REFEREE REPORT ON "ADAPTIVE APPROXIMATION ..." BY DING, HICKERNELL, KRITZER AND MAK

Summary. The authors study algorithms and complexity issues (in the IBC style) and assume that the set of admissible inputs $\mathcal{C}$ is a cone. Algorithms should work for any $f \in \mathcal{C}$ and the cost bounds (and complexity bounds) are proved for $\mathcal{C} \cap B$, where B is a ball. Three examples/cases are studied in more detail: The first cone $\mathcal{C}$ is defined in (16) and studied in Section 2, the second cone $\mathcal{C}$ is defined in (29) and studied in Section 3, and the third cone is studied in Section 4.

I find the results interesting and believe that the paper should be published.

I also have some critical remarks, see the following.

Remarks.

(1) abstract, 4: "for convex sets": this assumption is not enough, the stated result is true for convex and symmetric sets

(2) p2, 2-3: again: adaptive algor. are better also for certain convex sets

(3) p3, middle of the page "If one has a fixed budget ... is the best answer": This is not true in general. Optimality depends on the class $\mathcal{C}$. Your statement is true if $\mathcal{C}$ is a ball in F.

(4) Next line: delete comma after algorithm

(5) p3, -13: "This algorithm": So far you did not define an algorithm and also a class $\mathcal{C}$ is not defined. In the next line you seem to assume that you consider only a very special class of adaptive algorithms. Do you have in mind to fix the order of the info functionals? One only is allowed to construct a stopping rule?

(6) p3, -5: "this example": I cannot really see an example yet. The first example of a cone is given in formula (16) on page 12. For me it seems that the reader has to wait quite long.

(7) In your to do list on page 3 you say *first* that you want to construct an algorithm and *then* you want to choose the input class $\mathcal{C}$. For me it seems that you first should state the problem and then find a solution/algorithm!

(8) Page 4: The intro is quite long and not really an introduction. No cones yet! Do you believe that technical results (like Lemma 1) belong to the intro? I would prefer a kind of a summary in the introduction.

(9) p5, -10: "to keep this norm is finite": delete "is"

(10) Section 1.3, beginning: "The optimal approximation": Optimality depends on the input class. So far you did not define $\mathcal{C}$.

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(11) Theorem 1 is *not* about cones. I understand that such a result might be useful, but does it belong to the intro?
(12) p7, The section starting with "The key difficulty": You say that (12) depends on the norm while your algor. "only depend(s) on function data". This is not correct: You assume (as you say a little later) that $f \in \mathcal{C}$. Hence you *replace* balls by cones and not balls by "no assumption at all".
(13) The definition of $\mathcal{A}(\mathcal{C})$ is hidden in the text. Please emphasize it somehow!
(14) p9, -5: "available": this word should be deleted here
(15) Section 1.6: Again, this is not an example of a cone, there are not yet cones.
(16) The sentence with formula (16) reads like a statement but actually you *define* (finally) a cone $\mathcal{C}$ that you want to study. The definition of the problem is a little hidden here.
(17) The next cone is introduced with formula (28) and this time it is correctly stated as a definition (again it is a bit hidden in the text). Can you add a little motivation? Why is it an interesting cone?
(18) Section 4 does not really state a math result, or do I miss something? You define a cone with formula (42) and there is an Algorithm 4. But is it really an algorithm? The step "Compute data-driven POSD weights" seems to be rather vague, as the sentence around (41): "We choose ...". You admit this with the sentence "The optimization in (41) is nontrivial to solve numerically." Would you still call it an algorithm?