9.37) Un conductor cilíndrico de radio 10^{-2} m tiene un campo magnético interno

$$H = (4.77\times10^{4})\left(\frac{r}{2} - \frac{r^2}{3\times10^{-2}}\right) a_\phi \ (A/m)$$

¿Cuál es la corriente total en el conductor? Resp// 5.0 A

$$H_\phi = \frac{I_{enc}}{2\pi r}$$

$$H_\phi \cdot 2\pi \cdot r = I_{enc}$$

$$I_{enc} = (4.77\times10^{4})\left(\frac{r}{2} - \frac{r^2}{3\times10^{-2}}\right) \cdot 2\pi r$$

$r = 10^{-2}$ m

$$I_{enc} = (4.77\times10^{4})\left(\frac{10^{-2}}{2} - \frac{(10^{-2})^2}{3\times10^{-2}}\right) \cdot 2\pi(10^{-2})$$

$$I_{enc} = (4.77\times10^{-4})\left(\frac{1}{200} - \frac{1}{300}\right)\left(\frac{\pi}{50}\right)$$

$$\boxed{I_{enc} = 5.0\,A}$$

9.38) En coordenadas cilindricas, $J = 10^5 \cos^2(2r)\ \hat{a}_z$ en una cierta región. Obtenga H apartir de esta densidad de corriente y luego tome el rotacional de H y comparelo con J.

Resp// $H = 10^5 \left(\frac{r}{4} + \frac{\operatorname{sen}(4r)}{8} + \frac{\cos(4r)}{32r} - \frac{1}{32r} \right) \hat{a}_\phi$

$$H_\phi (2\pi r) = \iint \vec{J}\, d\vec{l}$$

$$H_\phi (2\pi r) = \int_0^{2\pi} \int_0^{r} 10^5 \cos^2(2r)\, r\, dr\, d\phi \ \ a_\phi$$

$$H_\phi = \frac{10^5}{2\pi r} \int_0^{2\pi} \int_0^{r} \left(\frac{1 - \cos 4r}{2} \right) r\, dr\, d\phi \ \ \hat{a}_\phi$$

$$H_\phi = \frac{10^5}{2\pi r} \int_0^{2\pi} \left[\int_0^{r} \frac{r}{2} dr + \frac{1}{2} \int_0^{r} r \cos 4r \right] d\phi \ \ \hat{a}_\phi$$

$u = r$

$du = dr$

$dv = \cos 4r\, dr$

$v = \frac{1}{4} \operatorname{sen} 4r$

$$H_\phi = \frac{10^5}{2\pi r} \int_0^{2\pi} \left[\frac{r^2}{4} \Big|_0^r + \frac{1}{2} \left[\frac{r}{4} \operatorname{sen} 4r - \frac{1}{4} \int_0^{r} \operatorname{sen} 4r\, dr \right] \right]_0^r d\phi \ \ \hat{a}_\phi$$

$$H_\phi = \frac{10^5}{2\pi r} \int_0^{2\pi} \left[\frac{r^2}{4} + \frac{1}{2} \left[\frac{r}{4} \operatorname{sen} 4r + \frac{1}{16} \cos 4r \right] \right]_0^r d\phi \ \ a_\phi$$

$$H_\phi = \frac{10^5}{2\pi r} \int_0^{2\pi} \left[\frac{r^2}{4} + \frac{r}{8} \operatorname{sen} 4r + \frac{1}{32} \cos 4r - \frac{1}{32} \right] d\phi \ \ a_\phi$$

$$H_\phi = \frac{10^5}{\cancel{2\pi} r} \left[\frac{r^2}{4} + \frac{r}{8} \operatorname{sen} 4r + \frac{1}{32} \cos 4r - \frac{1}{32} \right] \cancel{2\pi} \ \ a_\phi$$

$$H_\phi = 10^5 \left[\frac{r}{4} + \frac{1}{8} \operatorname{sen} 4r + \frac{1}{32r} \cos 4r - \frac{1}{32r} \right] a_\phi$$

Rotacional

$J = \nabla \times H$

$\vec{H} = 10^5 \left(\frac{r}{4} + \frac{\operatorname{sen} 4r}{8} + \frac{\cos 4r}{32r} - \frac{1}{32r} \right) a\phi$

Como solo tiene componente $\hat{a\phi}$ el rotacional solo sera en $\hat{az}$

$\therefore$

$$\nabla \times \vec{H} = \frac{1}{r} \left[\frac{\partial}{\partial r} \left[r \cdot \left(10^5 \left(\frac{r}{4} + \frac{\operatorname{sen} 4r}{8} + \frac{\cos 4r}{32r} - \frac{1}{32r} \right) \right) - \frac{\partial}{\partial \phi}(0) \right] \right]$$

$$= \frac{10^5}{r} \left[\frac{\partial}{\partial r} \left(\frac{r^2}{4} + \frac{r}{8} \operatorname{sen} 4r + \frac{\cos 4r}{32} - \frac{1}{32} \right) \right] \hat{az}$$

$$= \frac{10^5}{r} \left[\frac{r}{2} + \frac{1}{8} \left(\operatorname{sen} 4r + 4r \cos 4r \right) - \frac{4r \operatorname{sen} 4r}{32} \right]$$

$$= \frac{10^5}{r} \left[\frac{r}{2} + \cancel{\frac{1}{8} \operatorname{sen} 4r} + \frac{r}{2} \cos 4r - \cancel{\frac{1}{8} \operatorname{sen} 4r} \right]$$

$$= 10^5 \left[\frac{1}{2} + \frac{1}{2} \cos 4r \right]$$

$$\boxed{J = 10^5 \cos^2 (2r)}$$

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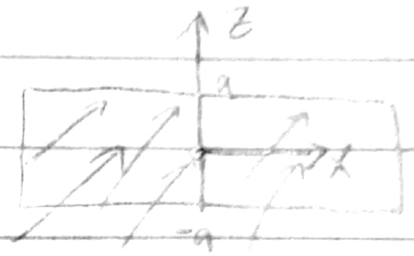
9.39) En coordenadas cartesianas, una densidad constante de corriente, $J = J_o \hat{a_y}$ existe en la región $-a \leq z \leq a$. Utilice la ley de Ampere para hallar H en todas las regiones. Obtenga el rotacional de H y comparelo con J.

Resp. $H = \begin{cases} J_o a \hat{a}_x & z > a \\ J_o z \hat{a}_x & -a \leq z \leq a \\ -J_o a \hat{a}_x & z < -a \end{cases}$ Rotacional H = J



$J = cte$

$J = \frac{I}{A}$

$i_{enc} = \frac{A_{enc}}{A} \cdot I$

$\oint H \cdot dl = i_{enc}$

$\frac{I_{enc}}{A_{enc}} = \frac{I}{A}$

$-a \leq z \leq a$

$$Rot = \nabla \times H = J = \left(\frac{\partial (J_o \cdot z)}{\partial z} - \frac{\partial (0)}{\partial x} \right) = J_o = J$$

$z > a$

$$Rot = \nabla \times H = J = \left(\frac{\partial (J_o \cdot a)}{\partial z} - \frac{\partial (0)}{\partial x} \right) = 0$$

$z < a$

$$Rot = \nabla \times H = J = \left(\frac{\partial (-J_o \cdot a)}{\partial z} - \frac{\partial (0)}{\partial x} \right) = 0$$