Stochastic collocation for nonlinear and non stationary equations

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We show convergence rates for a sparse grid approximation of the distribution of solutions of the stochastic Landau-Lifshitz-Gilbert equation. Beyond being a frequently studied equation in engineering and physics, the stochastic Landau-Lifshitz-Gilbert equation poses many interesting challenges that do not appear simultaneously in previous works on uncertainty quantification: The equation is strongly non-linear, time-dependent, and has a non-convex side constraint. Moreover, the parametrization of the stochastic noise features countably many unbounded parameters and low regularity compared to other elliptic and parabolic problems studied in uncertainty quantification. We use a novel technique to establish uniform holomorphic regularity of the parameter-to-solution map based on a Gronwall-type estimate and the implicit function theorem. This method is very general and based on a set of abstract assumptions. Thus, it can be applied beyond the Landau-Lifshitz-Gilbert equation as well. We demonstrate numerically the feasibility of approximating with sparse grid and show a clear advantage of a multi-level sparse grid scheme.

[1] An, X., Dick, J., Feischl, M., Scaglioni, A. and Tran, T. *Sparse grid approximation of the stochastic Landau-Lifshitz-Gilbert equation.* arXiv 2310.11225, 2024.