Quasi-Monte Carlo Kernel Density Estimation

Fred J. Hickernell
Department of Applied Mathematics and Center for Interdisciplinary Scientific Computation, Illinois Institute of Technology
hickernell@iit.edu

Coauthor(s): Guillem Grao I Grasa

Special session: Universality in QMC and related algorithms

Quasi-Monte Carlo (qMC) methods use low discrepancy (LD) sequences, such as lattice, digital, and Halton sequences to estimate multivariate integrals with greater computational efficiency than simple Monte Carlo. Given a random variable $Y = f(\boldsymbol{X})$, where $\boldsymbol{X} \sim \mathcal{U}[0,1]^d$, this equivalent to estimating $\mathbb{E}(Y)$.

QMC methods for estimating, ϱ, the density of Y, have been studied recently [1–4], in many cases employing conditional sampling. This talk explores qMC kernel density estimation without employing conditional sampling.

The error of the kernel density estimate is bounded above by the sum of two terms: the error in approximating the density by a smoothed version, and the error in approximating the smoothed density by a qMC cubature. Theoretical and numerical results are presented.

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[3] L'Ecuyer, P. and Puchhammer, F. (2022). Density estimation by Monte Carlo and quasi-Monte Carlo. In Keller, A., editor, *Monte Carlo and Quasi-Monte Carlo Methods: MCQMC, Oxford, England, August 2020*, Springer Proceedings in Mathematics and Statistics, pages 3–21. Springer, Cham.

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