Multilevel function approximation I: meta-theorems and PDE analysis

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Special session: Multilevel methods for function approximation

This and the next talk present new ideas on the approximation of functions $\xi \mapsto f(\xi)$, where each function evaluation corresponds to either a functional of the solution of a PDE, with parametric dependence on ξ, or the expected value of a functional of the solution of an SDE, again with a parametric dependence on ξ. In both cases, exact sampling of $f(\xi)$ is not possible, and greater accuracy comes at a higher computational cost.

The key idea to improve the computational cost for a given accuracy is a multilevel representation of the function f. Coarse levels use inaccurate approximations of $f(\xi)$ at a large number of points, whereas fine levels use very accurate approximations at a very limited number of points.

Building on prior research, the talk will present separate meta-theorems for the PDE and SDE case. Each meta-theorem determines the computational complexity if certain assumptions are satisfied. The presentation will then proceed to verify these assumptions for the PDE case, specifically when using finite difference approximations of $f(\xi)$. The analysis is supported by numerical results demonstrating the predicted savings.

[1] Heinrich, S. (2001). *Multilevel Monte Carlo methods.* Large-Scale Scientific Computing, 58–67.

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[3] Tempone, R., Wolfers, S. (2018). *Smolyak's algorithm: A powerful black box for the acceleration of scientific computations.* Sparse Grids and Applications - Miami 2016, 201–228.