

MATH 565 Monte Carlo Methods in Finance

Fred J. Hickernell

Fall 2007

Take-Home Part of Final Exam

Due 10:30 am, Thursday, December 13

Instructions:

- i. This take-home part of the final exam consists of THREE questions for a total possible of 50 marks. Answer all of them.
- ii. You may consult any book, web page, software repository or other inanimate object. You may not consult any other person face-to-face, by telephone, by email, or by any other means.
- iii. Show all your work to justify your answers. Submit hard copies of your derivations, programs, output, and explanations. Answers without adequate justification will not receive credit.

An insurance company receives a random variable X_i million dollars in premiums from customers for month i . The amount paid out in claims in month i is a random variable, Y_i million dollars, and $X_1, X_2, \dots, Y_1, Y_2, \dots$ are independent random variables. The X_i have a normal (Gaussian) distribution with mean 10 and variance 1, while the Y_i have an exponential distribution with a mean of 9, i.e., the probability density function for Y_i is $f(y) = e^{-y/9}/9$.

1. (15 marks)

The insurance company is required to have a reserve of at least \$100 million at the end of each month after receiving premiums and paying out claims. Assume an interest rate of zero. Use a simple Monte Carlo method to estimate the following:

- a) the probability to the nearest percent that the company will maintain at least \$100 million in reserves for 5 years (60 months) straight, assuming that it starts with \$150 million in reserves;
- b) the starting reserves required to the nearest \$10 million to maintain at least \$100 million in reserves for 5 years (60 months) straight with probability 95%.

You do not need to find the smallest sample size that works, but you need to compute an error estimate, and show that this error estimate is small enough for the sample size that you use.

2. (15 marks)

Repeat the previous problem, but now using Latin hypercube sampling with 20 replications and $n/20$ strata per variable, where n is the total sample size. How does the error change if the same total sample size is used for calculating the answer in this problem and the previous problem?

3. (20 marks)

A reinsurance company sells a derivative product. If the insurance company's claims in the coming one year, say Z million, exceed their premiums for the year, say W million, by \$10 million dollars or more, then the reinsurance company will pay the insurance company the excess amount, i.e., $Z - W - 10$ million. What is the fair price for this product, with an error of at most 5 cents on the dollar, using Monte Carlo simulation, again assuming zero interest rate?