Asignación #3

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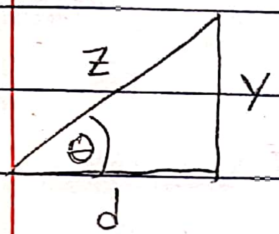
1.1) Una particula se emite desde un punto de forma isotropica en el plano. Colocamos una pantalla a una distancia d sobre el cual la particula impacta en la ordenada. La VA θ sigue una ley uniforme en el intervalo $[-\pi/2, \pi/2]$. Encuentre $f(z)$ y $f(y)$

$$\cos\theta = \frac{d}{z} \;;\quad \tan\theta = \frac{y}{d} \;;\quad \operatorname{sen}\theta = \frac{y}{z}$$



$$\theta_1 = \tan^{-1}\left(\frac{y}{d}\right)$$

$$y = d\tan\theta$$

$$y = \alpha(\theta)$$

$$f(y) = \frac{f(\theta_1)}{|\alpha'(\theta_1)|}$$

$$f(\theta_1) = \frac{1}{b-a} \quad \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$f(\theta_1) = \frac{1}{\left(\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right)} = \frac{1}{\pi}$$

$$y = \alpha(\theta)$$

$$\alpha(\theta) = d\tan\theta$$

$$\alpha'(\theta) = d\sec^2\theta$$

Remplazando la raiz

$$\alpha'(\theta) = d\sec^2\left(\tan^{-1}\left(\frac{y}{d}\right)\right)$$

$$\alpha'(\theta) = d\left(1 + \tan^2\left(\tan^{-1}\left(\frac{y}{d}\right)\right)\right)$$

$$\alpha'(\theta) = \frac{d^2 + y^2}{d}$$

$$f(y) = \frac{1}{\pi}\left(\frac{d}{d^2+y^2}\right)$$

$$\theta = \cos^{-1}\left(\frac{d}{z}\right)$$

$$z = \frac{d}{\cos\theta}$$

$$z = \alpha(\theta)$$

$$f(z) = \frac{f(\theta_2)}{|\alpha'(\theta_2)|}$$

$$z = \alpha(\theta)$$

$$\alpha(\theta) = \frac{d}{\cos\theta} = d \cdot \sec\theta$$

$$\alpha'(\theta) = d \cdot \operatorname{sen}(\theta) \cdot \sec^2(\theta)$$

$$\alpha'(\theta) = d \operatorname{sen}\left(\cos^{-1}\left(\frac{d}{z}\right)\right) \sec^2\left(\cos^{-1}\left(\frac{d}{z}\right)\right)$$

$$\alpha'(\theta) = d \operatorname{sen}\left(\cos^{-1}\left(\frac{d}{z}\right)\right) \frac{z^2}{d^2}$$

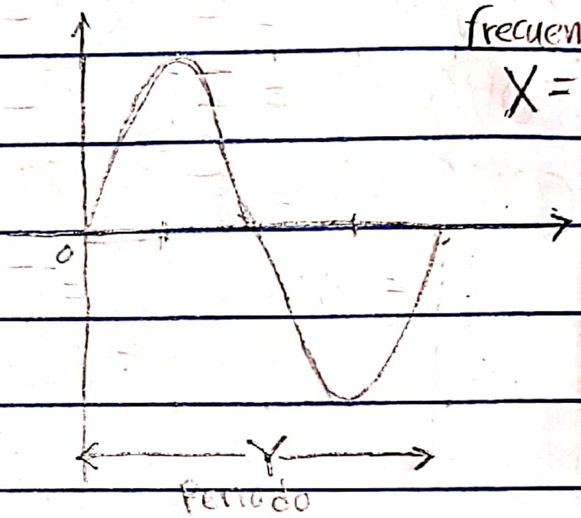
$$\alpha'(\theta) = \frac{\sqrt{z^2-d^2} \cdot z^2}{|z| \cdot d}$$

$$\alpha'(\theta) = \frac{|z| \cdot \sqrt{z^2-d^2}}{d}$$

$$f(z) = \frac{1}{\pi}\left(\frac{d}{|z| \cdot \sqrt{z^2-d^2}}\right)$$

Escaneado con CamScanner

4.2) Un transmisor envia una señal a una frecuencia X (en KHz). X es una VA con una distribución uniforme (continua) en el intervalo $[f_1, f_2]$. Determine $g(y)$ del periodo Y de la señal y el promedio $E[Y]$.



frecuencia $X = \frac{1}{Y}$

$$g(x_1) = \frac{1}{b-a} \quad ; \quad [f_1, f_2]$$

$$g(x_1) = \frac{1}{f_2 - f_1}$$

$$g(y) = \frac{g(x_1)}{|\alpha'(x_1)|}$$

$$Y = \alpha(x_1)$$

$$\frac{1}{X} = \alpha(x_1)$$

$$\alpha'(x_1) = \frac{-1}{x^2}$$

Remplazando la raiz

$$\alpha'(x_1) = \frac{-1}{\left(\frac{1}{Y}\right)^2}$$

$$\alpha'(x_1) = -Y^2$$

$$g(y) = \left(\frac{1}{f_2 - f_1}\right)\left|\frac{-1}{Y^2}\right|$$

$$g(y) = \frac{1}{Y^2(f_2 - f_1)}$$

$$E[Y] = \int_{f_1}^{f_2} Y\left[\frac{1}{Y^2(f_2 - f_1)}\right] dy$$

$$E[Y] = \int_{f_1}^{f_2} \frac{1}{Y(f_2 - f_1)} dy$$

$$E[Y] = \frac{\ln(Y)}{(f_2 - f_1)}\Bigg|_{f_1}^{f_2}$$

$$E[Y] = \frac{\ln(f_2) - \ln(f_1)}{(f_2 - f_1)}$$

$$E[Y] = \ln\left(\frac{f_2}{f_1}\right) \cdot \left(\frac{1}{f_2 - f_1}\right)$$

5.1) Dada dos VAs independientes X_1 y X_2 que pueden tomar valores de $\{-1, 1\}$ de probabilidad $\{1-p, p\}$ respectivamente. Es decir, $P(X_1=-1)=1-p$ y $P(X_1=1)=p$, igualmente para X_2. Si definimos $Y=X_1+X_2$ y $Z=X_1-X_2$. Determine el factor de correlación $r(Y,Z)$

$P(X_1,X_2)$	1,1	1,-1	-1,1	-1,-1
Y	2	0	0	-2
Z	0	2	-2	0

$Y=\{-2,0,2\}$

$Z=\{-2,0,2\}$

$P(X_1=1)\,P(X_2=1)=p^2$

$P(X_1=1)\,P(X_2=-1)=p(1-p)$

$P(X_1=-1)\,P(X_2=1)=p(1-p)$

$P(X_1=-1)\,P(X_2=-1)=(1-p)^2$

$P(Y)=\{(1-p)^2, 2p^2(1-p)^2, p^2\}$

$P(Z)=\{p(1-p), 2p^2(1-p)^2, p(1-p)\}$

$E(Y)=2(p^2)+(-2)(1-p)^2=4p-2$

$E(Z)=2(p(p-1))+(-2)(p(p-1))=0$

$E(Y^2)=4p^2+4(1-p)^2=8p^2-8p+4$

$E(Z^2)=4p(1-p)+4p(1-p)=8p-8p^2$

$E(YZ)=0$

$$V[Y] = \sigma_Y^2 = E[Y^2] - (E[Y])^2$$

$$V[Y] = \sigma_Y^2 = (8p^2 - 8p + 4) - (4p - 2)^2 = 8p - 8p^2$$

$$\sigma_Y = \sqrt{8p - 8p^2}$$

$$V[Z] = \sigma_Z^2 = E[Z^2] - (E[Z])^2$$

$$\sigma_Z^2 = (8p - 8p^2) - (0)^2 = 8p - 8p^2$$

$$\sigma_Z = \sqrt{8p - 8p^2}$$

$$COV(Y,Z) = E[YZ] - E[Y]E[Z]$$

$$COV(Y,Z) = (0) - (4p-2)(0)$$

$$COV(Y,Z) = 0$$

$$r(Y,Z) = \frac{COV(Y,Z)}{\sigma_Y \cdot \sigma_Z}$$

$$r(Y,Z) = \frac{(0)}{(\sqrt{8p-8p^2})(\sqrt{8p-8p^2})}$$

$$\boxed{r(Y,Z) = 0}$$

5.2) Dado que $COV(X,Y) = E[(X - E[X])(Y - E[Y])]$

demuestre que $COV(X,Y) = E[XY] - E[X]E[Y]$

$$COV(X,Y) = E[(X - E[X])(Y - E[Y])]$$

$$COV(X,Y) = E[XY - XE[Y] - YE[X] + E[X]E[Y]]$$

$$COV(X,Y) = E[XY] - E[X]E[Y] \cancel{- E[Y]E[X] + E[X]E[Y]}$$

$$COV(X,Y) = E[XY] - E[X]E[Y]$$