Spectral gap bounds for reversible hybrid Gibbs chains

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Hybrid Gibbs samplers represent a prominent class of approximated Gibbs algorithms that utilize Markov chains to approximate conditional distributions, with the Metropolis-within-Gibbs algorithm standing out as a well-known example. Despite their widespread use in both statistical and non-statistical applications, very little is known about their convergence properties. This article introduces novel methods for establishing bounds on the convergence rates of hybrid Gibbs samplers. In particular, we examine the convergence characteristics of hybrid random-scan Gibbs and data augmentation algorithms. Our analysis reveals that the absolute spectral gap of a reversible hybrid chain can be bounded based on the absolute spectral gap of the exact Gibbs chain and the absolute spectral gaps of the Markov chains employed for conditional distribution approximations. The new techniques are applied to four algorithms: a random-scan Metropolis-within-Gibbs sampler, a hybrid proximal sampler, random-scan Gibbs samplers with block updates, and a hybrid slice sampler.