Certifiable Low-Dimensional Structure in Bayesian Inference via Dimensional Logarithmic Sobolev and Poincaré Inequalities

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Inference in the Bayesian paradigm is completely characterized by the posterior distribution. However, accessing this high-dimensional probability measure poses significant computational challenges. To that end, we construct low-dimensional approximations to this distribution using a gradient-based algorithm derived from Markov semigroup inequalities. Our resulting approximation is furnished with certifiable and computable error guarantees. Of note, we identify linear low-dimensional subspaces which exert universal control over the 'worst-case' approximation error with respect to the Amari alpha-divergences. By leveraging recent advances in the study of functional inequalities, namely the dimensional logarithmic Sobolev and Poincaré inequalities, we also obtain refined subspaces which result in an order of magnitude improvement in error guarantees for the KL divergence and squared Hellinger metric, respectively.