Function reconstruction using determinantal sampling

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We study the approximation of a square-integrable function from a finite number of evaluations on a random set of nodes according to a well-chosen distribution. This is particularly relevant when the function is assumed to belong to a reproducing kernel Hilbert space (RKHS). This work proposes to combine several natural finite-dimensional approximations based two possible probability distributions of nodes. These distributions are related to determinantal point processes, and use the kernel of the RKHS to favor RKHS-adapted regularity in the random design. While previous work on determinantal sampling relied on the RKHS norm, we prove mean-square guarantees in L^2 norm. We show that determinantal point processes and mixtures thereof can yield fast convergence rates. Our results also shed light on how the rate changes as more smoothness is assumed, a phenomenon known as superconvergence. Besides, determinantal sampling generalizes i.i.d. sampling from the Christoffel function which is standard in the literature. More importantly, determinantal sampling guarantees the so-called instance optimality property for a smaller number of function evaluations than i.i.d. sampling.