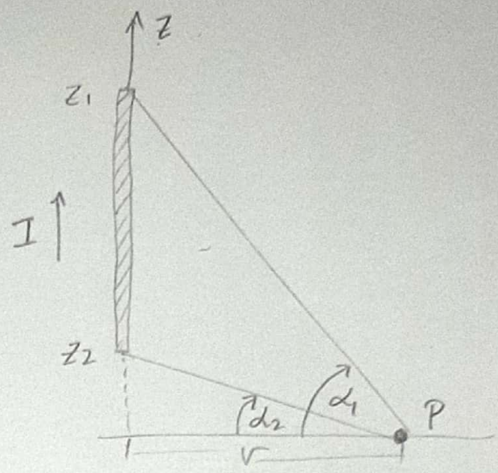
9.23) Demuestre que el campo magnético producido por el elemento finito de corriente que aparece en la figura 9-19 esta dado por:

$$H = \frac{I}{4\pi r}(\operatorname{Sen}\alpha_1 - \operatorname{Sen}\alpha_2)a\phi$$




$$H = \oint \frac{I\,\vec{dl} \times \vec{a_R}}{4\pi R^2}$$

$$\tan\alpha_1 = \frac{z_1}{r}$$

$$\boxed{z_1 = r\tan\alpha_1}$$

$$\boxed{z_2 = r\tan\alpha_2}$$

$$\vec{dl} = dz\,\hat{a_z}$$

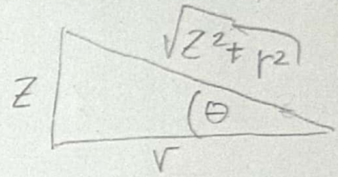
$$\vec{H} = \int_{z_2}^{z_1} \frac{I\,\vec{dl} \times \vec{a_R}}{4\pi R_2}$$

$$\vec{H} = \int_{z_2}^{z_1} \frac{I\,dz\,\hat{a_z} \times (r\hat{a_r} - z\hat{a_z})}{4\pi(r^2+z^2)^{3/2}}$$

$$\vec{H} = \int_{r\tan\alpha_2}^{r\tan\alpha_1} \frac{Ir\,dz}{4\pi(r^2+z^2)^{3/2}}\,\hat{a\phi}$$

$$\vec{H} = \frac{Ir}{4\pi}\int_{r\tan\alpha_2}^{r\tan\alpha_1} \frac{dz}{(r^2+z^2)^{3/2}}\,\hat{a\phi}$$

Sustitución trigonométrica




CV:

$z = r\tan\theta$

$dz = r\operatorname{Sec}^2\theta\,d\theta$

$\sqrt{z^2+r^2} = r\operatorname{Sec}\theta$

$$\vec{H} = \frac{Ir}{4\pi}\int_{r\tan\alpha_2}^{r\tan\alpha_1} \frac{r\operatorname{Sec}^2\theta\,d\theta}{(r\operatorname{Sec}\theta)^3}\,\hat{a\phi}$$

$$\vec{H} = \frac{Ir}{4\pi}\int_{r\tan\alpha_2}^{r\tan\alpha_1} \frac{\cancel{r}\operatorname{Sec}^2\theta\,d\theta}{r^{\cancel{3}}\operatorname{Sec}^3\theta}\,\hat{a\phi}$$

$$\vec{H} = \frac{I}{4\pi r}\int_{r\tan\alpha_2}^{r\tan\alpha_1} \operatorname{Cos}\theta\,d\theta\,\hat{a\phi}$$

$$\vec{H} = \frac{I}{4\pi r}\operatorname{Sen}\theta\Big|_{r\tan\alpha_2}^{r\tan\alpha_1}\hat{a\phi}$$

$$\vec{H} = \frac{I}{4\pi r}\cdot\frac{z}{\sqrt{z^2+r^2}}\Bigg|_{r\tan\alpha_2}^{r\tan\alpha_1} a\phi$$

$$\vec{H} = \frac{I}{4\pi r}\left(\frac{r\tan\alpha_1}{\sqrt{r^2\tan^2\alpha_1+r^2}} - \frac{r\tan\alpha_2}{\sqrt{r^2\tan^2\alpha_2+r^2}}\right)a\phi$$

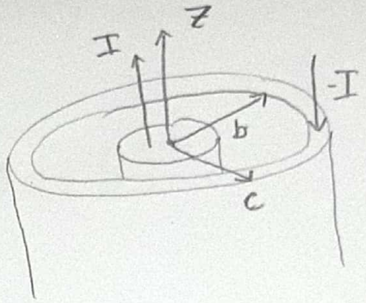
$$\vec{H} = \frac{I}{4\pi r}\left(\frac{\cancel{r}\tan\alpha_1}{\cancel{r}\sqrt{\operatorname{Sec}^2\alpha_1}} - \frac{\cancel{r}\tan\alpha_2}{\cancel{r}\sqrt{\operatorname{Sec}^2\alpha_2}}\right)\hat{a\phi}$$

$$\vec{H} = \frac{I}{4\pi r}\left(\frac{\tan\alpha_1}{\operatorname{Sec}\alpha_1} - \frac{\tan\alpha_2}{\operatorname{Sec}\alpha_2}\right)\hat{a\phi}$$

$$\vec{H} = \frac{I}{4\pi r}(\operatorname{Sen}\alpha_1 - \operatorname{Sen}\alpha_2)\hat{a\phi}$$

9.25) Las corrientes en los conductores interno y externo de la figura 9-20 estan uniformemente distribuidas. Utilice la ley de Ampere para demostrar que para $b \leq r \leq c$,

$$H = \frac{I}{2\pi r}\left(\frac{c^2 - r^2}{c^2 - b^2}\right)\hat{a\phi}$$




$$\oint H \cdot dl = I$$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{enc}$$

$$J = \frac{-I}{\pi c^2 - \pi b^2}$$

$$I_{enc} = \int J \, dA$$

$$J = \frac{-I}{\pi(c^2 - b^2)}$$

$$B(2\pi r) = \mu_0\left[I + \int J \, dA\right]$$

$$B(2\pi r) = \mu_0\left[I - \frac{I}{\pi(c^2 - b^2)} \cdot (\pi r^2 - \pi b^2)\right]$$

$$B(2\pi r) = \mu_0 I\left[1 - \frac{\cancel{\pi}(r^2 - b^2)}{\cancel{\pi}(c^2 - b^2)}\right]$$

$$B = \frac{\mu_0 I}{2\pi r}\left[1 - \frac{r^2 - b^2}{c^2 - b^2}\right]$$

$$B = \frac{\mu_0 I}{2\pi r}\left[\frac{c^2 - \cancel{b^2} - r^2 + \cancel{b^2}}{c^2 - b^2}\right]$$

$$B = \frac{\mu_0 I}{2\pi r}\left[\frac{c^2 - r^2}{c^2 - b^2}\right]$$

$$B = \mu_0 H$$

$$\boxed{H = \frac{B}{\mu_0} = \frac{I}{2\pi r}\left(\frac{c^2 - r^2}{c^2 - b^2}\right)}$$

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9.10) Calcule el flujo magnetico total $\Phi$ que cruza el plano $z=0$ en coordenadas cilindricas para $r \leq 5\times10^{-2}\,m$ si $B = \frac{0.2}{r}(\mathrm{Sen}^2\phi)\hat{a}_z$ (T)

$$\Phi = \int_S \vec{B}\cdot\vec{ds} \qquad \vec{ds} = r\,dr\,d\phi\;\hat{a}_z$$

$$\Phi = \int_0^{5\times10^{-2}}\int_0^{2\pi} \frac{0.2}{\cancel{r}}(\mathrm{Sen}^2\phi)(\cancel{\hat{a}_z})\cdot \cancel{r}\,d\phi\,dr\,(\cancel{\hat{a}_z})$$

$$\Phi = \int_0^{5\times10^{-2}}\int_0^{2\pi} 0.2\,\mathrm{Sen}^2\phi\,d\phi\,dr$$

$$\Phi = 0.2\int_0^{5\times10^{-2}}\int_0^{2\pi}\left(\frac{1}{2}-\frac{1}{2}\cos2\phi\right)d\phi\,dr$$

$$\Phi = 0.2\int_0^{5\times10^{-2}}\left[\int_0^{2\pi}\frac{1}{2}\,d\phi - \frac{1}{2}\int_0^{2\pi}\cos2\phi\,d\phi\right]dr$$

$$\Phi = 0.2\int_0^{5\times10^{-2}}\left[\frac{\phi}{2}\Big|_0^{2\pi} - \frac{1}{4}(\mathrm{Sen}\,2\phi)\Big|_0^{2\pi}\right]dr$$

$$\Phi = 0.2\int_0^{5\times10^{-2}}\left[(\pi-0) - \frac{1}{4}\left(\overset{0}{\mathrm{Sen}(4\pi)} - \overset{0}{\mathrm{Sen}(0)}\right)\right]dr$$

$$\Phi\;\; 0.2\cdot\pi\int_0^{5\times10^{-2}} dr$$

$$\Phi = 0.2\cdot\pi\cdot\; r\Big|_0^{5\times10^{-2}}$$

$$\Phi = 0.2\cdot\pi\cdot5\times10^{-2}$$

$$\boxed{\Phi = 3.14\times10^{-2}\ \mathrm{Wb}}$$

9.41)

Sea $\vec{B} = 2.5\ \text{Sen}\left(\frac{\pi x}{2}\right) \cdot e^{-2y}\ \hat{a_z}\ (T)$

Halle el flujo magnetico total que cruza la franja $z=0,\ y \geq 0,\ 0 \leq x \leq 2m$

Resp $\boxed{1.59\ Wb}$

$$\vec{dS} = dx\,dy\ \hat{a_z}$$

$$\Phi = \int_S \vec{B}\,ds$$

$$\Phi = \int_0^2\int_0^y 2.5\ \text{Sen}\left(\frac{\pi x}{2}\right) \cdot e^{-2y}\ (\hat{a_z}) \cdot dy\,dx\,(\hat{a_z})$$

$$\Phi = 2.5\int_0^2 \text{Sen}\left(\frac{\pi x}{2}\right) \cdot \left[-\frac{1}{2}e^{-2y}\right]\Big|_0^y dx$$

$$\Phi = 2.5\int_0^2 \text{Sen}\left(\frac{\pi x}{2}\right) \cdot \left(-\frac{1}{2}\right)\left[e^{-2y} - \cancel{e^0}^{\,1}\right] dx$$

$$y \geq 0$$

$$\Phi = 2.5\int_0^2 \text{sen}\left(\frac{\pi x}{2}\right)\left(-\frac{1}{2}\right)\left[\cancel{e^{-\infty}}^{\,0} - 1\right] dx$$

$$\Phi = 2.5\left(\frac{1}{2}\right)\int_0^2 \text{sen}\left(\frac{\pi x}{2}\right) dx$$

$$\Phi = 2.5\left(\frac{1}{\cancel{2}}\right)\left(-\frac{\cancel{2}}{\pi}\right)\left[\cos\left(\frac{\pi x}{2}\right)\right]\Big|_0^2$$

$$\Phi = 2.5\left(-\frac{1}{\pi}\right)\left[\cancel{\cos(\pi)}^{\,-1} - \cancel{\cos(0)}^{\,1}\right]$$

$$\Phi = 2.5 \cdot \left(-\frac{1}{\pi}\right)(-2)$$

$$\boxed{\Phi = 1.59\ Wb}$$

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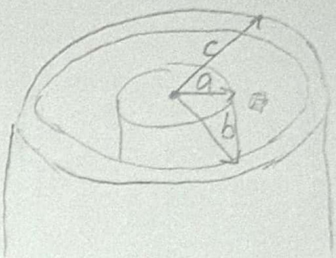
9.42) Un cable coaxial cuyo conductor interno tiene radio a y el externo tiene radios interno y externo b y c respectivamente, transporta una corriente I en el conductor interno. Halle el flujo magnetico por unidad de longitud que cruza un plano $\phi$ = cte entre los conductores Resp:// $\frac{\mu_0 I}{2\pi} \ln\left(\frac{b}{a}\right)$

$$\Phi = \int_s \vec{B} \cdot \vec{ds}$$

$$\boxed{\vec{B} = \frac{I \mu_0}{2\pi r}}\ \hat{a_z}$$ ← Campo en la region $a < r < b$ ①

$$\vec{ds} = dr dz$$




$$\Phi = \int_0^z \int_a^b \frac{I \mu_0}{2\pi r} \cdot dr dz$$

$$\Phi = \frac{I \mu_0}{2\pi} \int_0^z \left[ \ln(r) \Big|_a^b \right] dz$$

$$\Phi = \frac{I \mu_0}{2\pi} \int_0^z \left[ \ln(b) - \ln(a) \right] dz$$

$$\Phi = \frac{I \mu_0}{2\pi} \int_0^z \ln\left(\frac{b}{a}\right) dz$$

$$\Phi = \frac{I \mu_0 z}{2\pi} \ln\left(\frac{b}{a}\right)$$

$$\boxed{\Phi = \frac{I \mu_0}{2\pi} \ln\left(\frac{b}{a}\right) \cdot (z)}$$ donde z es la unidad de longitud

$$\oint \vec{B} \cdot \vec{ds} = \mu_0 I_{enc}$$

$$I_{enc} = \int J \, dA$$

$$B(2\pi r) = \mu_0 I$$

$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$ ①