Approximation of vectors using adaptive randomized information

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Abstract

Working in the framework of Information-Based Complexity we study approximation of the embedding $\ell_p^m \hookrightarrow \ell_q^m$, $1 \leq p < q \leq \infty$, based on randomized adaptive algorithms that use arbitrary linear functionals as information on a problem instance. In the case $p \leq 2$ and $q = \infty$ we show upper bounds for which the complexity n exhibits only a $(\log\log m)$-dependence. In particular we improve upon known results for $p = 1$ and $q = \infty$ and by this give an example for a gap of order $n/(\log n)^2$ for the error for adaptive vs. non-adaptive Monte Carlo methods which is the biggest possible gap for linear problems up to logarithmic factors.

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