

Constructive Sparsification of Finite Frames and Applications to Multivariate Function Recovery

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Abstract

We present a new constructive subsampling technique for finite frames to extract (almost) minimal plain (non-weighted) subsystems which preserve a good lower frame bound. The technique is based on a greedy type selection of frame elements to positively influence the spectrum of rank one updates of a matrix. It is a modification of the 2009 algorithm by Batson, Spielman, Srivastava and produces an optimal size subsystem (up to a prescribed oversampling factor) without additional weights. It moreover achieves this in polynomial time and avoids the Weaver subsampling (based on the Kadison-Singer theorem) which has been applied in earlier work, yielding rather bad oversampling constants. In the second part of the talk we give applications for multivariate function recovery. Here we consider the particular the problem of L_2 and L_∞ recovery from sample values. In this context, the presented subsampling technique allows to determine optimal (in cardinality) node sets even suitable for plain least squares recovery. It can be applied, for instance, to reconstruct functions in dominating mixed-smoothness Sobolev spaces, where we are able to discretize trigonometric polynomials with frequencies from a hyperbolic cross with nodes coming from an implementable subsampling procedure. In addition we may apply this to subspaces coming from hyperbolic cross wavelet subspaces. Numerical experiments illustrate the theoretical findings.