Asignación #2
Probabilidad y Procesos Aleatorios

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3.3) Una VA X tiene una densidad de Probabilidad $f(x) = 3x^2$ en el intervalo [0, 1]. Determine $F(x) = P(X < x)$. Determine $E(X)$, y la varianza $V(X)$

$$F(x) = P(X<x) = \int_0^x f(x)\,dx$$

$$= \int_0^x 3x^2\,dx$$

$$= x^3 \Big|_0^x$$

$$F(x) = P(X<x) = x^3 - 0 = x^3$$

Promedio

$$E(X) = m = \int_{-\infty}^{\infty} x \cdot f(x)\,dx$$

$$E(X) = m = \int_0^1 x \cdot 3x^2\,dx$$

$$= \frac{3}{4}x^4 \Big|_0^1 = \frac{3}{4}(1)^4 - 0 = \frac{3}{4}$$

$$\boxed{E(X) = m = \frac{3}{4}}$$

Varianza $V[X] = \sigma^2 = \int_{-\infty}^{\infty} (x-m)^2 \cdot f(x)\, dx$

$$= \int_0^1 (x - 3/4)^2 (3x^2)\, dx$$

$$= 3\int_0^1 \left(x^2 - \frac{3}{2}x + \frac{9}{16}\right)(x^2)\, dx$$

$$= 3\int_0^1 \left(x^4 - \frac{3}{2}x^3 + \frac{9}{16}x^2\right) dx$$

$$= 3\left[\frac{x^5}{5} - \frac{3}{8}x^4 + \frac{3x^3}{16}\right]_0^1$$

$$= 3\left[\frac{(1)^5}{5} - \frac{3}{8}(1)^4 + \frac{3(1)^3}{16}\right] - [0]$$

$$= 3\left[\frac{1}{5} - \frac{3}{8} + \frac{3}{16}\right]$$

$$\boxed{V[X] = \sigma^2 = \frac{3}{80}}$$

Escaneado con CamScanner

3.b) Sabiendo que la densidad de Probabilidad de una VA es $f(x) = 1/[2(b-a)]$ para el intervalo $[a;b]$ $f(x) = [1/(\beta-b)] - [x/(\beta^2-b^2)]$ para el intervalo $[b;\beta]$ y $f(x) = 0$ para el resto. Dado que $a<b<\alpha<\beta$ Calcule el valor de las siguientes probabilidades:

$P(b \leq X \leq \beta)$, $P(a \leq X \leq \alpha)$ y $P(a \leq X \leq \alpha \mid X > b)$

$$f(x) = \begin{cases} 0 & x<a \\ \dfrac{1}{2(b-a)} & a \leq x < b \\ \dfrac{1}{(\beta-b)} - \dfrac{x}{(\beta^2-b^2)} & b \leq x < \beta \\ 0 & x \geq \beta \end{cases}$$

a)

$$P(b \leq x < \beta) = \int_b^\beta \left[\frac{1}{(\beta-b)} - \frac{x}{(\beta^2-b^2)}\right] dx$$

$$= \left[\frac{x}{(\beta-b)} - \frac{x^2}{2(\beta^2-b^2)}\right]_b^\beta$$

$$= \frac{\beta}{(\beta-b)} - \frac{\beta^2}{2(\beta^2-b^2)} - \frac{b}{\beta-b} + \frac{b^2}{2(\beta^2-b^2)}$$

$$= \frac{\beta-b}{(\beta-b)} - \frac{\cancel{(\beta^2-b^2)}}{2\cancel{(\beta^2-b^2)}}$$

$$\boxed{P(b \leq x \leq \beta) = 1 - \frac{1}{2} = 1/2}$$

b)

$$P(a \leq x \leq \alpha) = \int_a^\alpha f(x)\,dx$$

$$= \int_a^b \frac{1}{2(b-a)}\,dx + \int_b^\alpha \left[\frac{1}{\beta - b} - \frac{x}{(\beta^2 - b^2)}\right] dx$$

$$= \left[\frac{x}{2(b-a)}\right]_a^b + \left[\frac{x}{\beta - b} - \frac{x^2}{2(\beta^2 - b^2)}\right]_b^\alpha$$

$$= \frac{b}{2(b-a)} - \frac{a}{2(b-a)} + \frac{\alpha}{\beta - b} - \frac{\alpha^2}{2(\beta^2 - b^2)} - \frac{b}{\beta - b} + \frac{b^2}{2(\beta^2 - b^2)}$$

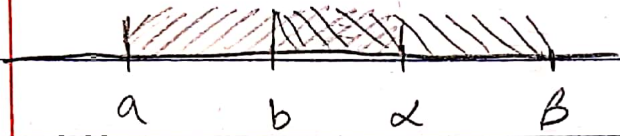
$$= \frac{\cancel{b-a}}{2(\cancel{b-a})} + \frac{\alpha - b}{\beta - b} - \frac{\alpha^2 - b^2}{2(\beta^2 - b^2)}$$

$$= \frac{1}{2} + \frac{(\alpha - b)(2)(\beta^2 - b^2) - (\alpha^2 - b^2)(\beta - b)}{2(\beta^2 - b^2)(\beta - b)}$$

$$P(a \leq x \leq \alpha) = 1 - \frac{(2b - \alpha + \beta)(\alpha - \beta)}{2(b + \beta)(b - \beta)}$$

c)

$$P(a\le x<\alpha \mid x>b) = \frac{P(a\le x\le \alpha \cap x>b)}{P(x>b)}$$



$$= \frac{P(b\le x\le \alpha)}{P(x>b)}$$

$$P(b\le x<\alpha)=\int_b^\alpha \left[\frac{1}{\beta-b} - \frac{x}{(\beta^2-b^2)}\right]dx$$

$$=\left[\frac{x}{\beta-b} - \frac{x^2}{2(\beta^2-b^2)}\right]\Bigg|_b^\alpha$$

$$=\frac{\alpha}{\beta-b}-\frac{\alpha^2}{2(\beta^2-b^2)}-\frac{b}{\beta-b}+\frac{b^2}{2(\beta^2-b^2)}$$

$$=\frac{\alpha-b}{\beta-b}-\frac{\alpha^2-b^2}{2(\beta^2-b^2)}$$

$$=\frac{b^2-2b\cdot(\alpha-\beta)+\alpha(\alpha-2\beta)}{2(\beta+b)(\beta-b)}$$

$$P(x>b)=P(b\le x\le\beta)=\int_b^\beta\left[\frac{1}{\beta-b}-\frac{x}{(\beta^2-b^2)}\right]dx=\frac{1}{2}$$

Calculado Anteriormente

$$P(a\le x<\alpha \mid x>b)=\frac{P(b\le x<\alpha)}{P(x>b)}=\frac{b^2-2b(\alpha-\beta)+\alpha(\alpha-2\beta)}{2(\beta+b)(\beta-b)}\cdot 2$$

$$\boxed{P(a\le x<\alpha \mid x>b)=\frac{b^2-2b(\alpha-\beta)+\alpha(\alpha-2\beta)}{(\beta+b)(\beta-b)}}$$

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