

A universal median quasi-Monte Carlo integration

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We study quasi-Monte Carlo (QMC) integration over the multi-dimensional unit cube in several weighted function spaces with different smoothness classes. We consider approximating the integral of a function by the median of several integral estimates under independent and random choices of the underlying QMC point sets (either linearly scrambled digital nets or infinite-precision polynomial lattice point sets). Even though our approach does not require any information on the smoothness and weights of a target function space as an input, we can prove a probabilistic upper bound on the worst-case error for the respective weighted function space, where the failure probability converges to 0 exponentially fast as the number of estimates increases. Our obtained rates of convergence are nearly optimal for function spaces with finite smoothness, and we can attain a dimension-independent super-polynomial convergence for a class of infinitely differentiable functions. This implies that our median-based QMC rule is *universal* in the sense that it does not need to be adjusted to the smoothness and the weights of the function spaces and yet exhibits the nearly optimal rate of convergence. Numerical experiments support our theoretical results.