Mixed Precision Multilevel Monte Carlo Method

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We present the analysis and application of mixed precision within the Multilevel Monte Carlo (MLMC) method. To begin, we state the model problem, an elliptic PDE with random coefficients and a random right-hand side. Such a problem arises, for example, in uncertainty quantification for groundwater flow. Our focus is on approximating a quantity of interest given as the expected value of a functional of the solution of the PDE problem. To this end, we use the conforming finite element method for approximation in the spatial variable and the MLMC method for approximation of the expected value. We provide a novel rigorous analysis of the MLMC method in finite precision arithmetic and based on this we formulate an adaptive algorithm which determines the optimal target numerical accuracy on each level of discretisation. Our theoretical results are then verified on multiple examples including an elliptic PDE with lognormal random coefficients. Using the MINRES solver a practical speedup of $2\times$ in terms of FLOPs is achieved compared to a reference value of error tolerance. To use mixed precision efficiently with a sparse direct solver, we employ the technique of iterative refienement. The $LDL^\top$ solver with iterative refinement seems to improve the energy-efficiency significantly compared to the reference double precision.