Report on "Quasi-Monte Carlo Methods: What, Why, and How?" by Hickernell, Kirk and Sorokin

This survey paper provides an overview of quasi-Monte Carlo methods based on the tutorial presented at MCQMC 2024 by the first author. The paper is very well written and provides a comprehensive view of the field that is accessible to researchers outside the community. Below are some comments for the authors.

Comments

1. It is unavoidable that surveys of this type are not comprehensive and will choose to focus on some areas and ignore others. It would be useful to mention something along those lines somewhere in the introduction.

2. page 8, equation (11) and corresponding description. This is an unusual way to define a digital sequence. While I'm open to see how this description may work in a way that bypasses the immediate need for introducing generating matrices, it is not obvious that this captures all digital sequences (in base 2). In particular, it is confusing to say "where $\mathbf{x}_1^{\text{dig}}, \mathbf{x}_2^{\text{dig}}, \mathbf{x}_3^{\text{dig}}, \ldots$ are carefully chosen". First, for i not a power of 2, (11) suggests that $\mathbf{x}_i^{\text{dig}}$ is completely determined by the points where i is a power of 2. So the only points $\mathbf{x}_i^{\text{dig}}$ that need to be carefully chosen are those for which i is a power of 2. In fact it is not possible to choose all of them carefully independently from each other. So this statement is misleading. Second, when the connection to generating matrices is made, the description suggests that, e.g., the third row of C_j would correspond to the digits of $x_{3,j}^{\text{dig}}$, but really it should be digits of $x_{4,j}^{\text{dig}}$, which corresponds to $i_2 = 1$ and all other i_r being 0.

   Please review this part carefully to make sure this description is accurate.

3. Page 8, 5-6 lines below (14): the definition of a digital sequence also requires segments between powers of 2 to be nets with parameter (at most) t. This also brings a minor point: especially when defining a $(t, d)-$sequence, one does not usually assume that the t parameter applies as the exact t for all segments, i.e., it relies on a definition of $(t, m, d)-$net where t is not understood as the exact t value but rather an upper bound on it. I.e., the authors previously defined the t of a (t, m, d)-net as being exact (2 lines below (14)), but as we can see this makes them run into problems once they start discussing $(t, d)-$sequences. To avoid this problem, they should not assume the t of a $(t, m, d)-$net is necessarily exact.

4. Page 10, top and caption of Figure 8: Halton sequences do exhibit specific equidistribution properties when n is a product of powers of the d bases used. So it is not completely accurate to say that Halton sequences do not have preferred sample sizes.

5. Page 11, about 2/3 down: it would be important to cite other works than [49] when discussing the Halton sequences. For example [18] which includes many other important references.

6. page 26, line 3 of Section 7.3: "One example" rather than "On example". Also in such cases d may be larger than the number of times the asset is monitored, e.g., when dealing with stochastic volatility models or multiple assets. So this should say something like "where d could denote..."

Minor Comments

1. Please review the entire manuscript to make sure hyphenation is used correctly. For example, in line 1 of the 3rd paragraph of the abstract, it should be "low-discrepancy sequences", and there are many other instances where an hyphen is missing. Similarly, line -6 on page 3 should be "low-dimensional problems"
2. page 3, 3 lines below (4), remove "is" before Φ
3. The formatting in (14) on page 8 needs to be changed. Some lines are centered and others are not.
4. page 16, equation (24): what represent the double dots after the first L in the final expression?
5. page 16, equation (26): what represent the double dots within some of the parentheses?
6. page 23, 3 lines below (42), remove "is" before "finds"
7. page 23, line below (43), not sure what "[(]" represents, perhaps a typo?
8. page 25, line 2 of Section 7.1, might be best to insert "joint" before density
9. page 25, end of Section 7.1: should add references to relevant papers, e.g., "A Comparison of Some Monte Carlo and Quasi Monte Carlo Techniques for Option Pricing" by Acworth, Broadie and Glasserman, in the Proceedings of MCQMC 1996.
10. pages 26-27, notation is introduced without being defined, e.g., $\mathbf{n}, \mathbf{d}, \mathbf{C}, \mathbf{V}$
11. page 27, in the definition of V_1 and V_l, it would be best to add multiplication signs between the different terms (assuming they are indeed being multiplied; otherwise please explain what this means)
12. page 29, please note that [86] also used qMC and not just MC