A calculus for Markov chain Monte Carlo: studying approximations in algorithms

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Markov chain Monte Carlo (MCMC) algorithms are based on the construction of a Markov Chain with transition probabilities $P_\mu(x, \cdot)$, where μ indicates an invariant distribution of interest. In this work, we look at these transition probabilities as functions of their invariant distributions, and we develop a notion of *derivative in the invariant distribution* of a MCMC kernel. We build around this concept a set of tools that we refer to as *Markov Chain Monte Carlo Calculus*. This allows us to compare Markov chains with different invariant distributions within a suitable class via what we refer to as mean value inequalities. We explain how MCMC Calculus provides a natural framework to study algorithms using an approximation of an invariant distribution, also illustrating how it suggests practical guidelines for MCMC algorithms efficiency. We conclude this work by showing how the tools developed can be applied to prove convergence of interacting and sequential MCMC algorithms, which arise in the context of particle filtering.

[1] Rocco Caprio, & Adam M. Johansen, A calculus for Markov chain Monte Carlo: studying approximations in algorithms. arXiv 2310.03853.