

# Guaranteed Automatic Algorithms with Relative Error

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## Abstract

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Let  $\mathbf{A}$  be an  $M \times N$  matrix, let  $K$  be a small number, and let

$$\mathcal{X} = \{\mathbf{Ac} : \|\mathbf{c}\| \leq K\}$$

be a set of elements that we want to estimate based on incomplete data. Here  $\|\cdot\|$  be a measure of size, e.g.,

$$\|\mathbf{c}\| = \|\mathbf{c}\|_0 = \sum_{j=1}^N |c_j|^0 \quad \text{or} \quad \|\mathbf{c}\| = \|\mathbf{c}\|_1 = \sum_{j=1}^N |c_j|$$

Given integers  $\widetilde{M} \leq M$  and  $\widetilde{N} \leq N$ , let  $\widetilde{\mathbf{A}}$  denote the upper left  $\widetilde{M} \times \widetilde{N}$  block of  $\mathbf{A}$ . For any  $\mathbf{x} \in \mathcal{X}$ , let  $\widetilde{\mathbf{x}}$  denote the upper  $\widetilde{M}$  rows of  $\mathbf{x}$ . Let  $\widetilde{\mathbf{c}}$  denote the solution of

$$\widetilde{\mathbf{c}} = \operatorname{argmin}_{\mathbf{b}, \|\mathbf{b}\| \leq K} \|\widetilde{\mathbf{x}} - \widetilde{\mathbf{A}}\mathbf{b}\|.$$

Our estimate of  $\mathbf{x}$  based on only  $\widetilde{M}$  data is  $\mathbf{A}\widetilde{\mathbf{c}}$ .

## Acknowledgements