

MATH 476 Statistics

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Test 1

Thursday, February 19, 2026

Instructions:

- i. This test has **FOUR** question(s). Attempt all. The maximum number of points is **100**.
- ii. The time allowed is 75 minutes.
- iii. This test is closed book, but you may use **4** double-sided letter-size sheets of notes.
- iv. No calculators or other electronic devices are allowed. Phones must be placed in your bags under your desks or at the front of the room. Hands must be on top of your desks.
- v. You are expected to simplify any expressions that can be simplified as whole numbers, e.g., $36/15 = 12/5$, but you may leave other expressions as is, e.g., $\sqrt{47}$ or $18/29$.
- vi. You will be provided all of the critical values that you need.
- vii. Show all your work to justify your answers. Answers without adequate justification will not receive credit.
- viii. Off-site students may contact the instructor as directed by your syllabus.

I understand these instructions and have not relied on any help for this exam beyond what is allowed, nor provided any help to anyone else beyond what is allowed.

Signature

Date

Distribution	Sample Space	$g(x)$	μ	σ^2
Uniform Unif[a, b]	$[a, b]$	$\frac{1}{b-a}$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
Geometric Geom(p)	$\{1, 2, \dots\}$	$p(1-p)^{x-1}$	$\frac{1}{p}$	$\frac{1-p}{p^2}$

Upper quantiles: q_α satisfies $\mathbb{P}(X > q_\alpha) = \alpha$

X	α	0.995	0.99	0.975	0.95	0.9	0.1	0.05	0.025	0.01	0.005
N(0, 1)	z_α	-2.58	-2.33	-1.96	-1.64	-1.28	1.28	1.64	1.96	2.33	2.58

1. (20 points) Given IID data, X_1, \dots, X_n , what is the maximum likelihood estimator (MLE) for the parameter p in the geometric distribution, Geom(p) (see above)?
2. (44 points) A machine gives readings that are randomly distributed Unif[$\theta, \theta + 2$] (see above) with unknown parameter θ .
 - a. (8 points) If \bar{X}_n is the sample mean of n IID samples, is \bar{X}_n an unbiased estimator of θ ? If not, modify it to make it one.
 - b. (20 points) Construct a large n approximate 95% confidence interval for θ in terms of \bar{X}_n .

- c. (8 points) If $\bar{x}_{75} = 35$ is observed, then what is the half-width of this confidence interval?
- d. (8 points) How large would n need to be to make the confidence interval have a half-width of no greater than 0.1?

3. (24 points) Let

- X_1, \dots, X_n be IID miles driven until replacement for cars with Michelin tires,
- Y_1, \dots, Y_n be IID miles driven until replacement for an independent group of cars with Goodyear tires, and
- $D_i = X_i - Y_i$, for $i = 1, \dots, n$

Let \bar{X} , \bar{Y} , and \bar{D} and S_X^2 , S_Y^2 , and S_D^2 , denote the respective sample means and variances of this random data.

Assuming large n , construct a 99% confidence interval in terms of this data for the correct population parameter that measures the difference between Michelin and Goodyear tires. Specify precisely the population parameter that your confidence interval is describing.

4. (12 points) You are an aide to the mayor. The mayor's draft speech states:

We directed our city managers to ask their staff by a show of hands, "Do you approve of the mayor's leadership?" Out of 250 responses, 92.47% answered, "Yes." We conclude that 92,470 out of our city's 100,000 residents approve of my leadership.

Explain to the mayor at least three things wrong with this statistical inference.