

**Report on: A Unified Treatment of Tractability for Approximation  
Problems Defined on Hilbert Spaces  
by: Onyekachi Emenike, Frederick John Hickernell and Peter Kritzer**

The paper deals with the tractability of approximation problems in Hilbert spaces in the worst case with the absolute error criterion. It unifies some of the proofs and generalizes a number of the results. The results are new and good. Nevertheless, I have a number of comments that need to be addressed.

1. p. 2, l. 13: Why such a long discussion of the normalized error criterion which you do not study? At the same time you spend less than a page and less than half of the Introduction discussing and motivating the specific topic of this paper, i.e., generalized tractability with the absolute error criterion. If the purpose is to say that similar results will presumably hold for the normalized error criterion, then do so in a couple of sentences.
2. p. 2, l. 25: In this paragraph you provide motivation for generalized tractability. Yet the text is not that inspiring and should be improved if the paper is to appeal to a broader range of readers beyond the few that have already worked on generalized tractability. You need to provide motivation, for example, as it's done in [6] and then give some background. Elaborate and don't save space.

It seems that you want  $T$  to be a "simple" function. Explain what you mean by this and why is it important. None of the proofs depends on the simplicity of  $T$ . Clearly, the conditions of your theorems are hard to evaluate when  $T$  has a complicated form.

So, on the one hand (3) may make it difficult to bound the complexity due to the ordering of the singular values, as you say, on the other hand you are providing an alternative way of obtaining complexity bounds whose practicality may be limited by the form of  $T$ . You need to present this comparison and then display Table 1 without the last column to illustrate the types of tractability that have been studied in the literature that have  $T$  with a simple form.

(You may duplicate Table 1 without the last column keeping the original one in its current location.)

3. p. 2, equation (4): Here  $T$  is defined to be a function of three variables but on line 25 it has been introduced as a function of two variables.
4. p. 2, l. 37: Illustrate the role of  $p$ , which you can do by referring to the exponents of  $\varepsilon$  and  $d$ , say in the case of polynomial tractability.
5. p. 2, l. 42: Describe when would it be desirable to consider  $\Omega$  as a proper subset of  $(0, \infty) \times \mathbb{N}$ . This is done very well in [6] and you could get some ideas from that paper.
6. p. 2, l. 43: Explain why do you consider  $\varepsilon \in (0, \infty)$  when usually  $\varepsilon \leq 1$ . When are large or huge values of  $\varepsilon$  relevant?
7. p. 2, l. 44:  $T(0, \mathcal{D}, p)$  has not been defined yet.
8. p. 3, l. 2: What are the “the additional layers of complexity?”
9. p. 3, equation (9): This “technical assumption” needs to be substantiated, say, by some example of  $T$  where it holds. Otherwise, it seems ad hoc, especially since it plays an important role in the proofs of the theorems.
10. p.4, l. -1: Remind the reader why the infinite sum exists. Since you are using this inequality in the proof of Theorem 2 as well, I suggest you number the displayed formula to refer to it later.
11. p. 5, l. 3: How do you conclude that equation (13) holds for all  $\mathcal{D}, n, N \in \mathbb{N}$ ? This is not true. The condition on the left hand side of the implication is satisfied for  $n$  large enough, which is what you need to bound the complexity in (14).
12. p. 5, l. 4: You explain why  $n \geq N$  although you have assumed it in the previous page.
13. p. 6, l. -1: Indeed this complexity bound follows from the material in the proof of Theorem 1, but be precise about which of the parts of the proof you are using by providing equation numbers.
14. p. 15, l. 13 and below it: You say that  $\mathcal{D}$  is as in Theorem 1. Why don't you set  $\mathcal{D} = 1$  since that's what you need and you do it at the end of the page.

15. p. 15, equation (40): For this implication you need  $n$  to be sufficiently large. See the comments in item 11 above.
16. p. 16, l. -9: State  $\tau > 1$ .
17. p. 17, l. -3: The brace under the summation refers to equation (39).  
Do you mean equation (45)?
18. p. 18, l. -14: State  $\tau > 1$ .