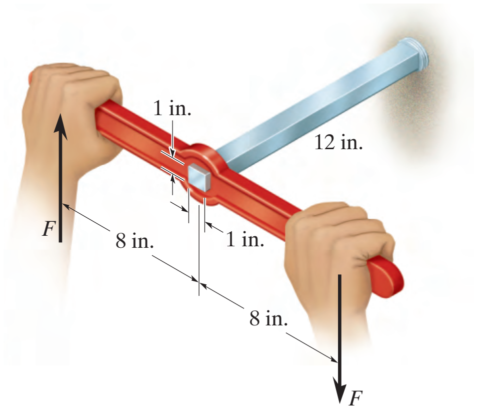
$$T = 1663.2 \text{ lb} \cdot \text{in.}$$

From Eq. (1),

$$F = 104 \text{ lb} \qquad \textbf{Ans.}$$



5–106. The steel shaft is 12 in. long and is screwed into the wall using a wrench. Determine the maximum shear stress in the shaft and the amount of displacement that each couple force undergoes if the couple forces have a magnitude of $F = 30$ lb, $G_{st} = 10.8(10^3)$ ksi.



$$T - 30(16) = 0$$

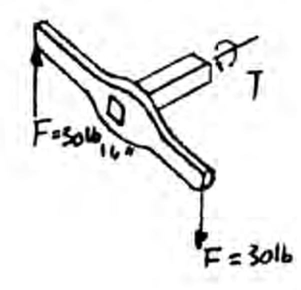
$$T = 480 \text{ lb} \cdot \text{in.}$$

$$\tau_{\max} = \frac{4.18T}{a^3} = \frac{4.81(480)}{(1)^3}$$

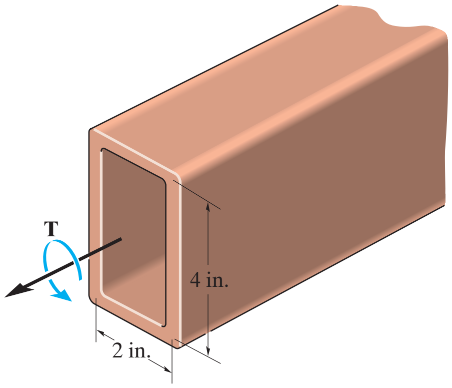
$$= 2.31 \text{ ksi} \qquad \textbf{Ans.}$$

$$\phi = \frac{7.10TL}{a^4G} = \frac{7.10(480)(12)}{(1)^4(10.8)(10^6)} = 0.00379 \text{ rad}$$

$$\delta_F = 8(0.00397) = 0.0303 \text{ in.} \qquad \textbf{Ans.}$$



5–107. Determine the constant thickness of the rectangular tube if the average shear stress is not to exceed 12 ksi when a torque of $T = 20$ kip · in. is applied to the tube. Neglect stress concentrations at the corners. The mean dimensions of the tube are shown.



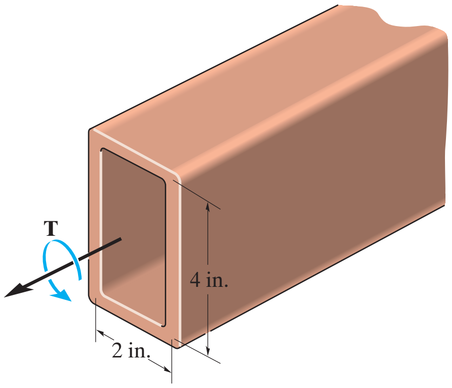
$$A_m = 2(4) = 8 \text{ in}^2$$

$$\tau_{avg} = \frac{T}{2\,t\,A_m}$$

$$12 = \frac{20}{2\,t\,(8)}$$

$t = 0.104$ in. **Ans.**

***5–108.** Determine the torque T that can be applied to the rectangular tube if the average shear stress is not to exceed 12 ksi. Neglect stress concentrations at the corners. The mean dimensions of the tube are shown and the tube has a thickness of 0.125 in.

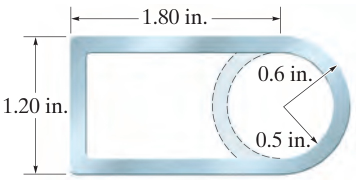


$$A_m = 2(4) = 8 \text{ in}^2$$

$$\tau_{avg} = \frac{T}{2\,t\,A_m}; \qquad 12 = \frac{T}{2(0.125)(8)}$$

$T = 24$ kip · in. $= 2$ kip · ft **Ans.**

•5–109. For a given maximum shear stress, determine the factor by which the torque carrying capacity is increased if the half-circular section is reversed from the dashed-line position to the section shown. The tube is 0.1 in. thick.



$$A_m = (1.10)(1.75) - \frac{\pi(0.55^2)}{2} = 1.4498 \text{ in}^2$$

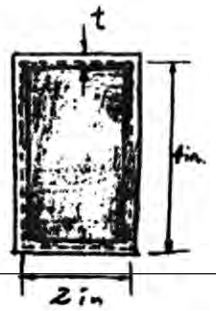
$$A_m' = (1.10)(1.75) + \frac{\pi(0.55^2)}{2} = 2.4002 \text{ in}^2$$

$$\tau_{max} = \frac{T}{2t\ A_m}$$

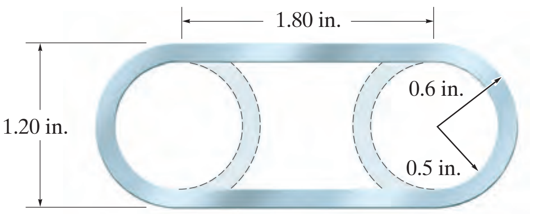
$$T = 2\,t\,A_m\,\tau_{max}$$

$$\text{Factor} = \frac{2t\ A_m'\ \tau_{max}}{2t\ A_m\,\tau_{max}}$$

$$= \frac{A_m'}{A_m} = \frac{2.4002}{1.4498} = 1.66$$ **Ans.**



5–110. For a given average shear stress, determine the factor by which the torque-carrying capacity is increased if the half-circular sections are reversed from the dashed-line positions to the section shown. The tube is 0.1 in. thick.



Section Properties:

$$A'_m = (1.1)(1.8) - \left[\frac{\pi\,(0.55^2)}{2}\right](2) = 1.02967 \text{ in}^2$$

$$A_m = (1.1)(1.8) + \left[\frac{\pi\,(0.55^2)}{2}\right](2) = 2.93033 \text{ in}^2$$

Average Shear Stress:

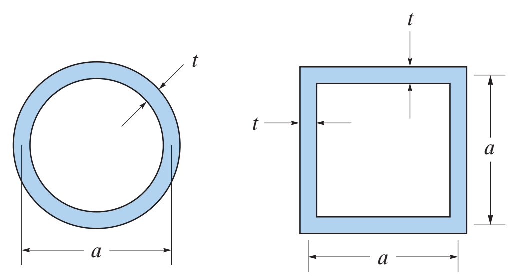
$$\tau_{avg} = \frac{T}{2\,t\,A_m}; \qquad T = 2\,t\,A_m\,\tau_{avg}$$

Hence, $$T' = 2\,t\,A'_m\,\tau_{avg}$$

$$\text{The factor of increase} = \frac{T}{T'} = \frac{A_m}{A'_m} = \frac{2.93033}{1.02967}$$

$$= 2.85$$ **Ans.**

5–111. A torque **T** is applied to two tubes having the cross sections shown. Compare the shear flow developed in each tube.



Circular tube:

$$q_{ct} = \frac{T}{2A_m} = \frac{T}{2\pi\,(a/2)^2} = \frac{2T}{\pi\, a^2}$$

Square tube:

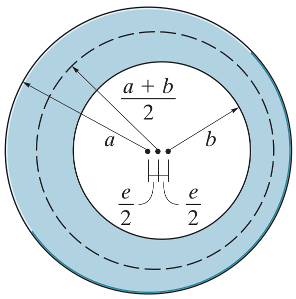
$$q_{st} = \frac{T}{2A_m} = \frac{T}{2a^2}$$

$$\frac{q_{st}}{q_{ct}} = \frac{T/(2a^2)}{2T/(\pi\, a^2)} = \frac{\pi}{4}$$

Thus;

$$q_{st} = \frac{\pi}{4}\, q_{ct} \qquad \textbf{Ans.}$$

***5–112.** Due to a fabrication error the inner circle of the tube is eccentric with respect to the outer circle. By what percentage is the torsional strength reduced when the eccentricity e is one-fourth of the difference in the radii?



Average Shear Stress:

For the aligned tube

$$\tau_{\text{avg}} = \frac{T}{2\,t\,A_m} = \frac{T}{2(a-b)(\pi)\left(\frac{a+b}{2}\right)^2}$$

$$T = \tau_{\text{avg}}\,(2)(a-b)(\pi)\left(\frac{a+b}{2}\right)^2$$

For the eccentric tube

$$\tau_{\text{avg}} = \frac{T'}{2\,t\,A_m}$$

$$t = a - \frac{e}{2} - \left(\frac{e}{2} + b\right) = a - e - b$$

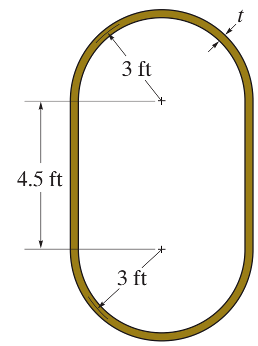
$$= a - \frac{1}{4}(a-b) - b = \frac{3}{4}(a-b)$$

$$T' = \tau_{\text{avg}}\,(2)\left[\frac{3}{4}(a-b)\right](\pi)\left(\frac{a+b}{2}\right)^2$$

$$\text{Factor} = \frac{T'}{T} = \frac{\tau_{\text{avg}}\,(2)\left[\frac{3}{4}(ab)\right](\pi)\left(\frac{a+b}{2}\right)^2}{\tau_{\text{avg}}\,(2)(a-b)(\pi)\left(\frac{a+b}{2}\right)^2} = \frac{3}{4}$$

$$\text{Percent reduction in strength} = \left(1 - \frac{3}{4}\right) \times 100\ \% = 25\ \% \qquad \textbf{Ans.}$$

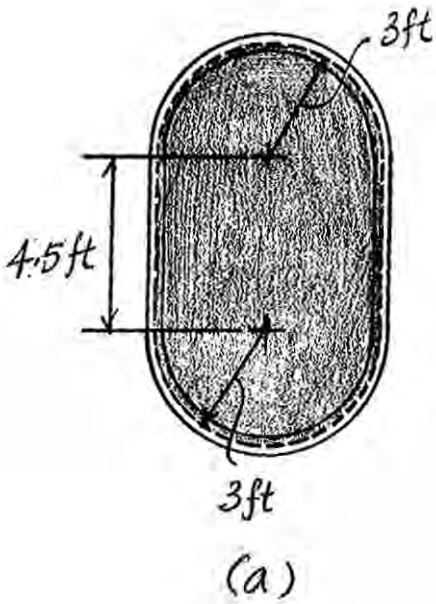
•5–113. The mean dimensions of the cross section of an airplane fuselage are shown. If the fuselage is made of 2014-T6 aluminum alloy having allowable shear stress of $\tau_{allow} = 18$ ksi, and it is subjected to a torque of 6000 kip · ft, determine the required minimum thickness t of the cross section to the nearest 1/16 in. Also, find the corresponding angle of twist per foot length of the fuselage.



Section Properties: Referring to the geometry shown in Fig. *a*,

$$A_m = \pi\left(3^2\right) + 4.5(6) = 55.2743 \text{ ft}^2\left(\frac{144 \text{ in}^2}{1 \text{ ft}^2}\right) = 7959.50 \text{ in}^2$$

$$\oint ds = 2\pi(3) + 2(4.5) = 27.8496 \text{ ft}\left(\frac{12 \text{ in.}}{1 \text{ ft}}\right) = 334.19 \text{ in.}$$



(a)

Allowable Average Shear Stress:

$$\left(\tau_{avg}\right)_{allow} = \frac{T}{2tA_m}; \qquad 18 = \frac{6000(12)}{2t(7959.50)}$$

$$t = 0.2513 \text{ in.} = \frac{5}{16} \text{ in.} \qquad \textbf{Ans.}$$

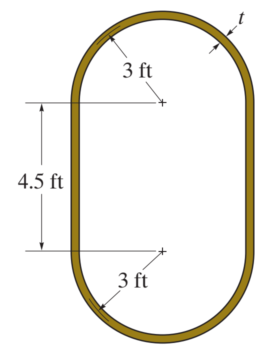
Angle of Twist: Using the result of $t = \frac{5}{16}$ in,

$$\phi = \Sigma \frac{TL}{4A_m{}^2G}\oint\frac{ds}{t}$$

$$= \frac{6000(12)(1)(12)}{4(7959.50^2)(3.9)(10^3)}\left(\frac{334.19}{5/16}\right)$$

$$= 0.9349(10^{-3}) \text{ rad} = 0.0536° \qquad \textbf{Ans.}$$

5–114. The mean dimensions of the cross section of an airplane fuselage are shown. If the fuselage is made from 2014-T6 aluminum alloy having an allowable shear stress of $\tau_{\text{allow}} = 18$ ksi and the angle of twist per foot length of fuselage is not allowed to exceed 0.001 rad/ft, determine the maximum allowable torque that can be sustained by the fuselage. The thickness of the wall is $t = 0.25$ in.



Section Properties: Referring to the geometry shown in Fig. *a*,

$$A_m = \pi\left(3^2\right) + 4.5(6) = 55.2743 \text{ ft}^2\left(\frac{144 \text{ in}^2}{1 \text{ ft}^2}\right) = 7959.50 \text{ in}^2$$

$$\oint ds = 2\pi(3) + 2(4.5) = 27.8496 \text{ ft}\left(\frac{12 \text{ in.}}{1 \text{ ft}}\right) = 334.19 \text{ in.}$$

Allowable Average Shear Stress:

$$\left(\tau_{\text{avg}}\right)_{\text{allow}} = \frac{T}{2tA_m}; \qquad 18 = \frac{T}{2(0.25)(7959.50)}$$

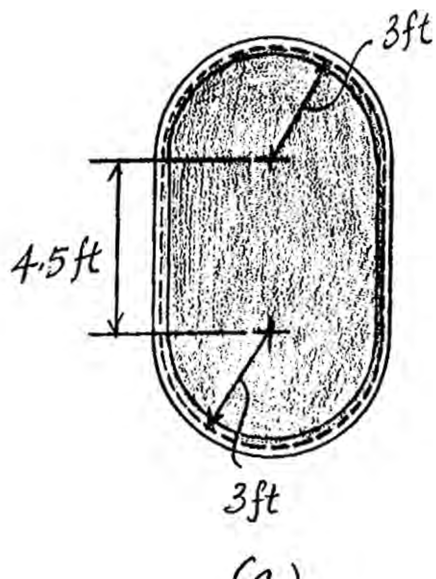
$$T = 71635.54 \text{ kip}\cdot\text{in}\left(\frac{1\text{ft}}{12 \text{ in.}}\right) = 5970 \text{ kip}\cdot\text{ft}$$

Angle of Twist:

$$\phi = \frac{TL}{4A_m{}^2G}\oint\frac{ds}{t}$$

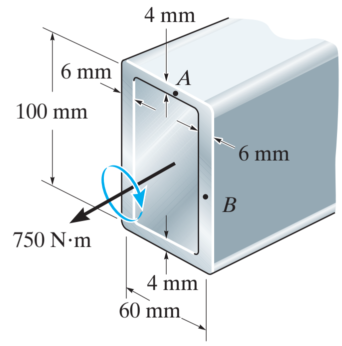
$$0.001 = \frac{T(1)(12)}{4(7959.50^2)(3.9)(10^3)}\left(\frac{334.19}{0.25}\right)$$

$$T = 61610.65 \text{ kip}\cdot\text{in}\left(\frac{1\text{ft}}{12 \text{ in.}}\right) = 5134 \text{ kip}\cdot\text{ft (controls)} \qquad \textbf{Ans.}$$



(a)

5–115. The tube is subjected to a torque of 750 N·m. Determine the average shear stress in the tube at points A and B.



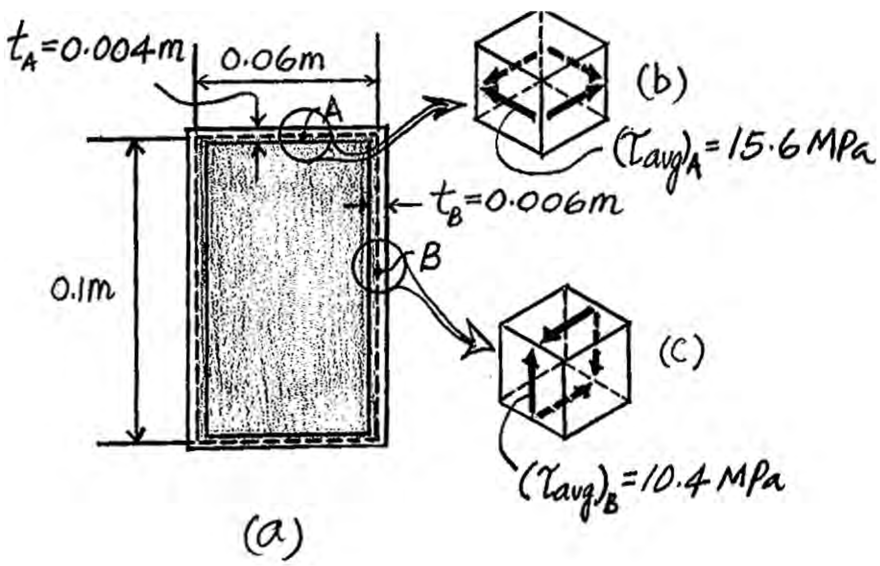
Referring to the geometry shown in Fig. a,

$$A_m = 0.06\,(0.1) = 0.006\ \text{m}^2$$

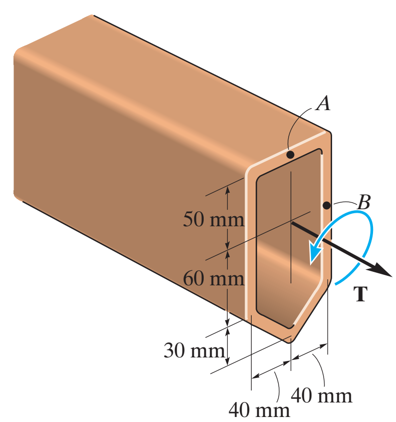
Thus,

$$(\tau_{\text{avg}})_A = \frac{T}{2t_A A_m} = \frac{750}{2(0.004)(0.006)} = 15.63(10^6)\text{Pa} = 15.6\ \text{MPa} \qquad \textbf{Ans.}$$

$$(\tau_{\text{avg}})_B = \frac{T}{2t_B A_m} = \frac{750}{2(0.006)(0.006)} = 10.42(10^6)\text{Pa} = 10.4\ \text{MPa} \qquad \textbf{Ans.}$$

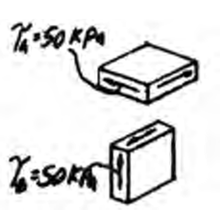


***5–116.** The tube is made of plastic, is 5 mm thick, and has the mean dimensions shown. Determine the average shear stress at points A and B if it is subjected to the torque of $T = 5$ N·m. Show the shear stress on volume elements located at these points.

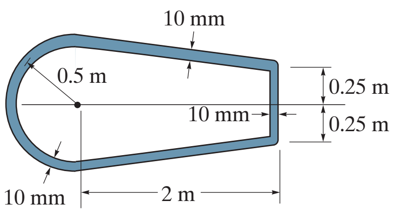


$$A_m = (0.11)(0.08) + \frac{1}{2}(0.08)(0.03) = 0.01\ \text{m}^2$$

$$\tau_A = \tau_B = \tau_{\text{avg}} = \frac{T}{2tA_m} = \frac{5}{2(0.005)(0.01)} = 50\ \text{kPa} \qquad \textbf{Ans.}$$



•5–117. The mean dimensions of the cross section of the leading edge and torsion box of an airplane wing can be approximated as shown. If the wing is made of 2014-T6 aluminum alloy having an allowable shear stress of $\tau_{\text{allow}} = 125$ MPa and the wall thickness is 10 mm, determine the maximum allowable torque and the corresponding angle of twist per meter length of the wing.



Section Properties: Referring to the geometry shown in Fig. *a*,

$$A_m = \frac{\pi}{2}\left(0.5^2\right) + \frac{1}{2}\left(1 + 0.5\right)(2) = 1.8927 \text{ m}^2$$

$$\oint ds = \pi(0.5) + 2\sqrt{2^2 + 0.25^2} + 0.5 = 6.1019 \text{ m}$$

Allowable Average Shear Stress:

$$\left(\tau_{\text{avg}}\right)_{\text{allow}} = \frac{T}{2tA_m}; \qquad 125(10^6) = \frac{T}{2(0.01)(1.8927)}$$

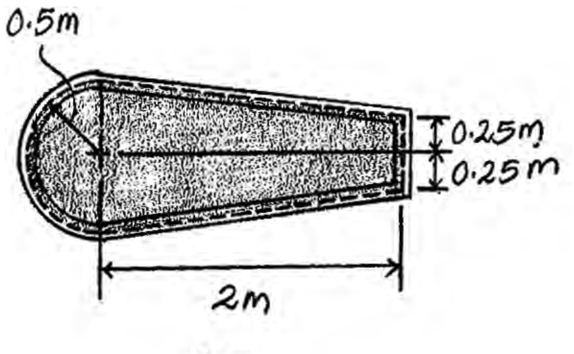
$$T = 4.7317(10^6)\text{N}\cdot\text{m} = 4.73 \text{ MN}\cdot\text{m} \qquad \textbf{Ans.}$$

Angle of Twist:

$$\phi = \frac{TL}{4A_m{}^2G}\oint\frac{ds}{t}$$

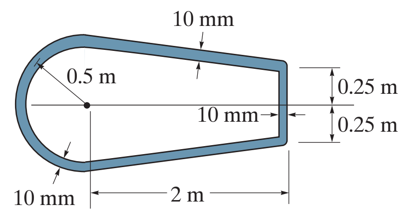
$$= \frac{4.7317(10^6)(1)}{4(1.8927^2)(27)(10^9)}\left(\frac{6.1019}{0.01}\right)$$

$$= 7.463(10^{-3}) \text{ rad} = 0.428°/\text{m} \qquad \textbf{Ans.}$$



(a)

5–118. The mean dimensions of the cross section of the leading edge and torsion box of an airplane wing can be approximated as shown. If the wing is subjected to a torque of 4.5 MN·m and the wall thickness is 10 mm, determine the average shear stress developed in the wing and the angle of twist per meter length of the wing. The wing is made of 2014-T6 aluminum alloy.



Section Properties: Referring to the geometry shown in Fig. *a*,

$$A_m = \frac{\pi}{2}\left(0.5^2\right) + \frac{1}{2}\left(1 + 0.5\right)(2) = 1.8927 \text{ m}^2$$

$$\oint ds = \pi(0.5) + 2\sqrt{2^2 + 0.25^2} + 0.5 = 6.1019 \text{ m}$$

Average Shear Stress:

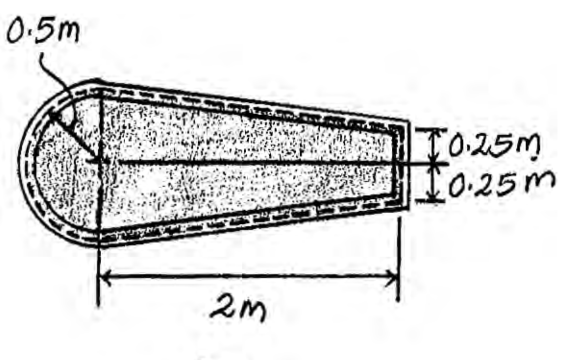
$$\tau_{\text{avg}} = \frac{T}{2tA_m} = \frac{4.5(10^6)}{2(0.01)(1.8927)} = 119 \text{ MPa} \qquad \textbf{Ans.}$$

Angle of Twist:

$$\phi = \frac{TL}{4A_m{}^2G}\oint\frac{ds}{t}$$

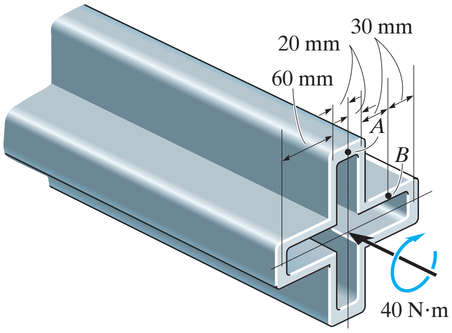
$$= \frac{4.5(10^6)(1)}{4(1.8927^2)(27)(10^9)}\left(\frac{6.1019}{0.01}\right)$$

$$= 7.0973(10^{-3}) \text{ rad} = 0.407°/\text{m} \qquad \textbf{Ans.}$$



(a)

5–119. The symmetric tube is made from a high-strength steel, having the mean dimensions shown and a thickness of 5 mm. If it is subjected to a torque of $T = 40\ \text{N}\cdot\text{m}$, determine the average shear stress developed at points A and B. Indicate the shear stress on volume elements located at these points.



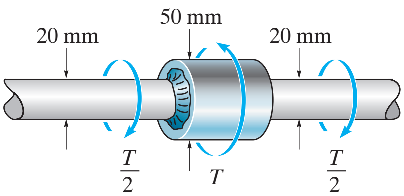
$$A_m = 4(0.04)(0.06) + (0.04)^2 = 0.0112\ \text{m}^2$$

$$\tau_{\text{avg}} = \frac{T}{2\,t\,A_m}$$

$$(\tau_{\text{avg}})_A = (\tau_{\text{avg}})_B = \frac{40}{2(0.005)(0.0112)} = 357\ \text{kPa}$$ **Ans.**

 $\tau_A = \tau_B = 357\ \text{kPa}$

***5–120.** The steel used for the shaft has an allowable shear stress of $\tau_{\text{allow}} = 8\ \text{MPa}$. If the members are connected with a fillet weld of radius $r = 4\ \text{mm}$, determine the maximum torque T that can be applied.



Allowable Shear Stress:

$$\frac{D}{d} = \frac{50}{20} = 2.5 \qquad \text{and} \qquad \frac{r}{d} = \frac{4}{20} = 0.20$$

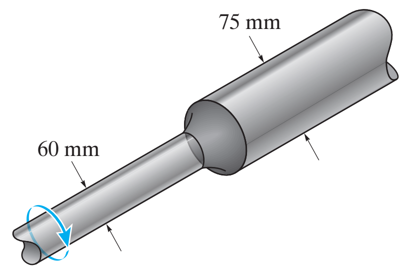
From the text, $K = 1.25$

$$\tau_{\max} = \tau_{\text{allow}} = K\frac{Tc}{J}$$

$$8(10)^4 = 1.25\left[\frac{\frac{T}{2}(0.01)}{\frac{\pi}{2}(0.01^4)}\right]$$

$$T = 20.1\ \text{N}\cdot\text{m}$$ **Ans.**

•5–121. The built-up shaft is to be designed to rotate at 720 rpm while transmitting 30 kW of power. Is this possible? The allowable shear stress is $\tau_{allow} = 12$ MPa.



$$\omega = 720 \frac{\text{rev}}{\text{min}}\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\frac{1 \text{ min}}{60 \text{ s}} = 24\,\pi \text{ rad/s}$$

$$T = \frac{P}{\omega} = \frac{30(10^3)}{24\,\pi} = 397.89 \text{ N}\cdot\text{m}$$

$$\tau_{max} = K\frac{Tc}{J}; \qquad 12(10^6) = K\left[\frac{397.89(0.03)}{\frac{\pi}{2}(0.03^4)}\right]; \qquad K = 1.28$$

$$\frac{D}{d} = \frac{75}{60} = 1.25$$

From Fig. 5-32, $\frac{r}{d} = 0.133$

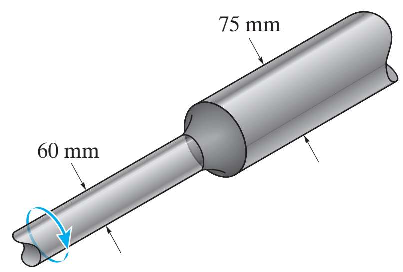
$$\frac{r}{60} = 0.133; \qquad r = 7.98 \text{ mm}$$

Check:

$$\frac{D-d}{2} = \frac{75-60}{2} = \frac{15}{2} = 7.5 \text{ mm} < 7.98 \text{ mm}$$

No, it is not possible. **Ans.**

5–122. The built-up shaft is designed to rotate at 540 rpm. If the radius of the fillet weld connecting the shafts is $r = 7.20$ mm, and the allowable shear stress for the material is $\tau_{allow} = 55$ MPa, determine the maximum power the shaft can transmit.



$$\frac{D}{d} = \frac{75}{60} = 1.25; \qquad \frac{r}{d} = \frac{7.2}{60} = 0.12$$

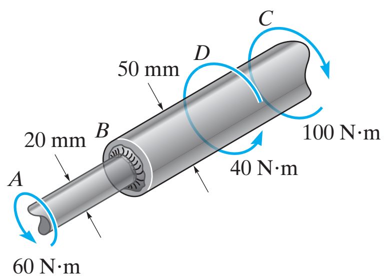
From Fig. 5-32, $K = 1.30$

$$\tau_{max} = K\frac{Tc}{J}; \qquad 55(10^6) = 1.30\left[\left[\frac{T(0.03)}{\frac{\pi}{2}(0.03^4)}\right]\right]; \qquad T = 1794.33 \text{ N}\cdot\text{m}$$

$$\omega = 540 \frac{\text{rev}}{\text{min}}\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\frac{1 \text{ min}}{60 \text{ s}} = 18\,\pi \text{ rad/s}$$

$$P = T\omega = 1794.33(18\pi) = 101466 \text{ W} = 101 \text{ kW}$$ **Ans.**

5–123. The steel shaft is made from two segments: AB and BC, which are connected using a fillet weld having a radius of 2.8 mm. Determine the maximum shear stress developed in the shaft.



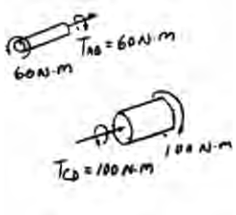
$$(\tau_{\max})_{CD} = \frac{T_{CD}c}{J} = \frac{100(0.025)}{\frac{\pi}{2}(0.025^4)}$$

$$= 4.07 \text{ MPa}$$

For the fillet:

$$\frac{D}{d} = \frac{50}{20} = 2.5; \qquad \frac{r}{d} = \frac{2.8}{20} = 0.14$$

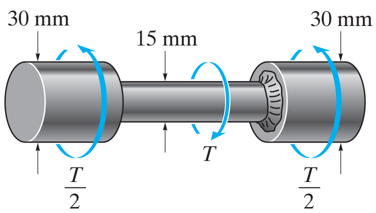
From Fig. 5-32, $K = 1.325$



$$(\tau_{\max})_f = K\frac{T_{AB}c}{J} = 1.325\left[\frac{60(0.01)}{\frac{\pi}{2}(0.01^4)}\right]$$

$$= 50.6 \text{ MPa (max)} \qquad \textbf{Ans.}$$

***5–124.** The steel used for the shaft has an allowable shear stress of $\tau_{\text{allow}} = 8$ MPa. If the members are connected together with a fillet weld of radius $r = 2.25$ mm, determine the maximum torque T that can be applied.



Allowable Shear Stress:

$$\frac{D}{d} = \frac{30}{15} = 2 \quad \text{and} \quad \frac{r}{d} = \frac{2.25}{15} = 0.15$$

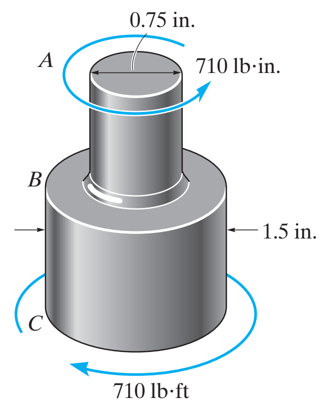
From the text, $K = 1.30$

$$\tau_{\max} = \tau_{\text{allow}} = K\frac{Tc}{J}$$

$$8(10^6) = 1.3\left[\frac{\left(\frac{T}{2}\right)(0.0075)}{\frac{\pi}{2}(0.0075^4)}\right]$$

$$T = 8.16 \text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$

•5–125. The assembly is subjected to a torque of 710 lb · in. If the allowable shear stress for the material is $\tau_{allow} = 12$ ksi, determine the radius of the smallest size fillet that can be used to transmit the torque.



$$\tau_{max} = \tau_{allow} = K\frac{Tc}{J}$$

$$12(10^3) = \frac{K(710)(0.375)}{\frac{\pi}{2}(0.375^4)}$$

$$K = 1.40$$

$$\frac{D}{d} = \frac{1.5}{0.75} = 2$$

From Fig. 5-32,

$$\frac{r}{d} = 0.1; \qquad r = 0.1(0.75) = 0.075 \text{ in.}$$ **Ans.**

Check:

$$\frac{D - d}{2} = \frac{1.5 - 0.75}{2} = 0.375 > 0.075 \text{ in.}$$ **OK**

5–126. A solid shaft is subjected to the torque T, which causes the material to yield. If the material is elastic plastic, show that the torque can be expressed in terms of the angle of twist ϕ of the shaft as $T = \frac{4}{3}T_Y(1 - \phi^3{}_Y/4\phi^3)$, where T_Y and ϕ_Y are the torque and angle of twist when the material begins to yield.

$$\phi = \frac{\gamma L}{\rho} = \frac{\gamma_Y}{\rho_Y}L$$

$$\rho_Y = \frac{\gamma_Y L}{\phi}$$ **(1)**

When $\rho_Y = c, \phi = \phi_Y$

From Eq. (1),

$$c = \frac{\gamma_Y L}{\phi_Y}$$ **(2)**

Dividing Eq. (1) by Eq. (2) yields:

$$\frac{\rho_Y}{c} = \frac{\phi_Y}{\phi}$$ **(3)**

Use Eq. 5-26 from the text.

$$T = \frac{\pi\,\tau_Y}{6}(4\,c^3 - \rho_Y^3) = \frac{2\pi\,\tau_Y c^3}{3}\left(1 - \frac{\rho_Y^3}{4\,c^3}\right)$$

Use Eq. 5-24, $T_Y = \frac{\pi}{2}\tau_Y c^3$ from the text and Eq. (3)

$$T = \frac{4}{3}T_Y\left(1 - \frac{\phi_Y^3}{4\,\phi^3}\right)$$ **QED**

5–127. A solid shaft having a diameter of 2 in. is made of elastic-plastic material having a yield stress of $\tau_Y = 16$ ksi and shear modulus of $G = 12(10^3)$ ksi. Determine the torque required to develop an elastic core in the shaft having a diameter of 1 in. Also, what is the plastic torque?

Use Eq. 5-26 from the text:

$$T = \frac{\pi\,\tau_Y}{6}(4\,c^3 - \rho_Y{}^3) = \frac{\pi\,(16)}{6}[4(1^3) - 0.5^3]$$

$$= 32.46 \text{ kip}\cdot\text{in.} = 2.71 \text{ kip}\cdot\text{ft} \qquad \textbf{Ans.}$$

Use Eq. 5-27 from the text:

$$T_P = \frac{2\pi}{3}\tau_Y c^3 = \frac{2\pi}{3}(16)(1^3)$$

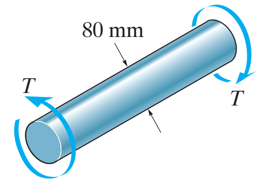
$$= 33.51 \text{ kip}\cdot\text{in.} = 2.79 \text{ kip}\cdot\text{ft} \qquad \textbf{Ans.}$$

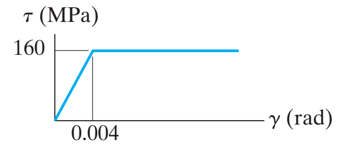
***5–128.** Determine the torque needed to twist a short 3-mm-diameter steel wire through several revolutions if it is made from steel assumed to be elastic plastic and having a yield stress of $\tau_Y = 80$ MPa. Assume that the material becomes fully plastic.

When the material becomes fully plastic then, from Eq. 5-27 in the text,

$$T_P = \frac{2\,\pi\,\tau_Y}{3}c^3 = \frac{2\,\pi\,(80)(10^6)}{3}(0.0015^3) = 0.565 \text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$

•5–129. The solid shaft is made of an elastic-perfectly plastic material as shown. Determine the torque T needed to form an elastic core in the shaft having a radius of $\rho_Y = 20$ mm. If the shaft is 3 m long, through what angle does one end of the shaft twist with respect to the other end? When the torque is removed, determine the residual stress distribution in the shaft and the permanent angle of twist.





Elastic-Plastic Torque: Applying Eq. 5-26 from the text

$$T = \frac{\pi\,\tau_Y}{6}\left(4c^3 - \rho_Y^3\right)$$

$$= \frac{\pi(160)(10^6)}{6}\left[4\left(0.04^3\right) - 0.02^3\right]$$

$$= 20776.40\ \text{N}\cdot\text{m} = 20.8\ \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$

Angle of Twist:

$$\phi = \frac{\gamma_Y}{\rho_Y}L = \left(\frac{0.004}{0.02}\right)(3) = 0.600\ \text{rad} = 34.4° \qquad \textbf{Ans.}$$

When the reverse $T = 20776.4\ \text{N}\cdot\text{m}$ is applied,

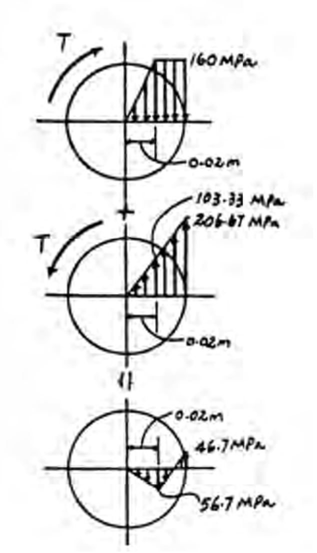
$$G = \frac{160(10^6)}{0.004} = 40\ \text{GPa}$$

$$\phi' = \frac{TL}{JG} = \frac{20776.4(3)}{\frac{\pi}{2}(0.04^4)(40)(10^9)} = 0.3875\ \text{rad}$$

The permanent angle of twist is,

$$\phi_r = \phi - \phi'$$

$$= 0.600 - 0.3875 = 0.2125\ \text{rad} = 12.2° \qquad \textbf{Ans.}$$



Residual Shear Stress:

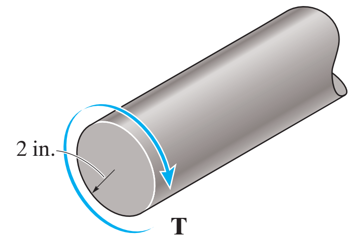
$$(\tau')_{\rho=c} = \frac{Tc}{J} = \frac{20776.4(0.04)}{\frac{\pi}{2}(0.04^4)} = 206.67\ \text{MPa}$$

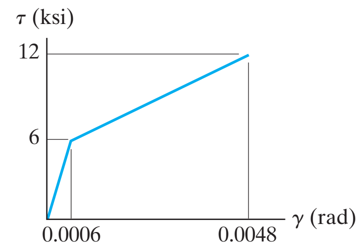
$$(\tau')_{\rho=0.02\ \text{m}} = \frac{Tc}{J} = \frac{20776.4(0.02)}{\frac{\pi}{2}(0.04^4)} = 103.33\ \text{MPa}$$

$$(\tau_r)_{\rho=c} = -160 + 206.67 = 46.7\ \text{MPa}$$

$$(\tau_r)_{\rho=0.02\text{m}} = -160 + 103.33 = -56.7\ \text{MPa}$$

5–130. The shaft is subjected to a maximum shear strain of 0.0048 rad. Determine the torque applied to the shaft if the material has strain hardening as shown by the shear stress–strain diagram.





From the shear - strain diagram,

$$\frac{\rho_Y}{0.0006} = \frac{2}{0.0048}; \qquad \rho_Y = 0.25 \text{ in.}$$

From the shear stress–strain diagram,

$$\tau_1 = \frac{6}{0.25}\rho = 24\rho$$

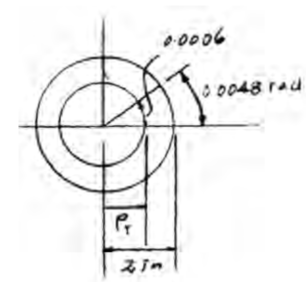
$$\frac{\tau_2 - 6}{\rho - 0.25} = \frac{12 - 6}{2 - 0.25}; \qquad \tau_2 = 3.4286\,\rho + 5.1429$$

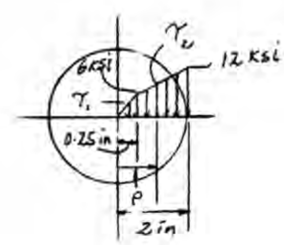
$$T = 2\pi \int_0^c \tau\,\rho^2\,d\rho$$

$$= 2\pi \int_0^{0.25} 24\rho^3\,d\rho + 2\pi \int_{0.25}^{2} (3.4286\rho + 5.1429)\rho^2\,d\rho$$

$$= 2\pi[6\rho^4]\Big|_0^{0.25} + 2\pi\left[\frac{3.4286\rho^4}{4} + \frac{5.1429\rho^3}{3}\right]\Big|_{0.25}^{2}$$

$$= 172.30 \text{ kip}\cdot\text{in.} = 14.4 \text{ kip}\cdot\text{ft}$$ **Ans.**





5–131. An 80-mm diameter solid circular shaft is made of an elastic-perfectly plastic material having a yield shear stress of $\tau_Y = 125$ MPa. Determine (a) the maximum elastic torque T_Y; and (b) the plastic torque T_p.

Maximum Elastic Torque.

$$T_Y = \frac{1}{2}\pi c^3 \tau_Y$$

$$= \frac{1}{2}\pi\left(0.04^3\right)(125)\left(10^6\right)$$

$$= 12\,566.37 \text{ N}\cdot\text{m} = 12.6 \text{ kN}\cdot\text{m}$$ **Ans.**

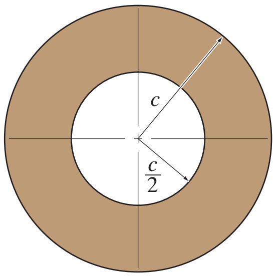
Plastic Torque.

$$T_P = \frac{2}{3}\pi c^3 \tau_Y$$

$$= \frac{2}{3}\pi\left(0.04^3\right)(125)\left(10^6\right)$$

$$= 16755.16 \text{ N}\cdot\text{m} = 16.8 \text{ kN}\cdot\text{m}$$ **Ans.**

***5–132.** The hollow shaft has the cross section shown and is made of an elastic-perfectly plastic material having a yield shear stress of τ_Y. Determine the ratio of the plastic torque T_p to the maximum elastic torque T_Y.



Maximum Elastic Torque. In this case, the torsion formula is still applicable.

$$\tau_Y = \frac{T_Y c}{J}$$

$$T_Y = \frac{J}{c}\tau_Y$$

$$= \frac{\frac{\pi}{2}\left[c^4 - \left(\frac{c}{2}\right)^4\right]\tau_Y}{c}$$

$$= \frac{15}{32}\pi c^3 \tau_Y$$

Plastic Torque. Using the general equation, with $\tau = \tau_Y$,

$$T_P = 2\pi\tau_Y \int_{c/2}^{c} \rho^2 d\rho$$

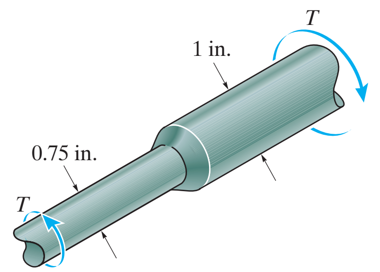
$$= 2\pi\tau_Y\left(\frac{\rho^3}{3}\right)\Bigg|_{c/2}^{c}$$

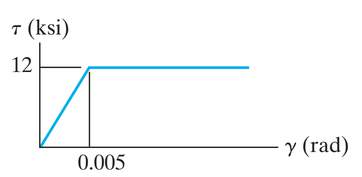
$$= \frac{7}{12}\pi c^3 \tau_Y$$

The ratio is

$$\frac{T_P}{T_Y} = \frac{\frac{7}{12}\pi c^3 \tau_Y}{\frac{15}{32}\pi c^3 \tau_Y} = 1.24 \qquad \textbf{Ans.}$$

5–133. The shaft consists of two sections that are rigidly connected. If the material is elastic plastic as shown, determine the largest torque T that can be applied to the shaft. Also, draw the shear-stress distribution over a radial line for each section. Neglect the effect of stress concentration.





0.75 in. diameter segment will be fully plastic. From Eq. 5-27 of the text:

$$T = T_p = \frac{2\pi\ \tau_Y}{3}(c^3)$$

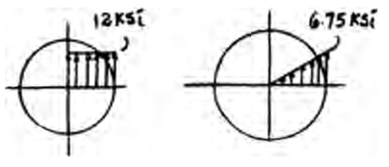
$$= \frac{2\pi\ (12)(10^3)}{3}(0.375^3)$$

$$= 1325.36 \text{ lb} \cdot \text{in.} = 110 \text{ lb} \cdot \text{ft} \qquad \textbf{Ans.}$$

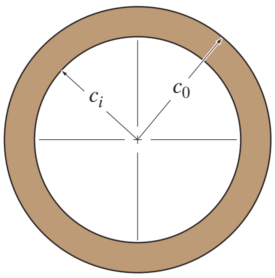
For 1 – in. diameter segment:

$$\tau_{max} = \frac{Tc}{J} = \frac{1325.36(0.5)}{\frac{\pi}{2}(0.5)^4}$$

$$= 6.75 \text{ ksi} < \tau_Y$$



5–134. The hollow shaft is made of an elastic-perfectly plastic material having a shear modulus of G and a yield shear stress of τ_Y. Determine the applied torque $\mathbf{T}_p$ when the material of the inner surface is about to yield (plastic torque). Also, find the corresponding angle of twist and the maximum shear strain. The shaft has a length of L.



Plastic Torque. Using the general equation with $\tau = \tau_Y$,

$$T_P = 2\pi\tau_Y \int_{c_i}^{c_o} \rho^2 d\rho$$

$$= 2\pi\tau_Y\left(\frac{\rho^3}{3}\right)\Bigg|_{c_i}^{c_o}$$

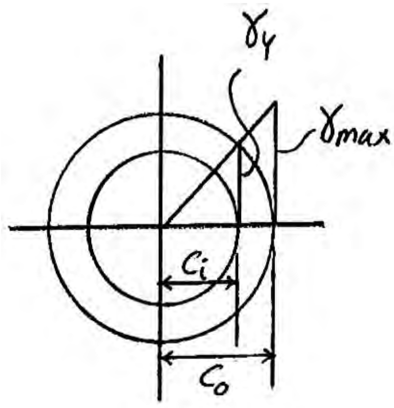
$$= \frac{2}{3}\pi\tau_Y\left(c_o^{\,3} - c_i^{\,3}\right) \qquad \textbf{Ans.}$$

Angle of Twist. When the material is about to yield at the inner surface, $\gamma = \gamma_Y$ at $\rho = \rho_Y = c_i$. Also, Hooke's Law is still valid at the inner surface.

$$\gamma_Y = \frac{\tau_Y}{G}$$

$$\phi = \frac{\gamma_Y}{\rho_Y}L = \frac{\tau_Y/G}{c_i}L = \frac{\tau_Y L}{c_i G} \qquad \textbf{Ans.}$$

Shear Strain. Since the shear strain varies linearly along the radial line, Fig. *a*,

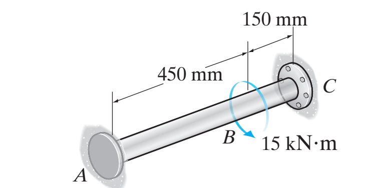


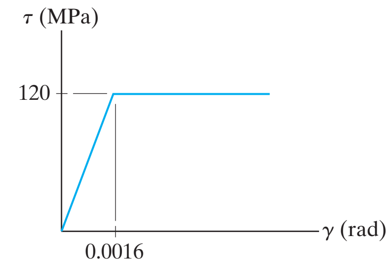
(a)

$$\frac{\gamma_{\max}}{c_o} = \frac{\gamma_Y}{c_i}$$

$$\gamma_{\max} = \left(\frac{c_o}{c_i}\right)\gamma_Y = \left(\frac{c_o}{c_i}\right)\left(\frac{\tau_Y}{G}\right) = \frac{c_o\tau_Y}{c_i G} \qquad \textbf{Ans.}$$

5–135. The hollow shaft has inner and outer diameters of 60 mm and 80 mm, respectively. If it is made of an elastic-perfectly plastic material, which has the $\tau-\gamma$ diagram shown, determine the reactions at the fixed supports A and C.





Equation of Equilibrium. Refering to the free - body diagram of the shaft shown in Fig. *a*,

$$\Sigma M_x = 0; \; T_A + T_C - 15\left(10^3\right) = 0 \qquad \textbf{(1)}$$

Elastic Analysis. It is required that $\phi_{B/A} = \phi_{B/C}$. Thus, the compatibility equation is

$$\phi_{B/A} = \phi_{B/C}$$

$$\frac{T_A L_{AB}}{JG} = \frac{T_C L_{BC}}{JG}$$

$$T_A(0.45) = T_C(0.15)$$

$$T_C = 3T_A \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2),

$$T_A = 3750 \text{ N}\cdot\text{m} \qquad T_C = 11\,250 \text{N}\cdot\text{m}$$

The maximum elastic torque and plastic torque in the shaft can be determined from

$$T_Y = \frac{J}{c}\tau_Y = \left[\frac{\frac{\pi}{2}\left(0.04^4 - 0.03^4\right)}{0.04}\right](120)\left(10^6\right) = 8246.68 \text{N}\cdot\text{m}$$

$$T_P = 2\pi\tau_Y \int_{c_i}^{c_o} \rho^2 d\rho$$

$$= 2\pi(120)\left(10^6\right)\left(\frac{\rho^3}{3}\right)\Bigg|_{0.03 \text{ m}}^{0.04 \text{ m}} = 9299.11 \text{N}\cdot\text{m}$$

Since $T_C > T_Y$, the results obtained using the elastic analysis are not valid.

Plastic Analysis. Assuming that segment BC is fully plastic,

$$T_C = T_P = 9299.11 \text{N}\cdot\text{m} = 9.3 \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$

Substituting this result into Eq. (1),

$$T_A = 5700 \text{ N}\cdot\text{m} = 5.70 \text{ kN}\cdot\text{m} \qquad \textbf{Ans.}$$

5–135. Continued

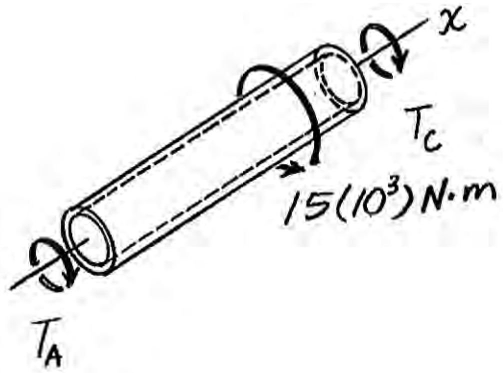
Since $T_A < T_Y$, segment AB of the shaft is still linearly elastic. Here, $G = \dfrac{120\left(10^6\right)}{0.0016} = 75\text{GPa}$.

$$\phi_{B/C} = \phi_{B/A} = \frac{T_A L_{AB}}{JG} = \frac{5700.89(0.45)}{\frac{\pi}{2}\left(0.04^4 - 0.03^4\right)(75)\left(10^9\right)} = 0.01244 \text{ rad}$$

$$\phi_{B/C} = \frac{\gamma_i}{c_i} L_{BC}; \qquad 0.01244 = \frac{\gamma_i}{0.03}(0.15)$$

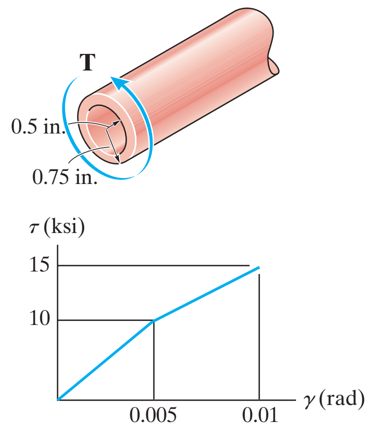
$$\gamma_i = 0.002489 \text{ rad}$$

Since $\gamma_i > \gamma_Y$, segment BC of the shaft is indeed fully plastic.



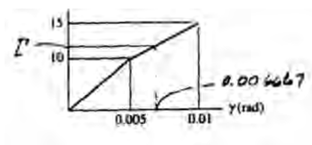
(a)

***5–136.** The tubular shaft is made of a strain-hardening material having a $\tau-\gamma$ diagram as shown. Determine the torque T that must be applied to the shaft so that the maximum shear strain is 0.01 rad.



From the shear–strain diagram,

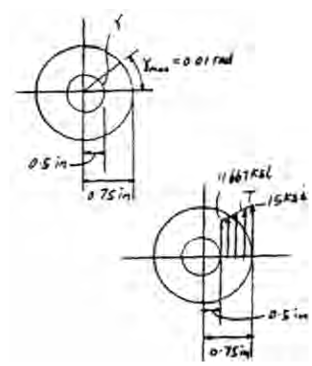
$$\frac{\gamma}{0.5} = \frac{0.01}{0.75}; \qquad \gamma = 0.006667 \text{ rad}$$



From the shear stress–strain diagram,

$$\frac{\tau - 10}{0.006667 - 0.005} = \frac{15 - 10}{0.01 - 0.005}; \quad \tau = 11.667 \text{ ksi}$$

$$\frac{\tau - 11.667}{\rho - 0.5} = \frac{15 - 11.667}{0.75 - 0.50}; \qquad \tau = 13.333\,\rho + 5$$



$$T = 2\pi \int_{c_i}^{c_o} \tau \rho^2 \, d\rho$$

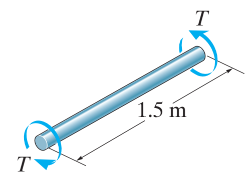
$$= 2\pi \int_{0.5}^{0.75} (13.333\rho + 5)\,\rho^2 \, d\rho$$

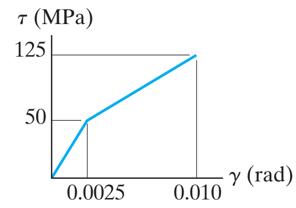
$$= 2\pi \int_{0.5}^{0.75} (13.333\rho^3 + 5\rho^2) \, d\rho$$

$$= 2\pi \left[\frac{13.333\rho^4}{4} + \frac{5\rho^3}{3} \right] \Bigg|_{0.5}^{0.75}$$

$$= 8.426 \text{ kip} \cdot \text{in.} = 702 \text{ lb} \cdot \text{ft}$$ **Ans.**

•5–137. The shear stress–strain diagram for a solid 50-mm-diameter shaft can be approximated as shown in the figure. Determine the torque T required to cause a maximum shear stress in the shaft of 125 MPa. If the shaft is 1.5 m long, what is the corresponding angle of twist?





Strain Diagram:

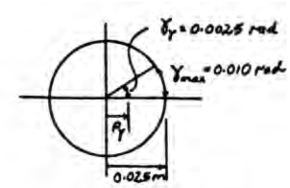
$$\frac{\rho_\gamma}{0.0025} = \frac{0.025}{0.01}; \qquad \rho_\gamma = 0.00625 \text{ m}$$

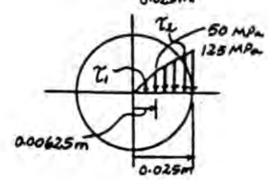
Stress Diagram:

$$\tau_1 = \frac{50(10^6)}{0.00625}\,\rho = 8(10^9)\,\rho$$

$$\frac{\tau_2 - 50(10^6)}{\rho - 0.00625} = \frac{125(10^6) - 50(10^6)}{0.025 - 0.00625}$$

$$\tau_2 = 4\left(10^9\right)\,\rho + 25\left(10^6\right)$$





The Ultimate Torque:

$$T = 2\pi \int_0^c \tau\,\rho^2 d\rho$$

$$= 2\pi \int_0^{0.00625\text{ m}} 8\left(10^9\right)\rho^3\,d\rho$$

$$+ 2\pi \int_{0.00625\text{ m}}^{0.025\text{ m}} \left[4\left(10^9\right)\rho + 25\left(10^6\right)\right]\rho^2 d\rho$$

$$= 2\pi\left[2\left(10^9\right)\rho^4\right]\Big|_0^{0.00625\text{ m}}$$

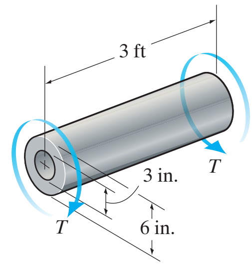
$$+ 2\pi\left[1\left(10^9\right)\rho^4 + \frac{25(10^6)\rho^3}{3}\right]\Bigg|_{0.00625\text{ m}}^{0.025\text{ m}}$$

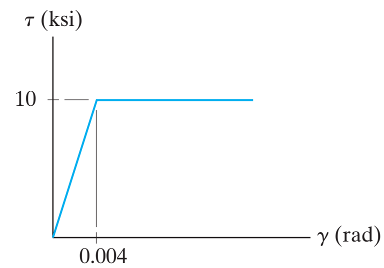
$$= 3269.30 \text{ N}\cdot\text{m} = 3.27 \text{ kN}\cdot\text{m}$$ **Ans.**

Angle of Twist:

$$\phi = \frac{\gamma_{max}}{c}L = \left(\frac{0.01}{0.025}\right)(1.5) = 0.60 \text{ rad} = 34.4°$$ **Ans.**

5–138. A tube is made of elastic-perfectly plastic material, which has the τ–γ diagram shown. If the radius of the elastic core is $\rho_Y = 2.25$ in., determine the applied torque T. Also, find the residual shear-stress distribution in the shaft and the permanent angle of twist of one end relative to the other when the torque is removed.





Elastic - Plastic Torque. The shear stress distribution due to **T** is shown in Fig. *a*. The linear portion of this distribution can be expressed as $\tau = \dfrac{10}{2.25}\rho = 4.444\rho$. Thus, $\tau_{\rho=1.5\text{ in.}} = 4.444(1.5) = 6.667$ ksi.

$$T = 2\pi \int \tau\rho^2 d\rho$$

$$= 2\pi \int_{1.5\text{ in.}}^{2.25\text{ in.}} 4.444\rho\left(\rho^2 d\rho\right) + 2\pi(10)\int_{2.25\text{ in.}}^{3\text{ in.}} \rho^2 d\rho$$

$$= 8.889\pi\left(\frac{\rho^4}{4}\right)\Bigg|_{1.5\text{ in.}}^{2.25\text{ in.}} + 20\pi\left(\frac{\rho^3}{3}\right)\Bigg|_{2.25\text{ in.}}^{3\text{ in.}}$$

$$= 470.50\text{ kip}\cdot\text{in} = 39.2\text{ kip}\cdot\text{ft} \qquad \textbf{Ans.}$$

Angle of Twist.

$$\phi = \frac{\gamma_Y}{\rho_Y}L = \frac{0.004}{2.25}(3)(12) = 0.064\text{ rad}$$

The process of removing torque **T** is equivalent to the application of T', which is equal magnitude but opposite in sense to that of **T**. This process occurs in a linear manner and $G = \dfrac{10}{0.004} = 2.5\left(10^3\right)$ ksi.

$$\phi' = \frac{T'L}{JG} = \frac{470.50(3)(2)}{\frac{\pi}{2}\left(3^4 - 1.5^4\right)(2.5)\left(10^3\right)} = 0.0568\text{ rad}$$

$$\tau'_{\rho=c_o} = \frac{T'c_o}{J} = \frac{470.50(3)}{\frac{\pi}{2}\left(3^4 - 1.5^4\right)} = 11.83\text{ ksi}$$

$$\tau'_{\rho=\rho_Y} = \frac{T'\rho_Y}{J} = \frac{470.50(2.25)}{\frac{\pi}{2}\left(3^4 - 1.5^4\right)} = 8.875\text{ ksi}$$

$$\tau'_{\rho=c_i} = \frac{T'c_i}{J} = \frac{470.50(1.5)}{\frac{\pi}{2}\left(3^4 - 1.5^4\right)} = 5.917\text{ ksi}$$

Thus, the permanent angle of twist is

$$\phi_P = \phi - \phi'$$

$$= 0.064 - 0.0568$$

$$= 0.0072\text{ rad} = 0.413° \qquad \textbf{Ans.}$$

5–138. Continued

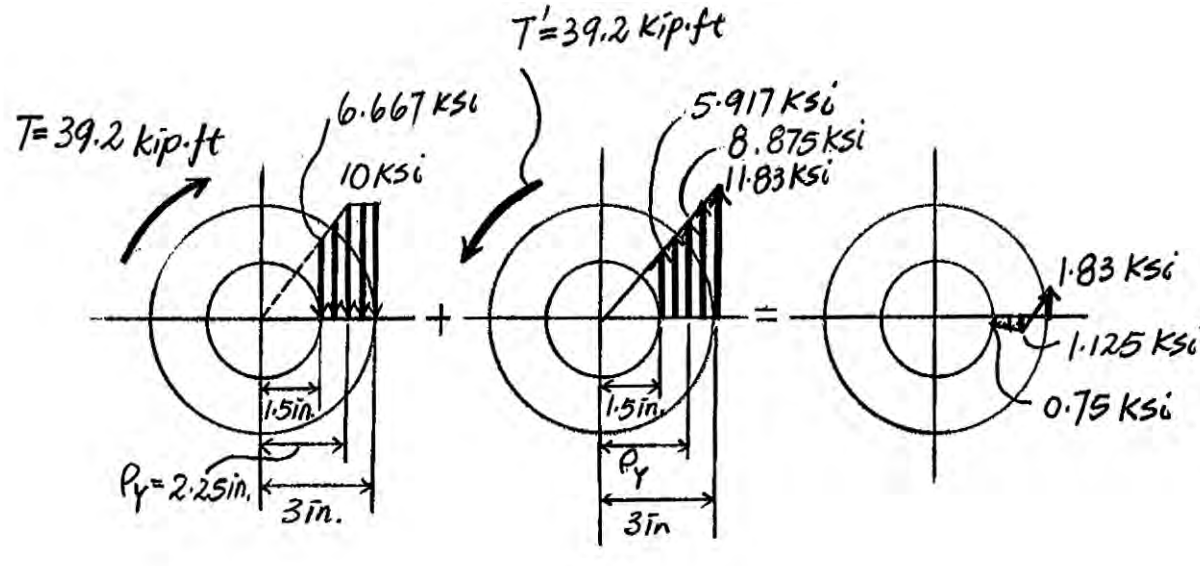
And the residual stresses are

$$(\tau_r)_{\rho=c_o} = \tau_{\rho=c} + \tau'_{\rho=c} = -10 + 11.83 = 1.83 \text{ ksi}$$

$$(\tau_r)_{\rho=\rho_Y} = \tau_{\rho=\rho_Y} + \tau'_{\rho=\rho_Y} = -10 + 8.875 = -1.125 \text{ ksi}$$

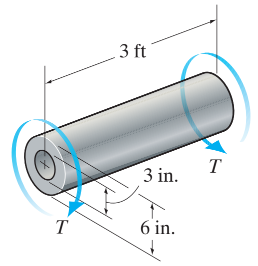
$$(\tau_r)_{\rho=c_i} = \tau_{\rho=c_i} + \tau'_{\rho=c_i} = -6.667 + 5.917 = -0.750 \text{ ksi}$$

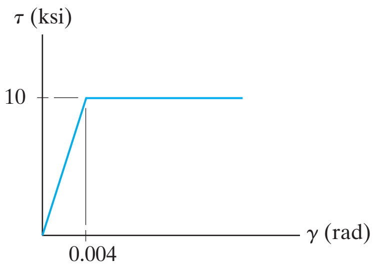
The residual stress distribution is shown in Fig. *a*.



(a)

5–139. The tube is made of elastic-perfectly plastic material, which has the $\tau-\gamma$ diagram shown. Determine the torque T that just causes the inner surface of the shaft to yield. Also, find the residual shear-stress distribution in the shaft when the torque is removed.





Plastic Torque. When the inner surface of the shaft is about to yield, the shaft is about to become fully plastic.

$$T = 2\pi \int \tau\rho^2 d\rho$$

$$= 2\pi\tau_Y \int_{1.5 \text{ in.}}^{3 \text{ in.}} \rho^2 d\rho$$

$$= 2\pi(10)\left(\frac{\rho^3}{3}\right)\Bigg|_{1.5 \text{ in.}}^{3 \text{ in.}}$$

$$= 494.80 \text{ kip} \cdot \text{in.} = 41.2 \text{ kip} \cdot \text{ft}$$ **Ans.**

Angle of Twist.

$$\phi = \frac{\gamma_Y}{\rho_Y} L = \frac{0.004}{1.5}(3)(12) = 0.096 \text{ rad}$$

The process of removing torque **T** is equivalent to the application of T', which is equal magnitude but opposite in sense to that of **T**. This process occurs in a linear manner and $G = \dfrac{10}{0.004} = 2.5\left(10^3\right)$ ksi.

$$\phi' = \frac{T'L}{JG} = \frac{494.80(3)(12)}{\frac{\pi}{2}\left(3^4 - 1.5^4\right)(2.5)\left(10^3\right)} = 0.05973 \text{ rad}$$

$$\tau'_{\rho=c_o} = \frac{T'c_o}{J} = \frac{494.80(3)}{\frac{\pi}{2}\left(3^4 - 1.5^4\right)} = 12.44 \text{ ksi}$$

$$\tau'_{\rho=c_i} = \frac{T'c_i}{J} = \frac{494.80(1.5)}{\frac{\pi}{2}\left(3^4 - 1.5^4\right)} = 6.222 \text{ ksi}$$

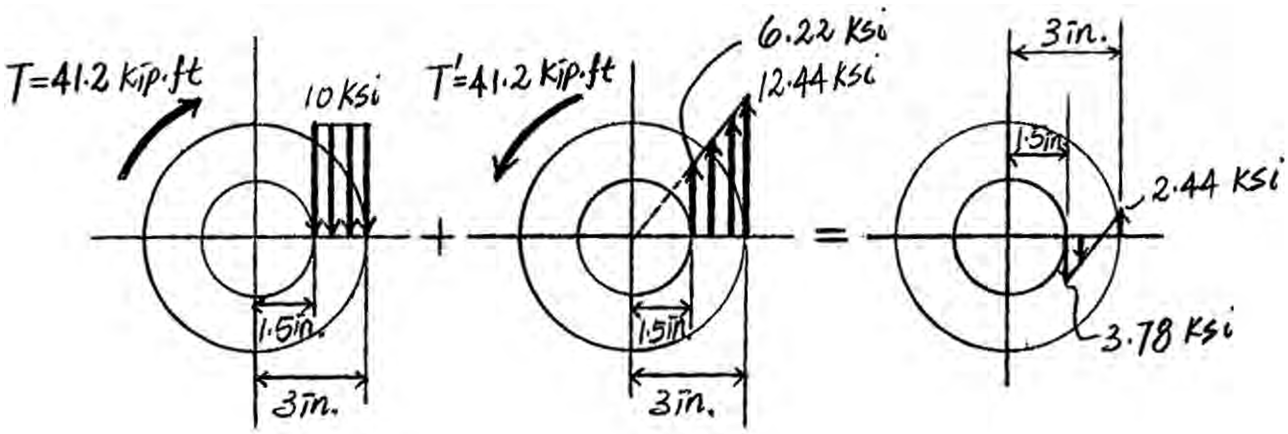
5–139. Continued

And the residual stresses are

$$(\tau_r)_{\rho=c_o} = \tau_{\rho=c} + \tau'_{\rho=c} = -10 + 12.44 = 2.44 \text{ ksi} \qquad \textbf{Ans.}$$

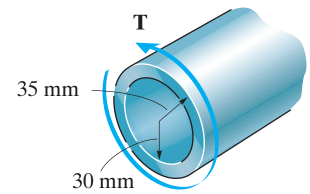
$$(\tau_r)_{\rho=c_i} = \tau_{\rho=c_i} + \tau'_{\rho=c_i} = -10 + 6.22 = -3.78 \text{ ksi} \qquad \textbf{Ans.}$$

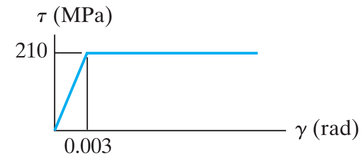
The shear stress distribution due to **T** and T' and the residual stress distribution are shown in Fig. *a*.



(a)

***5–140.** The 2-m-long tube is made of an elastic-perfectly plastic material as shown. Determine the applied torque T that subjects the material at the tube's outer edge to a shear strain of $\gamma_{\max} = 0.006$ rad. What would be the permanent angle of twist of the tube when this torque is removed? Sketch the residual stress distribution in the tube.

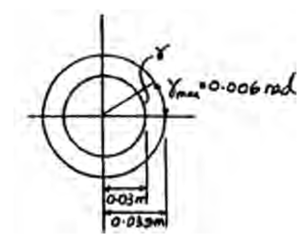




Plastic Torque: The tube is fully plastic if $\gamma_i \geq \gamma_r = 0.003$ rad.

$$\frac{\gamma}{0.03} = \frac{0.006}{0.035}; \qquad \gamma = 0.005143 \text{ rad}$$

Therefore the tube is fully plastic.



$$T_P = 2\pi \int_{c_i}^{c_o} \tau_Y \rho^2 \, d\rho$$

$$= \frac{2\pi\, \tau_Y}{3}\left(c_o^3 - c_i^3\right)$$

$$= \frac{2\pi(210)(10^6)}{3}\left(0.035^3 - 0.03^3\right)$$

$$= 6982.19 \text{ N}\cdot\text{m} = 6.98 \text{ kN}\cdot\text{m} \qquad \textbf{Ans.}$$

Angle of Twist:

$$\phi_P = \frac{\gamma_{\max}}{c_o} L = \left(\frac{0.006}{0.035}\right)(2) = 0.34286 \text{ rad}$$

When a reverse torque of $T_P = 6982.19$ N·m is applied,

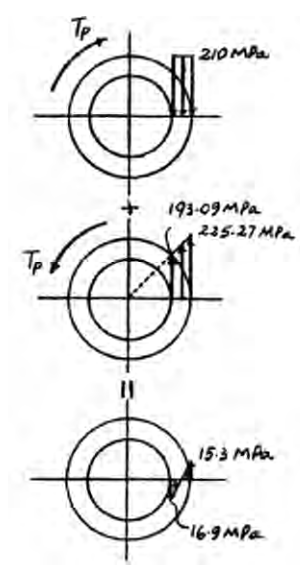
$$G = \frac{\tau_Y}{\gamma_Y} = \frac{210(10^6)}{0.003} = 70 \text{ GPa}$$

$$\phi'_P = \frac{T_P L}{JG} = \frac{6982.19(2)}{\frac{\pi}{2}(0.035^4 - 0.03^4)(70)(10^9)} = 0.18389 \text{ rad}$$

Permanent angle of twist,

$$\phi_r = \phi_P - \phi'_P$$

$$= 0.34286 - 0.18389 = 0.1590 \text{ rad} = 9.11^\circ \qquad \textbf{Ans.}$$



Residual Shear Stress:

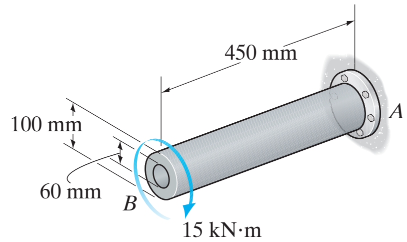
$$\tau'_{P_o} = \frac{T_P\, c}{J} = \frac{6982.19(0.035)}{\frac{\pi}{2}(0.035^4 - 0.03^4)} = 225.27 \text{ MPa}$$

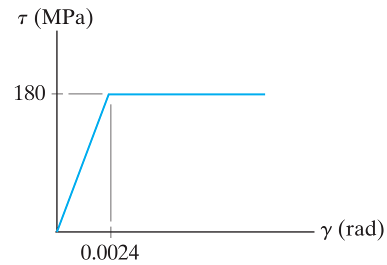
$$\tau'_{P_i} = \frac{T_P\, \rho}{J} = \frac{6982.19(0.03)}{\frac{\pi}{2}(0.035^4 - 0.03^4)} = 193.09 \text{ MPa}$$

$$(\tau_P)_o = -\tau_Y + \tau'_{P_o} = -210 + 225.27 = 15.3 \text{ MPa}$$

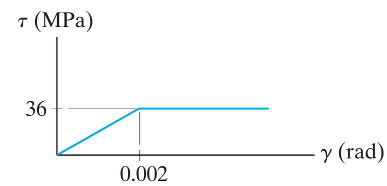
$$(\tau_P)_i = -\tau_Y + \tau'_{P_i} = -210 + 193.09 = -16.9 \text{ MPa}$$

•5–141. A steel alloy core is bonded firmly to the copper alloy tube to form the shaft shown. If the materials have the $\tau-\gamma$ diagrams shown, determine the torque resisted by the core and the tube.





Steel Alloy



Copper Alloy

Equation of Equilibrium. Refering to the free - body diagram of the cut part of the assembly shown in Fig. *a*,

$$\Sigma M_x = 0; \quad T_c + T_t - 15\left(10^3\right) = 0 \qquad \textbf{(1)}$$

Elastic Analysis. The shear modulus of steel and copper are $G_{st} = \dfrac{180\left(10^6\right)}{0.0024} = 75$ GPa and $G_{\infty} = \dfrac{36\left(10^6\right)}{0.002} = 18$ GPa. Compatibility requires that

$$\phi_C = \phi_t$$

$$\frac{T_c L}{J_c G_{st}} = \frac{T_t L}{J_t G_{\infty}}$$

$$\frac{T_c}{\frac{\pi}{2}\left(0.03^4\right)(75)\left(10^9\right)} = \frac{T_t}{\frac{\pi}{2}\left(0.05^4 - 0.03^4\right)(18)\left(10^9\right)}$$

$$T_c = 0.6204 T_t \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2),

$$T_t = 9256.95 \text{ N}\cdot\text{m} \qquad T_c = 5743.05 \text{ N}\cdot\text{m}$$

The maximum elastic torque and plastic torque of the core and the tube are

$$(T_Y)_c = \frac{1}{2}\pi c^3 (\tau_Y)_{st} = \frac{1}{2}\pi\left(0.03^3\right)(180)\left(10^6\right) = 7634.07 \text{ N}\cdot\text{m}$$

$$(T_P)_c = \frac{2}{3}\pi c^3 (\tau_Y)_{st} = \frac{2}{3}\pi\left(0.03^3\right)(180)\left(10^6\right) = 10\,178.76 \text{ N}\cdot\text{m}$$

and

$$(T_Y)_t = \frac{J}{c}\tau_Y = \left[\frac{\frac{\pi}{2}\left(0.05^4 - 0.03^4\right)}{0.05}\right]\left[(36)\left(10^6\right)\right] = 6152.49 \text{ N}\cdot\text{m}$$

$$(T_P)_t = 2\pi(\tau_Y)_{\infty}\int_{c_i}^{c_o}\rho^2 d\rho = 2\pi(36)\left(10^6\right)\left(\frac{\rho^3}{3}\right)\Bigg|_{0.03 \text{ m}}^{0.05 \text{ m}} = 7389.03 \text{ N}\cdot\text{m}$$

Since $T_t > (T_Y)_t$, the results obtained using the elastic analysis are not valid.

5–141. Continued

Plastic Analysis. Assuming that the tube is fully plastic,

$$T_t = (T_P)_t = 7389.03 \text{ N}\cdot\text{m} = 7.39 \text{ kN}\cdot\text{m} \qquad \textbf{Ans.}$$

Substituting this result into Eq. (1),

$$T_c = 7610.97 \text{ N}\cdot\text{m} = 7.61 \text{ kN}\cdot\text{m} \qquad \textbf{Ans.}$$

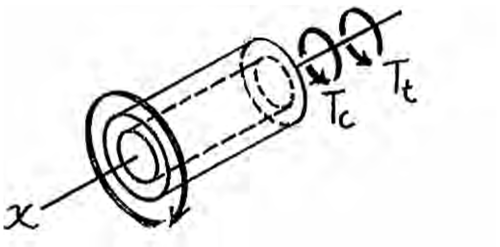
Since $T_c < (T_Y)_c$, the core is still linearly elastic. Thus,

$$\phi_t = \phi_{tc} = \frac{T_c L}{J_c G_{st}} = \frac{7610.97(0.45)}{\frac{\pi}{2}(0.03^4)(75)(10^9)} = 0.03589 \text{ rad}$$

$$\phi_t = \frac{\gamma_i}{c_i} L; \qquad 0.3589 = \frac{\gamma_i}{0.03}(0.45)$$

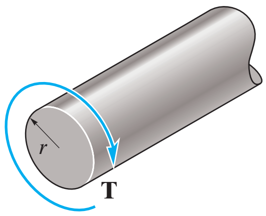
$$\gamma_i = 0.002393 \text{ rad}$$

Since $\gamma_i > (\gamma_Y)_\infty = 0.002$ rad, the tube is indeed fully plastic.



(a)

5–142. A torque is applied to the shaft of radius r. If the material has a shear stress–strain relation of $\tau = k\gamma^{1/6}$, where k is a constant, determine the maximum shear stress in the shaft.



$$\gamma = \frac{\rho}{c}\gamma_{\max} = \frac{\rho}{r}\gamma_{\max}$$

$$\tau = k\gamma^{\frac{1}{6}} = k\left(\frac{\gamma_{\max}}{r}\right)^{\frac{1}{6}}\rho^{\frac{1}{6}}$$

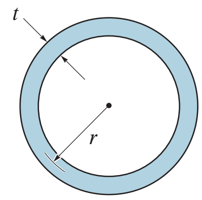
$$T = 2\pi\int_0^r \tau\rho^2\,d\rho$$

$$= 2\pi\int_0^r k\left(\frac{\gamma_{\max}}{r}\right)^{\frac{1}{6}}\rho^{\frac{13}{6}}\,d\rho = 2\pi k\left(\frac{\gamma_{\max}}{r}\right)^{\frac{1}{6}}\left(\frac{6}{19}\right)r^{\frac{19}{6}} = \frac{12\pi\,k\gamma_{\max}^{\frac{1}{6}}r^3}{19}$$

$$\gamma_{\max} = \left(\frac{19T}{12\pi\,kr^3}\right)^6$$

$$\tau_{\max} = k\gamma_{\max}^{\frac{1}{6}} = \frac{19T}{12\pi\,r^3}$$ **Ans.**

5–143. Consider a thin-walled tube of mean radius r and thickness t. Show that the maximum shear stress in the tube due to an applied torque T approaches the average shear stress computed from Eq. 5–18 as $r/t \to \infty$.



$$r_o = r + \frac{t}{2} = \frac{2r + t}{2}; \qquad r_i = r - \frac{t}{2} = \frac{2r - t}{2}$$

$$J = \frac{\pi}{2}\left[\left(\frac{2r + t}{2}\right)^4 - \left(\frac{2r - t}{2}\right)^4\right]$$

$$= \frac{\pi}{32}[(2r + t)^4 - (2r - t)^4] = \frac{\pi}{32}[64\,r^3\,t + 16\,r\,t^3]$$

$$\tau_{\max} = \frac{Tc}{J}; \qquad c = r_o = \frac{2r + t}{2}$$

$$= \frac{T\left(\frac{2r + t}{2}\right)}{\frac{\pi}{32}[64\,r^3\,t + 16\,r\,t^3]} = \frac{T\left(\frac{2r + t}{2}\right)}{2\pi\,r\,t[r^2 + \frac{1}{4}t^2]}$$

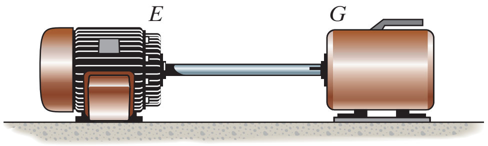
$$= \frac{T\left(\frac{2r}{2r^2} + \frac{t}{2\,r^2}\right)}{2\pi\,r\,t\left[\frac{r^2}{r^2} + \frac{1}{4}\frac{t^2}{r^2}\right]}$$

As $\frac{r}{t} \to \infty$, then $\frac{t}{r} \to 0$

$$\tau_{\max} = \frac{T\left(\frac{1}{r} + 0\right)}{2\pi\,r\,t(1 + 0)} = \frac{T}{2\pi\,r^2\,t}$$

$$= \frac{T}{2\,t\,A_m}$$ **QED**

***5–144.** The 304 stainless steel shaft is 3 m long and has an outer diameter of 60 mm. When it is rotating at 60 rad/s, it transmits 30 kW of power from the engine E to the generator G. Determine the smallest thickness of the shaft if the allowable shear stress is $\tau_{\text{allow}} = 150$ MPa and the shaft is restricted not to twist more than 0.08 rad.



Internal Torque:

$$P = 30(10^3)\ \text{W}\left(\frac{1\ \text{N}\cdot\text{m/s}}{\text{W}}\right) = 30(10^3)\ \text{N}\cdot\text{m/s}$$

$$T = \frac{P}{\omega} = \frac{30(10^3)}{60} = 500\ \text{N}\cdot\text{m}$$

Allowable Shear Stress: Assume failure due to shear stress.

$$\tau_{\max} = \tau_{\text{allow}} = \frac{Tc}{J}$$

$$150(10^6) = \frac{500(0.03)}{\frac{\pi}{2}(0.03^4 - r_i^4)}$$

$$r_i = 0.0293923\ \text{m} = 29.3923\ \text{mm}$$

Angle of Twist: Assume failure due to angle of twist limitation.

$$\phi = \frac{TL}{JG}$$

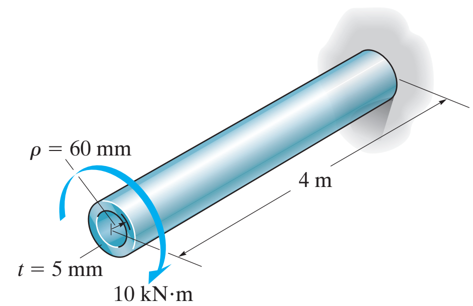
$$0.08 = \frac{500(3)}{\frac{\pi}{2}\left(0.03^4 - r_i^4\right)(75.0)(10^9)}$$

$$r_i = 0.0284033\ \text{m} = 28.4033\ \text{mm}$$

Choose the smallest value of $r_i = 28.4033$ mm

$$t = r_o - r_i = 30 - 28.4033 = 1.60\ \text{mm} \qquad \textbf{Ans.}$$

•5–145. The A-36 steel circular tube is subjected to a torque of 10 kN · m. Determine the shear stress at the mean radius $\rho = 60$ mm and compute the angle of twist of the tube if it is 4 m long and fixed at its far end. Solve the problem using Eqs. 5–7 and 5–15 and by using Eqs. 5–18 and 5–20.



Shear Stress:

Applying Eq. 5-7,

$$r_o = 0.06 + \frac{0.005}{2} = 0.0625 \text{ m} \qquad r_i = 0.06 - \frac{0.005}{2} = 0.0575 \text{ m}$$

$$\tau_{\rho=0.06\text{ m}} = \frac{T\rho}{J} = \frac{10(10^3)(0.06)}{\frac{\pi}{2}(0.0625^4 - 0.0575^4)} = 88.27 \text{ MPa} \qquad \textbf{Ans.}$$

Applying Eq. 5-18,

$$\tau_{\text{avg}} = \frac{T}{2\,t\,A_m} = \frac{10(10^3)}{29(0.005)(\pi)(0.06^2)} = 88.42 \text{ MPa} \qquad \textbf{Ans.}$$

Angle of Twist:

Applying Eq. 5-15,

$$\phi = \frac{TL}{JG}$$

$$= \frac{10(10^3)(4)}{\frac{\pi}{2}(0.0625^4 - 0.0575^4)(75.0)(10^9)}$$

$$= 0.0785 \text{ rad} = 4.495° \qquad \textbf{Ans.}$$

Applying Eq. 5-20,

$$\phi = \frac{TL}{4A_m^2 G}\int \frac{ds}{t}$$

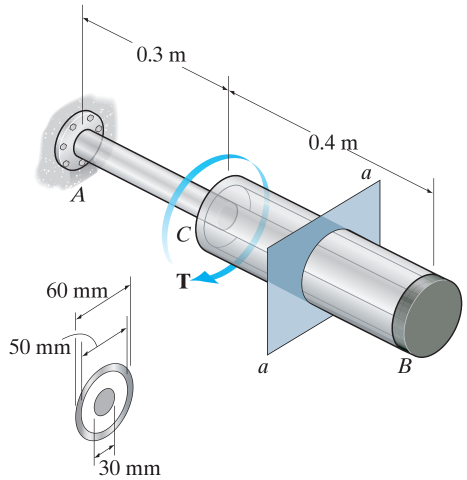
$$= \frac{TL}{4A_m^2 G\,t}\int ds \qquad \text{Where} \qquad \int ds = 2\pi\rho$$

$$= \frac{2\pi T L\rho}{4A_m^2 G\,t}$$

$$= \frac{2\pi(10)(10^3)(4)(0.06)}{4[(\pi)(0.06^2)]^2(75.0)(10^9)(0.005)}$$

$$= 0.0786 \text{ rad} = 4.503° \qquad \textbf{Ans.}$$

5–146. Rod AB is made of A-36 steel with an allowable shear stress of $(\tau_{\text{allow}})_{\text{st}} = 75\,\text{MPa}$, and tube BC is made of AM1004-T61 magnesium alloy with an allowable shear stress of $(\tau_{\text{allow}})_{\text{mg}} = 45\,\text{MPa}$. The angle of twist of end C is not allowed to exceed 0.05 rad. Determine the maximum allowable torque **T** that can be applied to the assembly.



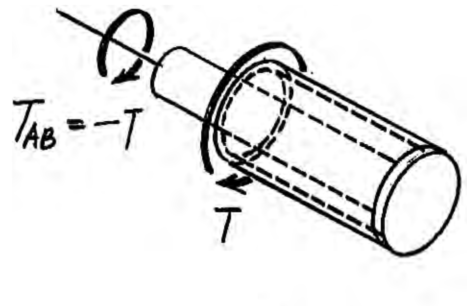
Section a–a

Internal Loading: The internal torque developed in rod AB and tube BC are shown in Figs. a and b, respectively.

Allowable Shear Stress: The polar moment of inertia of rod AB and tube BC are $J_{AB} = \frac{\pi}{2}\left(0.015^4\right) = 25.3125(10^{-9})\pi\ \text{m}^4$ and $J_{BC} = \frac{\pi}{2}\left(0.03^4 - 0.025^4\right) = 0.2096875(10^{-6})\pi\ \text{m}^4$. We have

$$\left(\tau_{\text{allow}}\right)_{st} = \frac{T_{AB}\,c_{AB}}{J_{AB}}; \qquad 75(10^6) = \frac{T(0.015)}{25.3125(10^{-9})\pi}$$

$$T = 397.61\ \text{N}\cdot\text{m}$$



(a)

and

$$\left(\tau_{\text{allow}}\right)_{mg} = \frac{T_{BC}\,c_{BC}}{J_{BC}}; \qquad 45(10^6) = \frac{T(0.03)}{0.2096875(10^{-6})\pi}$$

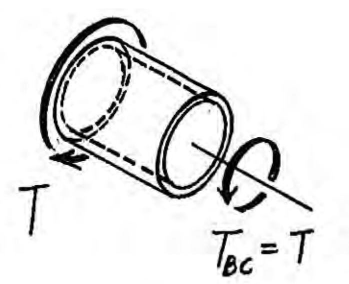
$$T = 988.13\ \text{N}\cdot\text{m}$$

Angle of Twist:

$$\phi_{B/A} = \frac{T_{AB}\,L_{AB}}{J_{AB}\,G_{st}} = \frac{-T(0.7)}{25.3125(10^{-9})\pi(75)(10^9)} = -0.11737(10^{-3})T = 0.11737(10^{-3})T$$

and

$$\phi_{C/B} = \frac{T_{BC}\,L_{BC}}{J_{BC}\,G_{mg}} = \frac{T(0.4)}{0.2096875(10^{-6})\pi(18)(10^9)} = 0.03373(10^{-3})T$$



(b)

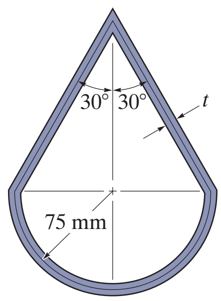
It is required that $\phi_{C/A} = 0.05$ rad. Thus,

$$\phi_{C/A} = \phi_{B/A} + \phi_{C/B}$$

$$0.05 = 0.11737(10^{-3})T + 0.03373(10^{-3})T$$

$$T = 331\ \text{N}\cdot\text{m}\ \text{(controls)}$$ **Ans.**

5–147. A shaft has the cross section shown and is made of 2014-T6 aluminum alloy having an allowable shear stress of $\tau_{\text{allow}} = 125$ MPa. If the angle of twist per meter length is not allowed to exceed 0.03 rad, determine the required minimum wall thickness t to the nearest millimeter when the shaft is subjected to a torque of $T = 15\,\text{kN}\cdot\text{m}$.



Section Properties: Referring to the geometry shown in Fig. *a*,

$$A_m = \frac{1}{2}(0.15)\left(\frac{0.075}{\tan 30°}\right) + \frac{1}{2}\pi\left(0.075^2\right) = 0.01858\ \text{m}^2$$

$$\oint ds = 2(0.15) + \pi(0.075) = 0.53562\ \text{m}$$

Allowable Shear Stress:

$$\left(\tau_{\text{avg}}\right)_{\text{allow}} = \frac{T}{2tA_m}; \qquad 125(10^6) = \frac{15(10^3)}{2t(0.01858)}$$

$$t = 0.00323\ \text{m} = 3.23\ \text{mm}$$

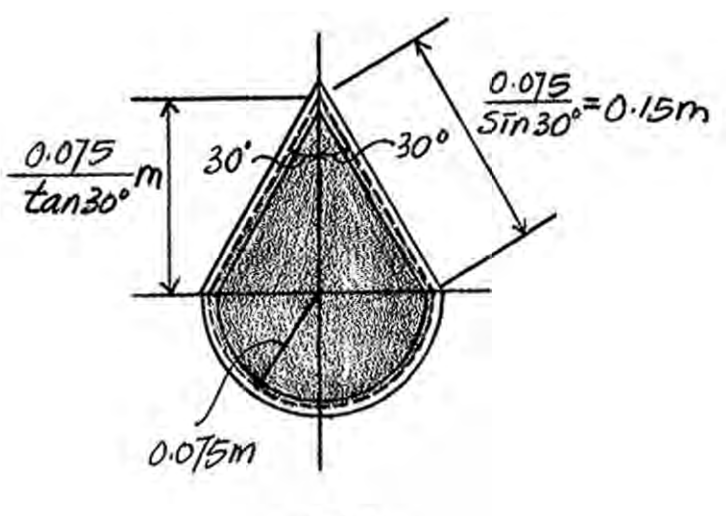
Angle of Twist:

$$\phi = \frac{TL}{4A_m{}^2G}\oint\frac{ds}{t}$$

$$0.03 = \frac{15(10^3)(1)}{4(0.01858^2)(27)(10^9)}\left(\frac{0.53562}{t}\right)$$

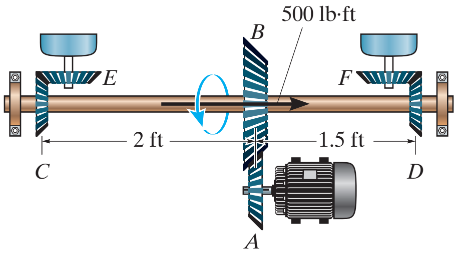
$$t = 0.007184\ \text{m} = 7.18\ \text{mm (controls)}$$

Use $t = 8$ mm **Ans.**



(a)

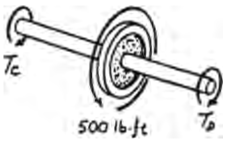
***5–148.** The motor *A* develops a torque at gear *B* of 500 lb·ft, which is applied along the axis of the 2-in.-diameter A-36 steel shaft *CD*. This torque is to be transmitted to the pinion gears at *E* and *F*. If these gears are temporarily fixed, determine the maximum shear stress in segments *CB* and *BD* of the shaft. Also, what is the angle of twist of each of these segments? The bearings at *C* and *D* only exert force reactions on the shaft.



Equilibrium:

$$T_C + T_D - 500 = 0 \qquad [1]$$

Compatibility:



$$\phi_{B/C} = \phi_{B/D}$$

$$\frac{T_C(2)}{JG} = \frac{T_D(1.5)}{JG}$$

$$T_C = 0.75T_D \qquad [2]$$

Solving Eqs. [1] and [2] yields:

$$T_D = 285.71 \text{ lb}\cdot\text{ft} \qquad T_C = 214.29 \text{ lb}\cdot\text{ft}$$

Maximum Shear Stress:

$$(\tau_{CB})_{\max} = \frac{T_C c}{J} = \frac{214.29(12)(1)}{\frac{\pi}{2}(1^4)} = 1.64 \text{ ksi} \qquad \textbf{Ans.}$$

$$(\tau_{BD})_{\max} = \frac{T_D c}{J} = \frac{285.71(12)(1)}{\frac{\pi}{2}(1^4)} = 2.18 \text{ ksi} \qquad \textbf{Ans.}$$

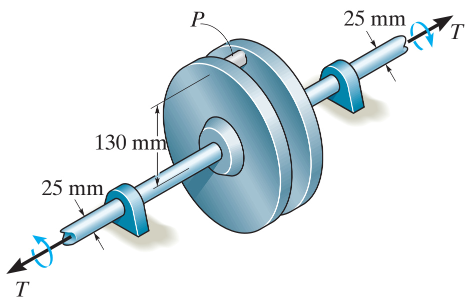
Angle of Twist:

$$\phi_{CB} = \phi_{BD} = \frac{T_D L_{BD}}{JG}$$

$$= \frac{285.71(12)(1.5)(12)}{\frac{\pi}{2}(1^4)(11.0)(10^6)}$$

$$= 0.003572 \text{ rad} = 0.205° \qquad \textbf{Ans.}$$

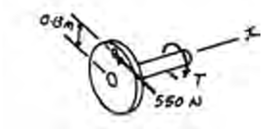
5–149. The coupling consists of two disks fixed to separate shafts, each 25 mm in diameter. The shafts are supported on journal bearings that allow free rotation. In order to limit the torque **T** that can be transmitted, a "shear pin" P is used to connect the disks together. If this pin can sustain an *average* shear force of 550 N before it fails, determine the maximum constant torque T that can be transmitted from one shaft to the other. Also, what is the maximum shear stress in each shaft when the "shear pin" is about to fail?



Equilibrium:

$$\Sigma M_x = 0; \qquad T - 550(0.13) = 0$$

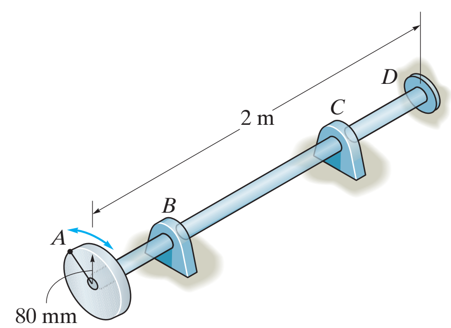
$$T = 71.5 \text{ N} \cdot \text{m} \qquad \textbf{Ans.}$$



Maximum Shear Stress:

$$\tau_{\max} = \frac{Tc}{J} = \frac{71.5(0.0125)}{\frac{\pi}{2}(0.0125^4)} = 23.3 \text{ MPa} \qquad \textbf{Ans.}$$

5–150. The rotating flywheel and shaft is brought to a sudden stop at D when the bearing freezes. This causes the flywheel to oscillate clockwise–counterclockwise, so that a point A on the outer edge of the flywheel is displaced through a 10-mm arc in either direction. Determine the maximum shear stress developed in the tubular 304 stainless steel shaft due to this oscillation. The shaft has an inner diameter of 25 mm and an outer diameter of 35 mm. The journal bearings at B and C allow the shaft to rotate freely.



Angle of Twist:

$$\phi = \frac{TL}{JG} \qquad \text{Where} \qquad \phi = \frac{10}{80} = 0.125 \text{ rad}$$

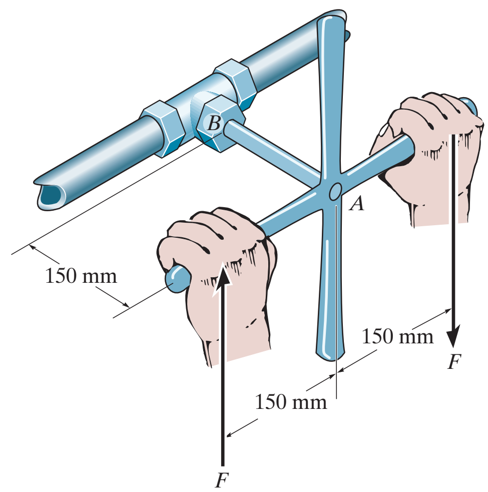
$$0.125 = \frac{T(2)}{\frac{\pi}{2}(0.0175^4 - 0.0125^4)(75.0)(10^9)}$$

$$T = 510.82 \text{ N} \cdot \text{m}$$

Maximum Shear Stress:

$$\tau_{\max} = \frac{Tc}{J} = \frac{510.82(0.0175)}{\frac{\pi}{2}(0.0175^4 - 0.0125^4)} = 82.0 \text{ MPa} \qquad \textbf{Ans.}$$

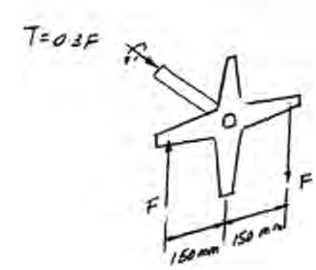
5–151. If the solid shaft AB to which the valve handle is attached is made of C83400 red brass and has a diameter of 10 mm, determine the maximum couple forces F that can be applied to the handle just before the material starts to fail. Take $\tau_{\text{allow}} = 40$ MPa. What is the angle of twist of the handle? The shaft is fixed at A.



$$\tau_{\max} = \tau_{\text{allow}} = \frac{Tc}{J}$$

$$40(10^6) = \frac{0.3F(0.005)}{\frac{\pi}{2}(0.005)^4}$$

$F = 26.18 \text{ N} = 26.2 \text{ N}$ **Ans.**



$T = 0.3F = 7.85 \text{ N} \cdot \text{m}$

$$\phi = \frac{TL}{JG} = \frac{7.85(0.15)}{\frac{\pi}{2}(0.005)^4(37)(10^9)}$$

$= 0.03243 \text{ rad} = 1.86°$ **Ans.**