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Rigged Hilbert space

A Hilbert space $\mathcal{H}$ containing a linear, everywhere-dense subset $\Phi \subseteq \mathcal{H}$, on which the structure of a topological vector space is defined, such that the imbedding is continuous. This imbedding generates a continuous imbedding of the dual space $\mathcal{H}' \subseteq \Phi'$ and a chain of continuous imbeddings $\Phi \subseteq \mathcal{H} \subseteq \Phi'$ (using the standard identification $\mathcal{H}' = \mathcal{H}$). The most interesting case is that in which Φ is a nuclear space. The following strengthening of the spectral theorem for self-adjoint operators acting on $\mathcal{H}$ is true: Any self-adjoint operator A mapping Φ continuously (in the topology of Φ) onto itself possesses a complete system of generalized eigenfunctions $(F_\alpha \mid \alpha \in \mathfrak{A})$ ($\mathfrak{A}$ is a set of indices), i.e. elements $F_\alpha \in \Phi'$ such that for any $\phi \in \Phi$,

$$F_\alpha(A\phi) = \lambda_\alpha F_\alpha(\phi), \qquad \alpha \in \mathfrak{A},$$

where the set of values of the function $\alpha \mapsto \lambda_\alpha$, $\alpha \in \mathfrak{A}$, is contained in the spectrum of A (cf. Spectrum of an operator) and has full measure with respect to the spectral measure $\sigma_f(\lambda), f \in \mathcal{H}$, $\lambda \in \mathbb{R}$, of any element $f \in \mathcal{H}$. The completeness of the system means that $F_\alpha(\phi) \neq 0$ for any $\phi \in \Phi$, $\phi \neq 0$, for at least one $\alpha \in \mathfrak{A}$. Moreover, for any element $\phi \in \Phi$, its expansion with respect to the system of generalized eigenfunctions $(F_\alpha \mid \alpha \in \mathfrak{A})$ exists and generalizes the known expansion with respect to the basis of eigenvectors for an operator with a discrete spectrum.

Example: The expansion into a Fourier integral

$$f(x) = \int_{\mathbb{R}} e^{isx} \tilde{f}(s) \, \mathrm{d}s, \qquad x \in \mathbb{R}, \qquad f, \tilde{f} \in L^2(\mathbb{R}),$$

$(x \mapsto e^{isx} \mid s \in \mathbb{R})$ is a system of generalized eigenfunctions of the differentiation operator, acting on $L^2(\mathbb{R})$, arising under the natural rigging of this space by the Schwartz space $S(\mathbb{R})$ (cf. Generalized functions, space of). The same assertions are also correct for unitary operators acting on a rigged Hilbert space.

References

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Comments

A rigged Hilbert space $\Phi \subseteq \mathcal{H} \subseteq \Phi'$ is also called a Gel'fand triple. Occasionally one also finds the phrases nested Hilbert space, or equipped Hilbert space.

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