Single-ensemble multilevel Monte Carlo for discrete interacting-particle methods

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Interacting-particle methods have found applications in domains such as filtering, optimization and posterior sampling. These parallelizable and often gradient-free algorithms use an ensemble of particles that evolve in time, based on the combination of well-chosen dynamics and interaction between the particles. For computationally expensive dynamics – for example, Bayesian inversion with an expensive forward model or optimization of an expensive objective function – the cost of attaining a high accuracy quickly becomes prohibitive.

In this talk, we discuss our recent work [1], in which we exploit a hierarchy of approximations to this forward model and apply multilevel Monte Carlo (MLMC) techniques. More specifically, we extend the approach suggested in [2, 3] for the ensemble Kalman filter: we use MLMC at each time step to estimate the interaction term within a single, globally-coupled ensemble. We formulate conditions on the method dynamics, interaction term, and forward model, under which we prove an asymptotic bound on the cost-to-error relation. This bound suggests that a multilevel simulation strategy is more efficient than a single-level one for a large family of interacting-particle methods. Numerical experiments corroborate our analysis of the improved asymptotic cost-to-error rate.

[1] A. Bouillon, T. Ingelaere, G. Samaey, Single-ensemble multilevel Monte Carlo for discrete interacting-particle methods, in preparation.

[2] H. Hoel, K. J. H. Law, and R. Tempone, Multilevel ensemble Kalman filtering, SIAM J. Numer. Anal., 54 (2016), pp. 1813–1839.

[3] A. Chernov, H. Hoel, K. J. H. Law, F. Nobile, and R. Tempone, Multilevel ensemble Kalman filtering for spatio-temporal processes, Numer. Math., 147 (2021), pp. 71–125.