Function space embeddings for non-tensor product spaces and application to high-dimensional approximation

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In tractability analysis one is interested in continuous approximation problems that are defined on a scale of function spaces $(H_d)_{d\in\mathbb{N}}$, where the parameter d typically denotes the number of variables the functions in H_d depend on. An important goal is to find algorithms that scale well with respect to the dimension parameter d and help to break the curse of dimensionality. Some specific features of the function spaces can be very helpful for the analysis of certain types of algorithms; e.g., for the analysis of unbiased randomized algorithms it may be extremely helpful if the norm on H_d induces an ANOVA decomposition on H_d, which can be used to analyze the error (= variance) of the algorithms. In general, a suitable embedding of scales of function spaces may help to transfer results from scales of function spaces with favourable features to other scales of interest. There are several general embedding results known in the case where H_d is the d-fold tensor product of a reproducing kernel Hilbert space (RKHS) H_1; examples include weighted RKHSs where the weights are product weights and so-called spaces of increasing smoothness.

In this talk we discuss a general embedding approach that still works in the case where H_d is not necessarily of tensor product form. This is, e.g., the case if we consider weighted RKHSs, where the weights are product and order-dependent weights or finite-order weights. As an application we study L^∞-approximation on a RKHS of functions that depend on infinitely many variables (which can be viewed as a limiting case of tractability analysis).