ANOVA-boosting for high-dimensional approximation

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Special session: Function spaces and algorithms for high-dimensional problems

We study the problem of scattered-data approximation on $\mathbb{R}^d$, where we have given sample points and the corresponding function evaluations of a function f. The random Fourier feature approach is a two-layer network with a randomized but fixed single hidden layer. The output layer is trained. In this talk we present algorithms from [1], which aim to adapt the randomized hidden layer to the function f.

We use the analysis of variance (ANOVA) decomposition

$$f(\mathbf{x}) = \sum_{\mathbf{u} \subseteq \{1,\dots,d\}} f_{\mathbf{u}}(\mathbf{x}_{\mathbf{u}})$$

for approximating high-dimensional functions of low effective dimension. Thereby we give a relation between the Fourier transform of the function f and the ANOVA terms $f_{\mathbf{u}}$.

In the case for dependent input variables, the ANOVA decomposition is generalized with the aim to detect the structure of the function. We use a least-squares algorithm with a regularization which penalizes the non-orthogonality of ANOVA terms to find which input variables and variable interactions are important. This information is then used to boost random Fourier feature algorithms.

[1] Potts, D., Weidensager, L. ANOVA-boosting for Random Fourier Features, *ArXiv e-print: 2404.03050*, 2024