

# Automatic Estimation of Sobol' Indices Based on Quasi-Monte Carlo Methods

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## Abstract

Given the independent uniformly distributed random variables  $X_1, \dots, X_d$ , we are interested in measuring what part of the variance of  $f(X_1, \dots, X_d)$  is explained by each input. For this study, we will consider the *global sensitivity* indices defined by Sobol' in [5].

*First order* and *total effect* indices provide most of the information about the variance of the model. However, since evaluating  $f$  is usually time consuming, the estimation of these indices might be costly. If we require exactly  $n$  data points to estimate each *first order* and *total effect* indice, the total number of function evaluations needed to estimate them, one by one, is  $4dn$ . This value can be easily reduced to  $(2+d)n$ . However, for the case of *first order* indices, one can estimate them with only  $2n$  function evaluations using the replication method [2]. We will explain how to use this method in conjunction with automatic quasi-Monte Carlo cubatures [3, 4] and provide some examples.

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