Application of quasi-Monte Carlo in Mine Countermeasure Simulations with a Stochastic Optimal Control Framework

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Modeling and simulating mine countermeasures (MCM) search missions performed by autonomous vehicles is a challenging endeavor. The goal of these simulations typically consists of calculating trajectories of autonomous vehicles in a designated zone such that the coverage of the zone is below a certain threshold. We started from the work of [1], and implemented the MCM search mission formulation in a stochastic optimal control framework, see [2]. Mathematically, the MCM problem is defined as minimizing the total mission time needed to survey a designated zone for a given non-coverage percentage of the considered zone Ω, i.e.,

$$\min T_f, \tag{1}$$

subjected to

$$\mathbb{E}[q\,(T_F)] := \int_{\Omega} \mathrm{e}^{-\int_0^{T_F} \gamma(\boldsymbol{x}(\tau),\boldsymbol{\omega})\,d\,\tau} \phi\,(\boldsymbol{\omega})\,d\,\boldsymbol{\omega} \leq \text{non-coverage percentage} \tag{2}$$

where the desired result consists of the position of the autonomous vehicle, $\boldsymbol{x}\,(t) := f(x(t), y(t), \psi(t), r(t))$. In order to compute the expected value of Eq. (2), we use a quasi-Monte Carlo (qMC) sampling scheme instead of the more traditional Monte Carlo (MC) sampling scheme. Our contributions and findings regarding the use of a qMC sampling scheme are twofold. First, we investigated if a qMC sampling scheme, where the points are based on a rank-1 lattice rule, would yield an advantage compared to a MC sampling scheme when considering a square domain Ω. We observed that when repeating the same simulation multiple times the sample variance, where the individual samples consisted of the calculated expected values of Eq. (2), exhibits a larger value when MC is used than when qMC is used. This result indicates that a lower value of T_f, see Eq. (1), can be found when using qMC than when using MC. This in turn can be leverage into a computational speedup. Second, we implemented the algorithm presented in [3], through which we obtained qMC points for use in a triangular domain. After which, we used the points to compute the expected value, see Eq. (2), when considering a triangular domain Ω. We plan to use this result for computing the expected value for non-square domains by partitioning the domain into four different triangles.

[1] S. Kragelund, C. Walton, I. Kaminer, and V. Dobrokhodov, "Generalized optimal control for autonomous mine countermeasures missions," IEEE Journal of Oceanic Engineering, vol. 46, no. 2, pp. 466–496, 2021.

[2] J. L. Pulsipher, W. Zhang, T. J. Hongisto, and V. M. Zavala, "A unifying modeling abstraction for infinite-dimensional optimization," Computers & Chemical Engineering, vol. 156, 2022.

[3] K. Basu, and A. B. Owen, "Low Discrepancy Constructions in the Triangle," SIAM Journal on Numerical Analysis, vol. 53, no. 2, pp 743-761, 2015