Quasi-Monte Carlo for Efficient Fourier Pricing of Multi-Asset Options

Michael Samet
Chair of Mathematics for Uncertainty Quantification, RWTH Aachen University, Aachen, Germany
samet@uq.rwth-aachen.de

Coauthor(s): Christian Bayer, Chiheb Ben Hammouda, Antonis Papapantoleon, and Raul Tempone

Efficiently pricing multi-asset options poses a significant challenge in quantitative finance. Fourier methods leverage the regularity properties of the integrand in the Fourier domain to accurately and rapidly value options that typically lack regularity in the physical domain. However, most of the existing Fourier approaches face hurdles in high-dimensional settings due to the tensor product (TP) structure of the commonly employed numerical quadrature techniques. To overcome this difficulty, this work advocates using the randomized quasi-MC (RQMC) quadrature to improve the scalability of Fourier methods with high dimensions. The RQMC technique benefits from the smoothness of the integrand and alleviates the curse of dimensionality while providing practical error estimates. Nonetheless, the applicability of RQMC on the unbounded domain, $\mathbb{R}^d$, requires a domain transformation to $[0,1]^d$, which may result in singularities of the transformed integrand at the corners of the hypercube, and hence deteriorate the performance of RQMC. To circumvent this difficulty, we design an efficient domain transformation procedure based on boundary growth conditions on the transformed integrand. The proposed transformation preserves sufficient regularity of the original integrand for fast convergence of the RQMC method. To validate our analysis, we demonstrate the efficiency of employing RQMC with an appropriate transformation to evaluate options in the Fourier space for various pricing models, payoffs, and dimensions. Finally, we highlight the computational advantage of applying RQMC over the TP quadrature in the Fourier domain and over the MC method in the physical domain for options with up to 15 assets.

[1] Christian Bayer, Chiheb Ben Hammouda, Antonis Papapantoleon, Michael Samet, and Raul Tempone (2024). *Quasi-Monte Carlo for efficient Fourier pricing of multi-asset options*. arXiv preprint.

[2] Christian Bayer, Chiheb Ben Hammouda, Antonis Papapantoleon, Michael Samet, and Raul Tempone (2024). *Optimal damping with a hierarchical adaptive quadrature for efficient Fourier pricing of multi-asset options in Lévy models*. Journal of Computational Finance, 27(3).