Hierarchical and Quasi Monte Carlo Techniques for McKean-Vlasov Equations

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In this work we improve the weak and strong convergence rates for particle system approximations of McKean-Vlasov equations using Quasi Monte Carlo. Given a coupled system of P particles it was shown in the literature that the strong error converges at rate $\mathcal{O}(P^{-\frac{1}{2}})$ and the weak error converges at rate $\mathcal{O}(P^{-1})$. We show numerically that the novel approach presented here achieves a strong convergence rate of $\mathcal{O}(P^{-1})$ and a weak convergence rate of $\mathcal{O}(P^{-2})$. This is shown partially based on simulations under knowledge of the exact solution for an Ornstein-Uhlenbeck-type mean field SDE and partially on simulations using a reference solution for the well-known Kuramoto model. It has been shown in the literature that a plain Monte Carlo particle system estimator for a quantity of the form $\mathbb{E}[g(Z(T))]$, where $Z(T)$ is the solution of a McKean-Vlasov equation at time T and g is a smooth function, admits a cost of order $\mathcal{O}(\mathrm{TOL}^{-4})$ to satisfy an error smaller than TOL. Our approach yields a cost reduction to $\mathcal{O}(\mathrm{TOL}^{-3})$. Later we extend the method to a hierarchical setting. The further improvements using hierarchical techniques bring the complexity down to $\mathcal{O}(\mathrm{TOL}^{-2})$. Additionally, we discuss theoretical convergence results in regards to the new particle system presented here.