

# Achieving High Order Convergence of Quasi-Monte Carlo Methods for Unbounded Integrands by Importance Sampling

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Many problems in finance and statistics can be formulated as high-dimensional integration with unbounded integrands. Monte Carlo (MC) and quasi-Monte Carlo (QMC) methods are popular approaches to approximate such integrals. However, the classical Koksma-Hlawka inequality cannot be directly used to obtain a valid error bound due to the unbounded variation of the integrands. We establish a novel framework to study the convergence rate of (randomized) QMC methods for smooth unbounded integrands based on the so-called projection method to avoid the singularities. We prove that under certain conditions on the integrands a convergence rate of  $O(N^{-1+\varepsilon})$  can be achieved with a  $N$ -points (randomized) QMC rule for an arbitrary small  $\varepsilon > 0$ . Furthermore, a higher convergence rate of  $O(N^{-3/2+\varepsilon})$  can be achieved by using a proper importance sampling that slows down the growth of the integrands with a randomized QMC rule based on scrambled digital nets. Numerical experiments are performed to support the theoretical findings.